Study on Integrator & Differentiator

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Motivation

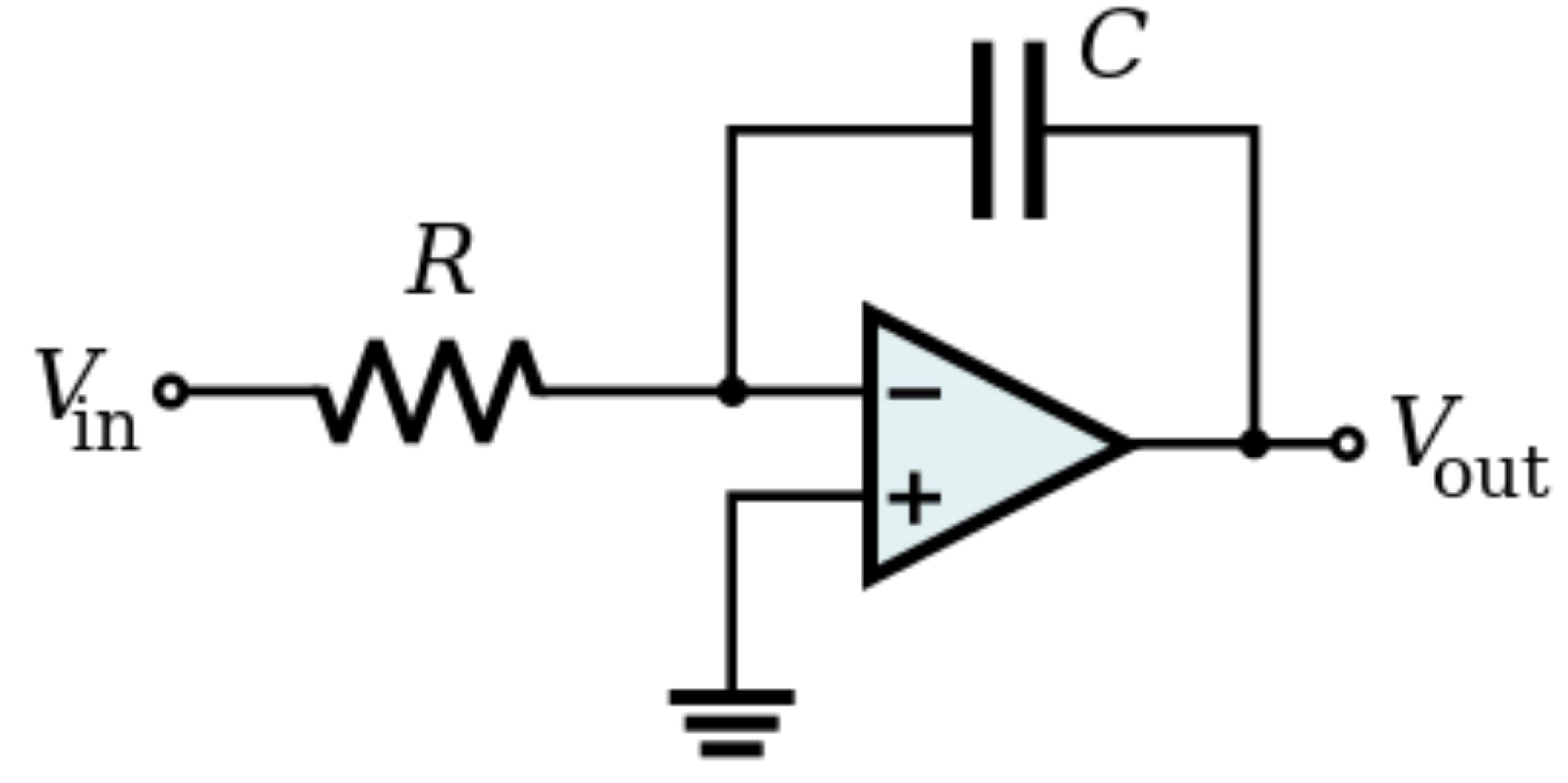
- To measure nano ampere scale current.

Initial Idea

- Using integrator.

Integrator

Integrator is also called “triangle wave generator”. Because a triangle wave is output when a square wave is input.

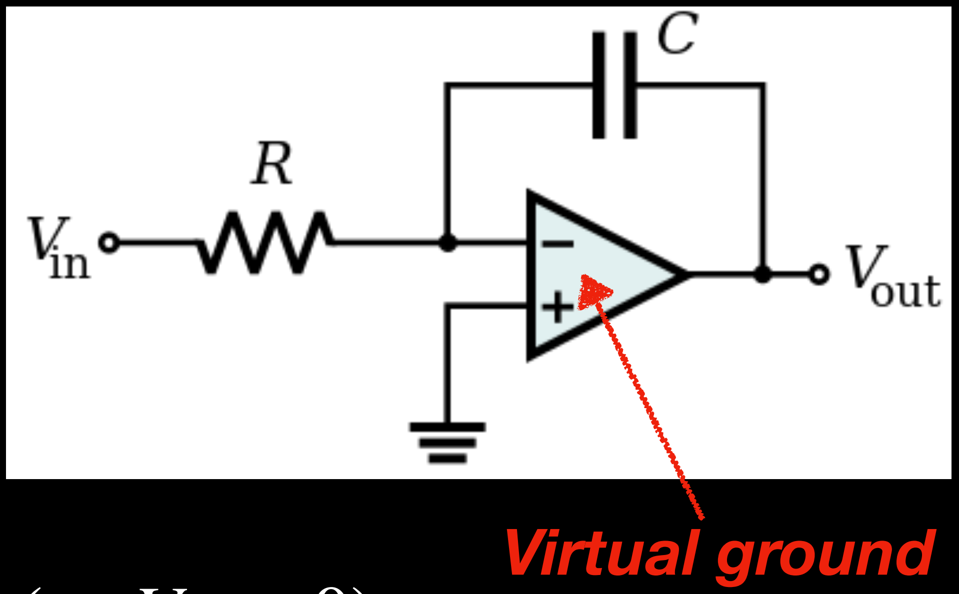


For ideal OP Amp.

Input impedance : ∞
Input current : 0
Input offset voltage : 0

V_- **&** V_+ **are virtually grounded**

$$Q = CV, \frac{dQ}{dt} = C\frac{dV}{dt}$$



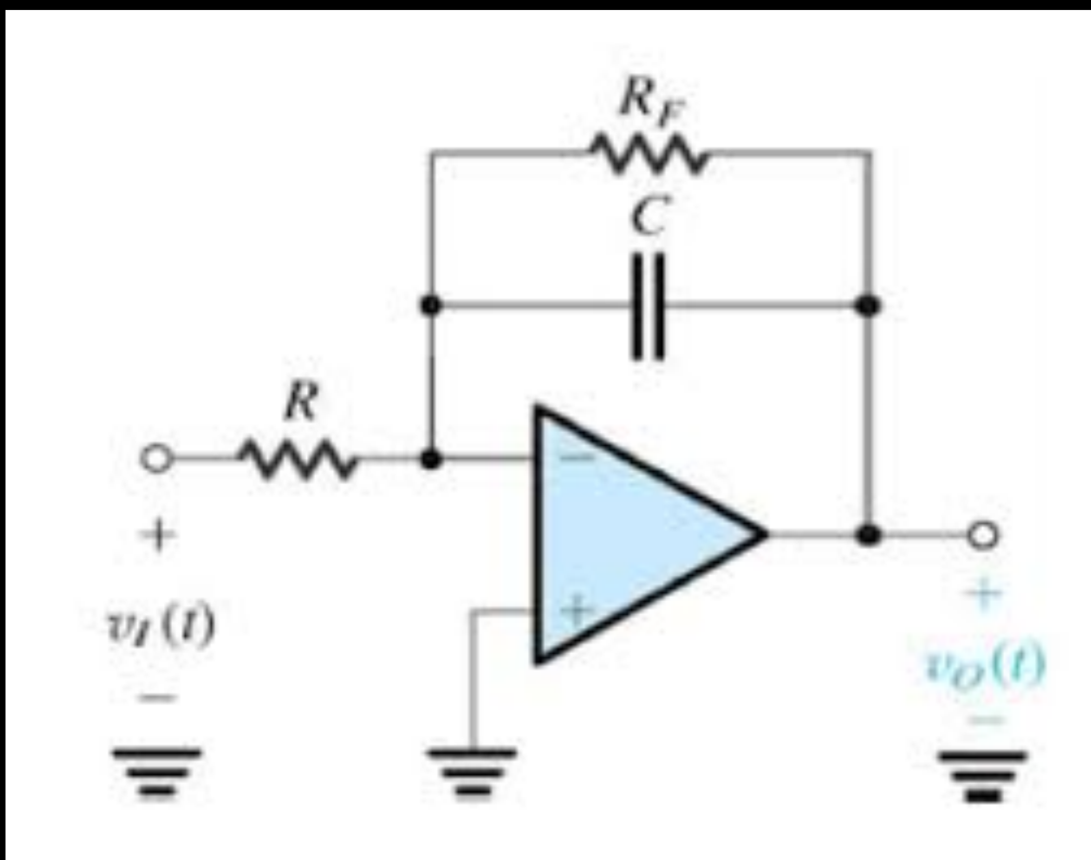
$$\frac{V_{in} - 0}{R} = C\frac{d}{dt}(0 - V_{out}) \qquad (\because V_{-} = 0)$$

$$V_{out} = -\frac{1}{RC}\int V_{in}dt \; + \; C \qquad (C = V_o(t = 0))$$

Actual Integrator

- Pulse may occur due to OP Amp. input offset voltage & bias current.
- Output voltage can be saturated if signal is not centered.
- Small noise can cause “Oscillation” through feedback loop.

Actual Integrator



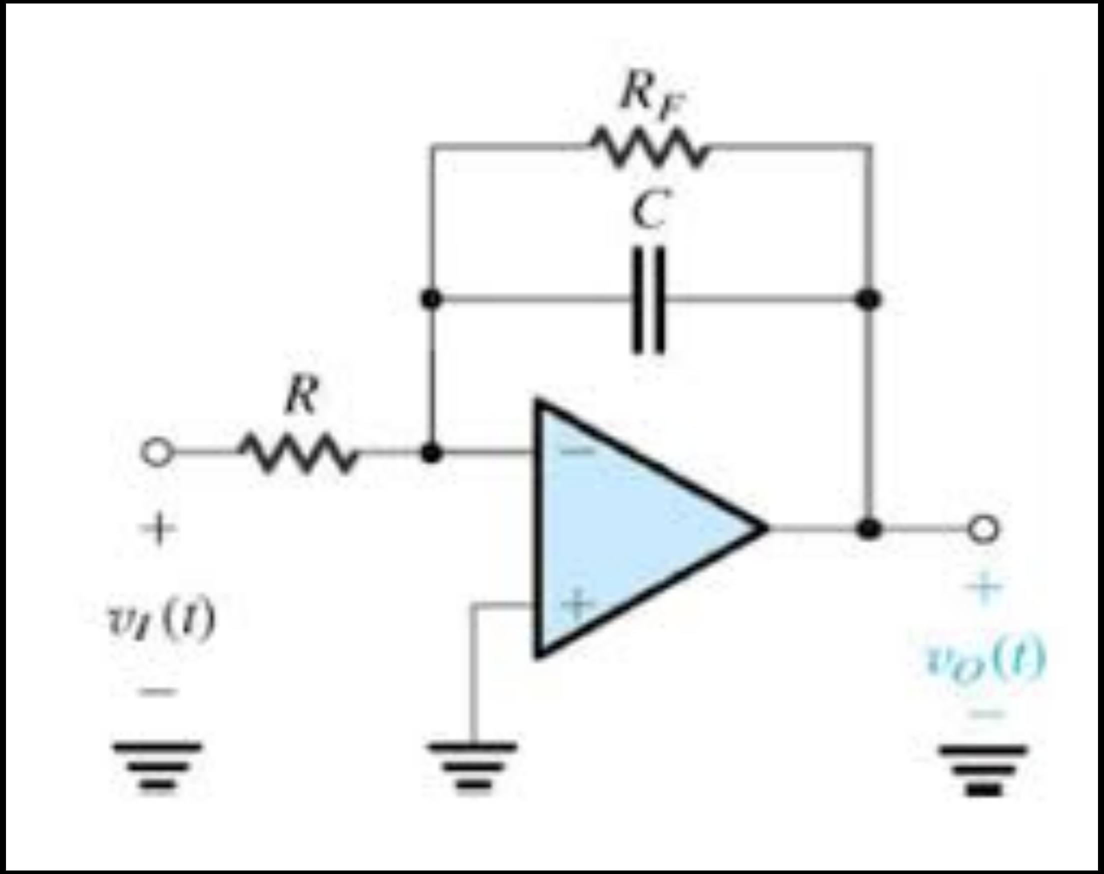
Offset problem is solved by connecting a resistor in parallel to the capacitor.

Decreasing gain can solve “Oscillation” problem.

Resistance of RC parallel circuit part is determined by R_C, the gain is calculated using the composition resistance.

This circuit works as integrator & low-pass filter.

Real Integrator



$$Gain = \frac{V_{in}}{V_{out}} = -\frac{R_1}{R}; \quad \frac{1}{R_1} = \frac{1}{R_C} + \frac{1}{R_F}$$

$$-\frac{R1}{R} = -\frac{1}{R}\frac{R_C R_F}{R_C + R_F} = -\frac{R_F}{R}\frac{1}{1 + R_F/R_C}$$

$$= -\frac{R_F}{R}\frac{1}{j\omega C R_F + 1}; \quad R_C = \frac{1}{j\omega C}$$

In complex plane,

Laplace tranceform

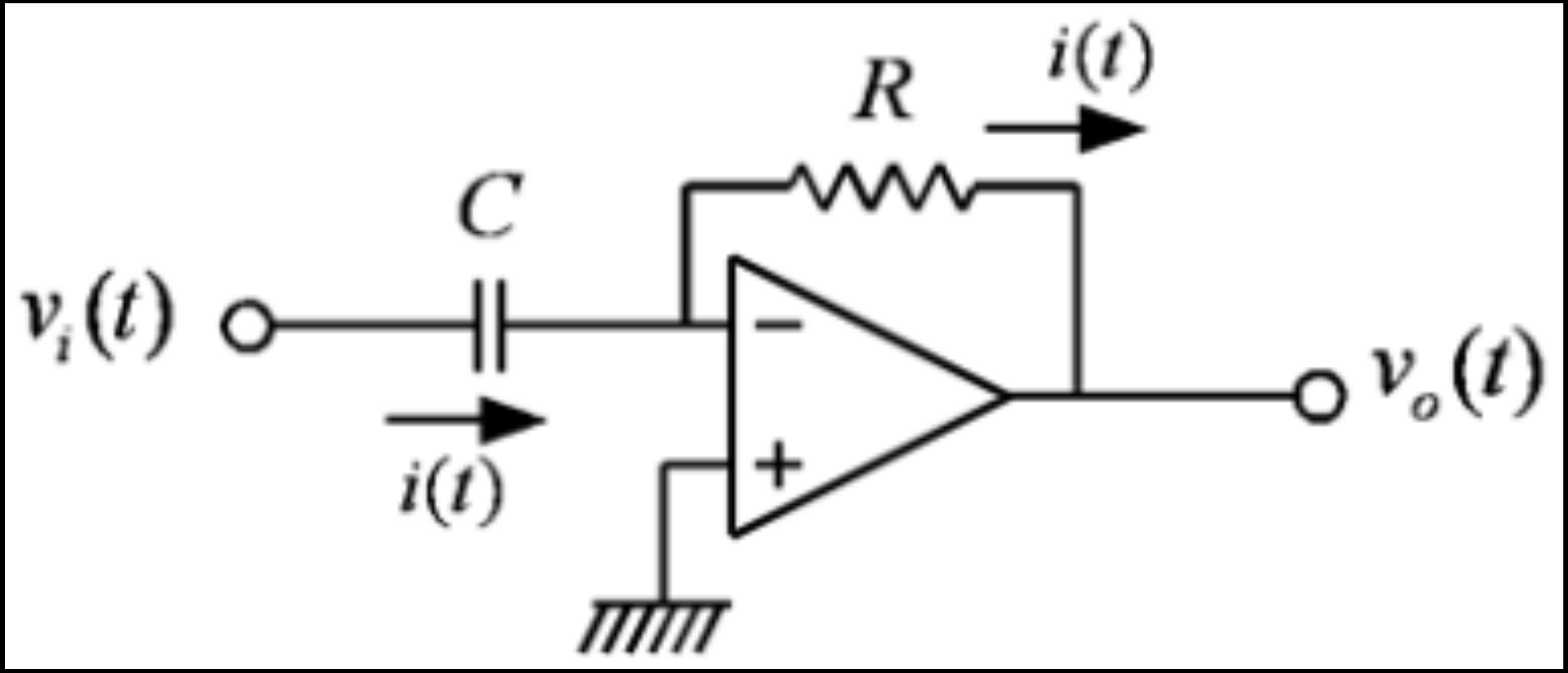
$$|Gain| = \frac{R_F}{R}\left|\frac{1}{j\omega C R_F + 1}\right|$$

$$= \frac{R_F}{R}\frac{1}{\sqrt{(\omega R_F C)^2 + 1}}$$

$$= \frac{R_F}{R}\frac{1}{\sqrt{(2\pi f R_F C)^2 + 1}}$$

If R_F & C are constant, gain at low frequency is emphasized.

Differentiator



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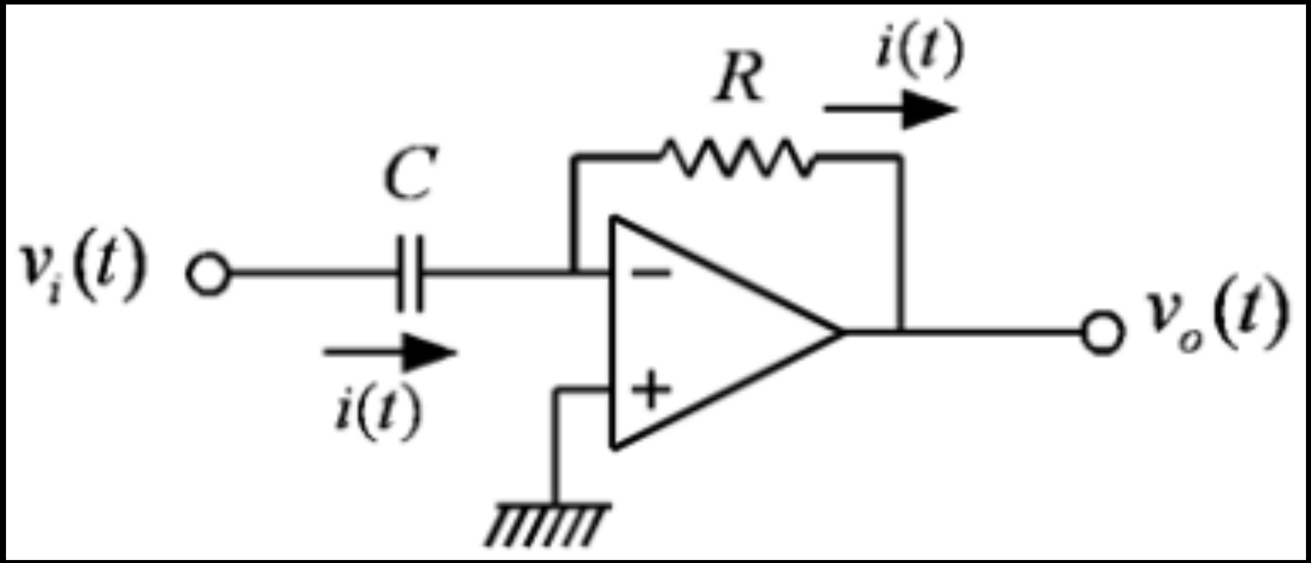
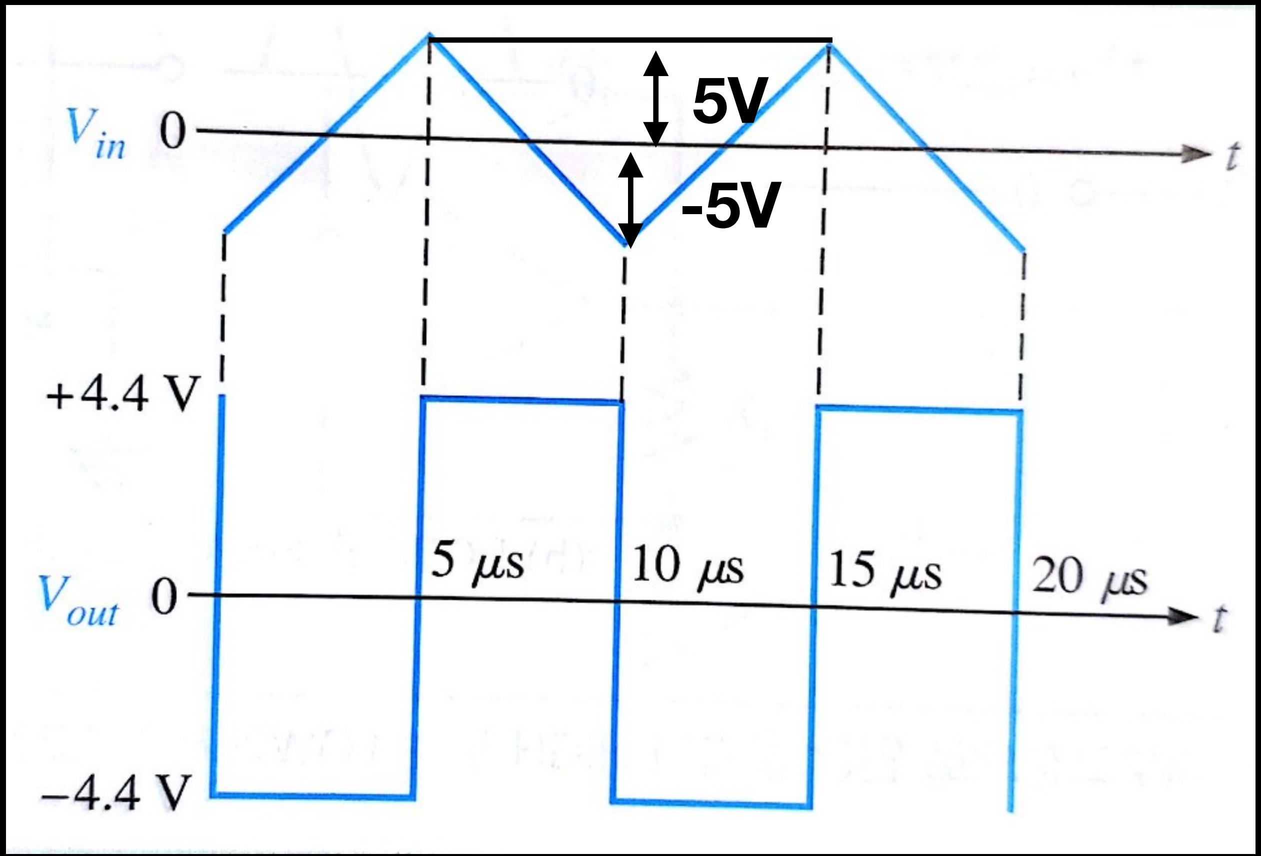
Differentiator





$I_C = I_R$ **(∵Input current = 0)**

$$I_C = \left(\frac{V_C}{t}\right)C; \quad V_{in} = V_C - 0$$

$$V_{out} = I_R R = I_C R = -\left(\frac{\Delta V_C}{\Delta t}\right)RC$$

Gain at high frequency is emphasized by the frequency characteristic of the capacitor.

If we find the area of output signal,

$V_{out}\Delta t = -\Delta V_C RC$

ex) $\Delta V_{in} = 10V$, $R = 2.2k\Omega$, $C = 0.001\mu F$

$V_{out}\Delta t = -4.4V \cdot 5\mu s = -22.0V\mu s$

$-\Delta V_C RC = -10V \cdot 2.2\mu s = -22.0V\mu s$

Conclusion

- In actual differentiator, a resistor is connected in series to the capacitor to remove the signal caused by high-frequency noise.
- However, it is expected that noise can be detected without connecting a resistor.
- Considering high-frequency characteristics of noise & actual integrator acting as a low-pass filter, it would be better to use differentiator without connecting resistor.
- Because pulse width of noise is thought to be very small, output voltage of integrator will be change slowly. But output voltage of differentiator will be change rapidly. (Time term of V_{out} is related to pulse width) So using differentiator will make it easier to understand the output signal.