

# $\pi N \rightarrow \pi N$ elastic scattering at finite density

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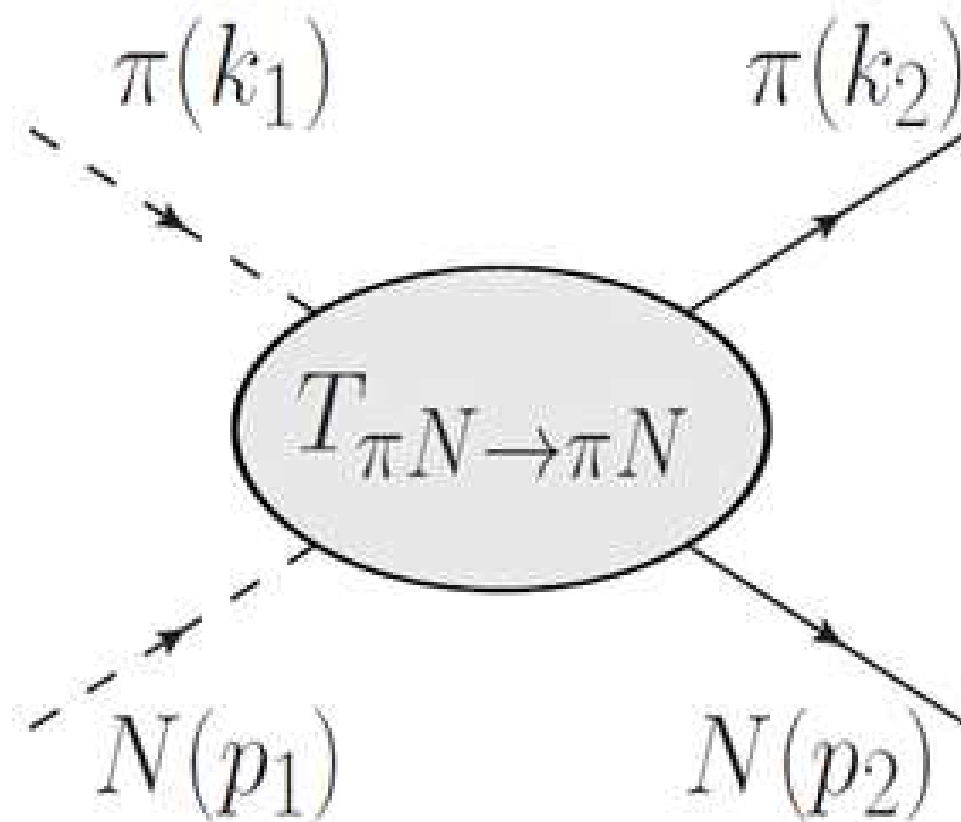
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# 1. Introduction : $\pi N \rightarrow \pi N$ elastic scattering at finite density



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**Pion-Nucleon Elastic Scattering.**

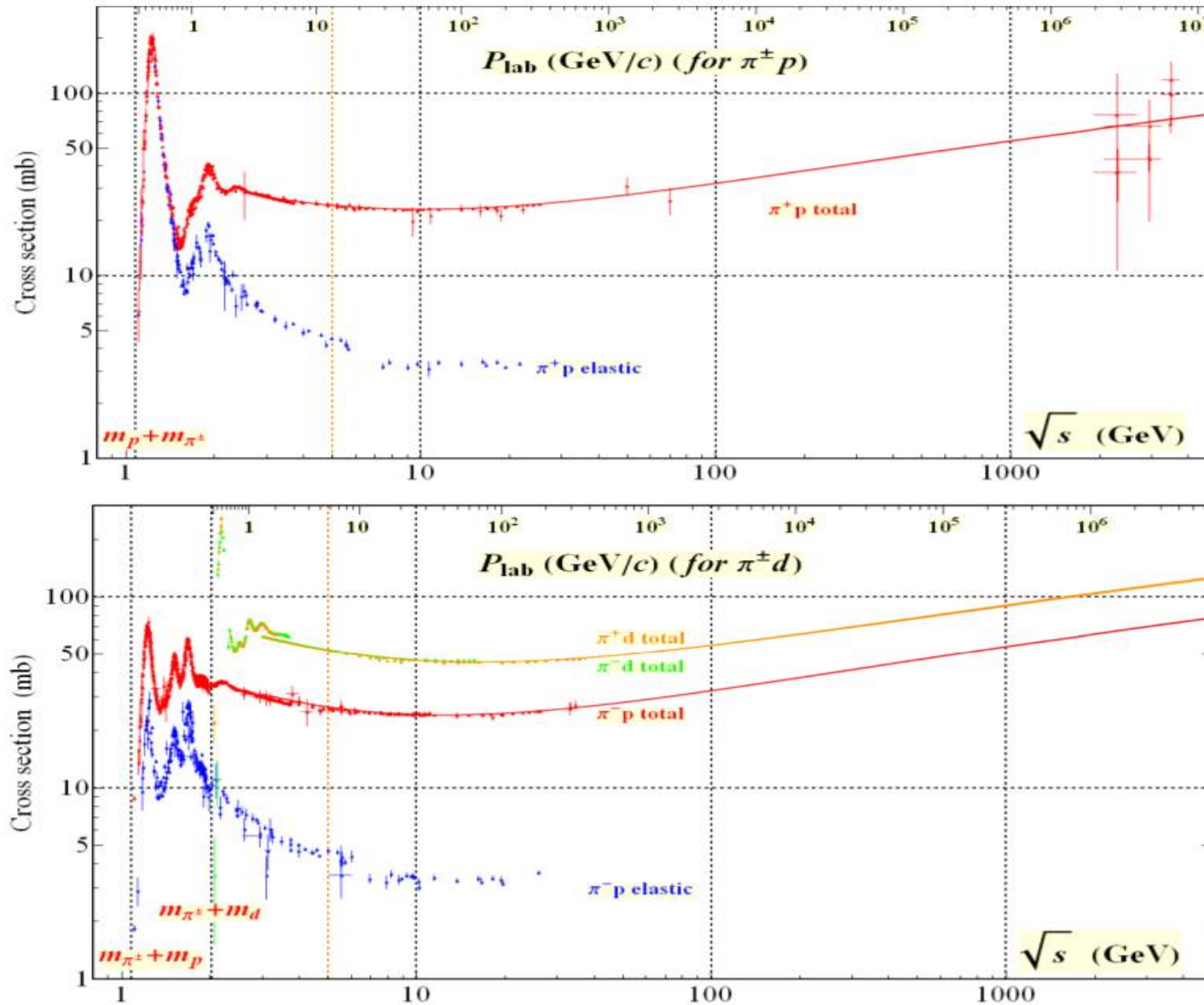
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(ricevuto il 30 Giugno 1970)

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# 1. Introduction : $\pi N \rightarrow \pi N$ elastic scattering at finite density

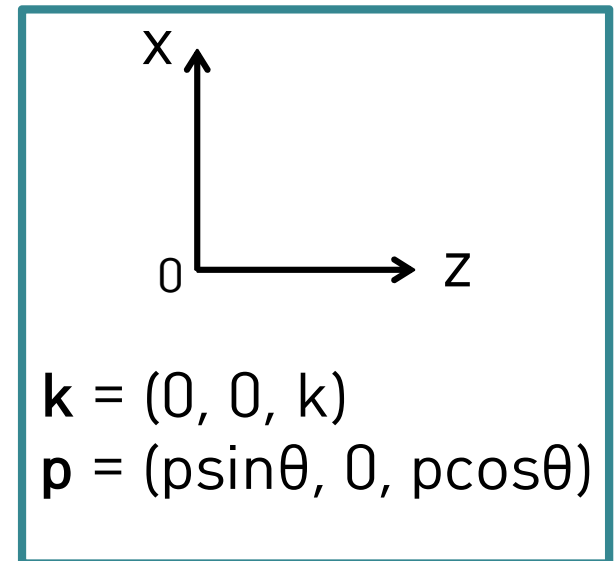
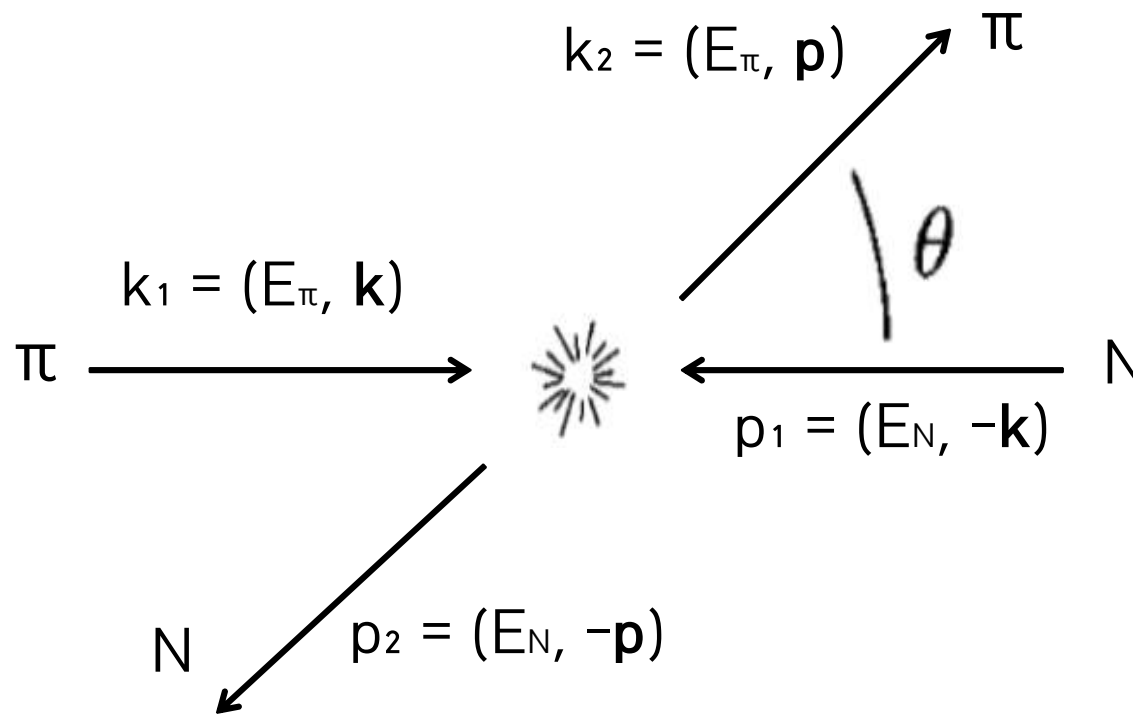


<http://pdg.lbl.gov/2020/hadronic-xsections/>

2nd CENuM Joint Workshop, 3-4 July 2020

# 1. Introduction : $\pi N \rightarrow \pi N$ elastic scattering at finite density

- $\pi N \rightarrow \pi N$  elastic scattering in center-of-mass frame

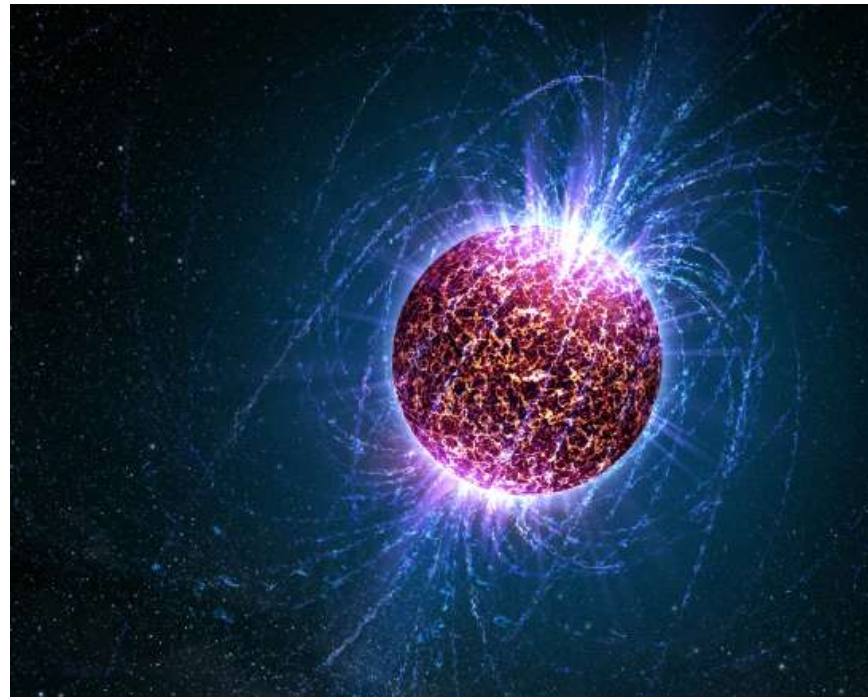


$$E_N = \sqrt{M_N^2 + |\vec{k}|^2}$$

$$E_\pi = \sqrt{M_\pi^2 + |\vec{k}|^2}$$

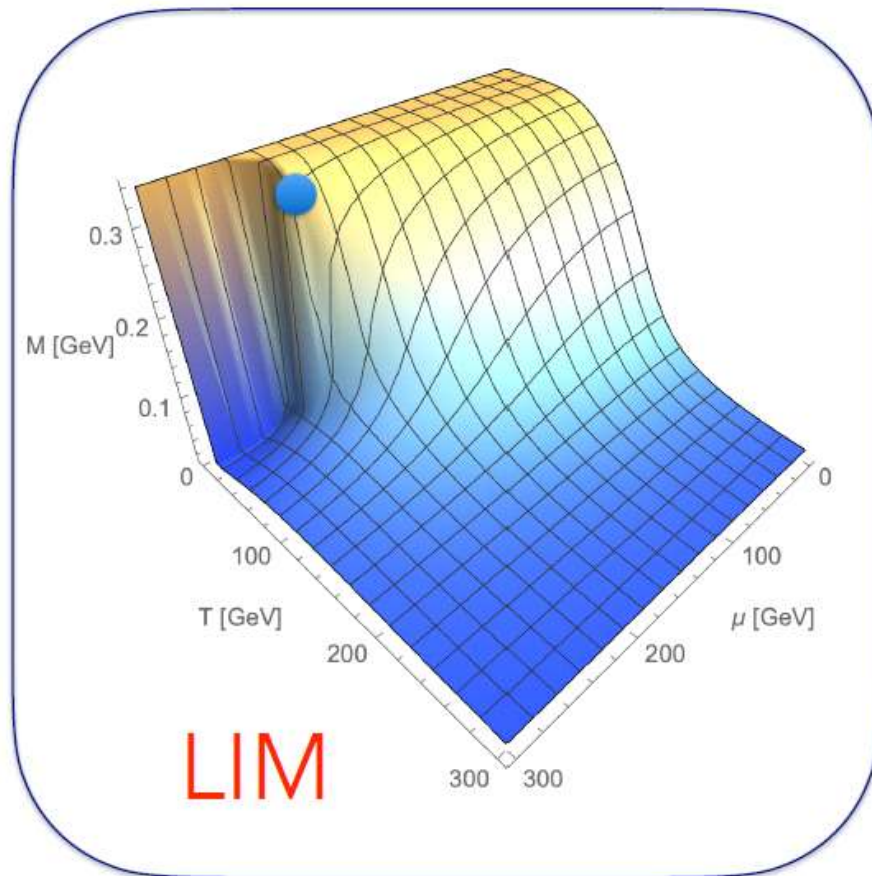
# 1. Introduction : $\pi N \rightarrow \pi N$ elastic scattering at finite density

$$M_\pi, M_N \rightarrow M_\pi(\mu), M_N(\mu)$$



# 1. Introduction : $\pi N \rightarrow \pi N$ elastic scattering at finite density

- Modified instanton model



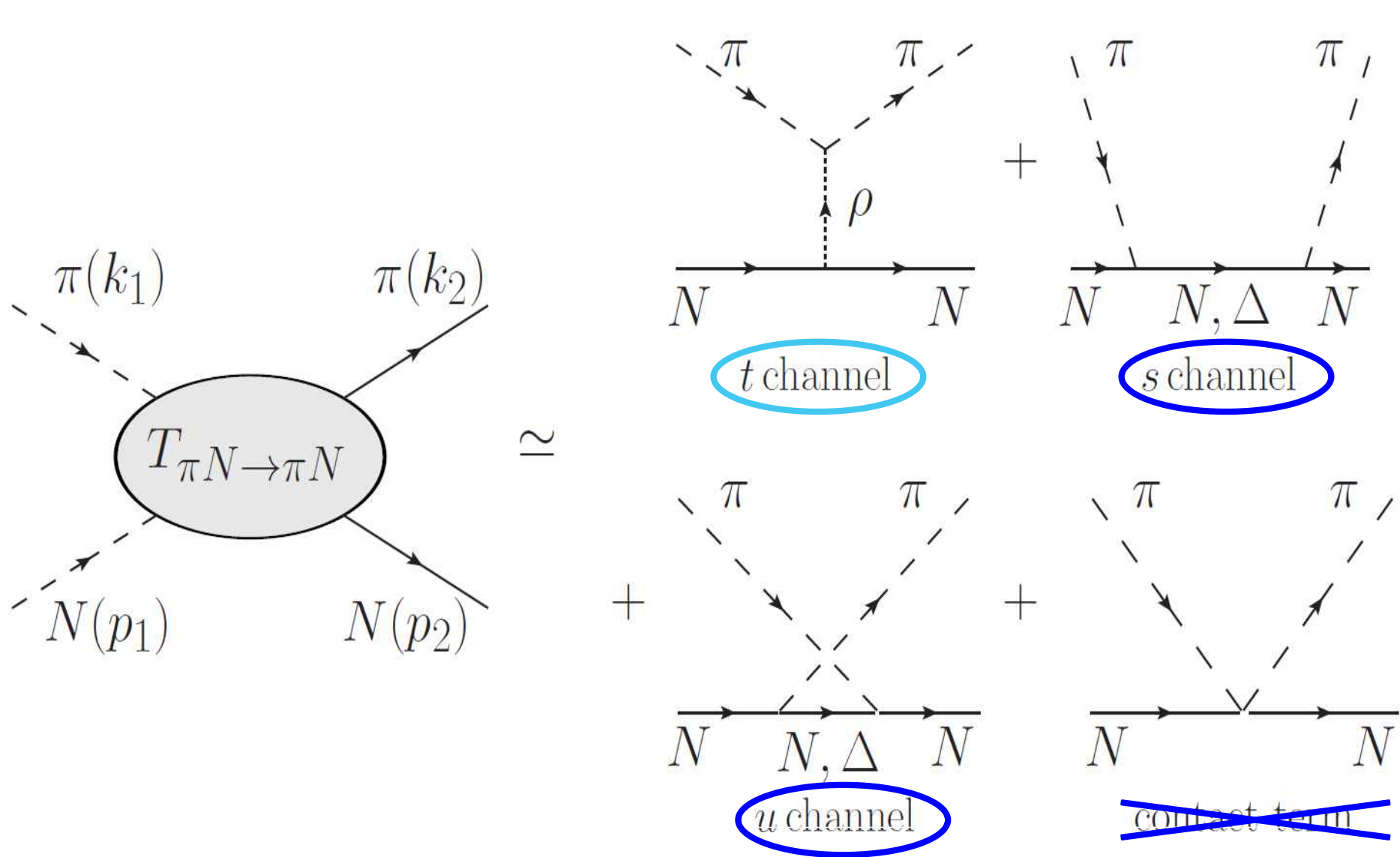
$$M_{quark}(\mu)$$



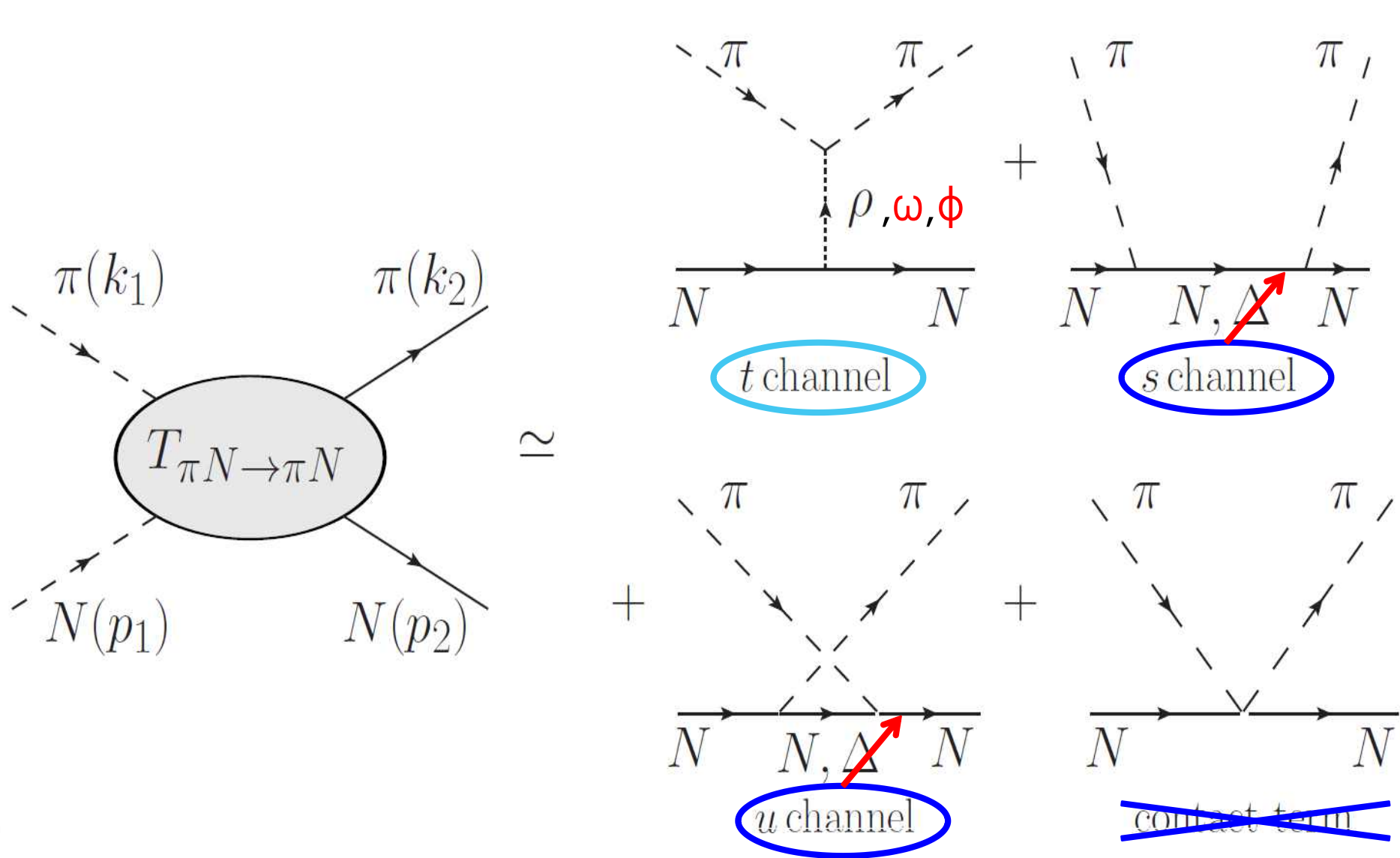
$$M_{\pi}(\mu), M_N(\mu)$$



## 2. Amplitude calculation : $\pi N \rightarrow \pi N$ elastic scattering



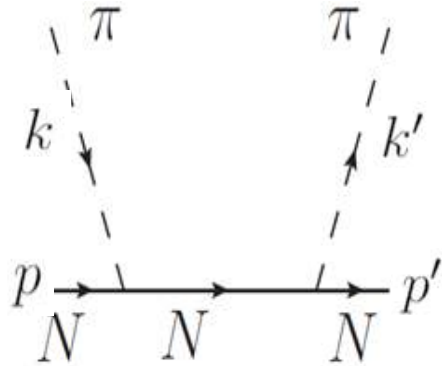
## 2. Amplitude calculation : $\pi N \rightarrow \pi N$ elastic scattering



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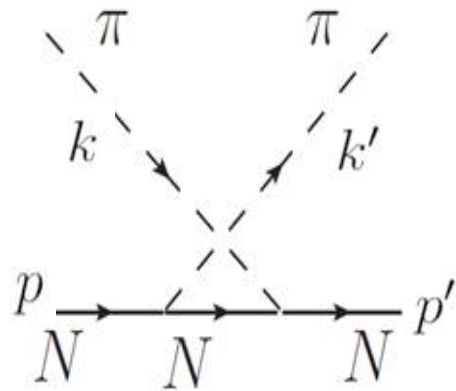
### ○ S-channel

$$\mathcal{L}_{\pi NN} = \frac{g_{\pi NN}}{2f_\pi} \bar{N} \gamma^5 \not{\partial} \pi N$$



$$\mathcal{M}_s = -\frac{g_{\pi NN}^2}{4f_\pi^2} \bar{u}(p') \gamma^5 \not{k}' \frac{\not{p} + \not{k} + m_N}{(p+k)^2 - m_N^2} \gamma^5 \not{k} u(p)$$

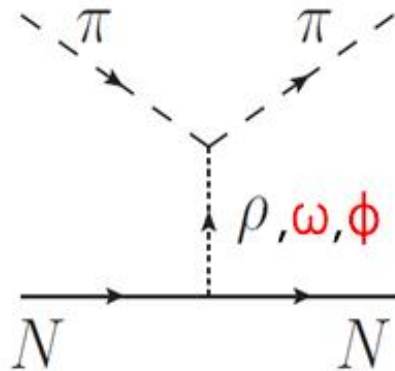
### ○ U-channel



$$\mathcal{M}_u = -\frac{g_{\pi NN}^2}{4f_\pi^2} \bar{u}(p') \gamma^5 \not{k} \frac{\not{p} - \not{k}' + m_N}{(p-k')^2 - m_N^2} \gamma^5 \not{k}' u(p)$$

## 2. Amplitude calculation : $\pi N \rightarrow \pi N$ elastic scattering

### ○ T-channel



$$\mathcal{L}_{\rho\pi\pi} = -ig_{\rho\pi\pi}\rho_\mu(\boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \partial^\mu \boldsymbol{\pi} \cdot \boldsymbol{\pi})$$

$$\mathcal{L}_{\rho NN} = -ig_{\rho NN}\bar{N}\left[\rho^\mu\gamma_\mu - \frac{\kappa_{\rho NN}}{2M_N}\partial^\nu\rho^\mu\sigma_{\mu\nu}\right]N + H.c.$$

⋮

$$\mathcal{M}_\rho, \mathcal{M}_\omega, \mathcal{M}_\phi$$

## 2. Amplitude calculation : $\pi N \rightarrow \pi N$ elastic scattering

- Form factor
  - s, u channel

$$F_x(q^2) = \frac{\Lambda^4}{\Lambda^4 + (x - M_x^2)^2}, \quad x = s, u$$

( $\Lambda$  = cut-off mass,  $M_x$  = exchange particle mass)

- t channel

$$F_V(q^2) = \frac{\Lambda^4}{\Lambda^4 + (t - M_V^2)^2}, \quad V = \rho, \omega, \phi$$

### 3. Cross-section calculation

#### ○ Differential Cross-Section (DCS)

$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_{CM} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|p_1|}{(2\pi)^2 4E_{CM}} |\mathcal{M}(p_A, p_B \rightarrow p_1, p_2)|^2$$

$$\Rightarrow \frac{1}{64\pi^2} \frac{p}{k} \frac{1}{E_{CM}^2} \frac{1}{2} \sum_{spins} |\mathcal{M}|^2$$

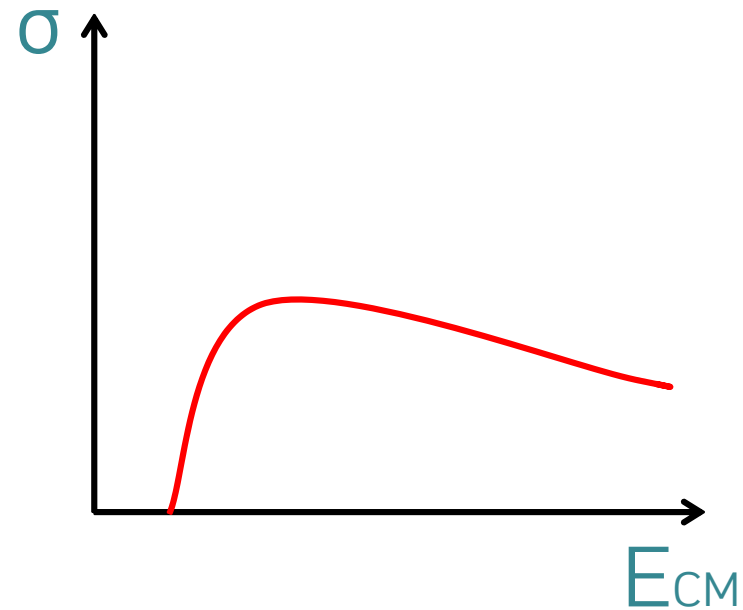
$$(|\mathcal{M}(p_A, p_B \rightarrow p_1, p_2)|^2 = \frac{1}{2} \sum_{spins} |\mathcal{M}_s F_s + \mathcal{M}_u F_u + \mathcal{M}_t F_t|^2)$$

#### ○ Total Cross-Section (TCS)

$$\sigma^{total} = 2\pi \int_0^\pi \left(\frac{\partial\sigma}{\partial\Omega}\right)_{CM} \sin\theta d\theta$$

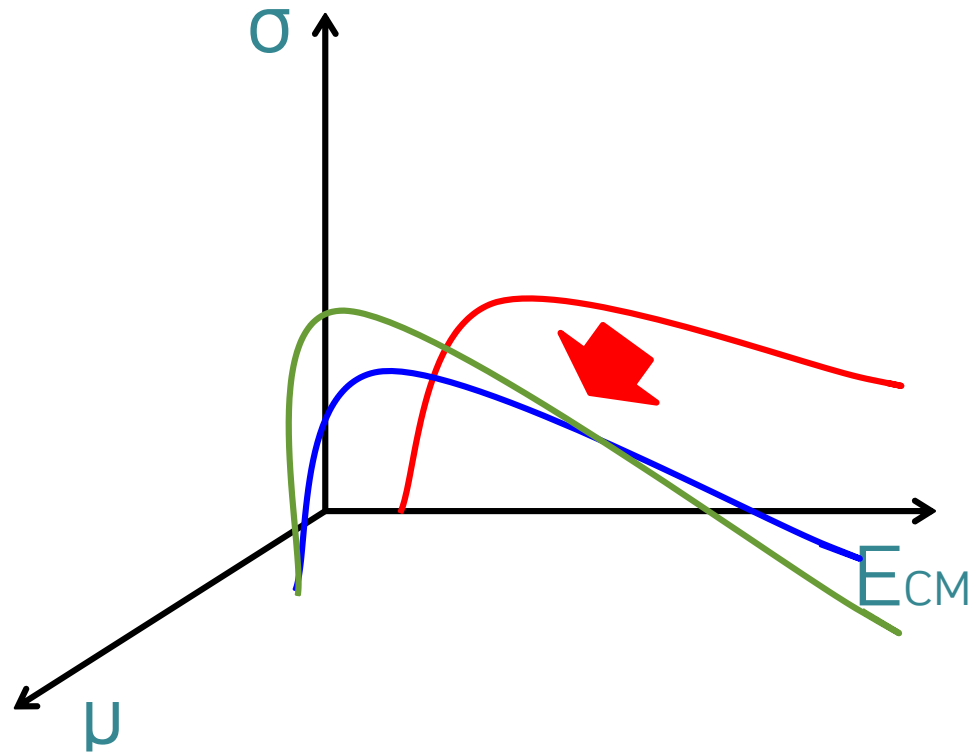
### 3. Cross-section calculation

- $\pi N \rightarrow \pi N$  elastic scattering at finite density



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- $\pi N \rightarrow \pi N$  elastic scattering at finite density





Thank you  
for  
listening!