Color-octet heavy quark potential from the instanton vacuum

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Motivation

- Properties of charmonia are often explained by phenomenological heavy-quark potentials.
- The potentials include the confining potential and Coulomb potential that arises from perturbative one-gluon exchange, nonperturbative effects have been often neglected.
- Recently, the nonperturbative heavy-quark potential has been investigated from the instanton vacuum, motivated by Diakonov et al [1].

[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)

Motivation

- The color-octet potential plays a very important role in examining various properties of the charmonia.

- Moreover, it provides essential information on the heavy pentaquarks that were observed by the LHCb Collaboration.

- In this talk, we will present recent investigation on the color-octet heavy-quark potential.

Color-Octet Static Potential of $Q\bar{Q}$

Potential definition in QFT

From QFT, we can use the gauge invariant form of the definition:

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle\Omega|\mathrm{Tr}_c W_P[A]|\Omega\rangle$$

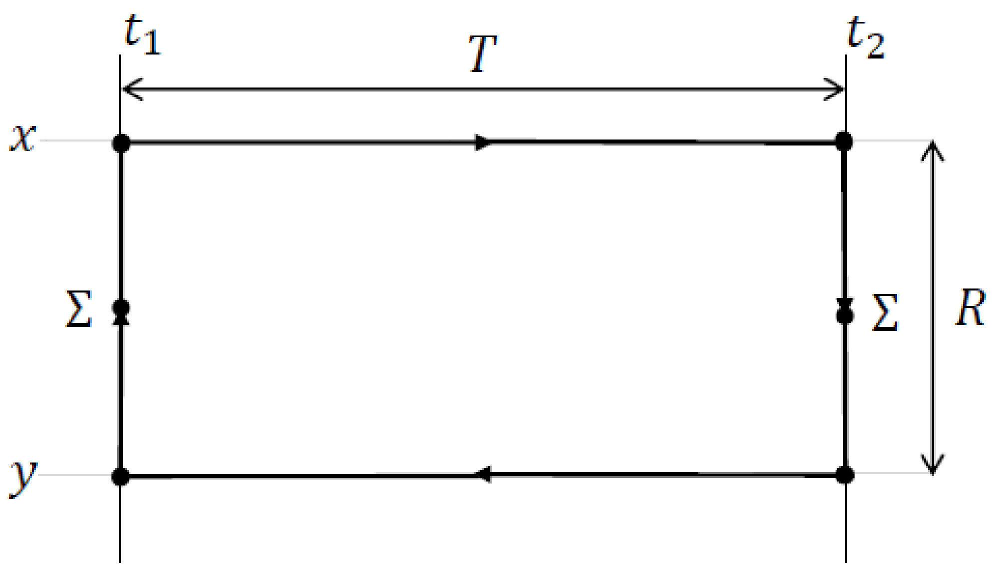
$$W_P = P\exp\left(ig_s \oint_P A^a_\mu T^a dx^\mu\right)$$

where Pexp denotes path-ordering exponential and P denotes the path of the loop.

$$V_\Sigma(R) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle\langle W_C^{(\Sigma)}[A]\rangle\rangle$$

Color-Octet Static Potential of $Q\bar{Q}$

Wilson line is gauge connection between two fields at two different point.



Wilson line $y \to z$:

$$U(x;y,z) \equiv P\exp\left(i\int_y^z dx_\mu A_{I\mu}\right)$$

$$U^\Sigma(x;y,z) \equiv U(x;y,s)\Sigma U(x;s,z),$$

Σ is SU(3) generator for color-exchange point.

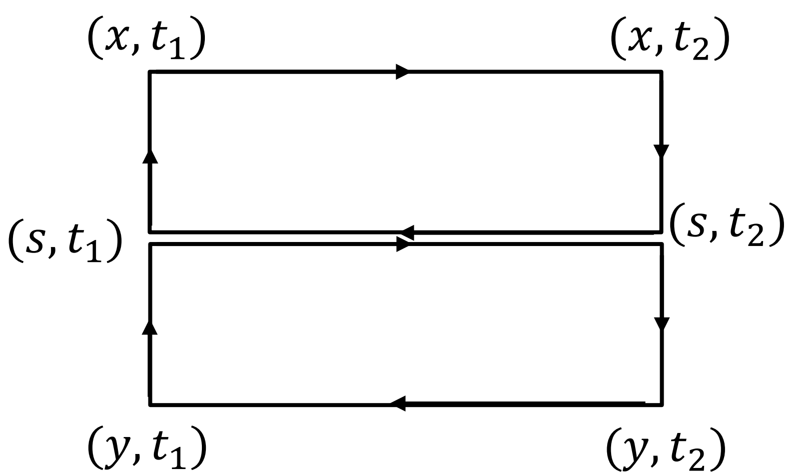
$$W_C^\Sigma[A] = U(\vec{x};t_1,t_2)U^\Sigma(t_2;\vec{x},\vec{y})U(\vec{y};t_2,t_1)U^\Sigma(t_1;\vec{y},\vec{x}),$$

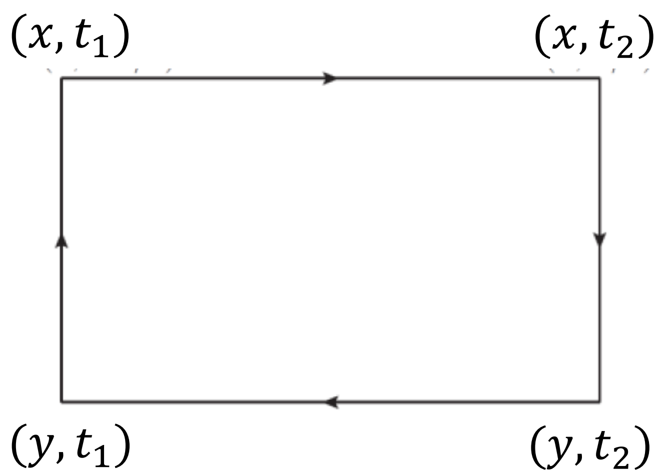
$$W_C^{(8)} = U(\vec{x};t_1,t_2)\frac{\lambda_a}{2}U(\vec{y};t_2,t_1)\frac{\lambda_a}{2},$$

$T \to \infty \gg R,\ \mathrm{m_Q} \to \infty, \mathrm{dx_i} \to 0$ (static state)

Color-Octet Static Potential of $Q\bar{Q}$

$$\langle\langle W_C^{(8)}\rangle\rangle = \frac{N_c^2}{2(N_c^2-1)}\,\langle\langle U(\vec{x};t_1,t_2)U(\vec{s};t_2,t_1)\rangle\,\langle U(\vec{s};t_1,t_2)U(\vec{y};t_2,t_1)\rangle\rangle$$
$$- \frac{1}{2(N_c^2-1)}\,\langle\langle U(\vec{x};t_1,t_2)U(\vec{y};t_2,t_1)\rangle\rangle\,,$$





The color-octet Wilson loop is separated two Wilson loop. First Wilson loop shows the doubled Wilson loop and the second one gives the same as the color-singlet Wilson loop.

Color-Octet Static Potential of $Q\bar{Q}$

- Instanton solution

Instanton is simply called 'classical trajectory', in other words, it is also called 'localized pseudo particle'.

To find the best tunneling trajectory having the largest amplitude one has thus to minimize the YM action

$$S = \frac{1}{4g^2} \int d^4x \; F^a_{\mu\nu} F^a_{\mu\nu} = \frac{8\pi^2}{g^2}.$$

$$F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}.$$

: self-duality condition

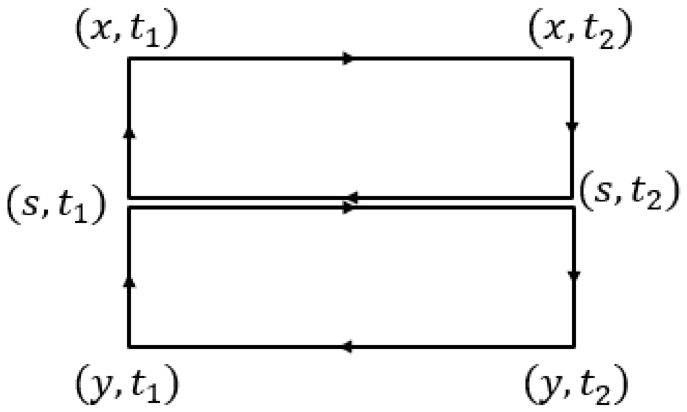
$$A_{I,\mu}(x, z_I) = \frac{\eta^{-a}_{\mu\nu}(x - z_I)_\mu \lambda^a \rho^2}{(x - z_I)^2((x - z_I)^2 + \rho^2)},$$

Color-Octet Static Potential of $Q\bar{Q}$

$$V_8(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle\langle W_C[A]\rangle\rangle$$
$$= \frac{N_c^2 G_2(r) - g_1(r)}{N_c^2 - 1} + \mathcal{O}\left(\left(\frac{N}{2VN_c}\right)^2\right)$$

where $G_2(r)$ is averaged value of g_2, respectively.

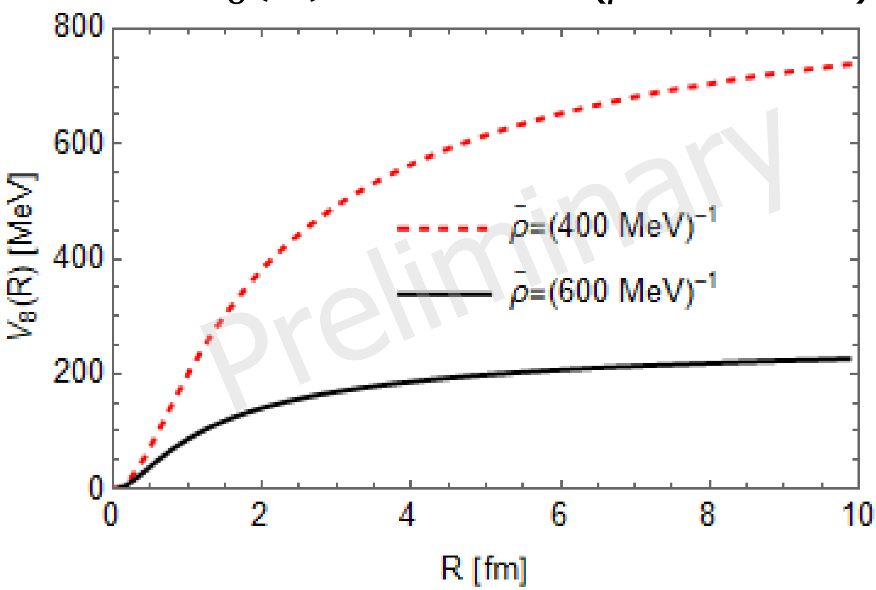
$$G_2(r) = \frac{1}{r}\int_0^r ds g_2(r, s)$$

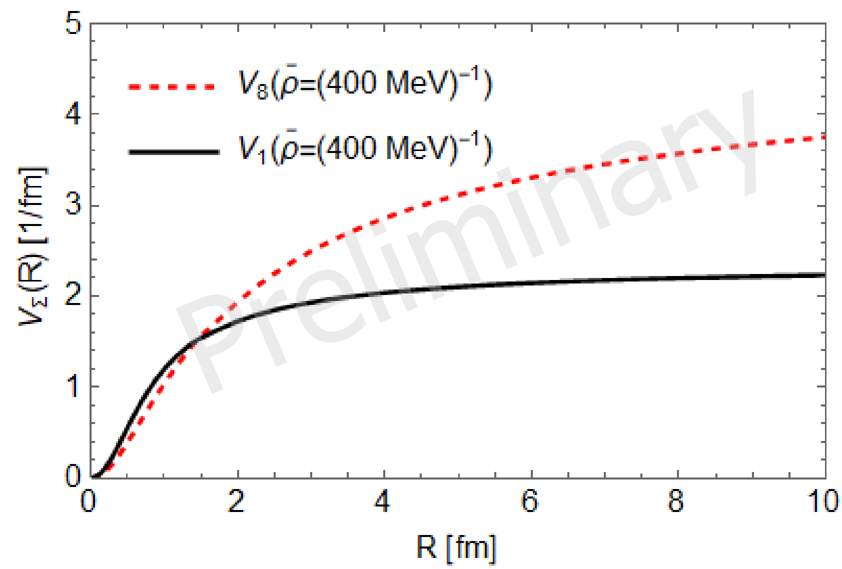


Results

$V_1(\infty) \simeq 140$ MeV ($\bar{\rho} \simeq 0.33$ fm) $V_1 \simeq 460$ MeV ($\bar{\rho} \simeq 0.48$ fm)

$V_8(\infty) \simeq 270$ MeV ($\bar{\rho} \simeq 0.33$ fm) $V_8 \simeq 923$ MeV ($\bar{\rho} \simeq 0.48$ fm)

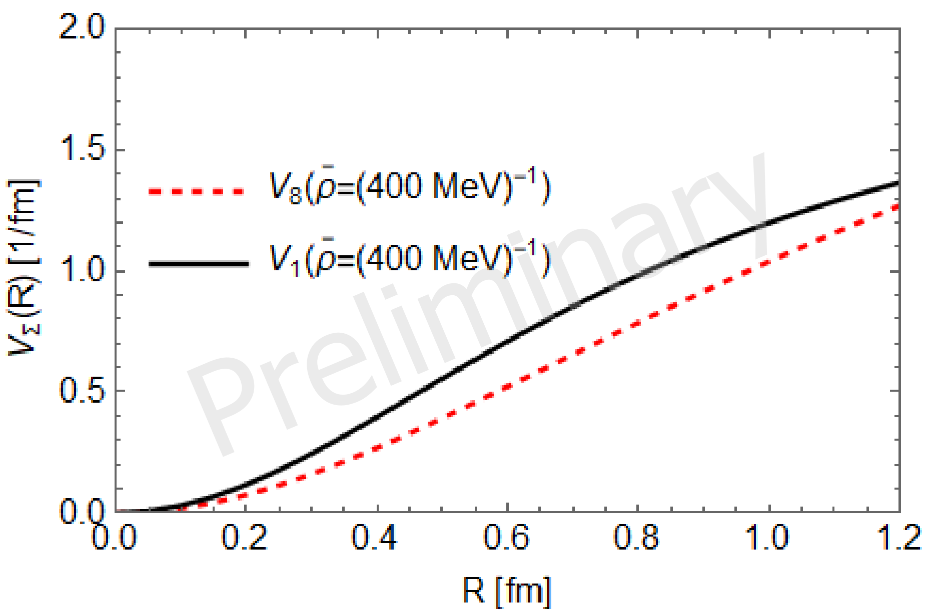


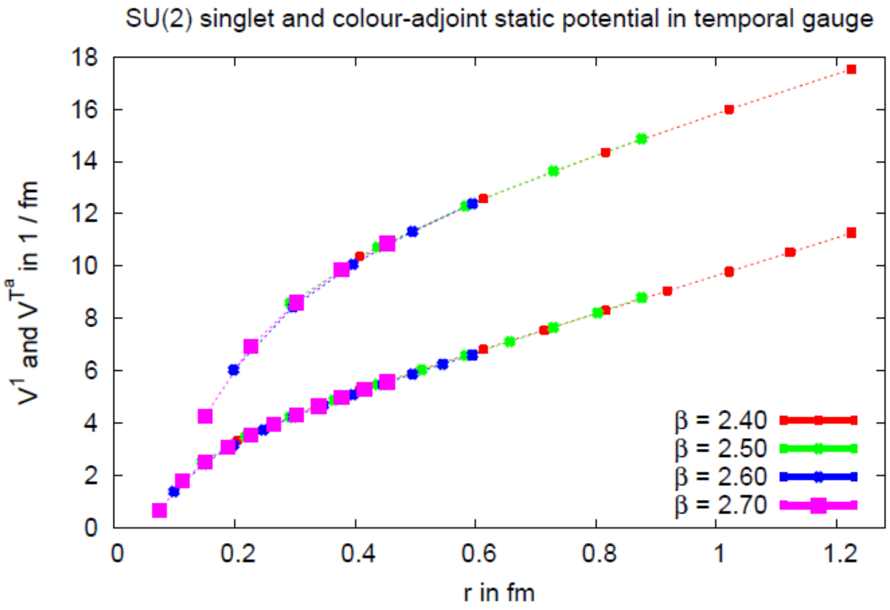


Comparison color-octet potential according to the average instanton size for $\bar{\rho} = 0.33$ fm and $\bar{\rho} = (400\ MeV)^{-1} = 0.48$ fm.

Graph of comparing between color-singlet and color-octet potential for average instanton size $\bar{\rho} = (400\ MeV)^{-1} = 0.48$ fm.

Results





Instanton effects contribute to the lattice result about 6 % ~ 12 %.

Lattice result from reference [2], V_1(lower curve) and V_8(upper curve).

[2] Philipsen et al, Phys. Rev. D 89, no. 1,014509 (2014)

Summary & Outlooks

Summary

- We got the color-octet potential using the Wilson loop based on instanton vacuum model.
- Instanton size affects the potential. If the instanton size is increased then potential is also increased.
- Instanton effects contribute to the lattice result about 6 % ~ 12 %.

Outlooks

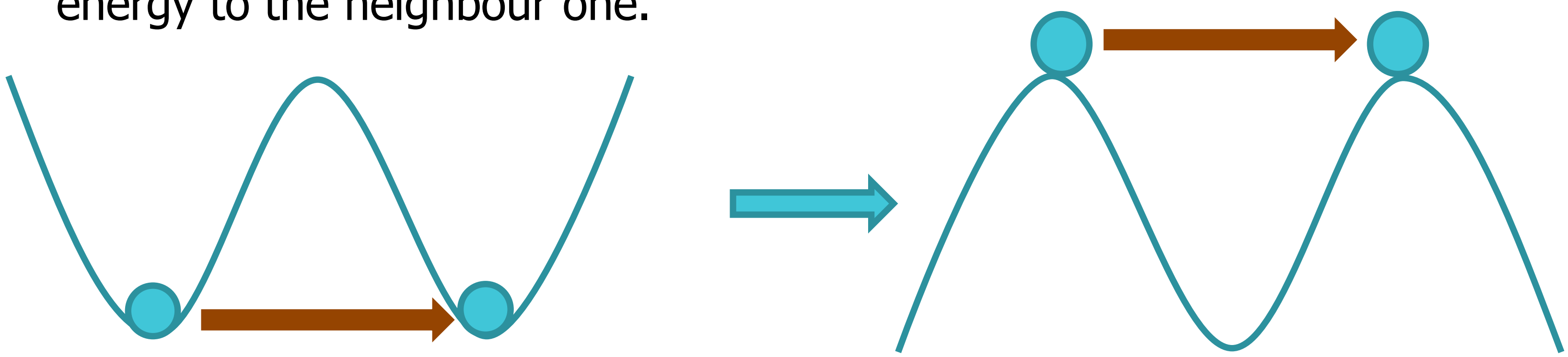
- We will use the color-octet potential to derive the chromoelectric polarizability of the heavy quarkonia, which can be used to describe the heavy pentaquarks, P_c's.

Thank you for your attention!

Back Up

Introduction

- Instanton is a large fluctuation of the gluon field in Euclidean time corresponding to quantum tunneling from one minimum of the potential energy to the neighbour one.



Minkowski space :
quantum tunneling

Euclidean space :
Classical tragectory

Color-Octet Static Potential of $Q\bar{Q}$ system

From lattice QCD, the corresponding potentials are represented by exponential decay of the Wilson loop

$$\langle\langle W_C^{\Sigma}(r,T)\rangle\rangle = \sum_{n=0}^{\infty} c_n \exp\left(-V_n^{\Sigma}(r)T\right) \underset{T\to\infty}{\propto} \exp\left(-V^{\Sigma}(r)T\right).$$

Color-orientation integration:

$$\int dU U_{a_1 i_1} U^{\dagger}_{j_1 b_1} U_{a_2 i_2} U^{\dagger}_{j_2 b_2}$$

$$= \frac{1}{N_c^2}\delta_{a_1 b_1}\delta_{i_1 j_1}\delta_{a_2 b_2}\delta_{i_2 j_2} + \frac{1}{4(N_c^2-1)}\lambda^i_{a_1 b_1}\lambda^i_{a_2 b_2}\lambda^j_{j_1 i_1}\lambda^j_{j_2 i_2},$$

$$\lambda^i_{a_1 b_1}\lambda^i_{a_2 b_2} = 2\left(\delta_{a_1 b_2}\delta_{a_2 b_1} - \frac{1}{N_c}\delta_{a_1 b_1}\delta_{a_2 b_2}\right)$$

$$\lambda^j_{j_1 i_1}\lambda^j_{j_2 i_2} = 2\left(\delta_{j_1 i_2}\delta_{j_2 i_1} - \frac{1}{N_c}\delta_{j_1 i_1}\delta_{j_2 i_2}\right).$$

Color-Octet Static Potential of $Q\bar{Q}$ system

$$g_8(r,s) = \frac{2\pi N\rho^3}{VN_c}\int_0^\infty dx \int_{-1}^{1} dt x^2 \left\{ \mathrm{Tr}_c f_1(x,r,s,t) + \mathrm{Tr}_c f_2(x,r,s,t) - \frac{1}{N_c}\mathrm{Tr}_c f_1(x,r,s,t)\mathrm{Tr}_c f_2(x,r,s,t)\right\}$$

$$g_1(r,s) = \frac{2\pi N\rho^3}{VN_c}\int_0^\infty dx \int_{-1}^{1} dt x^2 \mathrm{Tr}_c f_0(x,r,s,t),$$

$$\begin{aligned} f_0(x,r,s,t) = P_T \Bigg[1 &- \cos\left(\frac{\pi x}{\sqrt{x^2+1}}\right)\cos\left(\frac{\pi\sqrt{x^2+r^2+2xrt}}{\sqrt{x^2+r^2+2xrt+1}}\right) \\ &- \frac{x+rt}{\sqrt{x^2+r^2+2xrt}}\sin\left(\frac{\pi x}{\sqrt{x^2+1}}\right)\sin\left(\frac{\pi\sqrt{x^2+r^2+2xrt}}{\sqrt{x^2+r^2+2xrt+1}}\right)\Bigg] \\ f_1(x,r,s,t) = P_T \Bigg[1 &- \cos\left(\frac{\pi x}{\sqrt{x^2+1}}\right)\cos\left(\frac{\pi\sqrt{x^2+s^2+2xst}}{\sqrt{x^2+s^2+2xst+1}}\right) \\ &- \frac{x+st}{\sqrt{x^2+s^2+2xst}}\sin\left(\frac{\pi x}{\sqrt{x^2+1}}\right)\sin\left(\frac{\pi\sqrt{x^2+s^2+2xst}}{\sqrt{x^2+s^2+2xst+1}}\right)\Bigg] \\ f_2(x,r,s,t) = P_T \Bigg[1 &- \cos\left(\frac{\pi\sqrt{x^2+s^2+2xst}}{\sqrt{x^2+s^2+2xst+1}}\right)\cos\left(\frac{\pi\sqrt{x^2+r^2+2xst}}{\sqrt{x^2+r^2+2xrt+1}}\right) \\ &- \frac{x^2+xst+xrt+rs}{\sqrt{x^2+r^2+2xrt}\sqrt{x^2+s^2+2xst}} \\ &\times \sin\left(\frac{\pi\sqrt{x^2+s^2+2xst}}{\sqrt{x^2+s^2+2xst+1}}\right)\sin\left(\frac{\pi\sqrt{x^2+r^2+2xrt}}{\sqrt{x^2+r^2+2xrt+1}}\right)\Bigg] \end{aligned}$$

$$\therefore\ P_T = \frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Introduction

To investigate the heavy pentaquark systems P_c, we need to describe the effective Hamiltonian for interaction of $Q\bar{Q}$ pair with other hadrons.

$$H_{eff} = -\frac{1}{2}\beta\vec{E}^a(x)\cdot\vec{E}^a(x),$$

where β is chromoelectric polarizability of heavy quarkonium system, which can be written as

$$\beta = -\frac{1}{3N_c}\langle\phi|\vec{r}\frac{1}{E_\phi - h_o}\vec{r}|\phi\rangle$$

$|\phi\rangle$: 1S coulombic state, h_o: color-octet Hamiltonian

$$h_o = \frac{\vec{p}^2}{m_Q} + V_o = \frac{\vec{p}^2}{m_Q} + V_8 + \frac{\alpha_s}{6r}$$

Since gluons carry color charge, the intermediate states must be color-octet state.