Tetraquark mixing framework for light mesons in $J^P = 0^+$

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References:
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3. PRD (2018) 97:094005, Hungchong Kim, K.S.Kim, M.K.Cheoun, M.Oka
4. PRD (2019) 99:014005, Hungchong Kim, K.S.Kim, M.K.Cheoun, D.Jido, M.Oka
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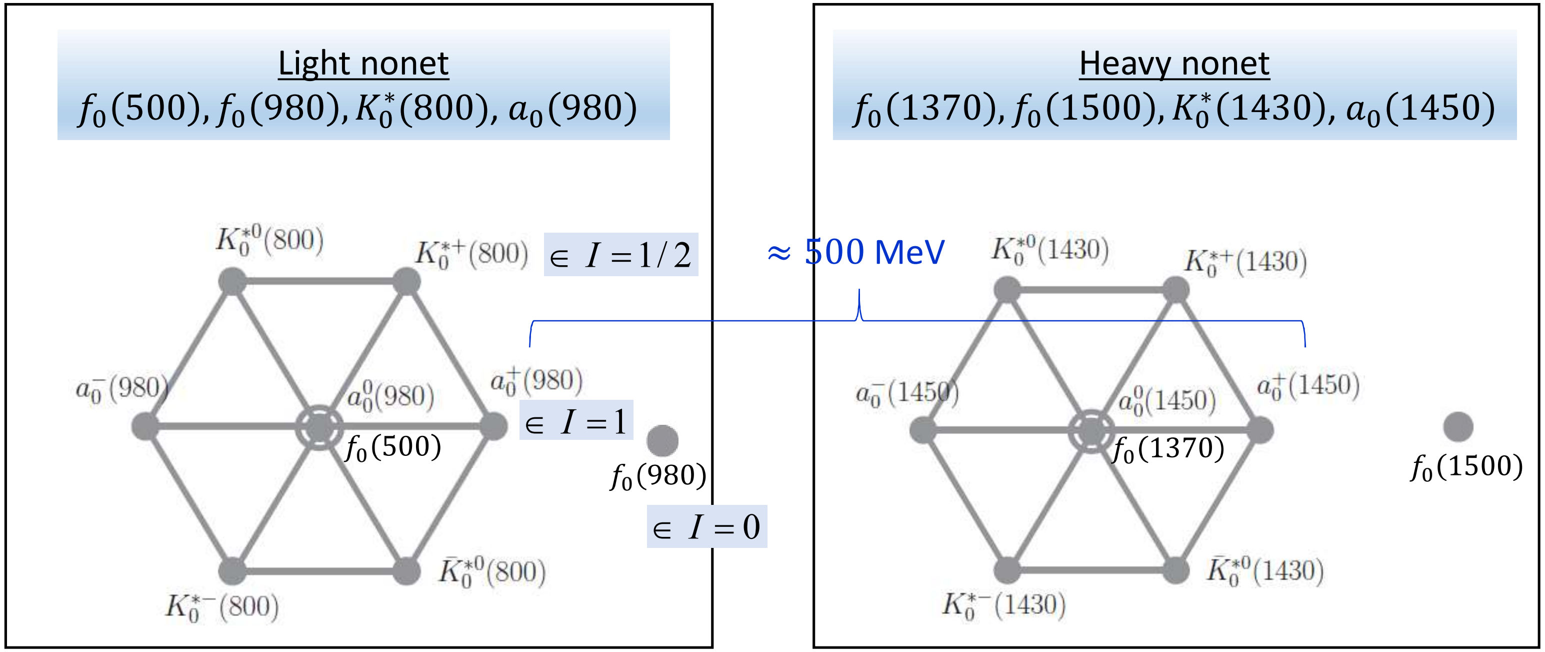
2nd CENuM Workshop, 3-4 July 2020 (On-line workshop)

- In PDG, there are two nonets with $J^P = 0^+$ which we call light and heavy nonet.
  - They seem to form SU(3) flavor nonet as they are composed of $I = 0, \frac{1}{2}, 1$ members.
  - Two nonets are separated by huge mass gap ($\gtrsim 500$ MeV)



※ $K_0^*(800)$ has been renamed as $K_0^*(700)$ in the latest PDG.

- In this work, we investigate tetraquark possibility from the two nonets.
- We introduce two types of tetraquark based on diquark-antidiquark model and their mixing in order to explain the two nonets in PDG.

According to the diquark-antidiquark model, tetraquarks are constructed by $qq \times \bar{q}\bar{q} \Longrightarrow qq\bar{q}\bar{q}$ assuming all the quarks are in an S-wave.

Two tetraquark types in $J^P = 0^+$

Type 1 :

$$[qq \in (J_{12} = 0, \bar{3}_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J_{34} = 0, 3_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J = 0, 1_c, 8_f \oplus 1_f)$$

- Spin: $|J\, J_{12}\, J_{34}\rangle = |\mathbf{0}00\rangle$
- Color: $\bar{3}_c \otimes 3_c \Rightarrow 1_c, \quad |1_c \bar{3}_c 3_c\rangle$

Jaffe's tetraquark originally introduced for the light nonet $f_0(500), f_0(980), K_0^*(800), a_0(980)$

Type 2 :

$$[qq \in (J_{12} = 1, 6_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J_{34} = 1, \bar{6}_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J = 0, 1_c, 8_f \oplus 1_f)$$

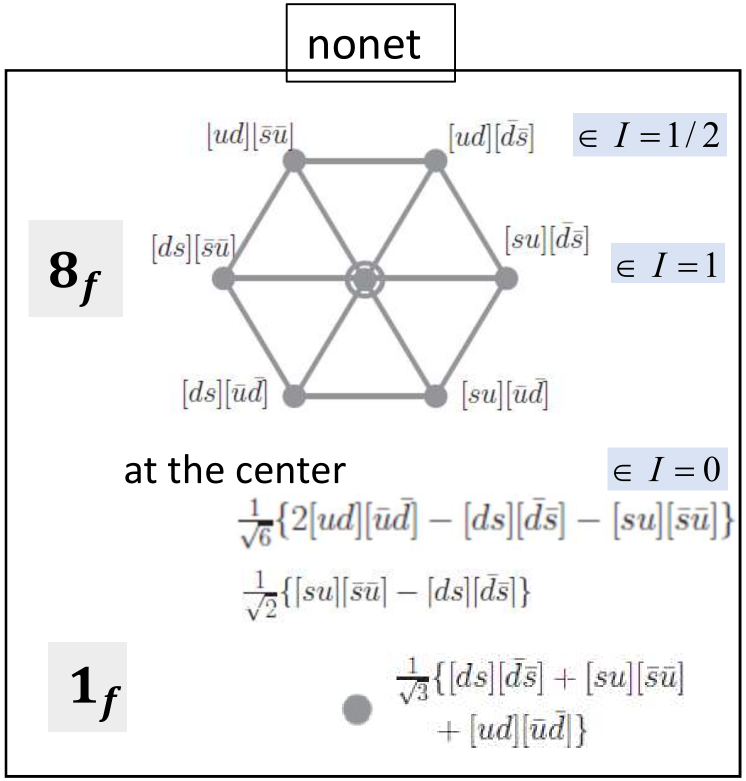
- Spin: $|J\, J_{12}\, J_{34}\rangle = |\mathbf{0}11\rangle$
- Color: $6_c \otimes \bar{6}_c \Rightarrow 1_c, \quad |1_c 6_c \bar{6}_c\rangle$

2nd tetraquark proposed to accommodate the heavy nonet $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

Flavor: Both types have the **same** flavor structure!

$\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$ forming a nonet.

Common flavor structure



Notation: $[ud] = \frac{1}{\sqrt{2}}(ud - du)$, etc.

Main characteristics of tetraquark

- The mass ordering among the octet members,
  $(I = 1) > \left(I = \frac{1}{2}\right) > (I = 0)$,
  ex) $M([su][\bar{d}\bar{s}]) > M([su][\bar{u}\bar{d}])$.

This is opposite to what expected from two-quark system ($q\bar{q}$), $(I = 0) > \left(I = \frac{1}{2}\right) > (I = 1)$.

- Two nonets in PDG satisfy the mass ordering !

Light nonet: $M[a_0(980)] > M[K_0^*(800)] > M[f_0(500)]$.

Heavy nonet: $M[a_0(1450)] > M[K_0^*(1430)]$ with $\Delta M \sim 50$ MeV, $M[K_0^*(1430)] \gtrsim M[f_0(1370)]$. (marginal mass ordering)

- First indication that the two nonets could be tetraquark.

We need to investigate further signatures for tetraquark from two nonets in PDG.

Question

How to describe

two nonets in PDG

- Light nonet: $f_0(500), f_0(980), K_0^*(800), a_0(980)$
- Heavy nonet: $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

by

two tetraquark types

- $|000\rangle_{\bar{3}_c,3_c} \Longrightarrow |000\rangle$
- $|011\rangle_{6_c,\bar{6}_c} \Longrightarrow |011\rangle$

that differ by the spin and color configurations ?

⇒Tetraquark mixing framework !

Crucial observation is that

- the two tetraquark types, $|000\rangle$, $|011\rangle$, in each isospin channel, mix through the color-spin interaction!

$$V_{CS} \propto -\sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

λ_i: Gell-Mann matrix for color
J_i: spin
m_i: constituent quark mass

- i.e., $\langle 011|V_{CS}|000\rangle \neq 0$,
☞ $\langle V_{CS}\rangle$ forms a 2x2 matrix constituting so called the hyperfine mass matrix.

The upshot is that

- physical states, the two nonets in PDG, can be identified by the eigenstates that diagonalize the 2x2 matrix,

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

This is our tetraquark mixing framework for the two nonets in $J^P = 0^+$.

The steps that I have taken in this research

- Construct the wave functions for $|000\rangle, |011\rangle$, in spin, color, flavor space.
- Calculate the hyperfine mass matrix of V_{CS} in the basis $|000\rangle, |011\rangle$.
- Then diagonalize the matrix to determine the hyperfine mass $\langle V_{CS}\rangle$ and the mixing parameters α, β, in

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

- Investigate the phenomenological signatures of tetraquark and some interesting consequences.

Example) In the $I = 1$ channel [corresponding to $a_0(980)$, $a_0(1450)$].

- Diagonalization leads to the physical hyperfine masses

$\langle V_{CS}\rangle$	$\lvert 000\rangle$	$\lvert 011\rangle$
$\lvert 000\rangle$	-173.9	-222.3
$\lvert 011\rangle$	-222.3	-331.5

$\longrightarrow$

$\langle V_{CS}\rangle$	$\lvert 0_A^{a_0}\rangle$	$\lvert 0_B^{a_0}\rangle$
$\lvert 0_A^{a_0}\rangle$	-16.8	0.0
$\lvert 0_B^{a_0}\rangle$	0.0	-488.5

and eigenstates corresponding to $a_0(980), a_0(1450)$,

$$|0_A^{a_0}\rangle = -0.817|000\rangle + 0.577|011\rangle \quad \Longrightarrow |a_0(1450)\rangle$$

$$|0_B^{a_0}\rangle = \underline{0.577}|000\rangle + \underline{0.817}|011\rangle \quad \Longrightarrow |a_0(980)\rangle\,.$$

$\hookrightarrow \beta$ $\qquad$ $\alpha \hookleftarrow$

This identification follows from $\langle 0_A^{a_0}|V_{CS}|0_A^{a_0}\rangle > \langle 0_B^{a_0}|V_{CS}|0_B^{a_0}\rangle$

Note, the **strong mixing** causes **large separation** in hyperfine masses [$\Delta\langle V_{CS}\rangle = -16.8 - (-488.5) \approx 471.7$ MeV].

Results: some features of our mixing model

$\langle V_{CS}\rangle$ in the diagonal basis.

Light nonet	M_{exp}	$\langle V_{CS}\rangle$	Heavy nonet	M_{exp}	$\langle V_{CS}\rangle$	α	β
$a_0(980)$	980	-488.5	$a_0(1450)$	1474	-16.8	0.8167	0.5770
$K_0^*(800)$	824	-592.7	$K_0^*(1430)$	1425	-26.9	0.8130	0.5822
$f_0(500)$	475	-667.5	$f_0(1370)$	1350	-29.2	0.8136	0.5814
$f_0(980)$	990	-535.1	$f_0(1500)$	1506	-20.1	0.8157	0.5784

1. α, β are almost independent of the isospin!

$\Rightarrow$ Support our identification of the two nonets in PDG as flavor nonet.

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

2. $\alpha > \beta$; Light nonet has more probability to stay in $|011\rangle$ rather than in $|000\rangle$!

$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

- Surprising ! But recent QCDSR [PRD(2019)] also supports this result.

Light nonet	M_{exp}	$\langle V_{CS} \rangle$	Heavy nonet	M_{exp}	$\langle V_{CS} \rangle$
$a_0(980)$	980	-488.5	$a_0(1450)$	1474	-16.8
$K_0^*(800)$	824	-592.7	$K_0^*(1430)$	1425	-26.9
$f_0(500)$	475	-667.5	$f_0(1370)$	1350	-29.2
$f_0(980)$	990	-535.1	$f_0(1500)$	1506	-20.1

3. Our $\langle V_{CS} \rangle$ help to understand the light nonet mass below 1 GeV.

- For the light nonet, $\langle V_{CS} \rangle$ is negatively huge ~ -500 MeV due to the strong mixing.

$\Rightarrow$ the light nonet even if viewed as a four-quark state can have mass below 1 GeV.

- On the other hand, for the heavy nonet, $\langle V_{CS} \rangle \sim -20$ MeV,

$\Rightarrow$ So the heavy nonet mass is not far from $\sim 4m_q$.

4. Our results satisfy the mass splitting formula $\Delta M \approx \Delta\langle V_{CS}\rangle$ relatively well.

Isospin	ΔM_{exp}	$\Delta\langle V_{CS}\rangle$
$I = 1$	**494**	**472**
$I = 1/2$	**601**	**566**
$I = 0\ (\sim\mathbf{8}_f)$	**875**	**612**
$I = 0\ (\sim\mathbf{1}_f)$	**515**	**515(fit)**

- Indeed, the **large separation** in hyperfine masses $\Delta\langle V_{CS}\rangle$ qualitatively explains the large mass gap ($\Delta M_{exp} \gtrsim 500$ MeV) between the two nonets!

⇒ It is the strong mixing that gives rise to this result also.

5. $\langle V_{CS}\rangle$ gives a partial explanation for the marginal mass ordering in the heavy nonet.

Isospin	Light	$\langle V_{CS}\rangle$	Heavy	$\langle V_{CS}\rangle$
$I=1$	$a_0(980)$	-488.5	$a_0(1450)$	-16.8
$I=1/2$	$K_0^*(800)$	-592.7	$K_0^*(1430)$	-26.9
$I=0\ (\sim\mathbf{8}_f)$	$f_0(500)$	-667.5	$f_0(1370)$	-29.2
$I=0\ (\sim\mathbf{1}_f)$	$f_0(980)$	-535.1	$f_0(1500)$	-20.1

~100 ~75 ~10

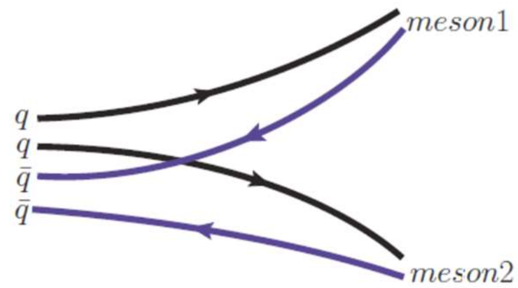
Expt. masses (increasing)

Light	Heavy
980	1474
824	1425
475	1350
990	1505

- For the octet, our hyperfine masses are ordered as the expt. masses,
  $\langle V_{CS}\rangle_{I=1} > \langle V_{CS}\rangle_{I=1/2} > \langle V_{CS}\rangle_{I=0} \overset{\text{the same ordering}}{\Longleftrightarrow} M[a_0] > M[K_0^*] > M[f_0]$.
  ☞ $\langle V_{CS}\rangle$ is also responsible for the mass ordering (in addition to m_q).
- $\Delta\langle V_{CS}\rangle \lesssim 100$ MeV for the light nonet, $\Delta\langle V_{CS}\rangle \lesssim 10$ MeV for the heavy nonet
- The hyperfine mass spitting is almost ineffective on the heavy nonet.
  It explains partially the marginal mass ordering,
  $M[a_0(1450)] - M[K_0^*(1430)] \approx 50$ MeV.

6. Signatures from the fall-apart modes into two mesons

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$



$qq\bar{q}\bar{q}$ fall-apart decay

- PP mode: suppressed in heavy nonet but enhanced in light nonet.
- VV mode: enhanced in heavy nonet but suppressed in light nonet.

opposite trend!

The PP prediction tested on isovector channel works quite well !

Based on expt. analysis

	Theory	Bugg	PDG
$\frac{\Gamma[a_0(980)\to\pi\eta]}{\Gamma[a_0(1450)\to\pi\eta]}$	2.51–2.54	2.53	2.93–3.9
$\frac{\Gamma[a_0(980)\to K\bar{K}]}{\Gamma[a_0(1450)\to K\bar{K}]}$	0.52–0.89	0.62	0.61–0.81

Bugg: PRD78,074023(2008)

PRD(2019) 99:014005

VV modes from the two nonets

- The enhancement factor is about $\sim$15 !
- But most channels are not allowed due to $M(\text{mother}) < M_1 + M_2(\text{daughters})$.

	Mode	$a_0^+(980)$	$a_0^+(1450)$	Ratio
$I=1$	$\bar{K}^{*0}K^{*+}$	−0.0449	−0.6439	14.33
	$\phi\rho^+$	0.0449	0.6439	
	Mode	$K_0^{*+}(800)$	$K_0^{*+}(1430)$	Ratio
$I=1/2$	ρ^+K^{*0}	−0.0408	−0.6442	15.78
	ρ^0K^{*+}	−0.0289	−0.4555	
	ωK^{*+}	0.0289	0.4555	
	Mode	$f_0(500)$	$f_0(1370)$	Ratio
$I=0$ ($\sim\mathbf{8}_f$)	$\rho^0\rho^0$	0.0185	0.2869	15.54
	$\bar{K}^{*0}K^{*0}$	−0.0133	−0.2069	
	$\phi\omega$	0.0188	0.2927	
	$\omega\omega$	−0.0185	−0.2869	
	Mode	$f_0(980)$	$f_0(1500)$	Ratio
$I=0$ ($\sim\mathbf{1}_f$)	$\rho^0\rho^0$	0.0100	0.1463	14.70
	$\bar{K}^{*0}K^{*0}$	0.0276	0.4057	
	$\phi\omega$	−0.0390	−0.5737	
	$\omega\omega$	−0.0100	−0.1463	

Thank you for your listening !

Extra slide: PDG full listing of the mesons in $J^P = 0^+$

	Category	Name	Isospin	J^{PC}	M(MeV)	Γ(MeV)
($m \le 1$ GeV)	Light nonet	$f_0(500)$	0	0^{++}	400-550	400-700
		$f_0(980)$	0	0^{++}	990	10-100
		$K_0^*(800)$	1/2	0^{+}	824	478
		$a_0(980)$	1	0^{++}	980	50-100
($1 < m \le 1.5$ GeV)	Heavy nonet	$f_0(1370)$	0	0^{++}	1200-1500	200-500
		$f_0(1500)$	0	0^{++}	1505	109
		$K_0^*(1430)$	1/2	0^{+}	1425	270
		$a_0(1450)$	1	0^{++}	1474	265
No nonet structure here	The rest higher	$f_0(1710)$	0	0^{++}	1723	139
		$f_0(2020)$	0	0^{++}	1992	442
		$f_0(2100)$	0	0^{++}	2101	224
		$f_0(2200)$	0	0^{++}	2189	238
		$f_0(2330)$	0	0^{++}	2314	144
		$K_0^*(1950)$	1/2	0^{+}	1945	201

Resonances in each isospin channel are separated by huge mass gap $\gtrsim 500$ MeV

※ $K_0^*(800)$ has been renamed as $K_0^*(700)$ in the latest PDG