Form factors of a Baryon

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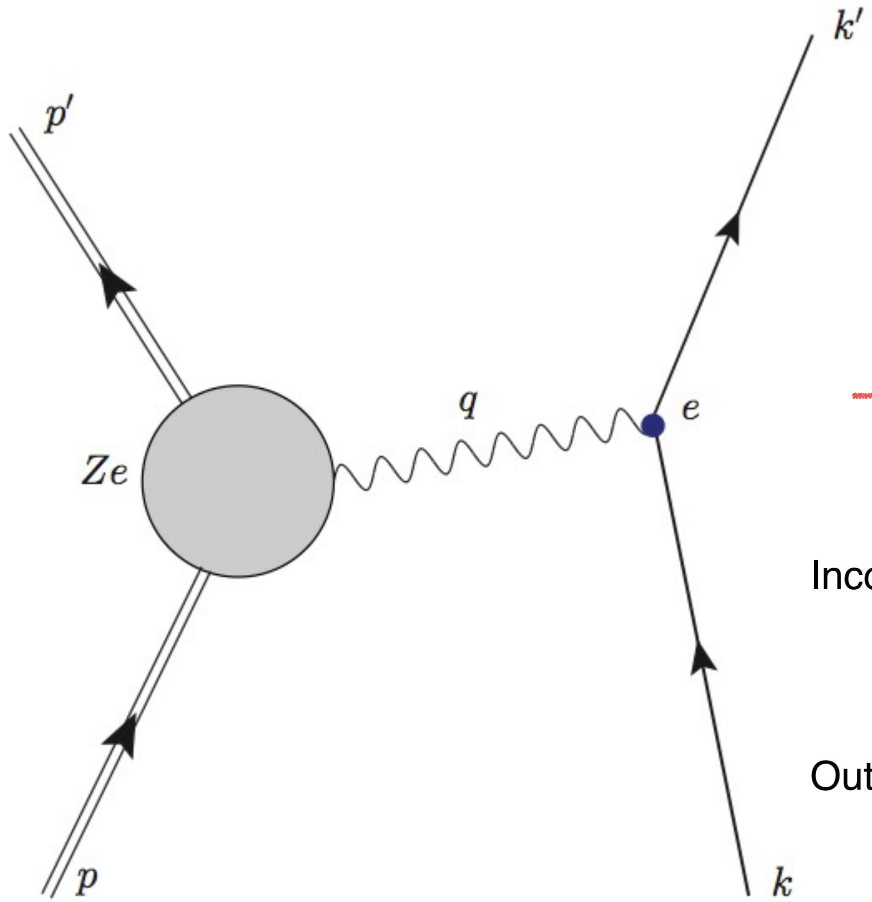
Form factors in general

What is a form factor?

Form factors tell you how the corresponding particle looks like in various aspects.

Historical example: Rutherford scattering



- Target is so heavy that the recoil effects are negligible.
- Elastic scattering

If $Z\alpha \ll 1 \, (\alpha \approx 1/137)$

→ Born approximation can be used.

Incoming wave for the electron $\psi_i = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}/\hbar}$

Outgoing wave for the electron $\psi_f = \frac{1}{\sqrt{V}} e^{i\mathbf{k}'\cdot\mathbf{r}/\hbar}$

What is a form factor?

Quantum mechanical definition of the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{V^2 E'^2}{(2\pi)^2 (\hbar c)^4} \left| \langle \psi_f \left| \mathcal{H}_{\text{int}} \right| \psi_i \rangle \right|^2$$

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$$\langle \psi_f | \mathcal{H}_{\text{int}} | \psi_i \rangle = \frac{e}{V} \int \phi(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar} \, d^3x$$

$$= -\frac{e\hbar^2}{V|\mathbf{q}|^2} \int \nabla^2 \phi(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar} \, d^3x$$

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$$\nabla^2\phi(\mathbf{r}) = -\frac{\rho_{\rm ch}(\mathbf{r})}{\varepsilon_0}$$

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$$\nabla^2\phi(\mathbf{r}) = -\frac{\rho_{\text{ch}}(\mathbf{r})}{\varepsilon_0}$$

$$\rho_{\text{ch}} = Ze\rho(\mathbf{r})$$

$$\langle \psi_f | \mathcal{H}_{\text{int}} | \psi_i \rangle = \frac{Z4\pi\alpha\hbar^3 c}{|\mathbf{q}|^2 \cdot V} \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, d^3x$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, d^3x$$

What is a form factor?

Rutherford scattering

$$\rho(\mathbf{r}) = \delta(\mathrm{r})$$

: The particle taken as a point-like one

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} = \frac{4Z^2\alpha^2(\hbar c)^2 E'^2}{\mathbf{q}^4 c^4}$$

It decrease very quickly as q becomes larger.

$$E = E', \quad E \approx |\mathbf{k}|c \qquad |\mathbf{q}| = 2|\mathbf{k}|\sin\frac{\theta}{2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4\theta/2}$$

What is a form factor?

Mott scattering

Electron spins are considered

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \left(1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right)$$

(Recoil effects are still neglected.)

It can be easily derived by using the interaction Lagrangian:

$$\mathcal{L}_{\text{int}} = -e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu^{\text{ext}}$$

HW. If you have time, please try to derive the Mott formula using the following S-matrix:

$$S_{fi} = -ie\int d^4x\bar{\psi}_f(x)\gamma^\mu\psi(x)_i A_\mu^{\text{ext}}(x)$$

Please, keep in mind that the Mott formula applies only for structureless particles such as electrons etc.

What is a form factor?

Charge distribution $f(r)$		Form Factor $F(\boldsymbol{q}^2)$	
point	$\delta(r)/4\pi$	1	constant
exponential	$(a^3/8\pi)\cdot\exp(-ar)$	$\left(1+\boldsymbol{q}^2/a^2\hbar^2\right)^{-2}$	dipole
Gaussian	$\left(a^2/2\pi\right)^{3/2}\cdot\exp\left(-a^2r^2/2\right)$	$\exp\left(-\boldsymbol{q}^2/2a^2\hbar^2\right)$	Gaussian
homogeneous sphere	$\begin{cases} 3/4\pi R^3 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$	$3\,\alpha^{-3}(\sin\alpha-\alpha\cos\alpha)$ with $\alpha=\lvert\boldsymbol{q}\rvert R/\hbar$	oscillating

Particles & Nuclei by Povh et al.

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An oscillating form factor corresponds to a homogeneous sphere with sharp edge!

Particles & Nuclei by Povh et al.

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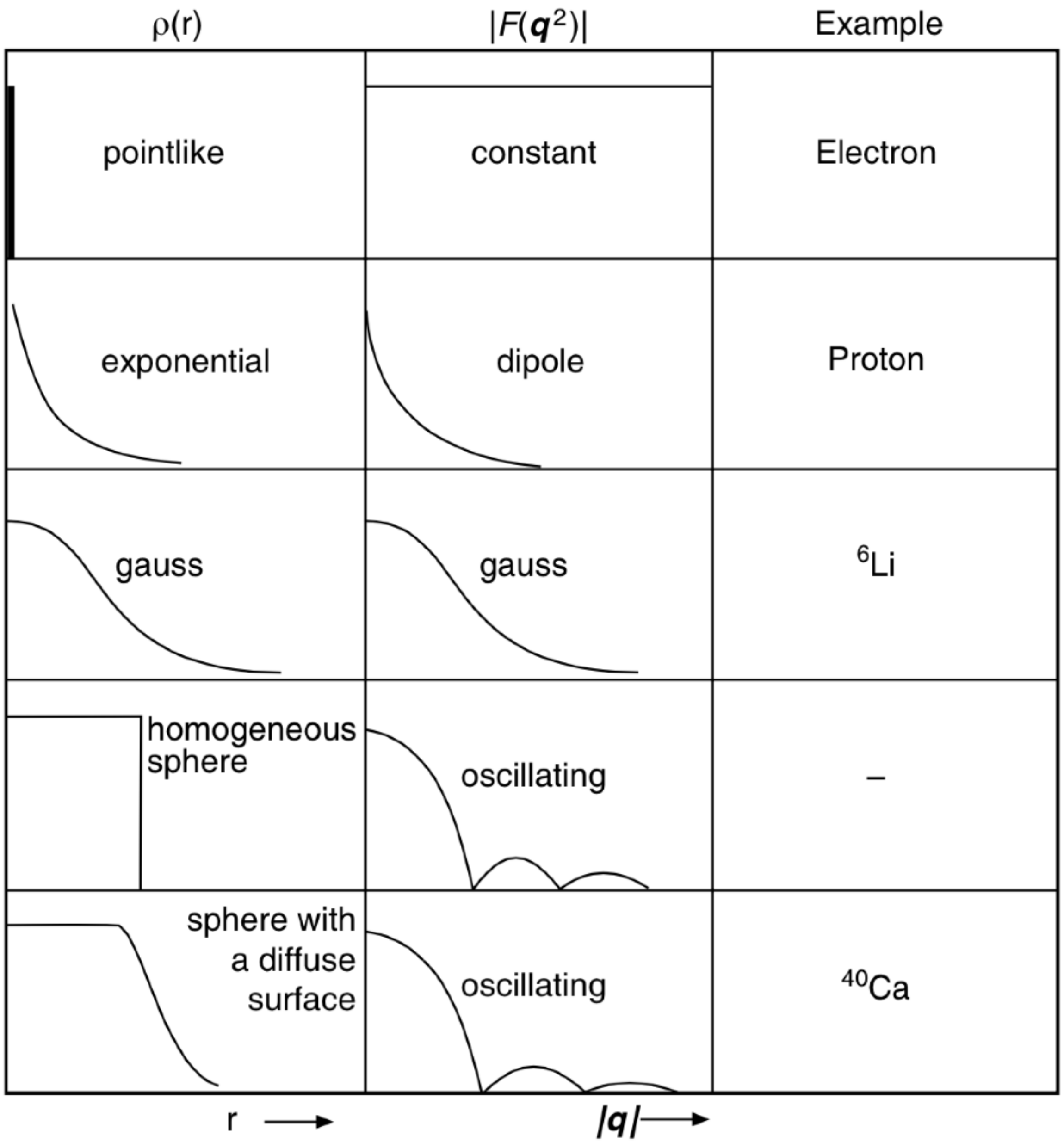
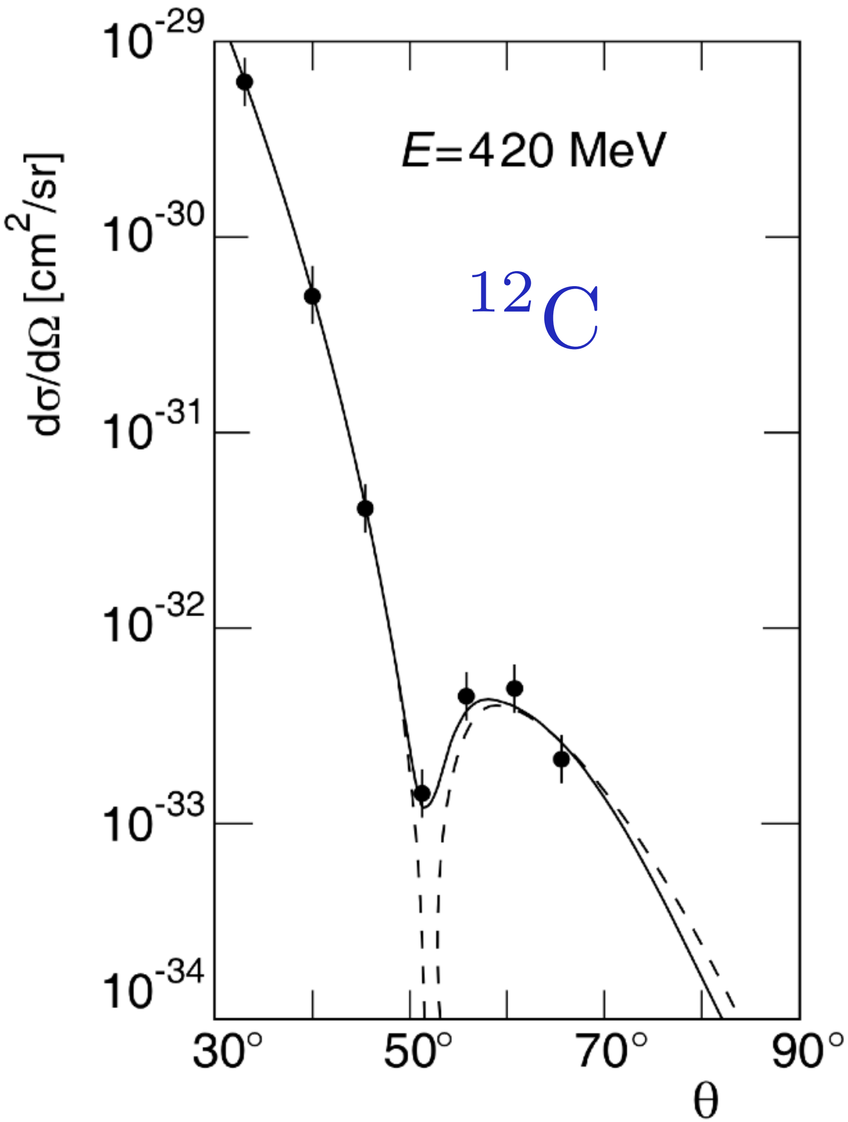


$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, d^3x$$

$$\langle r^2 \rangle = -6\hbar^2 \left. \frac{dF(\mathbf{q}^2)}{d\mathbf{q}^2} \right|_{\mathbf{q}^2=0}$$

Particles & Nuclei by Povh et al.

What is a form factor?

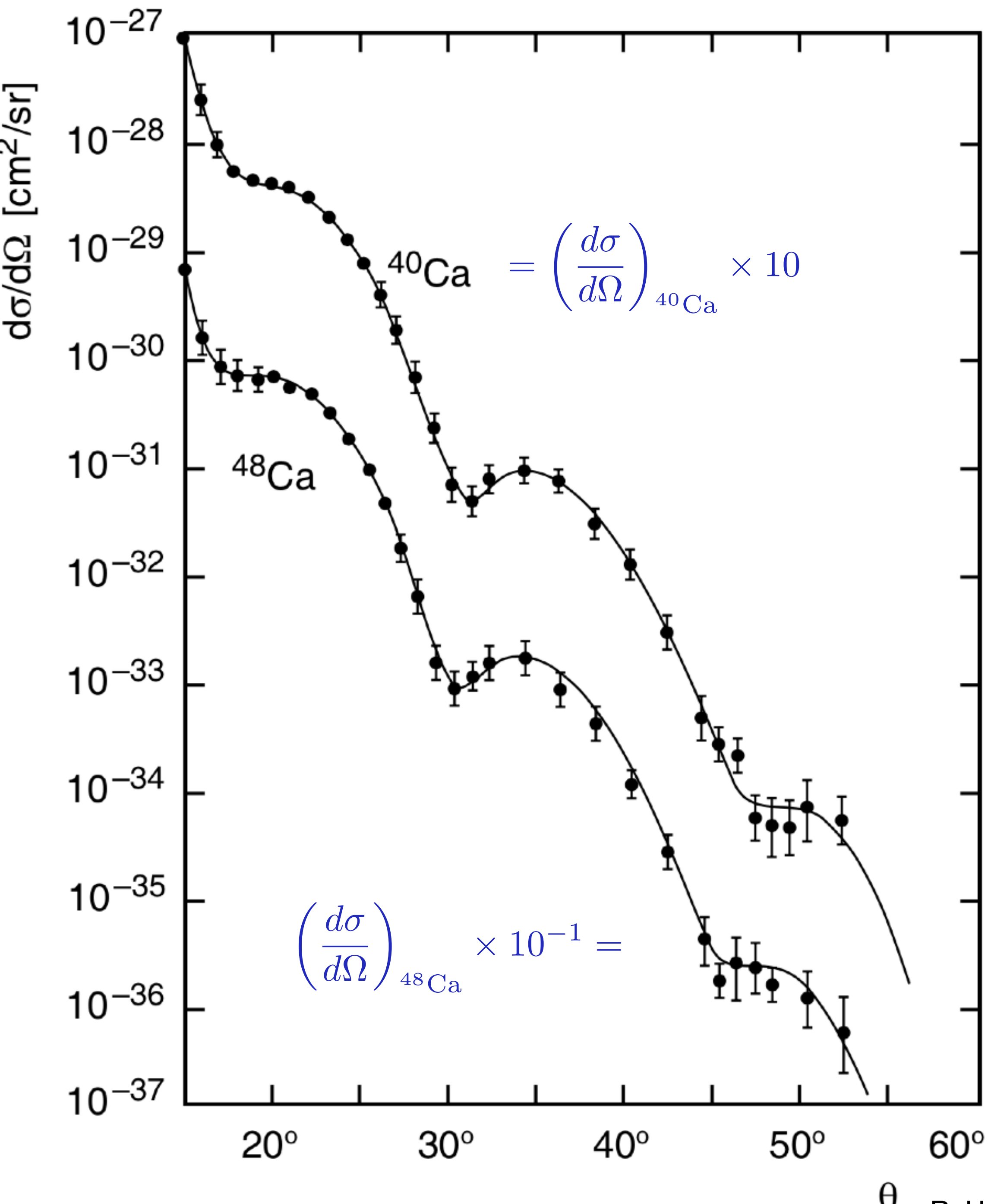


Measurement of the form factor of C12 by electron scattering.

—— Exact phase shift analysis

- - - - Scattering of a plane wave off an homogeneous sphere with a diffuse surface: Born approximation

R. Hofstadter, Ann. Rev. Nucl. Sci 7, 231 (1957)



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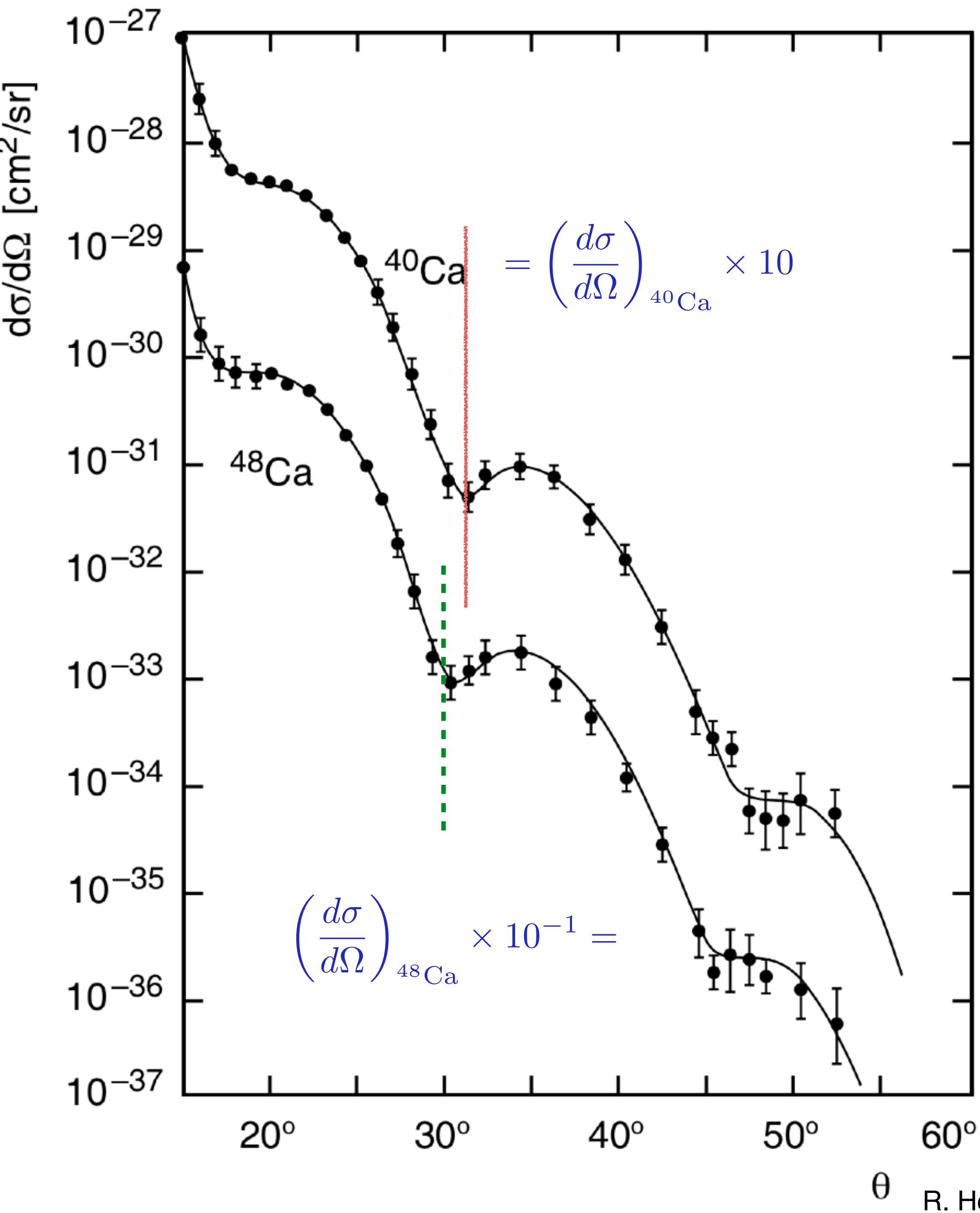
Those of which the form factors fall off faster, the corresponding sizes are larger!



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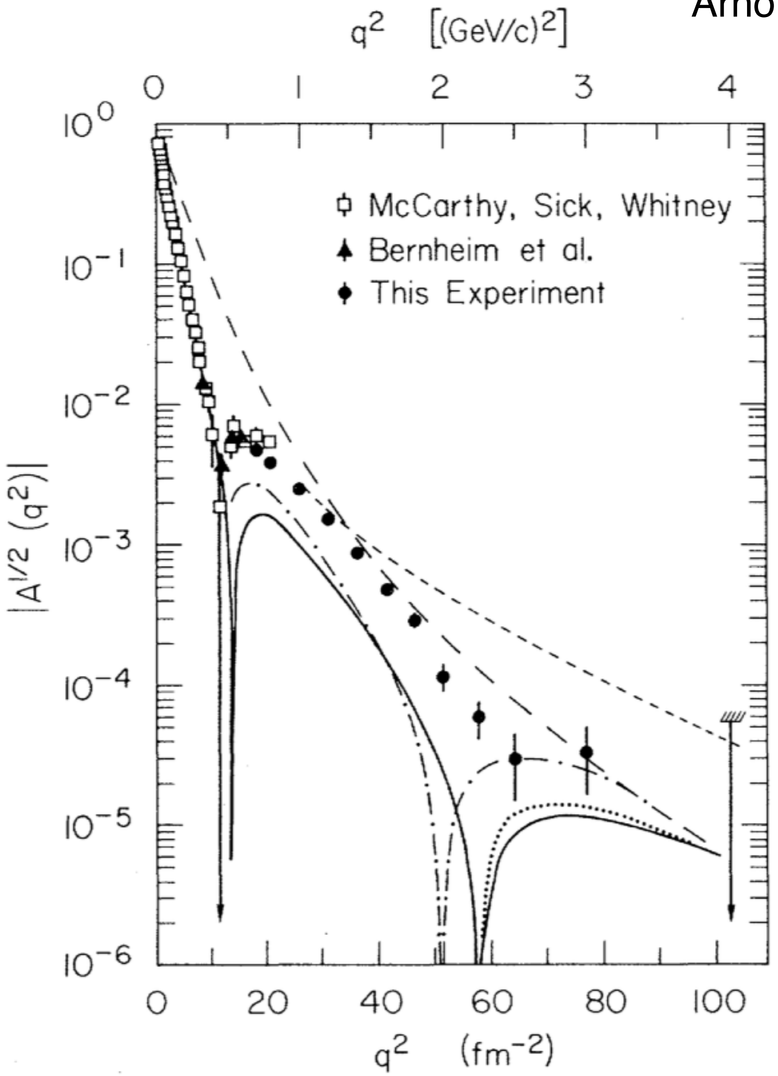
↓

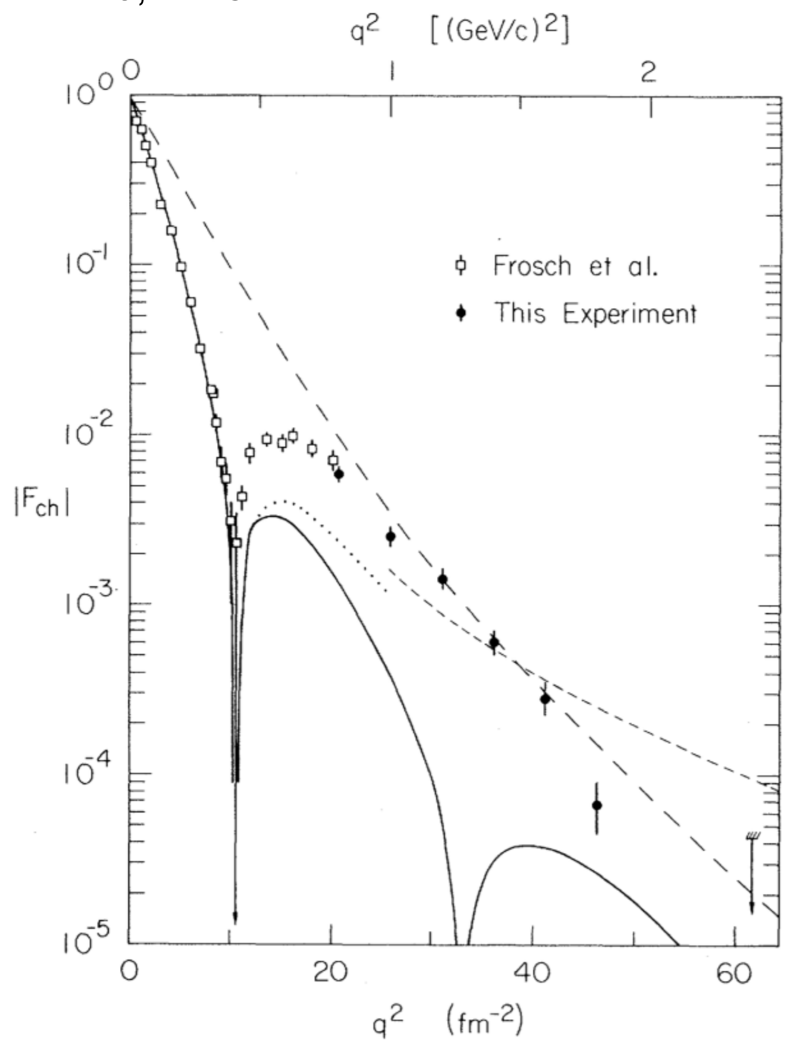
Ca48 is larger than Ca40

R. Hofstadter, Ann. Rev. Nucl. Sci 7, 231 (1957)

He3 & He4 Form factors

Arnold et al., PRL 40, 1429





$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} [A(q^2) + B(q^2)\tan^2\theta/2]$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} |F_{\rm ch}(q^2)|^2$$

Nucleon structure

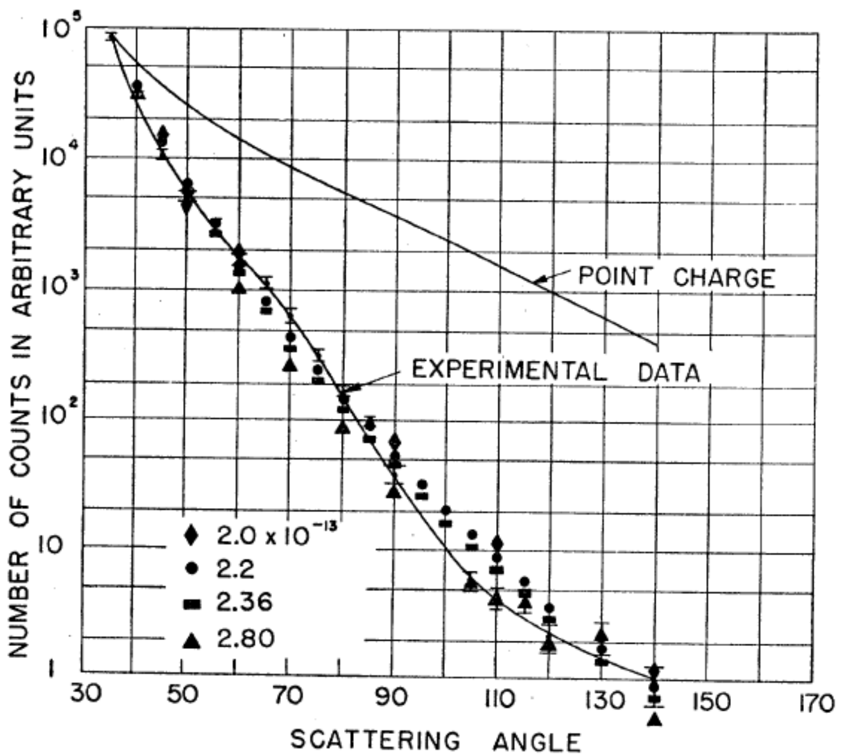


FIG. 11. The angular distribution of electrons scattered from a 2-mil gold foil at 125 Mev. The point charge calculation of Feshbach is indicated. Theoretical points based on the first Born approximation for exponential charge distributions are shown. Values of $\alpha = 2.0$, 2.2, 2.36, 2.8×10^{-13} cm are chosen to demonstrate the sensitivity of the angular distribution to change of radius. All curves are normalized arbitrarily at 35°.

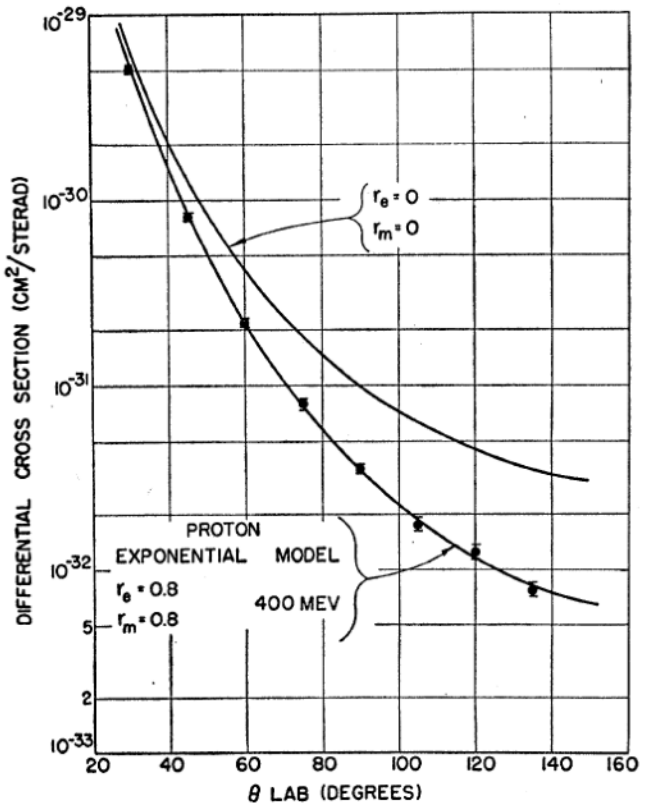
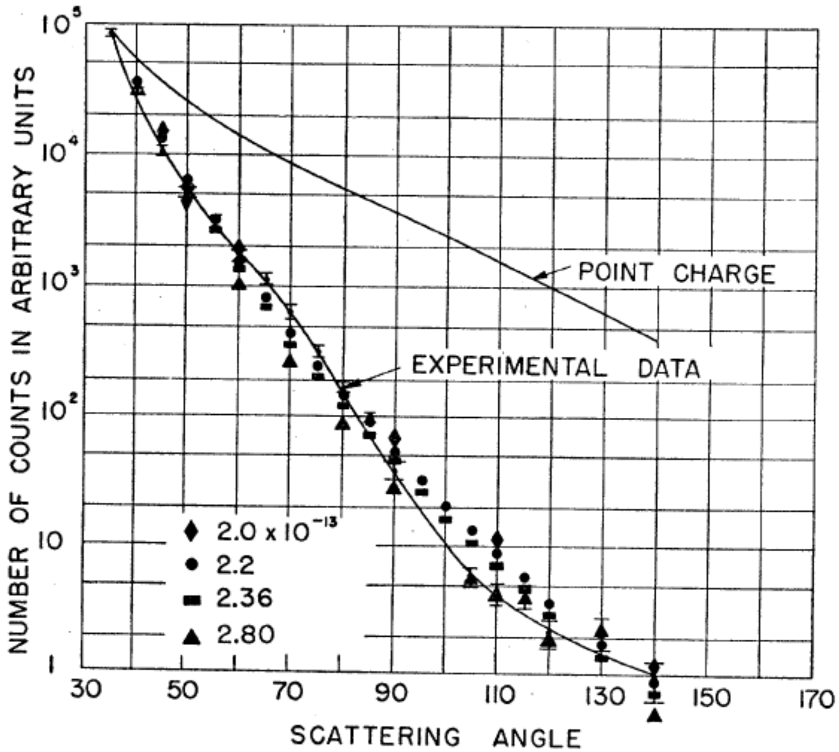


FIG. 26. Typical angular distribution for elastic scattering of 400-Mev electrons against protons. The solid line is a theoretical curve for a proton of finite extent. The model providing the theoretical curve is an exponential with rms radii $= 0.80 \times 10^{-13}$ cm.

Nucleon structure



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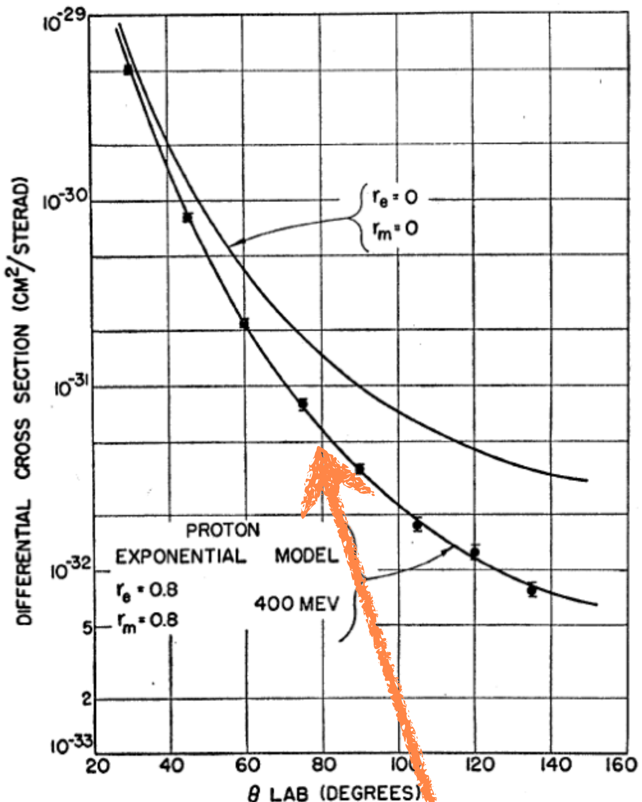

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Nucleon has a size!

Nucleus structure

The Nobel Prize in Physics 1961

Robert Hofstadter

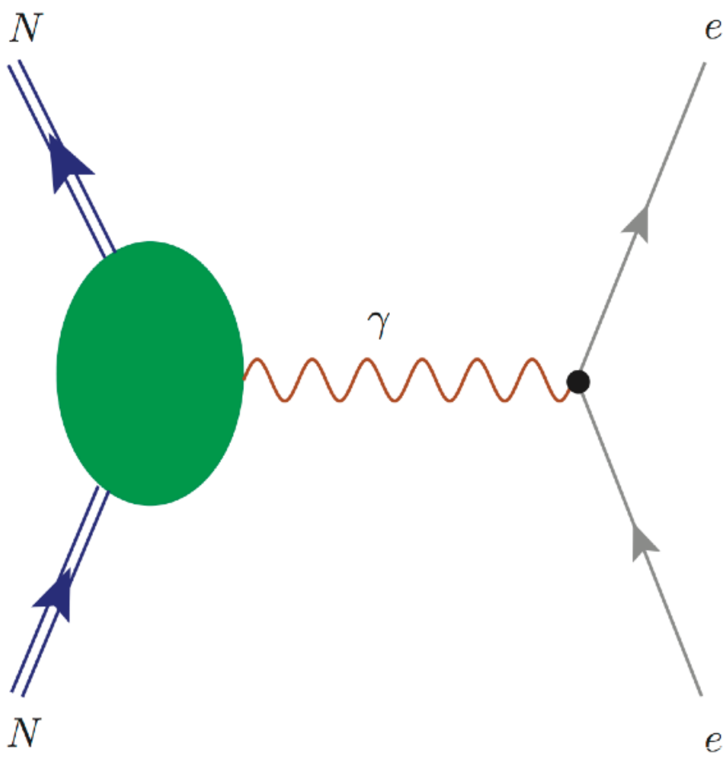
"For his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"

Electron scattering: A clean-cut probe to the nucleon

The electron is immune to the strong interaction
that contains a full of dirt.

Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



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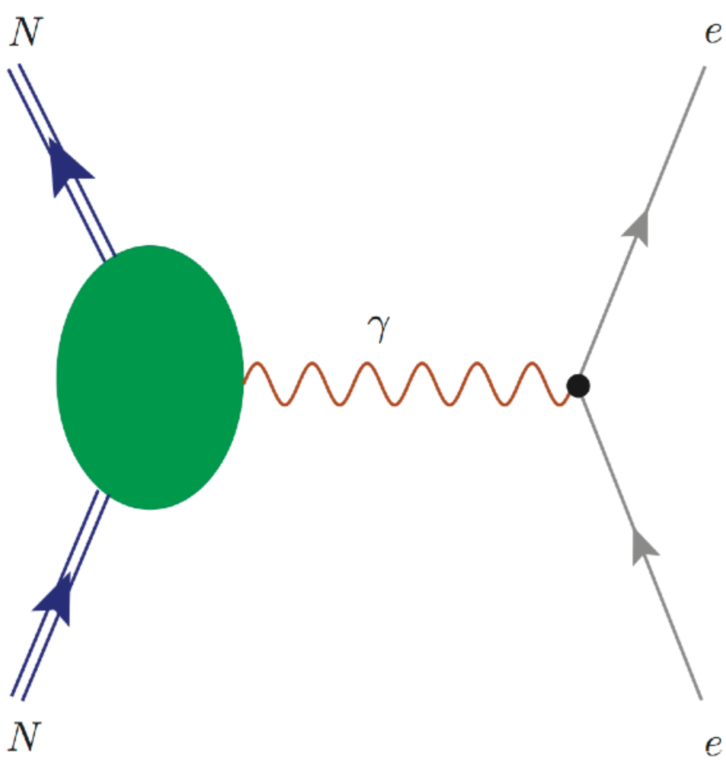

Schroedinger Eq. & Wave functions

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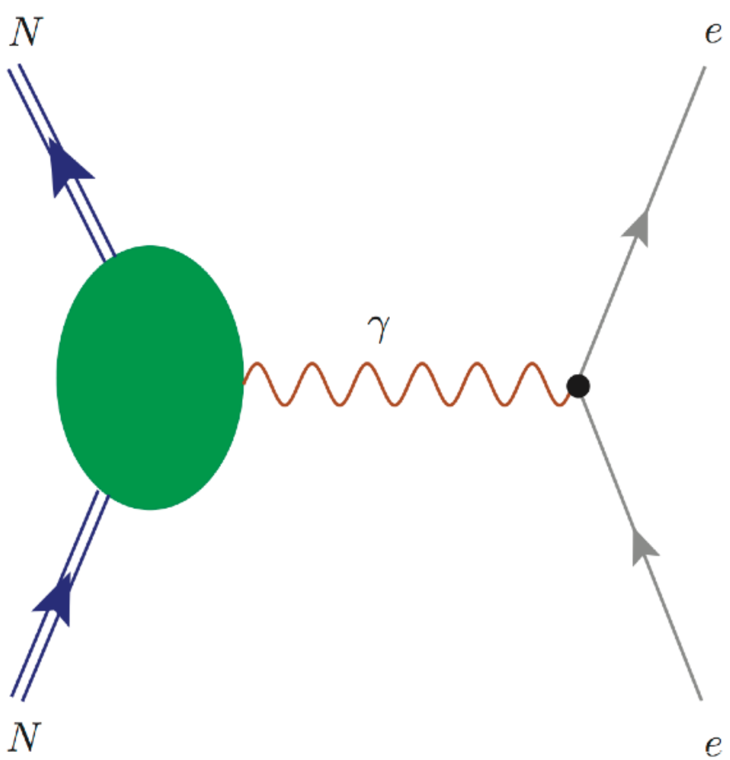

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions

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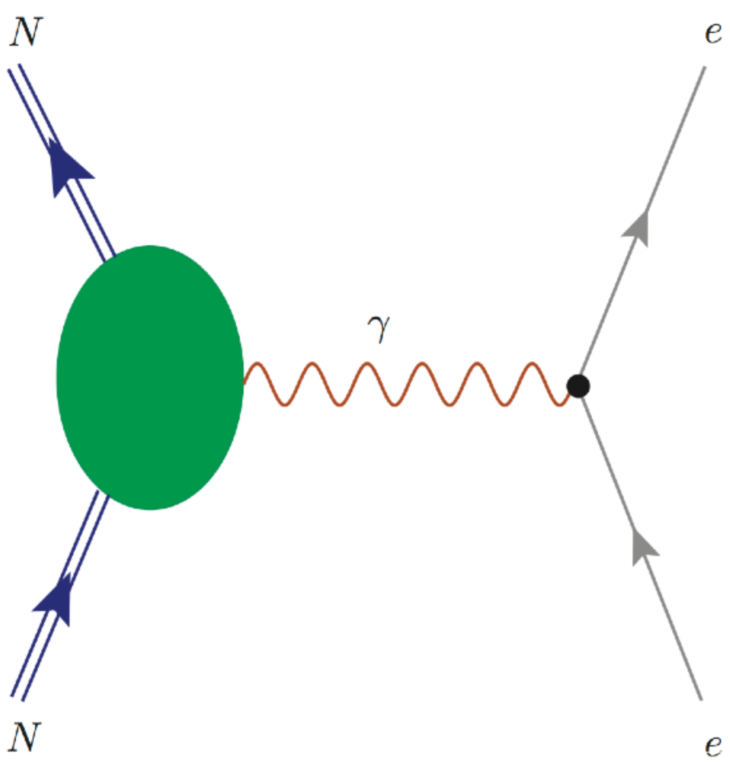

3-D Fourier Transform

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions

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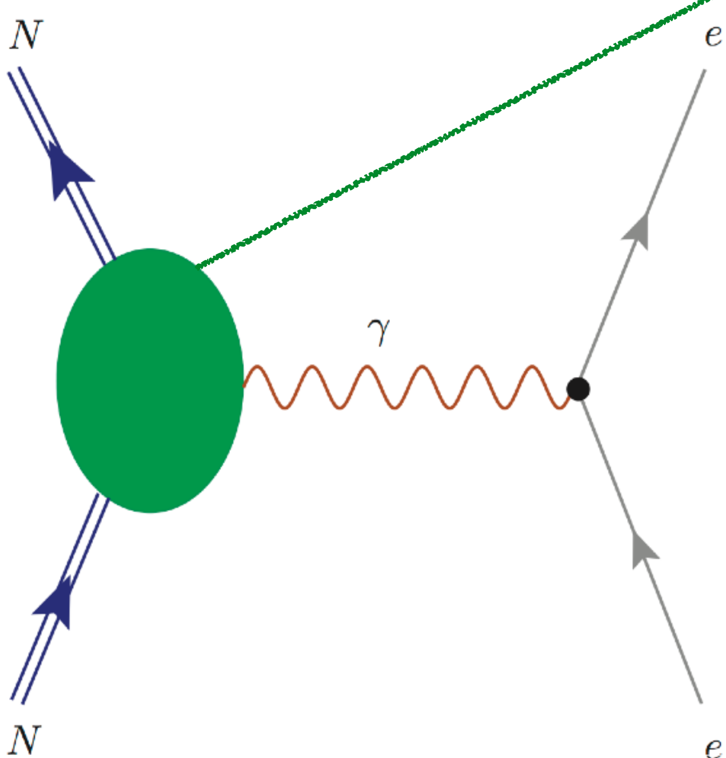


Electromagnetic form factors

↑

3-D Fourier Transform

↑

Charge & Magnetisation Densities

↑

Schroedinger Eq. & Wave functions

What is the nucleon?

Nucleon has internal structure!

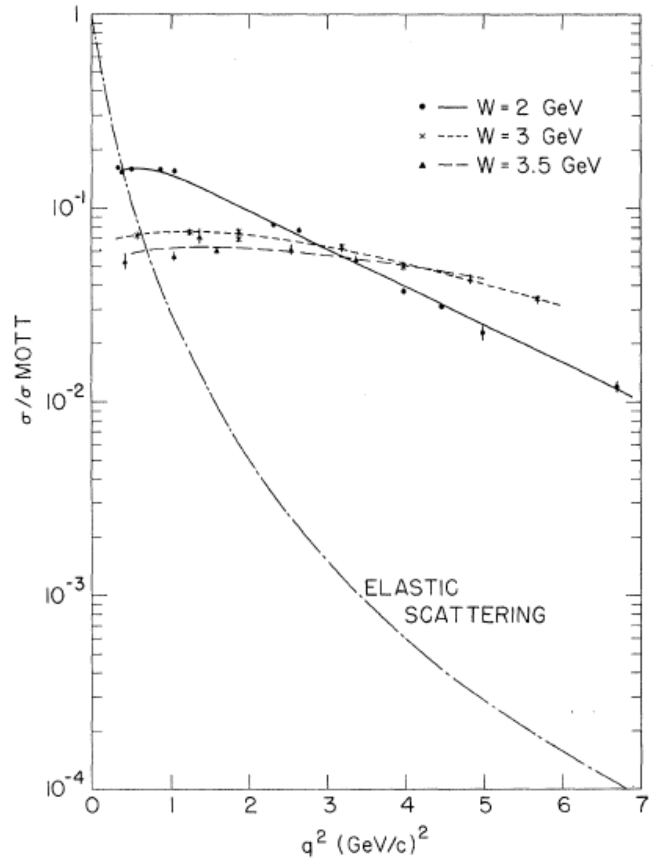
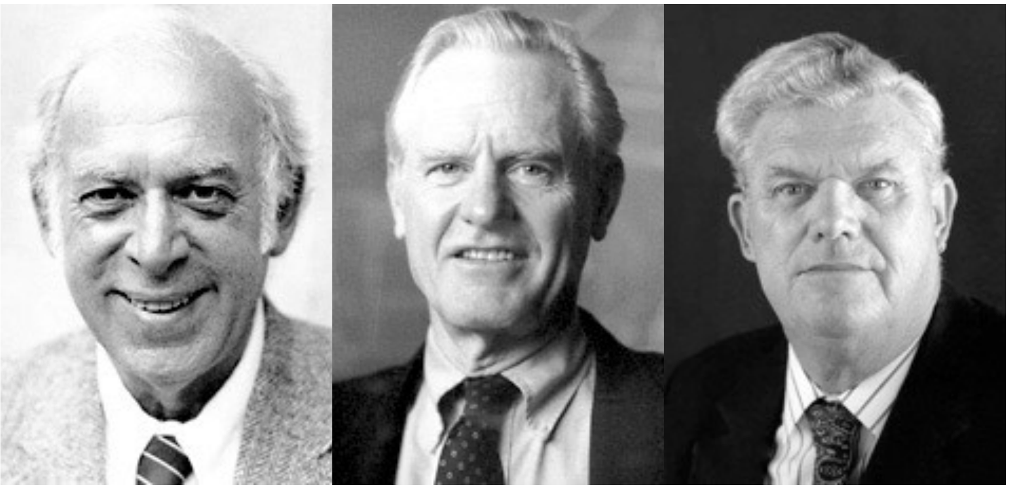


FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$, in GeV^{-1}, vs q^2 for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e–p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

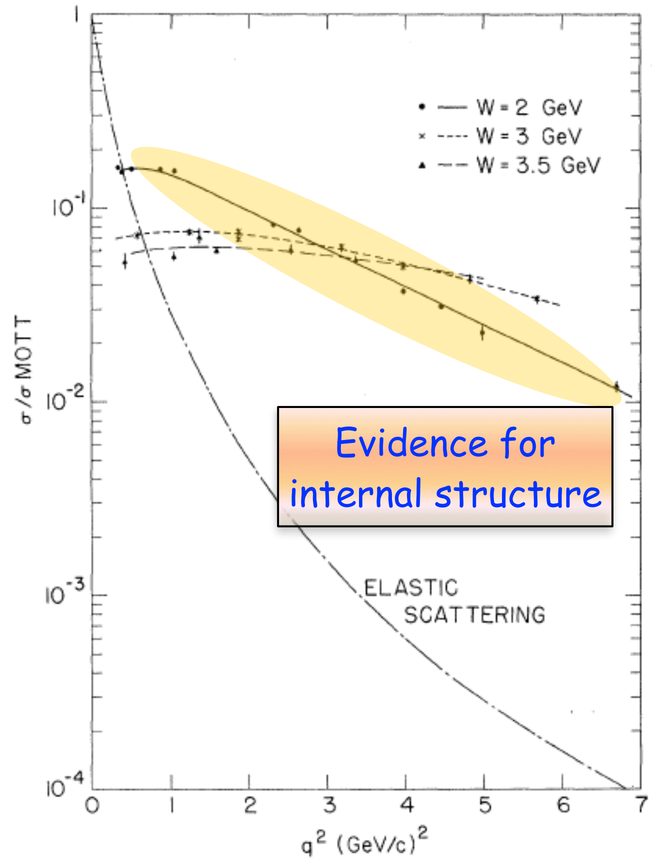
1990, Nobel Laureates



J. Friedman H. Kendall R. Taylor

"For their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

Nucleon has internal structure!



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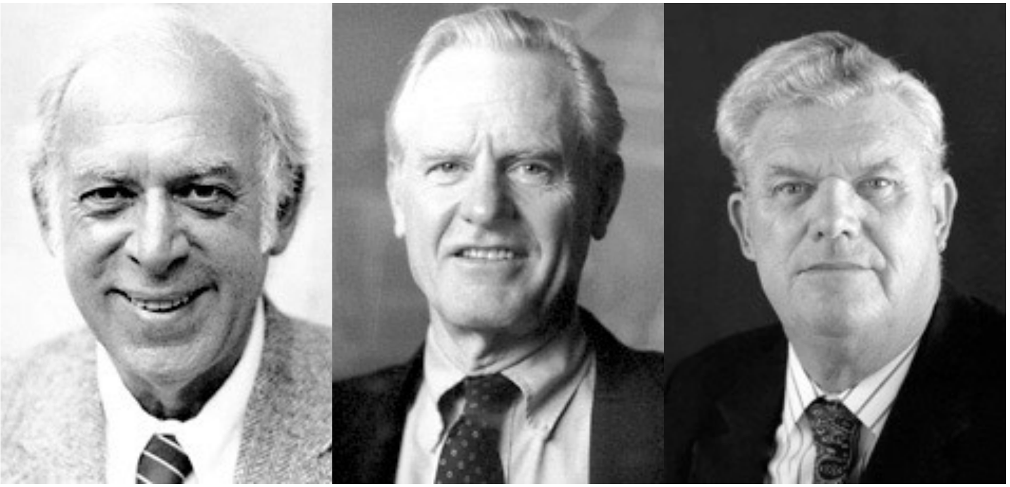

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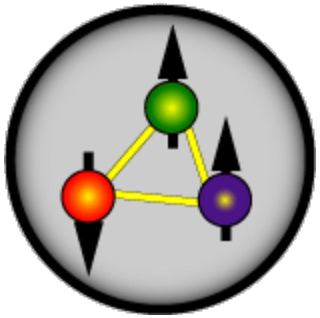
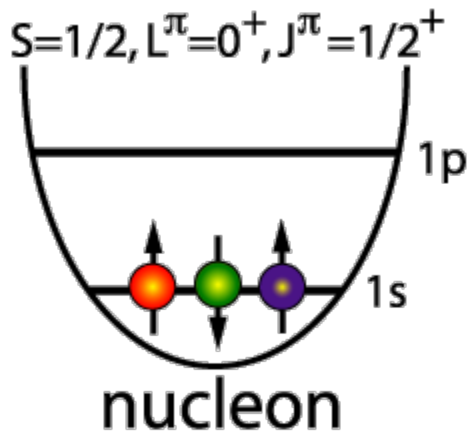
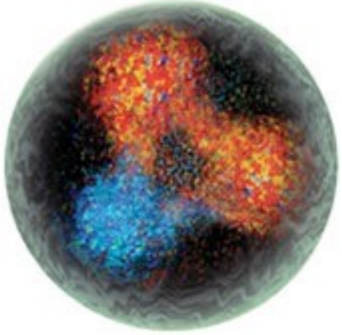
What we know about the Nucleon

- Charge
  - Proton: $Q_p = +1$
  - Neutron: $Q_n = 0$
- Miss: $M_p = 938.272046 \pm 0.000021\ MeV/c^2$
  $M_n = 939.565379 \pm 0.000021\ MeV/c^2$
  - Proton + neutron make up 99.9% of the mass of the visible universe

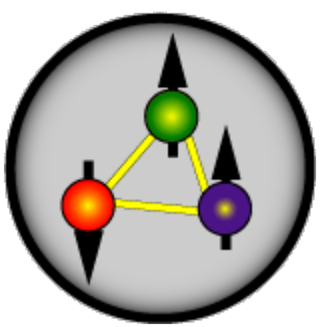


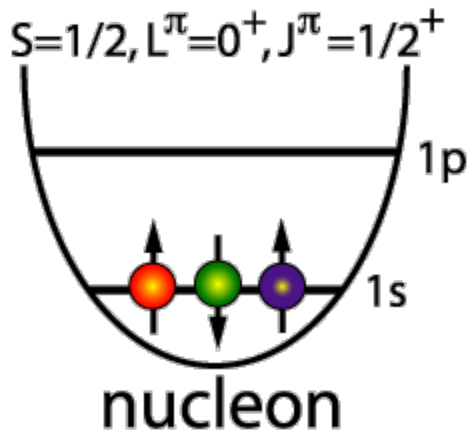


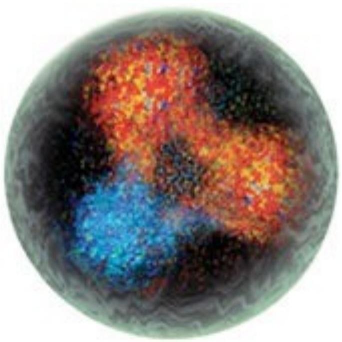


What we know about the Nucleon

- Spin: $s = \frac{1}{2}\hbar$
  - Magnetic moment $\mu_p = 2.79\mu_N$, $\mu_p = -1.91\mu_N$
  - Anomalous magnetic moment $\kappa_p = 1.79\mu_N$, $\kappa_n = -1.91\mu_N$

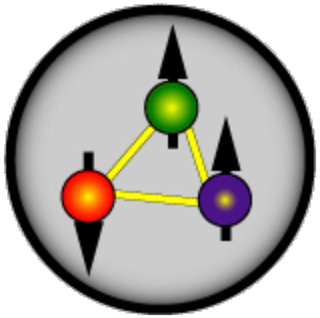






How the Nucleon looks like

The Non-Relativistic Quark Model



$$|N\rangle \sim |qqq\rangle$$

$$|N\rangle \sim |q_\uparrow q_\uparrow q_\downarrow\rangle$$

Amazingly successful!

SU(3) flavor Symmetry
+
Spin symmetry

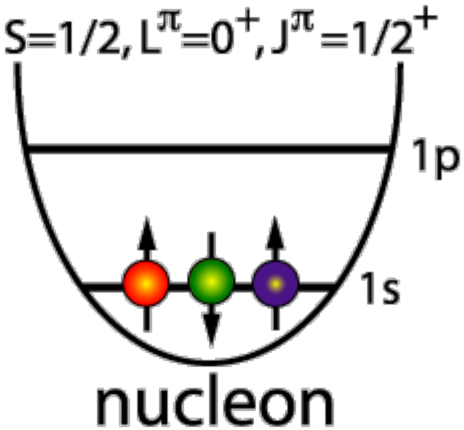
Constituent quark mass

$$M_q \simeq 350\,\mathrm{MeV}$$

$$M_N \approx 3M_q$$

- No explanation was given why the quark mass is so large.
- No interaction and dynamics were considered.

How the Nucleon looks like

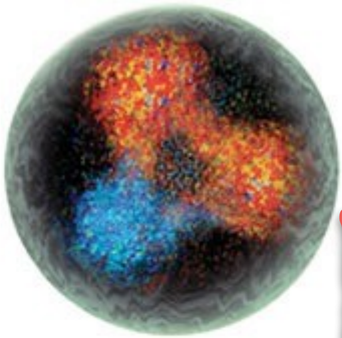


The Nucleon as three quarks
in an instantaneous potential

- Symmetry + Phenomenological dynamics
- Nucleon excited states can be described (confinement potential)
- Many properties were nicely explained.

- Potential originated from heavy-quark systems, not for the light-quark system.
- Failure of explaining strong decays (correct feature for resonances)
- Not fully relativistic (No sea quark, one needs field theory).

How the Nucleon looks like

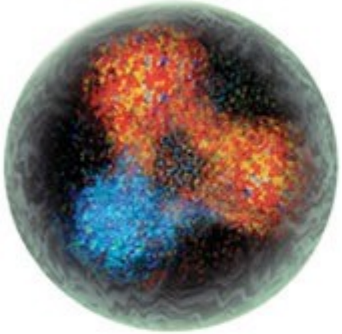


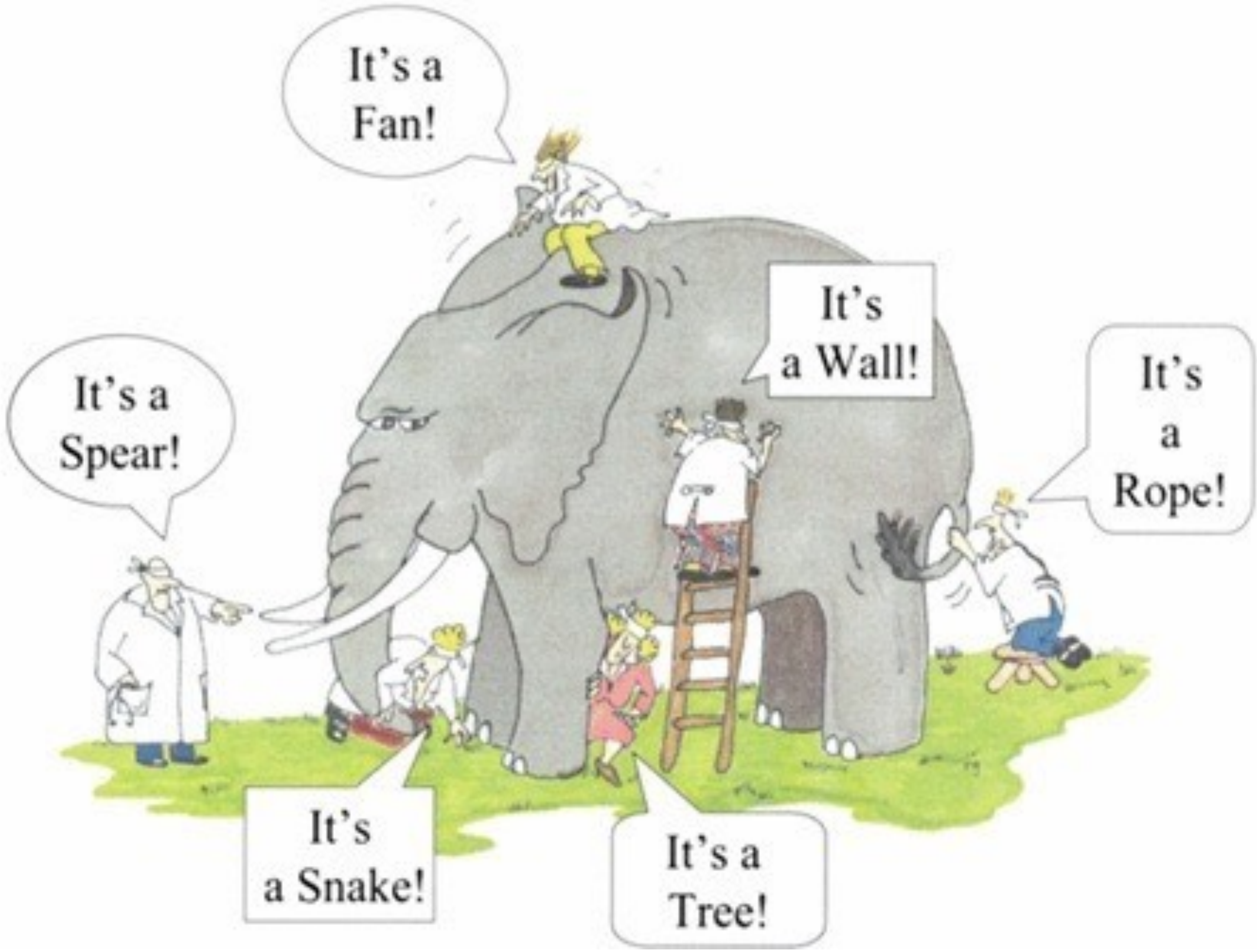
The Nucleon, the most messy object in the Universe

- Valence quarks + Sea quarks + gluons + …
- Too complicated to solve?
- Brute force way: Lattice QCD
- Hadrons as relevant degrees of freedom (Effective Field Theory)
- Holographic QCD (5D QCD)
- Instantons
- Monopoles
- Large Nc QCD
- Skyrme models, NJL models, Chiral quark models…

Each approach has pros and cons.

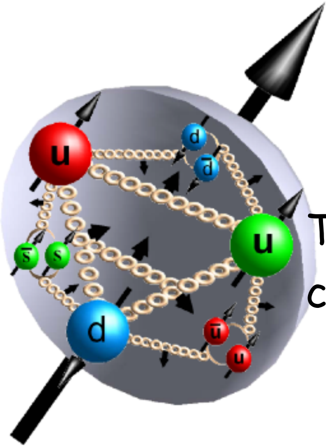
How the Nucleon looks like



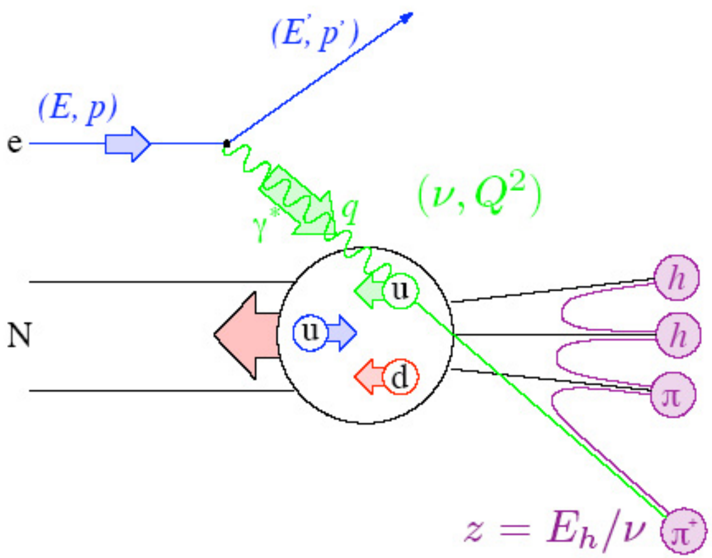


Strong interactions

Strong Interaction



The quarks are confined in the nucleon



Indication that the quarks live inside the nucleon! QCD

Nobel prize in Physics 2004

D.J. Gross H.D. Politzer F. Wilczek

"For the discovery of asymptotic freedom in the theory of the strong interaction"

QCD

Quantum Chromodynamics (QCD)

- Fundamental Theory for the strong interaction
- A Fundamental Mathematical problem: One of Millennium Prize Problems

QCD Partition function or Lagrangian

$$
\begin{aligned}
Z_{\rm QCD} &= \int DA_\mu D\psi D\psi^\dagger \exp\left[\sum_{f=1}^{N_f}\int d^4x \psi_f^\dagger (i\not{D} + im_f)\psi_f - \frac{1}{4g^2}\int d^4x G^2\right] \\
&= \int DA_\mu \exp\left[-\frac{1}{4g^2}\int d^4x G^2\right] {\rm Det}(i\not{D} + im_f)
\end{aligned}
$$

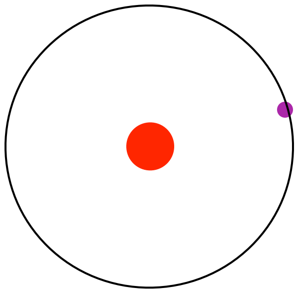
(No gauge fixing, no ghost field)

Most important Two Features of QCD

- Quark Confinement
- Spontaneous Breakdown of Chiral Symmetry

QED & QCD: Analogy and difference

to study structure of an atom…



neutral atom

QED Quantum Electro Dynamics

Christina Markert's talk

QED & QCD: Analogy and difference

to study structure of an atom…

electron

…separate constituents

nucleus

QED Quantum Electro Dynamics

Christina Markert's talk

QED & QCD: Analogy and difference

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QED Quantum Electro Dynamics

Christina Markert's talk

QCD: Quantum ChromoDymanics

QED & QCD: Analogy and difference

to study structure of an atom…

electron

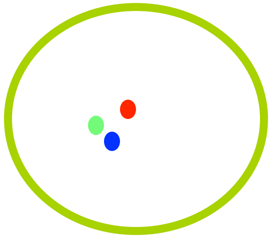
…separate constituents

nucleus

QED Quantum Electro Dynamics

Confinement: fundamental & crucial (but *not* understood!) feature of strong force

- colored objects (quarks) have ∞ energy in normal vacuum



“white” proton

Christina Markert’s talk

QCD: Quantum ChromoDymanics

QED & QCD: Analogy and difference

to study structure of an atom…

electron

…separate constituents

nucleus

QED Quantum Electro Dynamics

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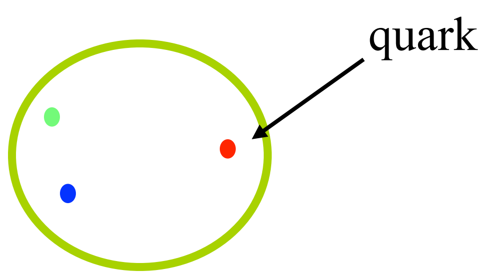

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Strong **color** field

Force *grows* with separation !!!

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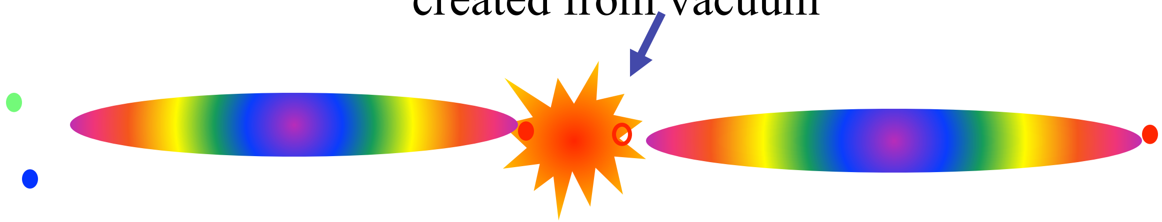
quark-antiquark pair

created from vacuum



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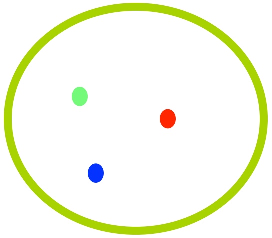
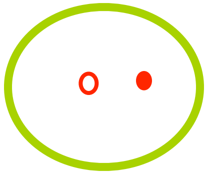


“white” proton (baryon)

(confined quarks)



“white” π^0 (meson)

(confined quarks)

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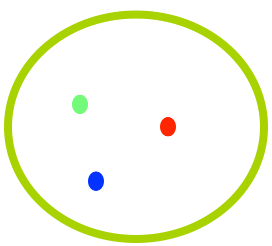
quarks

u,d, (s,c,t,b)



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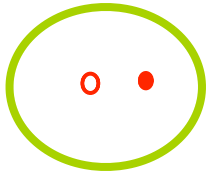

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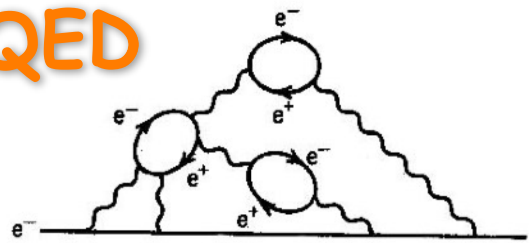
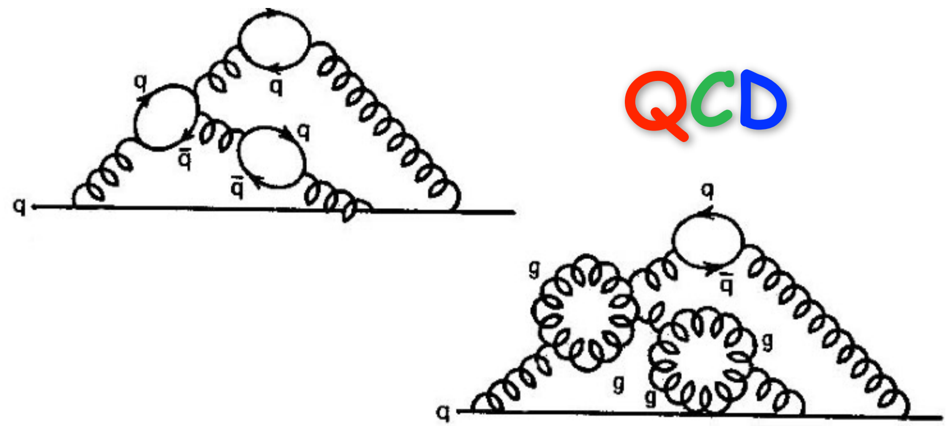
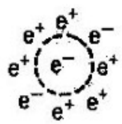
QED & QCD: Analogy and difference

QED

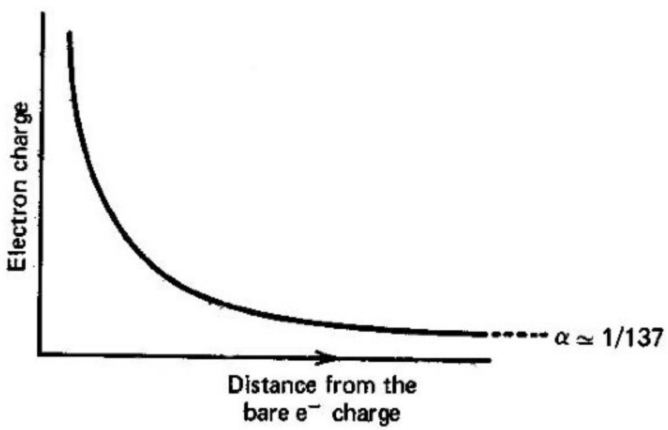


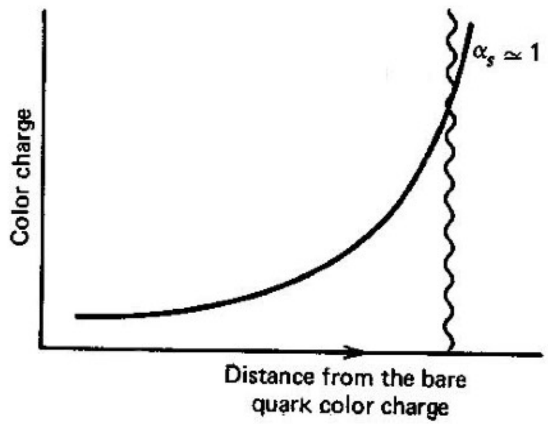
QCD





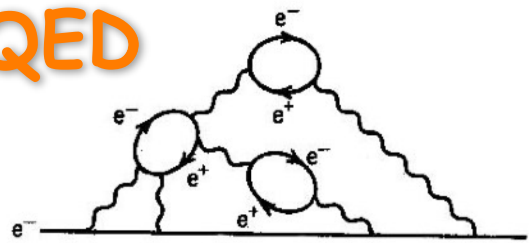
Vacuum polarization





QED & QCD: Analogy and difference

QED



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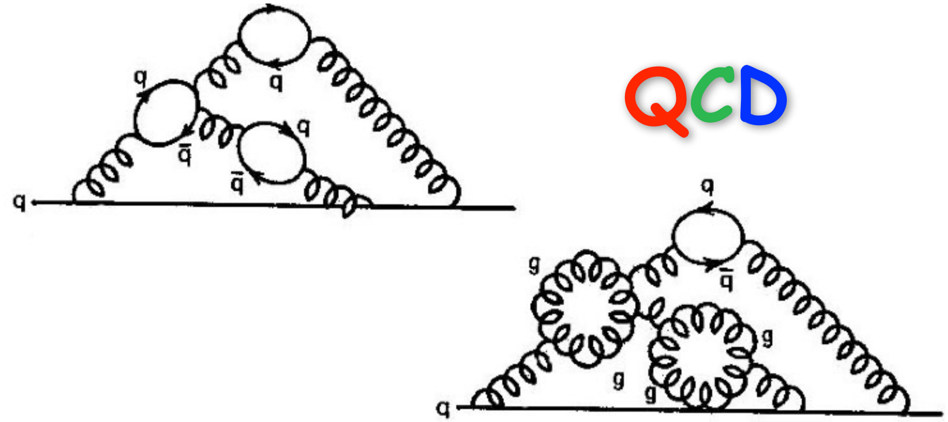

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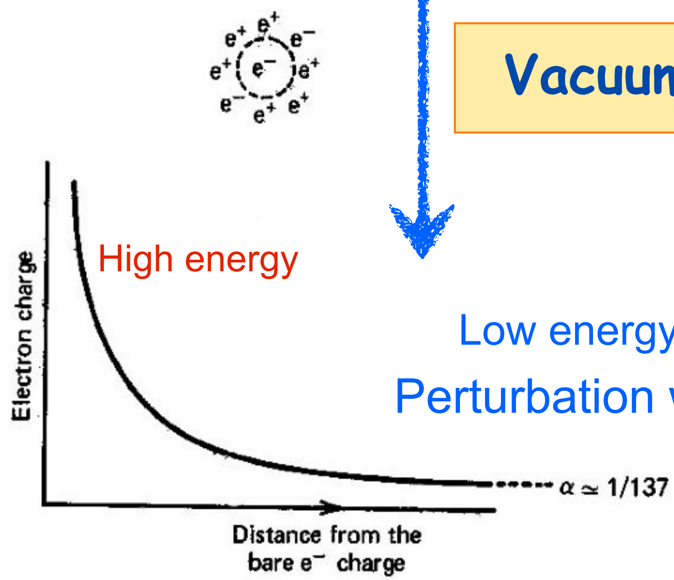
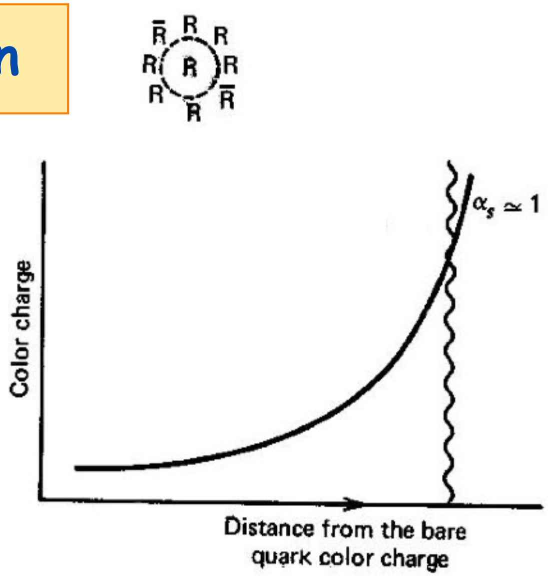


High energy

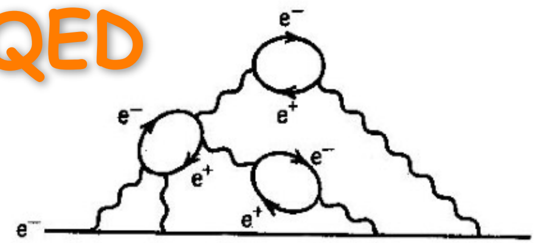
Low energy

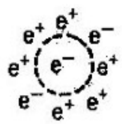
Perturbation works!



QED & QCD: Analogy and difference

QED





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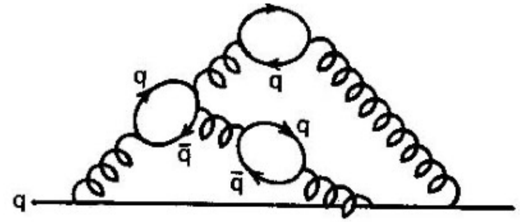
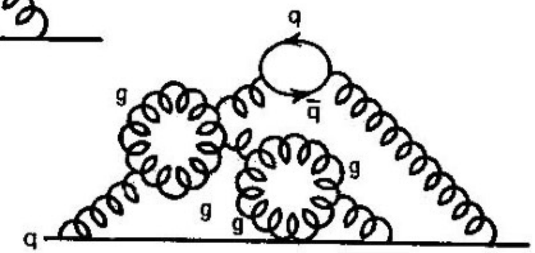
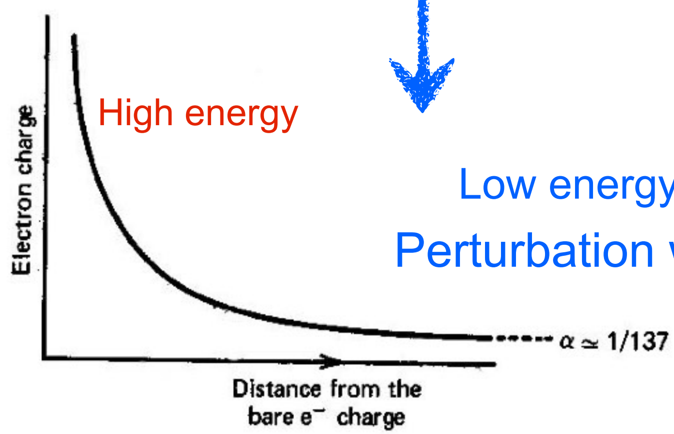




Vacuum polarization

Quark Confinement

Low energy



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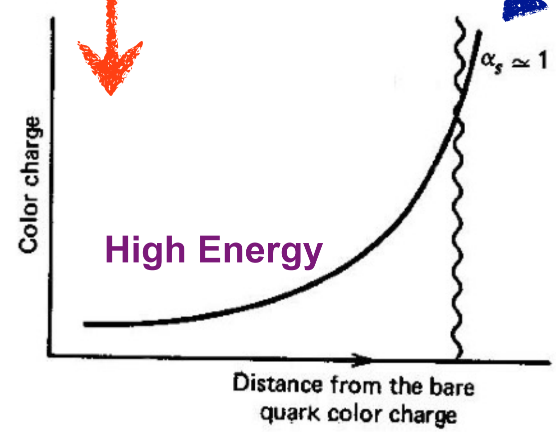


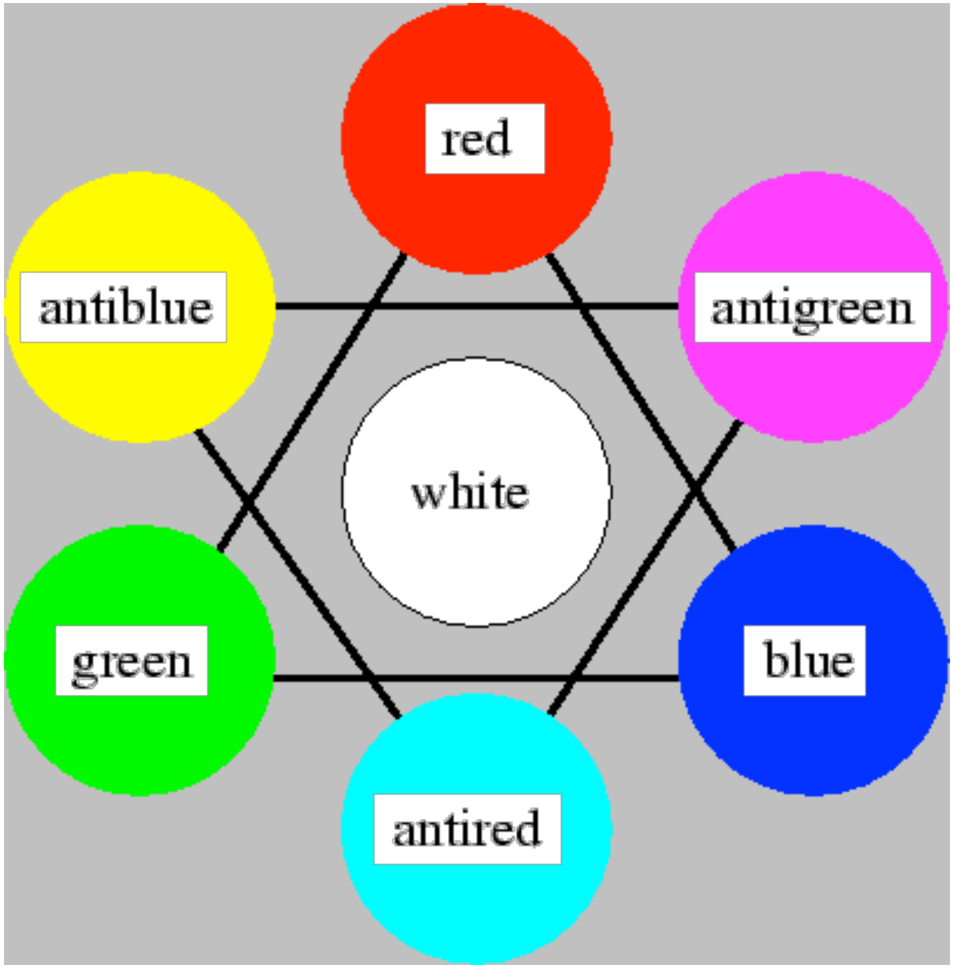
High Energy

Nonperturbative nature of QCD in low energies

Three different Charges in QCD

Quark Confinement

- You can not see the color charges, i.e. quarks in free space!
- You can only find hadrons (mesons and baryons), i.e. colorless (white) particles



SU(3) Color symmetry

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10}$$

Color Singlet

R, G, B

Proton Mass

Electron: $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$

Proton: $M_p = 1.67 \times 10^{-27}\,\mathrm{kg}$

A proton is about 2000 times heavier than an electron.

- A proton consists of three quarks (2 up + 1 down).

- quark mass: $m_q \sim 10^{-29}\,\mathrm{kg}$

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$$M_N \simeq 3m_q \simeq 3 \times 10^{-29}\,\mathrm{kg}\,???$$

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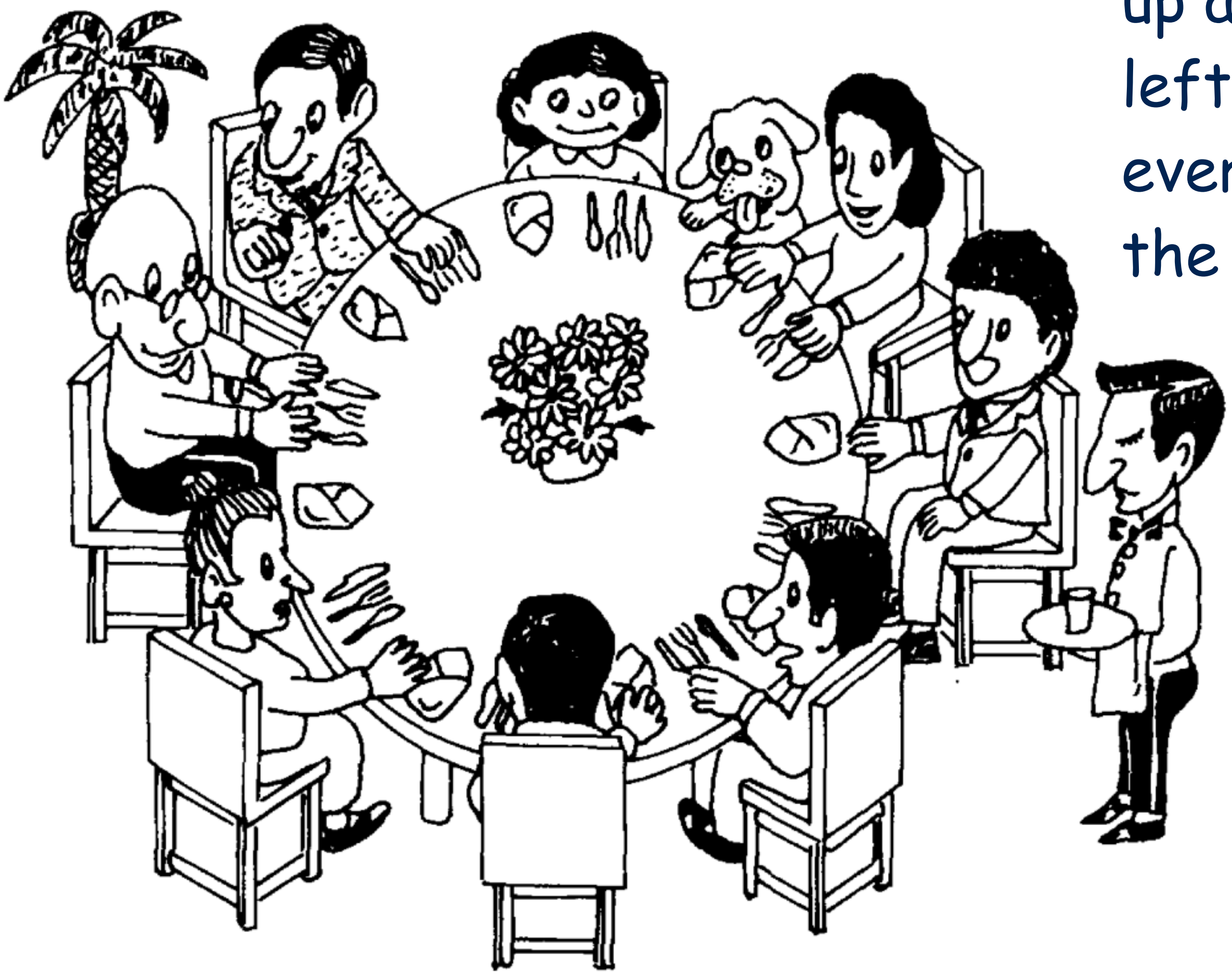
→ A possible answer comes from
the spontaneous breakdown of chiral symmetry.

Spontaneous breakdown of chiral symmetry

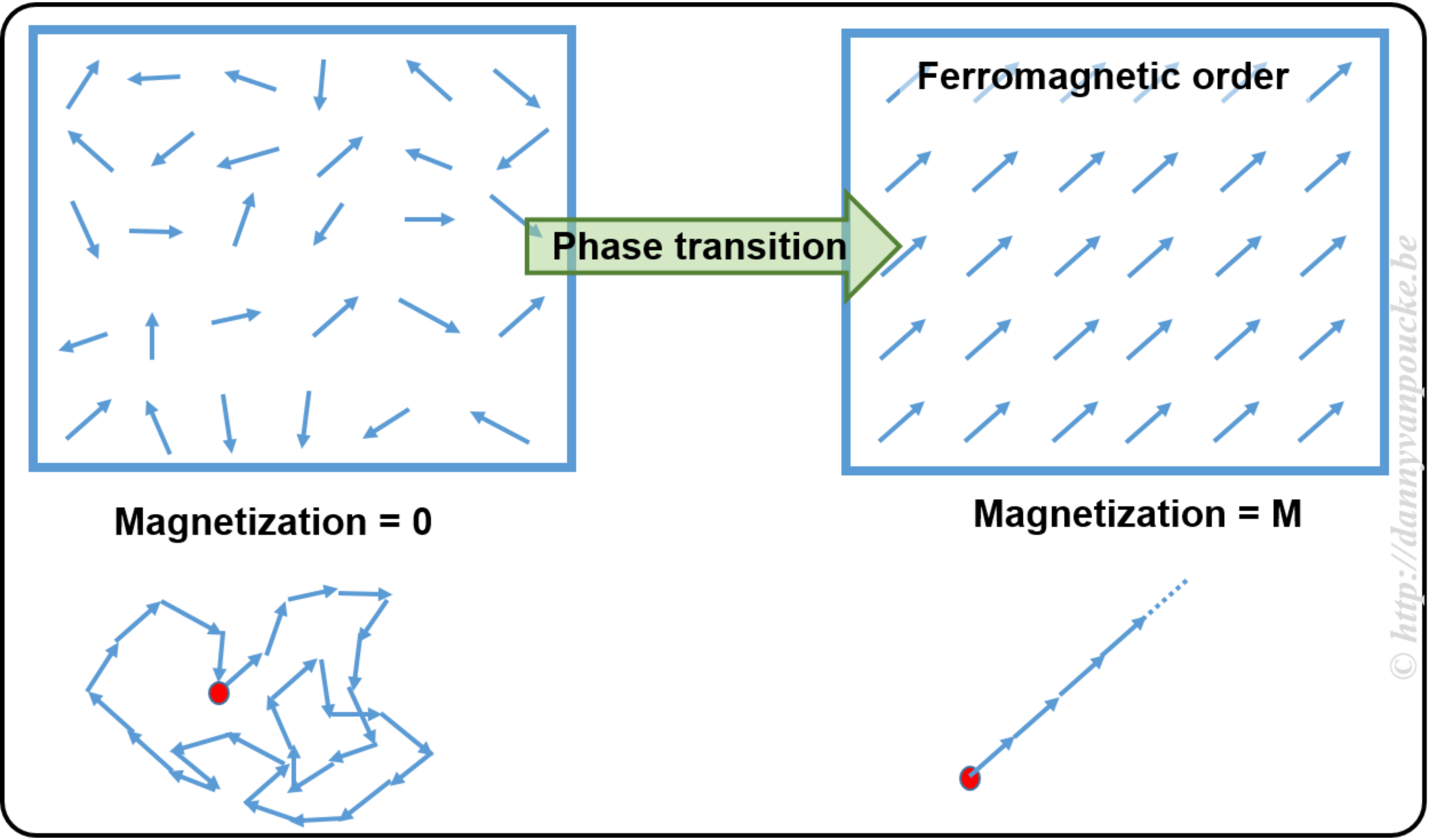
A. Salam's explanation



Once a guest picks up a napkin laid in the left handed side, everybody should take the left-lying ones.

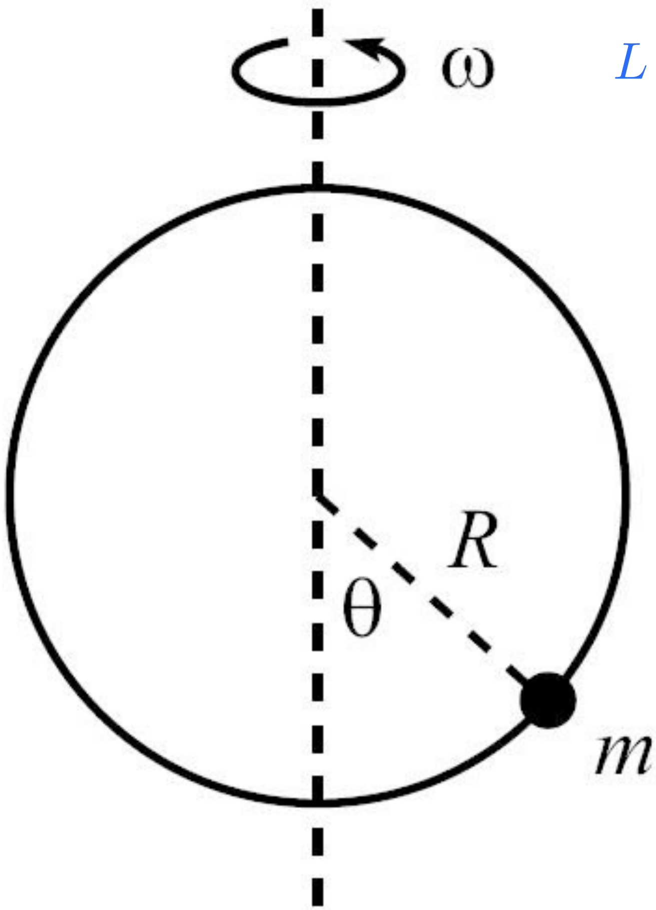
Taken from a book by Y. Nambu

Spontaneous breakdown of chiral symmetry



Spontaneous breakdown of chiral symmetry

Classical Example of spontaneous breakdown



$$L = \frac{1}{2} m R^2 \dot{\theta}^2 + \frac{1}{2} m R^2 \omega^2 \sin^2 \theta + mgR\cos\theta$$

Symon, Mechanics, pp.384

Spontaneous breakdown of chiral symmetry

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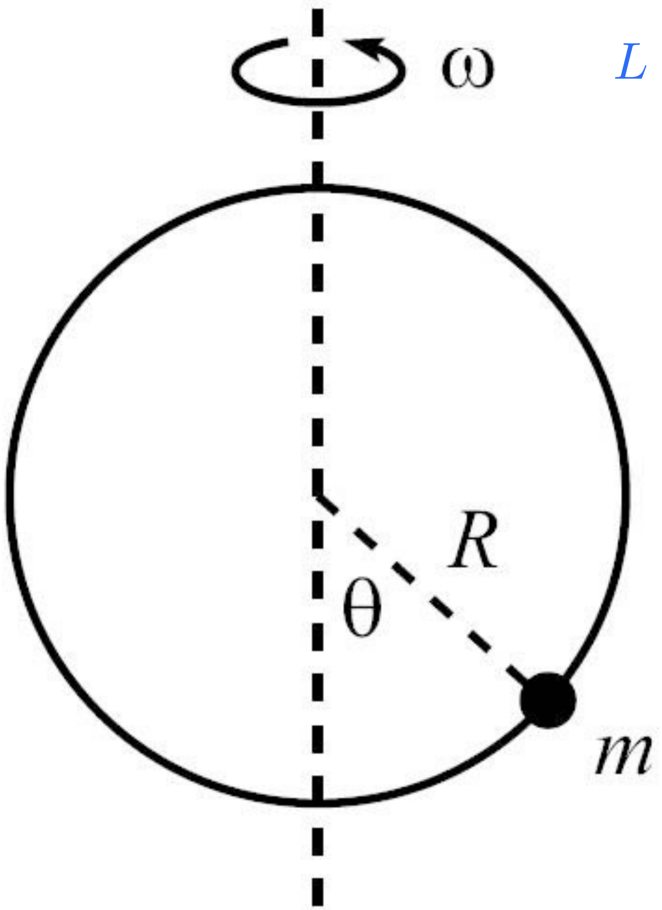

$$L = \frac{1}{2} m R^2 \dot{\theta}^2 + \frac{1}{2} m R^2 \omega^2 \sin^2\theta + mgR\cos\theta$$

Lagrangian is symmetric under

$$\theta \to -\theta$$

$$L(\theta) = L(-\theta)$$

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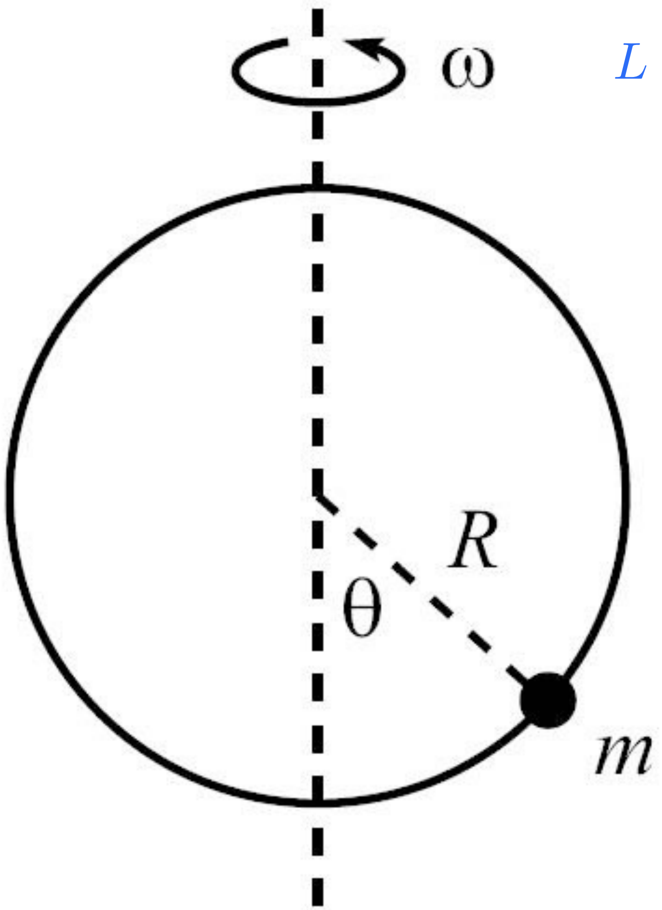

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Lagrangian is symmetric under

$$\theta \to -\theta$$

$$L(\theta) = L(-\theta)$$

However,

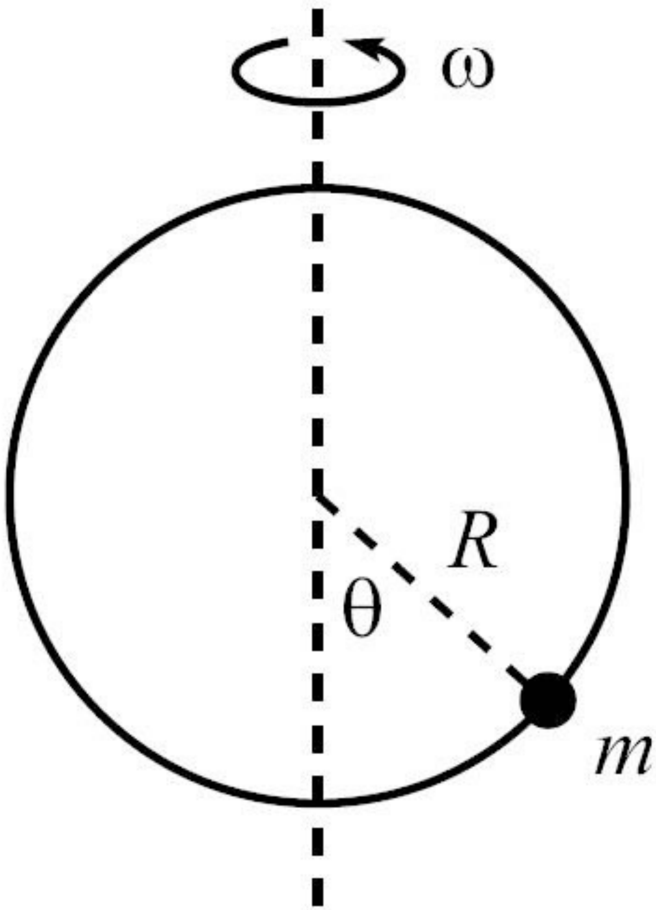
$$\frac{\partial L}{\partial \theta} = 0 \to \cos\theta - \frac{g}{\omega^2 R} = 0$$

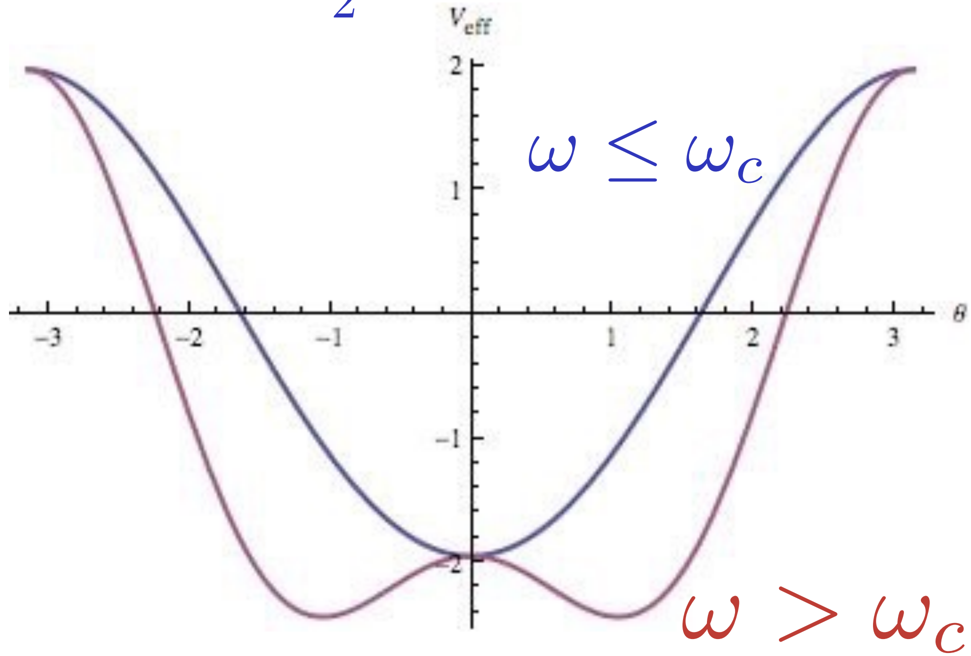
Symon, Mechanics, pp.384

Spontaneous breakdown of chiral symmetry

Classical Example of spontaneous breakdown

$$V_{\rm eff} = -\frac{1}{2} m R^2 \omega^2 \sin^2\theta - m g R \cos\theta$$





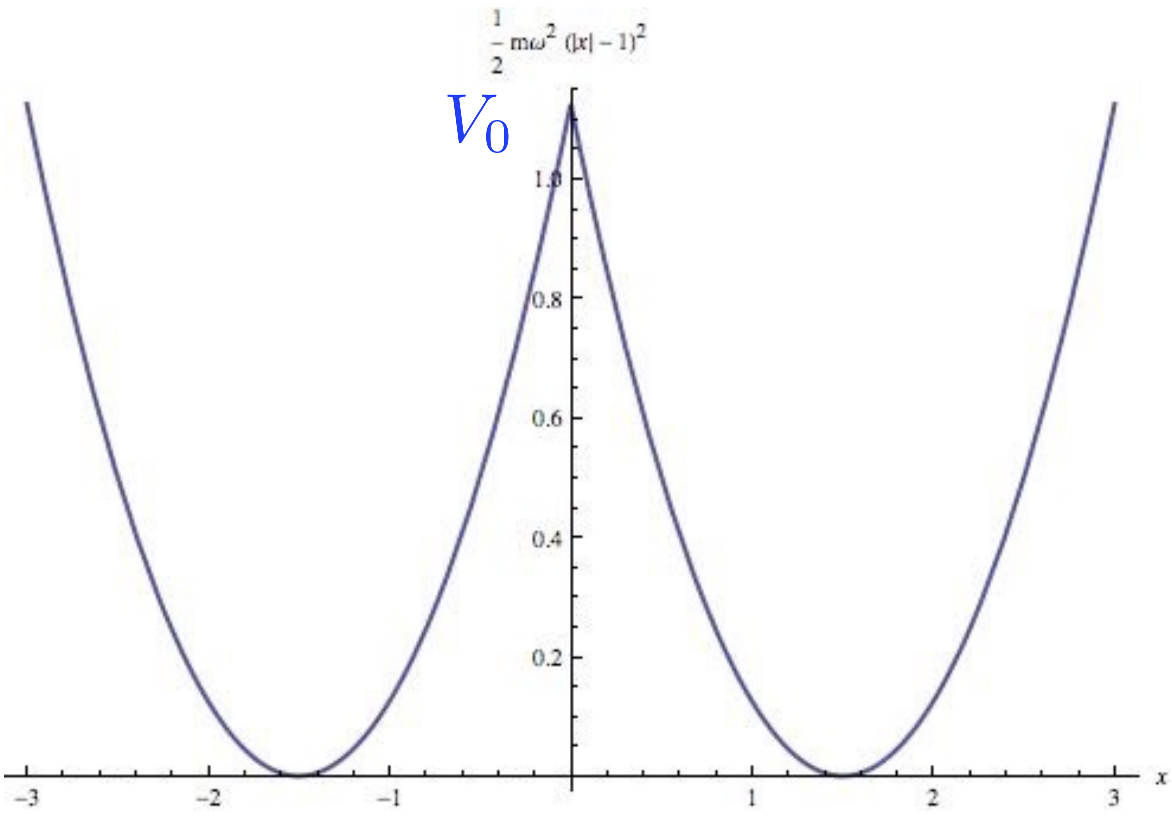
$$\omega_c = \sqrt{\frac{g}{R}}$$

$$L = \frac{1}{2} m R^2 \dot{\theta}^2 + \frac{1}{2} m R^2 \omega^2 \sin^2\theta + m g R \cos\theta$$

Symon, Mechanics, pp.384

Spontaneous breakdown of chiral symmetry

Quantum Example of spontaneous breakdown



$$V = \frac{1}{2} m\omega^2 (|x| - a)^2$$

Merzbacher, Quantum Mechanics, pp.150

Sakurai, Modern Quantum Mechanics, pp.257

Case II

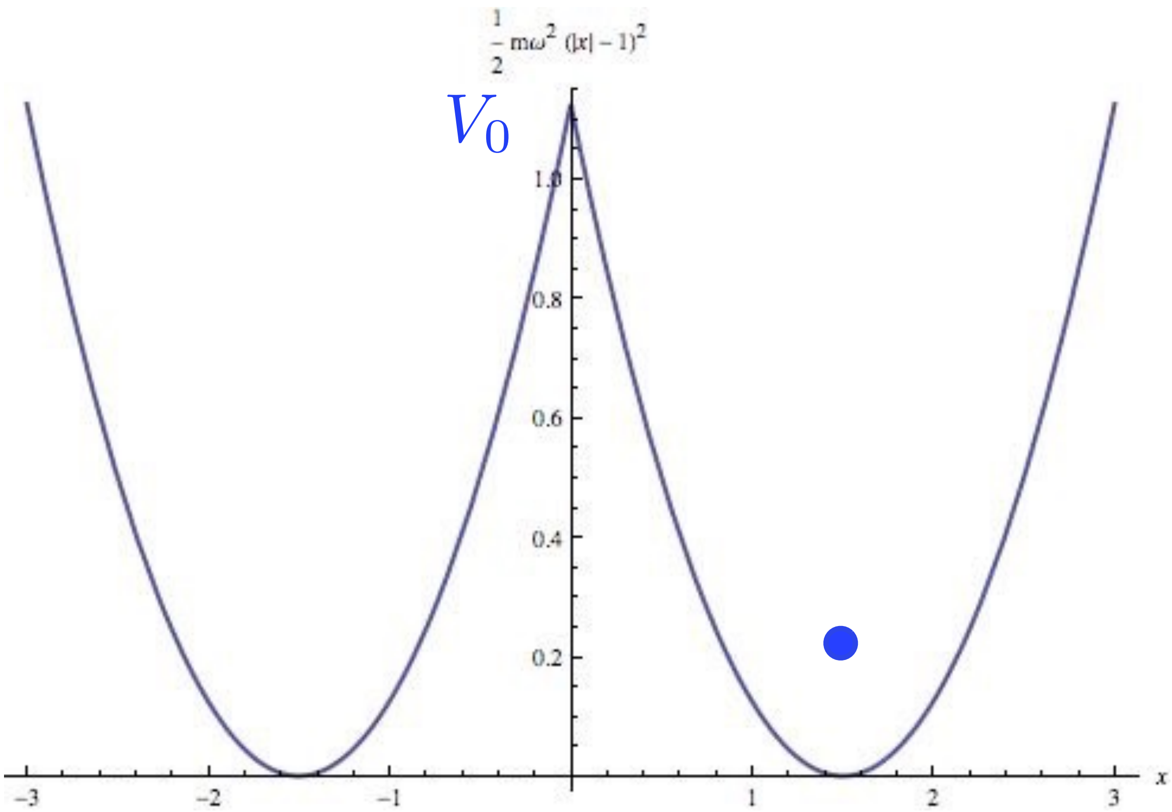
$$a \to \infty$$

$$V_0 \to \infty$$

$|S\rangle, |A\rangle$ are degenerate.

Spontaneous breakdown of chiral symmetry

Quantum Example of spontaneous breakdown



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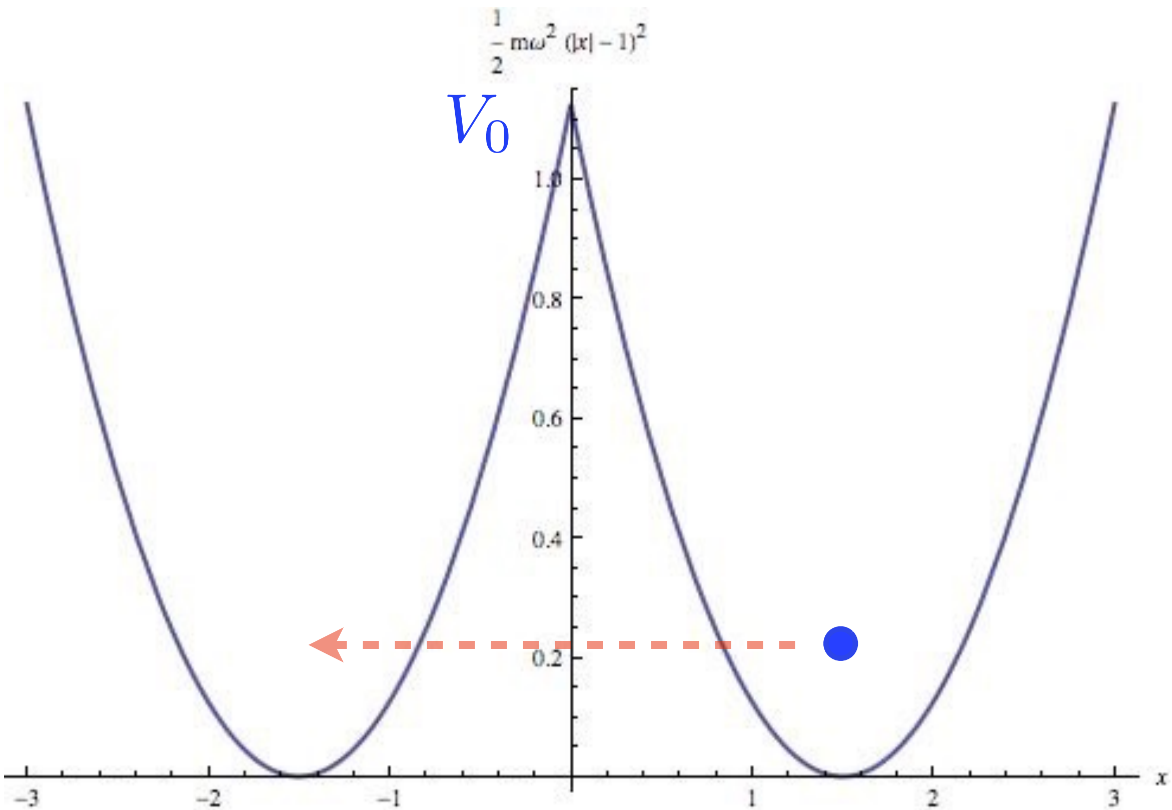

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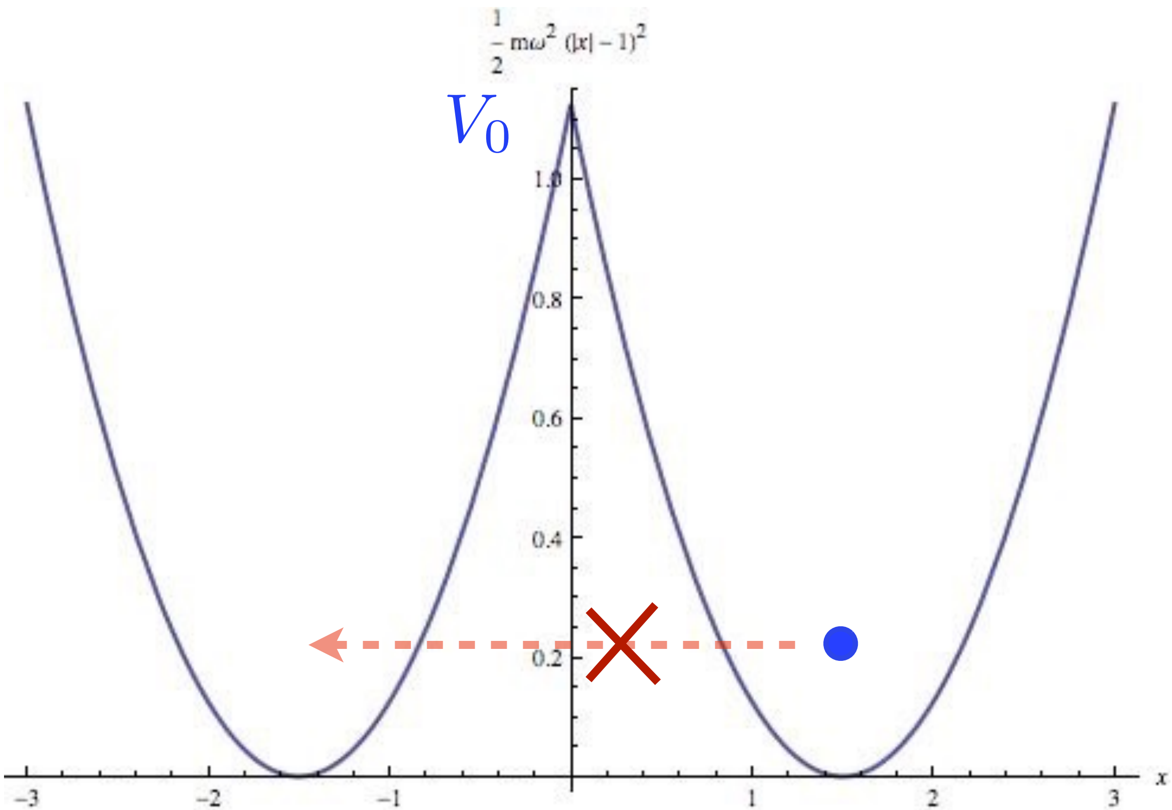

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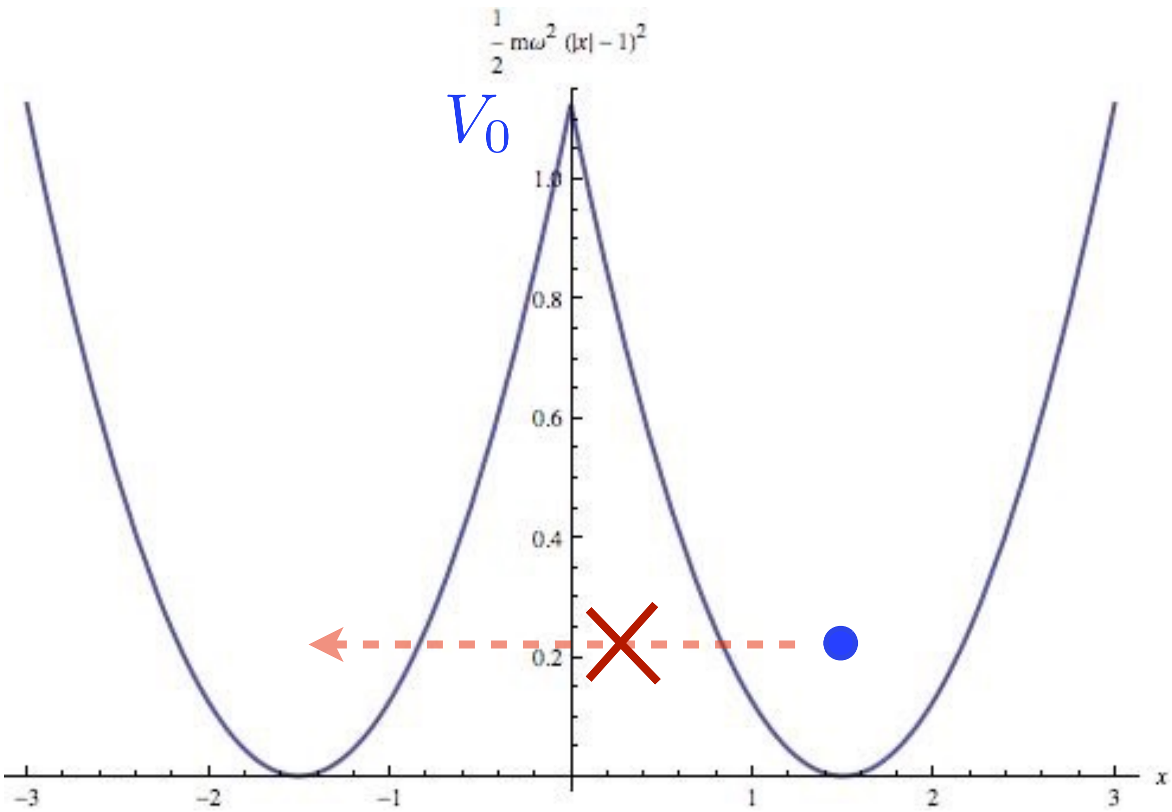

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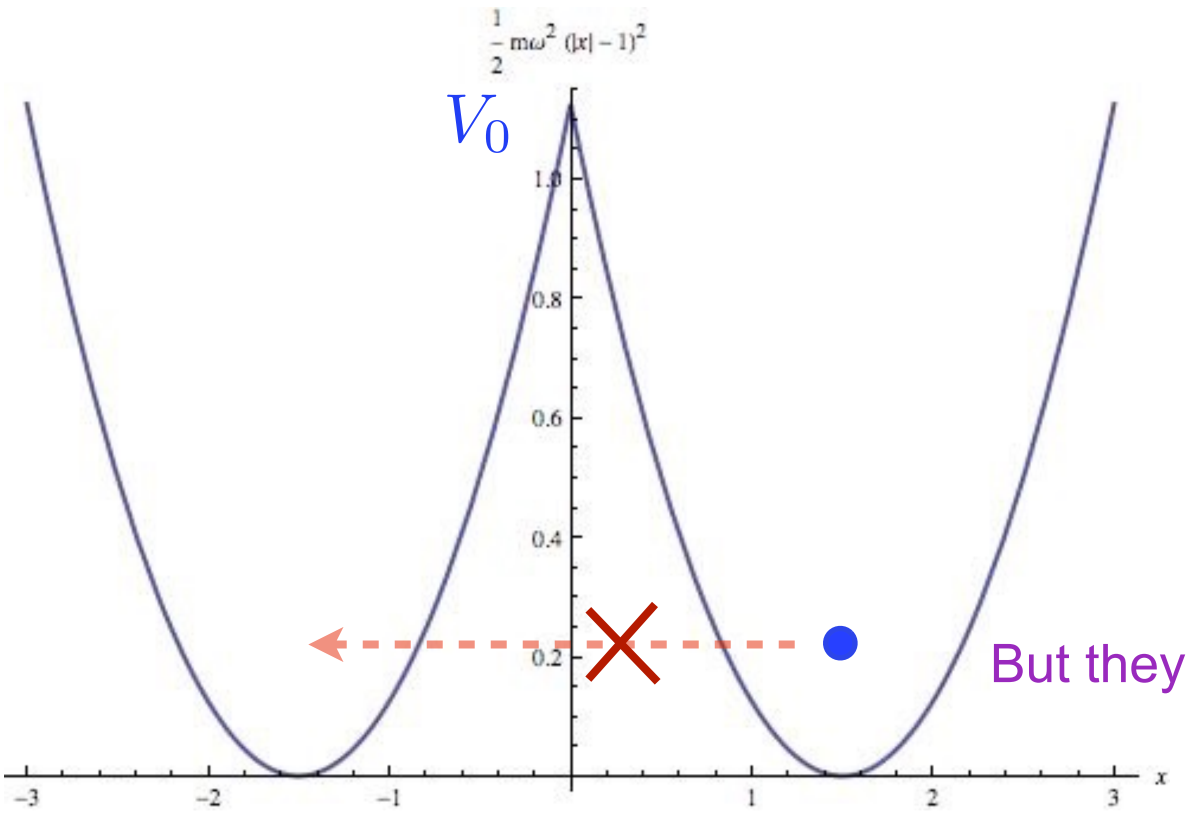
$|R\rangle, |L\rangle$

are also energy eigenstates!

Spontaneous breakdown of chiral symmetry

Quantum Example of spontaneous breakdown



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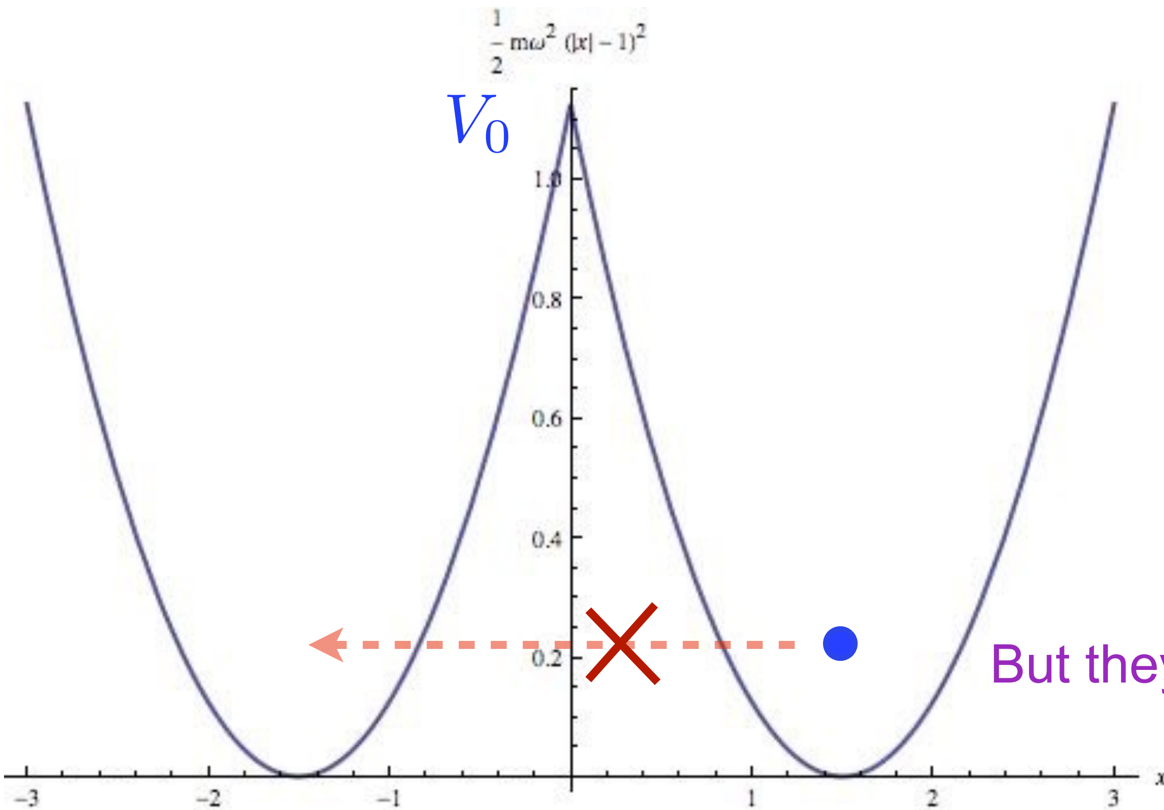
But they are not parity eigenstates!!!

Spontaneous breakdown of chiral symmetry

Quantum Example of spontaneous breakdown



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$|R\rangle, |L\rangle$

are also energy eigenstates!

But they are not parity eigenstates!!!

The Hamiltonian is invariant under parity but the ground state breaks the parity!

Spontaneous breakdown of chiral symmetry

Chiral symmetry: key ingredient for hadron physics

1. Spontaneous breaking of chiral symmetry:
   - quark condensate
   - dynamical quarks with finite mass
   - "massless" Goldstone bosons (pions, kaons, eta)

2. Quark Confinement: Important for excited states.

Spontaneous breakdown of chiral symmetry

Y. Nambu

M. Kobayashi

T. Maskawa

2008, Nobel Laureates

"For the discovery of the mechanism of spontaneous broken symmetry in subatomic physics", the other half jointly to Makoto Kobayashi and Toshihide Maskawa "for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Spontaneous breakdown of chiral symmetry

Spontaneous breakdown of chiral symmetry

QCD partition function

$$\mathcal{Z}_{\rm QCD} = \int DA_\mu D\psi D\psi^\dagger \exp\left[\sum_{f=1}^{N_f}\int d^4x\psi_f^\dagger(i\not{D}+im_f)\psi_f - \frac{1}{4g^2}\int d^4xG^2\right]$$

$$= \int DA_\mu \exp\left[-\frac{1}{4g^2}\int d^4xG^2\right]\mathrm{Det}(i\not{D}+im_f)$$

QCD Lagrangian is invariant under chiral symmetry but its vacuum is **infinitely** degenerate and is **not invariant** under that symmetry.

Spontaneous breakdown of chiral symmetry

Banks-Casher theorem $\longrightarrow$ Zero-mode spectrum $\nu(0)$

$$\mathrm{Det}(i\nabla\!\!\!/ + im) = \prod_n (\lambda_n + im) = \sqrt{\prod (\lambda_n^2 + m^2)} \qquad i D\!\!\!/ \Phi_n = \lambda_n \Phi_n$$

$$= \exp\left[\frac{1}{2}\sum_n \ln(\lambda_n^2 + m^2)\right] = \exp\left[\frac{1}{2}\int_{-\infty}^{\infty} d\lambda \nu(\lambda) \ln(\lambda_n^2 + m^2)\right]$$

$\nu(\lambda) := \sum_n \delta(\lambda - \lambda_n)$: Spectral density of the Dirac operator

Quark condensate, an order parameter
for spontaneous breakdown of chiral symmetry

$$\langle\bar{\psi}\psi\rangle = \frac{1}{V}\frac{\partial}{\partial m}\left[\frac{1}{2}\int d\lambda \nu(\bar{\lambda}) \ln(\lambda^2 + m^2)\right]_{m\to 0}$$

$$= -\frac{1}{V}\int_{-\infty}^{\infty} d\lambda \nu(\bar{\lambda}) \frac{m}{\lambda^2 + m^2}\bigg|_{m\to 0}$$

Spontaneous breakdown of chiral symmetry

Banks-Casher relation

$$\frac{m}{\lambda^2 + m^2} \to \mathrm{sign}(m)\pi\delta(\lambda)$$

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= -\frac{1}{V}\int_{-\infty}^{\infty} d\lambda\bar{\nu}(\lambda)\frac{m}{\lambda^2 + m^2}\Big|_{m\to 0} \\ &= -\frac{1}{V}\mathrm{sign}\pi\bar{\nu}(0)\end{aligned}$$

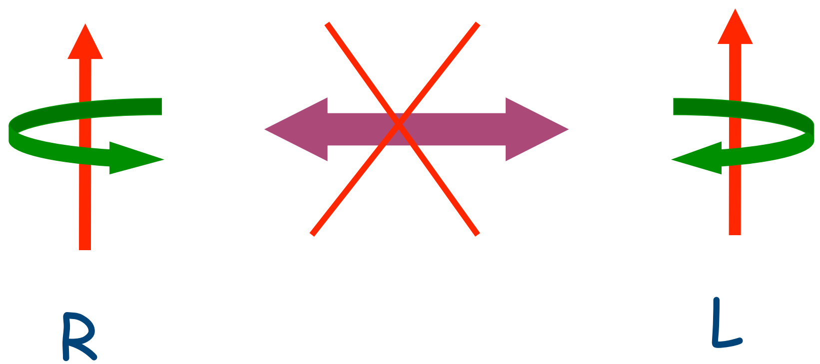
Quark condensate is proportional to the spectral density of the Dirac operator with zero eigenvalues(zero modes).

$\langle\bar{\psi}\psi\rangle \neq 0,$ Broken phase or Nambu-Goldstone phase,

$\langle\bar{\psi}\psi\rangle = 0,$ Unbroken phase or Weyl phase

Spontaneous breakdown of chiral symmetry

In the case of massless quarks



$$q_L := \frac{1}{2}(1-\gamma_5)\,q, \quad q_R := \frac{1}{2}(1+\gamma_5)\,q$$

Spontaneous breakdown of chiral symmetry

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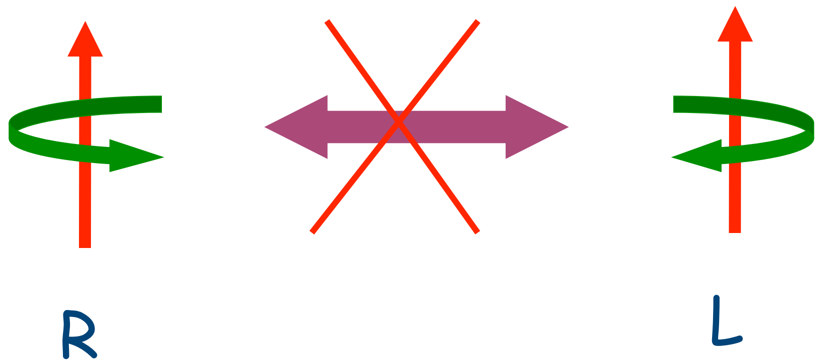
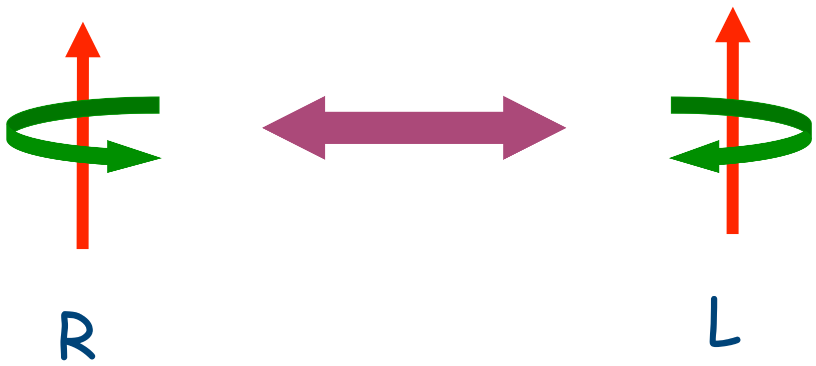


$$q_L := \frac{1}{2}(1-\gamma_5)\, q, \quad q_R := \frac{1}{2}(1+\gamma_5)\, q$$

→ QCD Lagrangian is invariant under chiral symmetry.

Spontaneous breakdown of chiral symmetry

Light quarks with small masses $m_q \neq 0$



$$\bar{q}_L q_R + \bar{q}_R q_L \,:\, (3, 3^*) + (3^*, 3)$$

Effective Partition function

QCD partition function

$$\mathcal{Z}_{\rm QCD} = \int DA_\mu D\psi D\psi^\dagger \exp\left[\sum_{f=1}^{N_f}\int d^4x\psi_f^\dagger(i\not{D}+im_f)\psi_f - \frac{1}{4g^2}\int d^4x G^2\right]$$

$$= \int DA_\mu \exp\left[-\frac{1}{4g^2}\int d^4x G^2\right] {\rm Det}(i\not{D}+im_f)$$

Integrating over gluons means averaging the partition function over (anti-)instantons

$$\Rightarrow \quad \mathcal{Z}_{\rm eff} = \overline{{\rm Det}(i\not{D}+im_f)}$$

Effective Partition function

QCD partition function

$$\mathcal{Z}_{\rm QCD} = \int DA_\mu D\psi D\psi^\dagger \exp\left[\sum_{f=1}^{N_f}\int d^4x \psi_f^\dagger (i\not{D} + i m_f)\psi_f - \frac{1}{4g^2}\int d^4x G^2\right]$$

$$= \int DA_\mu \exp\left[-\frac{1}{4g^2}\int d^4x G^2\right] \mathrm{Det}(i\not{D} + i m_f)$$

Integrating over gluons means averaging the partition function over (anti-)instantons

$$\Rightarrow \quad \mathcal{Z}_{\rm eff} = \overline{\mathrm{Det}(i\not{D} + i m_f)}$$

Zero-mode solution

Zero-mode equation

$$i\not{D}\Phi_n = \lambda_n \Phi_n$$

⇒ Zero modes $\lambda_0 = 0,\ \Phi_0$

Fourier transform of the zero mode will bring about the momentum dependent quark mass.

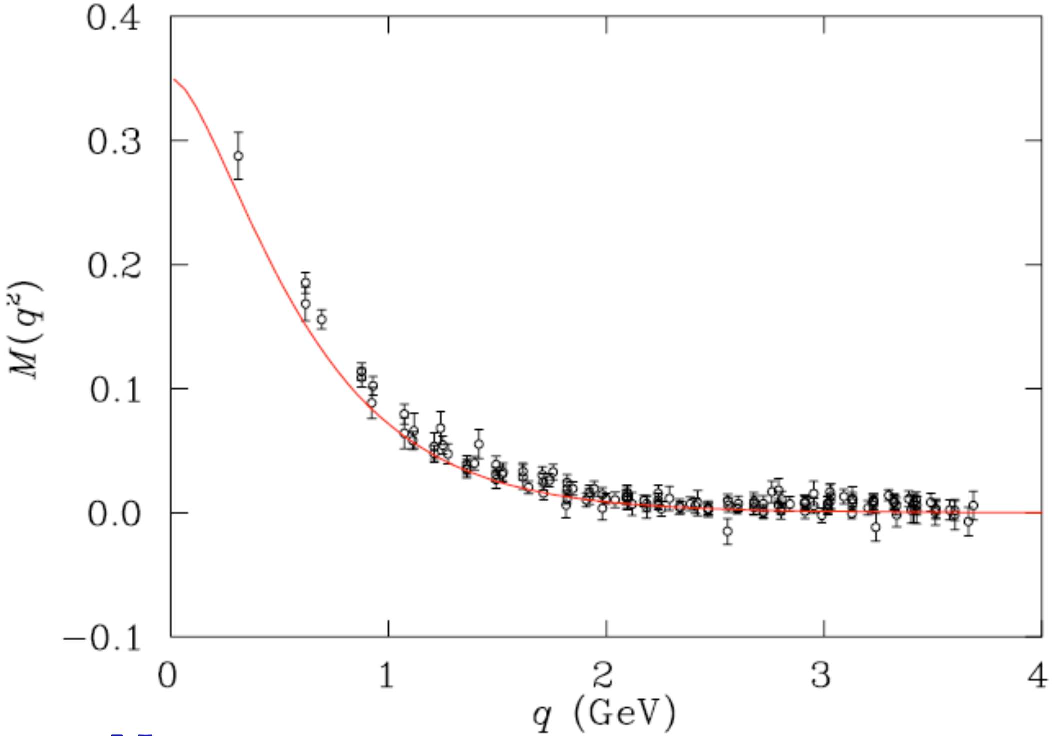
Momentum-dependent quark mass *M(k)*

$$F(k\rho) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right]\bigg|_{t=k\rho/2}$$

Spontaneous breakdown of chiral symmetry

Consequences Quark condensate: $\langle \bar{q} q \rangle \approx -(250\,\mathrm{MeV})^3$

Dynamic quark mass: $M(q^2)$

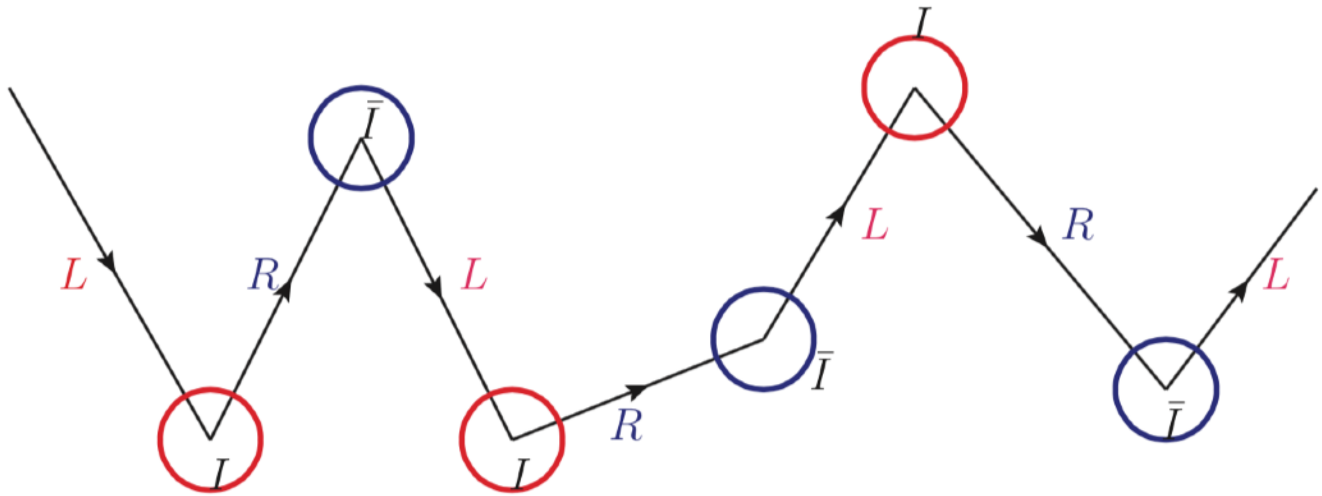


$$\frac{N}{V} \approx 1\,\mathrm{fm}^{-1}$$

$$\rho \approx 0.3\,\mathrm{fm}$$

$\Rightarrow$ $M(0) = 345\,\mathrm{MeV}$

Spontaneous breakdown of chiral symmetry



Helicity of a light quark is flipped by hoping from instants to anti-instantons and vice versa. By doing that, the quark acquires the dynamical quark mass *M(p)*.

$$S(p) = \frac{i}{\not{p} + iM(p^2)}$$

Nonzero quark condensate: $-i\langle\psi^\dagger\psi\rangle = 4N_c \int \frac{d^4p}{(2\pi)^4}\frac{M(p)}{p^2+M^2(p)} = -(253\,\mathrm{MeV})^3$

Eff. Chiral Action from the instanton vacuum

Effective QCD action from the instanton vacuum

$$\mathcal{Z} = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_{f=1}^{N_f} \bar{\psi}_f i\partial\!\!\!/ \psi^f\right) \left(\frac{Y_{N_f}^+}{VM_1^{N_f}}\right)^{N_+} \left(\frac{Y_{N_f}^-}{VM_1^{N_f}}\right)^{N_-}$$

$$Y_{N_f}^+ = \int d\rho\, d(\rho) \int dU \prod_{f=1}^{N_f} \left\{ \int \frac{d^4k_f}{(2\pi)^4} \left[2\pi\rho F(k_f\rho)\right] \int \frac{d^4 l_f}{(2\pi)^4} \left[2\pi\rho F(l_f\rho)\right] \right.$$

$$\left. \cdot (2\pi)^4 \delta(k_1 + \ldots + k_{N_f} - l_1 - \ldots - l_{N_f}) \cdot U^{\alpha_f}_{i'_f} U^{\dagger j'_f}_{\beta_f} \epsilon^{i_f i'_f} \epsilon_{j_f j'_f} \left[i\psi^\dagger_{Lf\alpha_f i_f}(k_f) \psi_L^{f\beta_f j_f}(l_f) \right] \right\}.$$

$d(\rho)$: instanton distribution, U: Color orientation

After integrating over zero modes and bosonizing, we get the effective chiral action:

$$S_{\rm eff} = -N_c \mathrm{Tr} \log \left[i\partial\!\!\!/ + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

Electromagnetic form factors of the Nucleon

EM Form factors of the nucleon

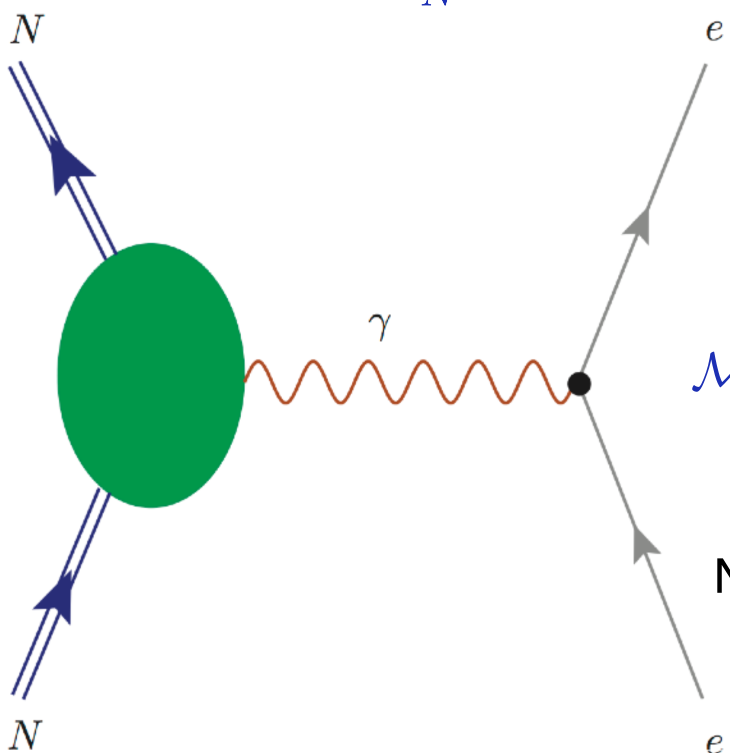
ep scattering (Rosenbluth formula)

$$\frac{d\sigma_{ep}}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{1}{(1+\tau)}\left(G_E^2 + \tau G_M^2\right) + 2\tan^2\frac{\theta}{2}\tau G_M^2\right]$$

$$\tau = \frac{Q^2}{4M_N^2}$$

Magnetic Sachs form factor

Electric Sachs form factor



$$\mathcal{M}_{ep} = e^2 \bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda)\frac{1}{q^2}\langle p', s'|J_\mu(0)|p, s\rangle$$

Nucleon Matrix element of the EM current

EM Form factors of the nucleon

Translational Invariance

$$\langle N(p')|J_\mu(x)|N(p)\rangle = e^{ix\cdot(p'-p)}\langle N(p')|J_\mu(0)|N(p)\rangle = \bar{u}(\mathbf{p}',s')\Gamma_\mu(p',p)u(\mathbf{p},s),$$

Ward Identity (Gauge invariance)

$$q^\mu \bar{u}(\mathbf{p}',s')\Gamma_\mu(p',p)u(\mathbf{p},s) = 0$$

Decomposition of the matrix elements

$$\bar{u}(\mathbf{p}',s')\Gamma^\mu(p',p)u(\mathbf{p},s) = \bar{u}(\mathbf{p}',s')\left[A^\mu + B^{\mu\nu}\gamma_\nu + C^{\mu\nu\rho}\sigma_{\nu\rho} + D^{\mu\nu}\gamma_\nu\gamma_5 + E^\mu\gamma_5\right]u(\mathbf{p},s)$$

Guideline: Gauge invariance, Lorentz invariance and parity invariance

Which term will survive?

EM Form factors of the nucleon

$$\bar{u}(\mathbf{p}', s')\Gamma^{\mu}(p', p)u(\mathbf{p}, s) = \bar{u}(\mathbf{p}', s')\left[A^{\mu} + B^{\mu\nu}\gamma_{\nu} + C^{\mu\nu\rho}\sigma_{\nu\rho} + D^{\mu\nu}\gamma_{\nu}\gamma_5 + E^{\mu}\gamma_5\right]u(\mathbf{p}, s)$$

$A,\ B,\ \cdots,\ E$ depend on $p,\ p'$

$$A^{\mu} = a_1 p^{\mu} + a_2 p'^{\mu}$$

a_1 and a_2 depend only on $p \cdot p'$, because $p^2 = p'^2 = M_N^2$.

⇒ $a_1(q^2),\ a_2(q^2)$ $\quad (q^2 = (p - p')^2 = 2M_N^2 - 2p \cdot p')$

$$B^{\mu\nu} = b_1 p^{\mu} p^{\nu} + b_2 p^{\mu} p'^{\nu} + b_3 p'^{\mu} p^{\nu} + b_4 p'^{\mu} p'^{\nu} + b_5 g^{\mu\nu} \qquad b_i := b_i(q^2)$$

$$C^{\mu\nu\rho} = c_1 p^{\mu}(p^{\nu}p'^{\rho} - p^{\rho}p'^{\nu}) + c_2 p'^{\mu}(p^{\nu}p'^{\rho} - p^{\rho}p'^{\nu}) + c_3(g^{\mu\nu}p^{\rho} - g^{\mu\rho}p^{\nu}) + c_4(g^{\mu\nu}p'^{\rho} - g^{\mu\rho}p'^{\nu})$$

$\sigma_{\nu\rho} = -\sigma_{\rho\nu}$ ⇒ $C^{\mu\nu\rho} = -C^{\mu\rho\nu}$

EM Form factors of the nucleon

$$\bar{u}(\mathbf{p}', s')\Gamma^\mu(p', p)u(\mathbf{p}, s) = \bar{u}(\mathbf{p}', s')\left[A^\mu + B^{\mu\nu}\gamma_\nu + C^{\mu\nu\rho}\sigma_{\nu\rho} + D^{\mu\nu}\gamma_\nu\gamma_5 + E^\mu\gamma_5\right]u(\mathbf{p}, s)$$

$$D^{\mu\nu} = d\epsilon^{\mu\nu\rho\sigma}p_\rho p'_\sigma$$

$$E^\mu = 0$$

There is no way to express the pseudo-vector in terms of two vectors

How to determine $a_i,\ b_i,\ c_i,\ d$

EM Form factors of the nucleon

Using the Dirac equations

$$
\begin{aligned}
(i\overleftarrow{\partial\!\!\!/} - M)u(\boldsymbol{p}, s) &= 0, \\
\bar{u}(\boldsymbol{p}, s)(i\overleftarrow{\partial\!\!\!/} - M) &= 0,
\end{aligned}
$$

we can show that a_1, a_2, b_1 and b_2 can be related:

$$(b_1 p^\mu p^\mu + b_2 p^\mu p'^\nu)\bar{u}(\boldsymbol{p}', s')\gamma_\nu u(\boldsymbol{p}, s) = (b_1 + b_2) M p_\mu \bar{u}(\boldsymbol{p}', s') u(\boldsymbol{p}, s).$$

Thus,

$$a_1 = (b_1 + b_2)M.$$

Similarly,

$$(b_3 p'^\mu p^\nu + b_4 p'^\mu p'^\nu)\bar{u}(\boldsymbol{p}', s')\gamma_\nu u(\boldsymbol{p}, s) = (b_3 + b_4) M p'^\mu \bar{u}(\boldsymbol{p}', s') u(\boldsymbol{p}, s),$$

from which we get

$$a_2 = (b_3 + b_4)M.$$

EM Form factors of the nucleon

We can reduce the number of functions further by doing the similar procedure:

$$\begin{aligned}(p^\nu p'^\rho - p^\rho p'^\nu)\bar{u}(\boldsymbol{p}', s')\sigma_{\nu\rho}u(\boldsymbol{p}, s) &= i\bar{u}(\boldsymbol{p}', s')(\not{p}\not{p}' - \not{p}'\not{p})u(\boldsymbol{p}, s) \\ &= \frac{i}{2}\bar{u}(\boldsymbol{p}', s')(\not{p}\not{p}' - \not{p}'\not{p} - \not{p}'\not{p} + \not{p}\not{p}')u(\boldsymbol{p}, s) \\ &= 4i(p\cdot p' - M^2)\bar{u}(\boldsymbol{p}', s')u(\boldsymbol{p}, s), \\ (g^{\mu\nu}p^\rho - g^{\mu\rho}p^\nu)\bar{u}(\boldsymbol{p}', s')\sigma_{\nu\rho}u(\boldsymbol{p}, s) &= i\bar{u}(\boldsymbol{p}', s')(\gamma^\mu\not{p} - \not{p}\gamma^\mu)u(\boldsymbol{p}, s) \\ &= 2i(M\gamma^\mu - p^\mu)\bar{u}(\boldsymbol{p}', s')u(\boldsymbol{p}, s).\end{aligned}$$

Thus, $C^{\mu\nu\rho}$ can be related to A^μ.

Using the relation

$$i\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_5 = i\epsilon^{\rho\sigma\mu\nu}\gamma_\nu\gamma_5 = g^{\rho\sigma}\gamma^\mu + g^{\sigma\mu}\gamma^\rho - g^{\mu\rho}\gamma^\sigma - \gamma^\rho\gamma^\sigma\gamma^\mu,$$

we can show that the term with $D^{\mu\nu}$ can be written as

$$\begin{aligned}\bar{u}(\boldsymbol{p}', s')\epsilon^{\rho\sigma\mu\nu}\gamma_\nu\gamma_5 u(\boldsymbol{p}, s)p_\rho p'_\sigma &= \bar{u}(\boldsymbol{p}', s')(\gamma^\mu p\cdot p' - g^{\sigma\mu}\gamma^\rho + g^{\mu\rho}\gamma^\sigma + \gamma^\rho\gamma^\sigma\gamma^\mu)u(\boldsymbol{p}, s)p_\rho p'_\sigma \\ &= \bar{u}(\boldsymbol{p}', s')(\gamma^\mu p\cdot p' + \not{p}p^{\mu'} - p^\mu\not{p}' - \not{p}\not{p}'\gamma^\mu)u(\boldsymbol{p}, s) \\ &= (-p\cdot p' - M^2)\bar{u}(\boldsymbol{p}', s')\gamma^\mu u(\boldsymbol{p}, s) + MP^\mu\bar{u}(\boldsymbol{p}', s')u(\boldsymbol{p}, s).\end{aligned}$$

EM Form factors of the nucleon

$$\bar{u}(\mathbf{p}', s')\Gamma^\mu u(\mathbf{p}, s) = \bar{u}(\mathbf{p}', s')\left[a\gamma^\mu + bP^\mu + cq^\mu\right]u(\mathbf{p}, s)$$

If we use the Gordan decomposition

$$\begin{aligned} i\bar{u}(\boldsymbol{p}', s')\sigma^{\mu\nu}u(\boldsymbol{p}, s)q^\nu &= -\frac{1}{2}\bar{u}(\boldsymbol{p}', s')(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)u(\boldsymbol{p}, s)q^\nu \\ &= \bar{u}(\boldsymbol{p}', s')[2M\gamma^\mu - P^\mu]u(\boldsymbol{p}, s), \end{aligned}$$

⇓

$$\langle N(p')|J_\mu(0)|N(p)\rangle = \bar{u}(\mathbf{p}', s')\left[F_1(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(q^2)\right]u(\mathbf{p}, s)$$

EM Form factors of the nucleon

$$\begin{aligned}\langle N(p')|J_\mu(0)|N(p)\rangle &= \bar{u}(\boldsymbol{p}',s')\left[F_1(q^2)\gamma_\mu + \frac{2M\gamma_\mu - P_\mu}{2M}F_2(q^2)\right]u(\boldsymbol{p},s)\\ &= \bar{u}(\boldsymbol{p}',s')\left[(F_1(q^2)+F_2(q^2))\gamma_\mu - \frac{F_2(q^2)}{2M}P_\mu\right]u(\boldsymbol{p},s)\end{aligned}$$

$$\Rightarrow \quad \langle N(p')|J_0(0)|N(p)\rangle = \bar{u}(\boldsymbol{p}',s')\left[(F_1(q^2)+F_2(q^2))\gamma_0 - \frac{F_2(q^2)}{2M}P_0\right]u(\boldsymbol{p},s)$$

In the Breit frame $\mathbf{p} = -\mathbf{p}'$

$$q^2 = -4\boldsymbol{p}^2 = -4(p_0^2 - M^2), \quad \frac{p_0^2}{M^2} = 1 - \frac{q^2}{4M^2}. \quad \bar{u}(\boldsymbol{p}',s')u(\boldsymbol{p},s) = \frac{p_0}{M}\bar{u}(\boldsymbol{p}',s')\gamma_0 u(\boldsymbol{p},s).$$

$$\begin{aligned}\langle N(p')|J_0(0)|N(p)\rangle &= \bar{u}(\boldsymbol{p}',s')\left[(F_1(q^2)+F_2(q^2))\gamma_0 - \frac{F_2(q^2)}{2M}P_0\right]u(\boldsymbol{p},s)\\ &= (F_1(q^2)+F_2(q^2))\bar{u}(\boldsymbol{p}',s')\gamma_0 u(\boldsymbol{p},s) - F_2(q^2)\frac{p_0}{M}\bar{u}(\boldsymbol{p}',s')u(\boldsymbol{p},s)\\ &= \left[(F_1(q^2)+F_2(q^2)) - F_2(q^2)\left(1-\frac{q^2}{4M^2}\right)\right]\bar{u}(\boldsymbol{p}',s')\gamma_0 u(\boldsymbol{p},s).\end{aligned}$$

EM Form factors of the nucleon

Sachs Form factors

$$G_E(q^2) = F_1 + \frac{q^2}{4M^2} F_2,$$

$$G_M(q^2) = F_1 + F_2$$

$$\begin{aligned} \langle N(p')|J_0(0)|N(p)\rangle &= G_E(q^2)\bar{u}(\boldsymbol{p}', s')\gamma_0 u(\boldsymbol{p}, s) \\ &= 2G_E E\delta_{s's}. \end{aligned}$$

$$\langle N(p')|J_i(0)|N(p)\rangle = \bar{u}(\boldsymbol{p}', s')\left[(F_1(q^2) + F_2(q^2))\gamma_i\right] u(\boldsymbol{p}, s) = G_M(q^2)\bar{u}(\boldsymbol{p}', s')\gamma_i u(\boldsymbol{p}, s)$$

Homework I: Think about it!

Pion Electromagnetic form factor

$$\langle \pi^a(p_f)|\bar{\psi}(0)\gamma^\mu\psi(0)|\pi^b(p_i)\rangle = (p_f + p_i)^\mu \delta^{ab} F_\pi(q^2)$$

Can you justify this expression?

Homework II: Think about it!

$$\langle B_8|J_\mu(0)|B_{10}\rangle = \bar{u}_{B_8}(\boldsymbol{p}', s')\Gamma_{\beta\mu}u^\beta_{B_{10}}(\boldsymbol{p}, s)$$

$$u_\mu(\boldsymbol{p}, s) = \sum_{\lambda_\alpha\lambda_\beta}\left\langle 1\lambda_\alpha\frac{1}{2}\lambda_\beta\middle|\frac{3}{2}\Lambda\right\rangle u(\boldsymbol{p}, \lambda_\beta)\epsilon_\mu(\boldsymbol{p}, \lambda_\alpha)$$

Rarita-Schwinger field

$$\Gamma_{\beta\mu} = i\sqrt{\frac{2}{3}}\left[G^*_M(q^2)\mathcal{K}^M_{\beta\mu} + G^*_E(q^2)\mathcal{K}^E_{\beta\mu} + G^*_C(q^2)\mathcal{K}^C_{\beta\mu}\right]$$

$$\begin{aligned}
\mathcal{K}^M_{\beta\mu} &= -i\frac{3(M_{10}+M_8)}{2M_8[(M_{10}+M_8)^2-q^2]}\epsilon_{\beta\mu\lambda\sigma}P^\lambda q^\sigma, \\
\mathcal{K}^E_{\beta\mu} &= -\mathcal{K}^M_{\beta\mu} - i\frac{6(M_{10}+M_8)}{M_8\Delta(q^2)}\epsilon_{\beta\sigma\lambda\rho}P^\lambda q^\rho\epsilon^\sigma_{\mu\kappa\delta}P^\kappa q^\delta\gamma^5, \\
\mathcal{K}^C_{\beta\mu} &= -i\frac{3(M_{10}+M_8)}{M_8\Delta(q^2)}q_\beta(q^2P_\mu - q\cdot Pq_\mu)\gamma^5
\end{aligned}$$

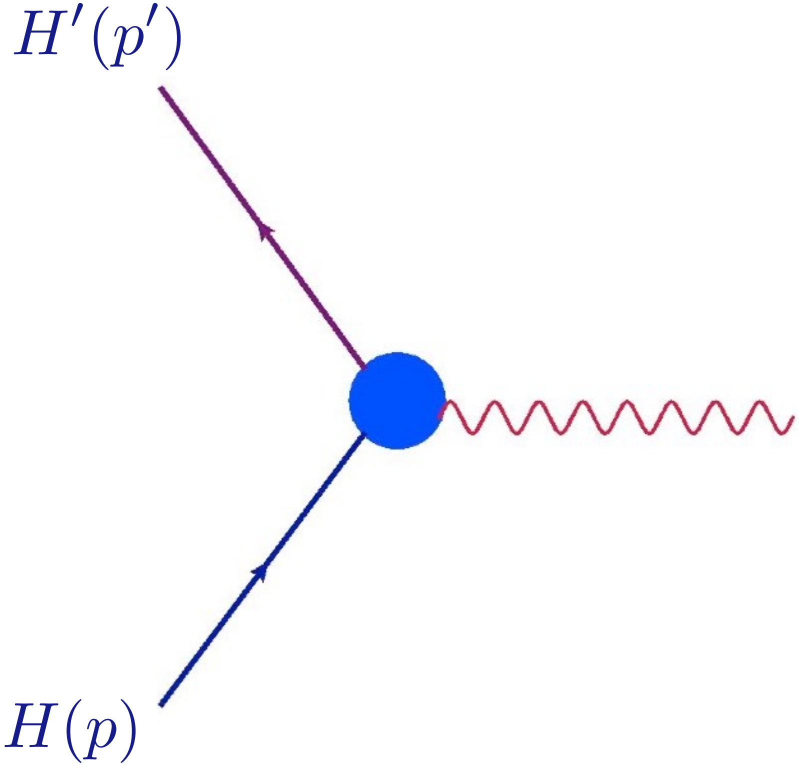
$$\Delta(q^2) = [(M_{10}+M_8)^2-q^2][(M_{10}-M_8)^2-q^2]$$

Can you justify this decomposition?

Modern Concept
of
the Form factors

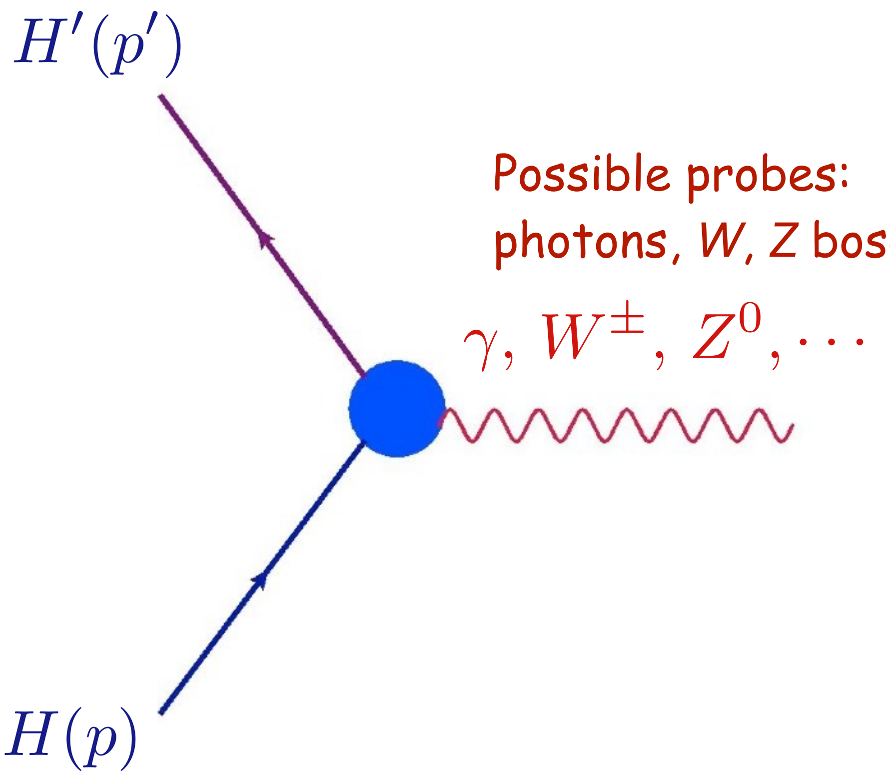
Traditional way of a hadron structure

Traditional way of studying structures of hadrons



Traditional way of a hadron structure

Traditional way of studying structures of hadrons

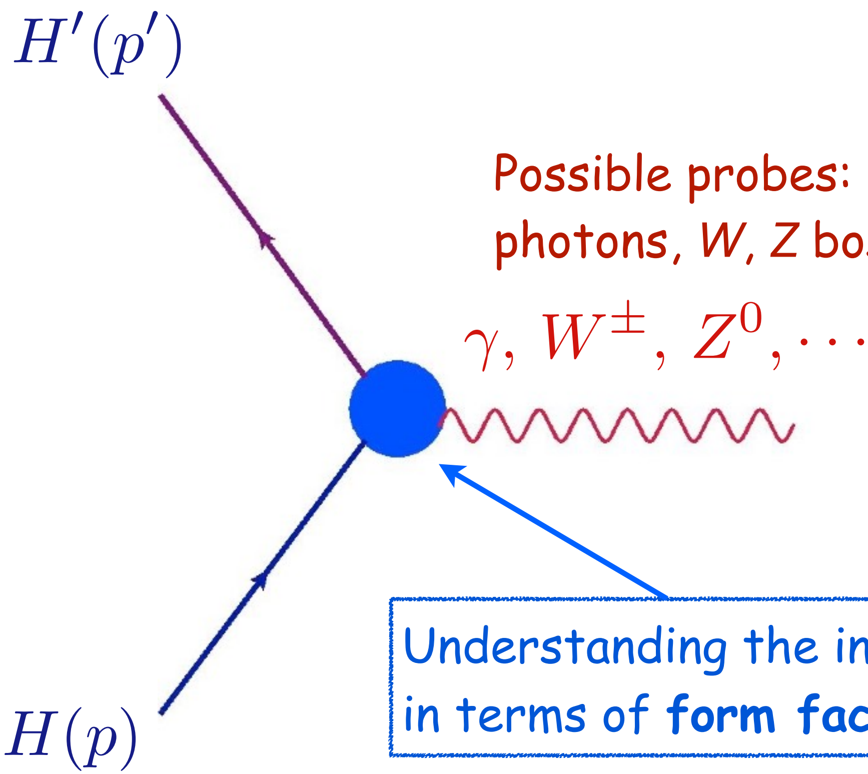


Possible probes:
photons, *W*, *Z* bosons, mesons, nucleons......

Traditional way of a hadron structure

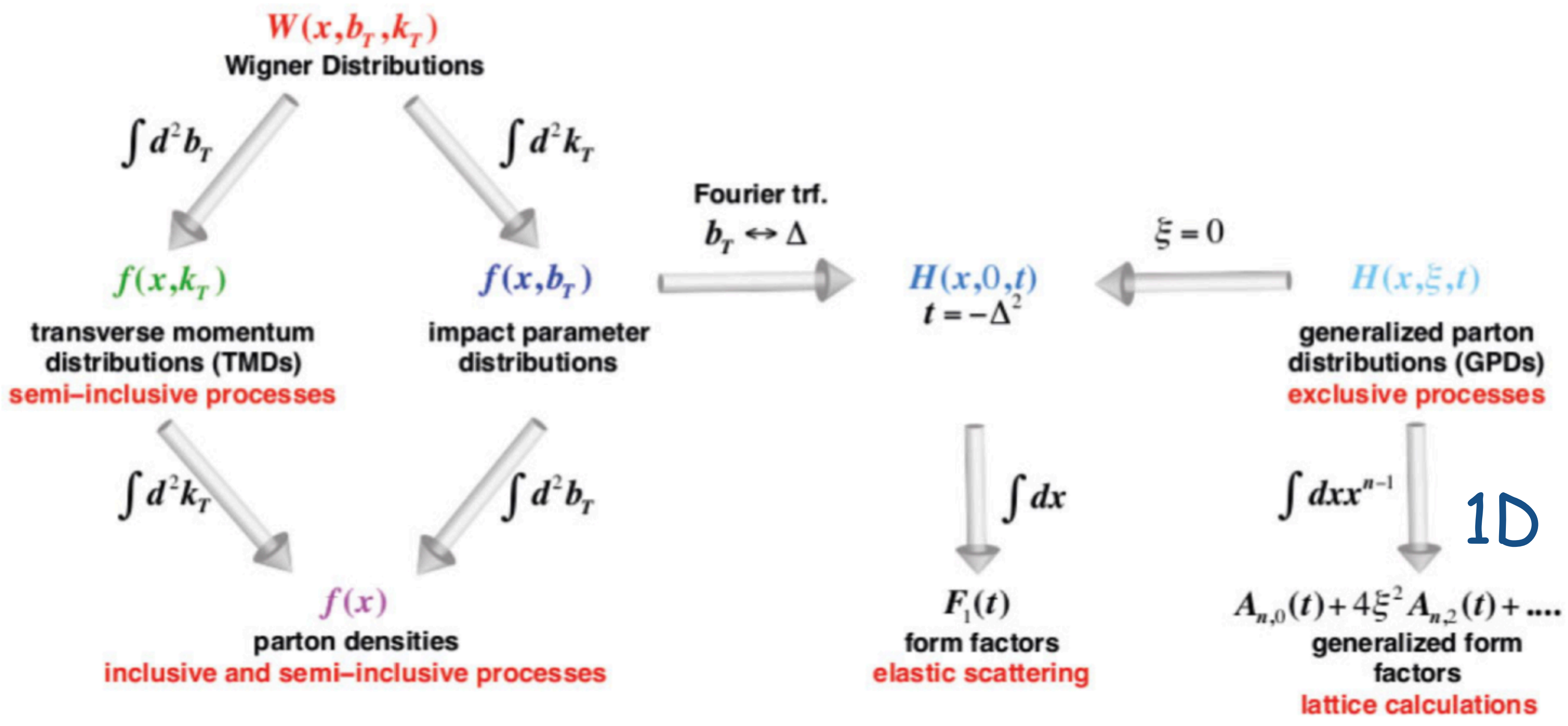
Traditional way of studying structures of hadrons

Possible probes:
photons, *W*, *Z* bosons, mesons, nucleons......

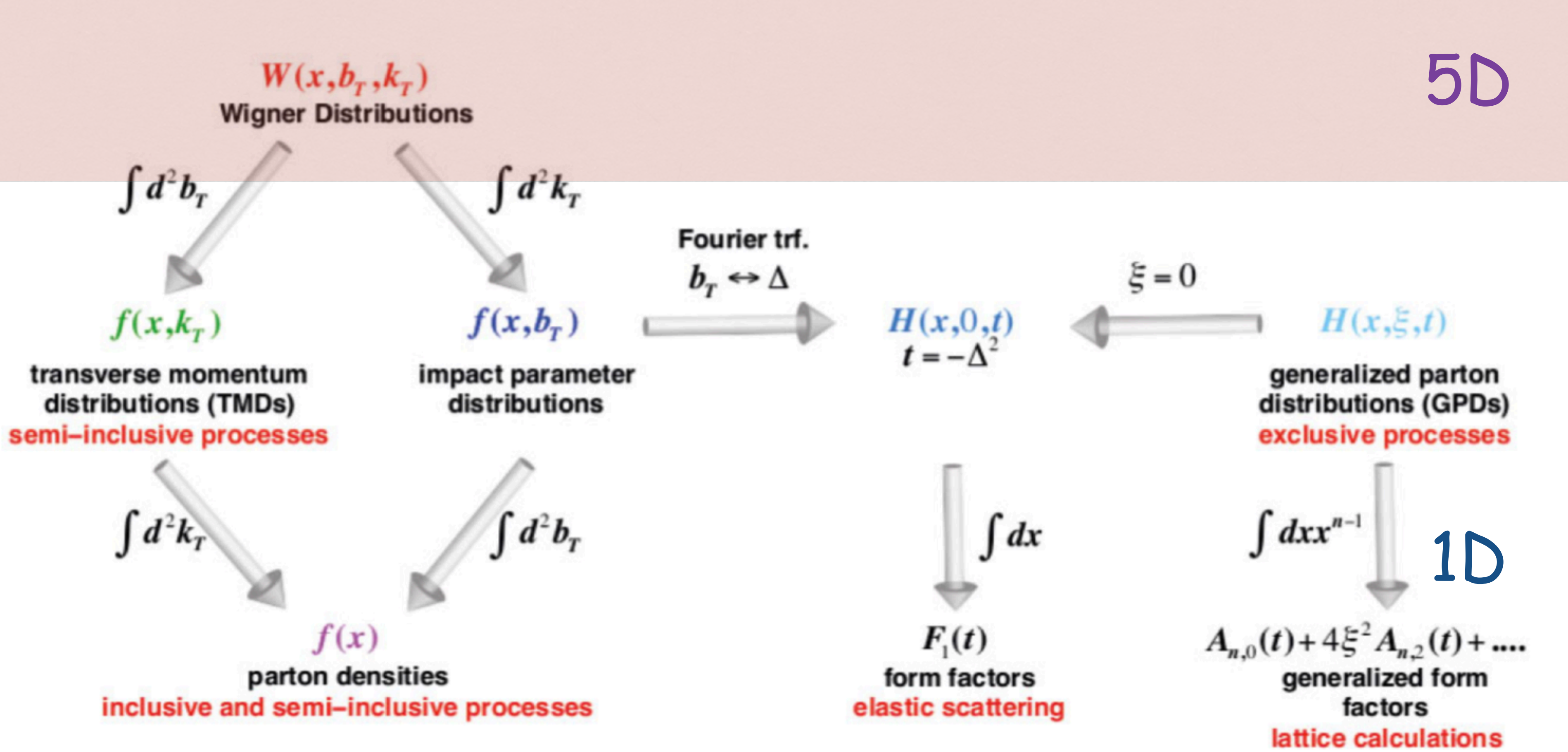


Understanding the internal structures of hadrons *H* in terms of **form factors**

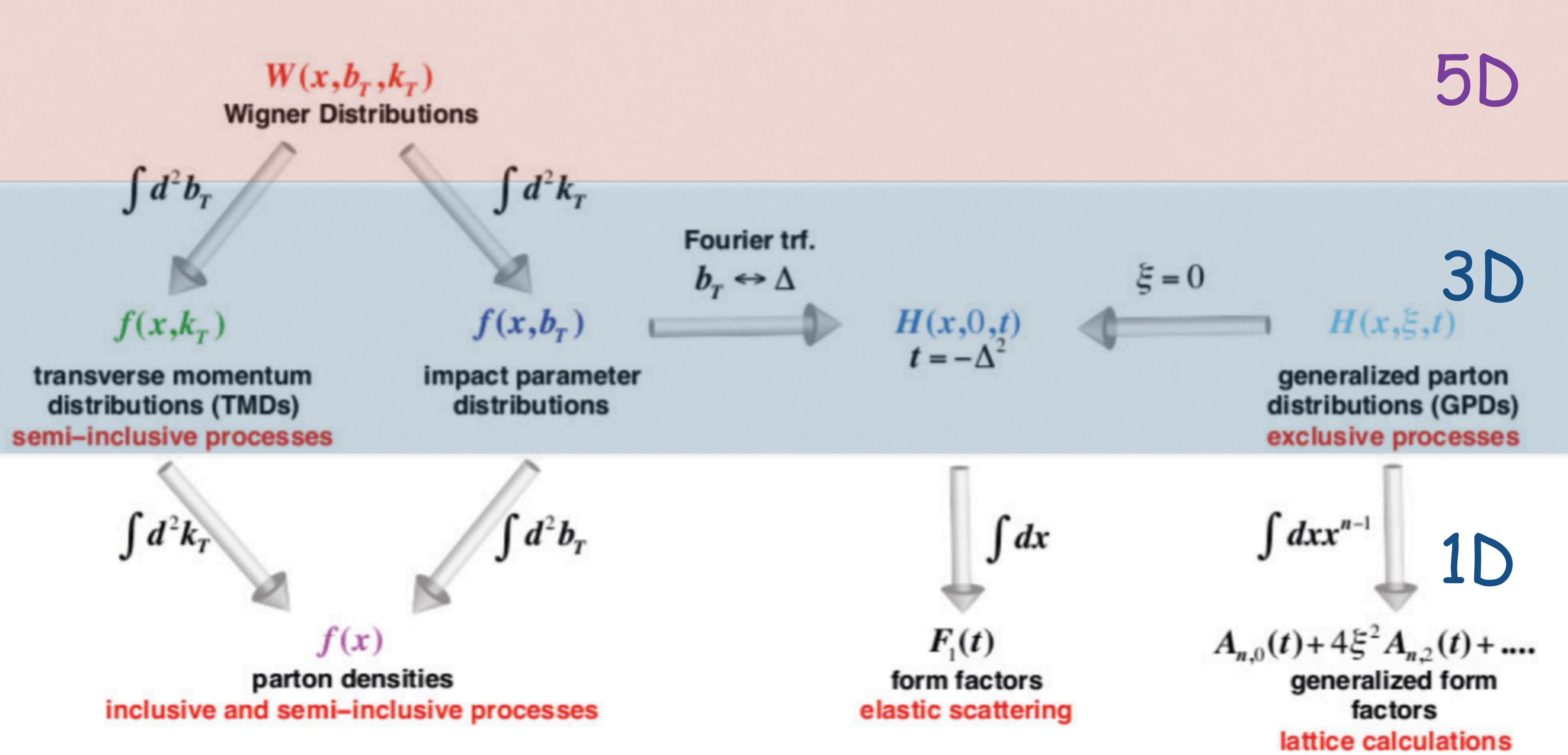
Modern understanding of a baryon structure



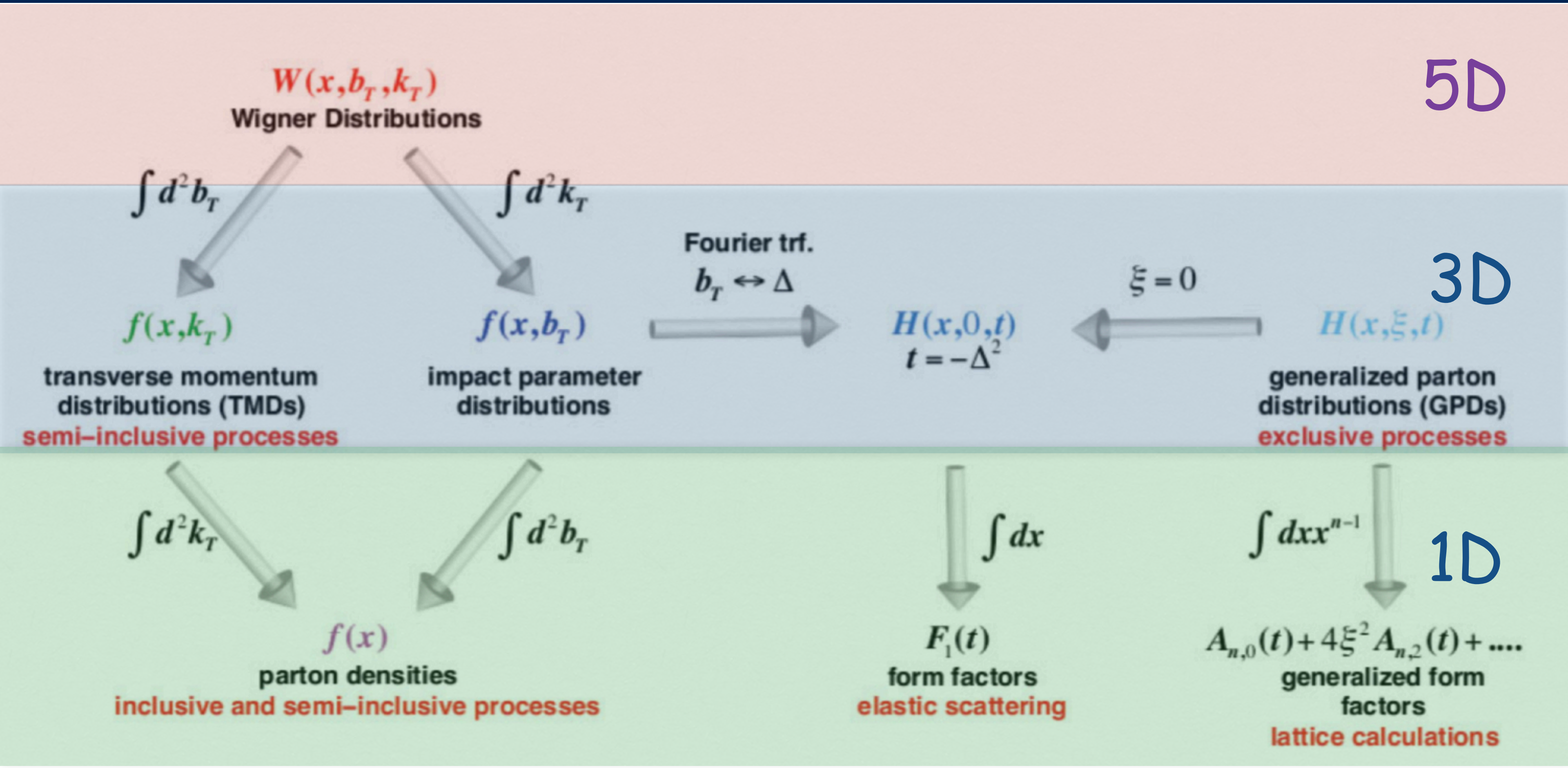
Modern understanding of a baryon structure



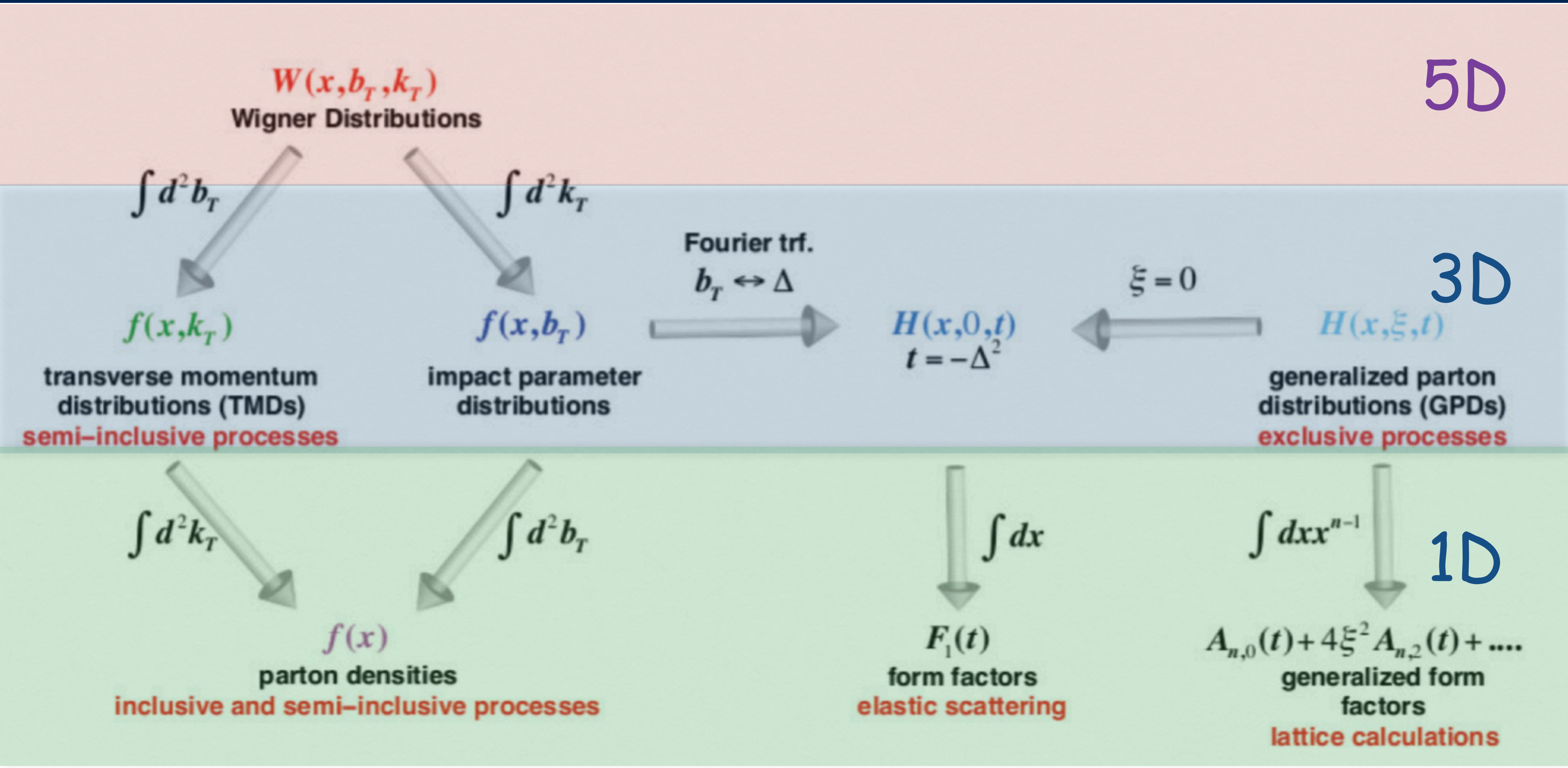
Modern understanding of a baryon structure



Modern understanding of a baryon structure



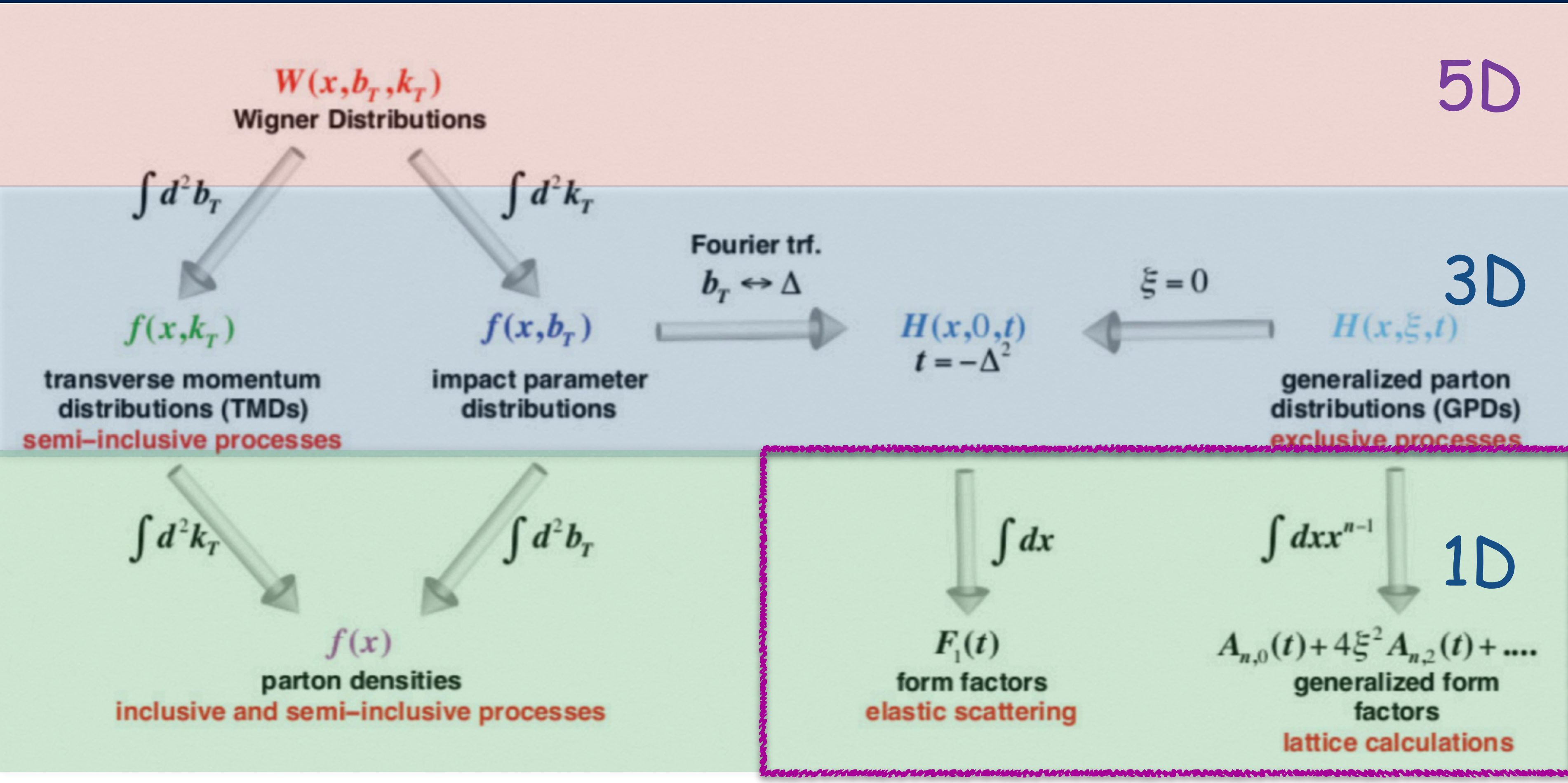
Modern understanding of a baryon structure



State of the art of the nucleon tomography

Figure taken from Eur. Phys. J. A (2016) 52: 268

Modern understanding of a baryon structure



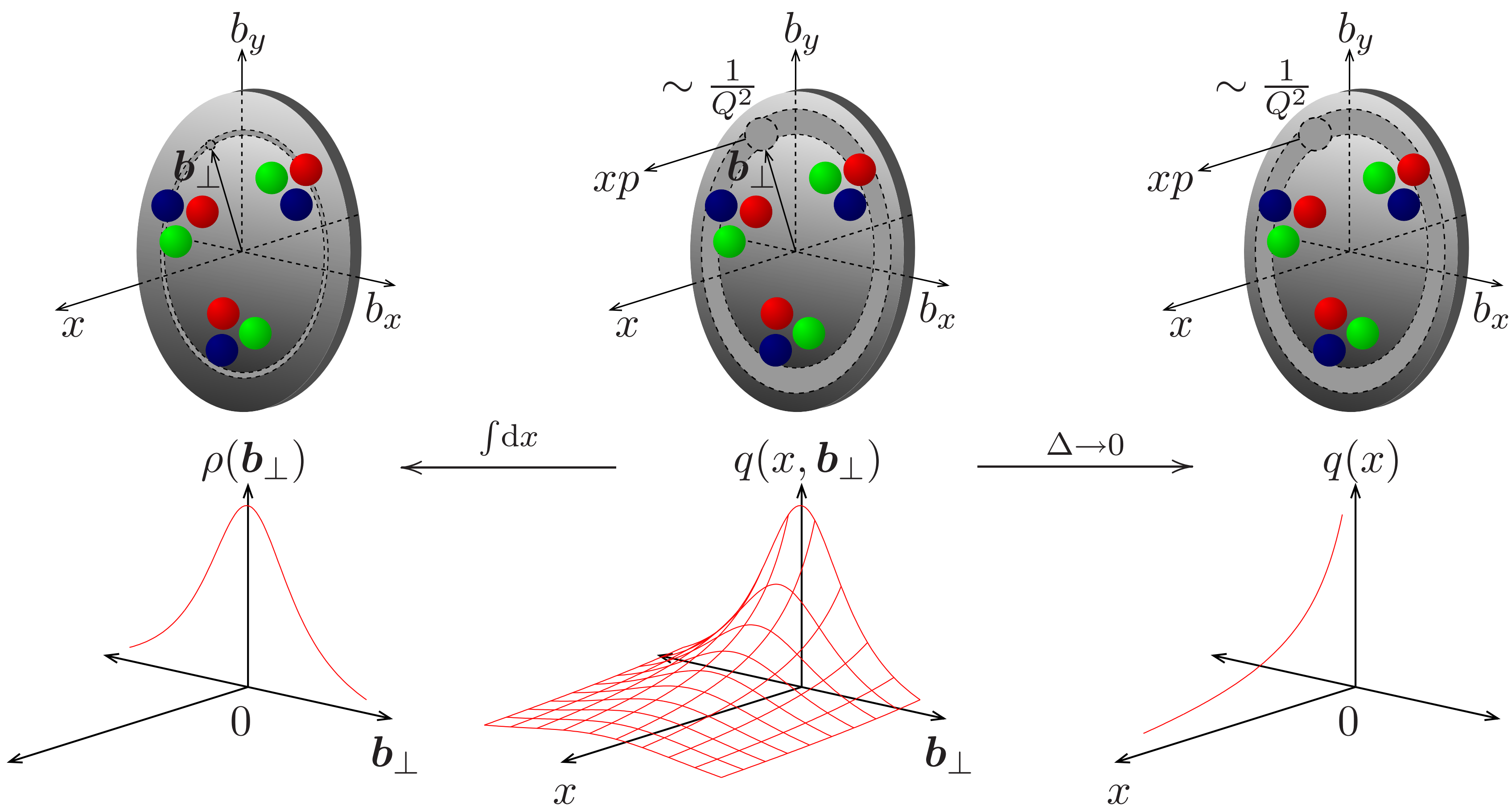
Today's topic to discuss

State of the art of the nucleon tomography

Figure taken from Eur. Phys. J. A (2016) 52: 268

Modern understanding of a baryon structure

3D Nucleon Tomography



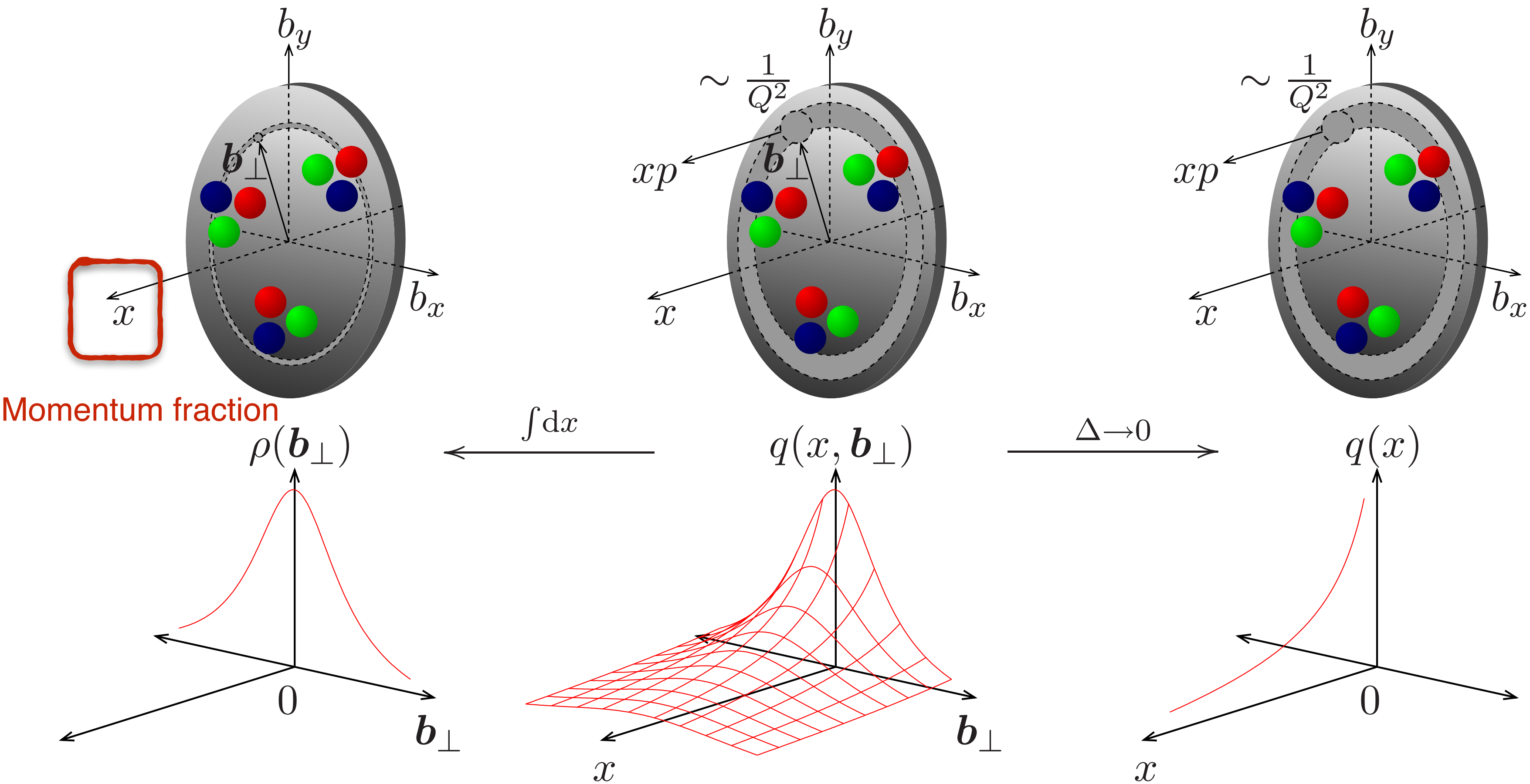
Transverse densities of Form factors

GPDs
Nucleon Tomography

Structure functions
Parton distributions

Modern understanding of a baryon structure

3D Nucleon Tomography



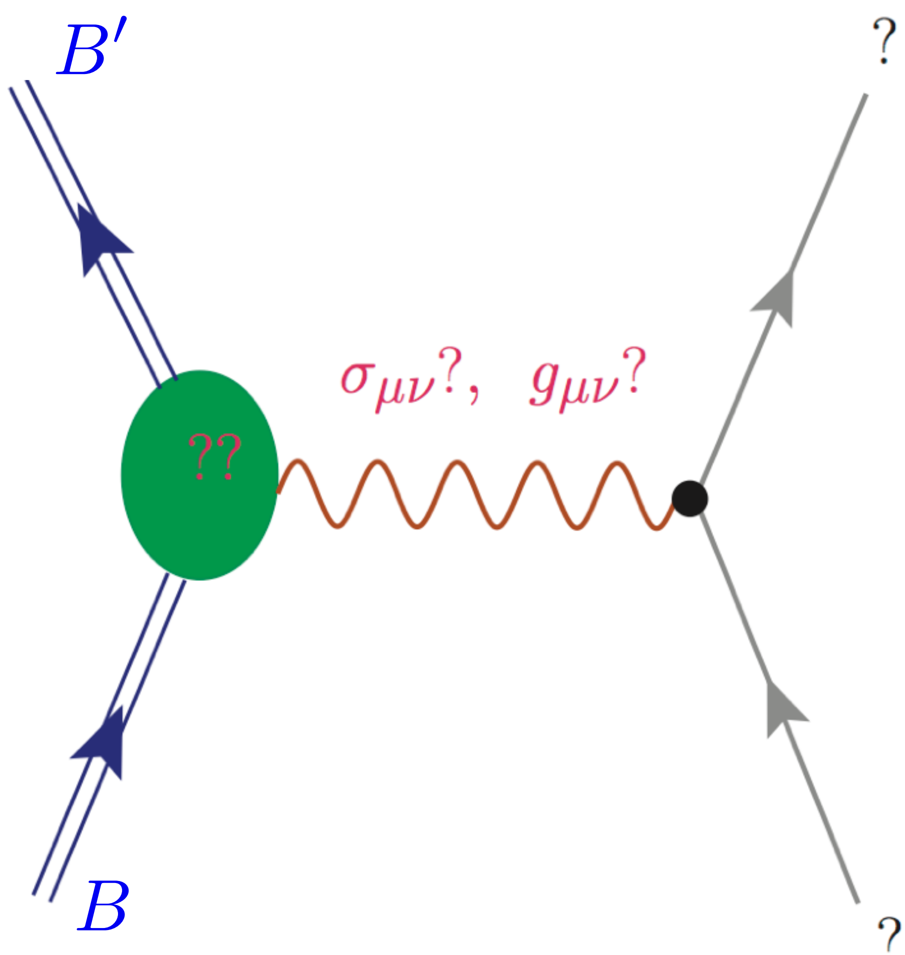
Transverse densities of Form factors

GPDs
Nucleon Tomography

Structure functions
Parton distributions

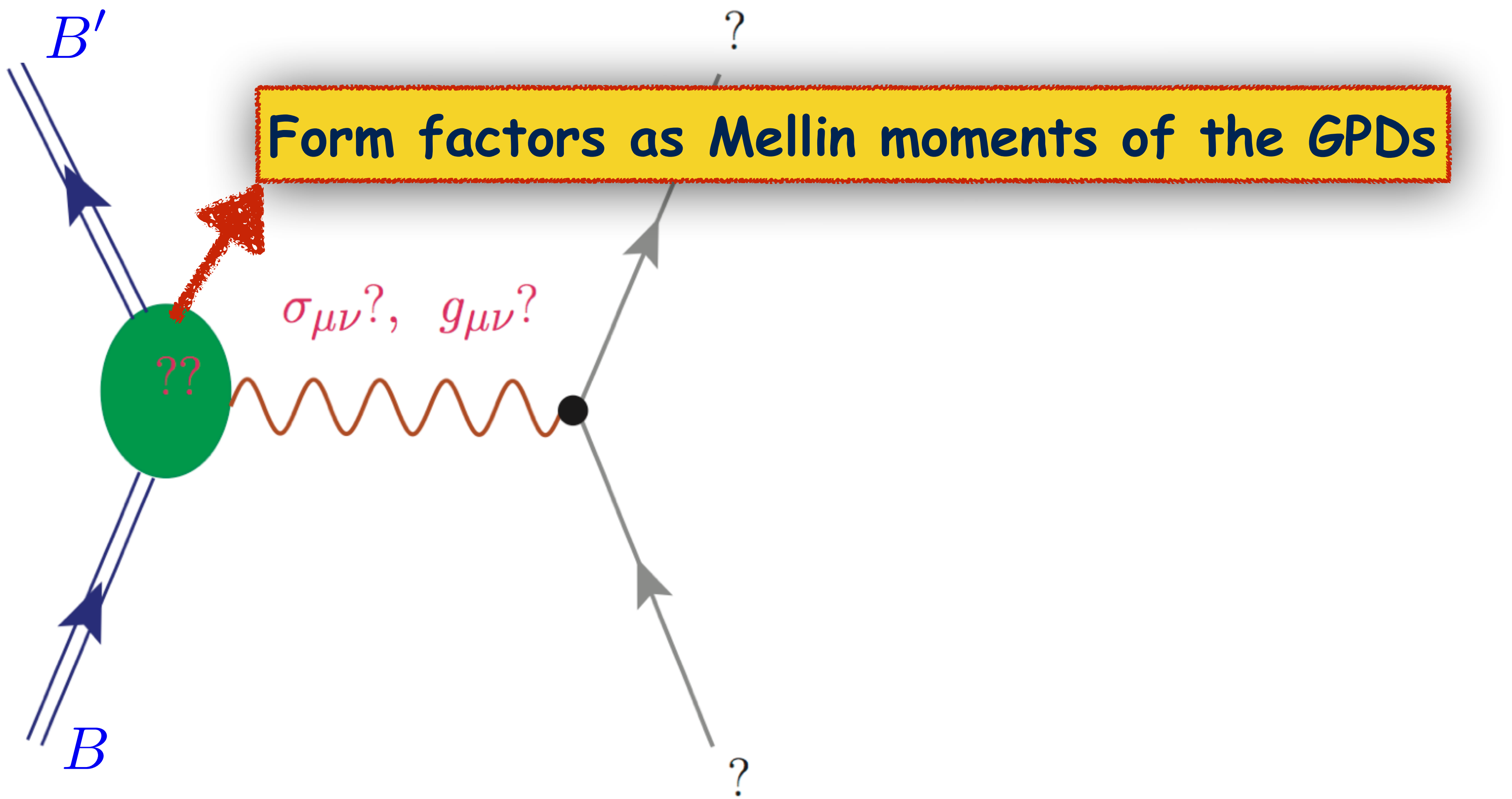
Modern understanding of a baryon structure

Probes are unknown for **Tensor form factors**
and the **Energy-Momentum Tensor form factors!**



Modern understanding of a baryon structure

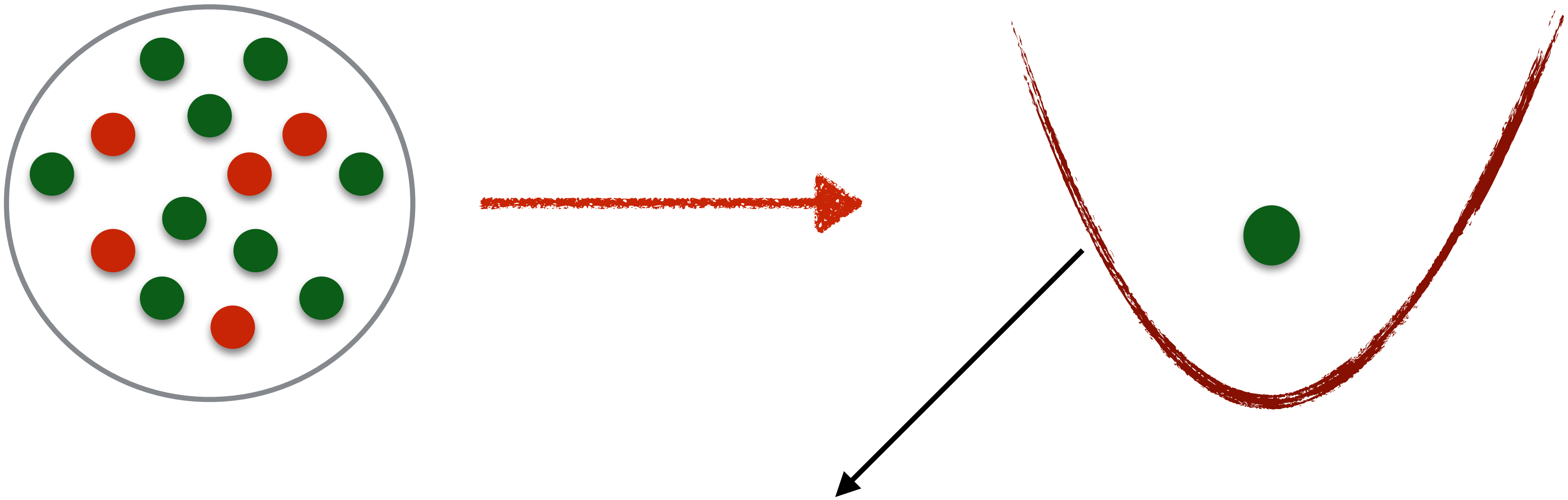
Probes are unknown for **Tensor form factors**
and the **Energy-Momentum Tensor form factors!**



Nucleon as Nc quarks bound by the pion mean fields

Mean-Field Approximation

Simple picture of a mean-field approximation



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

Mean-Field Approximation

More theoretically defined mean fields

Given action, $S[\phi]$

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

Key point: Ignore the quantum fluctuation.

→ How we can understand the structure of baryons,
based on this mean field approach,
this is the subject of the present talk.

Baryon in pion mean fields

* A baryon can be viewed as a state of Nc quarks bound by mesonic **mean fields** (E. Witten, NPB, 1979 & 1983).

Its mass is proportional to *Nc*, while its width is of order *O(1)*.

→ Mesons are weakly interacting (Quantum fluctuations are suppressed by 1/Nc: *O(1/Nc)*.

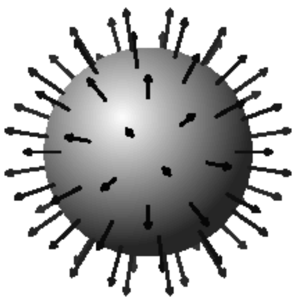
Meson mean-field approach (Chiral Quark-Soliton Model)

* Baryons as a state of Nc quarks bound by mesonic mean fields.

$$S_{\rm eff} = -N_c \mathrm{Tr}\ln\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\, 2,\, 3 \\ 0, & a = 4,\, 5,\, 6,\, 7,\, 8. \end{cases}$$


hedgehog

→ It breaks spontaneously $\mathrm{SU(3)_{flavor}} \otimes \mathrm{O(3)_{space}} \to \mathrm{SU(2)_{isospin+space}}$

Baryon in pion mean fields

✱ Merits of the Chiral Quark-Soliton Model

- It is directly related to nonperturbative QCD via the Instanton vacuum.

  → Natural scale of the model given by the instanton size:

  $$\rho \approx (600\,\mathrm{MeV})^{-1}$$

- Fully relativistic quantum-field theoretic model (we have a QCD vacuum):

  It explains almost all properties of the lowest-lying baryons.

- It describes the light & heavy baryons on an equal footing

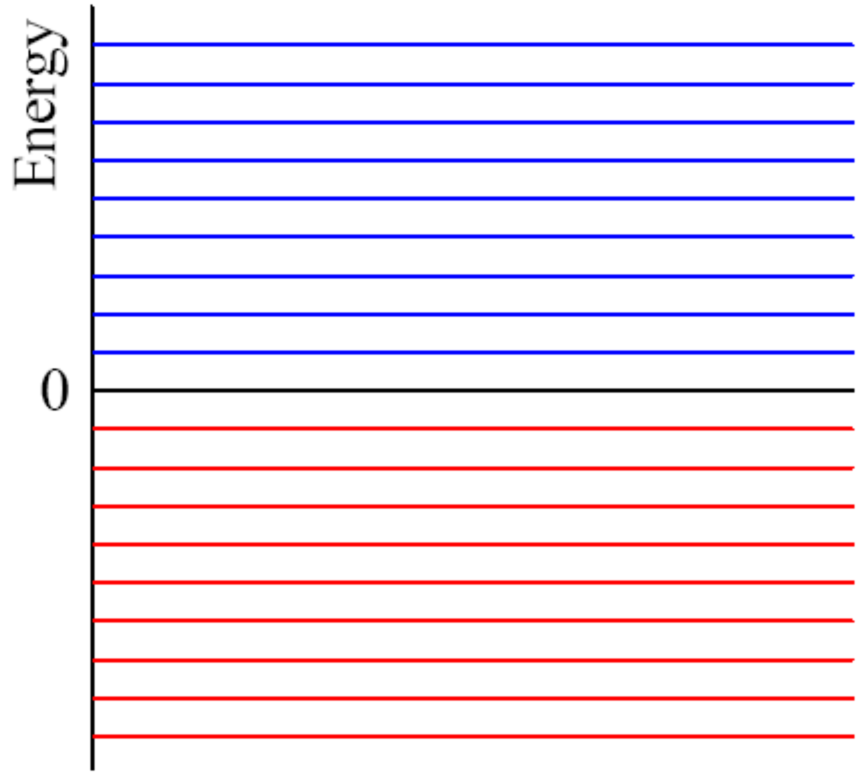
  (Advantage of the mean-field approach) .

- Basically, no free parameter to fit the experimental data.
  Cutoff parameter is fixed by the pion decay constant, and
  Dynamical quark mass (M=420 MeV) is fixed by the proton
  radius.

Baryon in pion mean fields

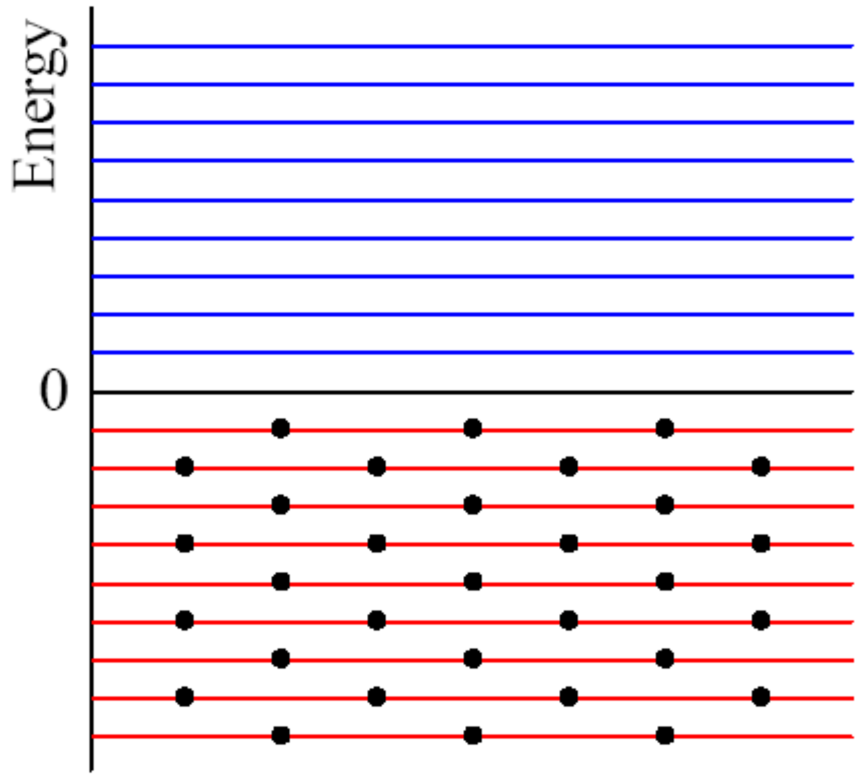
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



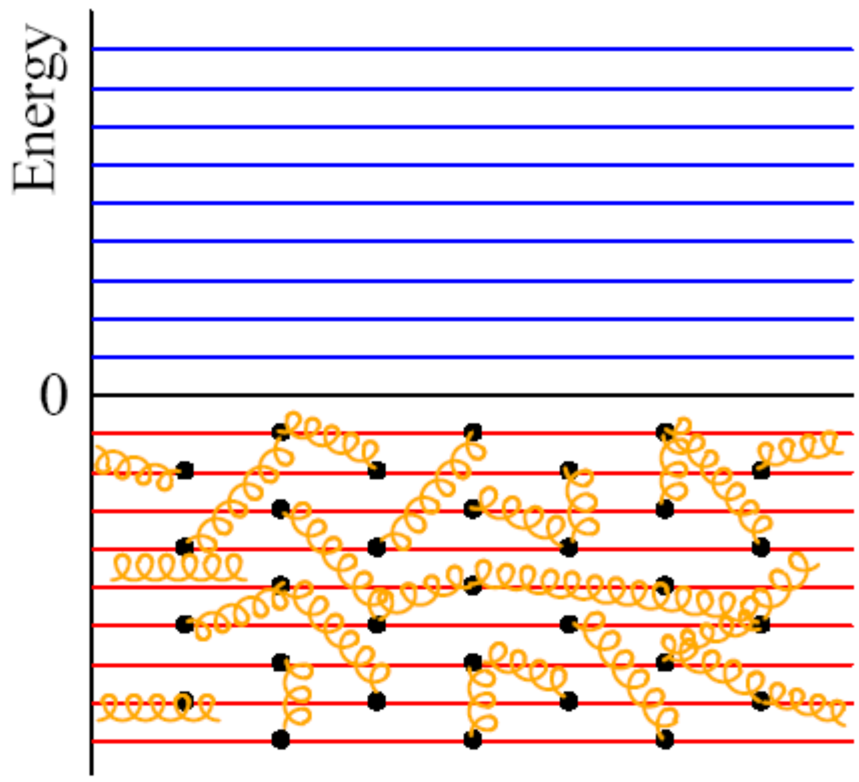
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



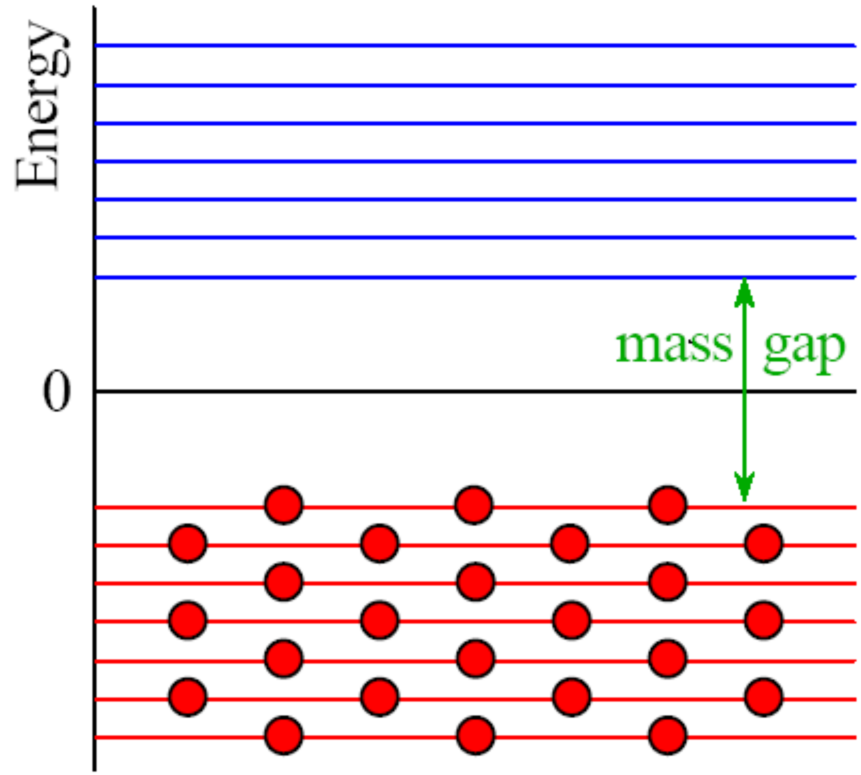
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



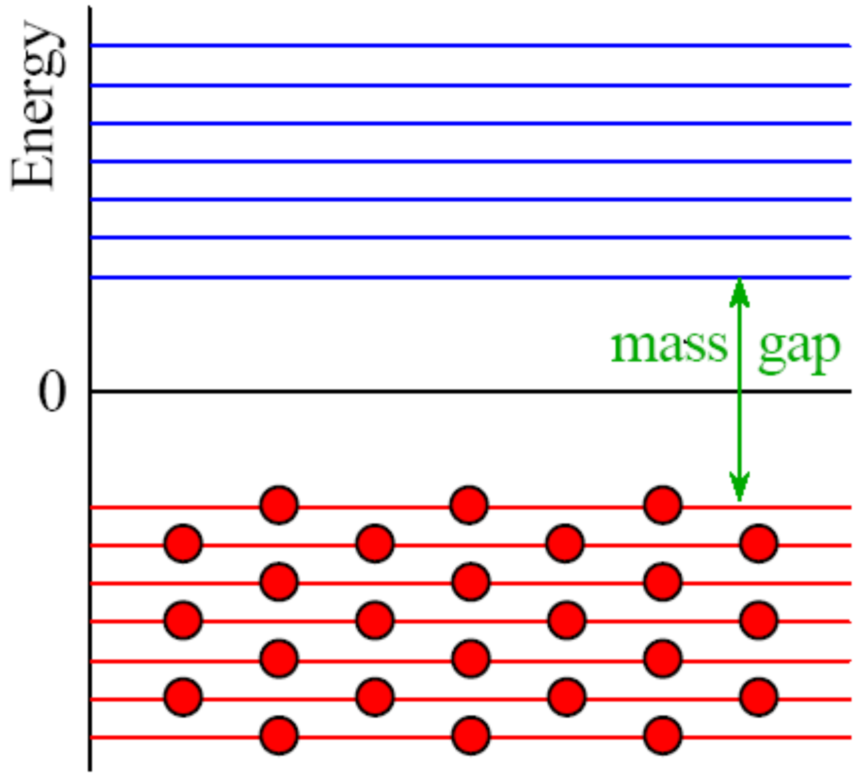
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



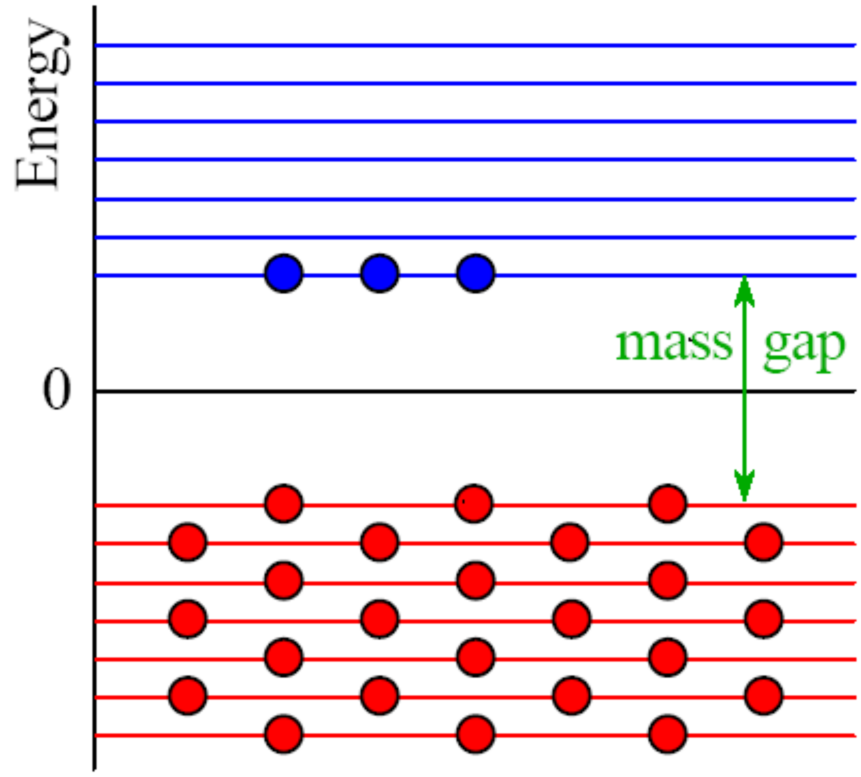
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



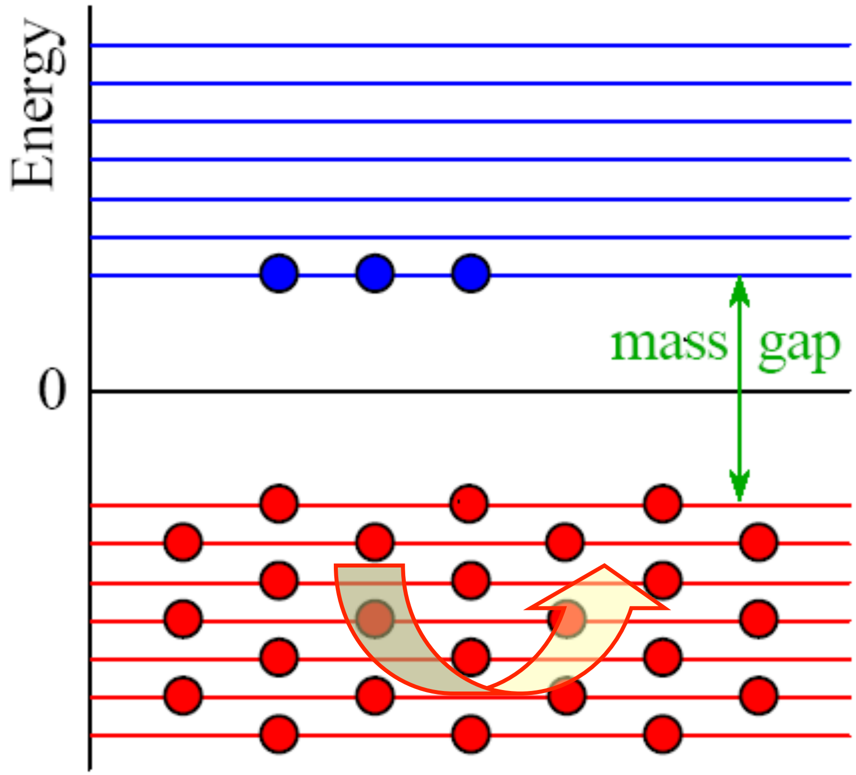
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



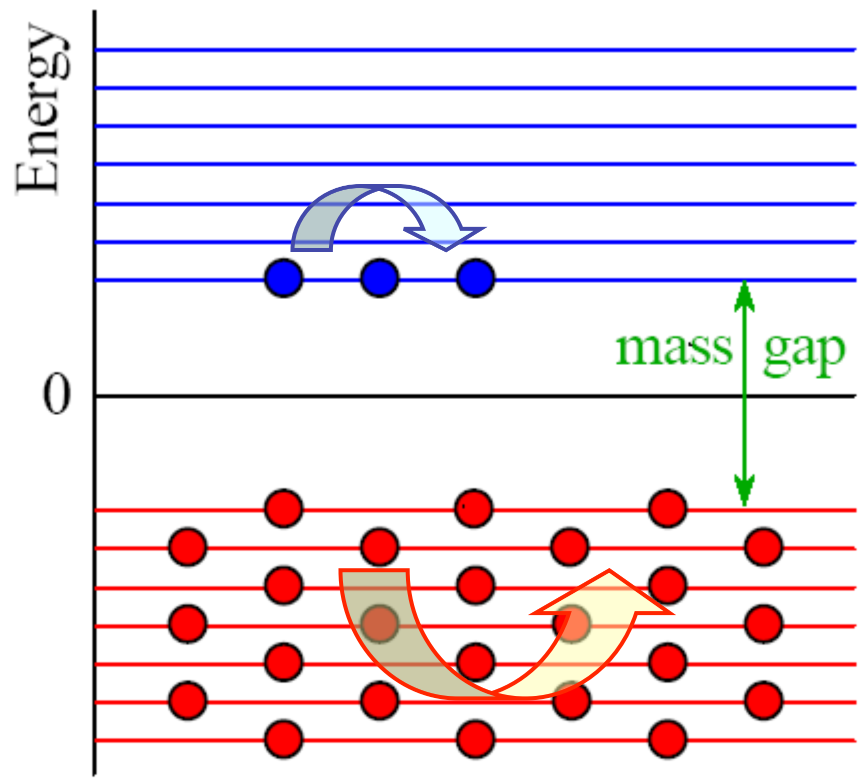
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



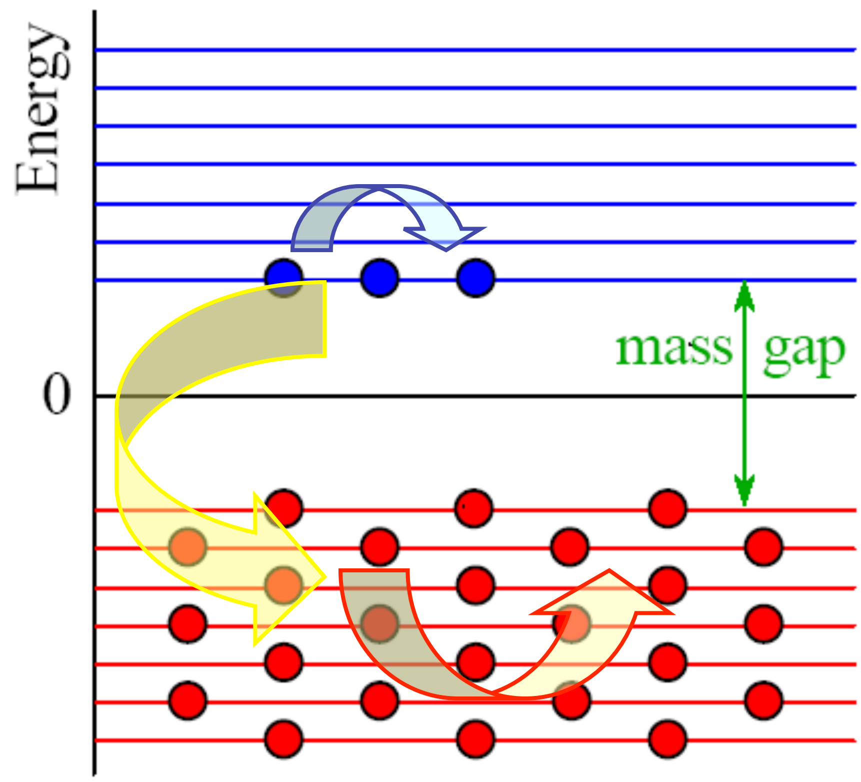
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



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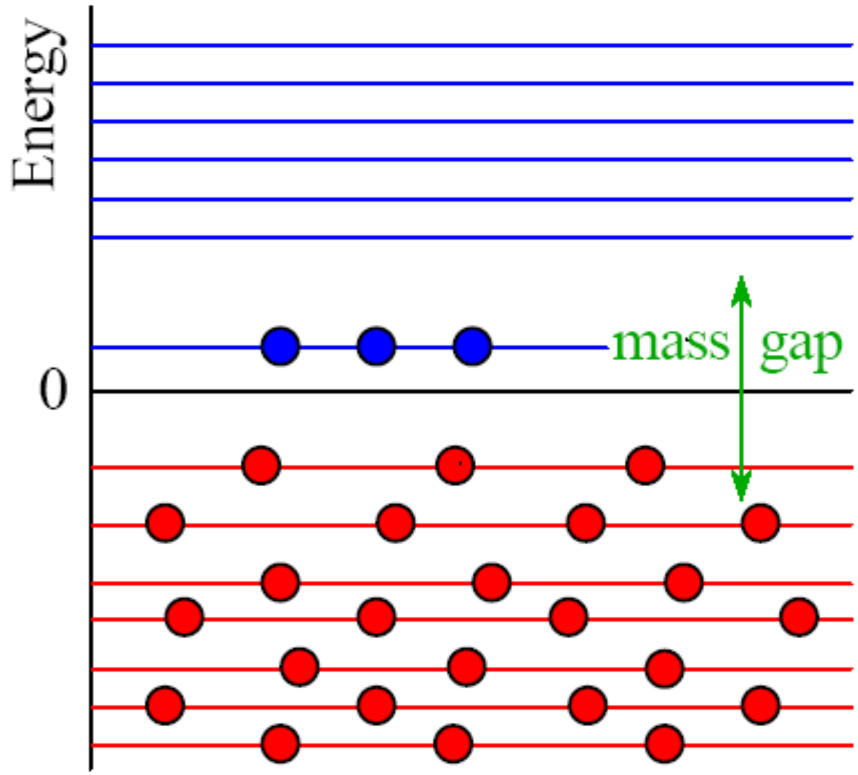
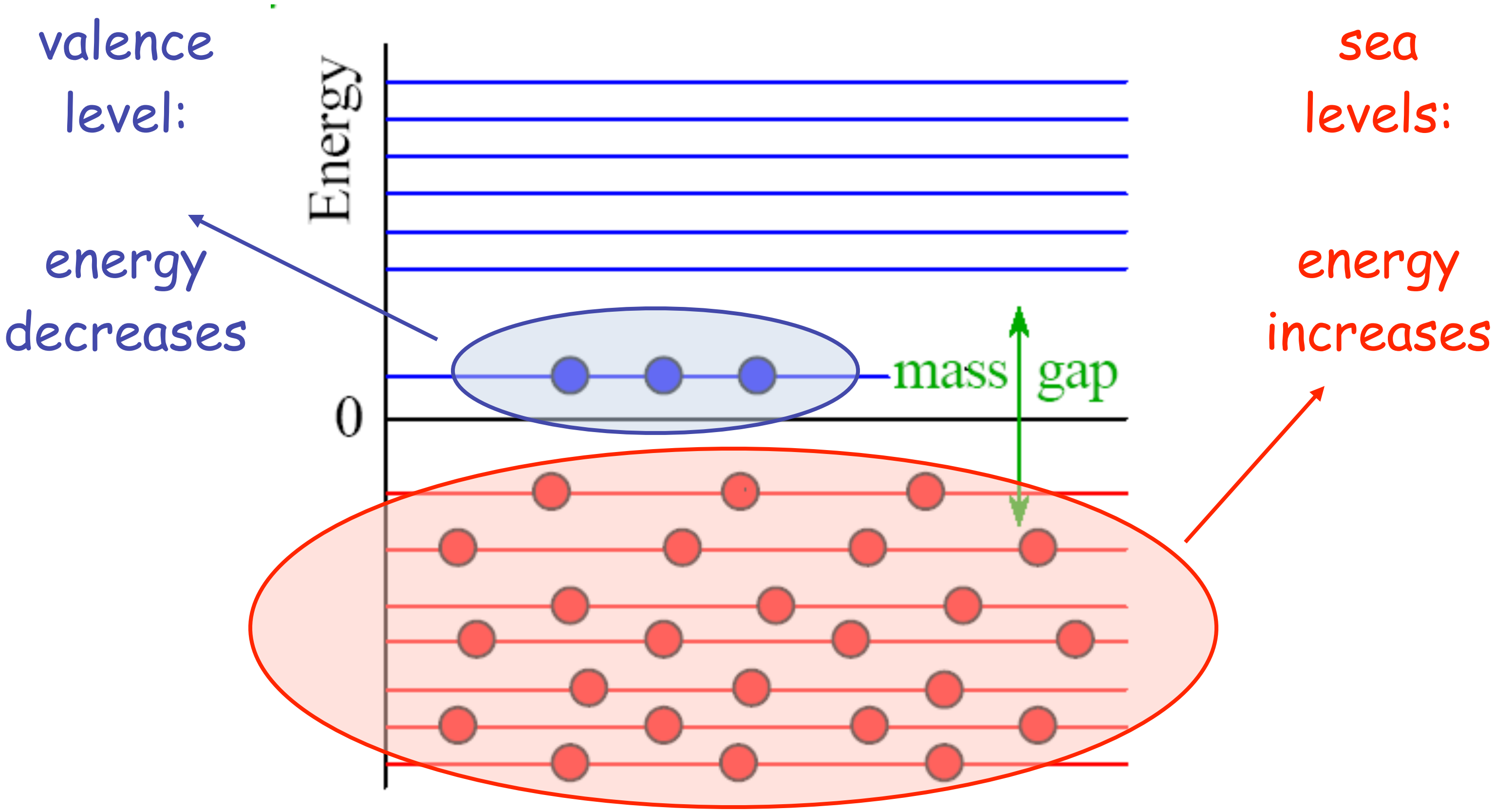
Baryon in pion mean fields



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

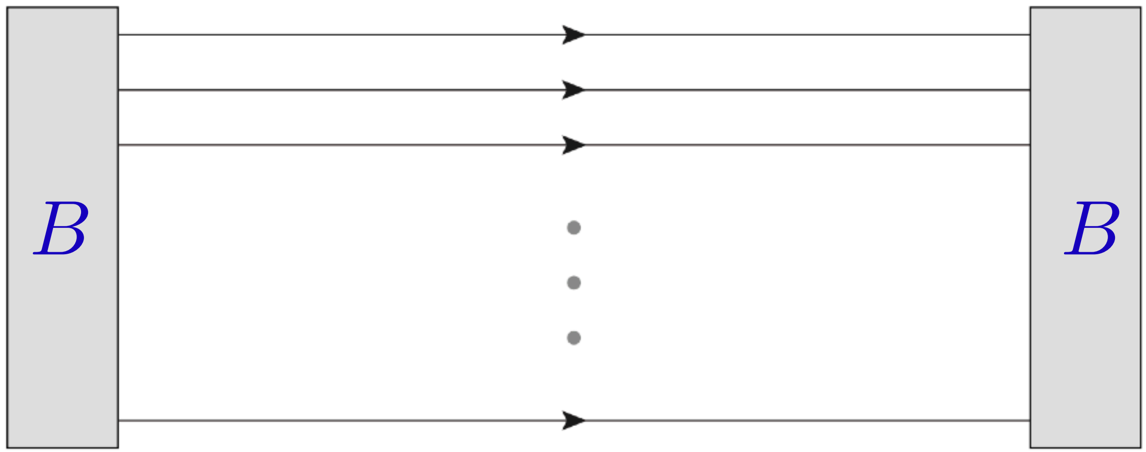
Baryon in pion mean fields



system is stabilized

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

A light baryon in pion mean fields

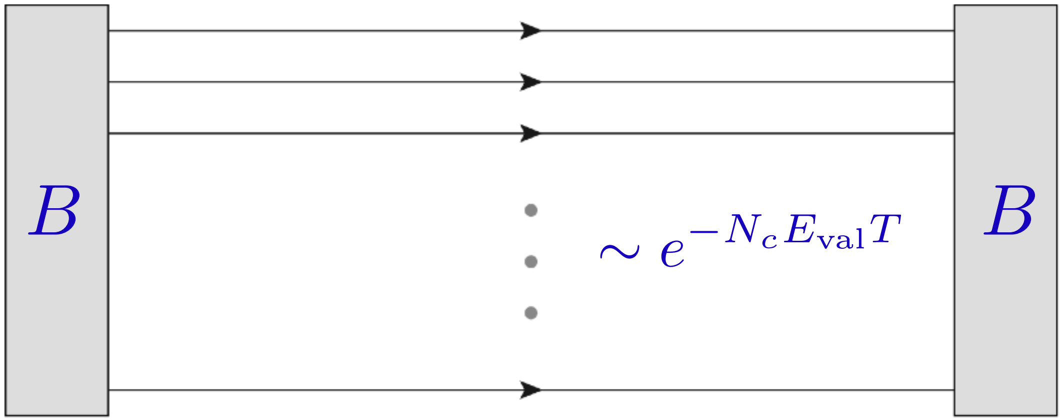


$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

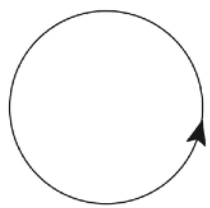
Presence of Nc quarks will polarize the vacuum or create mean fields.

Nc valence quarks → Vacuum polarization or meson mean fields

A light baryon in pion mean fields



$$E_{\rm cl} = N_c E_{\rm val} + E_{\rm sea}$$



$$\sim e^{-E_{\rm sea} T}$$

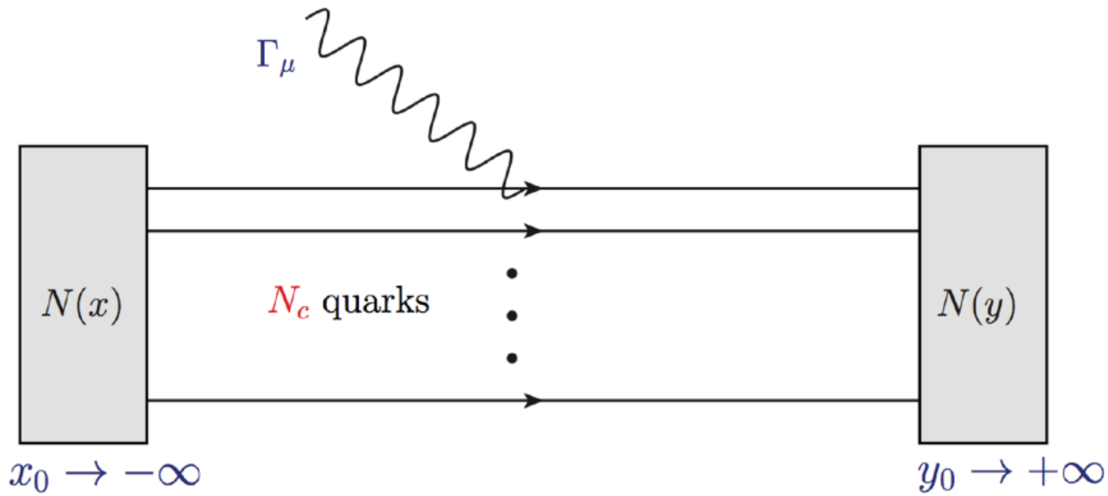
Classical Nucleon mass is described by the Nc valence quark energy and sea-quark energy.

$$\frac{\delta E_{cl}}{\delta U} = 0 \quad \longrightarrow \quad M_{\rm cl} \longrightarrow P(r)$$

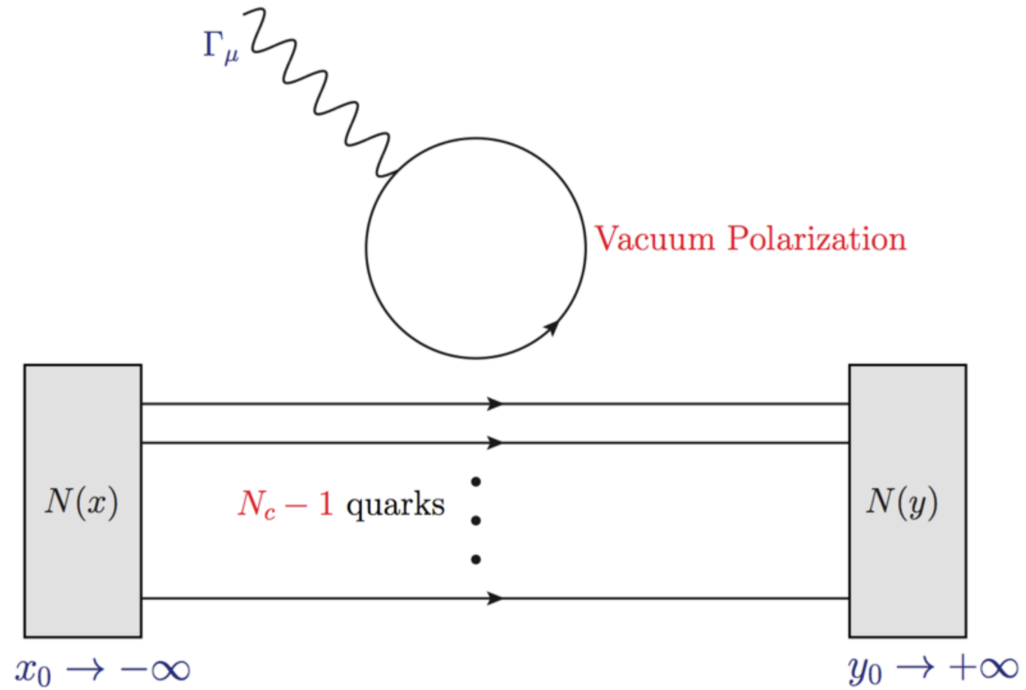
P(r): Soliton profile function
or Soliton field

An observable for the light baryon

Valence part

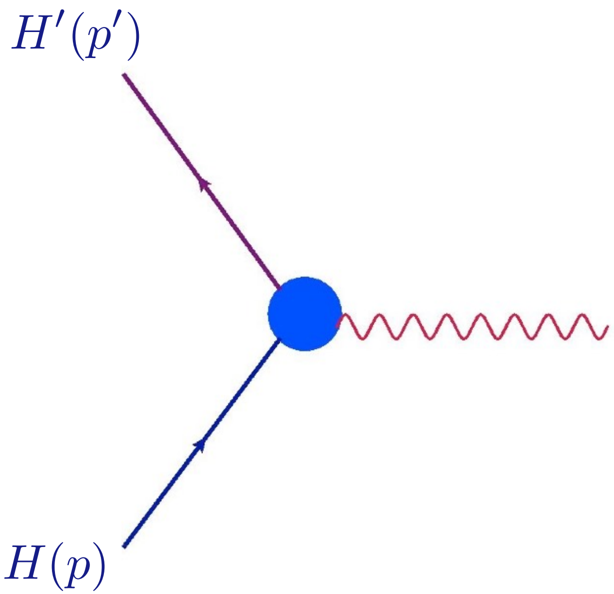


Sea part



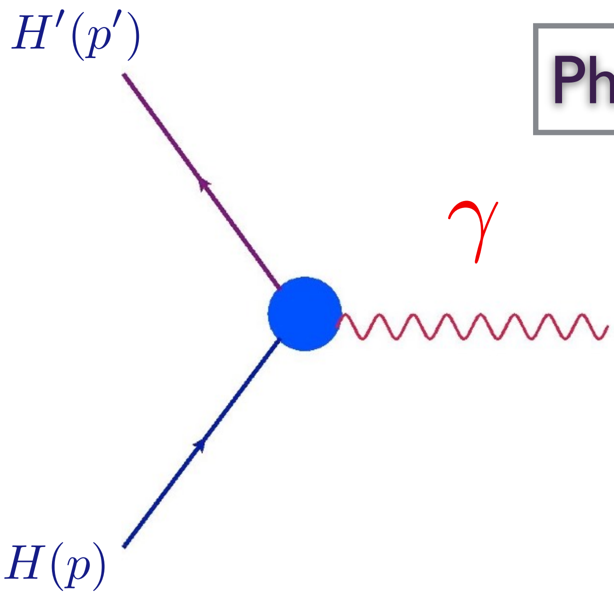
EM Form factors of the Nucleon

Traditional definition of form factors



No probe for the tensor & EMT (grav.) form factors!

Traditional definition of form factors



Photons → Charge & magnetic densities

ν, e_{PV}

Neutrino or PV electron scattering

↓

Axial-Vector properties

No probe for the tensor & EMT (grav.) form factors!

$$\langle N(P')|\bar{q}(0)\gamma^{\mu}q(0)|N(P)\rangle = \bar{U}(P')\left\{\gamma^{\mu}F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m_N}F_2(t)\right\}U(P),$$

$$\langle N(P')|\bar{q}(0)\gamma^{\mu}\gamma_5 q(0)|N(P)\rangle = \bar{U}(P')\left\{\gamma^{\mu}\gamma_5 G_A(t) + \frac{\gamma_5\Delta^{\mu}}{2m_N}G_P(t)\right\}U(P),$$

Traditional definition of form factors

$$G_E^{p,n}(Q^2) \Longleftrightarrow \rho_{\mathrm{ch}}^{p,n}(r^2)$$

Fourier transform

Textbook physics since 1950s.

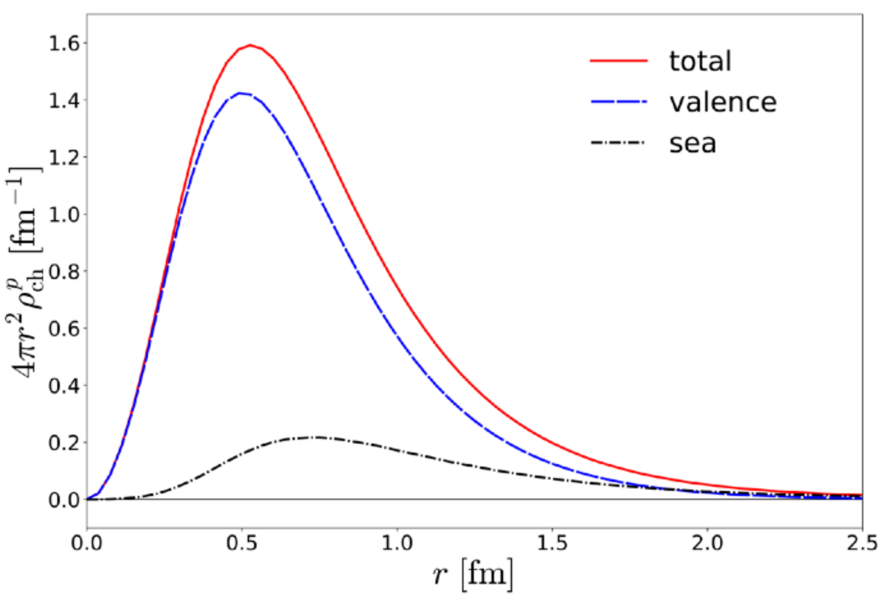
Traditional definition of form factors

$$G_E^{p,n}(Q^2) \Longleftrightarrow \rho_{\rm ch}^{p,n}(r^2)$$

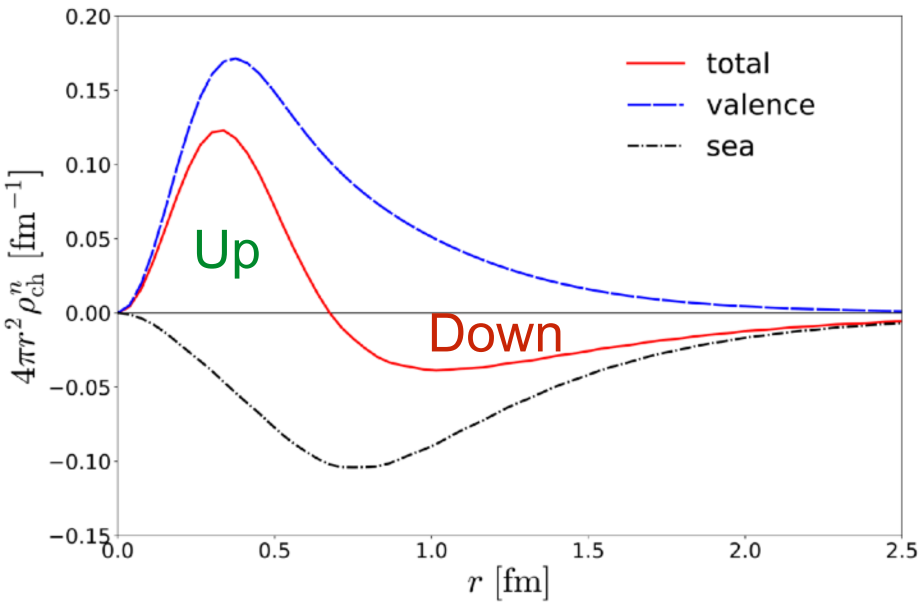
Fourier transform

Textbook physics since 1950s.

Proton



Neutron



New Definition

New Definition

Generalized Parton Distributions

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^{\mu}H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m_N}E(x,\xi,t)\right\}U(P) + \mathrm{ht},$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^{\mu}\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^{\mu}}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \mathrm{ht},$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \mathrm{ht}$$

New Definition

Generalized Parton Distributions

Melin transform

Generalized Form factors

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht},$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht},$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

$$F_1(t) = \int_{-1}^{1} dx H(x,\xi,t), \qquad F_2(t) = \int_{-1}^{1} dx E(x,\xi,t),$$

$$G_A(t) = \int_{-1}^{1} dx \widetilde{H}(x,\xi,t), \qquad G_P(t) = \int_{-1}^{1} dx \widetilde{E}(x,\xi,t),$$

$$A_{T10}(t) = \int_{-1}^{1} dx H_T(x,\xi,t), \qquad B_{T10}(t) = \int_{-1}^{1} dx E_T(x,\xi,t), \qquad \widetilde{A}_{T10}(t) = \int_{-1}^{1} dx \widetilde{H}_T(x,\xi,t)$$

New Definition

Generalized Parton Distributions

↓ Melin transform

Generalized Form factors

↓ 2D Fourier transform

Transverse charge densities

Quark probabilities inside a nucleon

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^{\mu}H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht},$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^{\mu}\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^{\mu}}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht},$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

$$F_1(t) = \int_{-1}^{1} dx H(x,\xi,t), \qquad F_2(t) = \int_{-1}^{1} dx E(x,\xi,t),$$

$$G_A(t) = \int_{-1}^{1} dx \widetilde{H}(x,\xi,t), \qquad G_P(t) = \int_{-1}^{1} dx \widetilde{E}(x,\xi,t),$$

$$A_{T10}(t) = \int_{-1}^{1} dx H_T(x,\xi,t), \qquad B_{T10}(t) = \int_{-1}^{1} dx E_T(x,\xi,t), \qquad \widetilde{A}_{T10}(t) = \int_{-1}^{1} dx \widetilde{H}_T(x,\xi,t)$$

New Definition

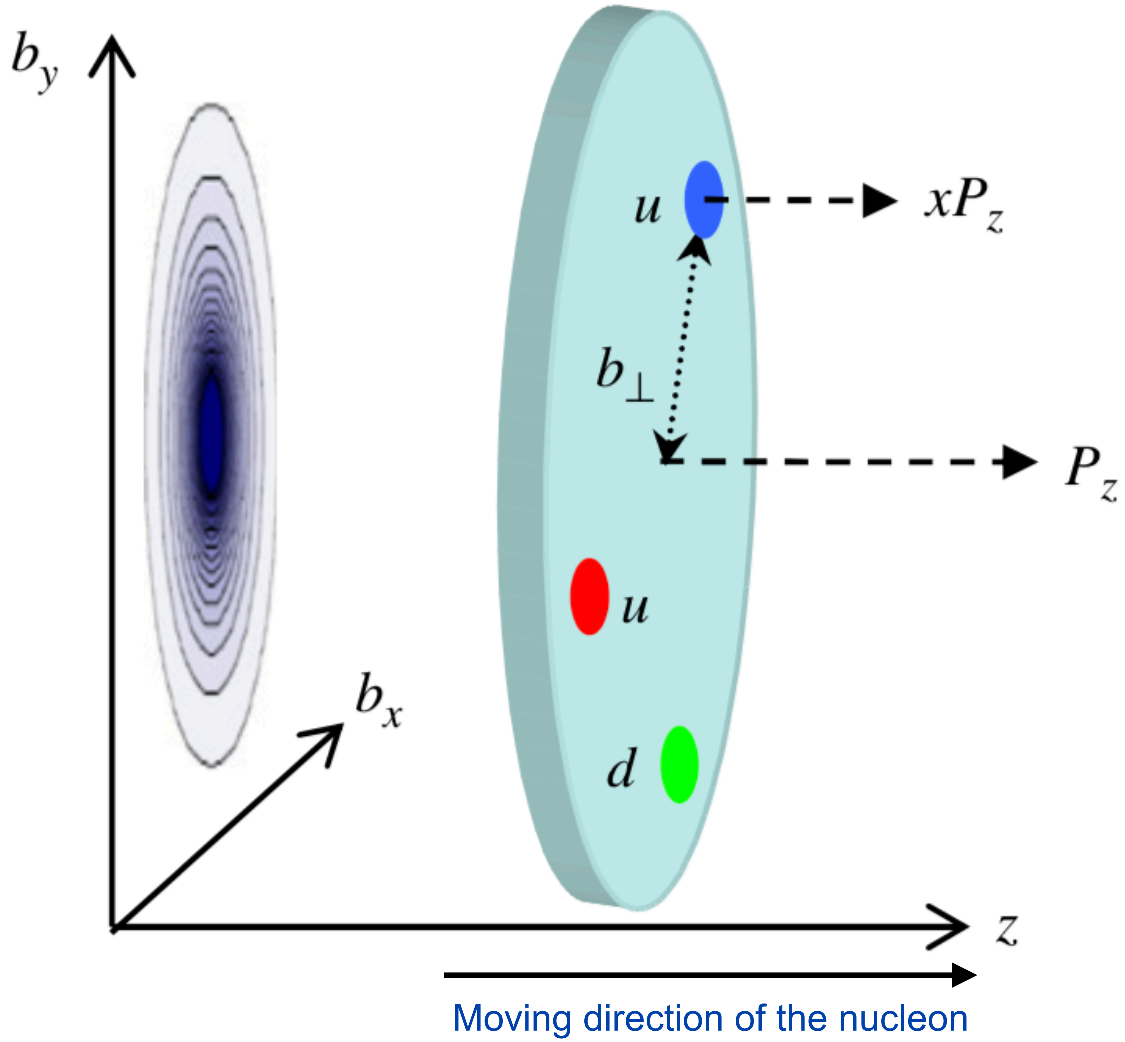
Generalized Parton Distributions

Melin transform

Generalized Form factors

2D Fourier transfor

Transverse charge densities



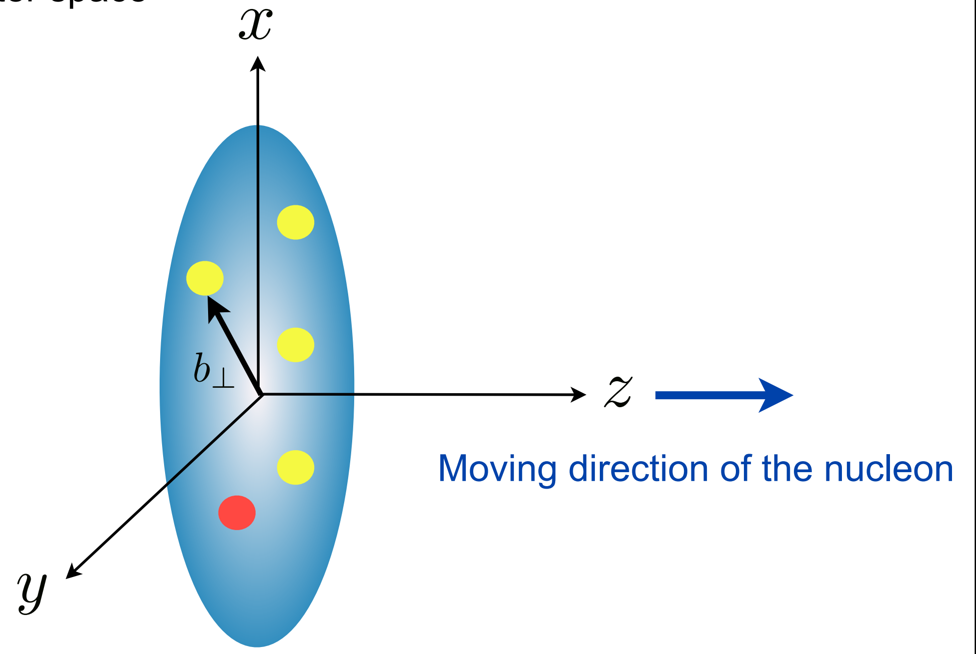
Quark probabilities inside a nucleon

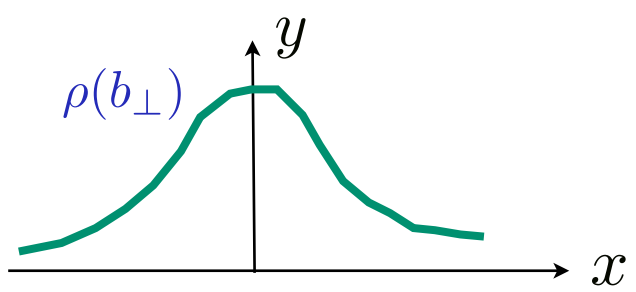
Transverse charge density

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$q(x,\,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x, -\mathbf{q}^2)$$





Transverse charge density

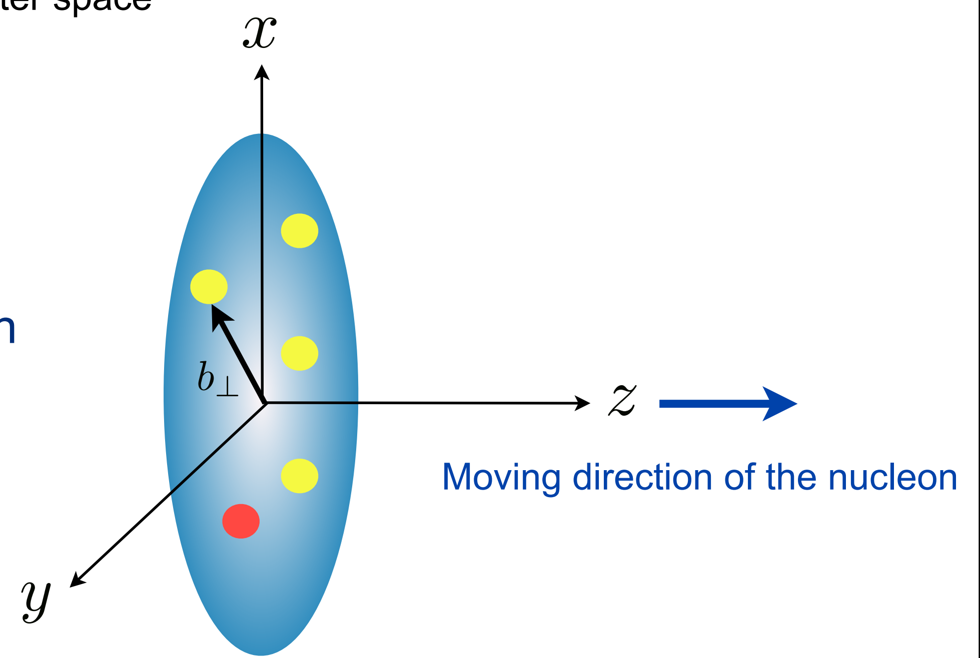
Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

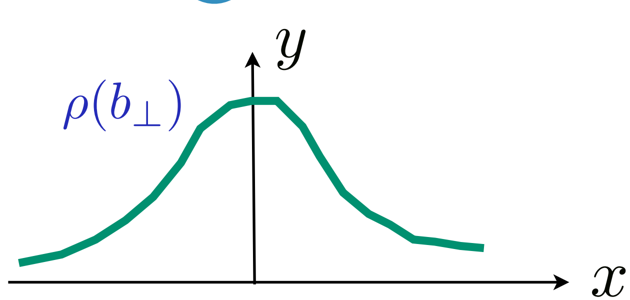
$$q(x,\,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x,-\mathbf{q}^2)$$

⇨ It can be interpreted as the probability distribution of a quark in the transverse plane.

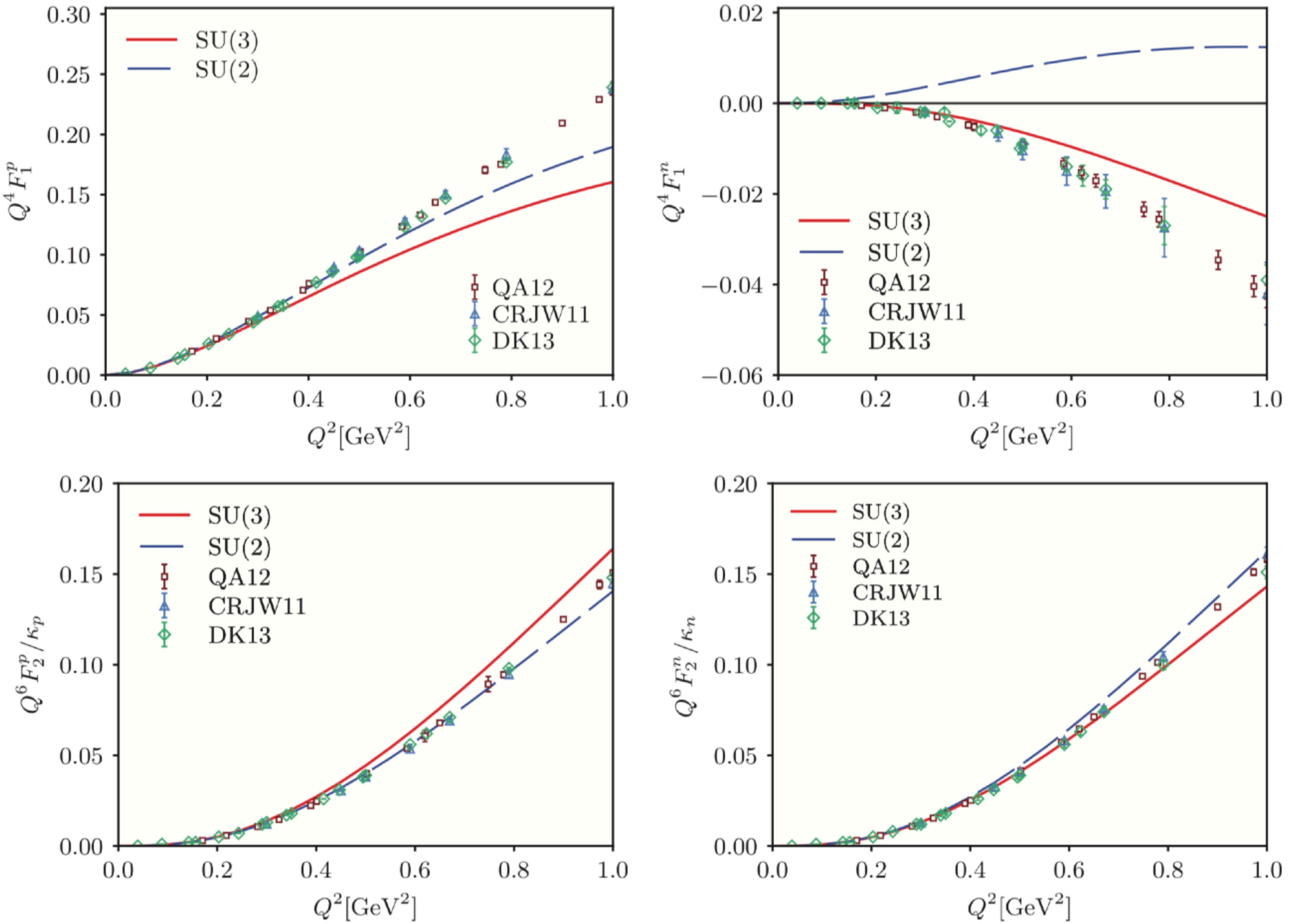
M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).



$$\rho(\mathbf{b}) := \sum_q e_q \int dx q(x,\,\mathbf{b})$$
$$= \int \frac{d^2q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$



Proton & neutron EM fom factors



Silva, Urbano, HChK, PTEP, 2018

Transverse charge density

Inside an unpolarized nucleon

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

G.A. Miller, PRL **99,** 112001 (2007)

$$\rho_{\rm ch}^{\chi}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) F_1^{\chi}(Q^2)$$

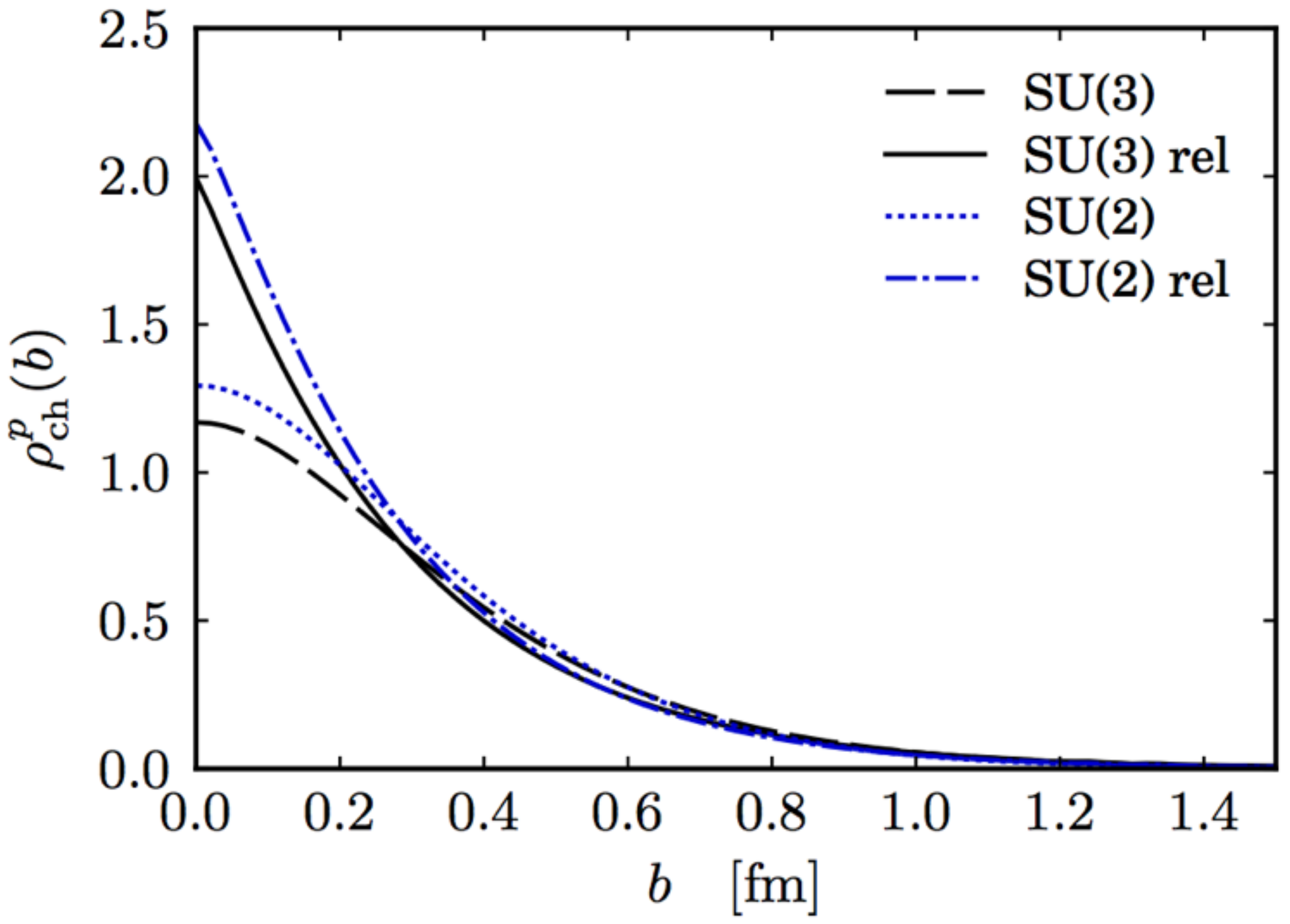
Inside an polarized nucleon

Carlson and Vanderhaeghen, PRL **100**, 032004

$$\rho_T^{\chi}(b) = \rho_{\rm ch}^{\chi}(b) - \sin(\phi_b - \phi_S)\frac{1}{2M_N}\int_0^\infty \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^{\chi}(Q^2)$$

Proton & neutron transverse charge densities

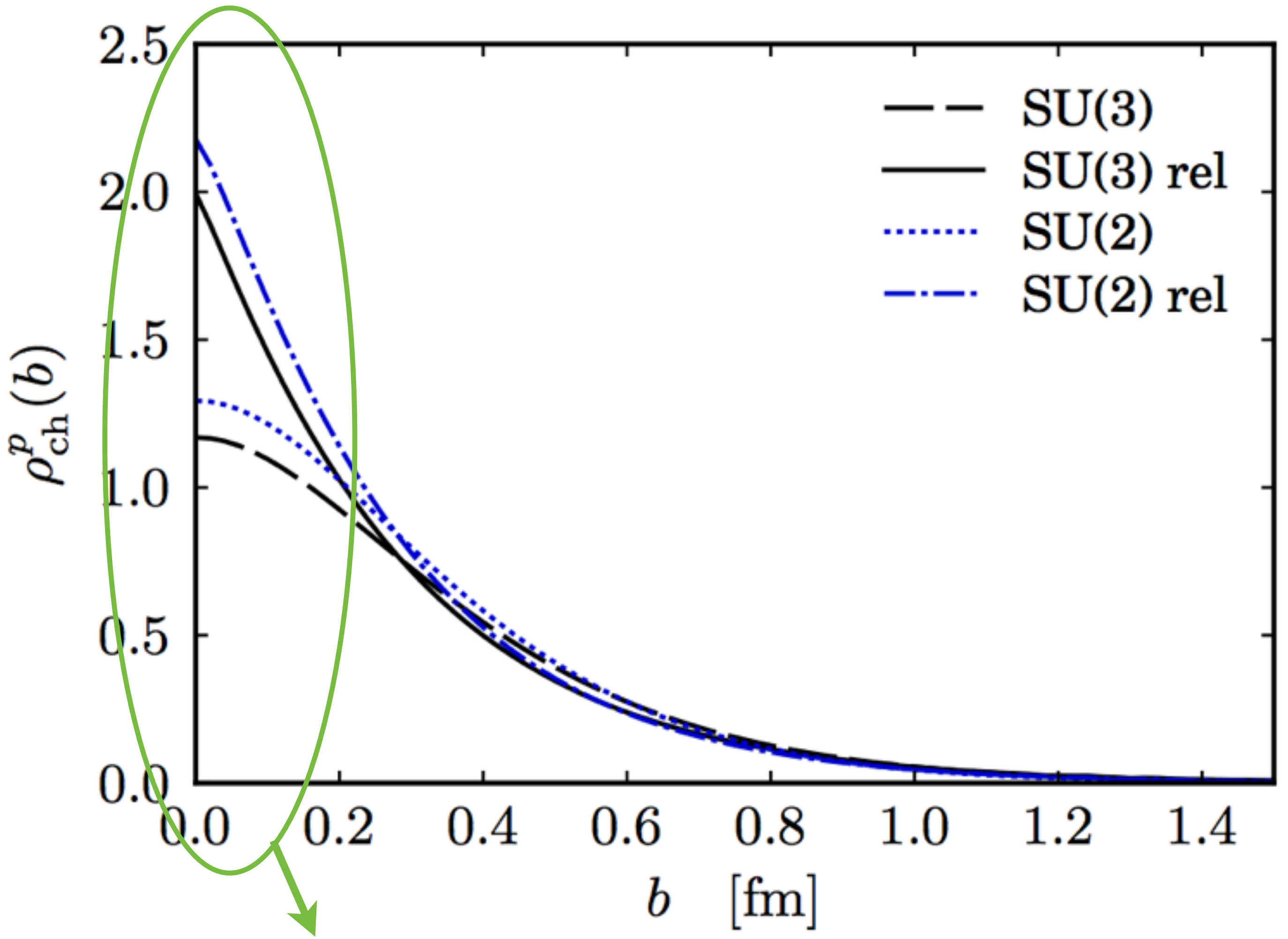
Transverse charge densities inside an unpolarized proton



Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

Transverse charge densities inside an unpolarized proton

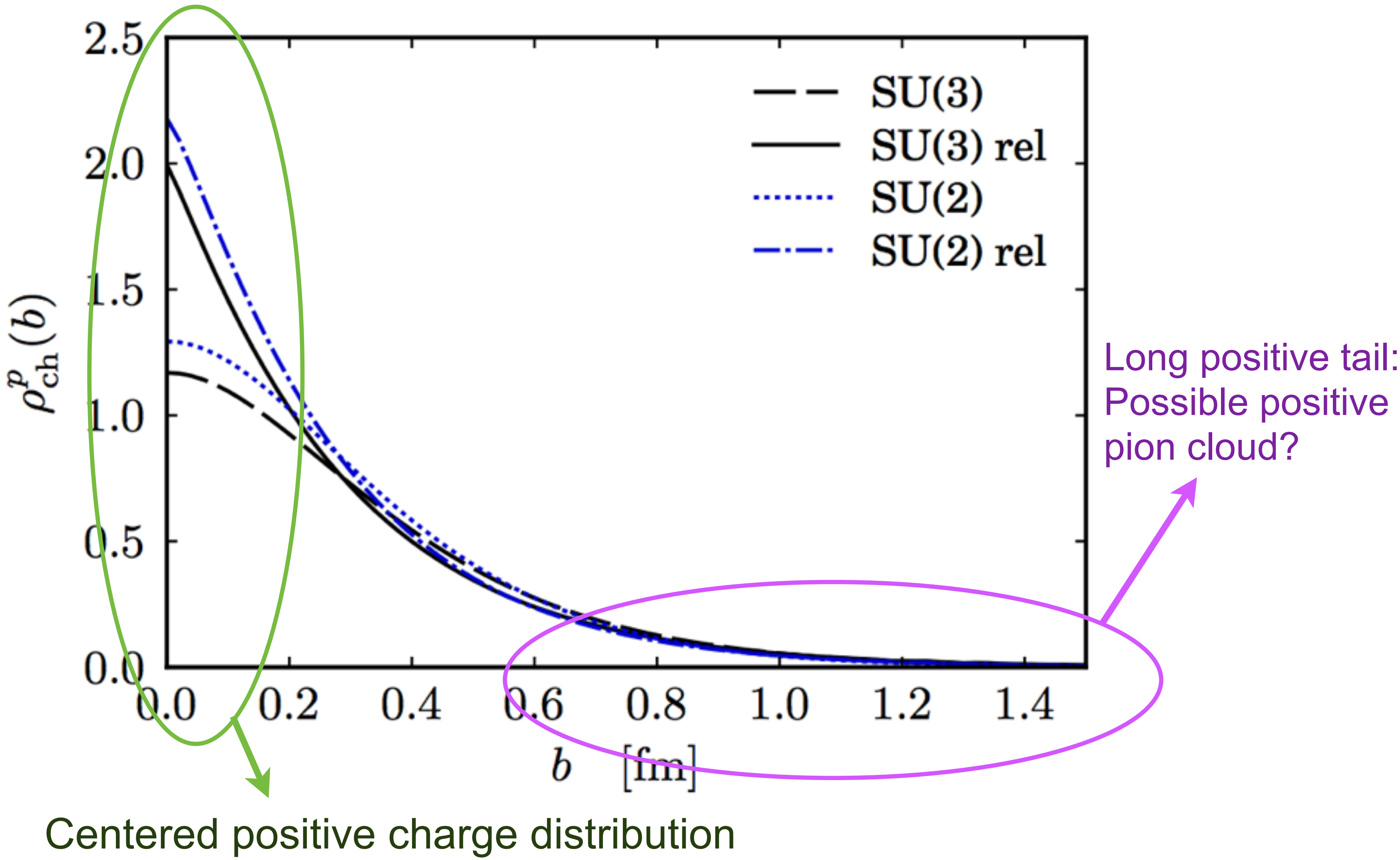


Centered positive charge distribution

Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

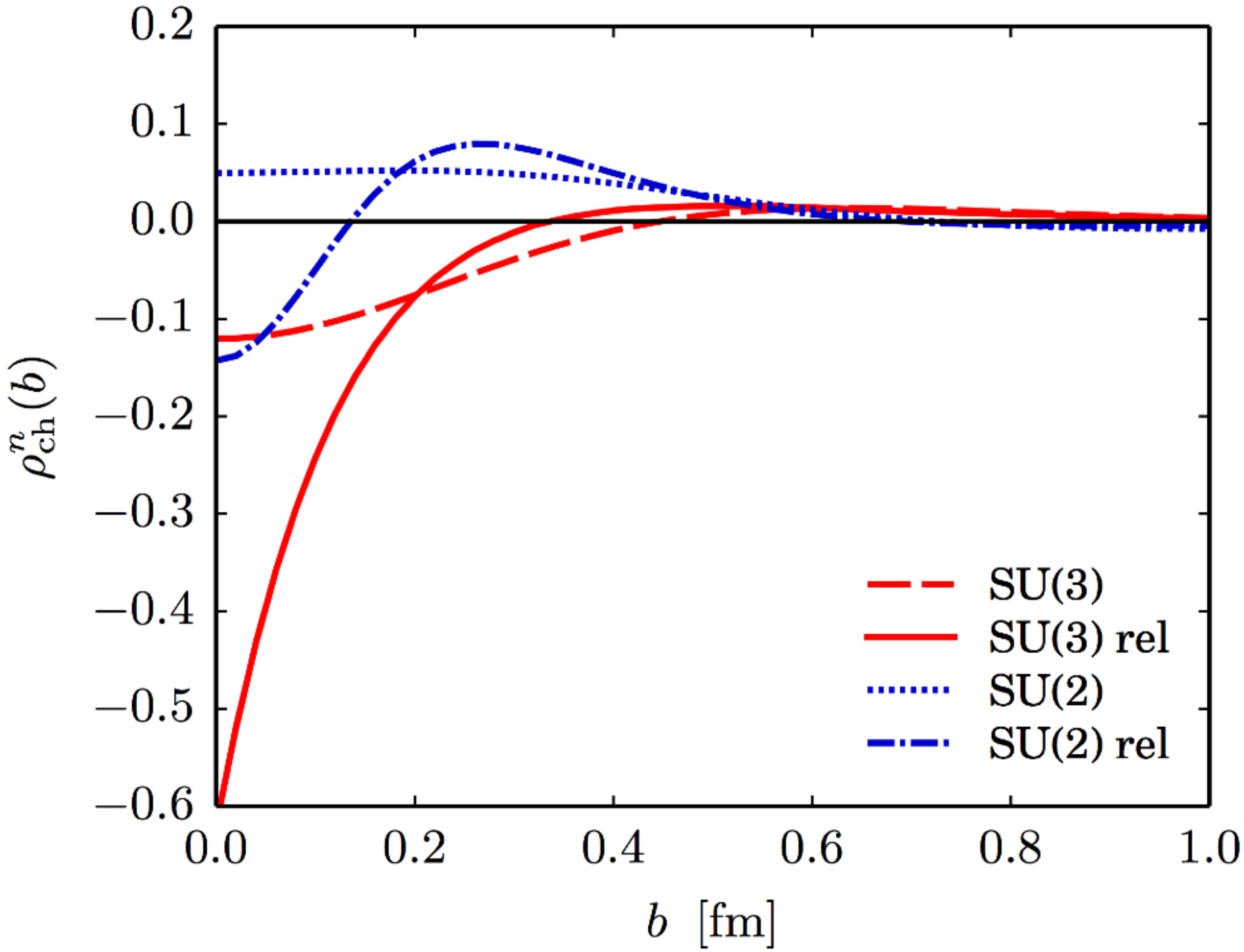
Transverse charge densities inside an unpolarized proton



Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

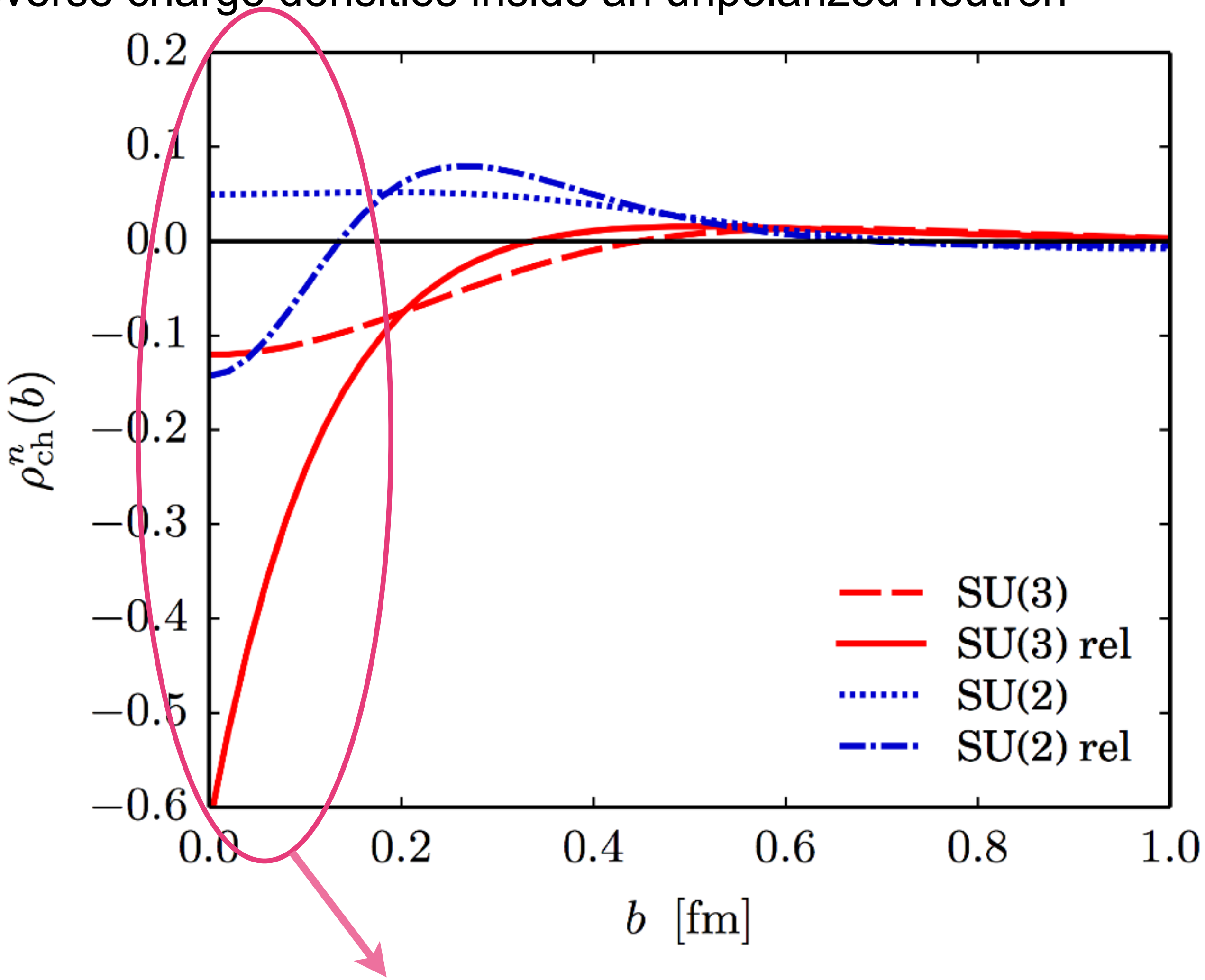
Transverse charge densities inside an unpolarized neutron



Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

Transverse charge densities inside an unpolarized neutron

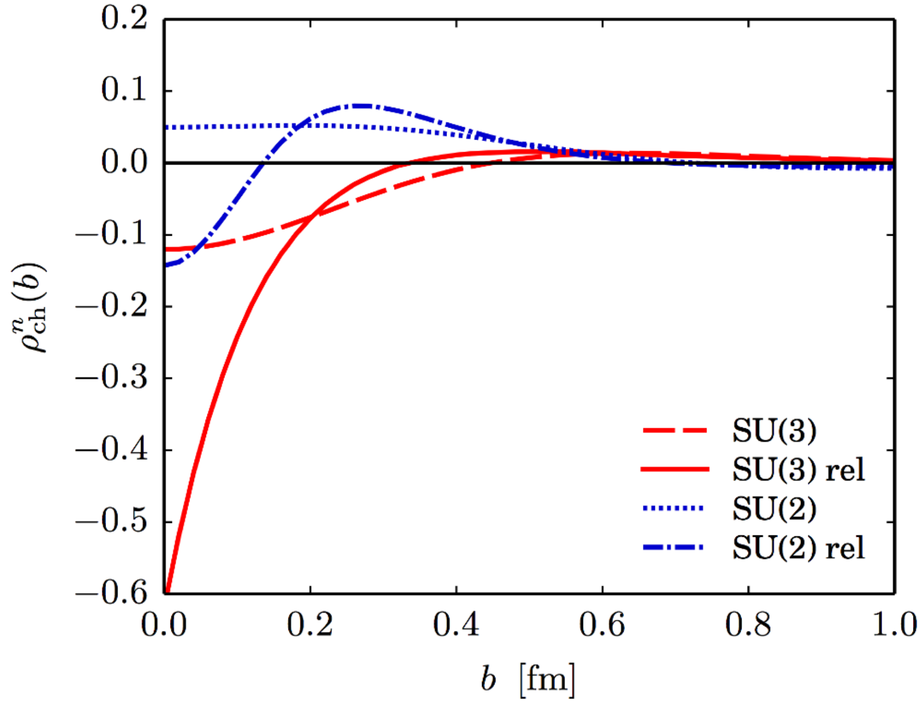


Surprisingly, negative charge distribution in the center of the neutron!

Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

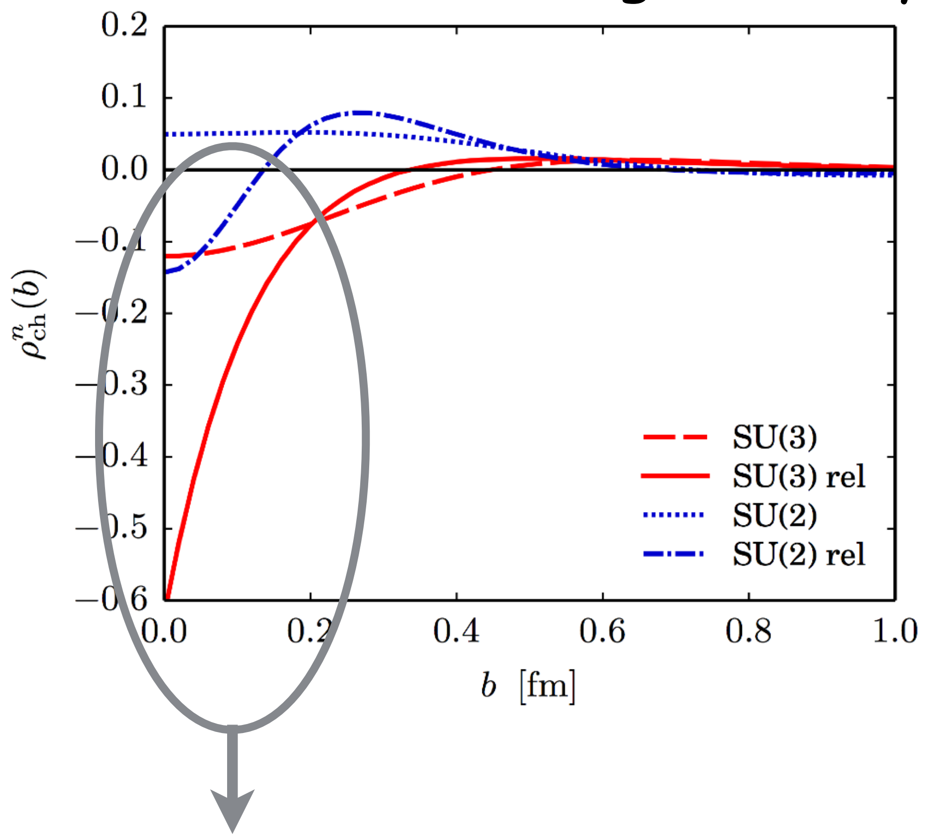
2D transverse charge density



Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

2D transverse charge density



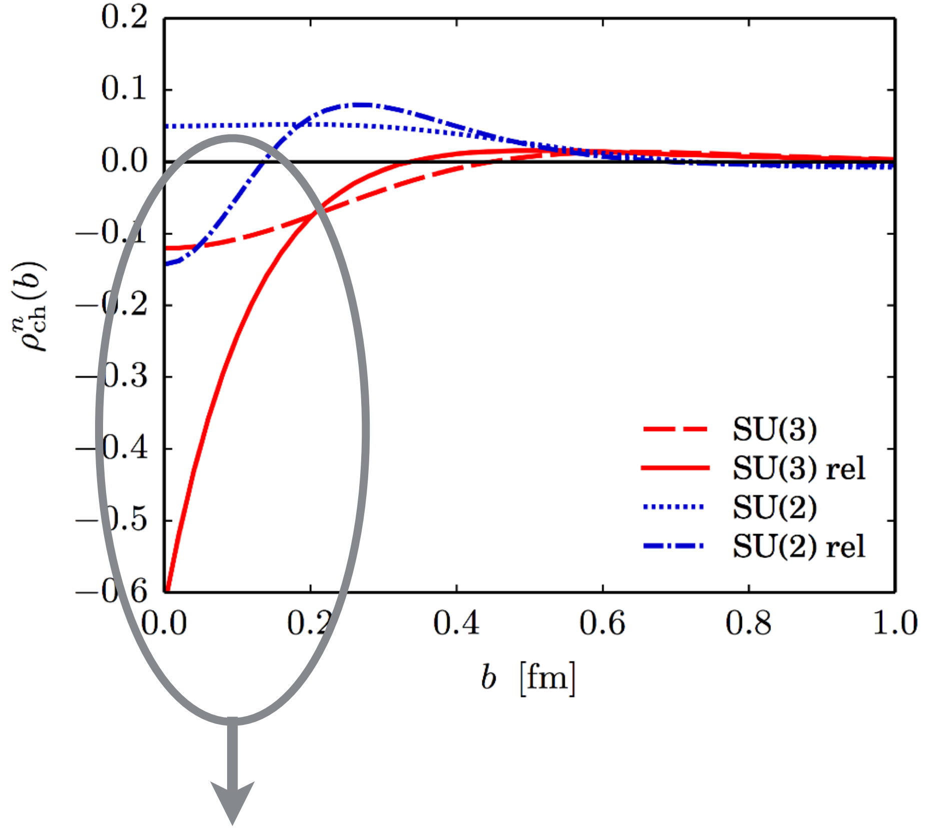
Negative!

Relativistically invariant!

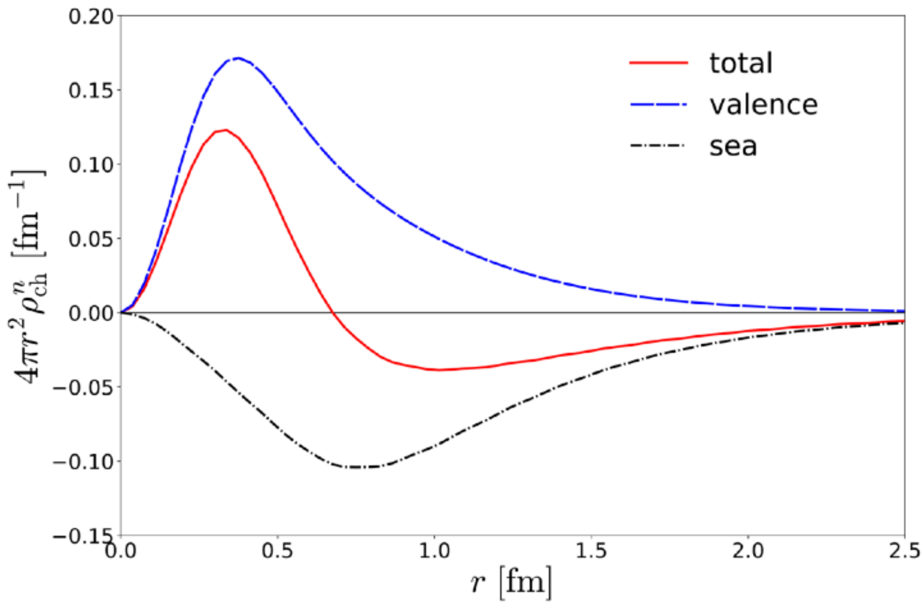
Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

2D transverse charge density



3D charge density



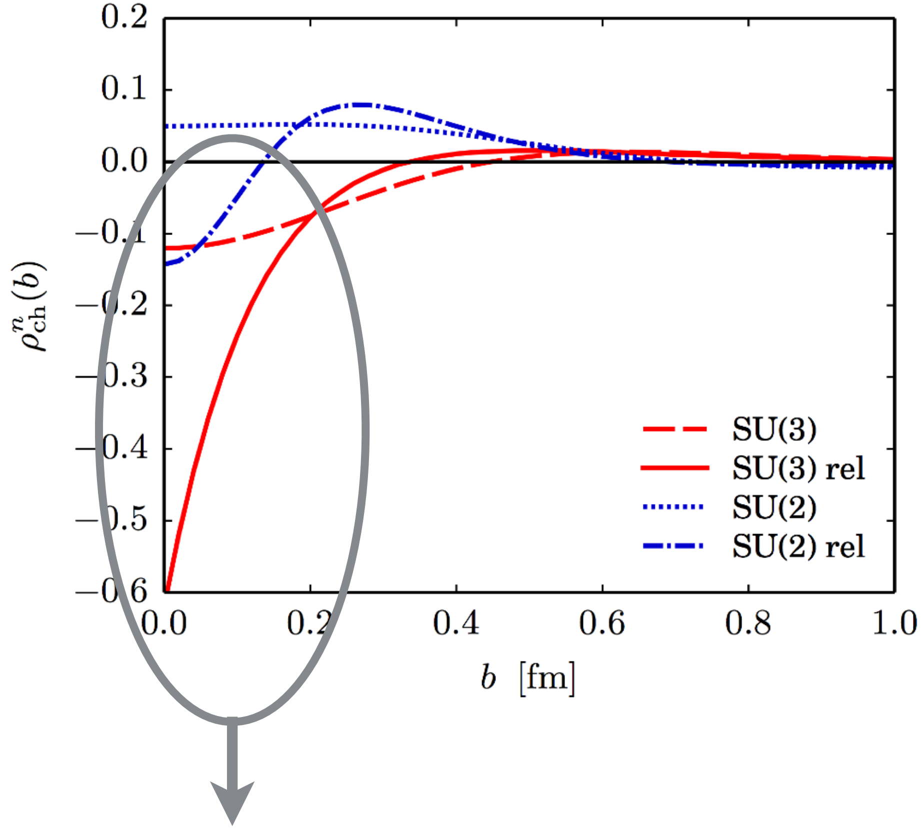
Negative!

Relativistically invariant!

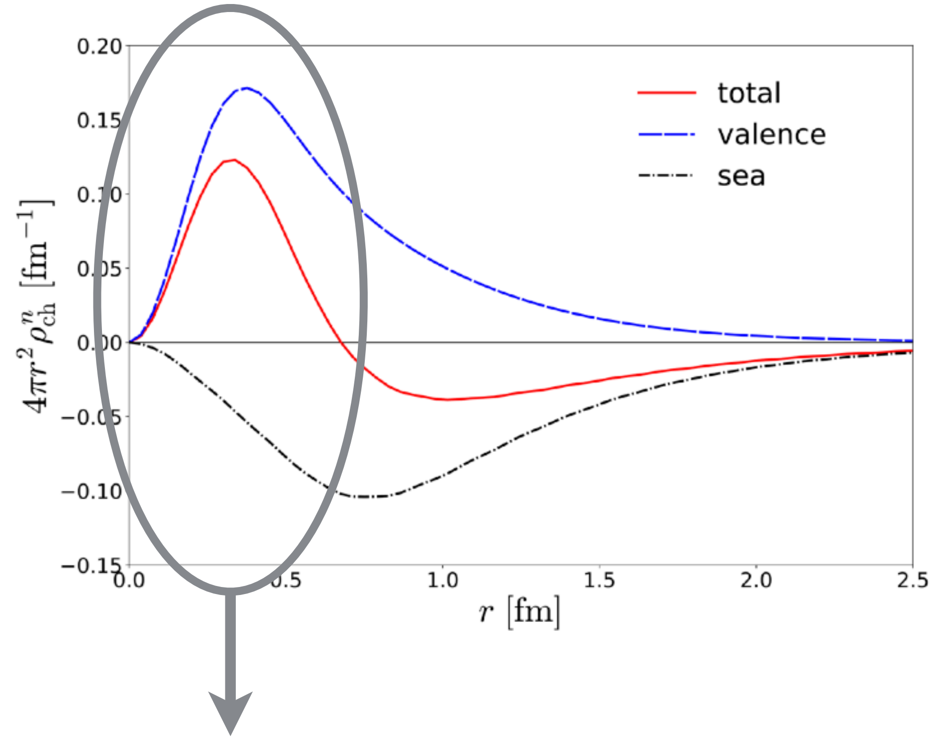
Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

2D transverse charge density



3D charge density



Negative!

Relativistically invariant!

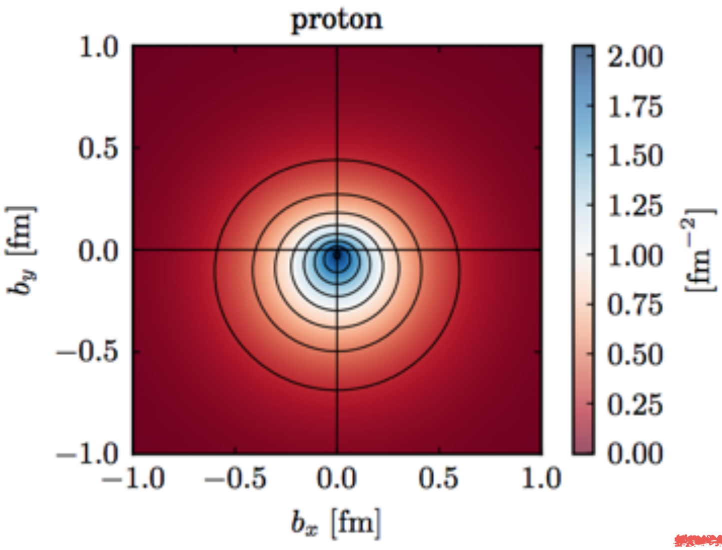
Positive!

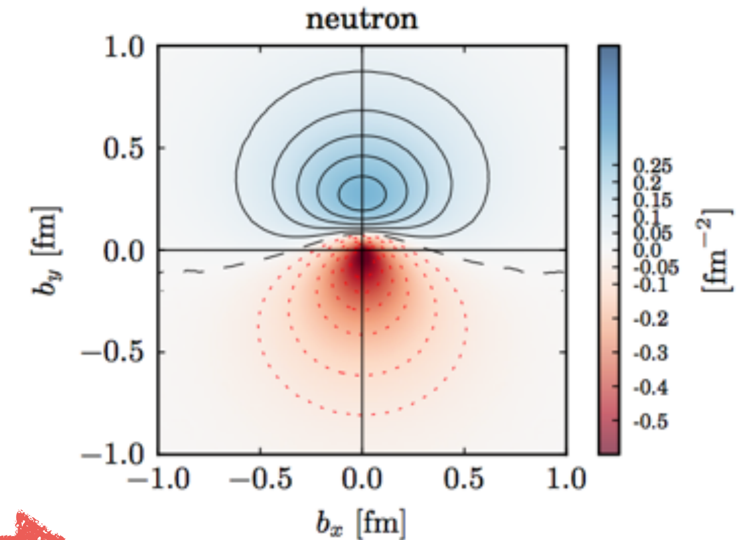
Nonrelativistic!

Silva, Urbano, HChK, PTEP, (2018)

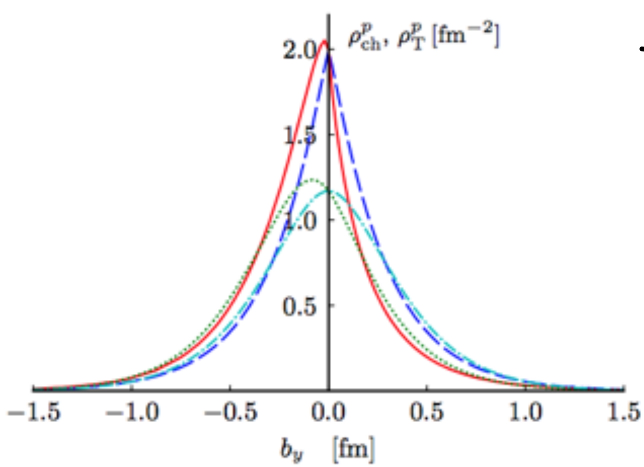
Proton & neutron transverse charge densities

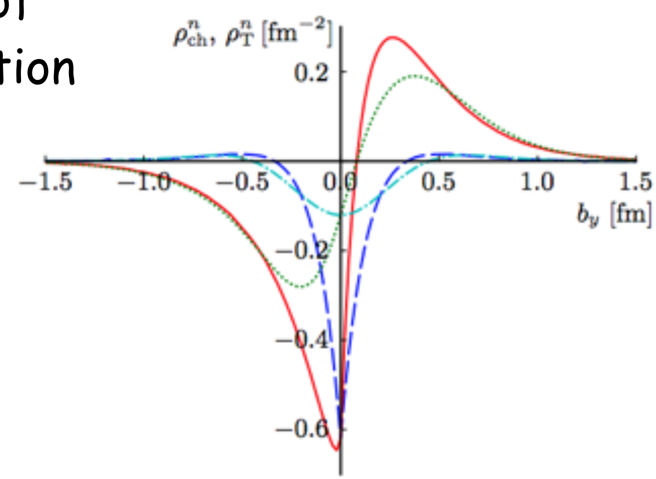
Transverse charge densities inside an **polarized** nucleon





Direction of
the polarization

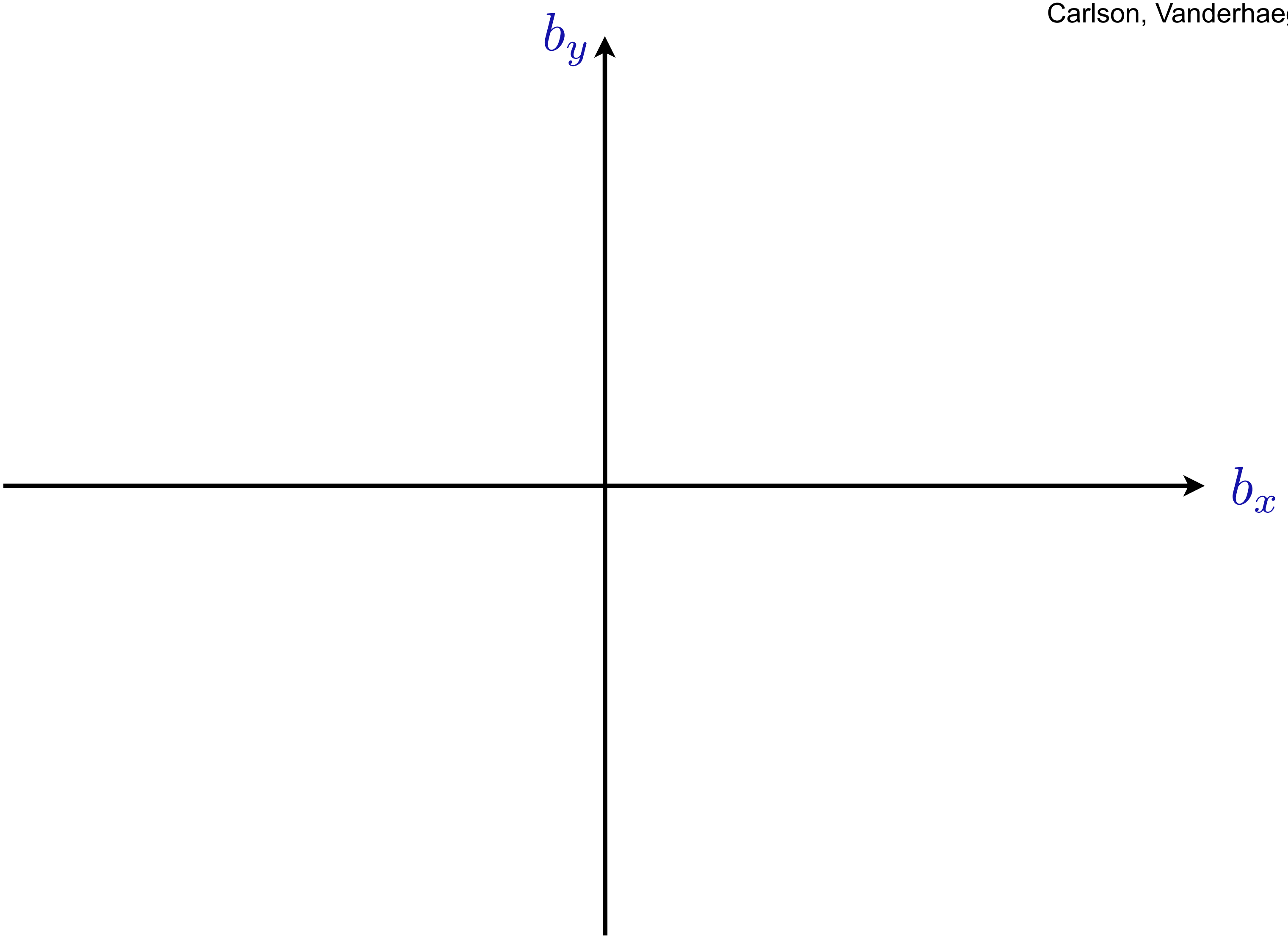




Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

Carlson, Vanderhaeghen, PRL **100**, 032004



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Proton & neutron transverse charge densities

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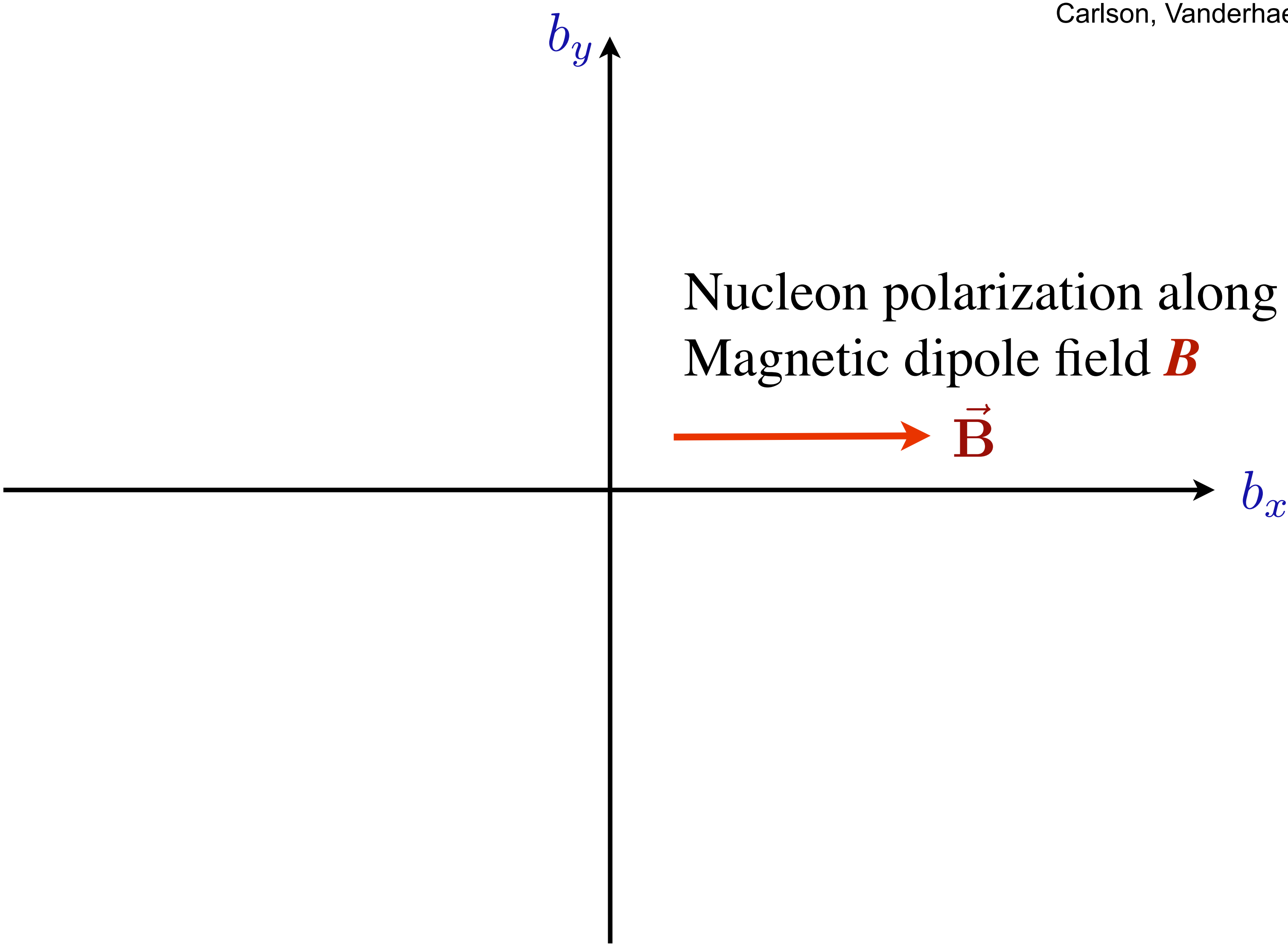
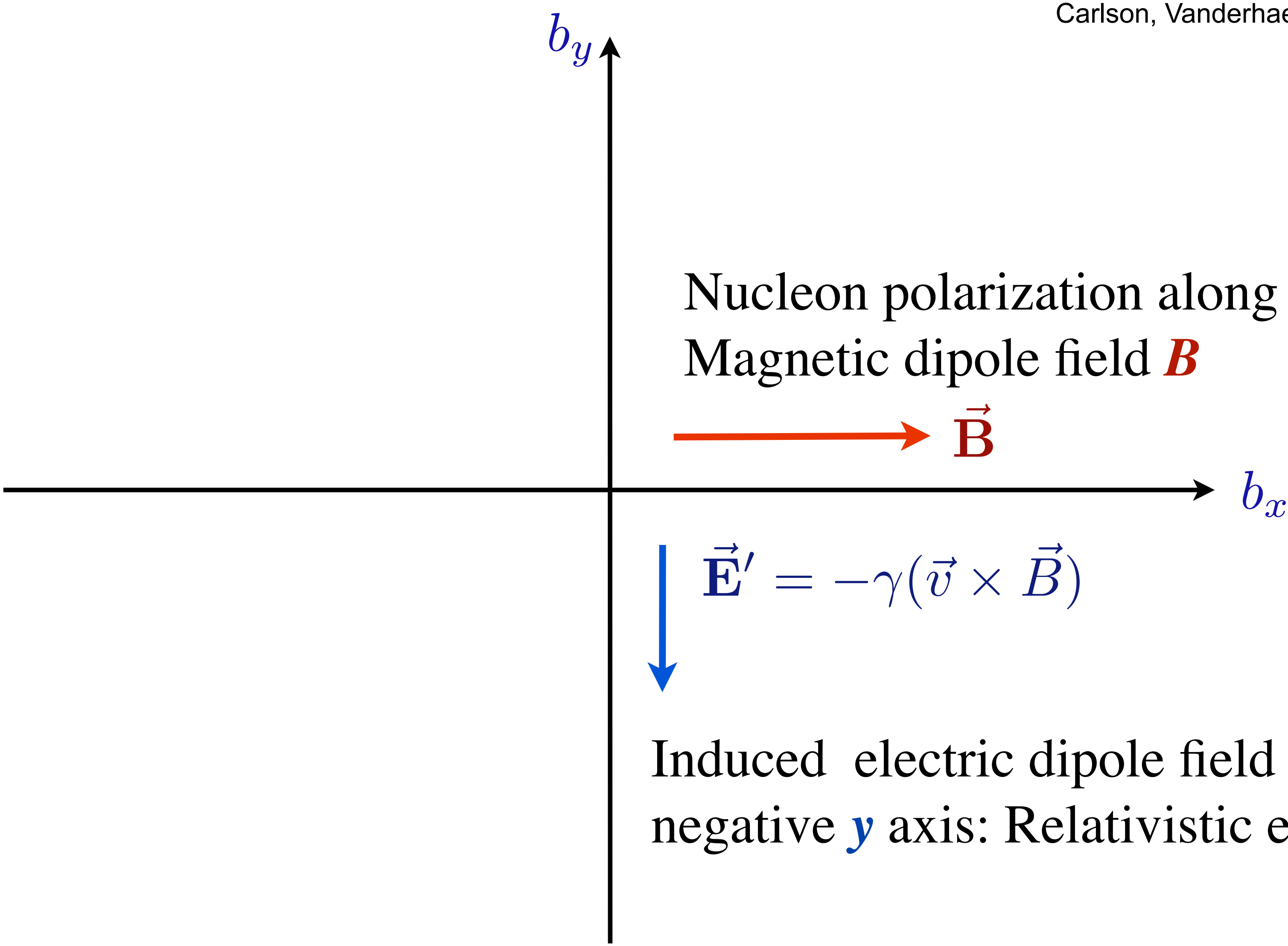


Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

Carlson, Vanderhaeghen, PRL **100**, 032004

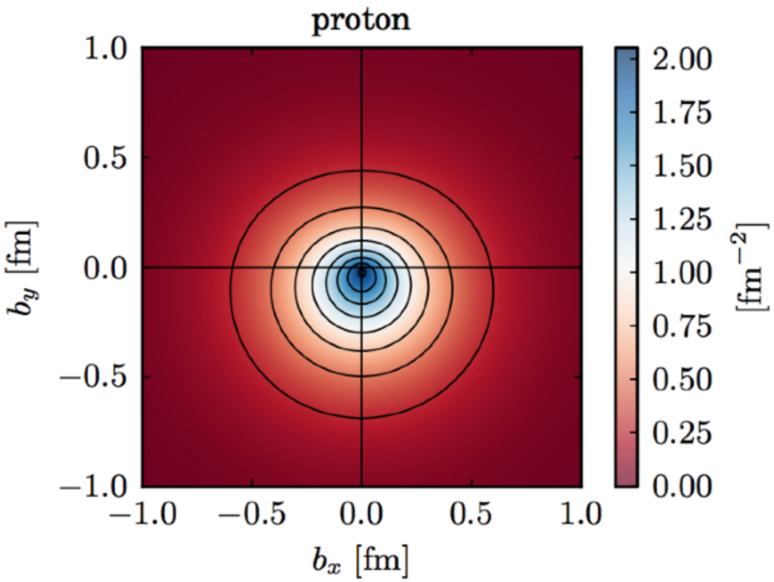


Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities



Carlson, Vanderhaeghen, PRL **100**, 032004

Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

$\vec{\mathrm{B}}$

b_x

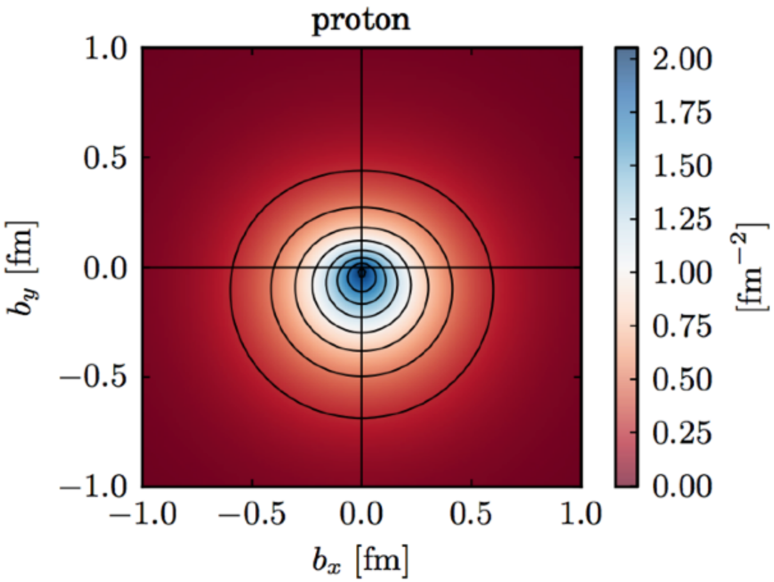
$$\vec{\mathbf{E}}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Silva, Urbano, HChK, PTEP, (2018)

Proton & neutron transverse charge densities

Carlson, Vanderhaeghen, PRL **100**, 032004



Nucleon polarization along the ***x*** axis:
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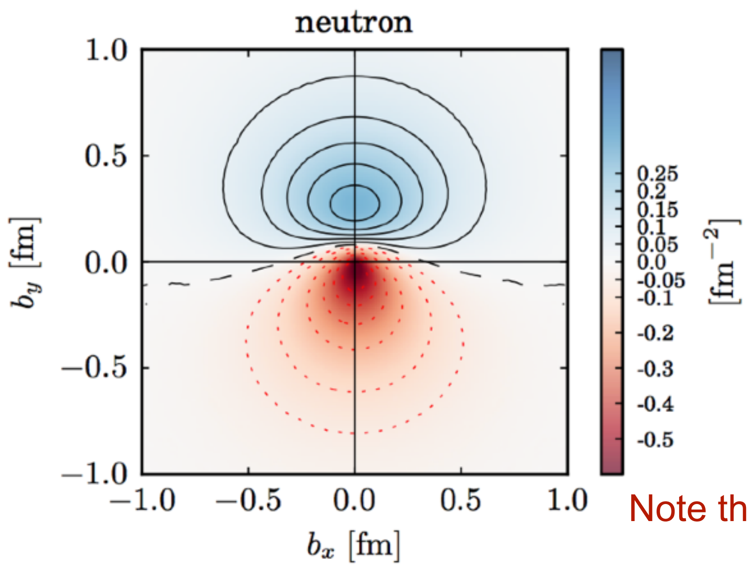
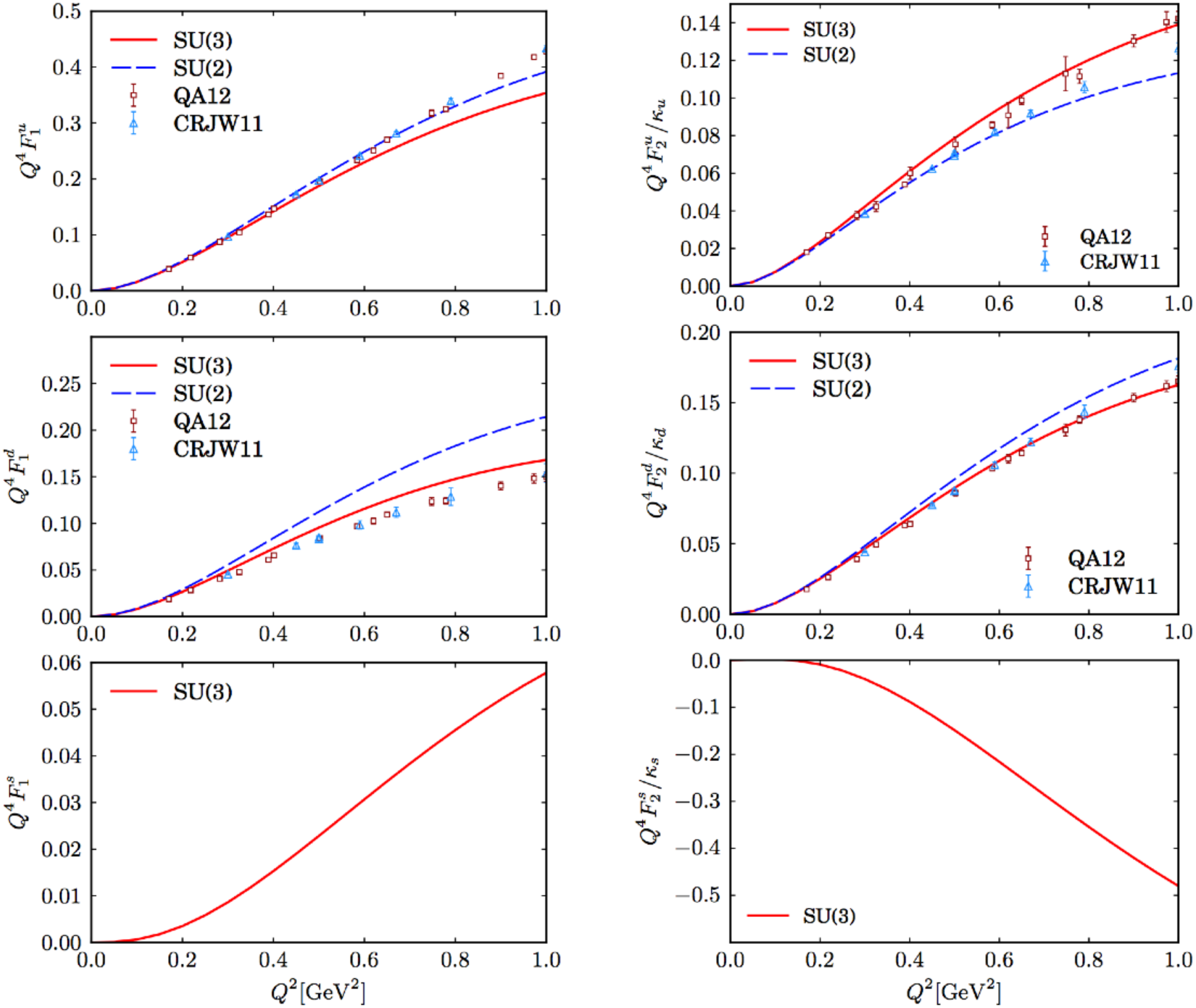


Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Note that the neutron anomalous magnetic moment is negative!

Silva, Urbano, HChK, PTEP, (2018)

Flavor structure



Up quark FFs

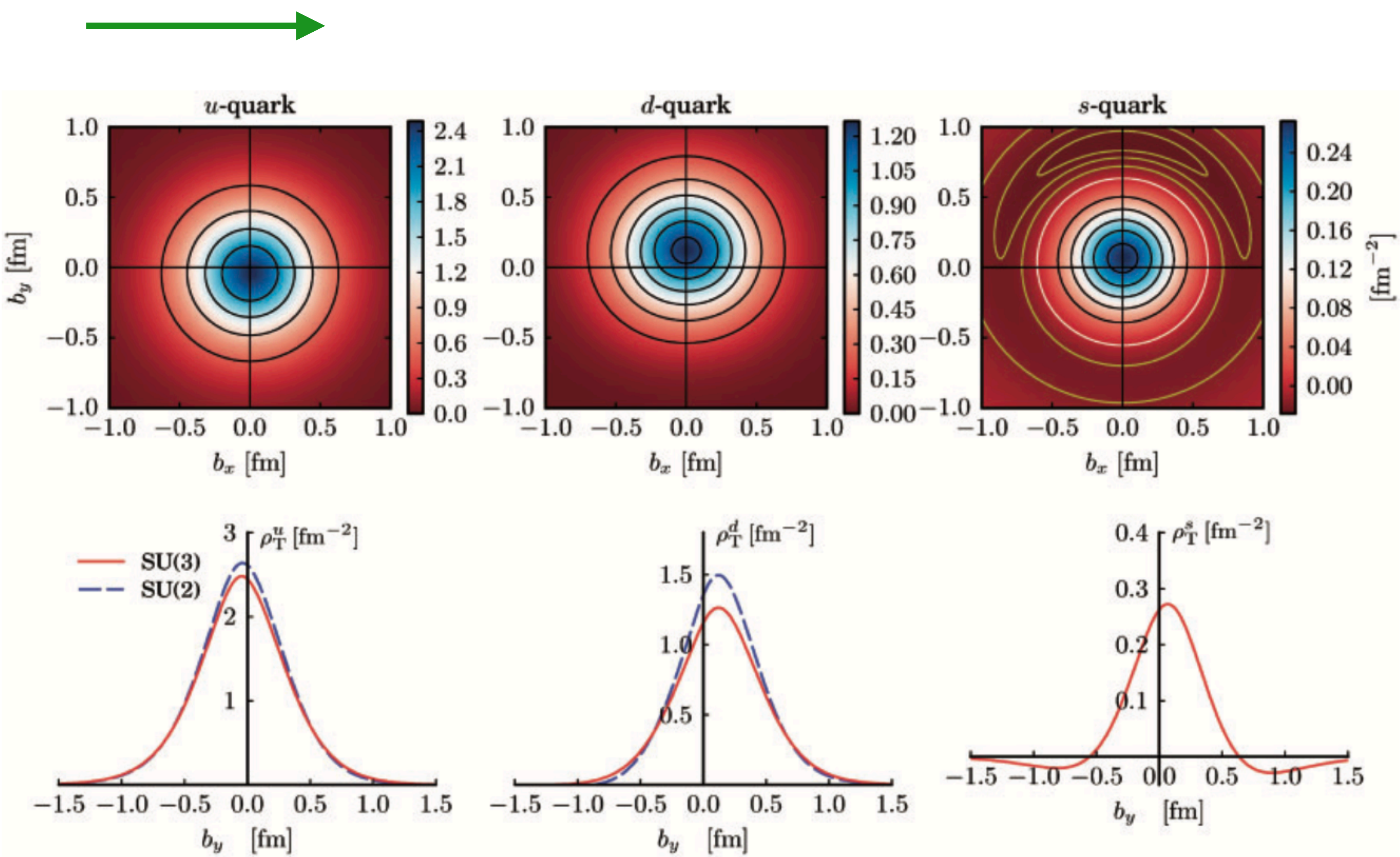
Down quark FFs

Strange quark FFs

Silva, Urbano, HChK, PTEP, (2018)

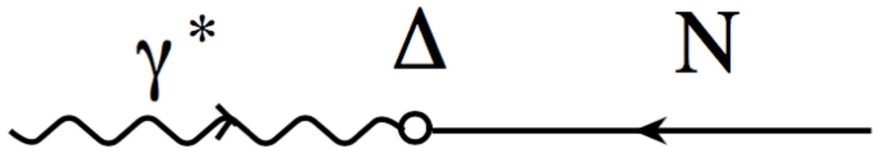
Flavor structure

Nucleon polarized along the x direction



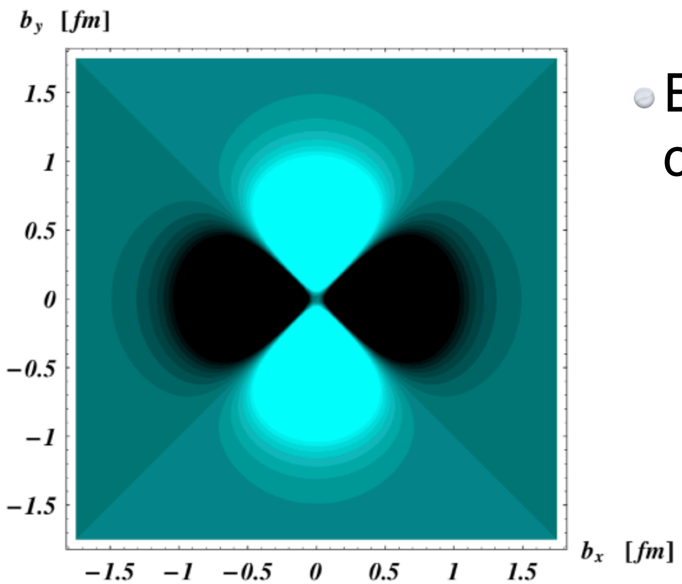
Silva, Urbano, HChK, PTEP, (2018)

EM transition form factors of the decuplet



$(\omega, \boldsymbol{q}) \qquad (E_\Delta, \mathbf{0}) \qquad (E_N, -\boldsymbol{q})$

- EM transition FFs provide information on how the Delta looks like.



- EM transition FFs are related to the VBB coupling constants through VDM & CFI.

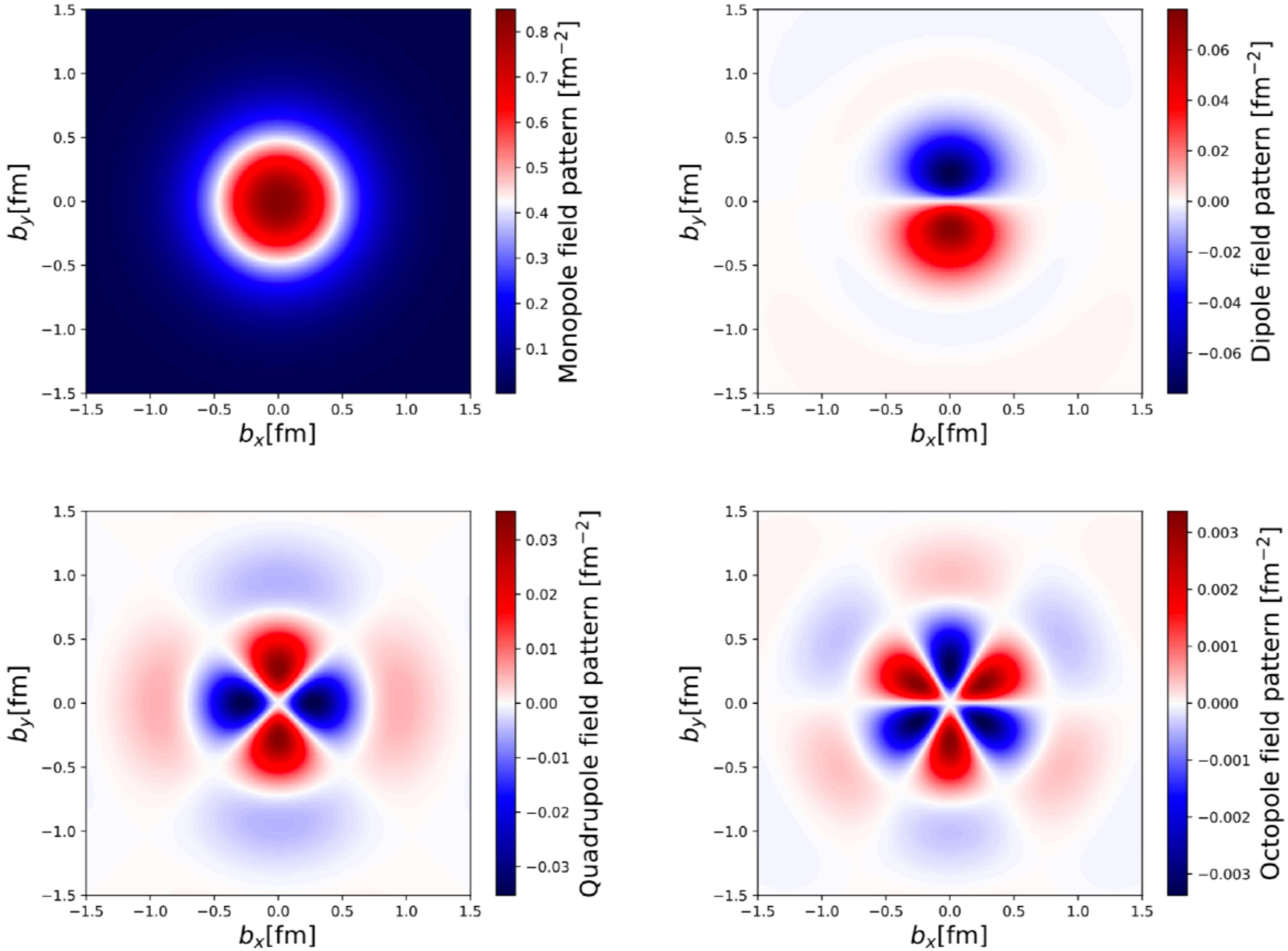
→ Essential to understand a production mechanism of hadrons.

Carlson & Vanderhaeghen, PRD 100 (2008) 032004

Multipole patter in the transverse plane

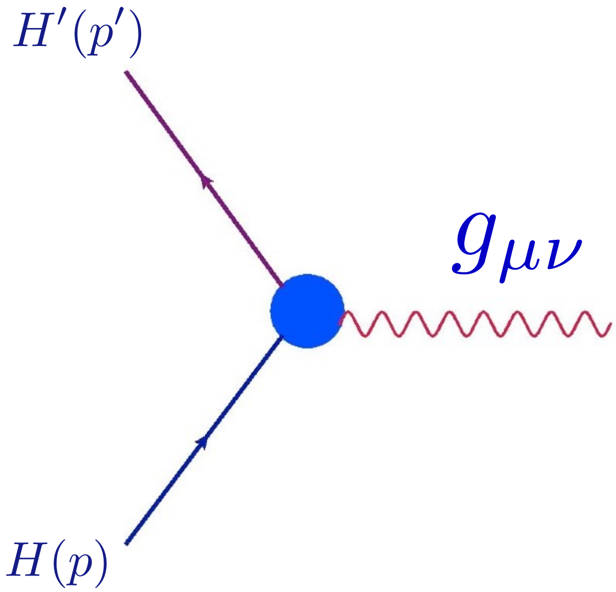
Δ^+

Preliminary results (J.-Y. Kim & HChK)



Gravitational Form factors
of
the Nucleon

Gravitational form factors



Graviton: To weak to probe the EMT structure of a hadron

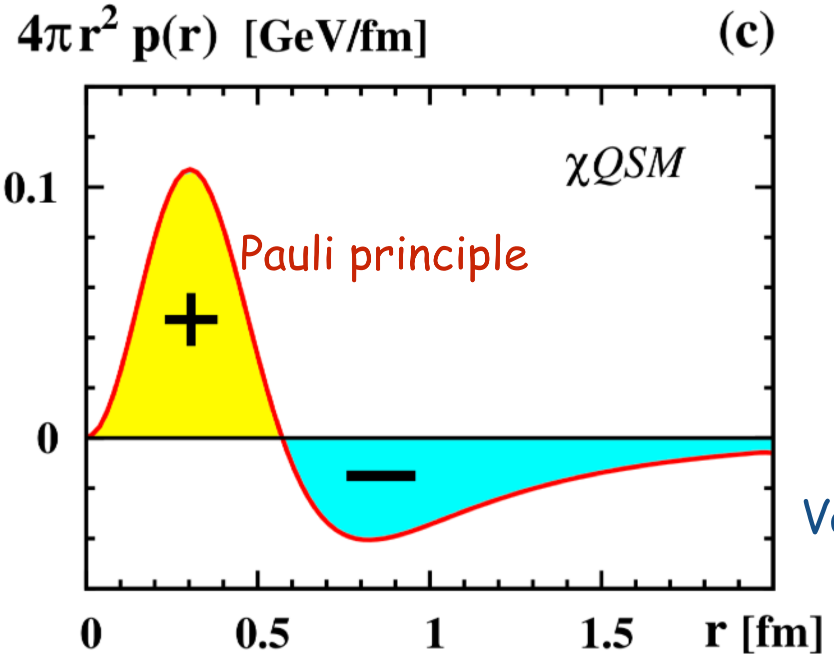
Given an action,

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0$$ or

$\delta S = 0$ under Poincaré transform

Stability

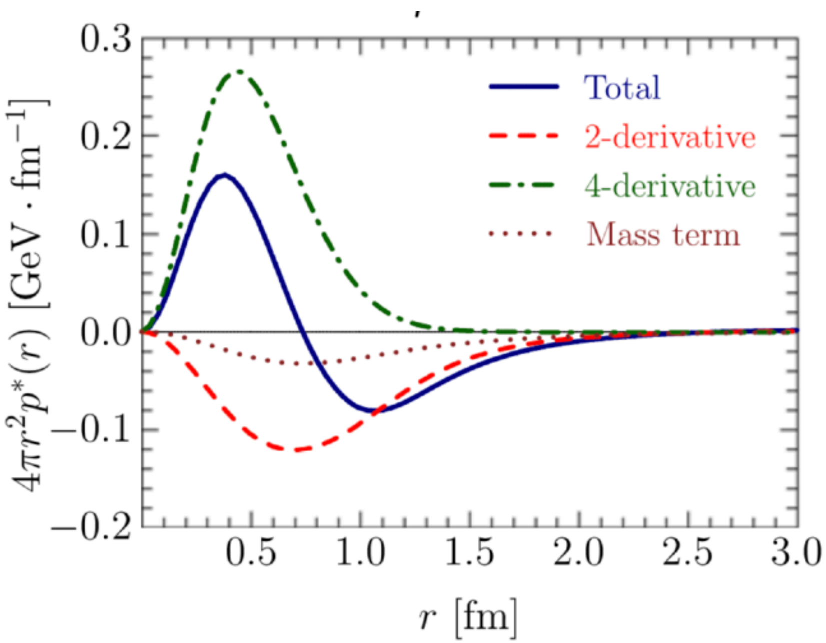
- Nucleon: The stability is guaranteed by the balance between the core valence quarks and the sea quarks (XQSM).

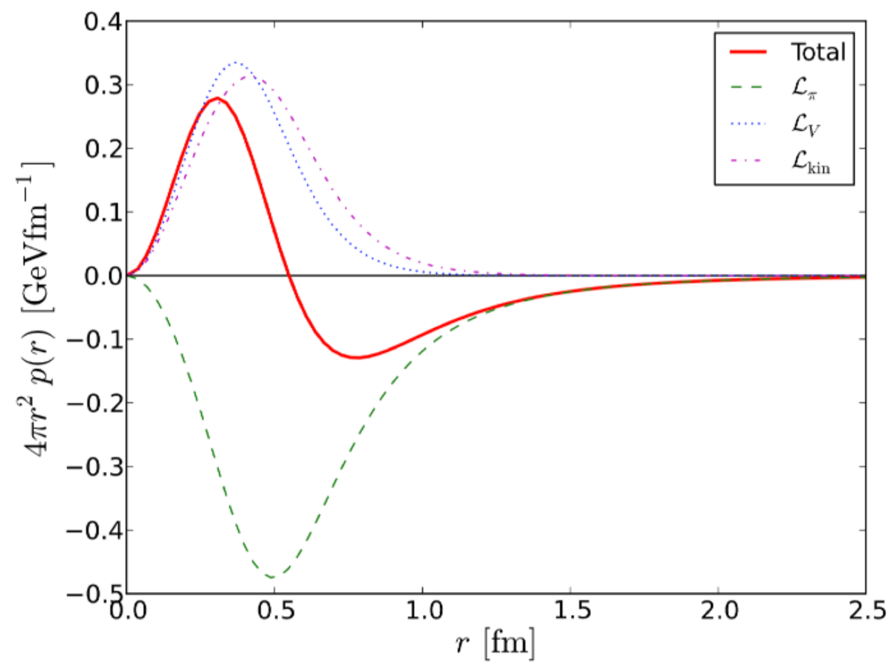


K. Goeke et al., PRD75 (2007) 094021

Stability

- Nucleon: The stability is guaranteed by the balance between the pion and rho mesons (Skyrme picture).





Original Skyrme model

pi-rho-omega model

Cebulla et al., NPA794 (2007) 87

HChK, P. Schweitzer, U. Yakhshiev, PLB 718 (2012) 625

J.H. Jung, U. Yakhshiev, HChK, J.Phys. G41 (2014) 055107

Summary & Outlook

Summary

- In the present talk, we aimed at reviewing a certain aspect on the form factors of the nucleon.
- We discussed first the transverse charge densities and its conceptual difference from the traditional 3D charge densities.
- We briefly discussed the gravitational form factors of the nucleon.

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Thank you very much for the attention!