Proton mass and RAON

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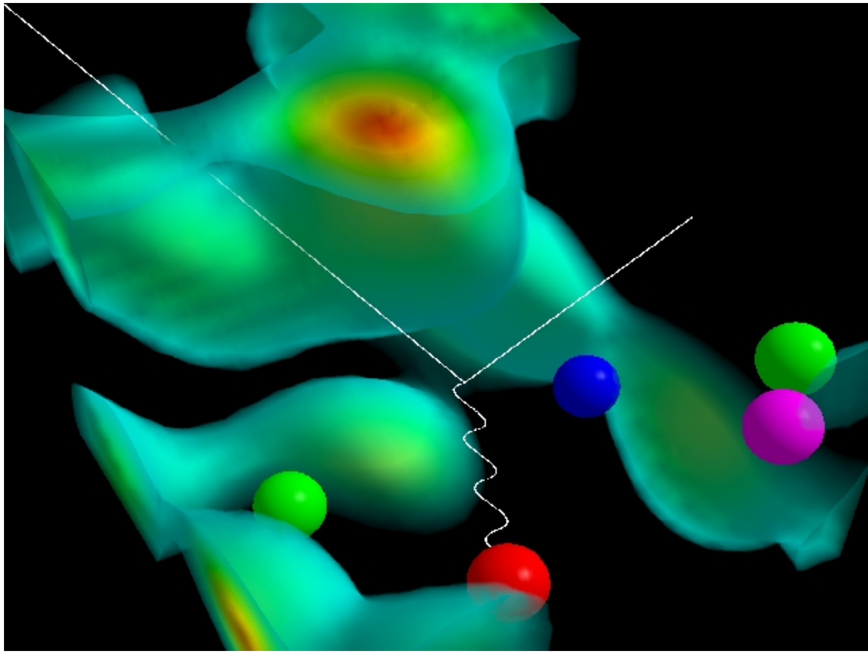
QCD vacuum and RAON: proton mass as an example

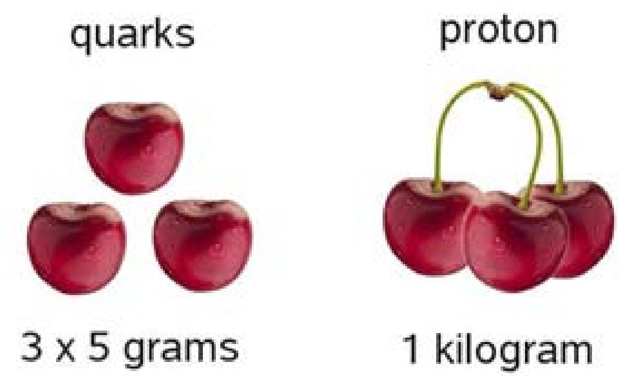
- **Accurate, reliable and microscopic theoretical description of nuclear properties should be the top priority.**
- **At the same time, one has to try to propose a (relatively new) topic to enrich RAON science.**
- **Anyhow, to prove/falsify the new topic, the microscopic theory should be timely developed.**

Artist's Rendition of a Proton

- Three quarks indicated by red, green and blue spheres (lower left) are localized by the gluon field.
- A quark-antiquark pair created from the gluon field is illustrated by the green-antigreen (magenta) quark pair on the right. These quark pairs give rise to a *meson* cloud around the proton.
- The masses of the quarks illustrated in this diagram account for only 3% of the proton mass. The gluon field is responsible for the remaining 97% of the proton's mass and is the origin of mass in most everything around us.
- Experimentalists probe the structure of the proton by scattering electrons (white line) off quarks which interact by exchanging a quantum of light (wavy line) known as a photon.



http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/



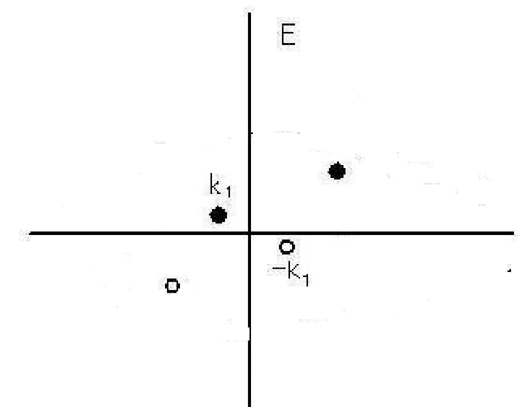
Origin of nucleon mass?

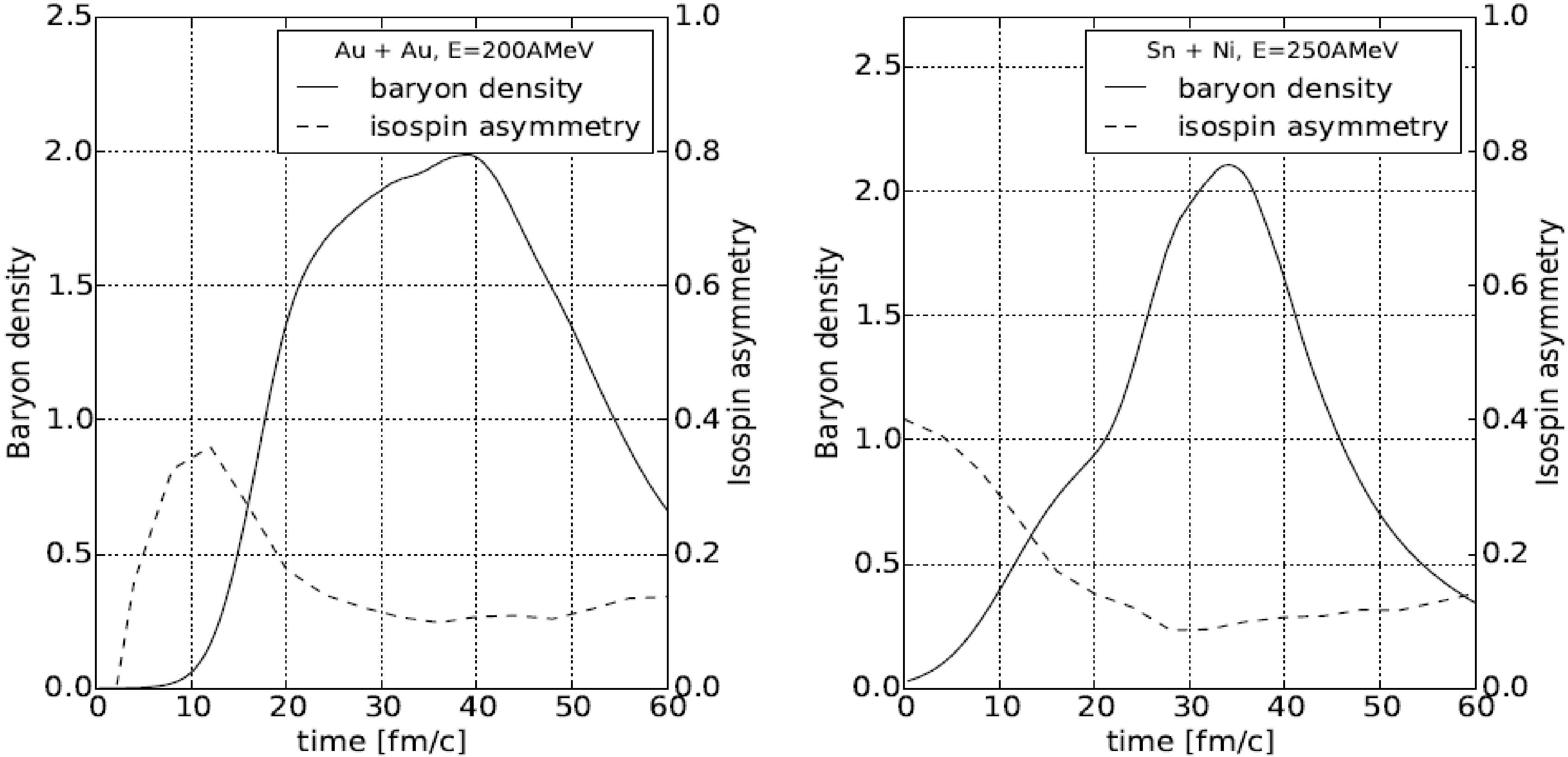
Can nuclear matter and nuclei do anything for this?

Nucleon mass (in the chiral limit) in the linear sigma model

$$\delta\mathcal{L} = -g_\pi \left[(i\bar{\psi}\gamma_5\vec{\tau}\psi)\,\vec{\pi} + (\bar{\psi}\psi)\,\sigma \right]$$

$$\begin{aligned} <\sigma> &= \sigma_0 = f_\pi \\ <\pi> &= 0 \\ M_N &= g_\pi\sigma_0 = g_\pi f_\pi \end{aligned}$$





Yujeong Lee, Chang-Hwan Lee, YK, T. Gaitanos, JKPS 69 1430 (2016)

From RBUU simulation

Contents

- QCD at low energy (quark-antiquark condensate, etc)
- In terms of hadrons
- In terms of quarks
- Summary

Low energy QCD

- Mesons and baryons
- (spontaneous) Chiral symmetry breaking
- Condensates
- Various EFTs

Chiral symmetry at low energy

$$\mathcal{L} = i\bar{\psi}_j \partial\!\!\!/ \psi_j$$

Note that gluons are flavor-blind

$$\Lambda_V : \ \psi \longrightarrow e^{-i\frac{\vec{\tau}}{2}\vec{\Theta}}\psi \simeq (1 - i\frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

: vector transform

$$\begin{aligned} i\bar{\psi}\partial\!\!\!/\psi &\longrightarrow i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi}i\partial\!\!\!/\frac{\vec{\tau}}{2}\psi - \bar{\psi}\frac{\vec{\tau}}{2}i\partial\!\!\!/\psi\right) \\ &= i\bar{\psi}\partial\!\!\!/\psi \end{aligned}$$

$$V^a_\mu = \bar{\psi}\gamma_\mu\frac{\tau^a}{2}\psi$$

$$\Lambda_A : \quad \psi \longrightarrow e^{-i\gamma_5\frac{\vec{\tau}}{2}\vec{\Theta}}\psi = (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

: axial-vector transform

$$\begin{aligned} i\bar{\psi}\partial\!\!\!/\psi &\longrightarrow i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi}\, i\partial_\mu\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\,\psi + \bar{\psi}\,\gamma_5\frac{\vec{\tau}}{2}i\partial_\mu\gamma^\mu\,\psi\right) \\ &= i\bar{\psi}\partial\!\!\!/\psi \end{aligned}$$

$$A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau}{2}\psi$$

Chiral symmetry breaking

$$\delta \mathcal{L} = -m\left(\bar{\psi}\psi\right)$$

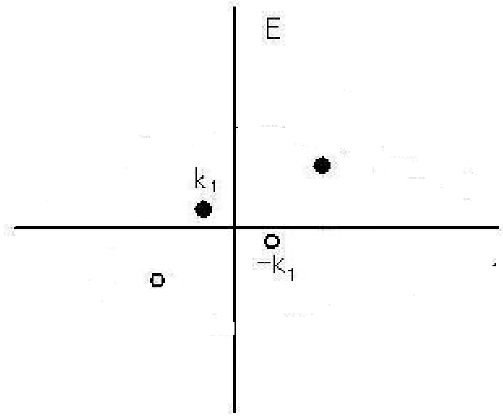
$$\Lambda_A : \; m\left(\bar{\psi}\psi\right) \longrightarrow m\bar{\psi}\psi - 2im\vec{\Theta}\left(\bar{\psi}\frac{\vec{\tau}}{2}\gamma_5\psi\right)$$

→ Explicit chiral symmtery breaking

$$\frac{m}{\Lambda_{\rm QCD}} \sim 0.05$$ → chiral limit: m=0

$$< \bar{q}q >^{1/3} / \Lambda_{\rm QCD} \sim 1$$ → SSB of chiral symmetry

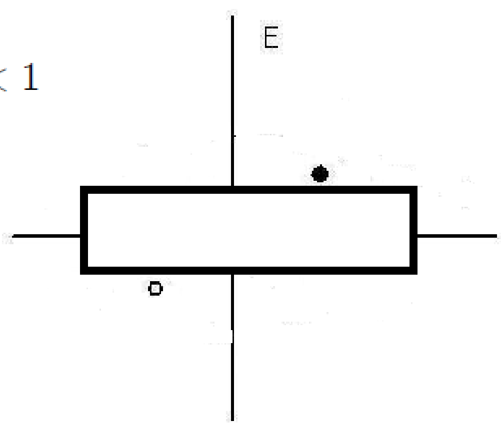
$$m \sim (5-10)\ {\rm MeV},\ \ \Lambda_{\rm QCD} \sim 200\ {\rm MeV}, < \bar{q}q >^{1/3} \simeq -240\ {\rm MeV}$$



$$< 0|\bar{q}q|0 > \neq 0$$

A BCS-like trial ground state --> true vacuum contains chiral (quark-antiquark) condensates **in free space** [Finger & Mandula, NPB 199, 168 (1982)]

$$\frac{< F|\bar{q}q|F >}{< 0|\bar{q}q|0 >} < 1$$



In dense matter, chiral condensates will be reduced as low energy phase space is already occupied by the fermions in Fermi sea.

Therefore, nucleon mass from spontaneous chiral symmetry is supposed to be reduced in dense matter.

Mesons and chiral symmetry

pion-like state: $\vec{\pi} \equiv i\bar{\psi}\vec{\tau}\gamma_5\psi;$ sigma-like state: $\sigma \equiv \bar{\psi}\psi$

rho-like state: $\vec{\rho}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\psi;$ a_1-like state: $\vec{a_1}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\gamma_5\psi$

$$\begin{aligned} \pi_i : \quad i\bar{\psi}\tau_i\gamma_5\psi \quad &\longrightarrow \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_j \left(\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi + \bar{\psi}\gamma_5\frac{\tau_j}{2}\tau_i\gamma_5\psi\right) \\ &= \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_i\bar{\psi}\psi \end{aligned}$$

→ $\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta}\sigma$

$\sigma \longrightarrow \sigma - \vec{\Theta}\vec{\pi}$ $\vec{\rho}_\mu \longrightarrow \vec{\rho}_\mu + \vec{\Theta} \times \vec{a_1}_\mu$

Linear sigma-model

$$\Lambda_V: \quad \pi^2 \longrightarrow \pi^2; \qquad \sigma^2 \longrightarrow \sigma^2 \qquad \Lambda_A: \quad \vec{\pi}^2 \longrightarrow \vec{\pi}^2 + 2\sigma\Theta_i\pi_i; \qquad \sigma^2 \longrightarrow \sigma^2 - 2\sigma\Theta_i\pi_i$$

$$\left(\vec{\pi}^2 + \sigma^2\right) \stackrel{\Lambda_V, \Lambda_A}{\longrightarrow} \left(\vec{\pi}^2 + \sigma^2\right)$$

* SSB → $$V = V(\pi^2 + \sigma^2) = \frac{\lambda}{4}\left((\pi^2 + \sigma^2) - f_\pi^2\right)^2$$

$$\mathcal{L}_{L.S.} = \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \; - \frac{\lambda}{4}\left((\pi^2 + \sigma^2) - f_\pi^2\right)^2$$

Parity doublet model in dense matter

Introduce two nucleon fields that transform in a mirror way under chiral transformations:

$$\mathrm{SU}_L(2) \times \mathrm{SU}(2)_R$$

$$\psi_{1R} \to R\psi_{1R}, \quad \psi_{1L} \to L\psi_{1L},$$
$$\psi_{2R} \to L\psi_{2R}, \quad \psi_{2L} \to R\psi_{2L}.$$

$$m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2)$$
$$= m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L})$$

$$\mathcal{L} = \bar{\psi}_1 i \partial\!\!\!/ \psi_1 + \bar{\psi}_2 i \partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2)$$
$$+ a\bar{\psi}_1 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_1 + b\bar{\psi}_2 (\sigma - i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_2$$

$$m_{N\pm} = \frac{1}{2}\left(\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right)$$

The state N+ is the nucleon N(938). while N- is its parity partner conventionally identified with N(1500).

the decay width $\Gamma_{N\pi}$ for $N^*(1535) \rightarrow N + \pi$, $m_0 = 270$ MeV

"Linear sigma model with parity doubling," C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)

Cold, dense nuclear matter in a SU(2) parity doublet model

$$
\begin{aligned}
\mathcal{L} = \; & \bar{\psi}_1 i \partial\!\!\!/ \psi_1 + \bar{\psi}_2 i \partial\!\!\!/ \psi_2 + m_0 (\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2) \\
& + a \bar{\psi}_1 (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_1 + b \bar{\psi}_2 (\sigma - i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_2 \\
& - g_\omega \bar{\psi}_1 \gamma_\mu \omega^\mu \psi_1 - g_\omega \bar{\psi}_2 \gamma_\mu \omega^\mu \psi_2 + \mathcal{L}_M ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_M = \; & \frac{1}{2} \partial_\mu \sigma^\mu \partial^\mu \sigma_\mu + \frac{1}{2} \partial_\mu \vec{\pi}^\mu \partial^\mu \vec{\pi}_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + g_4^4 (\omega_\mu \omega^\mu)^2 \\
& + \frac{1}{2} \bar{\mu}^2 (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + \epsilon \sigma ,
\end{aligned}
$$

D. Zschiesche, L. Tolos, Jurgen Schaffner-Bielich, Robert D. Pisarski, Phys.Rev. C75 (2007) 055202

PHYSICAL REVIEW C **92**, 025201 (2015)

Asymmetric nuclear matter in a parity doublet model with hidden local symmetry

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(Received 11 May 2015; published 3 August 2015)

We construct a model to describe dense hadronic matter at zero and finite temperatures, based on the parity doublet model of DeTar and Kunihiro [C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)], including the isosinglet scalar meson σ as well as ρ and ω mesons. We show that, by including a six-point interaction of the σ meson, the model reasonably reproduces the properties of normal nuclear matter with the chiral invariant nucleon mass m_0 in the range from 500 to 900 MeV. Furthermore, we study the phase diagram based on the model, which shows that the value of the chiral condensate drops at the liquid-gas phase transition point and at the chiral phase transition point. We also study asymmetric nuclear matter and find that the first-order phase transition for the liquid-gas phase transition disappears in asymmetric matter and that the critical density for the chiral phase transition at nonzero density becomes smaller for larger asymmetry.

Y. Motohiro, M. Harada, YK, Erratum: Phys.Rev. C95 (2017) 059903

Delta matter in a parity doublet model (within MFA)

Yusuke Takeda, YK, Masayasu Harada, Phys. Rev. C97 (2018) 065202

* In symmetric matter, Delta enters into matter at (1-4) times the saturation density. The stable Δ-nucleon matter is realized around 1.5 times the saturation density, and the phase transition from nuclear matter to Δ-nucleon matter is of first order in the wide parameter region.

* In asymmetric matter, the phase transition from the nuclear matter to the stable Δ-nucleon matter can be of the second order for most parameter region. The onset density is smaller than that in symmetric matter.

* In symmetric dense matter, larger chiral invariant nucleon mass tends to lower the transition density to the stable N-Δ phase.

* Partial restoration of chiral symmetry is enhanced by Delta matter.

	BE (MeV)			R_C (fm)	
	PDM	PC-PK1	Exp.	PDM	Exp.
^{16}O	8.04	7.96	7.98	2.76	2.70
^{24}O	7.06	7.12	7.04	2.82	–
^{32}Mg	7.83	7.91	7.80	3.14	–
^{38}Si	7.59	7.90	7.89	3.28	–
^{38}Ar	8.51	8.62	8.61	3.39	3.40
^{40}Ca	8.57	8.58	8.55	3.46	3.48
^{48}Ca	8.42	8.65	8.67	3.52	3.48
^{42}Ti	8.16	8.32	8.26	3.58	–
^{58}Ni	8.12	8.69	8.73	3.84	3.78
^{72}Kr	8.22	8.31	8.43	4.11	4.16
^{208}Pb	7.86	7.87	7.87	5.53	5.50

We observed that our results, especially the binding energies, are closest to the experiments when we take m_0 = 700 MeV.

Ik Jae Shin, Won-Gi Paeng, Masayasu Harada, YK, 1805.03402 in nucl-th

Constraint to chiral invariant masses of nucleons from GW170817 in an extended parity doublet model

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(Received 18 January 2019; published 12 August 2019)

We construct nuclear matter based on an extended parity doublet model including four light nucleons, $N(939)$, $N(1440)$, $N(1535)$, and $N(1650)$. We exclude some values of the chiral invariant masses by requiring the saturation properties of normal nuclear matter: saturation density, binding energy, incompressibility, and symmetry energy. We find a further constraint on the chiral invariant masses from the tidal deformability determined by the observation of the gravitational waves from neutron star merger GW170817. Our result shows that the chiral invariant masses are larger than about 600 MeV. We also give some predictions on the symmetry energy and the slope parameters in the high density region, which will be measured in future experiments.

Extended parity doublet model with a new transport code

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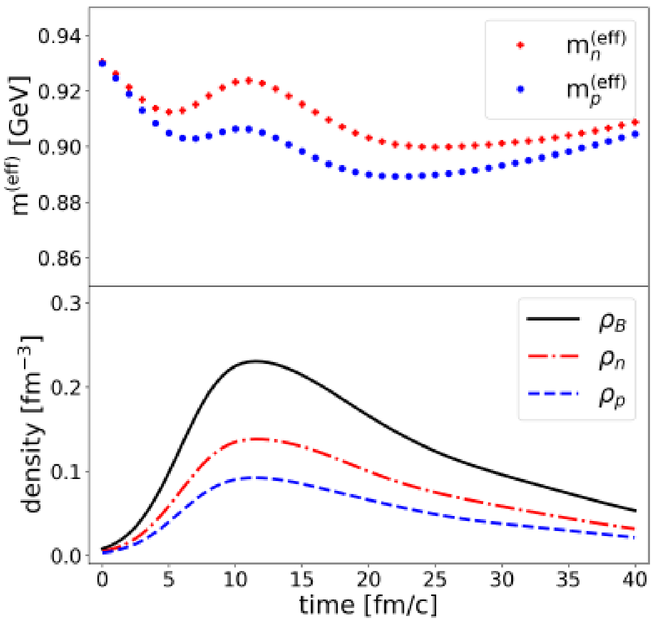
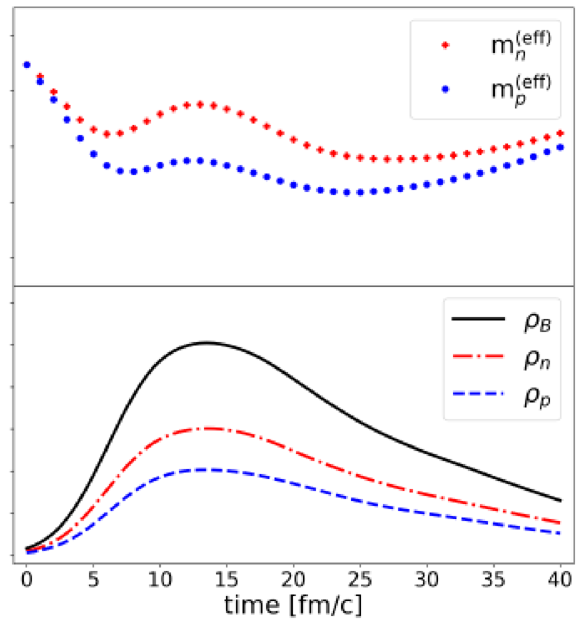
(Dated: September 17, 2019)

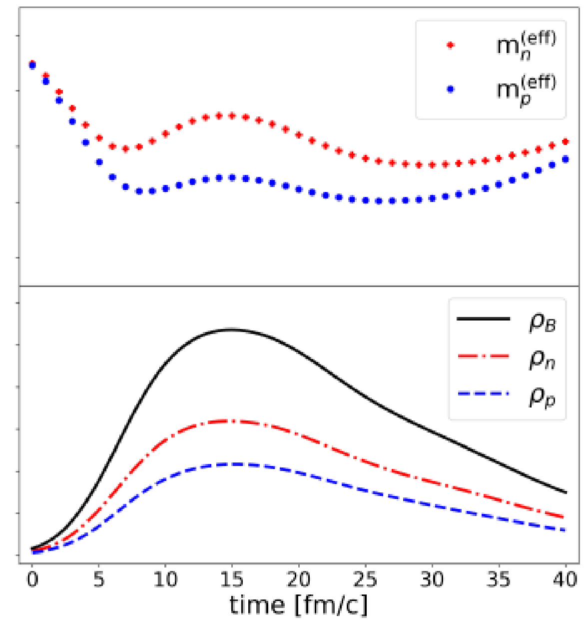
A new transport code "DaeJeon Boltzmann-Uehling-Uhlenbeck (DJBUU)" had been developed and enables to describe the dynamics of heavy-ion collisions in low-energy region. To confirm the validity of the new code, we first calculate Au + Au collisions at $E_{\mathrm{beam}} = 100$ and $400A$ MeV and perform the box calculation to check the detail of collisions and Pauli blocking without mean-field potential as suggested by the transport-code-comparison project (TCCP). After confirming the validity of new transport code, we study low-energy heavy-ion collisions with an extended parity doublet model. Since the distinctive feature of the parity doublet model is the existence of the chiral invariant mass that contributes to the nucleon mass, we investigate how physical quantities depend on the chiral invariant mass in heavy ion collisions at low energies. For this, we calculate physical quantities such as the effective nucleon mass in central collisions and transverse flow in semi-central collisions of Au + Au at $E_{\mathrm{beam}} = 400A$ MeV with different values of the chiral invariant masses.



(a) $m_0 = 600$ MeV



(b) $m_0 = 700$ MeV



(c) $m_0 = 800$ MeV

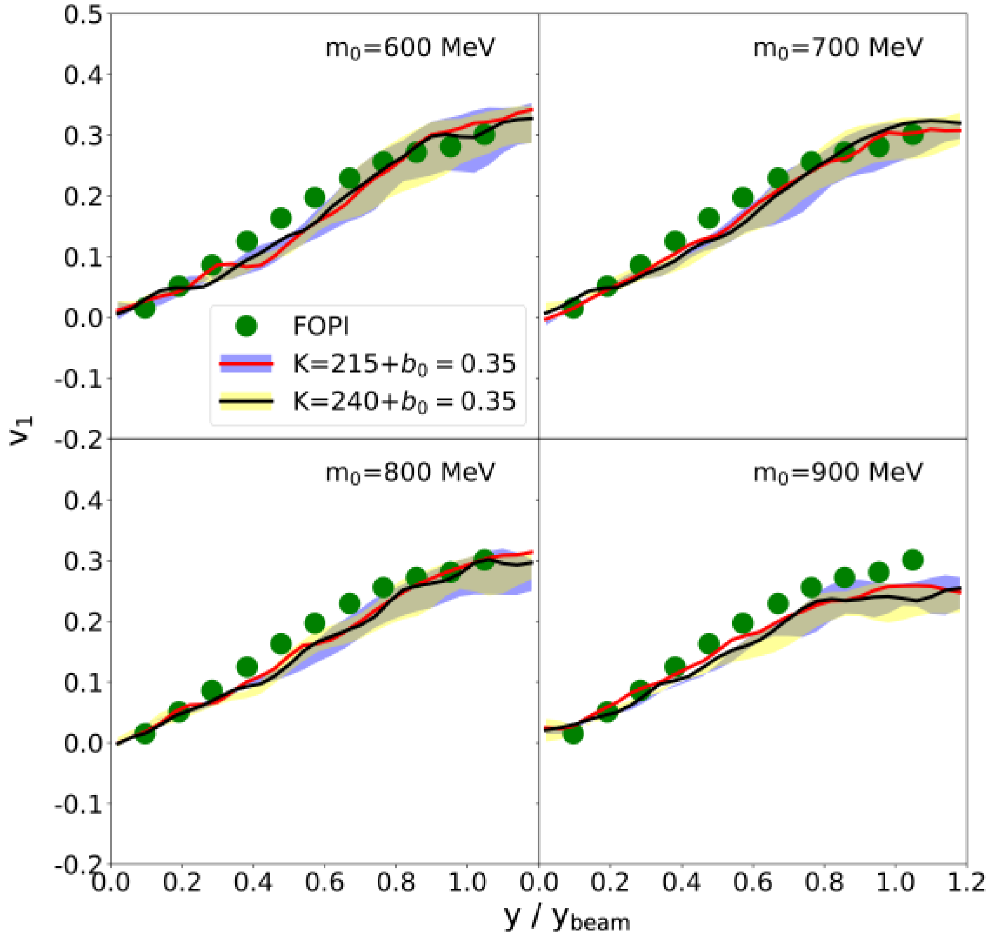
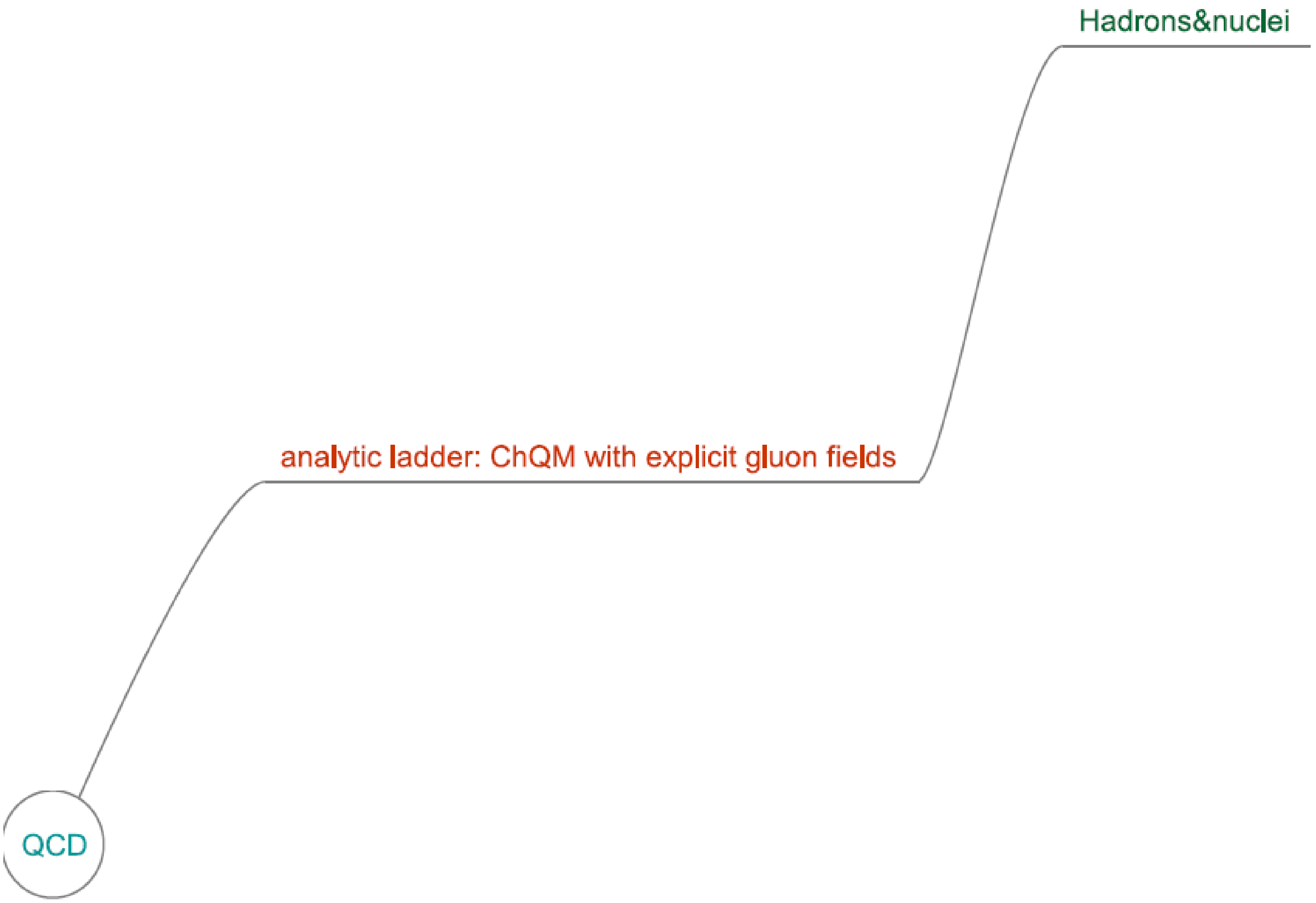


FIG. 11. Proton directed flow as a function of reduced rapidity for ^{197}Au+^{197}Au collisions with $0.25 < b_0 < 0.45$ at $E_{\rm beam} = 400A$ MeV. Two values of comppressibility, $K = 215$ MeV (purple shaded area) and $K = 240$ MeV (yellow shaded are), are considered. Upper and lower limits of each shaded area correspond to the upper and lower limits of impact parameter b_0, and the solid line correspond to the mean value $b_0 = 0.35$. FOPI data are taken from Ref. [39].

In terms of quark degrees of freedoms

- **QCD vacuum is often characterized by non-trivial gluon background configuration.**
- **Hadrons does not carry color charges.**
- **Back to old wisdom: constituent quark picture, chiral symmetry, explicit gluon field.**
- $M_p = 3m_Q$
- **There is an adequate model, called chiral quark model.**
- **We assume that the mass of constituent quark mass in the model has two components.**

Chiral quarks in Savvidy vacuum



Nuclear Physics B234 (1984) 189–212

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

The success of the NR QM?

First of all the quarks should be massive, having constituent mass.

So, we assume that the bulk of the light quark mass is the effect of chiral symmetry breaking.

The leading contribution to the baryon mass is just sum of the constituent quark masses.

$$\mathcal{L} = \bar{\psi}(i \not{D} + \not{V})\psi + g_A \bar{\psi} \not{A} \gamma_5 \psi - m\bar{\psi}\psi$$
$$+ \frac{1}{4} f^2 \mathrm{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \ldots , \quad (5)$$

where

$$V_\mu \equiv \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \quad (6)$$
$$A_\mu \equiv \frac{1}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) ,$$
$$D_\mu \equiv \partial_\mu + igG_\mu .$$

The field-strength tensor of the gluon field is given by

$$F_{\mu\nu} \equiv \partial_\mu G_\nu - \partial_\nu G_\mu + ig[G_\mu, G_\nu] .$$

$$\pi = c \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix},$$

where c$= 1/\sqrt{2}$. In terms of the matrix π, Σ and ξ are defined by

$$\Sigma = \xi\xi\,, \quad \xi = e^{i\pi/f}\,, \tag{7}$$

where $f \simeq 93$ MeV.

Pions are perturbative!

$$(2\pi)^4\delta^4(\Sigma p_i)\left(\frac{\pi}{f}\right)^A\left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B\left(\frac{gG_\mu}{\Lambda}\right)^C\left(\frac{p}{\Lambda}\right)^D f^2\Lambda^2\left[f^{-2L}(4\pi)^{-2L}\Lambda^{2L}\right].$$

How about gluons?

$$\alpha_s \simeq 0.28\,,$$

which was computed by looking at the ratio of color and electromagnetic hyperfine splittings of the baryon spectrum

- Then no room for the gluon in this model to develop any non-trivial v.e.v?
- Yes, if the constituent mass is not fully from quark-antiquark condensate.
- After all, Copenhagen (Spagetti) vacuum has also a small gauge coupling constant!

Background gluon field?

Reminder: fermions in external field

$$(i\not{D} - m)\psi = 0.$$

$$\not{D} = \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu + ieA_\mu) = \gamma^0(\partial_0 + ieA_0) + \vec{\gamma}(\vec{\nabla} - ie\vec{A}).$$

$$\partial_\mu = (\partial_0, \vec{\nabla}), A_\mu = (A_0, -\vec{A}).$$

γ-matrices in Weyl (chiral) representation

$$\gamma^0 = \begin{pmatrix} 0 & -1_2 \\ -1_2 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}.$$

$$A_0 = A_1 = A_3 = 0, A_2 = -Bx_1.$$

$$\psi(x) = \exp(-iEt + ip_2x_2 + ip_3x_3) \begin{pmatrix} f_1(x_1) \\ f_2(x_1) \\ f_3(x_1) \\ f_4(x_1) \end{pmatrix}.$$

$$\begin{pmatrix} -m & 0 & -(E+p_3) & -\xi_- \\ 0 & -m & -\xi_+ & -(E-p_3) \\ -(E-p_3) & \xi_- & -m & 0 \\ \xi_+ & -(E+p_3) & 0 & -m \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = 0,$$

where $\xi_+ = -i\frac{\partial}{\partial x_1} + i(p_2 + eBx_1)$, $\xi_- = -i\frac{\partial}{\partial x_1} - i(p_2 + eBx_1)$.

$$I_{np_2}(x_1) = \left(\frac{eB}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{1}{2}eB\left(x_1 + \frac{p_2}{eB}\right)^2\right)\frac{1}{\sqrt{n!}}H_n\left(\sqrt{2eB}\left(x_1 + \frac{p_2}{eB}\right)\right),$$

where $H_n(x)$ are "probabilistic" Hermite polynomials. Their Rodrigues formula

$$H_n = (-1)^n e^{x^2/2}\frac{d^n}{dx^n}e^{-x^2/2},$$

orthogonality is

$$\int_{-\infty}^{\infty} H_n(x)H_{n'}(x)e^{-x^2/2}dx = \sqrt{2\pi}n!\delta_{nn'}.$$

$\xi_\pm$ act to I_{np_2} by following rules:

$$\xi_+ I_{np_2} = i\sqrt{2eB(n+1)}I_{n+1,p_2}, \quad \xi_- I_{np_2} = -i\sqrt{2eBn}I_{n-1,p_2}$$

$$\psi^{(+)}_{1,np_2p_3} = \frac{\exp(-iEt + ip_2x_2 + ip_3x_3)}{\sqrt{2E_n(E_n - p_3)}} \begin{pmatrix} -i\sqrt{2eBn}I_{n-1,p_2}(x_1) \\ (E_n - p_3)I_{np_2}(x_1) \\ 0 \\ -mI_{np_2}(x_1) \end{pmatrix},$$

$$E_n = \sqrt{m^2 + p_3^2 + 2eBn}.$$

Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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(Received 11 May 1994)

It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

$$m_{\rm dyn} = \bar{\sigma} \sim |eB|^{1/2}.$$

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RG analysis of magnetic catalysis in dynamical symmetry breaking

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We perform the renormalization group analysis on dynamical symmetry breaking under a strong external magnetic field studied recently by Gusynin, Miransky, and Shovkovy. We find that any attractive four-Fermi interaction becomes strong at low energy, thus leading to dynamical symmetry breaking. When the four-Fermi interaction is absent, the β function for the electromagnetic coupling vanishes in the leading order in $1/N$. By solving the Schwinger-Dyson equation for the fermion propagator, we show that in the $1/N$ expansion, for any electromagnetic coupling, dynamical symmetry breaking occurs due to the presence of a Landau energy gap by the external magnetic field. [S0556-2821(96)03724-1]

$$k_z \to s k_z, \quad \omega \to s\omega \ \text{with} \ s<1,$$

$$\psi_0(k_z,\omega) \to s^{-3/2}\psi_0(\omega,k_z),$$

$$\psi_n(k_z,\omega) \to s^{-1}\psi_n(\omega,k_z) \ \text{for} \ n>0.$$

$$m_{\rm dyn} \sim \sqrt{|eB|}\, e^{-4\pi^2/Ng(|eB|)}.$$

Fermions at the LLL are 1+1 dimensional !

INFRARED INSTABILITY OF THE VACUUM STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

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A constant chromomagnetic field is a non-trivial classical solution of the SU(2) Yang-Mills equation of motion. The real part of the one-loop vacuum energy in a homogeneous chromomagnetic field is given by

$$\mathrm{Re}\,\epsilon = \frac{1}{2}H^2 + \frac{11}{48\pi^2}g^2H^2\left(\ln\frac{gH}{\Lambda^2} - \frac{1}{2}\right)$$

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AN UNSTABLE YANG-MILLS FIELD MODE

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Soon after this interesting finding, a subsequent study showed that the Savvidy vacuum is unstable due to the imaginary part in the one-loop vacuum energy

$$\mathrm{Im}\,\epsilon = -\frac{(gH)^2}{8\pi^2}\,.$$

It is straightforward to identify the origin of the instability [4]. For SU(2) Yang-Mills theory, with a specific choice of the constant chromomagnetic field $A_y^3 = Hx$, the eigenvalue of the gluon field, $W_\mu = (A_\mu^1 + A_\mu^2)/\sqrt{2}$ becomes

$$E_n = \sqrt{2gH(n+\frac{1}{2}) + k_3^2 \pm 2gH}\,.$$

Extension to SU(3) and SU(4) were done in Ref. [5].

It is obvious that the constant magnetic field cannot be the true QCD vacuum because it is unstable and breaks rotational and Lorentz invariance. It is argued that the instability can be removed by the formation of domains.

It was shown in Ref. [6] that the chromomagnetic field has locally a domain-like structure: the field has different orientation in different domains. At long distance the orientations of domains is random so that both gauge and rotational symmetries of the vacuum can be restored in the infrared regime.

The domain structure introduces an infrared cutoff that prevents the momenta from taking the smaller values causing the instability (it is still controversial, though …)

In Ref. [7], an interesting observation was made that the coupling constant that minimizes the vacuum energy is unexpectedly small $\alpha_s = g^2/4\pi = 0.37$.

[4] N. K. Nielsen and P. Olesen, Nucl. Phys. B **144**, 376 (1978).
[5] H. Flyvbjerg, Nucl. Phys. B **176**, 379 (1980).
[6] H. B. Nielsen and P. Olesen, Nucl. Phys. B **160**, 380 (1979).
[7] J. Ambjorn and P. Olesen, Nucl. Phys. B **170**, 60 (1980).

As a first step, we consider a simple choice of the constant chromomagnetic field $A_y^3 = Hx$ to calculate the effective action with finite chemical potential for the equation of state and the shift of the quark mass to discuss the chiral invariant nucleon mass, etc.

At least for the evaluation of the effective action, we can use the results from dense matter with external U(1) magnetic field.
The difference is that the gluon field has a color matrix T^3 whose eigenvalue is +1/2, -1/2.

$$\mathcal{L}_{eff} = \frac{4gH}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=0}^{2} \int_{-\infty}^{\infty} d\omega f_F(\omega)$$

$$\int_0^{\infty} dk k^2 \delta(\omega^2 - k^2 - 2gH(n + \lambda - 1) - m^2) \, .$$

P. Elmfors, D. Persson, B.-S. Skagerstam, Astroparticle Physics (1994),
D. Persson, V. Zeitlin, Phys. Rev. D 51 (1995), ...

Explicit evaluations of the mass corrections at one-loop with H in dense matter is underway with Igor Mazru (CENuM).

Summary

- ChQM + (improved) Savvidy vacuum may be a way to connect QCD vacuum with real world.
- Since certain gluon background fields ensure confinement, the combined one may also provide a handy platform for QM and NM.
- As a concrete attempt; ChQM in Savvidy vacuum (H) → chiral invariant mass → parity doublet model→ dense matter, finite nuclei, observables in HIC through nuclear transport.
- Hoping to enrich RI physics!