

Bench test of $\text{LaBr}_3(\text{Ce})$ Detector array for fast-timing gamma-ray measurements and GCD method

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Jaehwan Lee



Lifetime measurement for Nuclear Structure study

- Nuclear Deformation

$$R(\theta, \phi) = R_0 [1 + \sum_{\lambda} \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi)]$$

$\lambda=2$: quadrupole deformation

in principal axis and $O(\lambda>3)=0$,

$$\begin{aligned} R(\theta, \phi) &\approx R_0 [1 + \alpha_{0,0} Y_{0,0} + \alpha_{2,2} (Y_{2,2} + Y_{2,-2})] \\ &= R_0 [1 + \beta_2 \cos \gamma Y_{0,0} - \frac{1}{\sqrt{2}} \beta_2 \sin \gamma (Y_{2,2} + Y_{2,-2})] \end{aligned}$$

- Transition matrix (Wigner Eckart Theorem)

$$\langle I_2 M_2 | \hat{O}_{\lambda\mu} | I_1 M_1 \rangle = \frac{1}{\sqrt{2I_2+1}} \langle I_1 M_1 \lambda\mu | I_2 M_2 \rangle \langle I_2 || \hat{O}_{\lambda} || I_1 \rangle$$

→ Reduced transition probability $B(O_{\lambda}; I_i \rightarrow I_f) = \frac{1}{2I_i} \left| \langle I_f || \hat{O}_{\lambda} || I_i \rangle \right|^2$

and transition rate $\frac{1}{\tau} = T(O_{\lambda}) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{k^{2\lambda+1}}{\hbar} B(O_{\lambda})$

Lifetime measurement for Nuclear Structure study

For E2 transition,

$$B(E2; I \rightarrow I - 2) = \{5/16\pi\} Q_0^2 | \langle I, K, 2, 0 | I - 2, K \rangle |^2$$

$$\tau[\text{ps}] = \frac{0.0816}{E_\gamma^5[\text{MeV}] \times B[e^2 b^2]}$$

Provided lifetime of first 2+ state, electric quadrupole moment(Q_0), quadrupole deformation(beta_2) are directly calculated!

LaBr₃(Ce) gamma-ray detector array

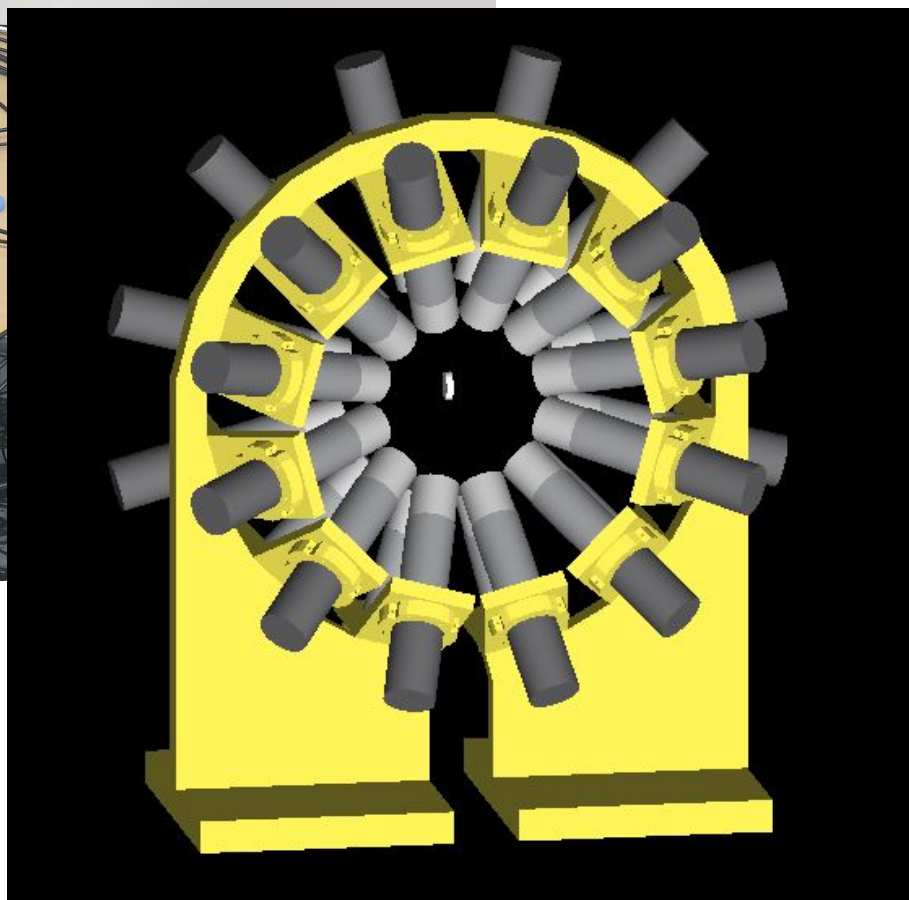


Bench test configuration

LaBr₃(Ce) gamma-ray detector array



Bench test configuration



24 detectors with supporting structure, in CAD

LaBr₃(Ce) Scintillator

- Technical specification

	LaBr ₃ (Ce)	NaI(Tl)
Light yield*	63 photons/keV γ	55 photons/keV γ
wavelength of maximum emission	380nm	415nm
Energy resolution @662keV	2.6% FWHM	6.5% FWHM
Density	5.08g/cm ³	3.67g/cm ³
Radiation length	1.8cm	2.6cm
Decay time*	16ns	250ns

PMT FADC

- R13408 PMT, Hamamatsu

Tube size	Dia.38 mm
wavelength	300~650nm, peak 420nm
Dynode stages	8
Anode-cathode supply voltage	1500V
Gain typ.	5.3×10^5
Dark current (after 30min.)	Typ. 3nA, Max. 30nA
Rise time typ.	1.2ns
Transit time typ.	13ns (spread 0.19ns)
Pulse linearity	2%dev 20mA, 5%dev 50mA

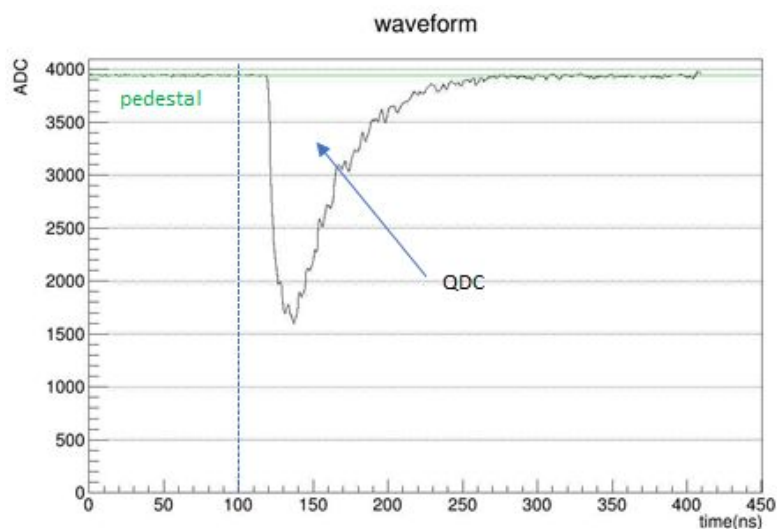
- V1742 FADC, CAEN

Channels	32+2
Sampling rate	5, 2.5, 1, 0.75 GSa/s selectable
Sampling length	1024, 520, 256, 136 selectable
Dead-time due to conversion	110 μ s, 181 μ s
Input dynamic range	1Vpp
Resolution	4096ch (12bit)

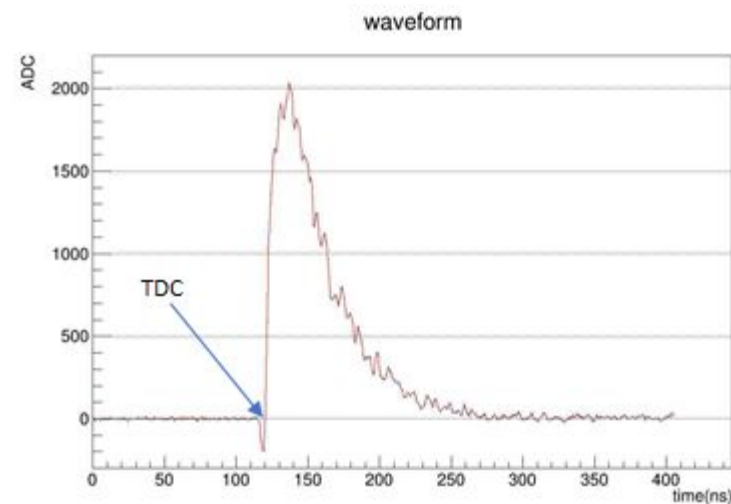
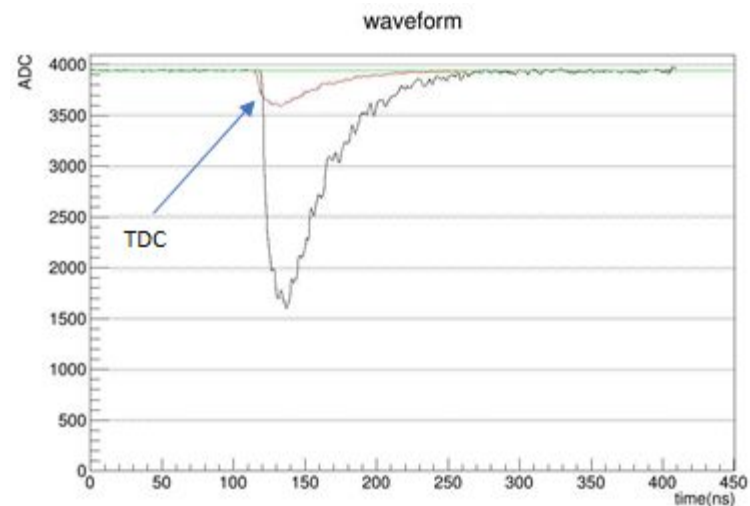
Pulse analysis

- QDC

$$QDC = \sum_i (pedestal - ADC[i])$$

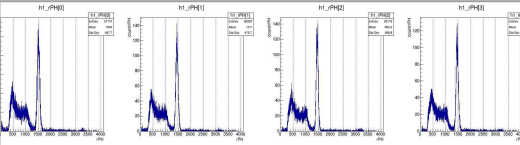
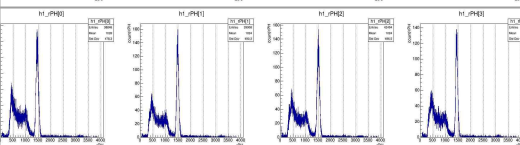
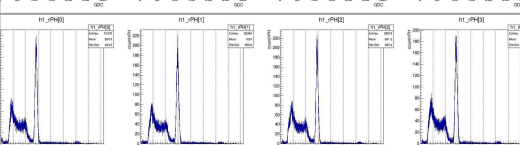


- TDC : CFD method



linear approximation ->

Energy resolution with Cs-137 (individual)

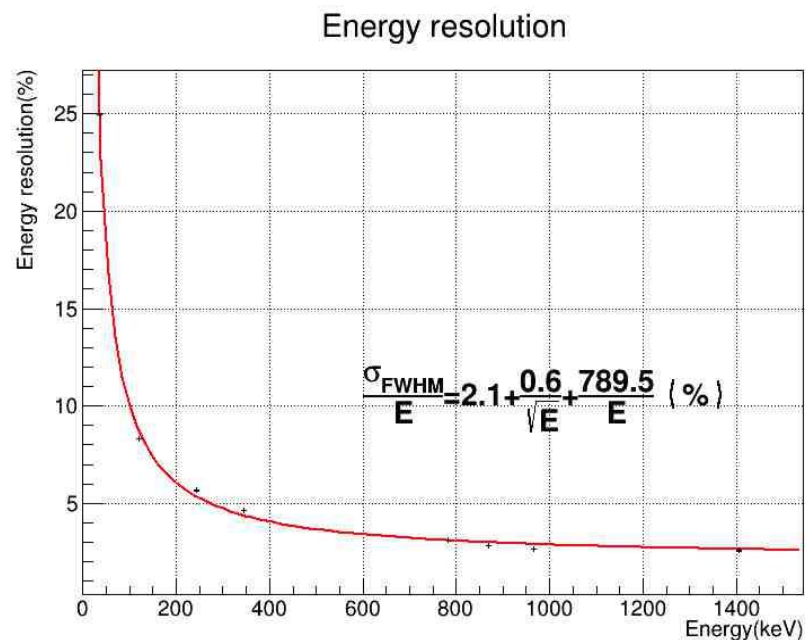
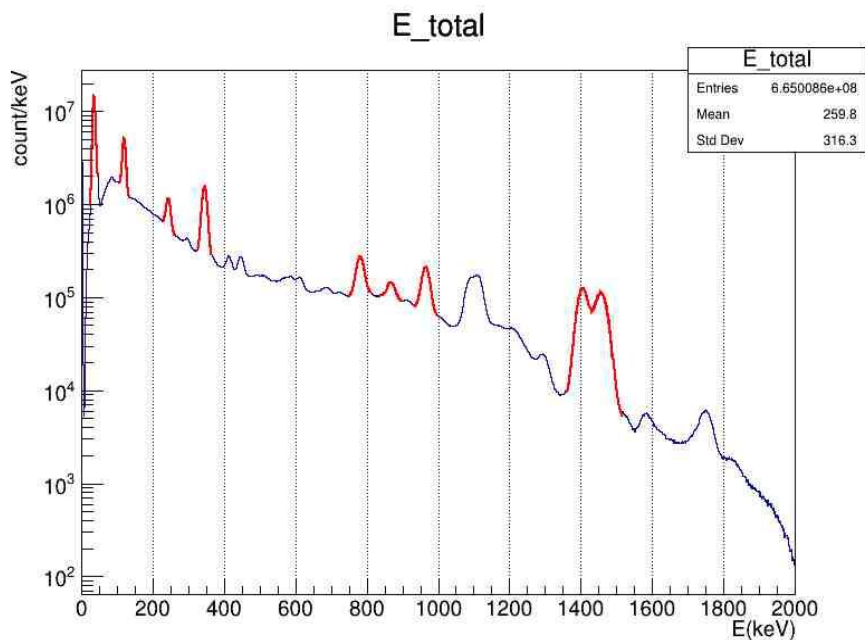
				HV(V)	E res(%)
	Set0	Ch0	#0	1130	3.45
		Ch1	#1	1010	3.32
		Ch2	#2	1050	3.27
		Ch3	#3	1040	3.26
	Set1	Ch0	#4	1090	3.43
		Ch1	#5	1170	3.33
		Ch2	#6	1060	3.19
		Ch3	#7	1100	3.27
	Set2	Ch0	#8	1130	3.25
		Ch1	#9	1030	3.18
		Ch2	#10	1030	3.16
		Ch3	#11	1090	3.32

HV lowered to 1000~1200V,
rPH=1500ADC for 662keV

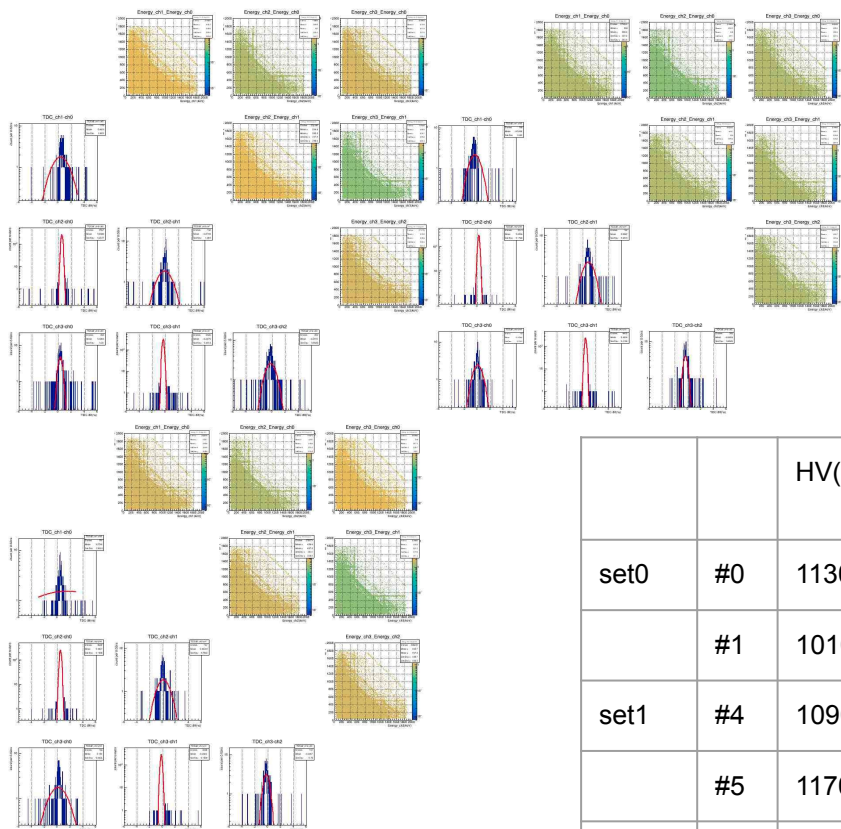
Energy resolution(FWHM)
(fit sig/mean*2.35)
: 3.2%~3.5% @662keV

cf)prototype: 3.9% @662keV

Energy resolution with Eu-152 (12ea array)



Time resolution with Na-22



- Na-22 : positron annihilation
- 511keV gamma in opposite direction
- T resolution~240ps @511keV

		HV(V)	E res(%)		HV(V)	E res(%)	sig(ps) FWHM	sig/1.414(ps) FWHM
set0	#0	1130	3.91	#2	1050	3.67	342.0(40)	241.9(28)
	#1	1010	3.71	#3	1040	3.94	337.6(34)	238.7(24)
set1	#4	1090	3.85	#6	1060	3.65	329.9(34)	233.3(24)
	#5	1170	3.83	#7	1100	3.91	347.2(39)	245.6(27)
set2	#8	1130	3.82	#10	1030	3.73	350.5(37)	247.9(26)
	#9	1030	3.72	#11	1090	3.79	321.8(36)	227.6(25)

Lifetime decision - conv

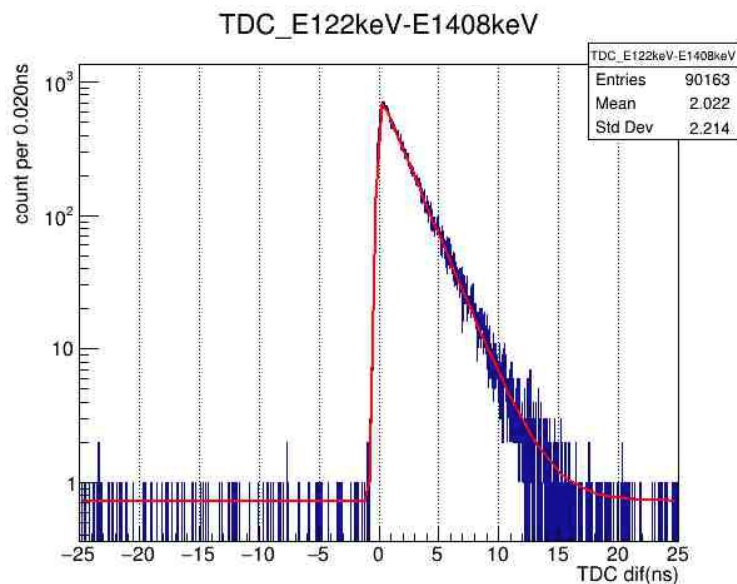
Time difference distribution of gamma cascade

has visible tail for $\tau > \sigma$

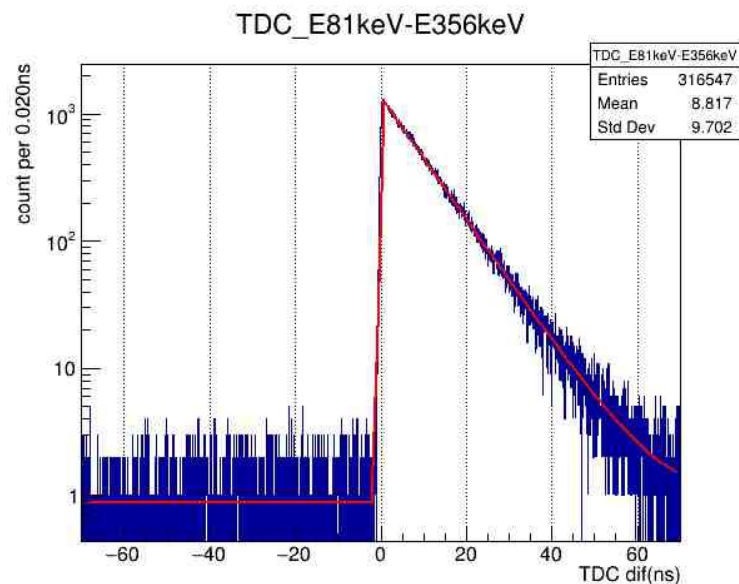
->fitting is possible with conv. function

$$(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)g(t - \tau) d\tau$$

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}, g(t > 0) = Ae^{-\lambda t}$$



Eu-152 122keV | 1408keV cascade
ref $\tau=2024(16)$ ps, measured $\tau=2018(8)$ ps



Ba-133 81keV | 356keV cascade
ref $\tau=9064(20)$ ps, measured $\tau=9023(18)$ ps

Lifetime decision - conv

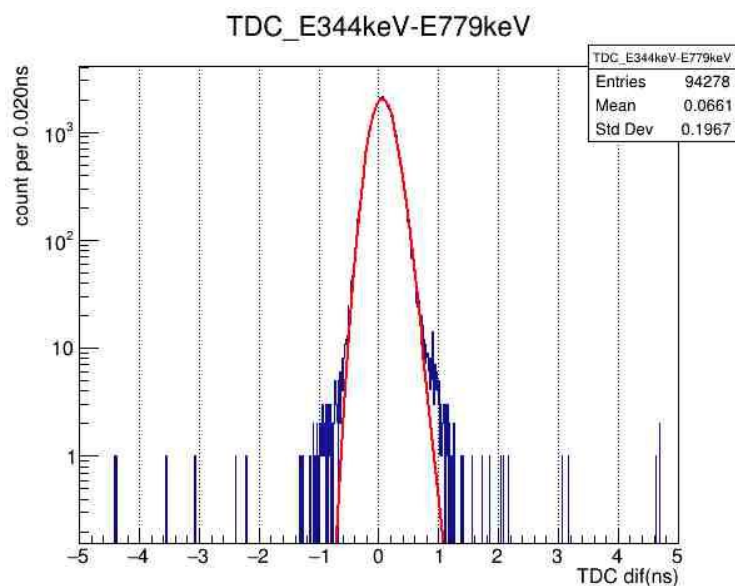
Time difference distribution of gamma cascade

: tail is not distinguishable for $\tau < \sigma$

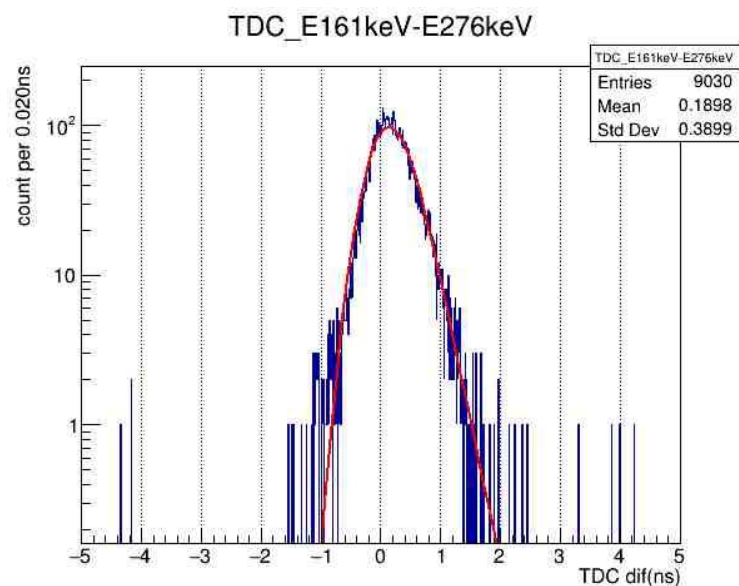
-> fitting is impossible with conv. function

$$(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)g(t - \tau) d\tau$$

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}, g(t > 0) = Ae^{-\lambda t}$$



Eu-152 344keV | 779keV cascade
ref $\tau=46.7(25)$ ps, measured $\tau=\text{NaN}$



Ba-133 161keV | 276keV cascade
ref $\tau=248(6)$ ps, measured $\tau=231(7)$ ps?

Lifetime decision - GCD

“Mean” lifetime

$$\frac{\int_0^\infty tP(t) dt}{\int_0^\infty P(t) dt} = \frac{1}{\lambda} = \tau$$

In reality,

$$\frac{\int_0^\infty tP(t) dt}{\int_0^\infty P(t) dt} = \frac{\sum^{count} t_{measure}}{count} = \tau + D$$

By averaging combinations of start-stop channel, energy independent offsets vanish.

$$TDC_i(E) = T(E) + D_i(E) + P_i$$

$$\Delta_{ij}TDC(E_d, E_f) = [T(E_d) + D_i(E_d) + P_i] - [T(E_f) + D_j(E_f) + P_j] \\ = \{T(E_d) - T(E_f)\} + \{D_i(E_d) - D_j(E_f)\} + \{P_i - P_j\}$$

$$C_{ij}(E_d, E_f) = \{\text{mean of } \Delta_{ij}TDC(E_d, E_f)\} \\ = \tau(E_d, E_f) + \{D_i(E_d) - D_j(E_f)\} + \{P_i - P_j\}$$

$$\bar{C}(E_d, E_f) = \{\text{avg for all combination of } i, j\} \\ = \tau(E_d, E_f) + \frac{(N_{ch}-1)\{\sum_i^N D_i(E_d) - \sum_j^N D_j(E_f)\}}{N_{ch}(N_{ch}-1)} + \frac{(N_{ch}-1)\{\sum_i^N P_i - \sum_j^N P_j\}}{N_{ch}(N_{ch}-1)} \\ = \tau(E_d, E_f) + \frac{\sum_i^N \{D_i(E_d) - D_i(E_f)\}}{N_{ch}}$$

Assuming energy dependent delay $D(E) = 0$,

$$\tau(E_d, E_f) = \bar{C}(E_d, E_f)$$

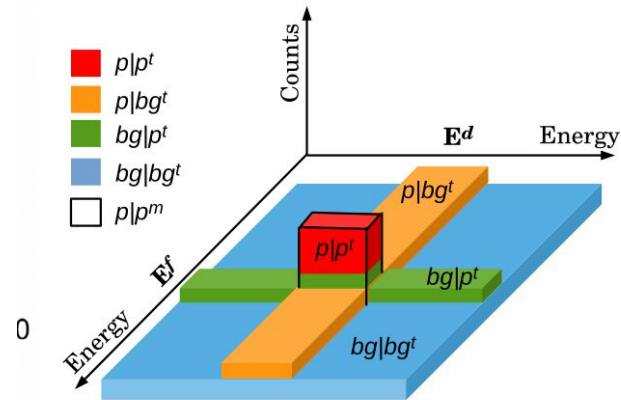
*linear approx in CFD

Background analysis

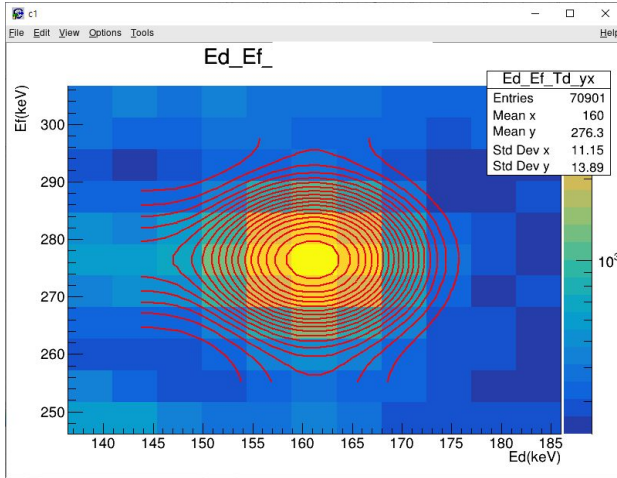
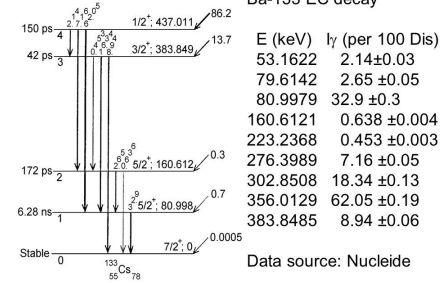
$$\sum^{count} t_{measure} = \bar{C}_{exp} \times (\text{total count}) \\ = \bar{C}_{FEP} \times (p = FEP \text{ count}) + \bar{C}_{bg} \times (b = bg \text{ count})$$

$$\tau = \bar{C}_{FEP} = \bar{C}_{exp} \cdot \frac{p+b}{p} - \bar{C}_{bg} \cdot \frac{b}{p} \\ = \bar{C}_{exp} + \frac{\bar{C}_{exp} - \bar{C}_{bg}}{p/b}$$

(b)



Lifetime decision - GCD



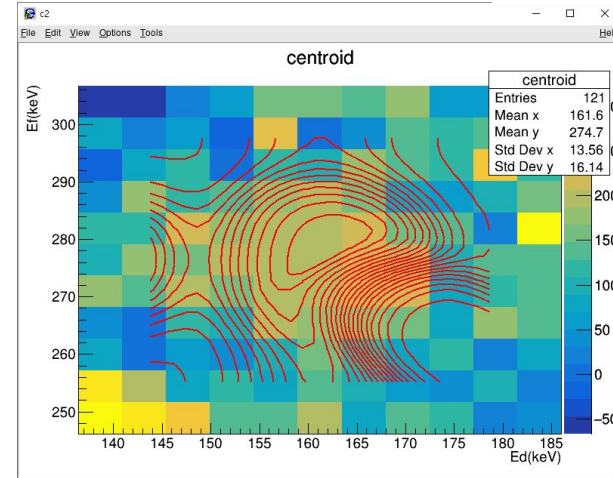
*bin size=sigma

xy gaus(p_0 =amp0, p_1 =xm, p_2 =xs, p_3 =ym, p_4 =ys)
 +gaus(amp=A(x), ym, ys)+gaus(amp=B(y), xm, xs)+BG

$A(x) = \text{amp0} * p_5 * (0.5(1 - \text{erf}(x_m/x_s)) + p_6 + p_7 * (x - x_m))$

$B(x) = \text{amp0} * p_8 * (0.5(1 - \text{erf}(y_m/y_s)) + p_9 + p_{10} * (y - y_m))$

$BG = \text{amp0} * p_{11}$



$C(p|p) = p_{16}$

$C(p|bg) = p_{17} + p_{18} * (x - x_m)$

$C(bg|p) = p_{19} + p_{20} * (y - y_m)$

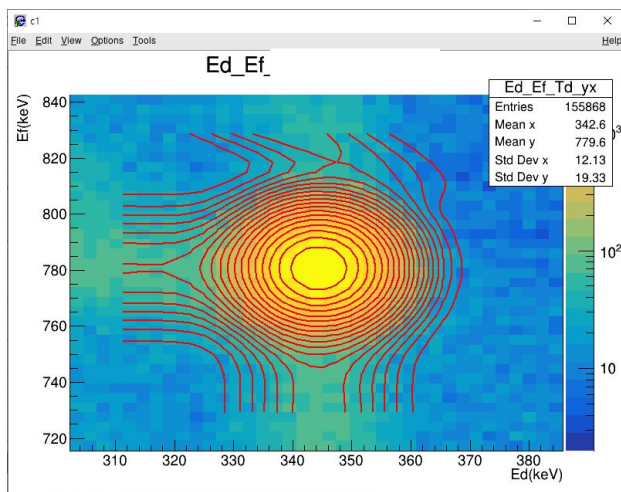
$C(p|p) = p_{21}$

Ba-133 -> Cs-133

161keV | 276keV

Lifetime ref=248(6)ps, measured=254(7)ps

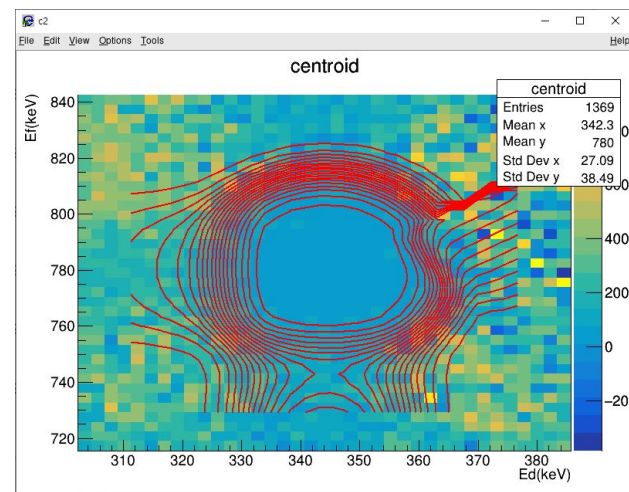
Lifetime decision - GCD



*bin size=sigma

xy gaus(p0=amp0, p1=xm, p2=xs, p3=ym, p4=ys)
 +gaus(amp=A(x), ym, ys)+gaus(amp=B(y), xm, xs)+BG

$A(x) = \text{amp0} \cdot p5 \cdot (0.5(1 - \text{erf}(x - xm, xs)) + p6 + p7 \cdot (x - xm))$
 $B(x) = \text{amp0} \cdot p8 \cdot (0.5(1 - \text{erf}(y - ym, ys)) + p9 + p10 \cdot (y - ym))$
 $BG = \text{amp0} \cdot p11$



$C(p|p) = p16$
 $C(p|bg) = p17 + p18 \cdot (x - xm)$
 $C(bg|p) = p19 + p20 \cdot (y - ym)$
 $C(p|p) = p21$

Eu-152 -> Gd-152

344keV | 779keV

Lifetime ref=46.7(25)ps, measured=48.1(08)ps

Summary

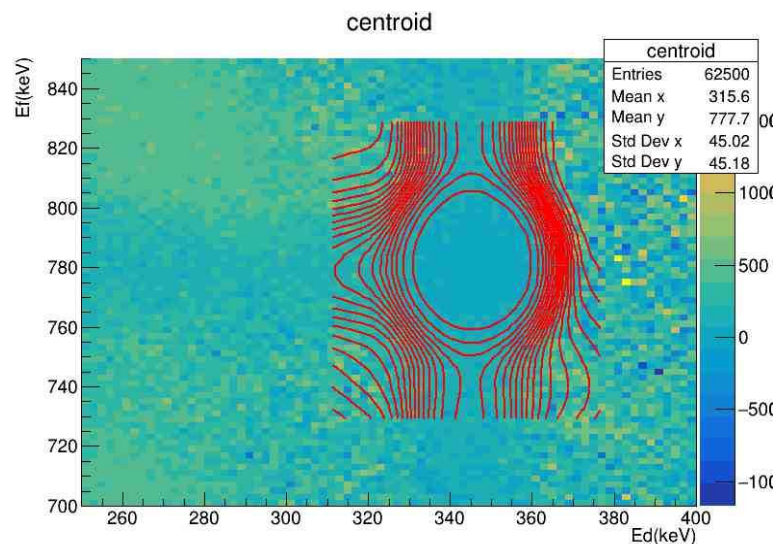
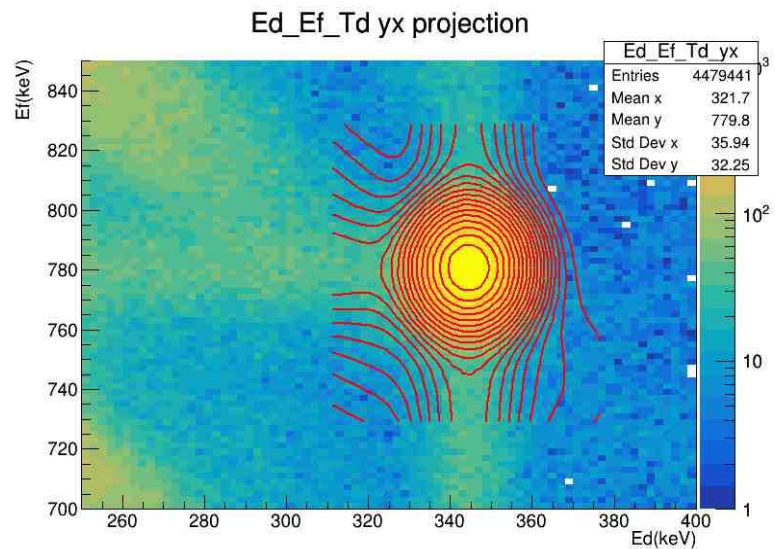
- Lifetime is one of the significant observable for studying nuclear structure
- LaBr₃(Ce) detectors array is under development for fast-timing gamma-ray measurement
 - Energy resolution: 3.3% FWHM @662keV
 - Time resolution: 240ps FWHM @511keV
- Lifetime($\tau > \sigma$) can be decided by convolution function fitting, provided time offset calibration.
- Lifetime($\tau < \sigma$) can be decided by GCD method, provided large number of statistics, BG interpolation.

Future

- Another 12 modules ordered: will be delivered Oct(?)
- Check reliability of GCD method with Ba-133

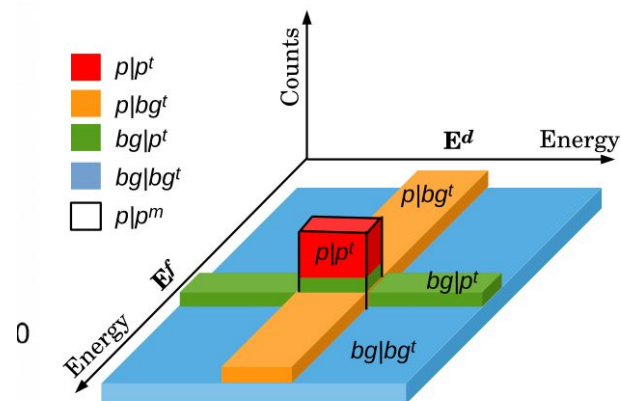
Backup following

Lifetime decision - GCD



p/b ratio at each bin of 2D-histogram
 ->centroid of background interpolated

(b)



Eu-152-> Gd-152 2+

344keV | 779keV

Lifetime ref=46.7(25)ps, measured 46.8(9)ps