Axial-vector form factors of the baryon decuplet

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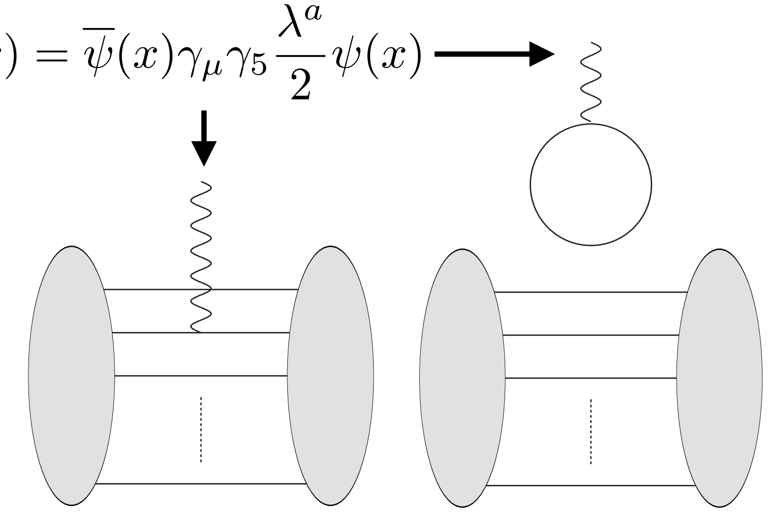
Contents

Introduction

- The axial-vector current examines various sides of baryon properties (e.g. spin content, decay width, etc)
- The axial-vector structures of the baryon decuplet are not well known.
- Mass difference of light baryons, electromagnetic properties etc were described well in the chiral quark-soliton model.
- We would like to explain the axial-vector form factors of the baryon decuplet within the chiral quark-soliton model.

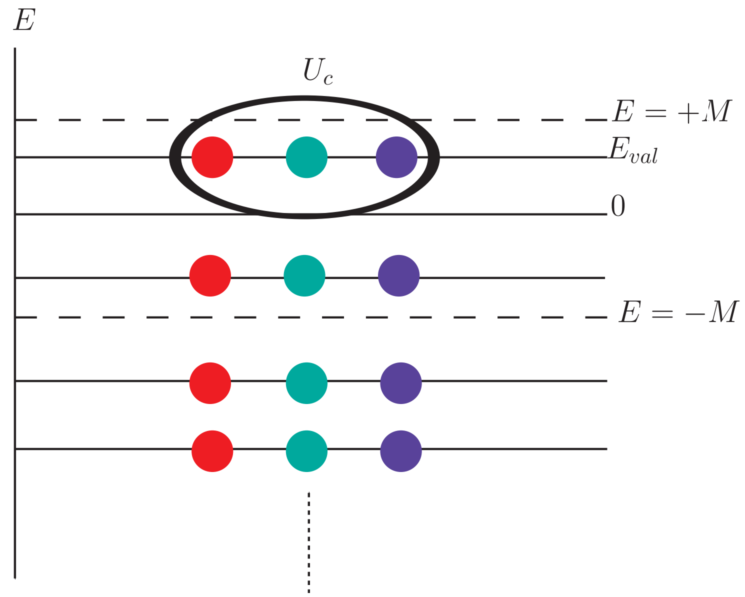
$$A^a_\mu(x) = \overline{\psi}(x)\gamma_\mu\gamma_5\frac{\lambda^a}{2}\psi(x)$$

Valence quark contribution



Sea quark contribution

Framework



- Baryons can be considered as Nc valence quarks, which are bound by the mesonic mean-fields at large Nc (E. Witten, NPB160, 57 (1979)).

Effective chiral action: $$S_{\rm eff} = -N_c \mathrm{Tr}\ln\left[i\gamma^\mu\partial_\mu + i\hat{m} + iMU^{\gamma_5}\right]$$

- The Hedgehog ansatz is applied to the pseudo-Nambu-Goldston boson field.

$$U_{SU(2)}(r) = e^{i\mathbf{n}\cdot\boldsymbol{\tau}P(r)/f_\pi}$$

- The mean-field can be found by solving the equations of motion self-consistently.

$$\frac{\delta E_{cl}}{\delta U} = 0 \longrightarrow E_{cl} \rightleftarrows P(r)$$

- Witten's trivial embedding is used to preserve the hedgehog symmetry in the flavor SU(3) (E. Witten, NPB223, 422 (1983)).

$$U(r) = \begin{pmatrix} U_{SU(2)}(r) & 0 \\ 0 & 1 \end{pmatrix}.$$

- We take into account the zero-mode quantization of the soliton (Rotational and translational zero modes).

$$U(\boldsymbol{x},t) = A(t)U_c(\boldsymbol{x} - \boldsymbol{Z}(t))A^\dagger(t)$$

Framework

$$\langle B'^{(10)}_{J'}(p')\,|A^{\mu}_{a}(0)|\,B^{(10)}_{J}(p)\rangle$$

$$-\,\overline{u}^{\alpha}(p',J')\left[\gamma_{\mu}\left\{g_1^{(a)B}(q^2)\eta_{\alpha\beta}+h_1^{(a)B}(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right.\\ \left.+\frac{q_\mu}{2M_B}\left\{g_3^{(a)B}(q^2)\eta_{\alpha\beta}+h_3^{(a)B}(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right]\gamma^5 u^{\beta}(p,J)$$

$$\int dA\int d^3z\, e^{i\vec{q}\cdot\vec{z}}\langle B'^{(10)}_{J'}|A\rangle\mathcal{F}^a_\mu(\vec{z},A)\langle A|B^{(10)}_J\rangle$$

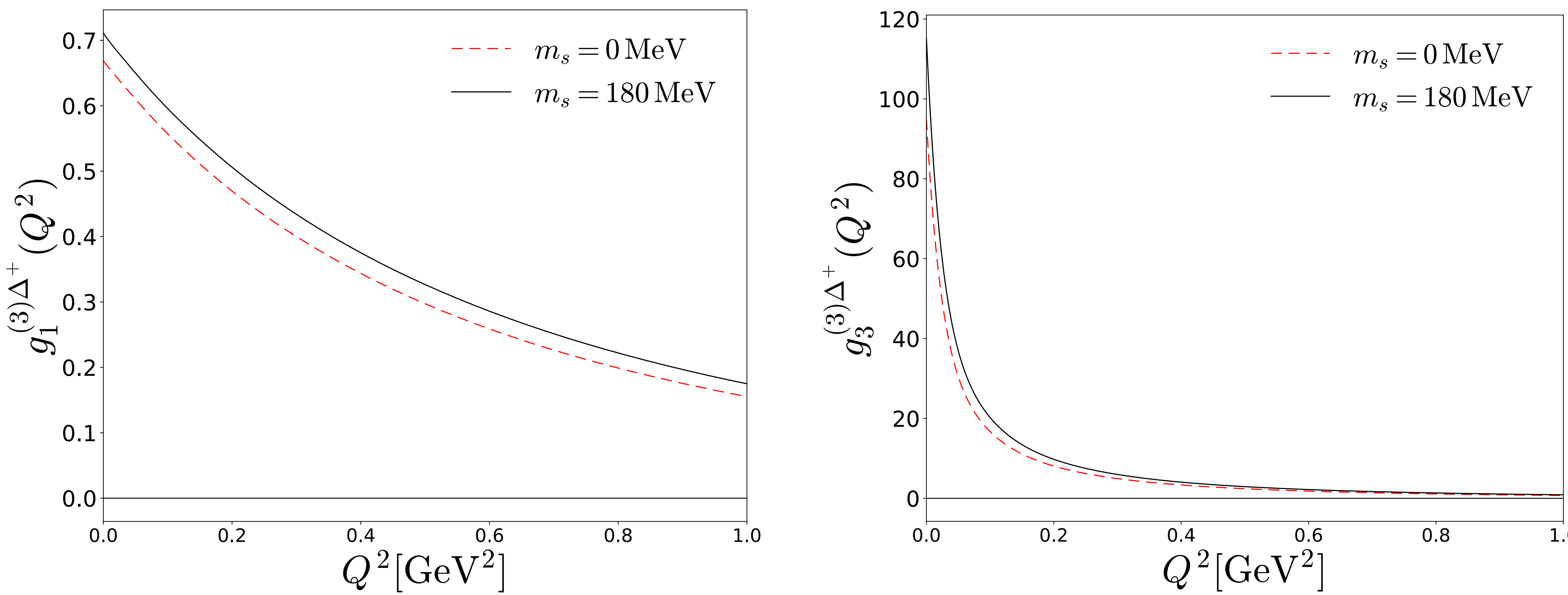
$$g_1^{(a)B}(Q^2)=\frac{M_B}{E}\int d^3r\langle B(p',\frac{3}{2})|\left[j_0(Q|\mathbf{r}|)\{\hat{\mathcal{F}}^a_A(\mathbf{r})\}_{10}-j_2(Q|\mathbf{r}|)\left\{\sqrt{2\pi}Y_2\otimes\hat{\mathcal{F}}^a_A(\mathbf{r})\right\}_{10}\right]|B(p,\frac{3}{2})\rangle$$

$$g_3^{(a)B}(Q^2)=-\frac{4M_B^2}{EQ^2}\int d^3r\langle B(p',\frac{3}{2})|\left[(E-M_B)j_0(Q|\mathbf{r}|)\{\hat{\mathcal{F}}^a_A(\mathbf{r})\}_{10}\right.\\ \left.+(2E+M_B)j_2(Q|\mathbf{r}|)\left\{\sqrt{2\pi}Y_2\otimes\hat{\mathcal{F}}^a_A(\mathbf{r})\right\}_{10}\right]|B(p,\frac{3}{2})\rangle$$

The form factor $h_{1,3}^{(a)B}(q^2)$ are in fact the same as $g_{1,3}^{(a)B}(q^2)$ apart from the kinematical factors

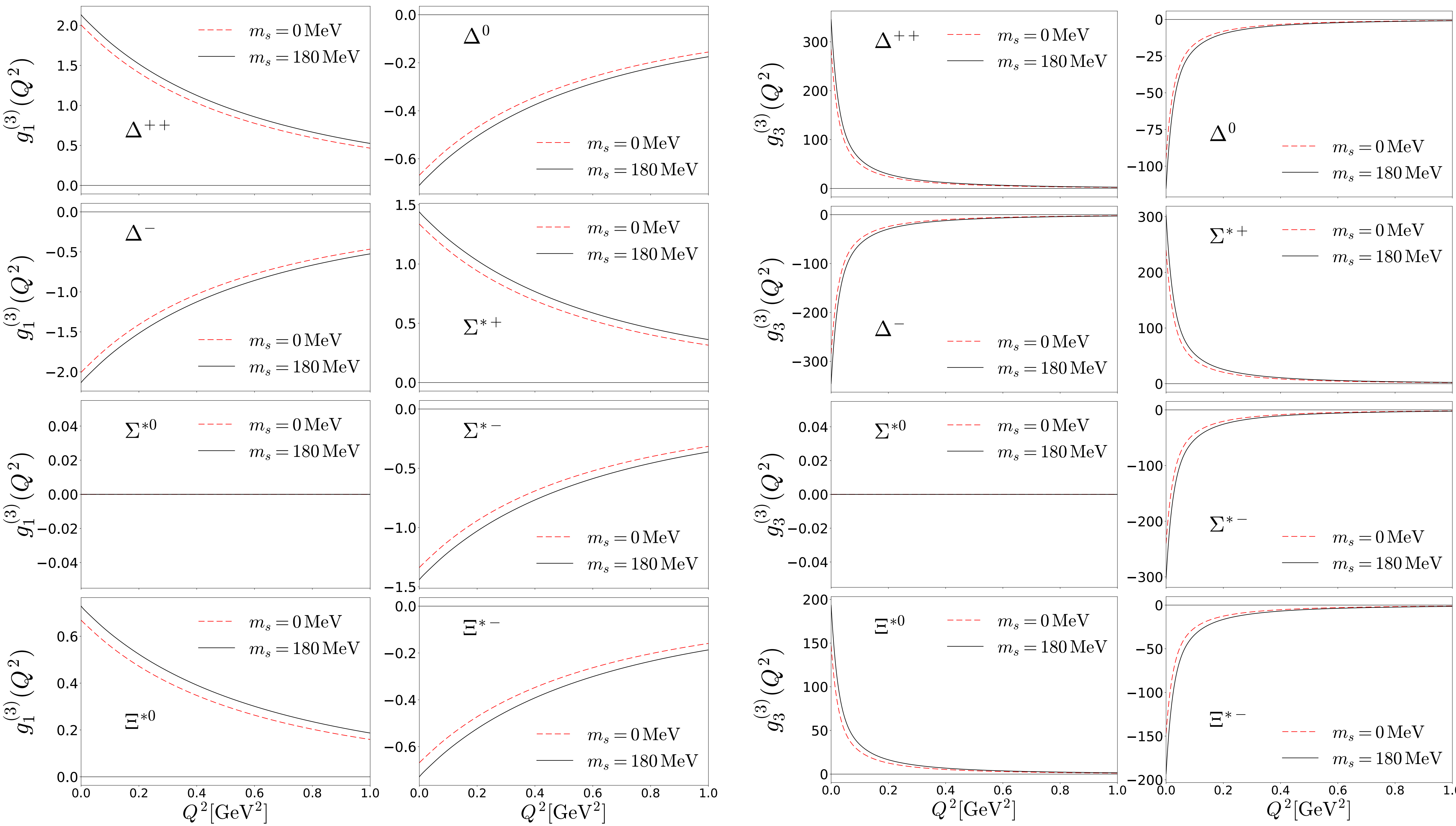
$$g_{1(3)}^{(a)B}(Q^2)=(g_{1(3)}^{(a)B}(Q^2))^{(\mathrm{sym})}+\underbrace{(g_{1(3)}^{(a)B}(Q^2))^{(\mathrm{op})}+(g_{1(3)}^{(a)B}(Q^2))^{(\mathrm{wf})}}_{\text{The flavor SU(3) symmetry breaking contributions}}$$

Axial-vector form factors for the baryon decuplet(Triplet)



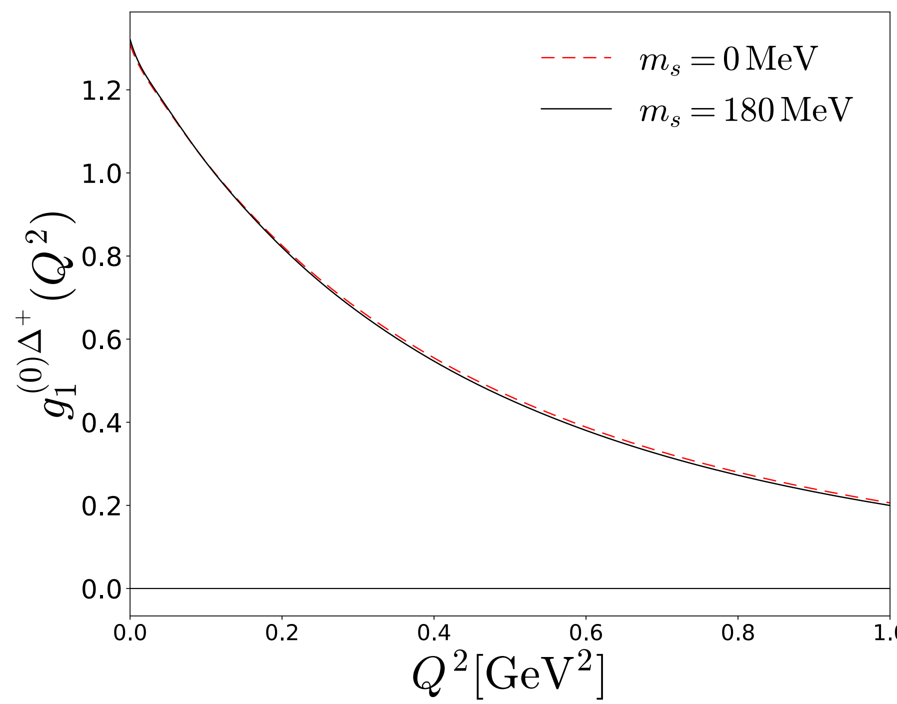
The triplet axial-vector form factors, a=3

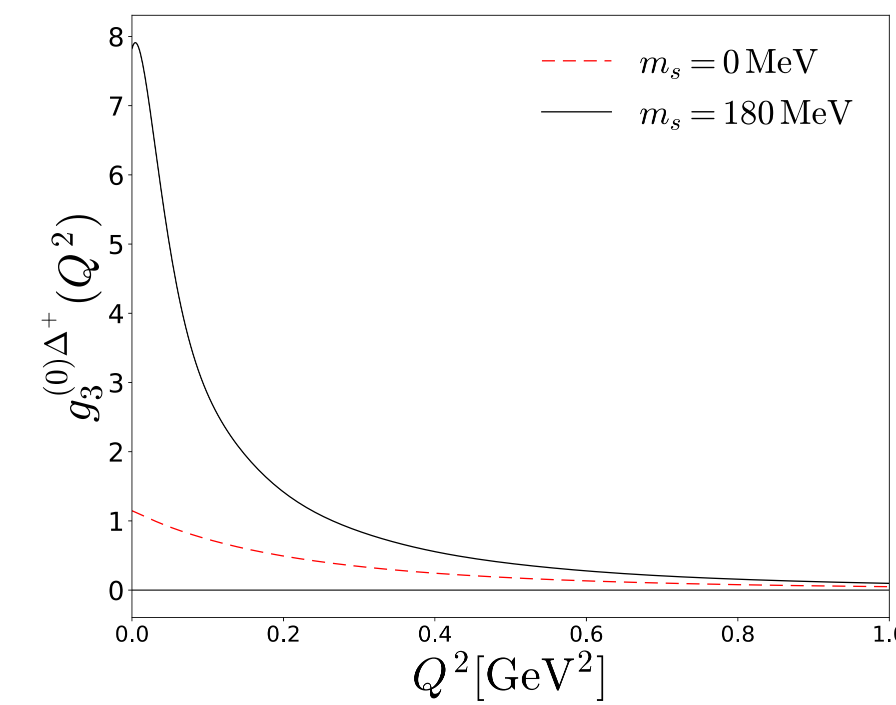
Axial-vector form factors for the baryon decuplet(Triplet)



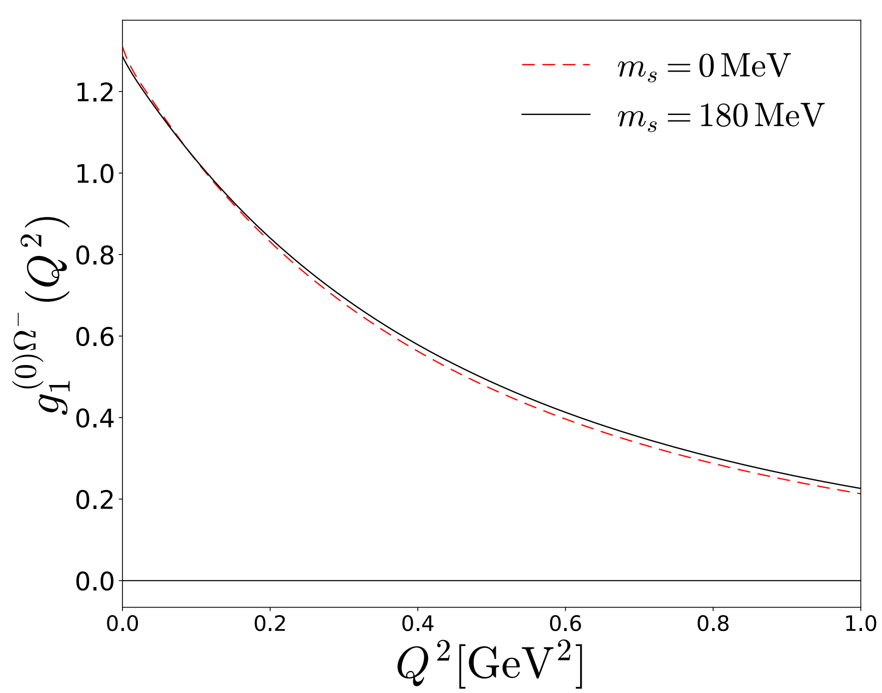
Axial-vector form factors for the baryon decuplet(Singlet)

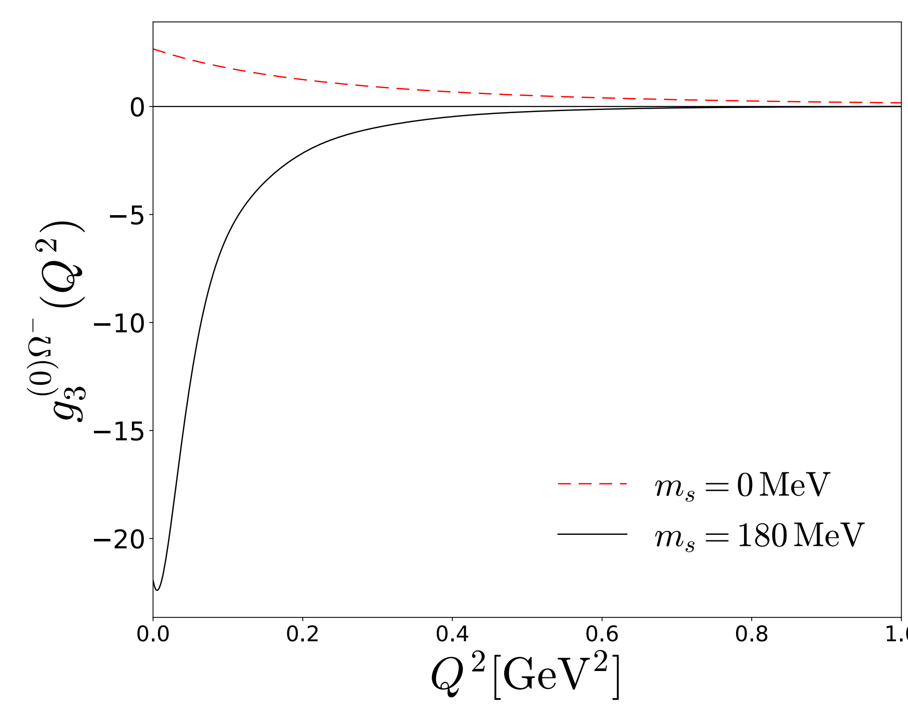
Δ^{+}





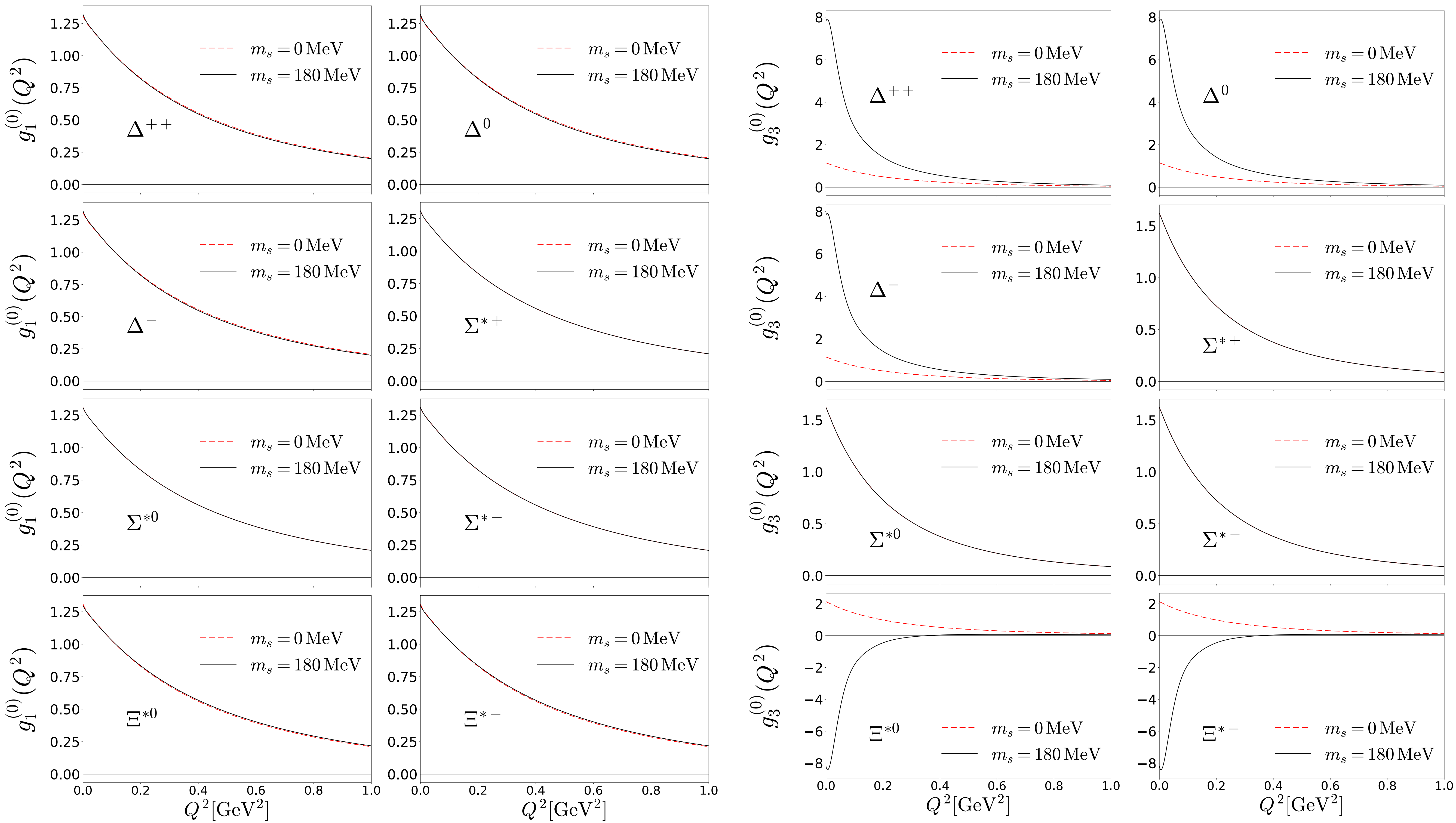
Ω^{-}





The singlet axial-vector form factors, a=0

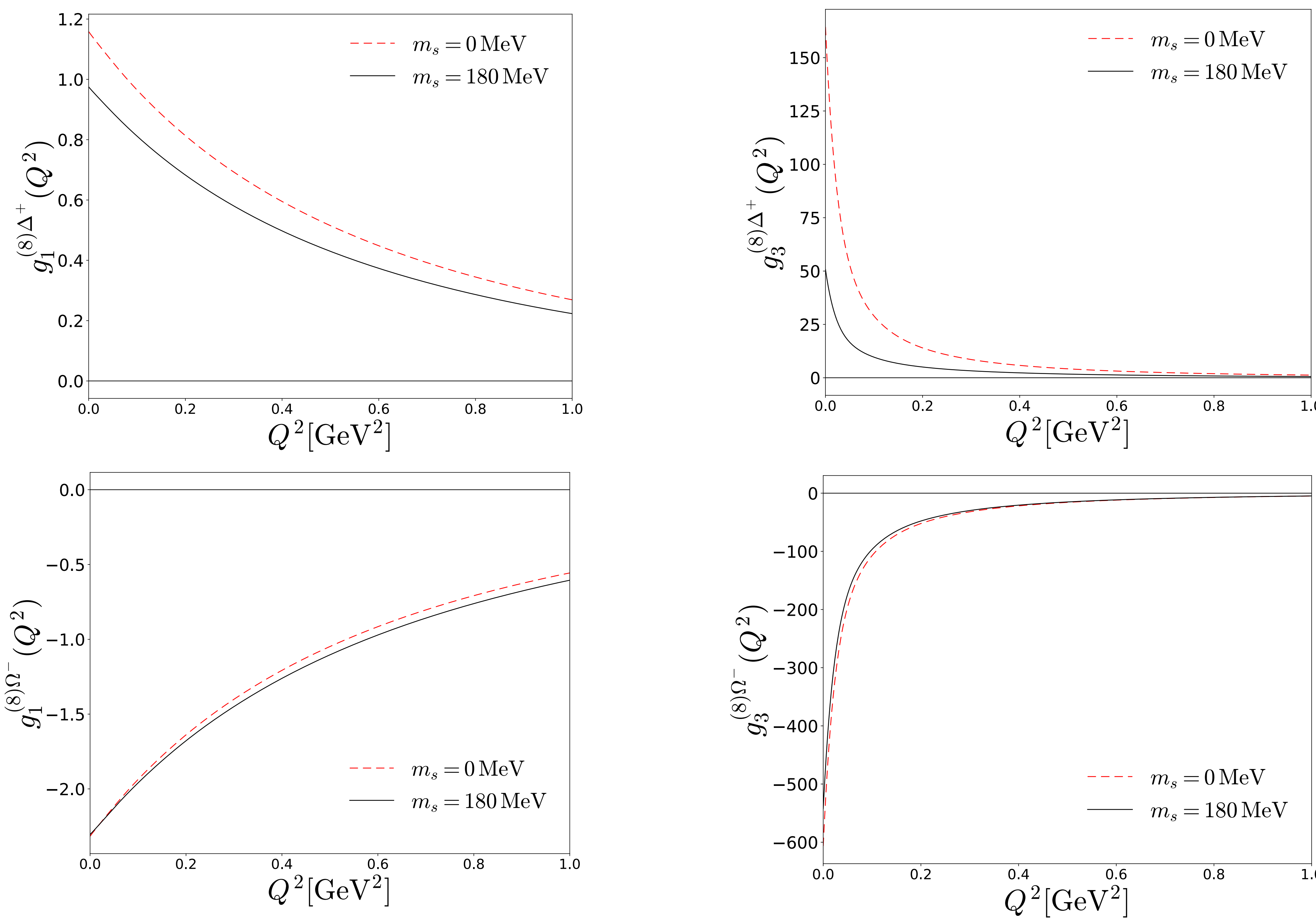
Axial-vector form factors for the baryon decuplet(Singlet)



Axial-vector form factors for the baryon decuplet(Octet)

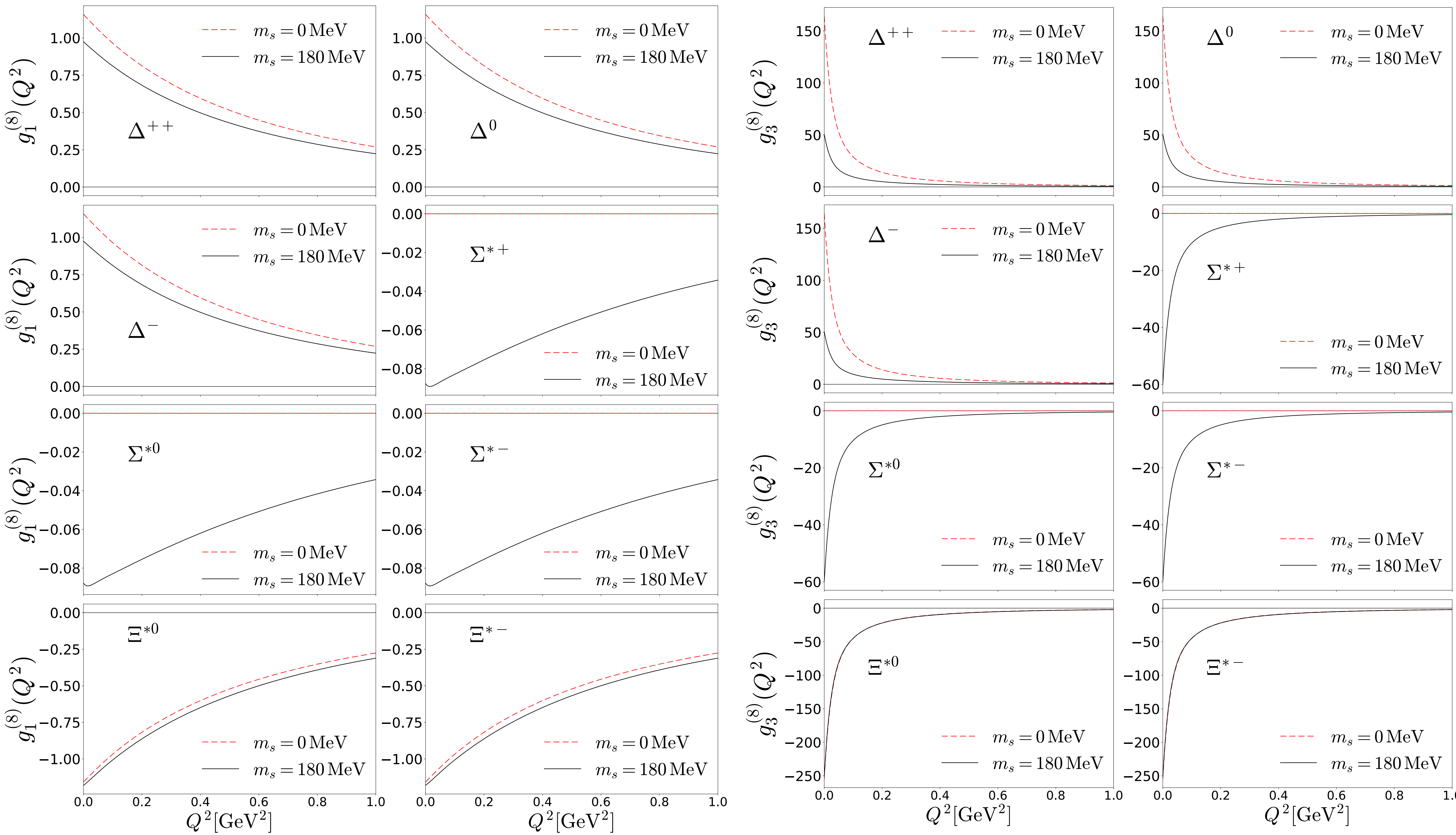
Δ^+

Ω^-

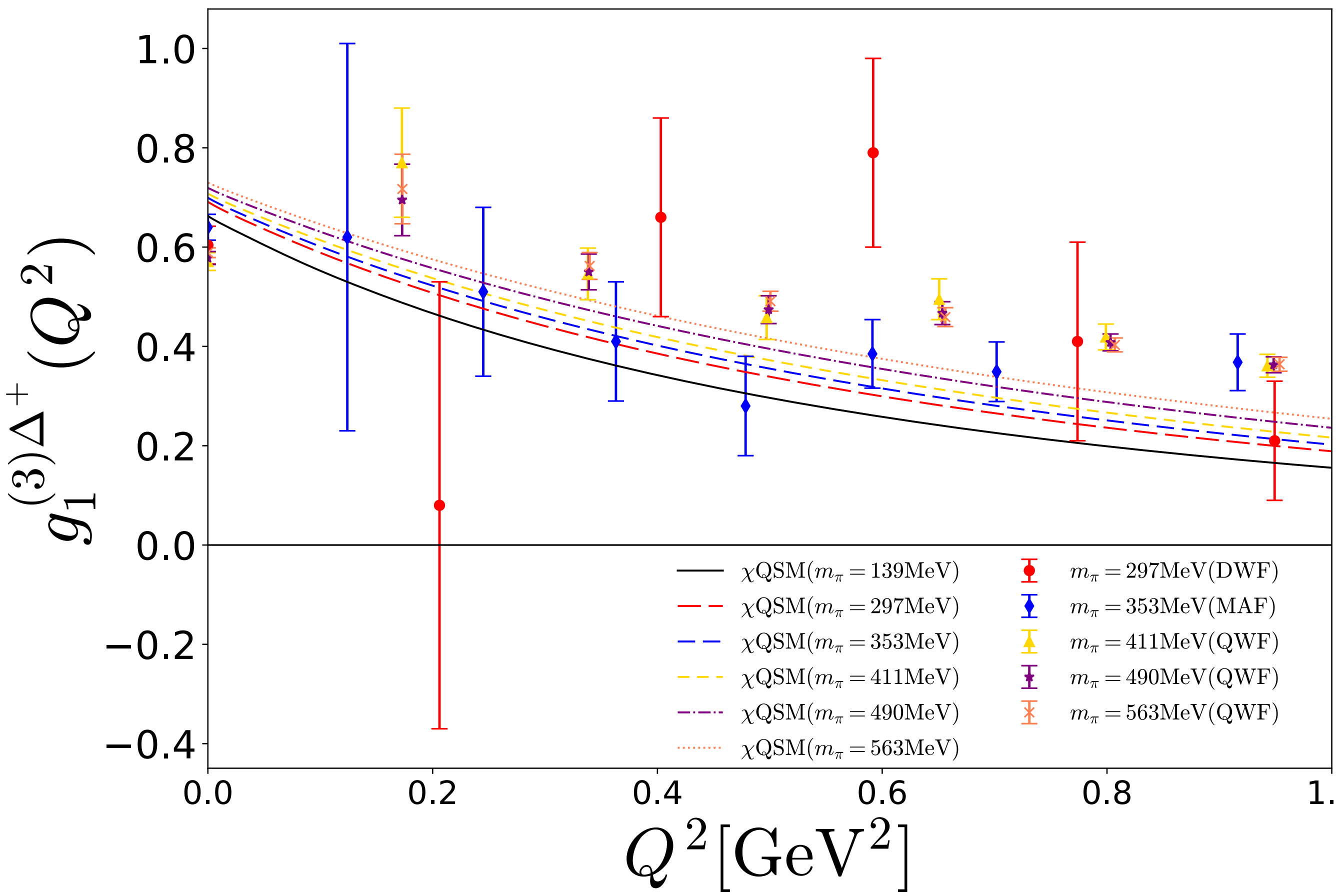


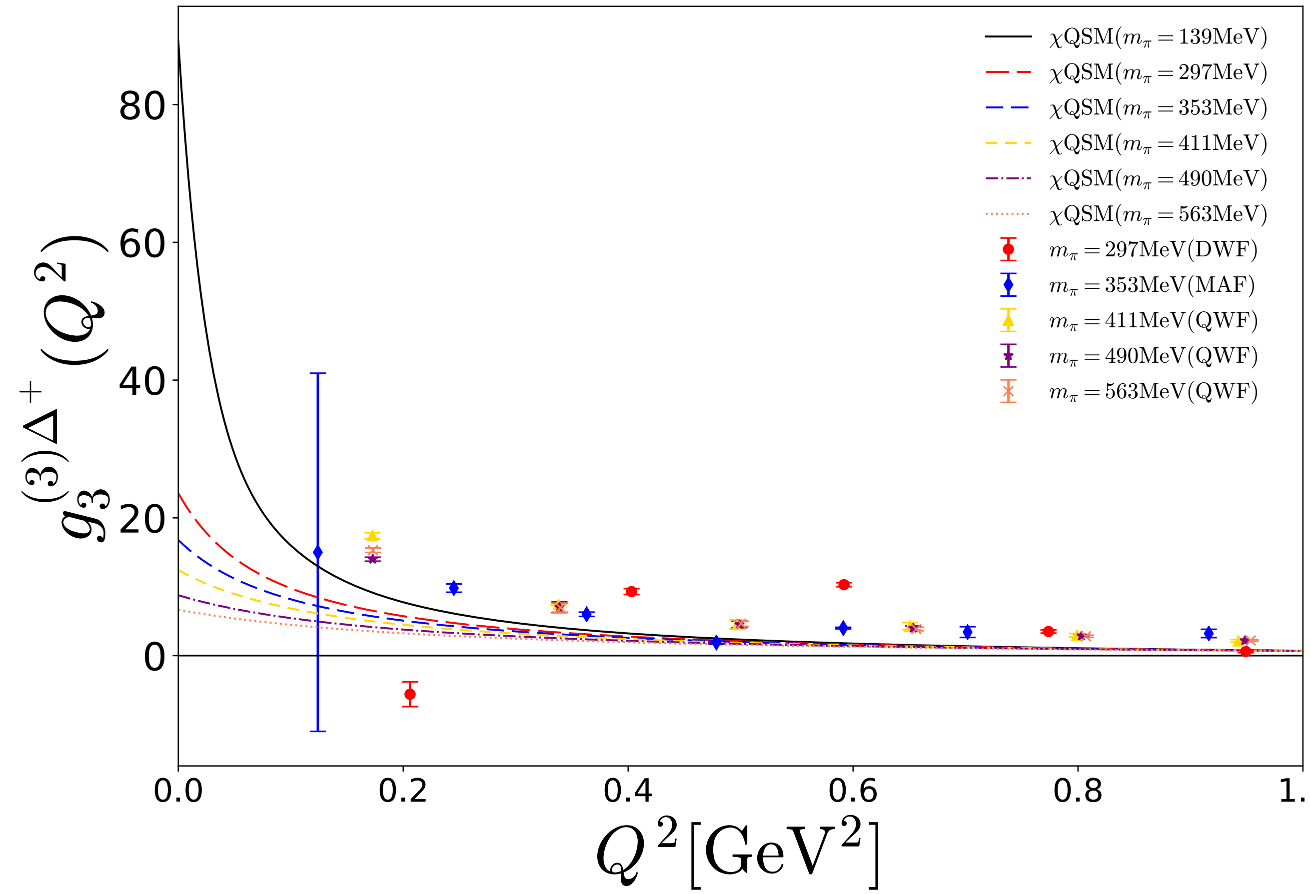
The octet axial-vector form factors, a=8

Axial-vector form factors for the baryon decuplet(Octet)

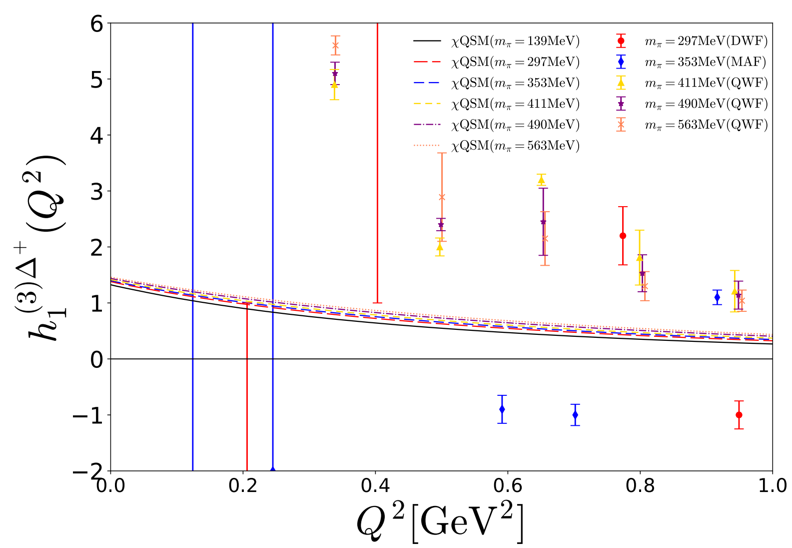


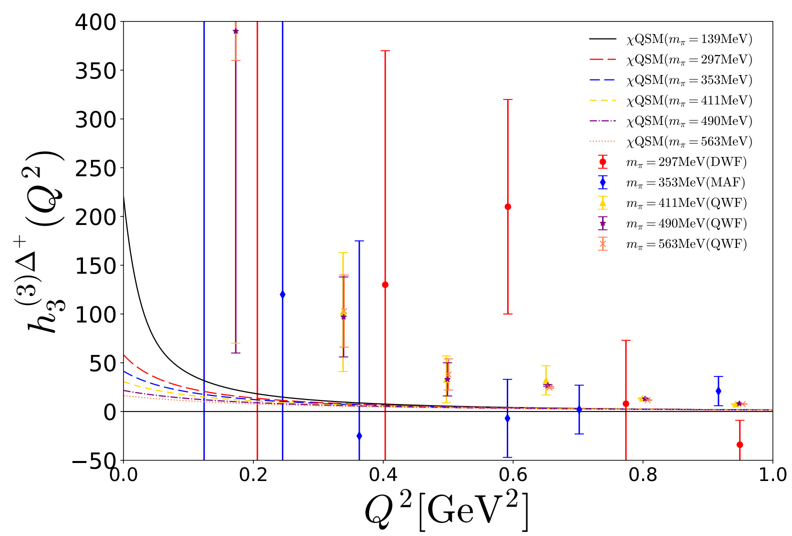
Comparison with other calculations





Alexandrou et al, PRD87, 114513, 2013





Comparison with other calculations

$g_1^{(3)B}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
$m_s = 0$ MeV	2.0064	0.6688	−0.6688	−2.0064	1.338	0	−1.338	0.669	−0.669	0
$m_s = 180$ MeV	2.1333	0.7111	−0.7111	−2.1333	1.440	0	−1.440	0.729	−0.729	0
LQCD [4] ($m_\pi = 131.2(13)$ MeV)	–	–	–	–	1.1740(380)	–	–	0.5891(198)	–	–
LQCD [4] ($m_\pi = 213$ MeV)	1.9777(1458)	0.5181(981)	−0.6499(973)	−1.7090(1422)	1.1929(521)	−0.1367(685)	−1.2633(516)	0.5869(216)	−0.6682(382)	–
LQCD [4] ($m_\pi = 256$ MeV)	1.6956(1897)	0.5670(1479)	−0.5929(1167)	−1.7322(1718)	1.1462(720)	0.0148(542)	−1.0646(661)	0.5785(278)	−0.5424(303)	–
LQCD [3] ($m_\pi = 297$ MeV)	–	0.604(38)	–	–	–	–	–	–	–	–
LQCD [4] ($m_\pi = 302$ MeV)	1.9574(1552)	0.6374(976)	−0.4798(1063)	−1.4374(1331)	1.2839(636)	0.0654(444)	−1.0423(619)	0.6204(256)	−0.5459(299)	–
LQCD [3] ($m_\pi = 353$ MeV)	–	0.640(26)	–	–	–	–	–	–	–	–
LQCD [4] ($m_\pi = 373$ MeV)	1.7602(1035)	0.5215(639)	−0.5676(635)	−1.5872(1270)	1.1478(558)	−0.0130(323)	−1.1139(485)	0.5741(243)	−0.5702(230)	–
LQCD [3] ($m_\pi = 411$ MeV)	–	0.571(18)	–	–	–	–	–	–	–	–
LQCD [4] ($m_\pi = 432$ MeV)	1.8520(875)	0.6129(478)	−0.5949(489)	−1.8108(868)	1.2228(473)	0.0124(244)	−1.1765(450)	0.6059(213)	−0.5885(223)	–
LQCD [3] ($m_\pi = 490$ MeV)	–	0.578(13)	–	–	–	–	–	–	–	–
LQCD [3] ($m_\pi = 563$ MeV)	–	0.5887(98)	–	–	–	–	–	–	–	–
χPT [6]	2.25*	–	–	–	–	–	–	–	–	–
RCQM[GBE] [7, 8]	2.24*	–	–	–	1.499†	–	–	0.75†	–	–
LCSR [9]	2.70 ± 0.6*	–	–	–	–	–	–	–	–	–
PCQM [10]	1.863*	–	–	–	1.242†	–	–	0.621†	–	–

[3] C. Alexandrou et al, PRD 87, 114513 (2013).
[4] C. Alexandrou et al, PRD 94, 034502 (2016).
[6] F. Jiang and B. C. Tiburzi, PRD 78, 017504 (2008).
[7,8] Ki-Seok Choi et al, PRD 82, 014007 (2010), FBS54, 1055 (2013).
[9] A. Kucukarslan et al, PRD 90, 054002 (2014).
[10] X. Y. Liu et al, PRC 97, 055206 (2018).

Additional numerical results

$m_\mathrm{s} = 180$ MeV	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
$g_3^{(3)B}(0)$	346.1	115.4	−115.4	−346.1	303.9	0	−303.9	193.7	−193.7	0
$g_3^{(0)B}(0)$	7.822	7.822	7.822	7.822	1.622	1.622	1.622	−8.204	−8.204	−21.936
$g_3^{(8)B}(0)$	50.8	50.8	50.8	50.8	−60.0	−60.0	−60.0	−251.9	−251.9	−542.8
$\langle r_A^2\rangle_B$ [fm^2]	0.447	0.447	0.447	0.447	0.438	–	0.438	0.431	0.431	–
M_A [GeV]	1.023	1.023	1.023	1.023	1.033	–	1.033	1.041	1.041	–

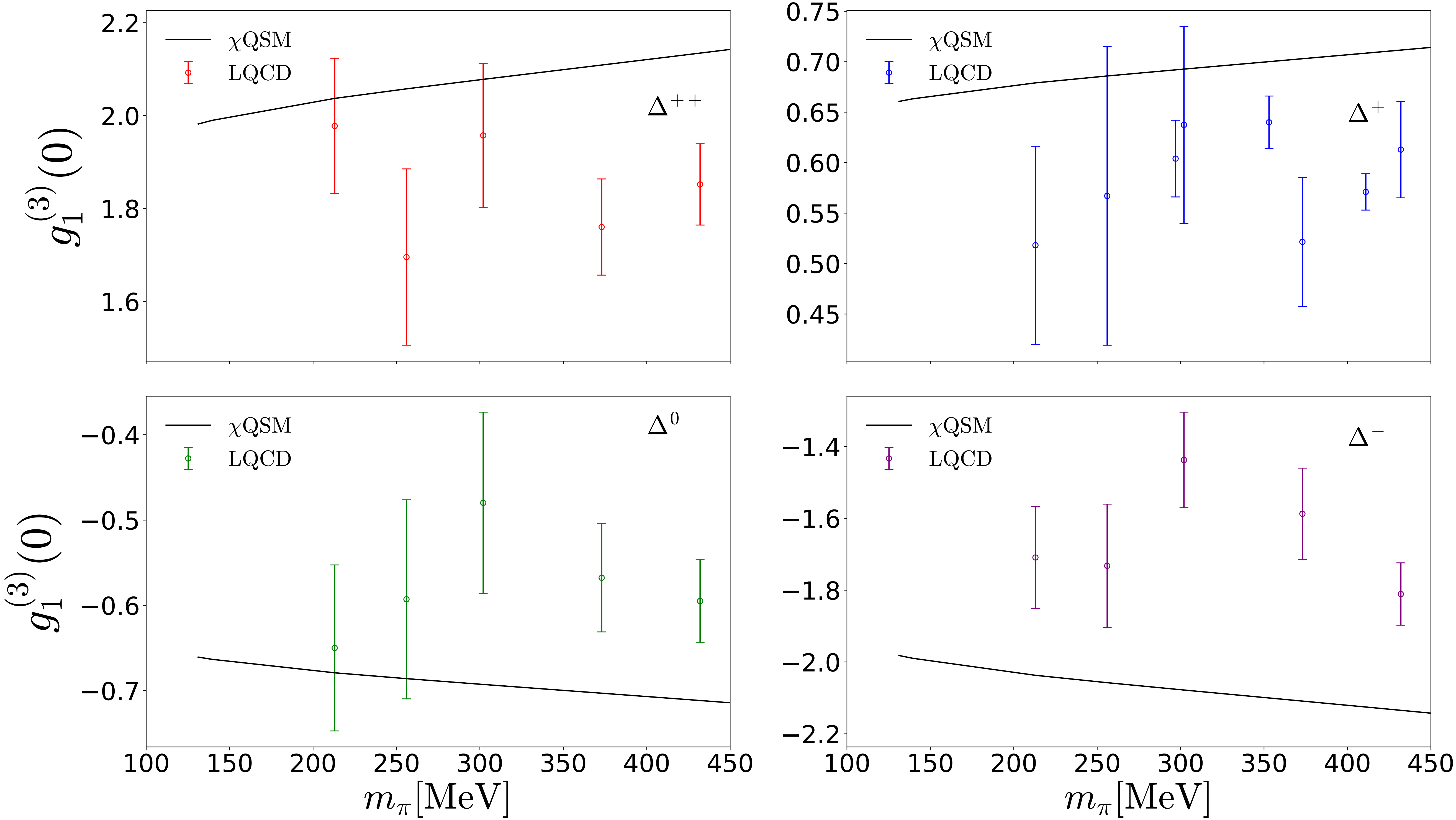
$$g_1^{(3)B}(Q^2) = \frac{g_1^{(3)B}(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}, \quad \langle r_A^2\rangle_B = \frac{-6}{g_1^{(3)B}(0)}\,\frac{\partial g_1^{(3)B}(Q^2)}{\partial Q^2}\bigg|_{Q^2=0}$$

Summary & Outlook

- Summary
  - We performed the axial-vector form factor calculations within the chiral quark-soliton model.
  - We discussed the flavor SU(3) symmetry breaking contributions to the axial-vector form factors.
  - We compared the axial-vector form factors and constants with Lattice data and other model calculations.
  - We have calculated the axial radii and masses for baryon decuplet as well.
- Outlook
  - The axial-vector form factors with the quark contents.
  - Transition form factors between the light baryons.
  - The Axial-vector form factors for the heavy baryons.
  - Tensor form factors.

Thank you for listening!!!

Comparison with other calculations



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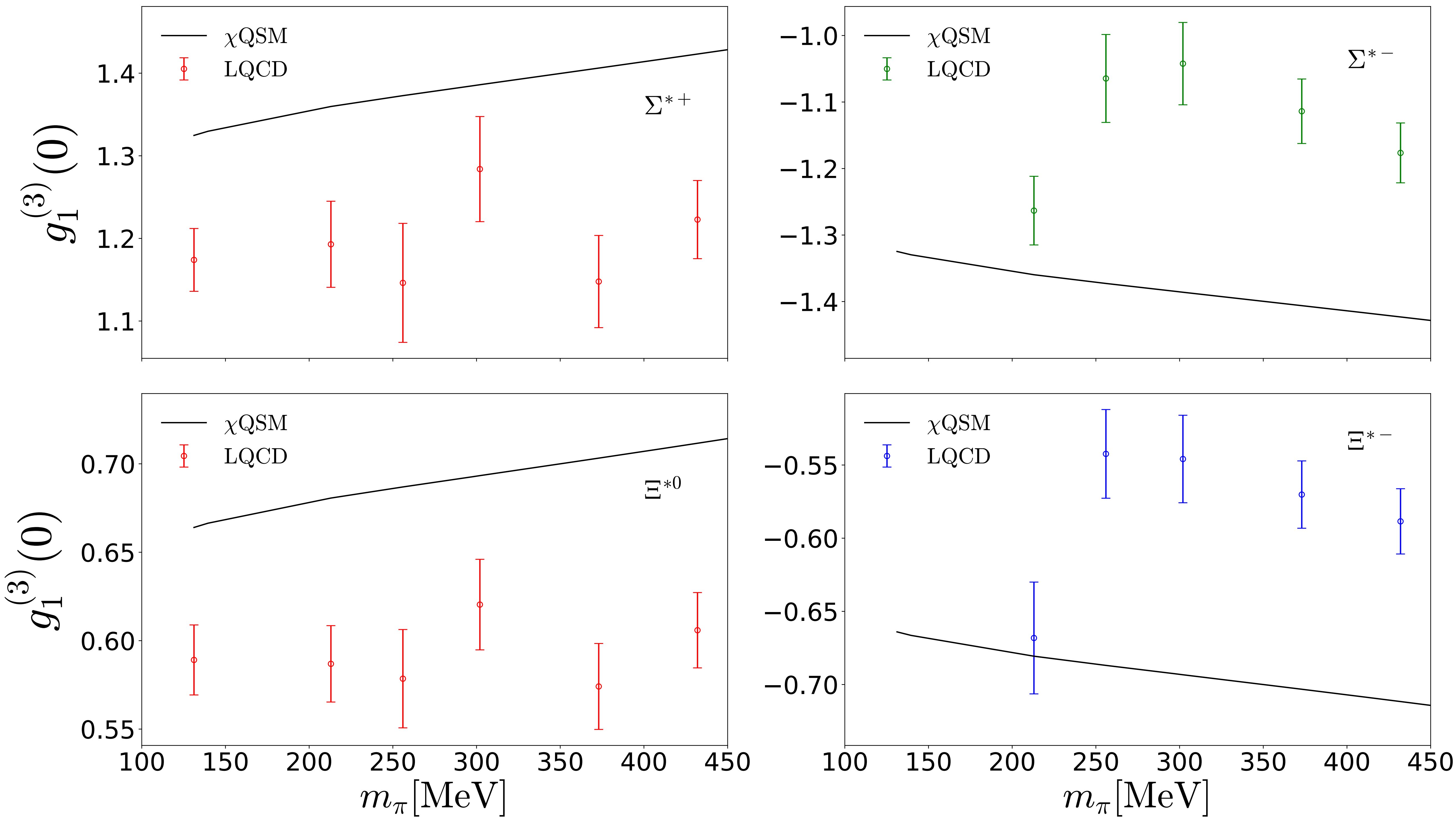

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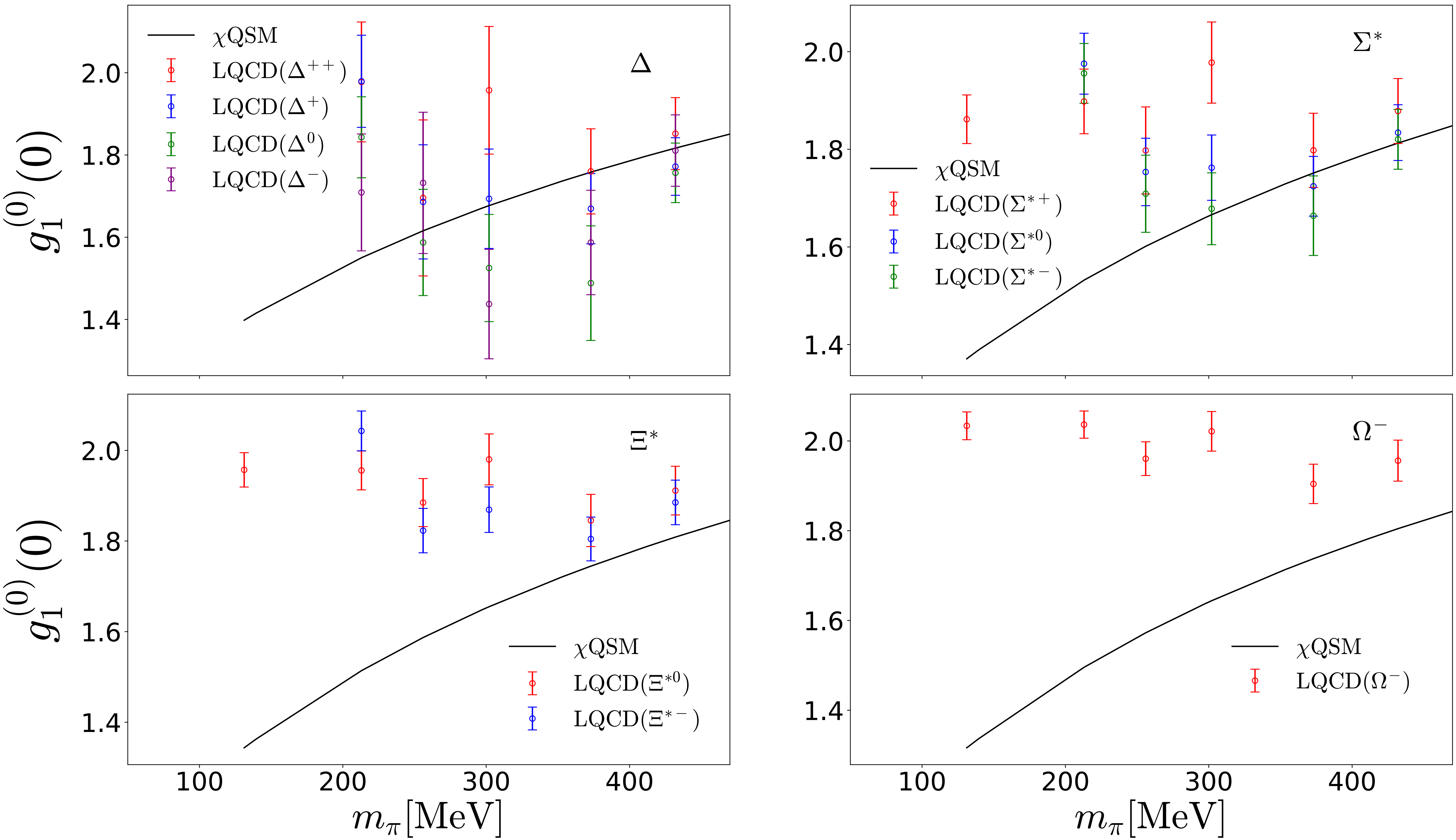

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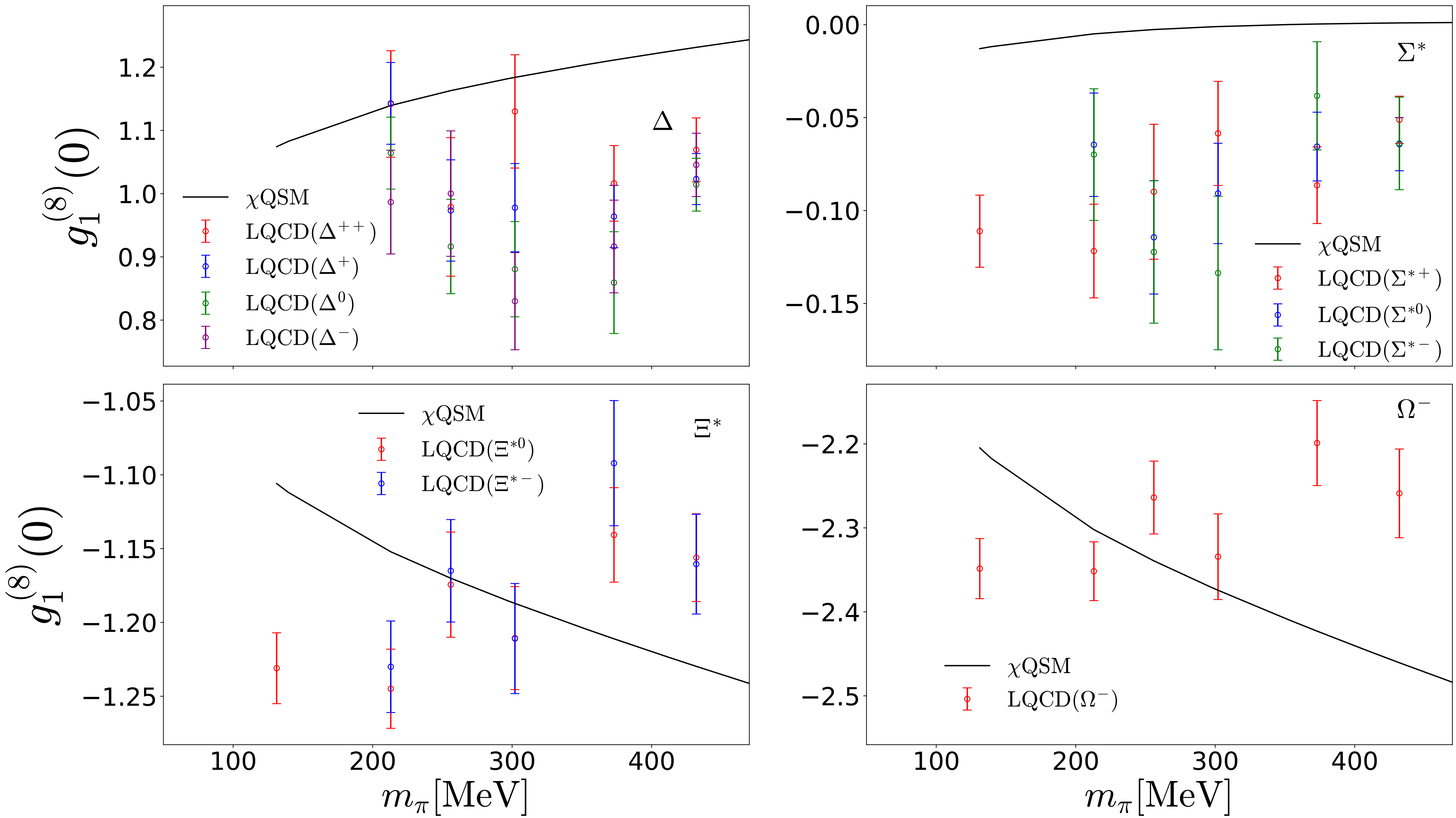

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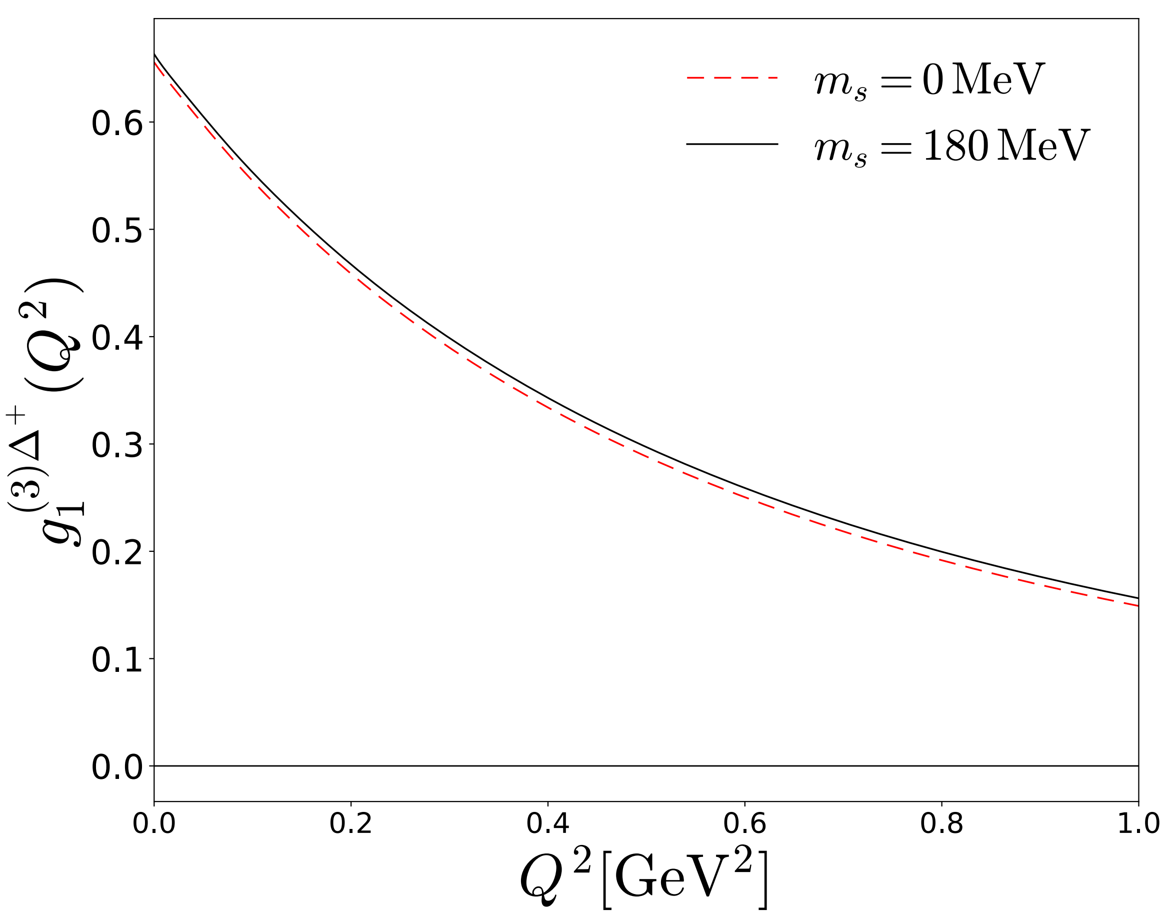
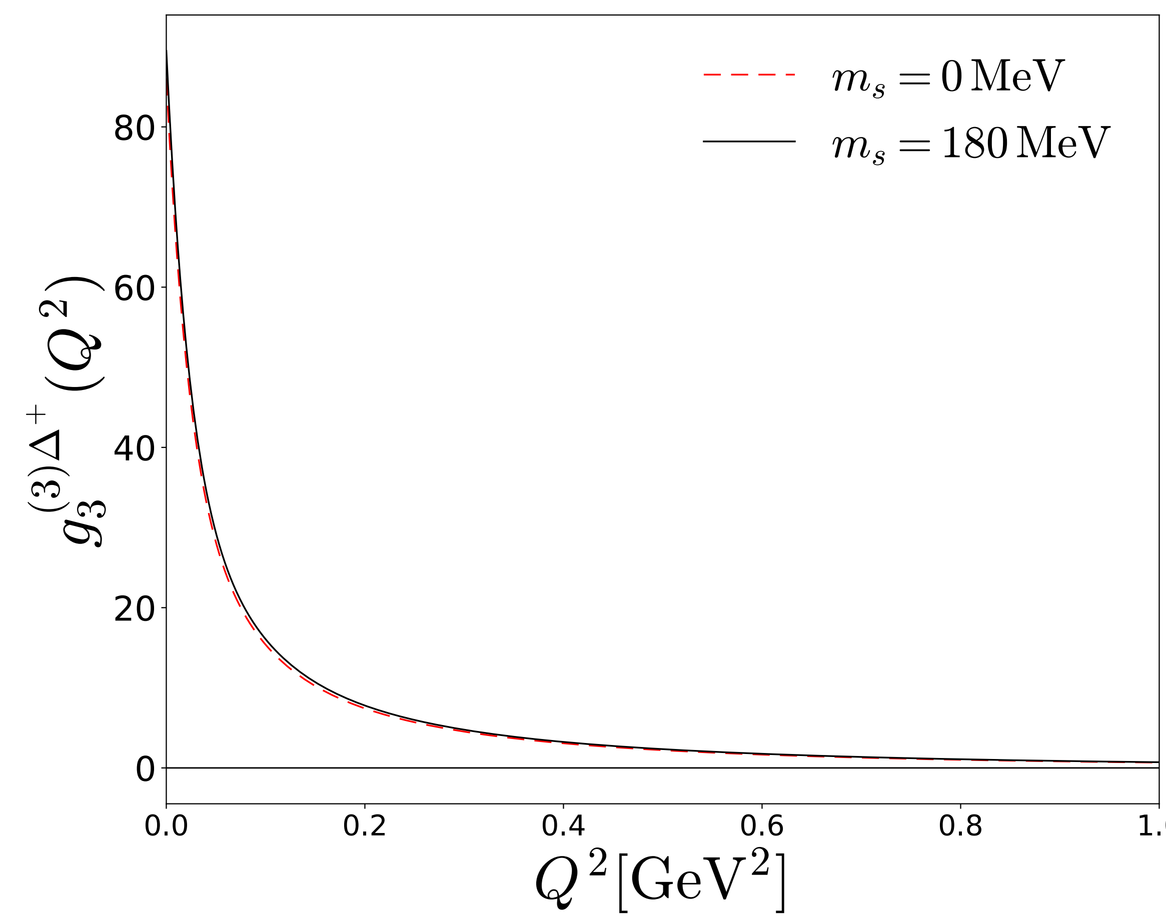
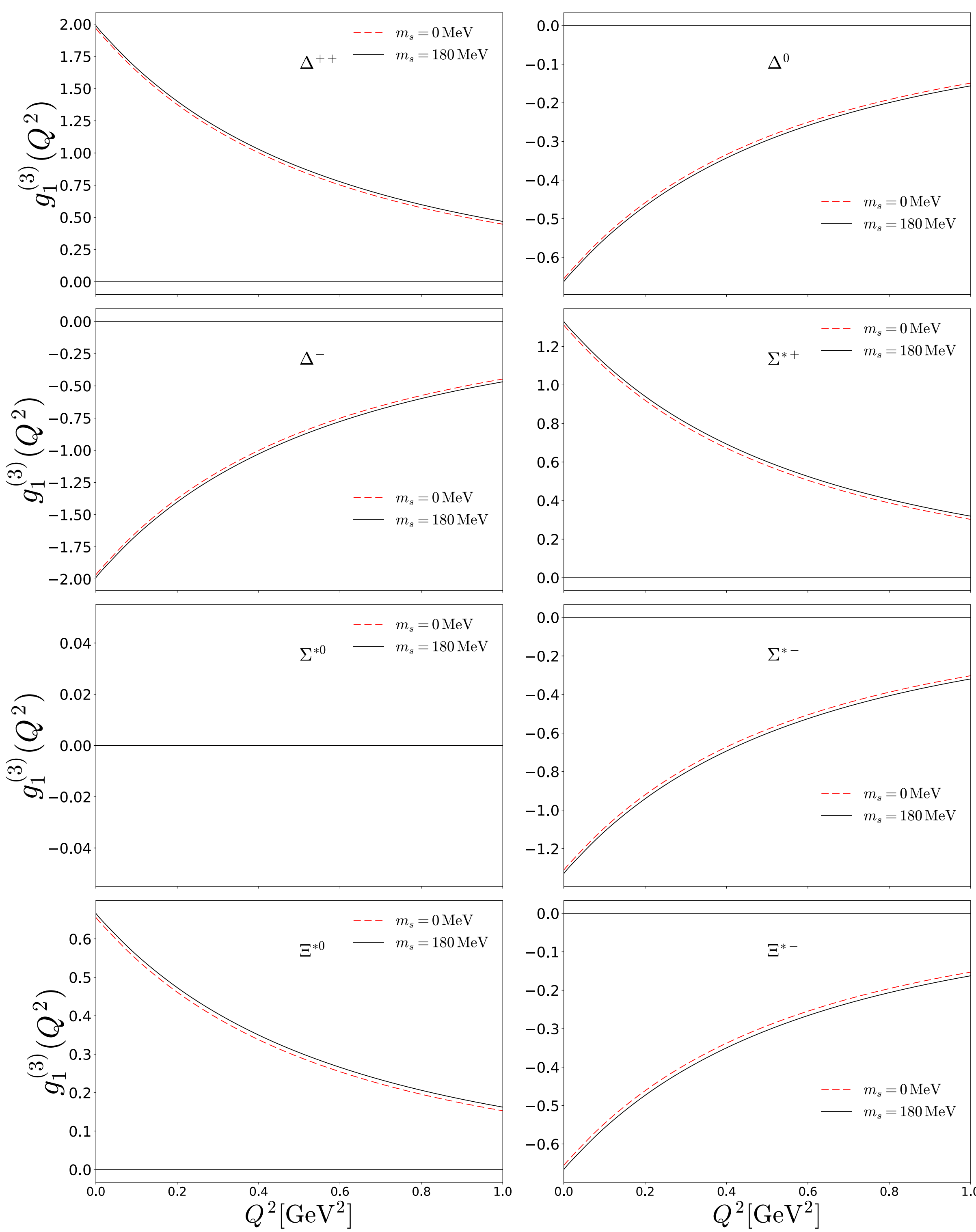
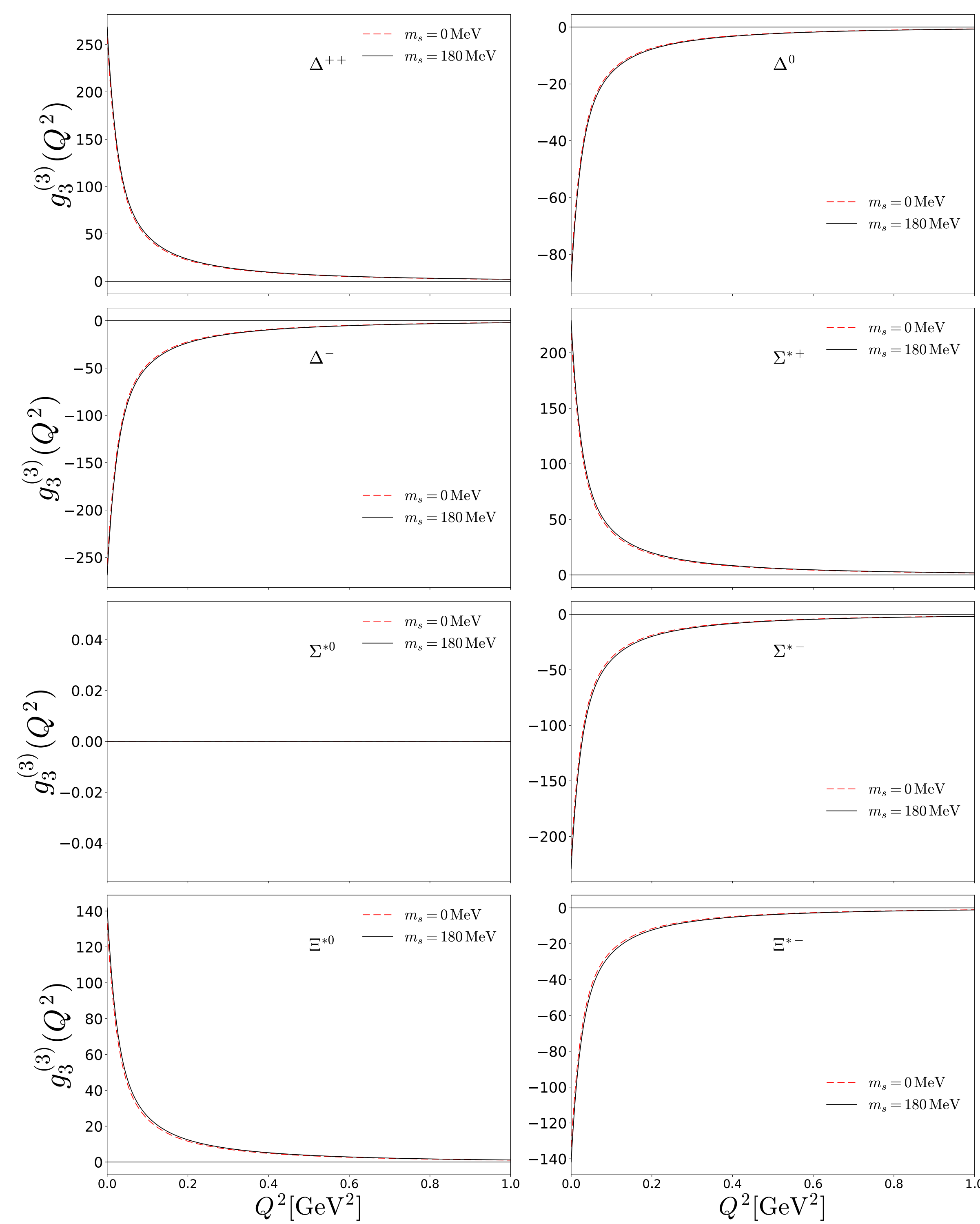




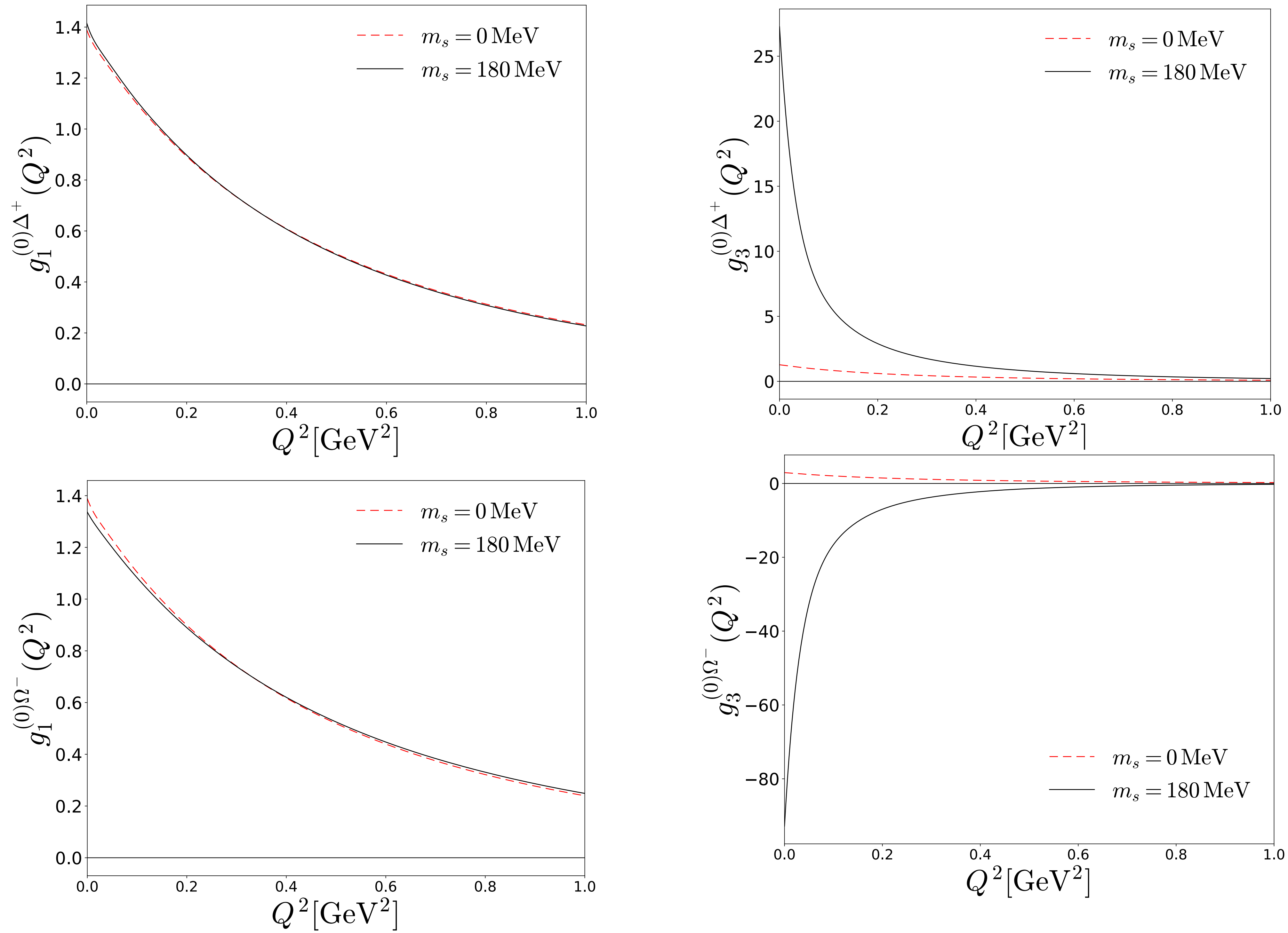
Triplet axial-vector form factors

Axial-vector form factors for the baryon decuplet



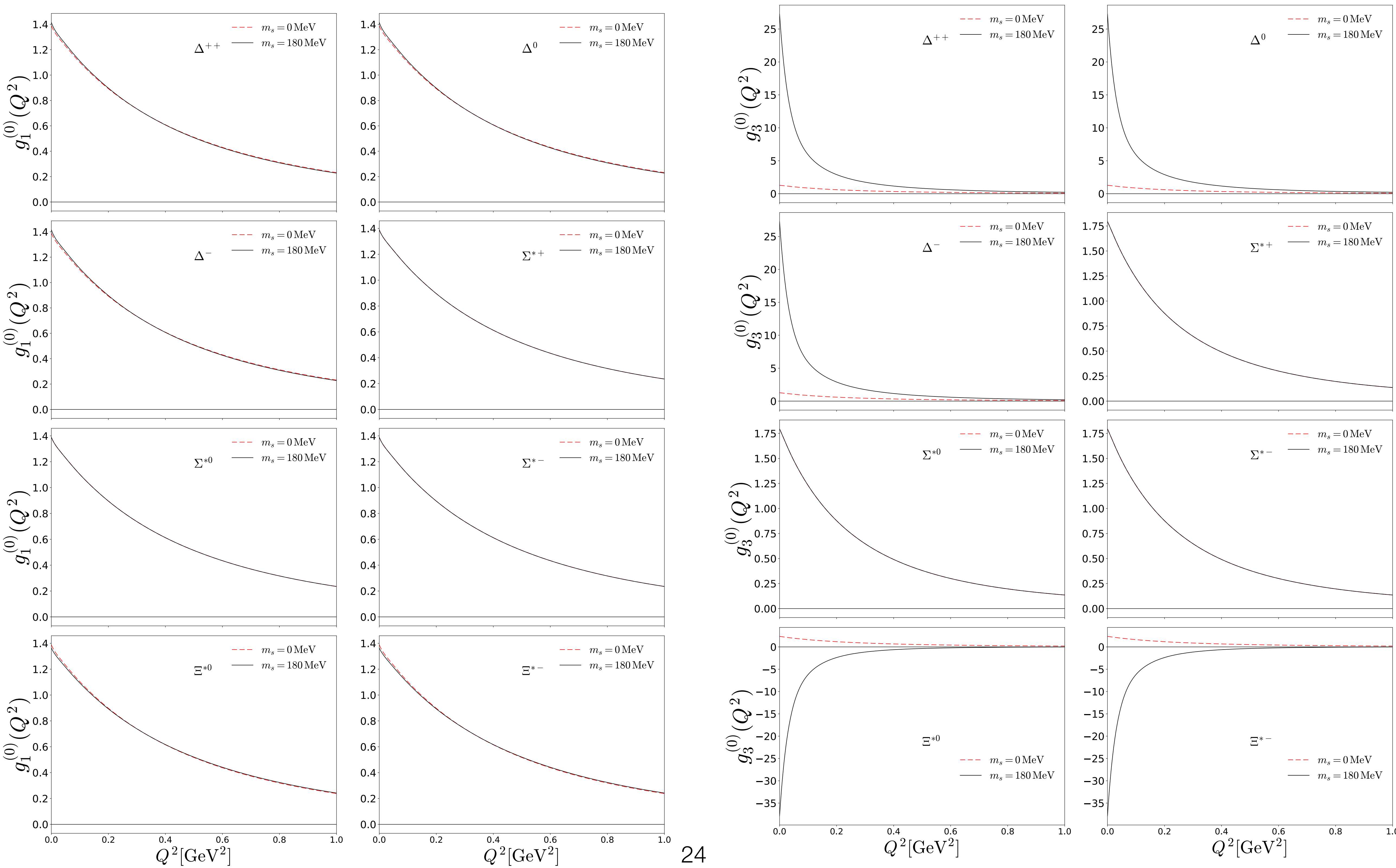


Axial-vector form factors for the baryon decuplet

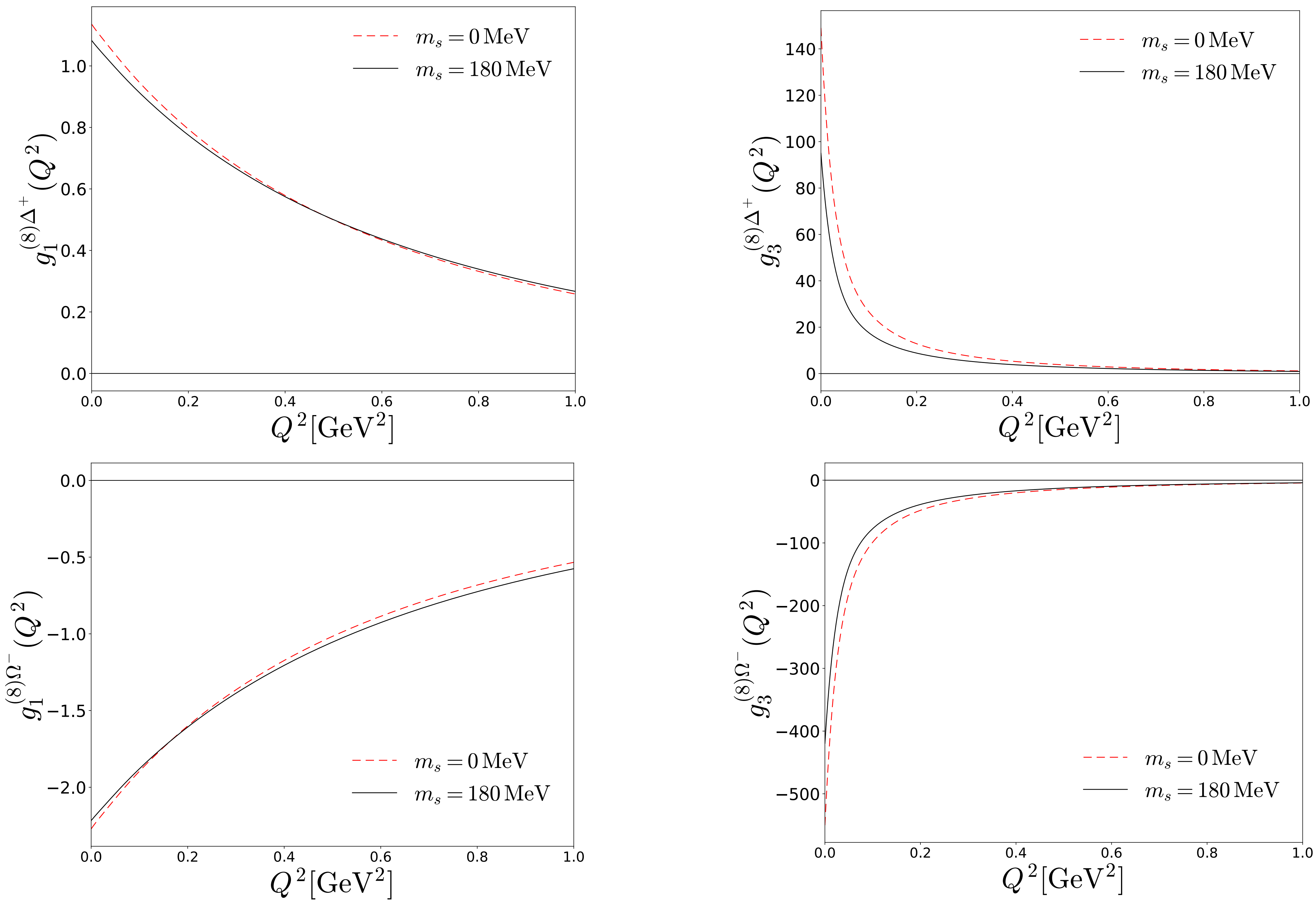


Singlet axial-vector form factors

Axial-vector form factors for the baryon decuplet

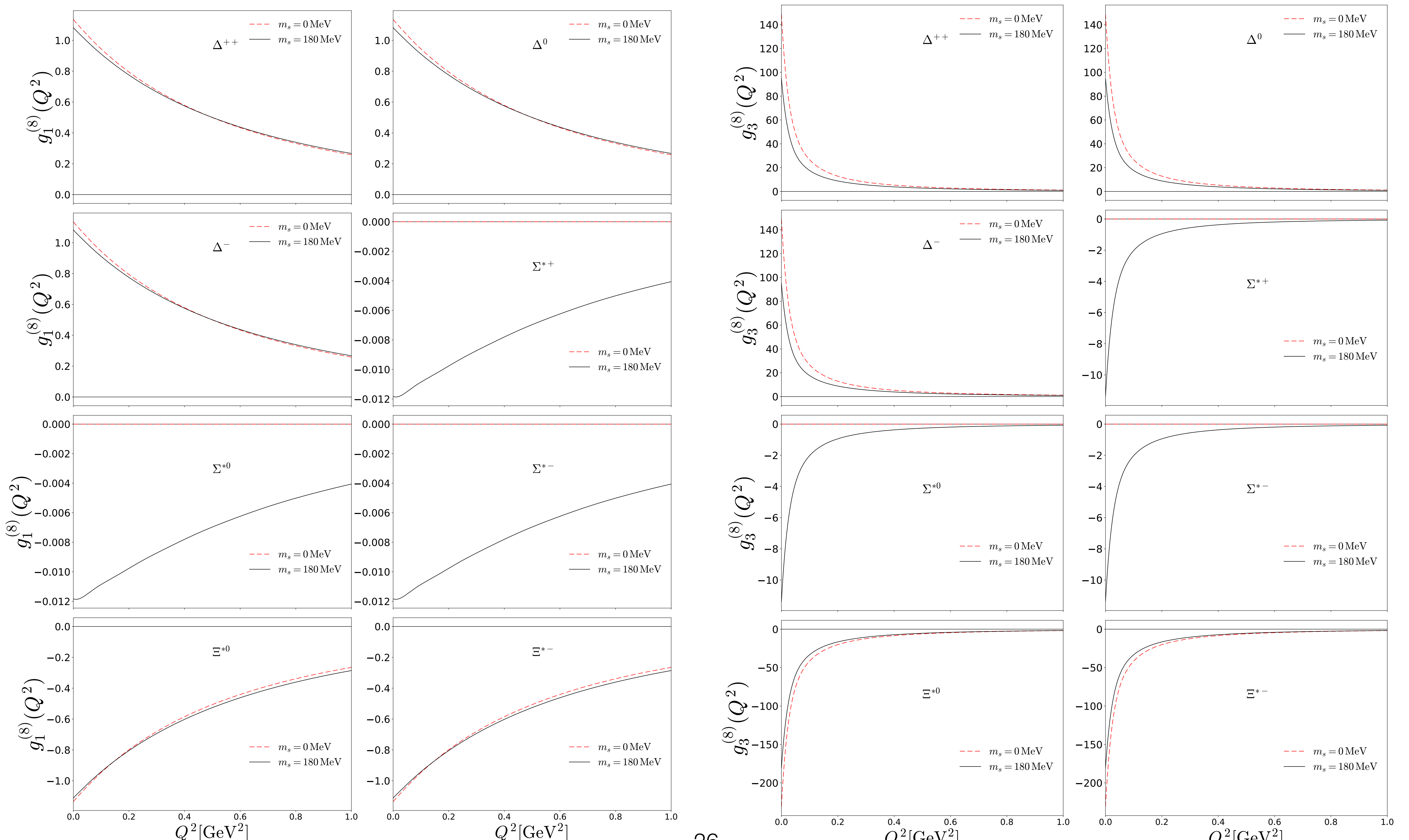


Axial-vector form factors for the baryon decuplet



Octet axial-vector form factors

Axial-vector form factors for the baryon decuplet



Axial-vector constants for the baryon decuplet

$g_1^{(3)B}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
$m_s = 0$ MeV	1.9671	0.6557	−0.6557	−1.9671	1.3114	0	−1.3114	0.6557	−0.6557	0
$m_s = 180$ MeV	1.9876	0.6625	−0.6625	−1.9876	1.3279	0	−1.3279	0.6654	−0.6654	0
LQCD [4] ($m_\pi = 131.2(13)$ MeV)	–	–	–	–	1.1740(380)	–	–	0.5891(198)	–	–
LQCD [4] ($m_\pi = 213$ MeV)	1.9777(1458)	0.5181(981)	−0.6499(973)	−1.7090(1422)	1.1929(521)	−0.1367(685)	−1.2633(516)	0.5869(216)	−0.6682(382)	–
LQCD [4] ($m_\pi = 256$ MeV)	1.6956(1897)	0.5670(1479)	−0.5929(1167)	−1.7322(1718)	1.1462(720)	0.0148(542)	−1.0646(661)	0.5785(278)	−0.5424(303)	–
LQCD [3] ($m_\pi = 297$ MeV)	–	0.604(38)	–	–	–	–	–	–	–	–
LQCD [4] ($m_\pi = 302$ MeV)	1.9574(1552)	0.6374(976)	−0.4798(1063)	−1.4374(1331)	1.2839(636)	0.0654(444)	−1.0423(619)	0.6204(256)	−0.5459(299)	–
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LQCD [3] ($m_\pi = 411$ MeV)	–	0.571(18)	–	–	–	–	–	–	–	–
LQCD [4] ($m_\pi = 432$ MeV)	1.8520(875)	0.6129(478)	−0.5949(489)	−1.8108(868)	1.2228(473)	0.0124(244)	−1.1765(450)	0.6059(213)	−0.5885(223)	–
LQCD [3] ($m_\pi = 490$ MeV)	–	0.578(13)	–	–	–	–	–	–	–	–
LQCD [3] ($m_\pi = 563$ MeV)	–	0.5887(98)	–	–	–	–	–	–	–	–
χPT [6]	2.25^*	–	–	–	–	–	–	–	–	–
RCQM[GBE] [7, 8]	2.24^*	–	–	–	$1.499^\dagger$	–	–	$0.75^\dagger$	–	–
LCSR [9]	$2.70 \pm 0.6^*$	–	–	–	–	–	–	–	–	–
PCQM [10]	1.863^*	–	–	–	$1.242^\dagger$	–	–	$0.621^\dagger$	–	–

[3] C. Alexandrou et al, PRD 87, 114513 (2013).
[4] C. Alexandrou et al, PRD 94, 034502 (2016).
[6] F. Jiang and B. C. Tiburzi, PRD 78, 017504 (2008).
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[9] A. Kucukarslan et al, PRD 90, 054002 (2014).
[10] X. Y. Liu et al, PRC 97, 055206 (2018).

Additional numerical results

$m_s = 180$ MeV	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
$g_3^{(3)B}(0)$	267.7	89.2	-89.2	-267.7	228.0	0	-228.0	141.1	-141.1	0
$g_3^{(0)B}(0)$	24.76	24.76	24.76	24.76	1.795	1.795	1.795	-34.01	-34.01	-83.59
$g_3^{(8)B}(0)$	100.4	100.4	100.4	100.4	-10.29	-10.29	-10.29	-185.8	-185.8	-432.6
$\langle r_A^2\rangle_B$ [fm^2]	0.507	0.507	0.507	0.507	0.498	–	0.498	0.490	0.490	–
M_A [GeV]	0.960	0.960	0.960	0.960	0.969	–	0.969	0.976	0.976	–

$$g_1^{(3)B}(Q^2) = \frac{g_1^{(3)B}(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}, \quad \langle r_A^2\rangle = \frac{12}{M_A^2}$$

Preparation for the form factor calculation

- **Using parameter**

$$m_\pi = 139.57\mathrm{MeV},\ f_\pi = 93\mathrm{MeV},\ M_c = 420\mathrm{MeV},\ m_s = 180\mathrm{MeV}$$

$$D = 8\mathrm{fm},\ \Lambda_1 = 0.377\mathrm{GeV},\ \Lambda_2 = 1.428\mathrm{GeV},\ m_0 = 6.13\mathrm{MeV}$$

- **Calculated values**

$M_{\mathrm{cl}}(\mathrm{GeV})$	$I_1(\mathrm{fm})$	$I_2(\mathrm{fm})$	$K_1(\mathrm{fm})$	$K_2(\mathrm{fm})$	$\Sigma_{\pi N}(\mathrm{MeV})$	$\langle\bar{q}q\rangle(\mathrm{MeV}^3)$
1.2957	1.084	0.519	0.410	0.264	43.89	$-(239.51)^3$