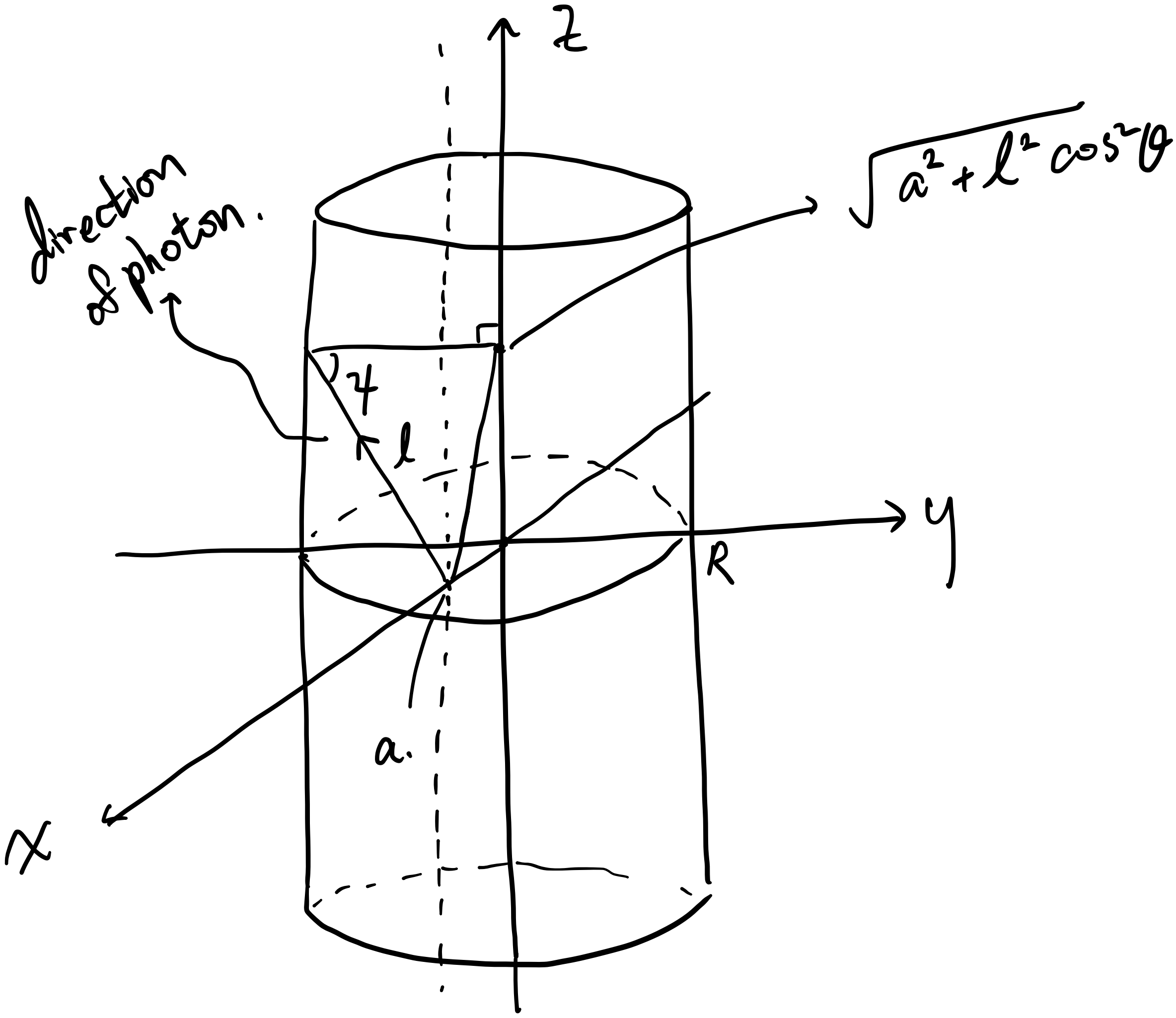


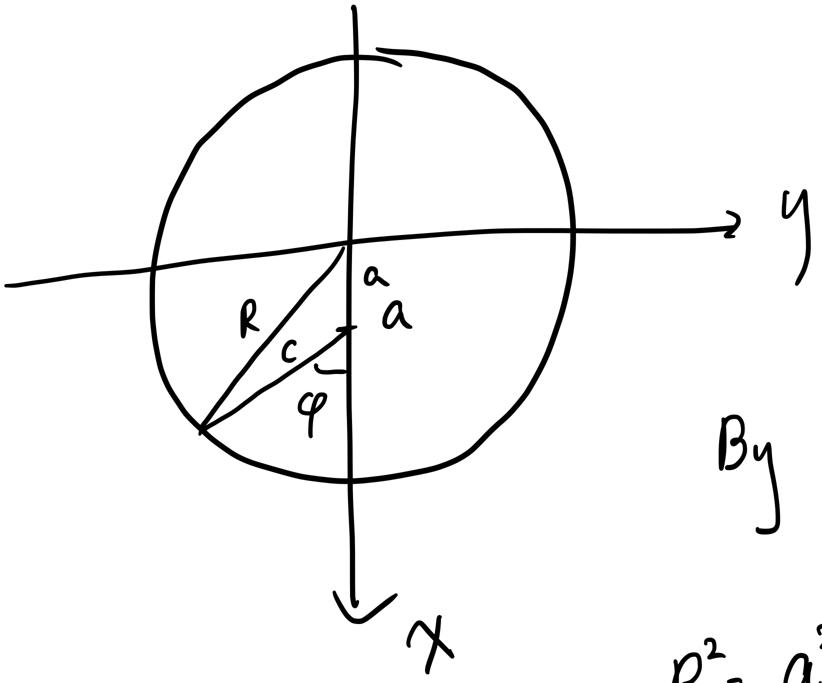

$\Rightarrow$ For total reflection,

$\underbrace{\psi > 69.57^\circ}$ $= \psi_c$

direction of photon이 z축과 이루는 각 : $\theta$

" " 의 xy평면 정사영이

x축과 이루는 각 : $\varphi$




By 2nd cosine law

$$R^2 = a^2 + c^2 - 2ac\underbrace{\cos(\pi - \varphi)}_{= -\cos\varphi}$$

$$\therefore \ c^2 + (2a\cos\varphi)\, c + a^2 - R^2 = 0$$

$$\therefore \ C = -a\cos\varphi \pm \sqrt{a^2\cos^2\varphi - a^2 + R^2}$$

$$= -a\cos\varphi \pm \sqrt{R^2 - a^2\sin^2\varphi}$$

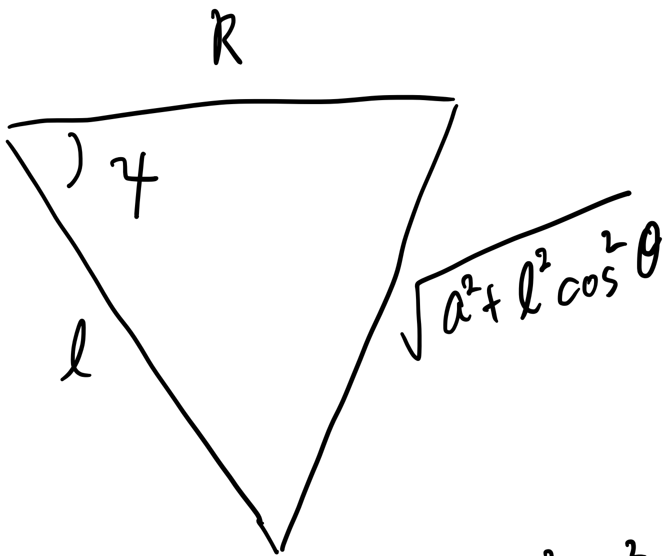
$$= C(\varphi).$$

because $C(\varphi) > 0$

$$\therefore \ C(\varphi) = -a\cos\varphi + \sqrt{R^2 - a^2\sin^2\varphi}$$

And $l\sin\theta = C(\varphi)$ 이므로

$$l = \frac{C(\varphi)}{\sin\theta}$$




$$\therefore \cos\psi = \frac{R^2+l^2-(a^2+l^2\cos^2\theta)}{2Rl}$$

$$= \frac{R^2+l^2\sin^2\theta - a^2}{2Rl} < \cos\psi_c$$

and $l = \dfrac{C(\varphi)}{\sin\theta}$ 이므로

$$\frac{(R^2+C(\varphi)^2-a^2)\sin\theta}{2RC(\varphi)} < \cos\psi_c$$

$$\therefore \theta < \sin^{-1}\left\{ \frac{\frac{R}{a}\cos\psi_c\left(\sqrt{\left(\frac{R}{a}\right)^2-\sin^2\varphi}-\cos\varphi\right)}{\left(\frac{R}{a}\right)^2-\sin^2\varphi-\cos\varphi\sqrt{\left(\frac{R}{a}\right)^2-\sin^2\varphi}} \right\}$$

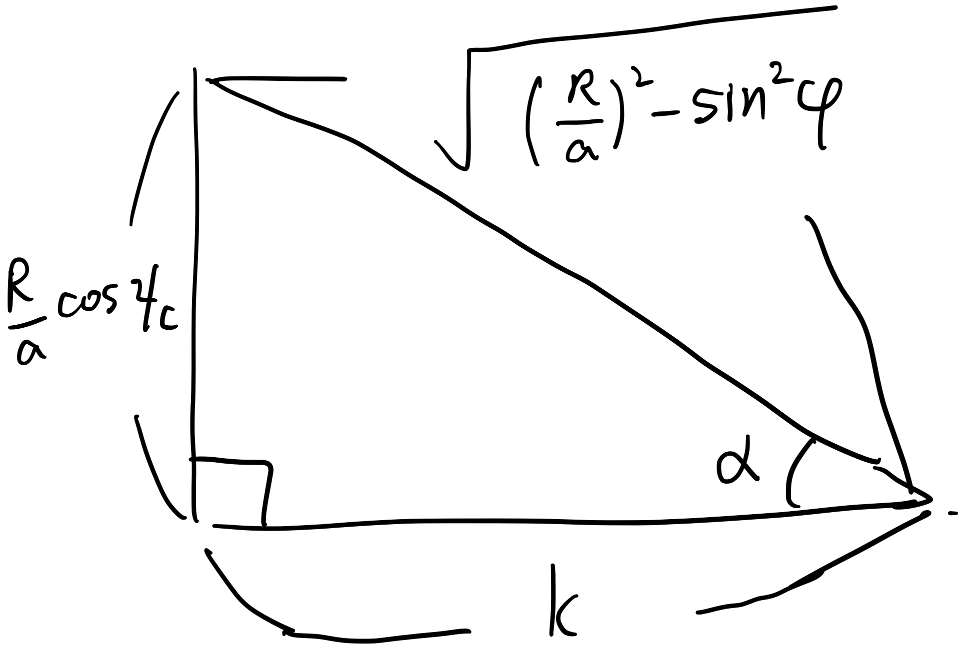
$$= \sin^{-1}\left\{ \frac{\frac{R}{a}\cos\psi_c}{\sqrt{\left(\frac{R}{a}\right)^2-\sin^2\varphi}} \right\} = f(\varphi)$$

$\therefore$ At point $(a, 0, 0)$

probability for total reflection

$$= \frac{\int_0^{2\pi}\int_0^{f(\varphi)} \sin\theta \, d\theta \, d\varphi}{\iint d\Omega}$$

$$= \frac{1}{4\pi}\int_0^{2\pi} 1 - \cos\left[\sin^{-1}\left(\frac{\frac{R}{a}\cos\psi_c}{\sqrt{\left(\frac{R}{a}\right)^2-\sin^2\varphi}}\right)\right] d\varphi$$




$$k=\sqrt{\left(\frac{R}{a}\right)^2-\sin^2\varphi-\left(\frac{R}{a}\right)^2\cos^2\psi_c}$$

$$\therefore\quad \cos\alpha=\sqrt{\frac{\left(\frac{R}{a}\right)^2-\sin^2\varphi-\left(\frac{R}{a}\right)^2\cos^2\psi_c}{\left(\frac{R}{a}\right)^2-\sin^2\varphi}}$$

$$\therefore\quad \frac{1}{4\pi}\int_0^{2\pi} 1-\sqrt{\frac{\left(\frac{R}{a}\right)^2\sin^2\psi_c-\sin^2\varphi}{\left(\frac{R}{a}\right)^2-\sin^2\varphi}}\,d\varphi$$