The σ and ρ coupling constants for charmed and beauty mesons from dispersive theoretical approach

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@2020 CeNuM Work shop

Outline

- Motivation
- DD→$\pi\pi$ and DD→$\pi\pi$ amplitudes
- Rescattering equation
- Spectral function
- Dispersion relation
- Extracting coupling constants
- Results
- Conclusion

Exotic heavy hadrons

- Understanding exotic mesons is one of the most important subject in hadronic physics, for example:
  - A charmonium-like state X(3872) is interpreted as tetra quark state or hybrid exotic state, meson-molecular states…

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$$M_{D^0} + M_{D^{*0}} - M_{X(3872)} = 0.00 \pm 0.18 \text{ MeV}$$

Exotic heavy hadrons

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$$Z_c(3900),\ X(3940),\ Z_c(4020),\ Z_b(10610),\ Z_b(10650)\dots$$

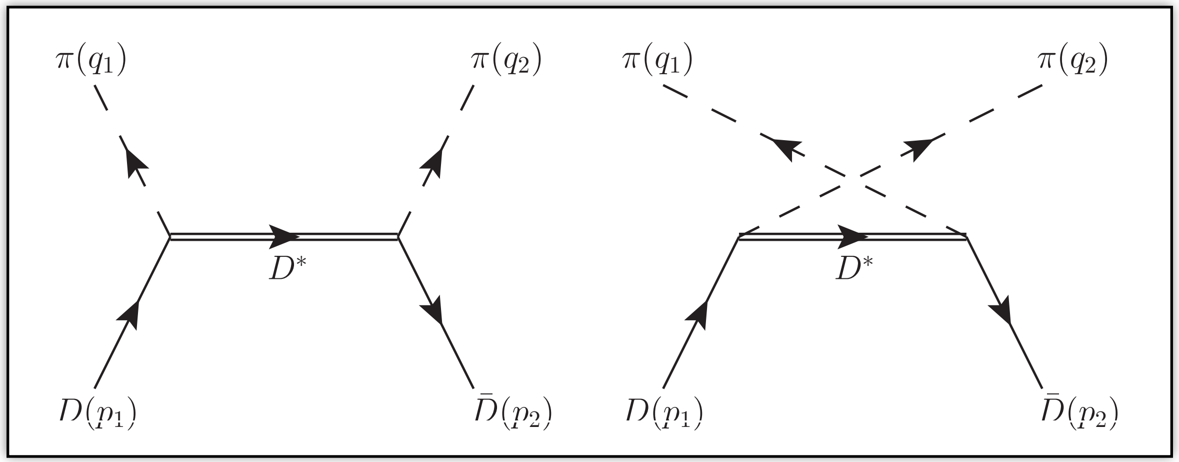
nearby thresholds : $D^0 D^{*+}$, $D_s D_s$, $D^{*0} D^{*+}$, $B^+ B^{*0}$, $B^{*+} B^{*0}$

Meson-molecular state

- Two-meson bound state through OBE → **Magnitude of coupling constants**
- The correlated 2π exchange could be approximated to OBE
  - One pion exchange gives a long-range interaction
  - The multi-pion exchange can be dominated by **multi-pion resonances**
  - The mid-range contribution may governed by light mesons : ***σ*, *ρ*, *ω* …**

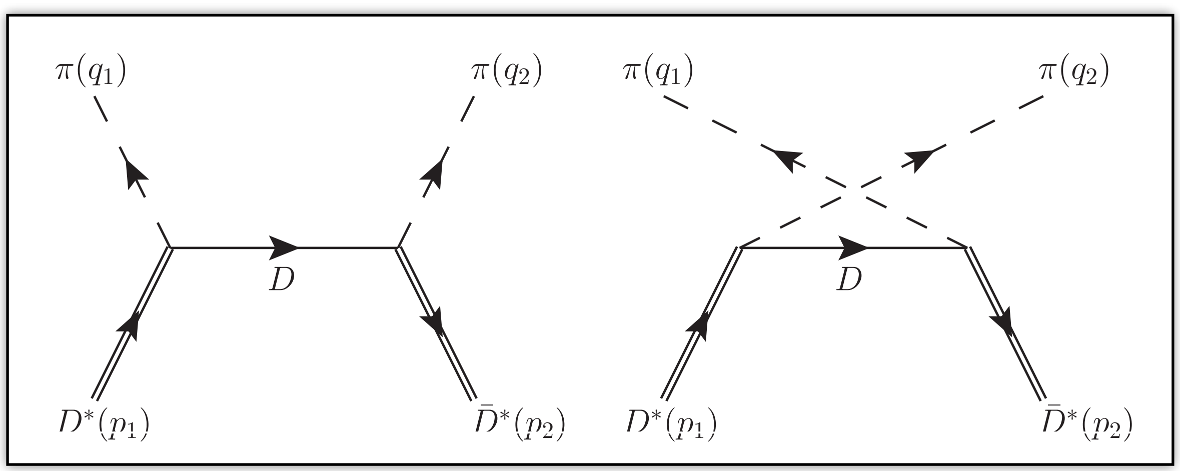
$D\bar{D} \to \pi\pi$ and $D^*\bar{D}^* \to \pi\pi$ amplitudes

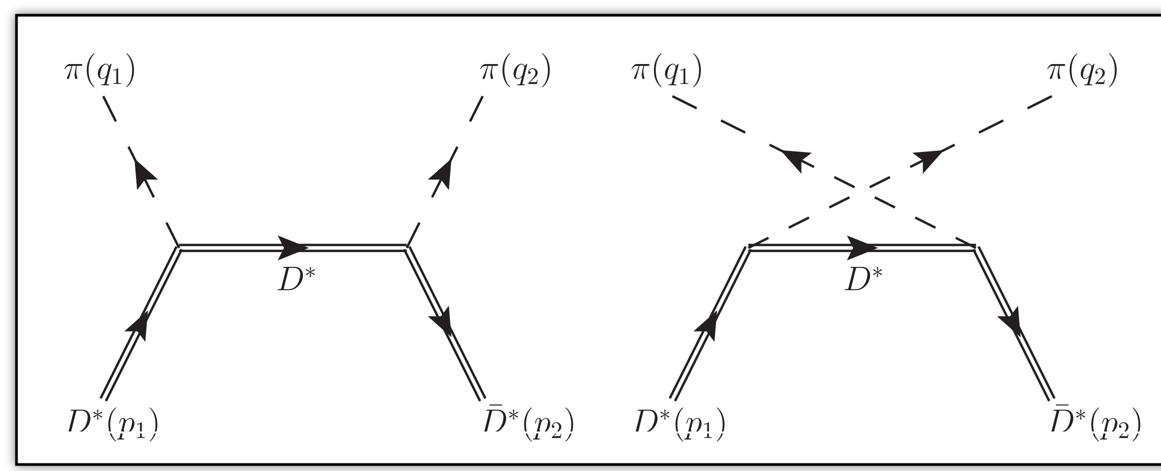
- $D\bar{D} \to \pi\pi$



D*-exchange

- $D^*\bar{D}^* \to \pi\pi$





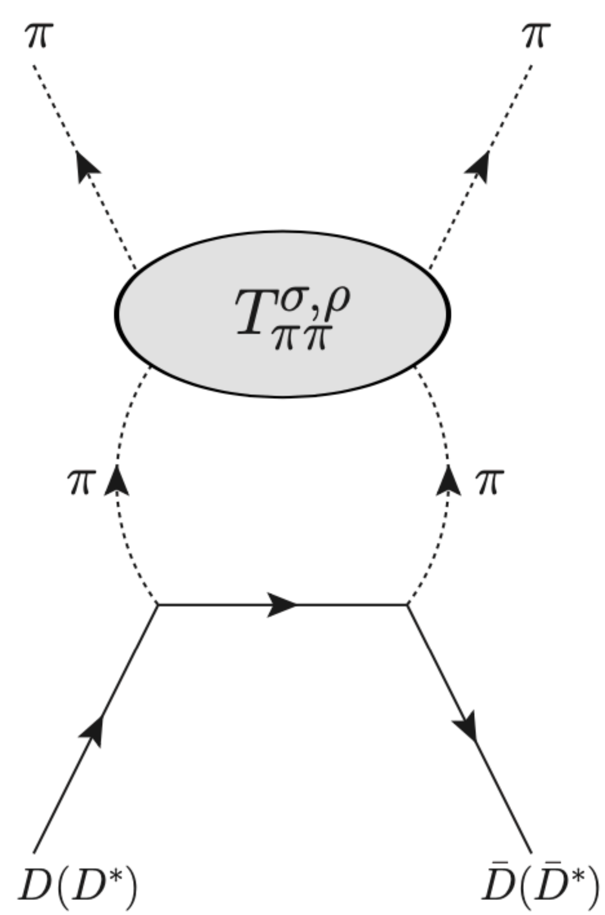
D-exchange

D*-exchange

Rescattering equation

- We construct the 2π-correlated $D\bar{D}\to\pi\pi$ amplitudes by combining with the $\pi\pi$ transition amplitude.

$$\mathcal{M}^{J}_{D\bar{D}\to\pi\pi} = \mathcal{M}^{\mathrm{Born},J}_{D\bar{D}\to\pi\pi} + \int dq\, q^2 \frac{\mathcal{M}^{\mathrm{Born},J}_{D\bar{D}\to\pi\pi}\,\mathcal{T}^{J}_{\pi\pi\to\pi\pi}}{(2\pi)^3 2\omega_q (s - 4\omega_q^2 + i\varepsilon)}$$



Spectral function

Spectral function of $D\bar{D} \to D\bar{D}$ for J, I channel

$$\rho^{J,I}_{D\bar{D}} = \frac{1}{32\pi}\sqrt{\frac{t-4m_\pi^2}{t}}\,|\mathcal{M}^{J,I}_{D\bar{D}\to\pi\pi}|^2$$

Spectral function

Spectral function of DD̄→DD̄ for J, I channel

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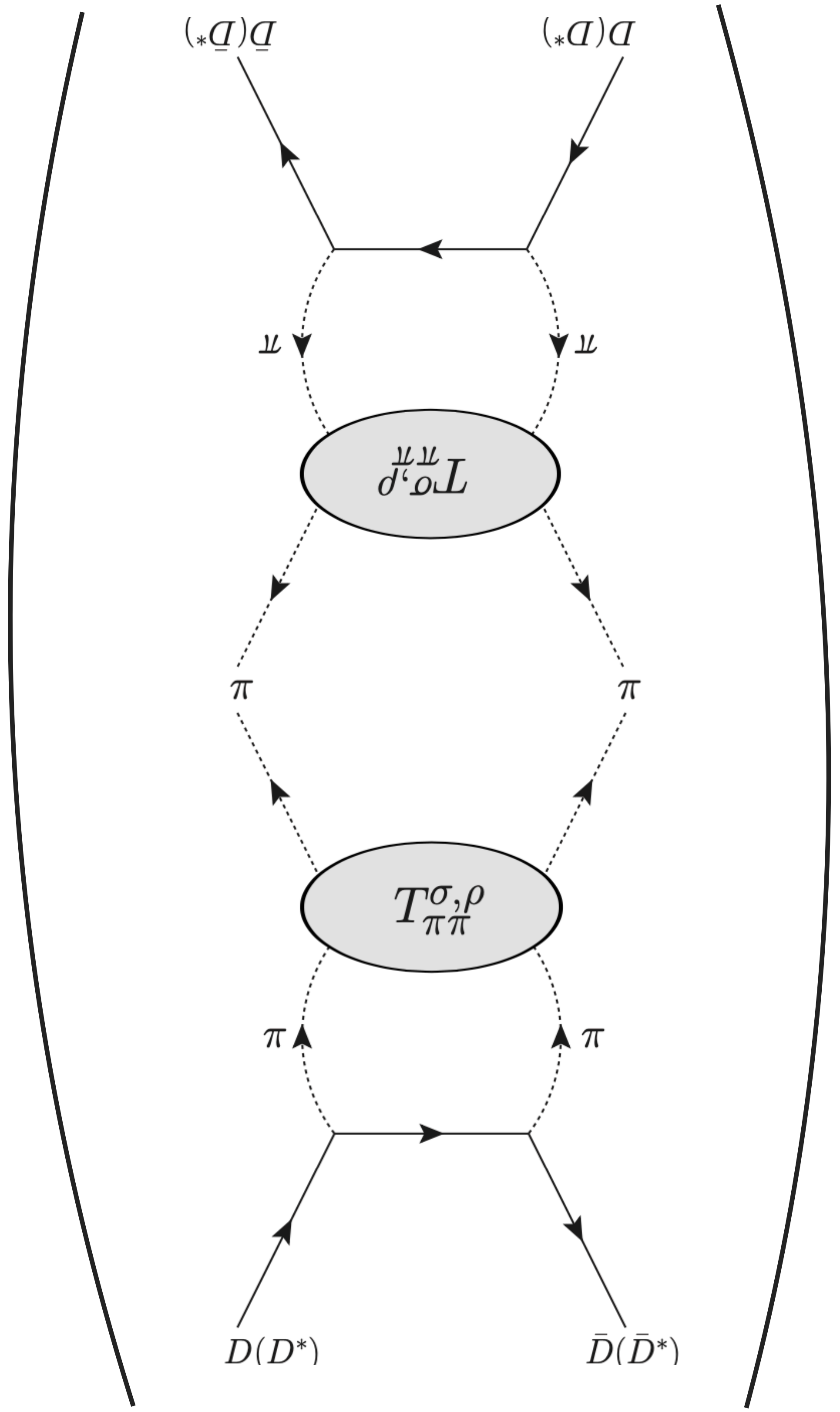
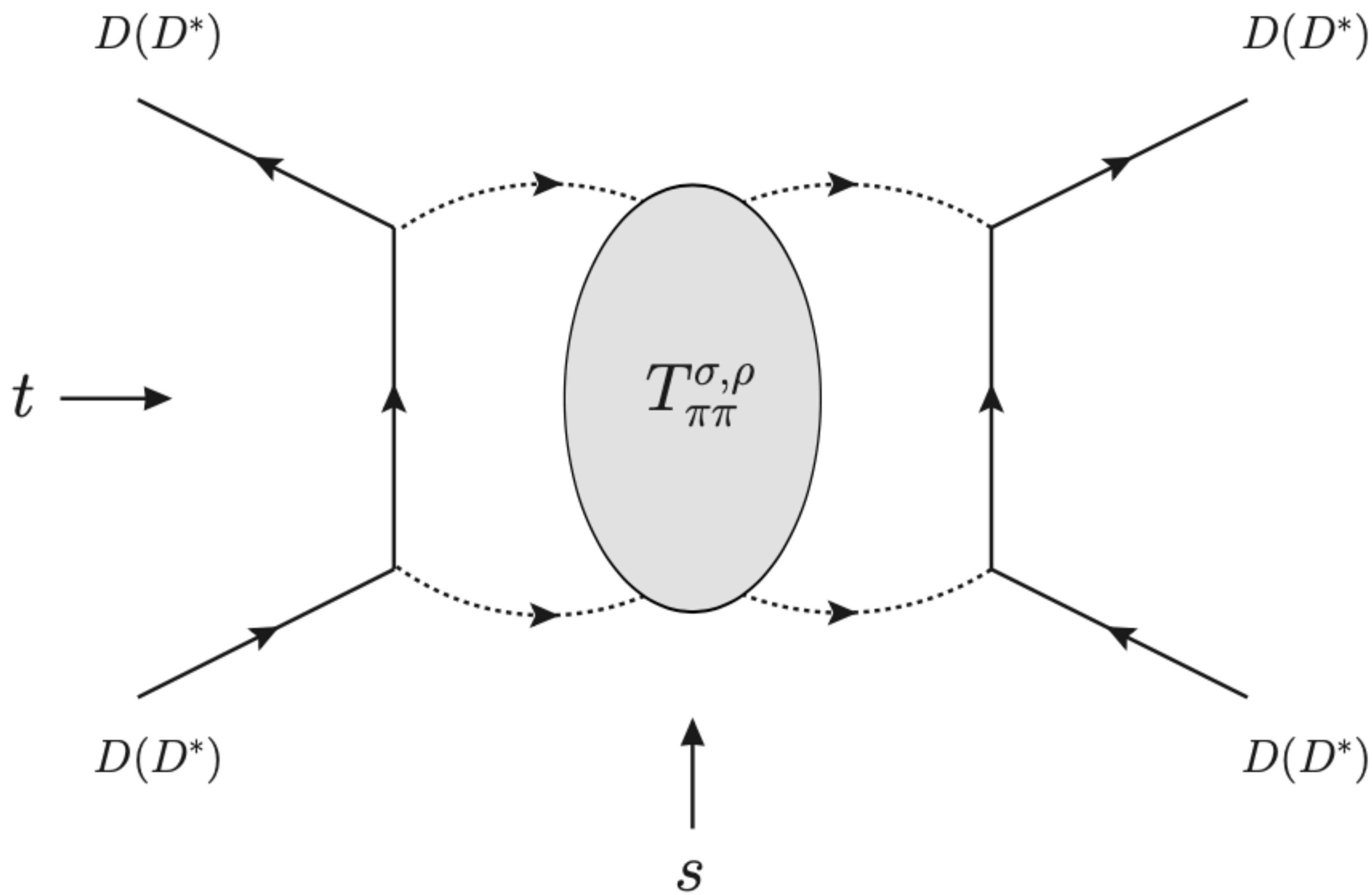


Dispersion relation

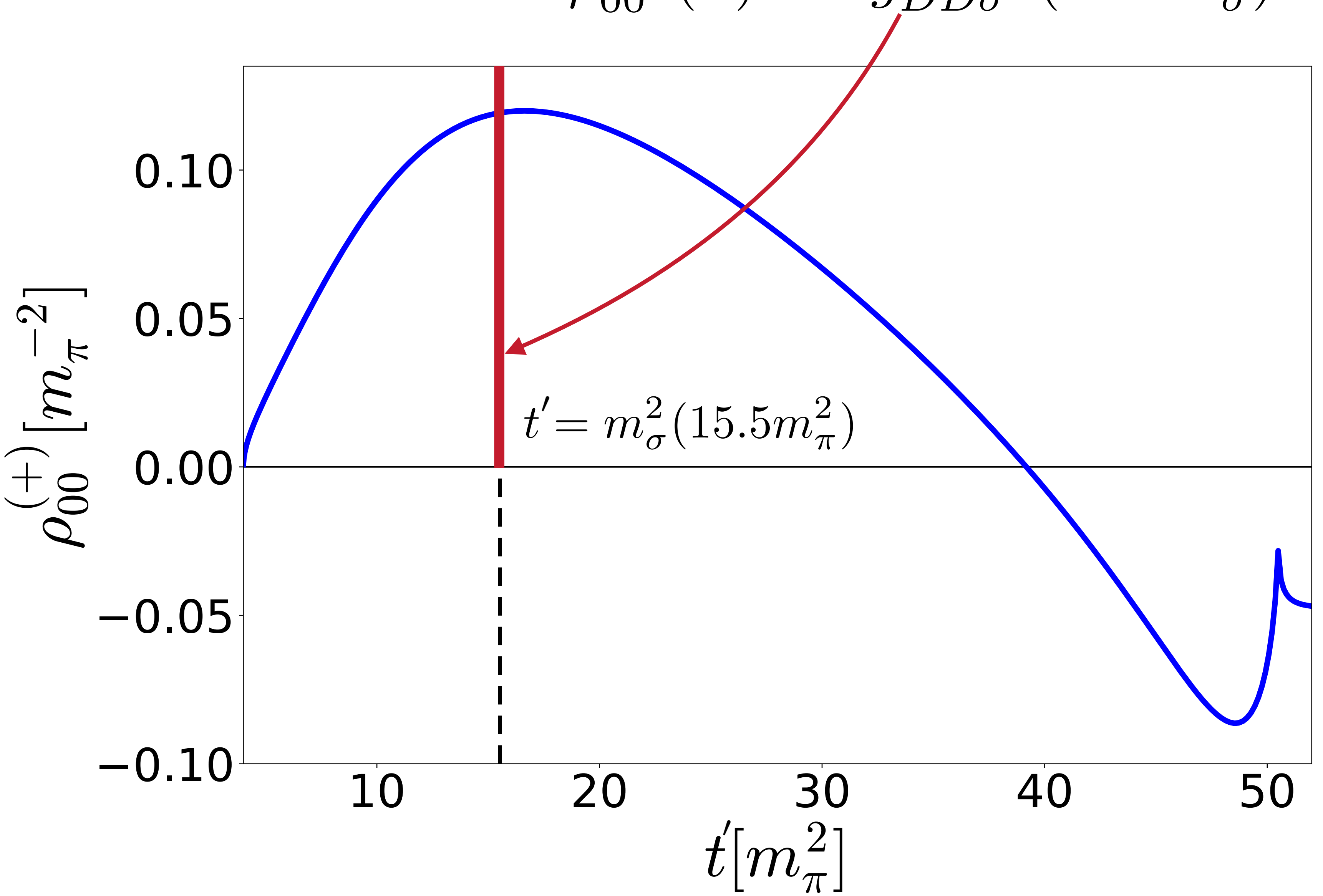
- Analyticity of the S-matrix
  - The s-channel process(DD) is analytically continued from that of t-channel(D$\bar{D}$)



Extracting the coupling constants

- Pole approximation
  - We assume that the spectral function have a zero-width distribution:

$$\rho_{00}^{(+)}(t') = \pi g_{DD\sigma}^2 \delta(t' - m_\sigma^2)$$



Extracting the coupling constants

- Coupling constants from the pole approximation

$$g^2_{DD\sigma} \approx \frac{t - m^2_\sigma}{\pi} \int_{4m^2_\pi}^{\infty} \frac{\rho^{(+)}_{00}(t')\, dt'}{t - t'}$$

- Off-mass-shell coupling constants
  - Coupling constant as a phenomenological form factor

$$g^2_{DD\sigma}(t) = \frac{t - m^2_\sigma}{\pi} \int_{4m^2_\pi}^{\infty} \frac{\rho^{(+)}_{00}(t')}{t - t'} \left(\frac{\Lambda^2_\sigma - t'}{\Lambda^2_\sigma - t}\right)^2 dt' \quad \text{for} \quad t \le 0$$

  - Fitting the t-dependent coupling constants to monopole-type form factor:

    We can determine the value of the on-mass-shell coupling constants

The extracted coupling constants will describe very well the DD(D*D*) amplitudes

DD coupling constants

- Coupling constants

	Present work	[29–31]	[32]
$g_{DD\sigma}$	1.50	0.76	3.4
$g_{DD\rho}$	1.65	3.71	2.6
$g_{D^*D^*\sigma}$	5.21	0.76	3.4
$g_{D^*D^*\rho}$	6.47	3.71	2.6
$f_{D^*D^*\rho}$	6.37	4.64	11.7

[29] X. Liu, Z. G. Luo, Y. R. Liu and S. L. Zhu, Eur. Phys. J. C **61** (2009) 411 [arXiv:0808.0073 [hep-ph]].
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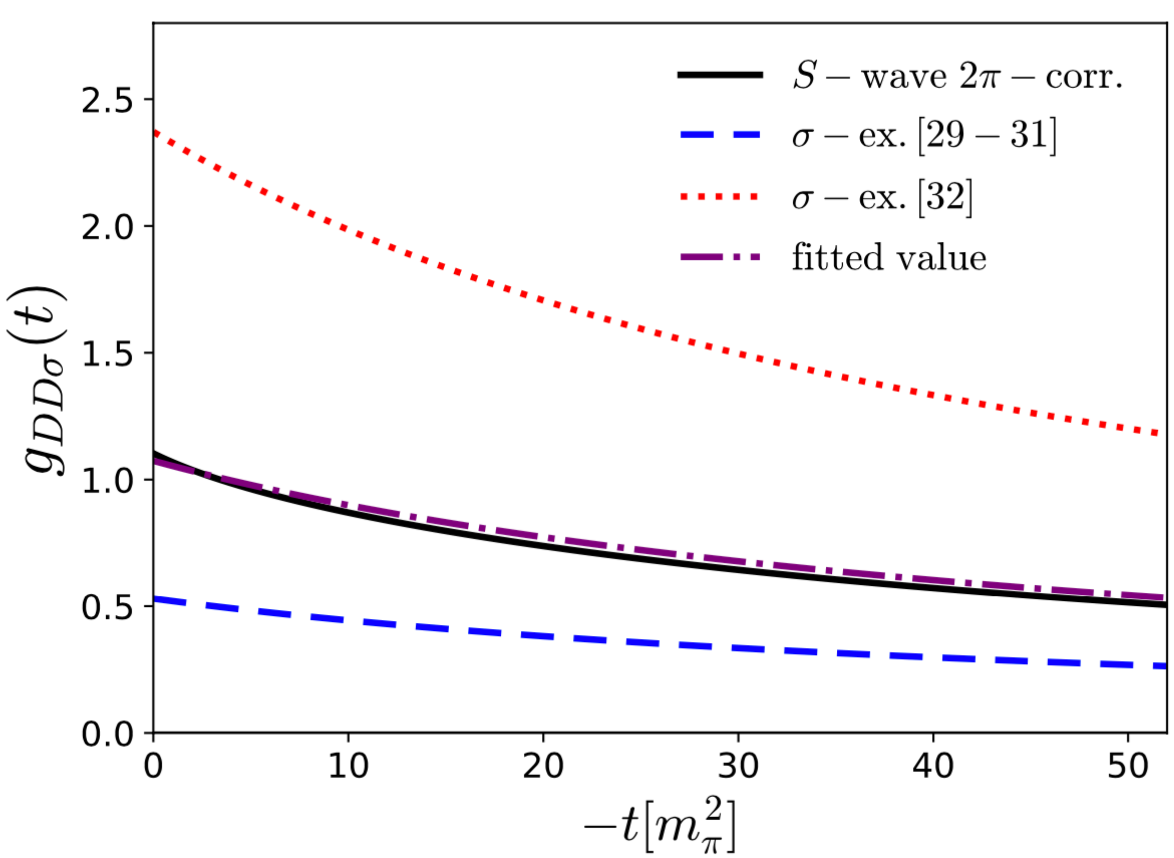

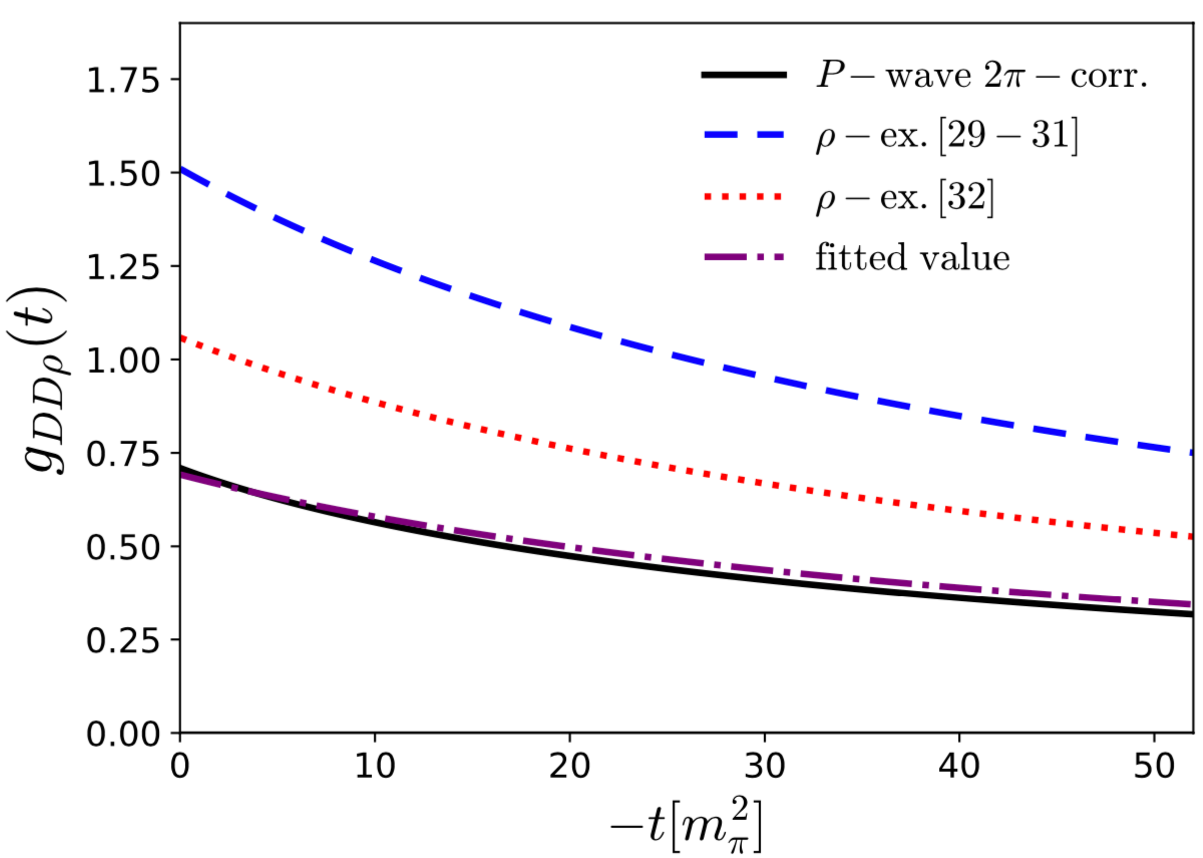

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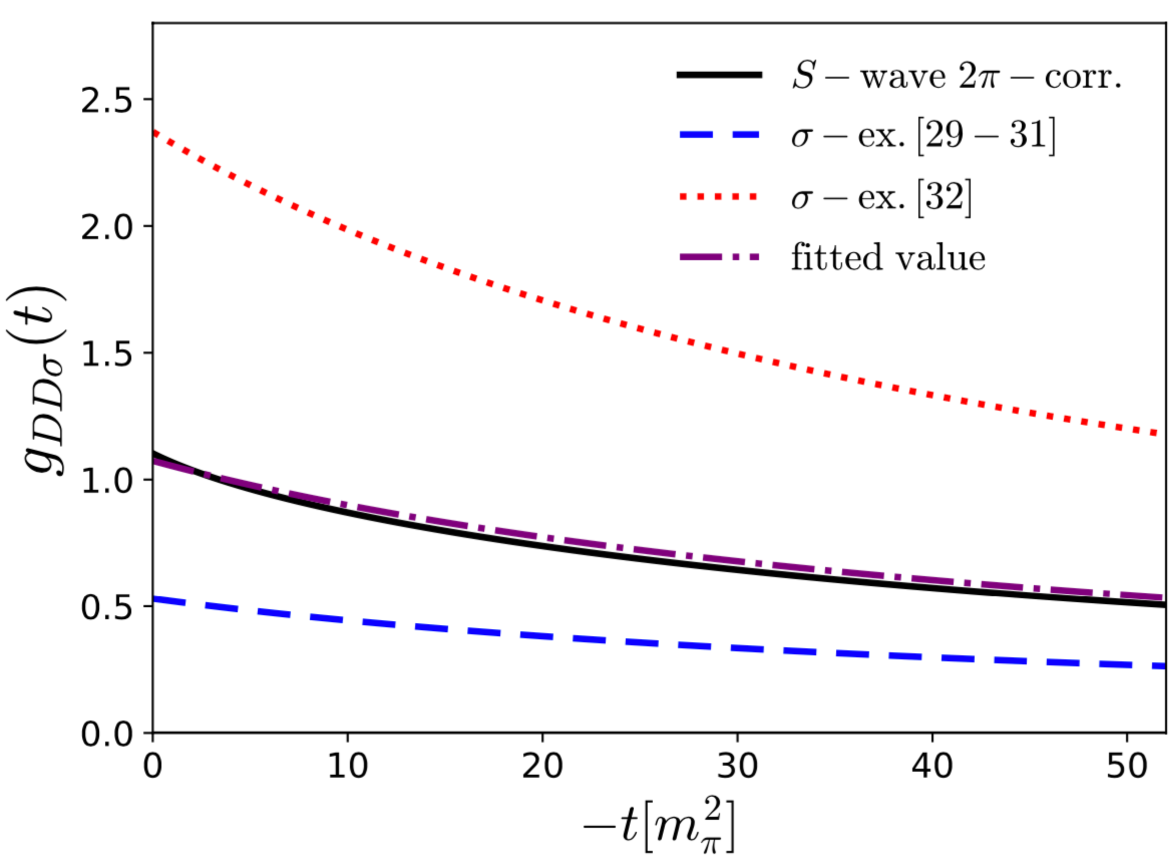
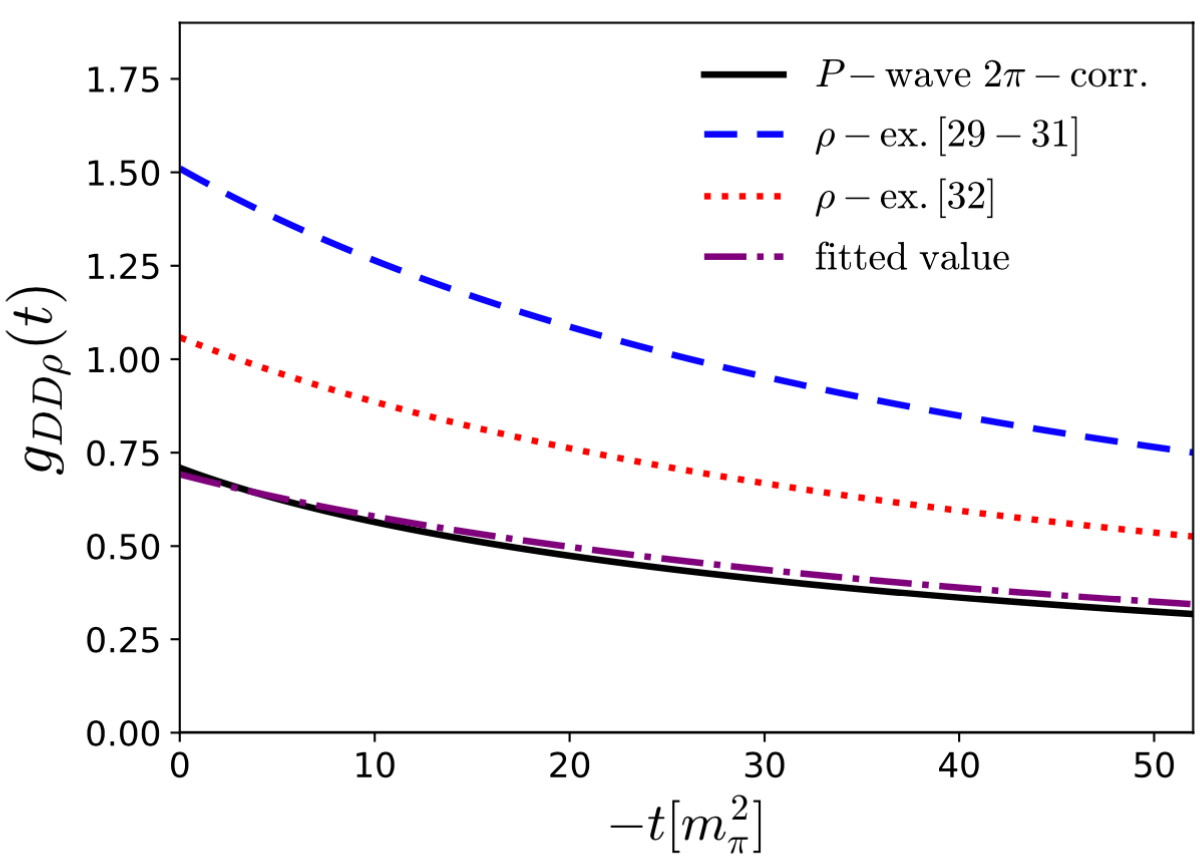
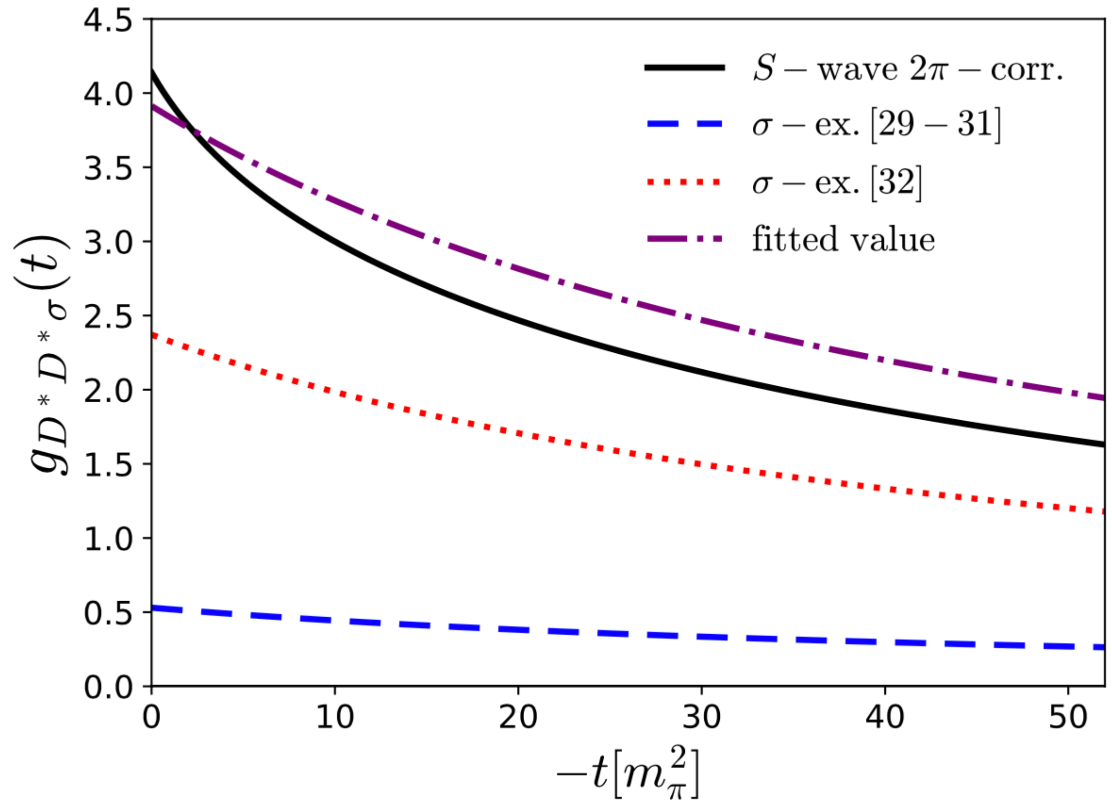
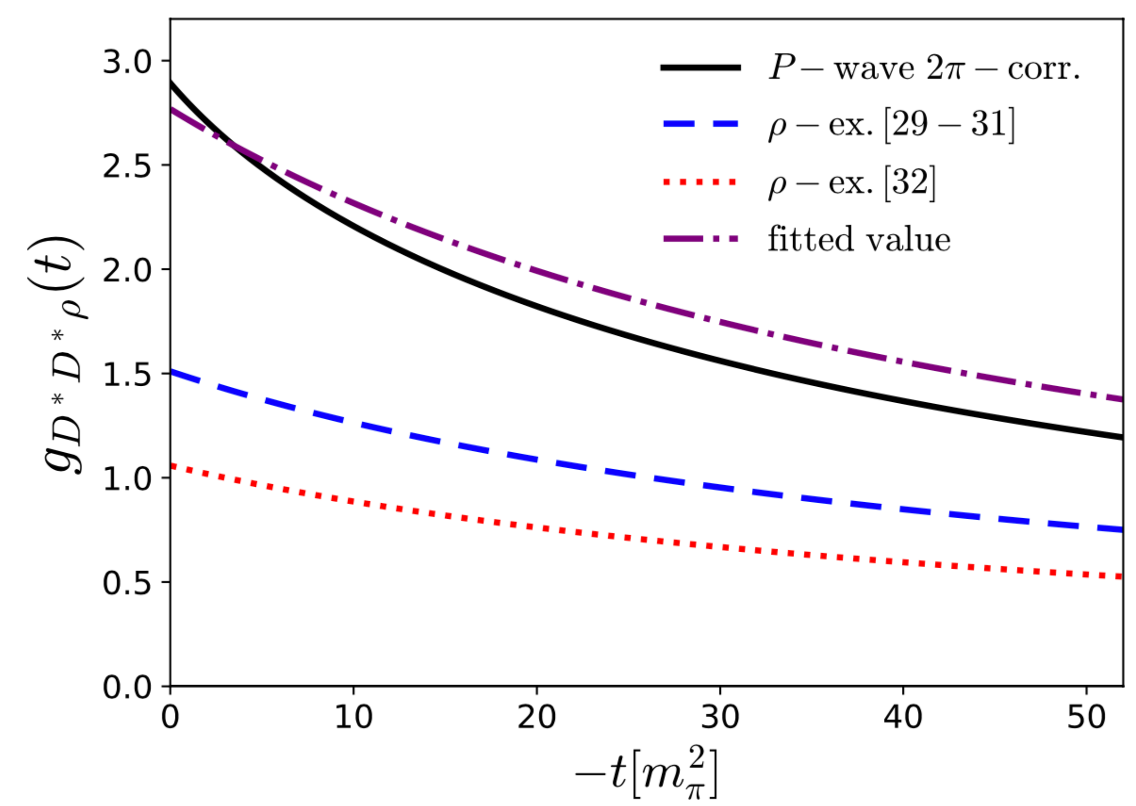
$$F(t) = \frac{\Lambda^2 - m_{\sigma(\rho)}}{\Lambda^2 - t}, \quad \Lambda = 1.0 \text{ GeV},$$
$$m_\sigma = 0.55 \text{ GeV}, \quad m_\rho = 0.70 \text{ GeV}$$

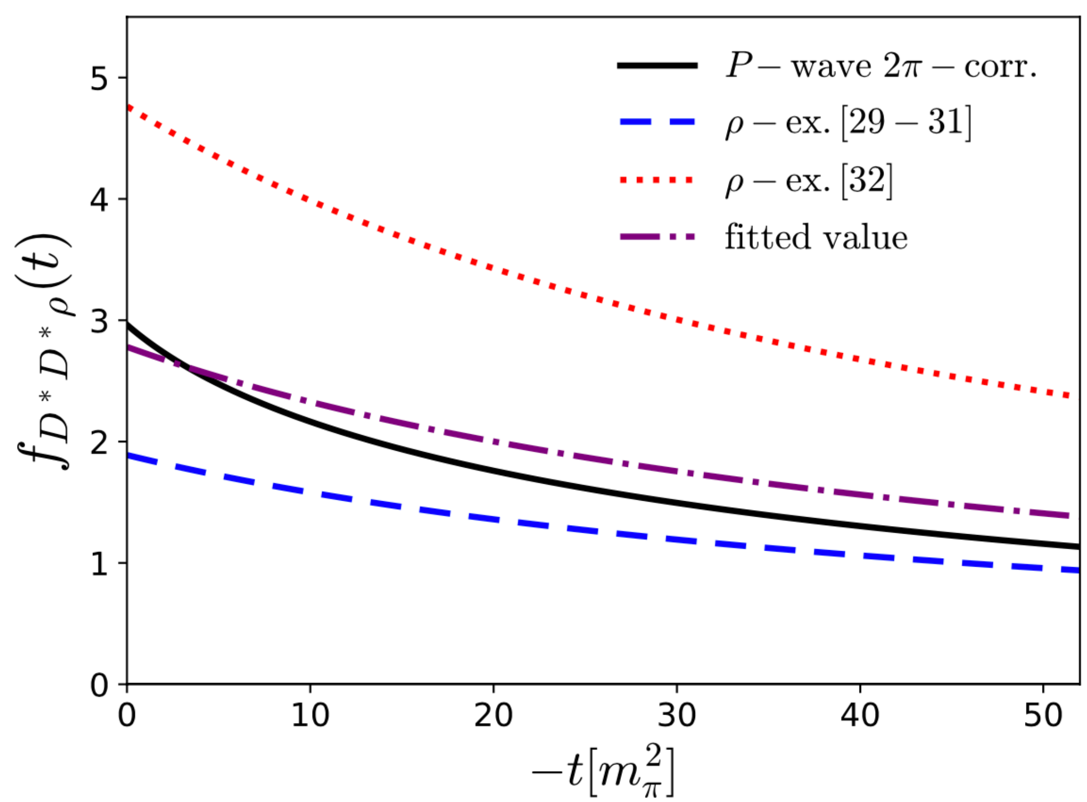




D*D* coupling constants







- Coupling constants

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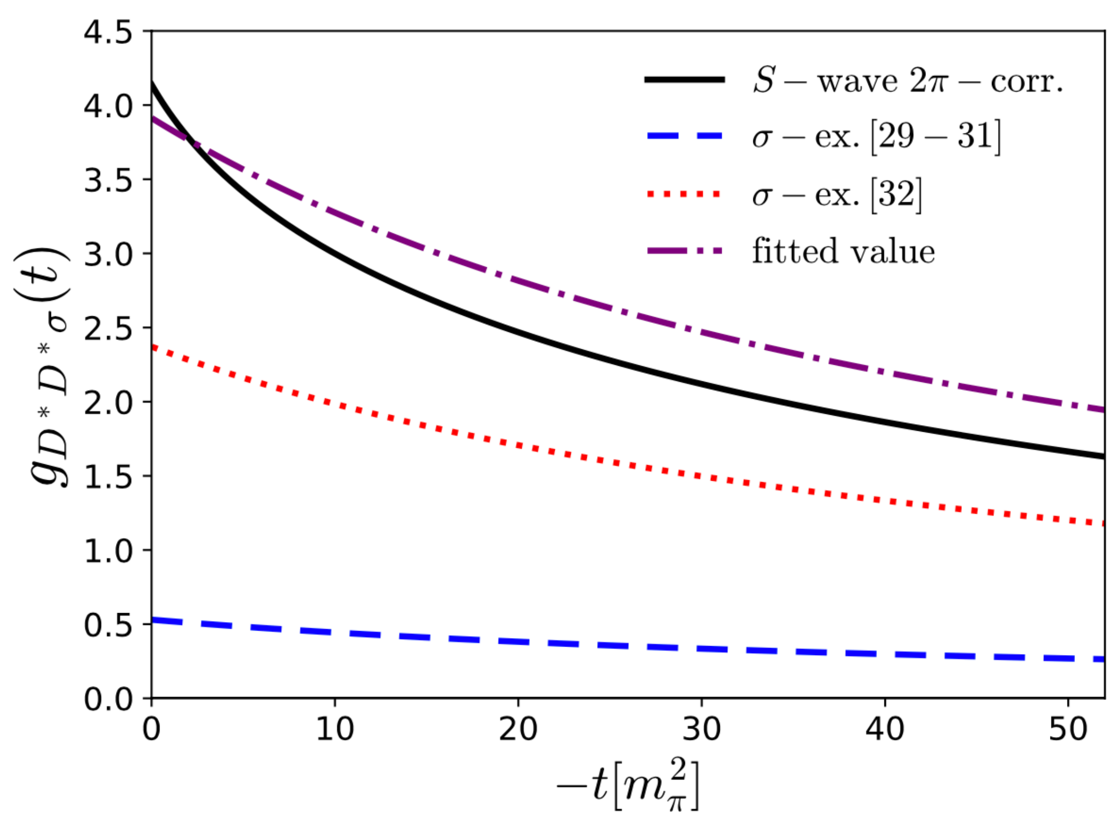

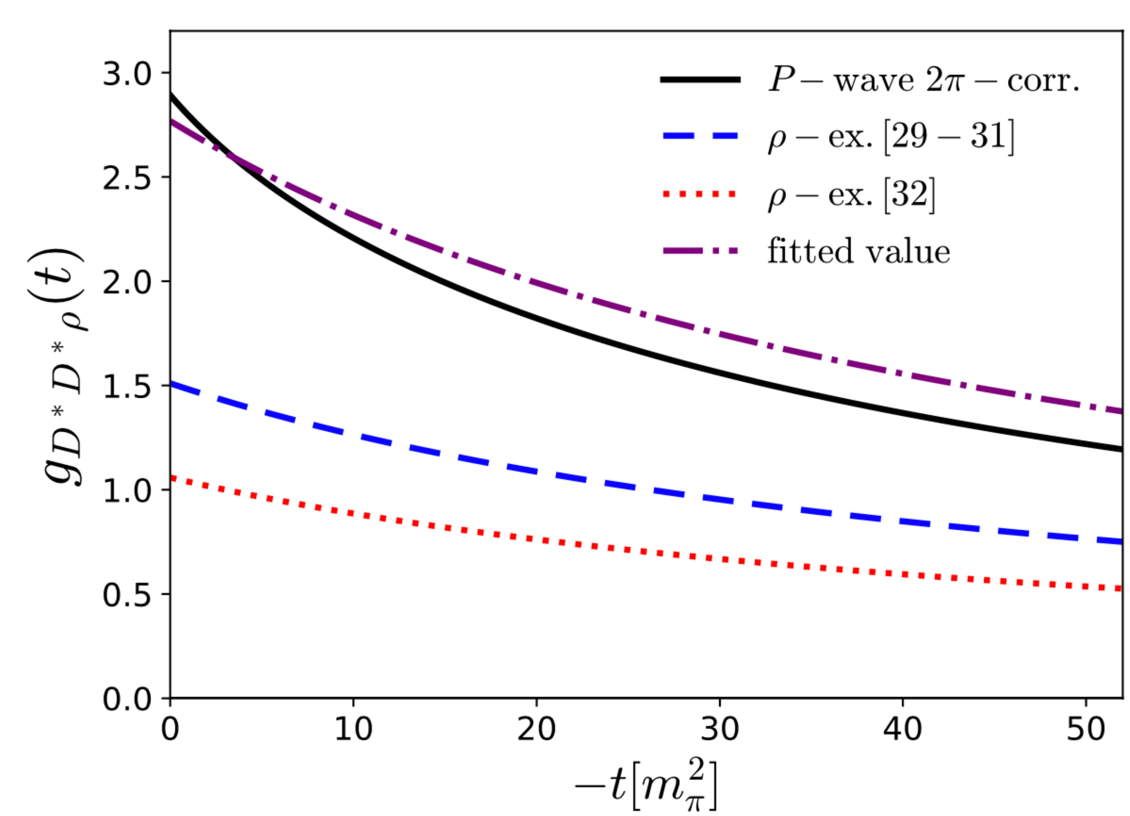

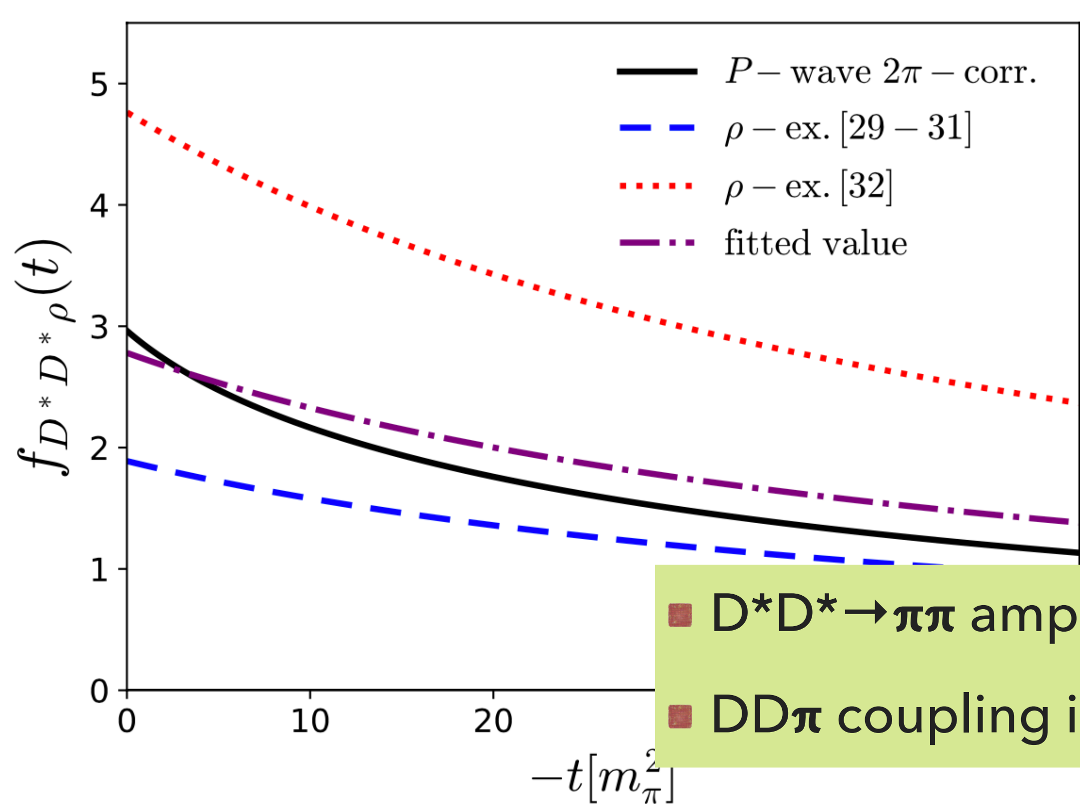

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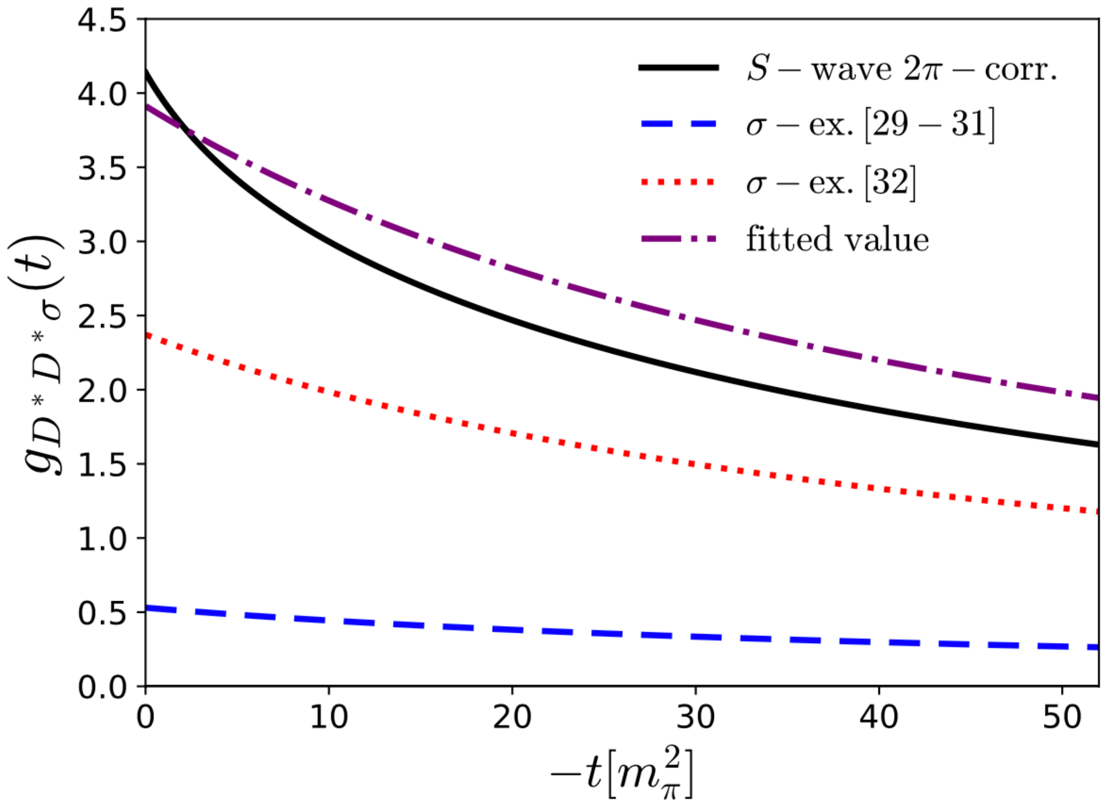
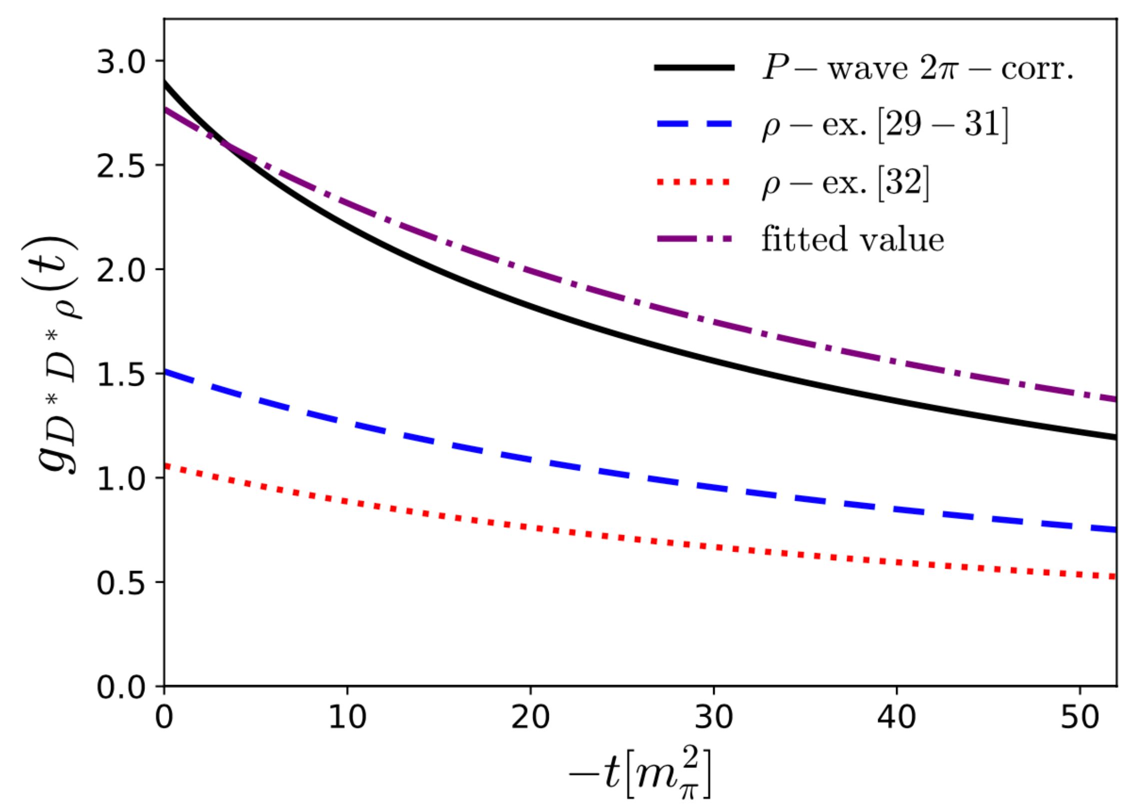
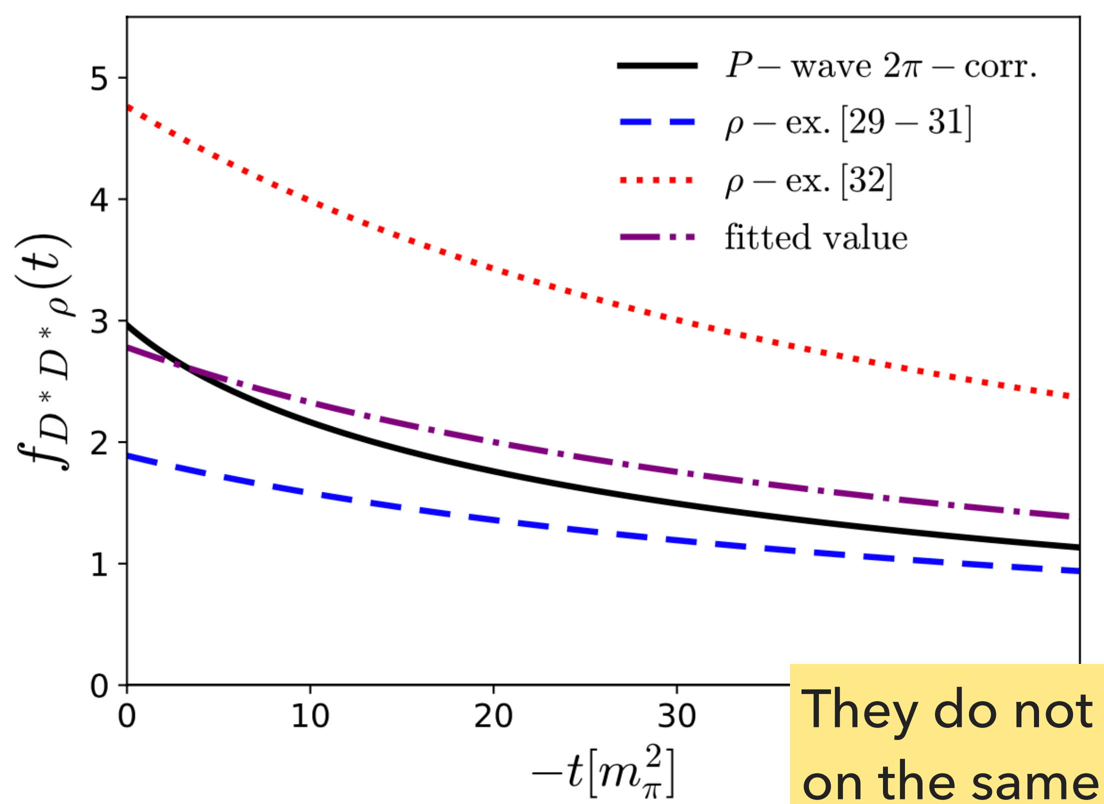
- D*D*→**ππ** amplitudes have stronger momentum dependence
- DD**π** coupling is forbidden due to parity conservation

D*D* coupling constants







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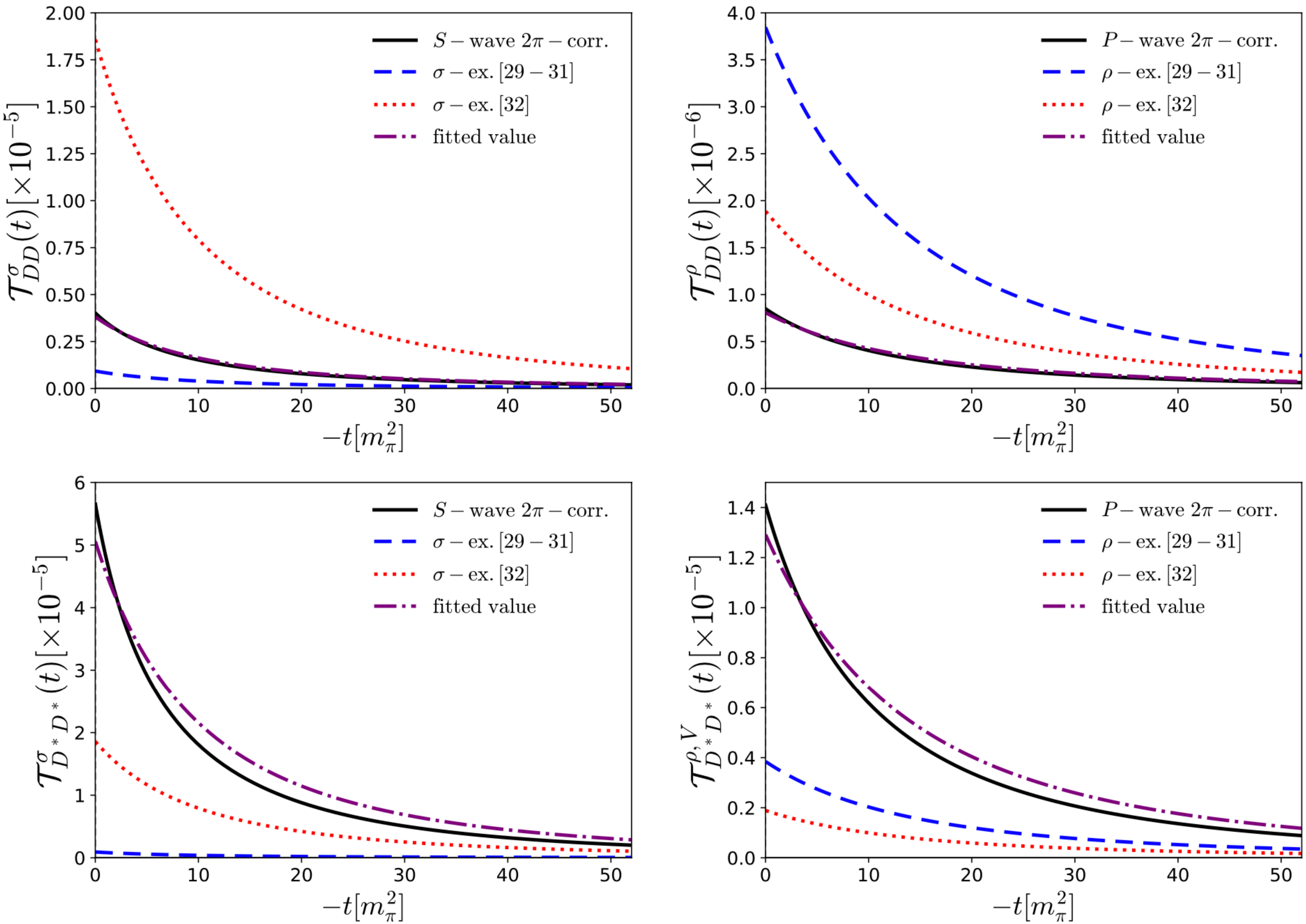
They do not agree with each other, even these models are based on the same heavy effective Lagrangians.

DD and D*D* amplitudes



BB and B*B* coupling constants

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$g_{BB\sigma}$	7.05	0.76	3.4
$g_{BB\rho}$	8.92	3.71	2.6
$g_{B^*B^*\sigma}$	9.47	0.76	3.4
$g_{B^*B^*\rho}$	10.1	3.71	2.6
$f_{B^*B^*\rho}$	29.8	4.64	11.7

In those models, the coupling constants for charmed and beauty mesons are considered to be coincident with each other.

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In those models, the coupling constants for charmed and beauty mesons are considered to be coincident with each other.

However, according to our calculation B and B* coupling constants are much larger than those for D and D* :

$$g_{DD^*\pi} = \frac{2g}{f_\pi}\sqrt{M_D M_{D^*}} = 17.3$$

$$g_{BB^*\pi} = \frac{2g}{f_\pi}\sqrt{M_B M_{B^*}} = 45.0$$

Conclusion

- We derived the spectral functions for DD and D*D* from the pseudo-physical $D\bar{D}(D^*\bar{D}^*)\rightarrow\pi\pi$ amplitudes
- We extracted those coupling constants through the pole approximation
- Having introduced a monopole-like form factor, we examined successfully the off-mass-shell coupling constants.
- Coupling constants for the beauty mesons are much larger than D and D* coupling constants.
- These coupling constants will be useful for understanding the exotic heavy mesons.


THANK YOU