

Decay Spectroscopy of Mo Isotopes

A Quest to Exotic Nuclear Shapes

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- New Levels, Lifetime of 2^+ States

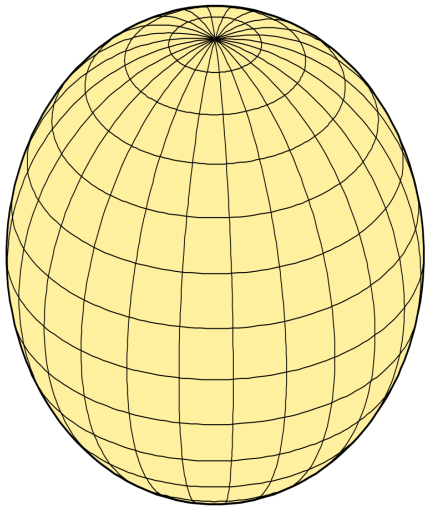
Discussion

- Quest to Exotic Nuclear Shape

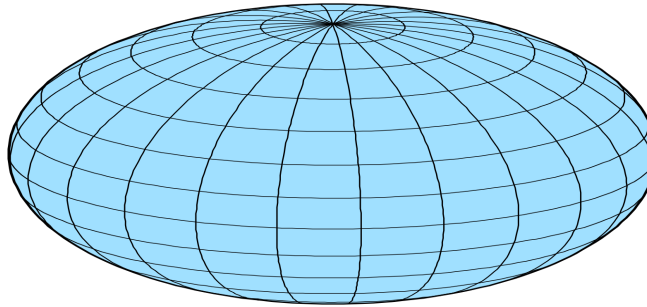
Summary

Shapes of Atomic Nuclei

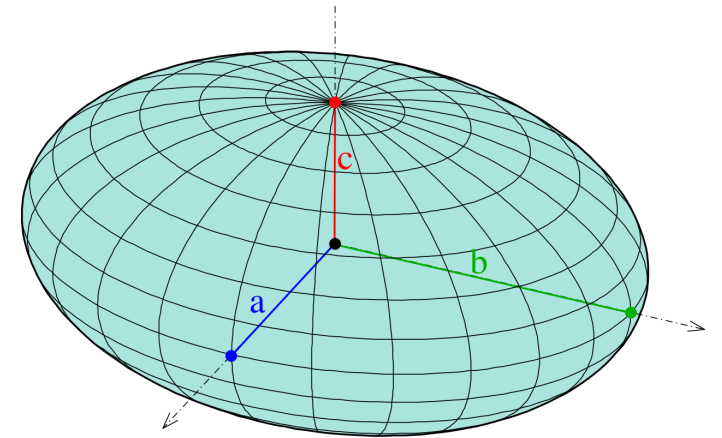
Possible shapes of deformed nuclei



Prolate



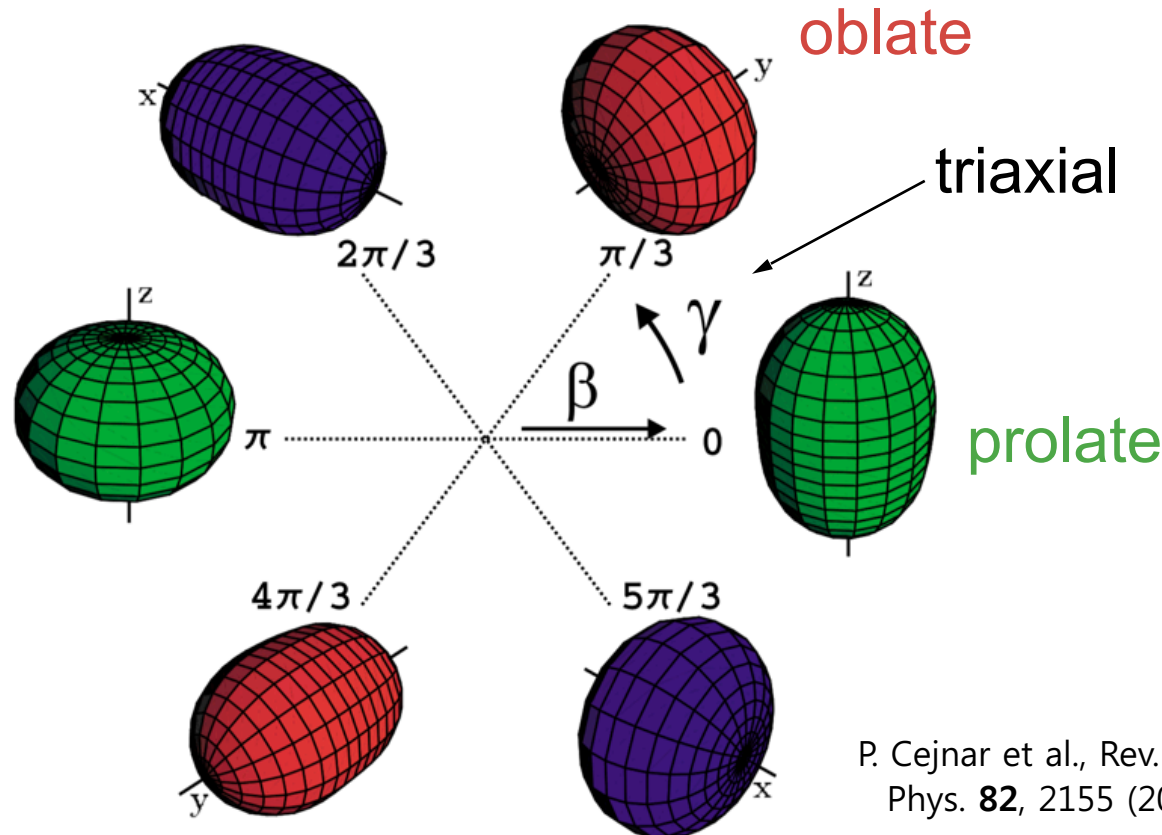
Oblate



Triaxial

Shapes of Atomic Nuclei

Coordinate: 'Hill-Wheeler' coordinate



Parametrization of
Quadrupole Deformation

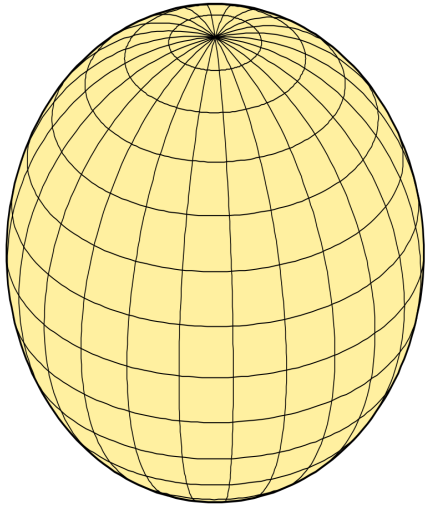
$$R = R_0(1 + a_{2m}Y_{2m}(\theta, \phi))$$

$$a_{20} = \beta \cos \gamma, \quad a_{22} = \frac{1}{\sqrt{2}}\beta \sin \gamma$$

P. Cejnar et al., Rev. Mod.
Phys. **82**, 2155 (2010)

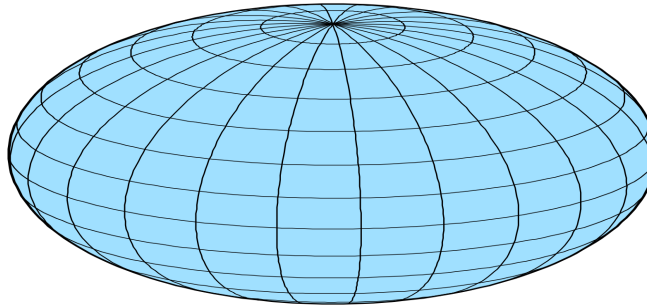
Shapes of Atomic Nuclei

Possible shapes of deformed nuclei



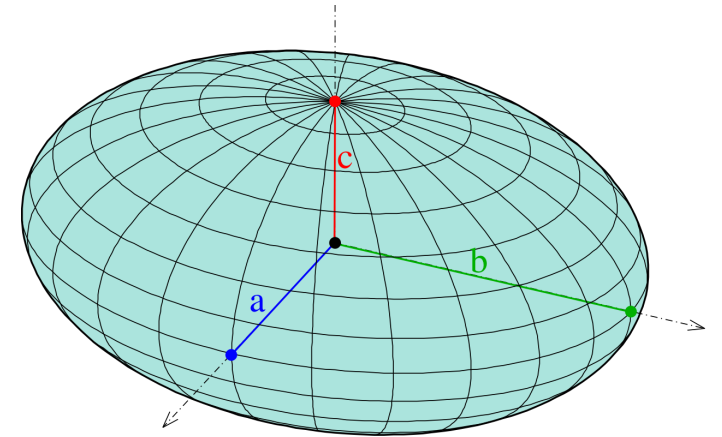
Prolate

$$\gamma = 0^\circ$$



Oblate

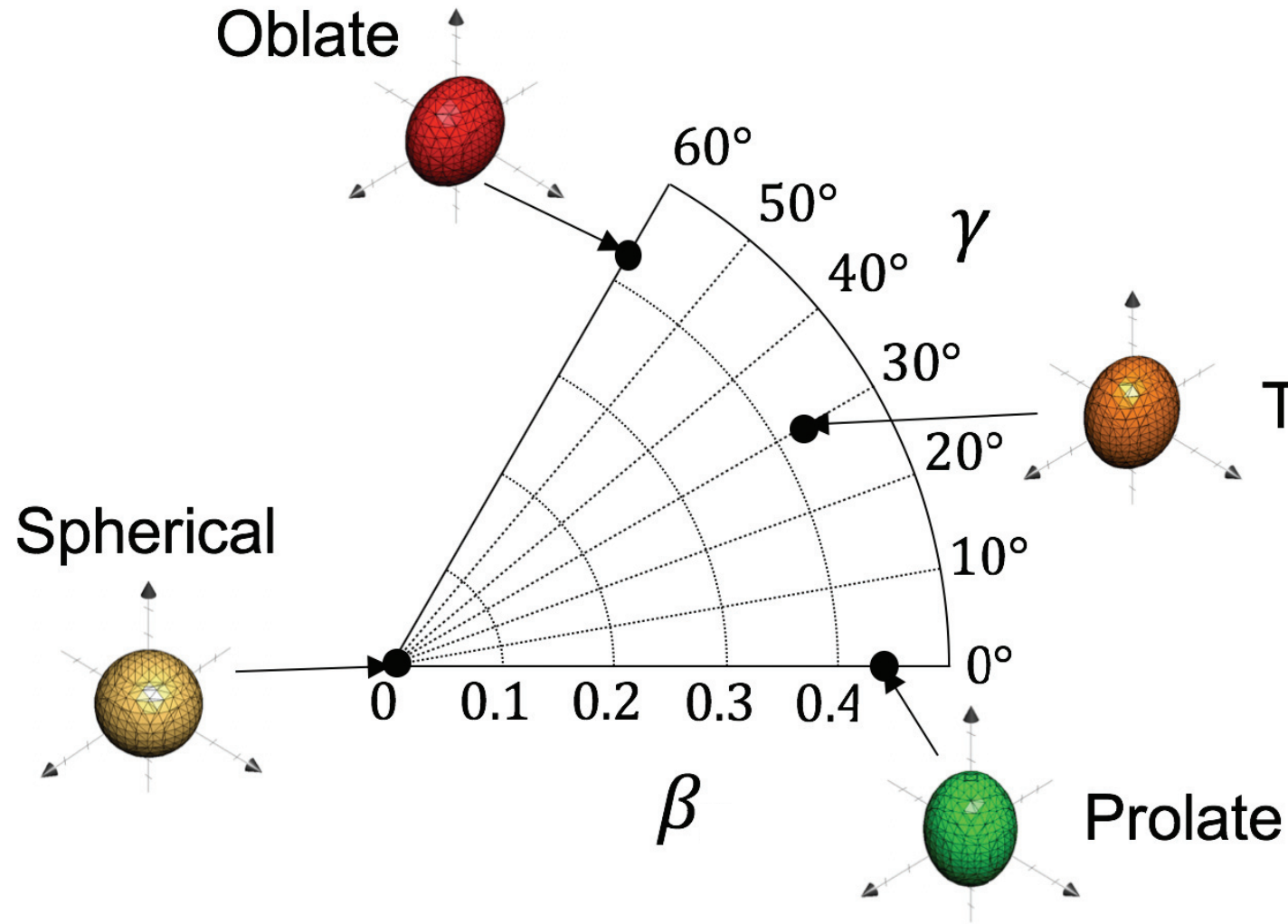
$$\gamma = 60^\circ$$



Triaxial

Degree of deformation: β , degree of asymmetry: γ

Shapes of Atomic Nuclei



Degree of deformation: β

Degree of asymmetry: γ

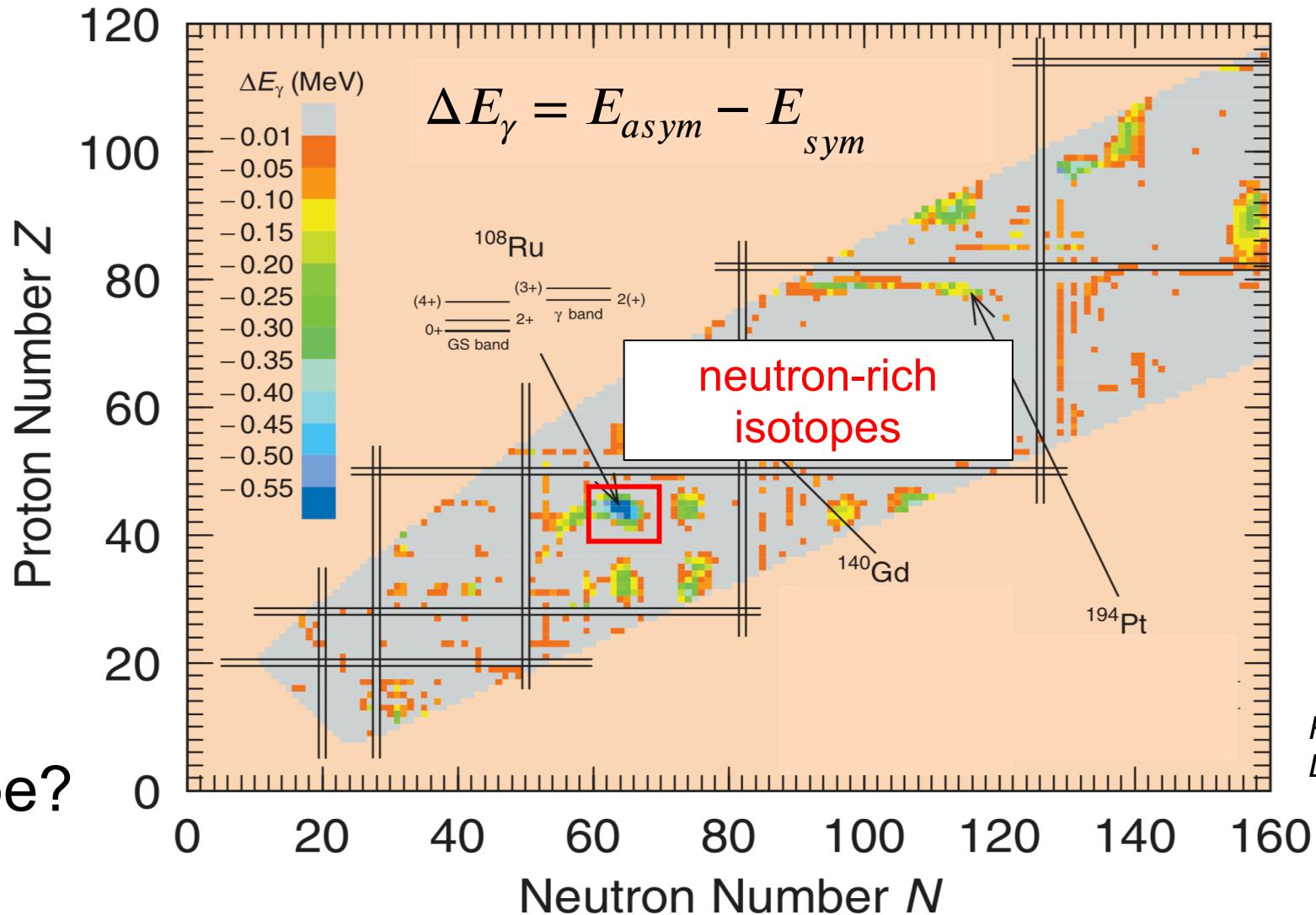
Triaxial

$$R = R_0(1 + a_{2m}Y_{2m}(\theta, \phi))$$

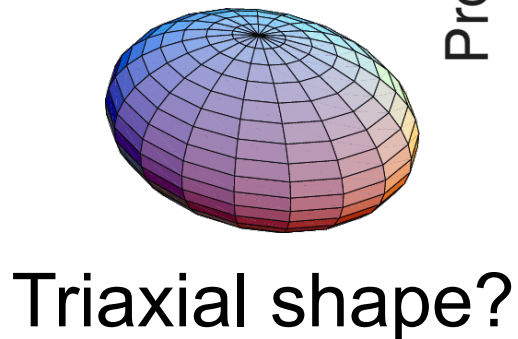
$$a_{20} = \beta \cos \gamma, \quad a_{22} = \frac{1}{\sqrt{2}}\beta \sin \gamma$$

Shapes of Atomic Nuclei

Candidates of the triaxial ground-state shape

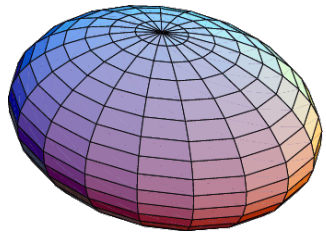


*P. Moller et al., Phys. Rev. Lett. **97**, 162502 (2006)*

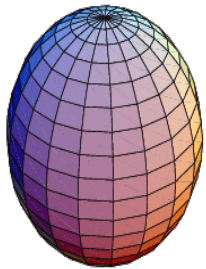


Shapes of Atomic Nuclei

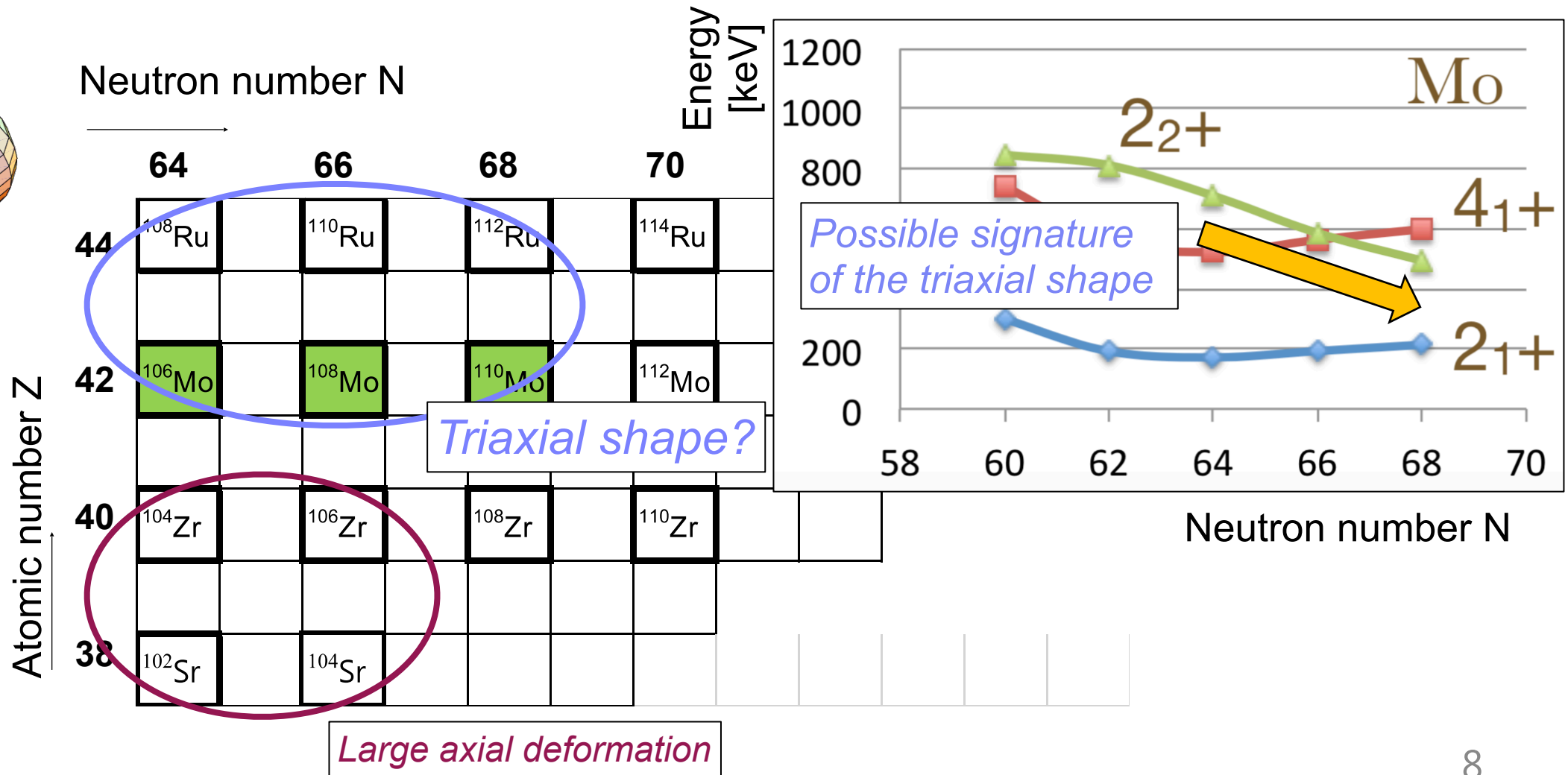
Nuclear chart around $A = 110$



Triaxial
shape?

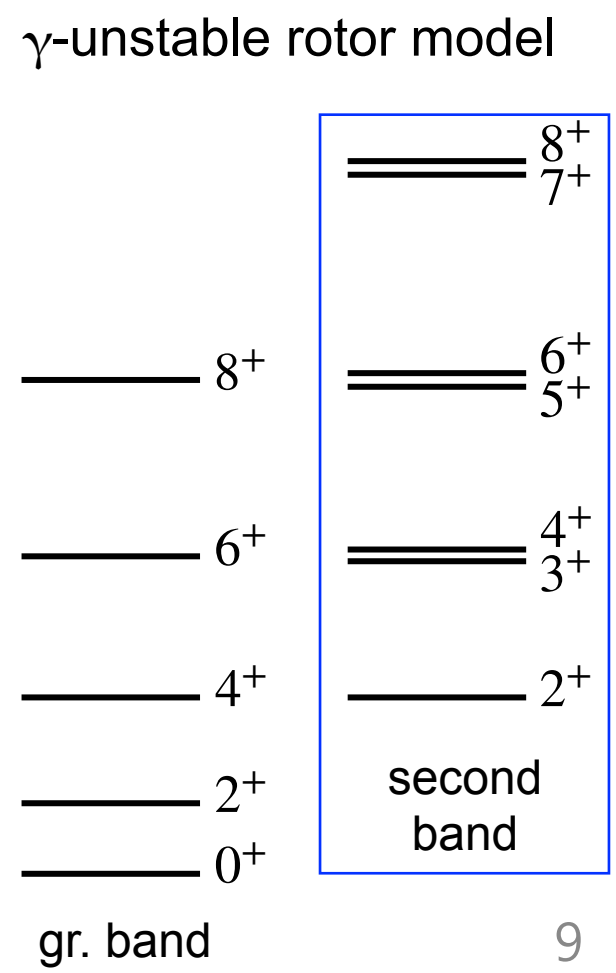
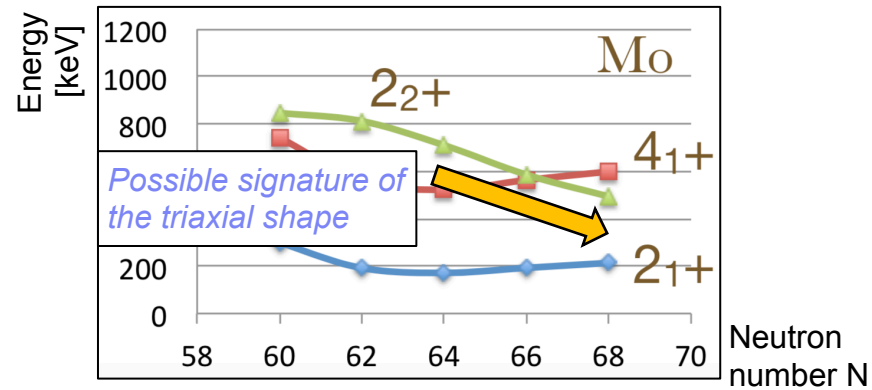
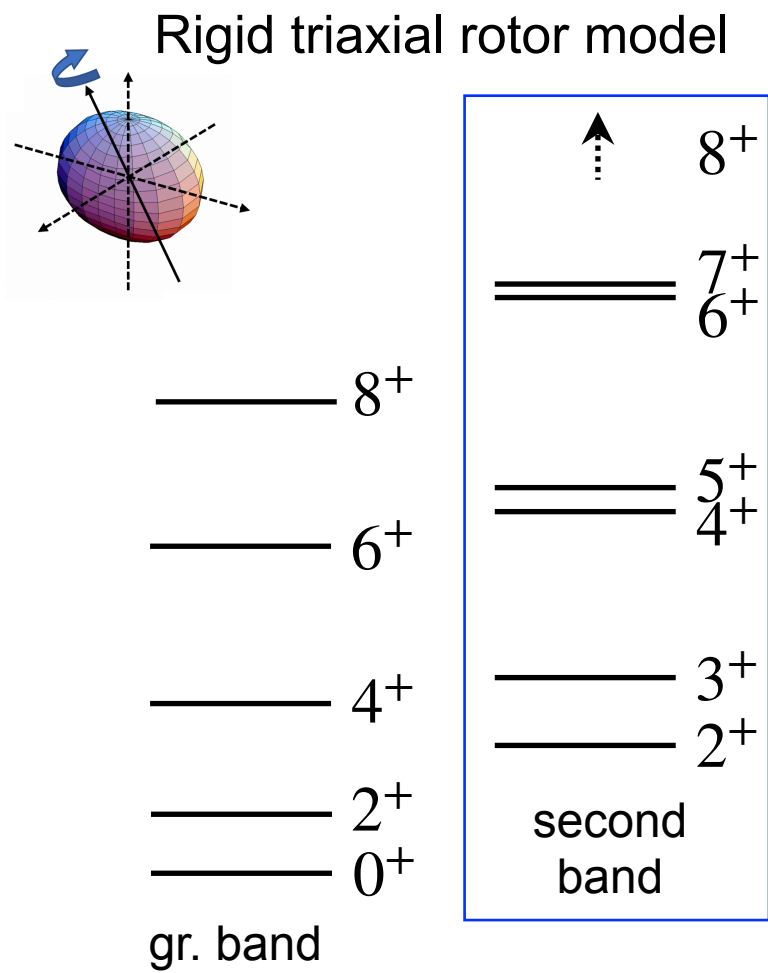
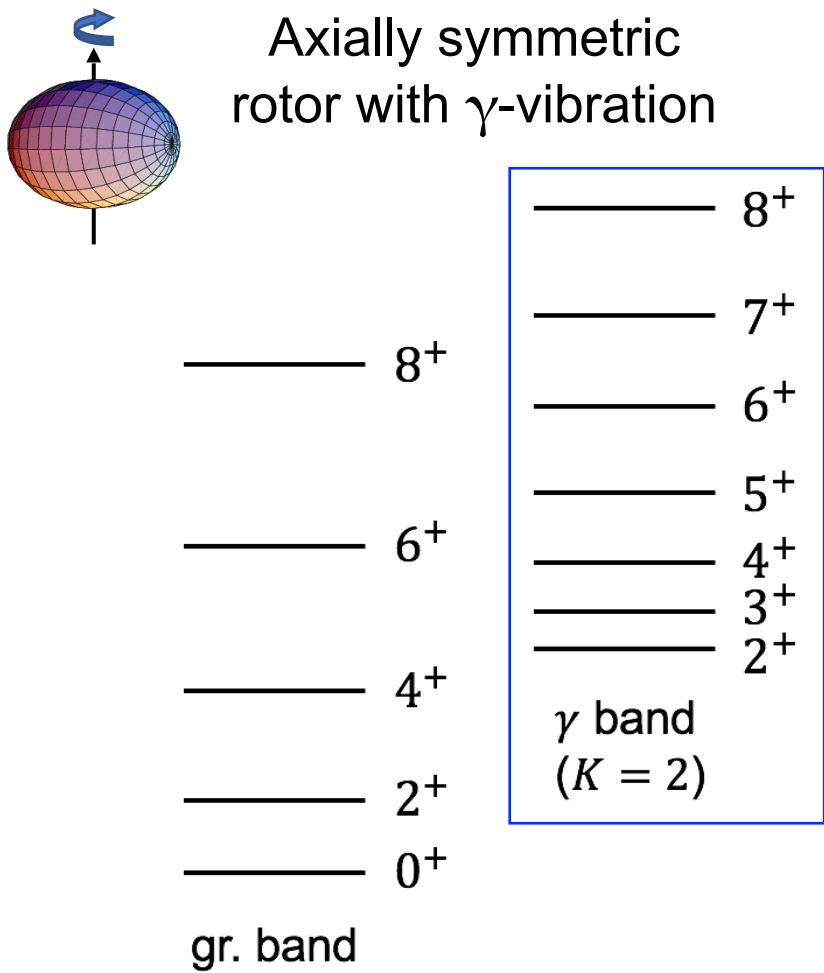


Axially
symmetric
shape

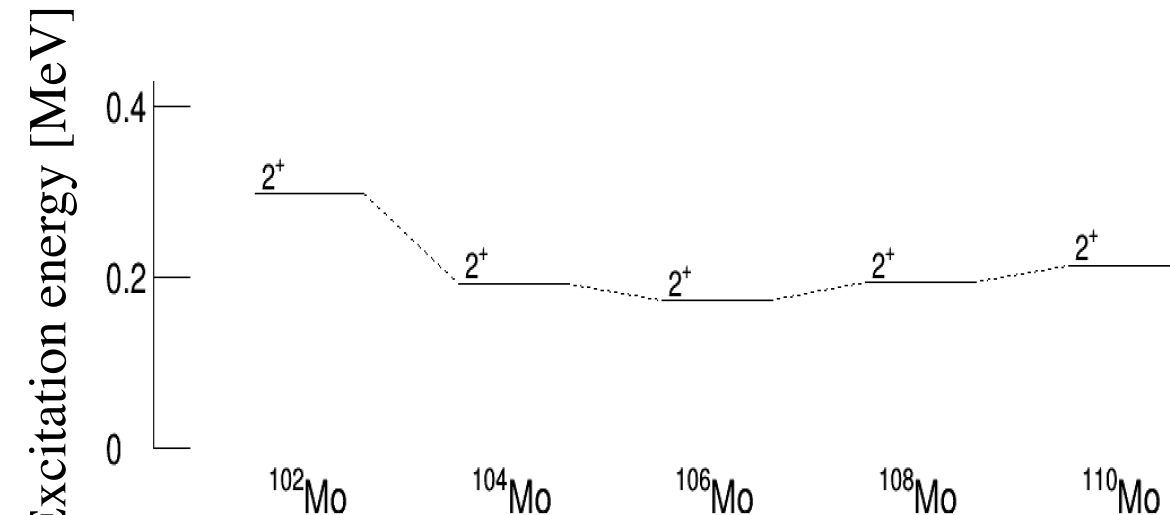
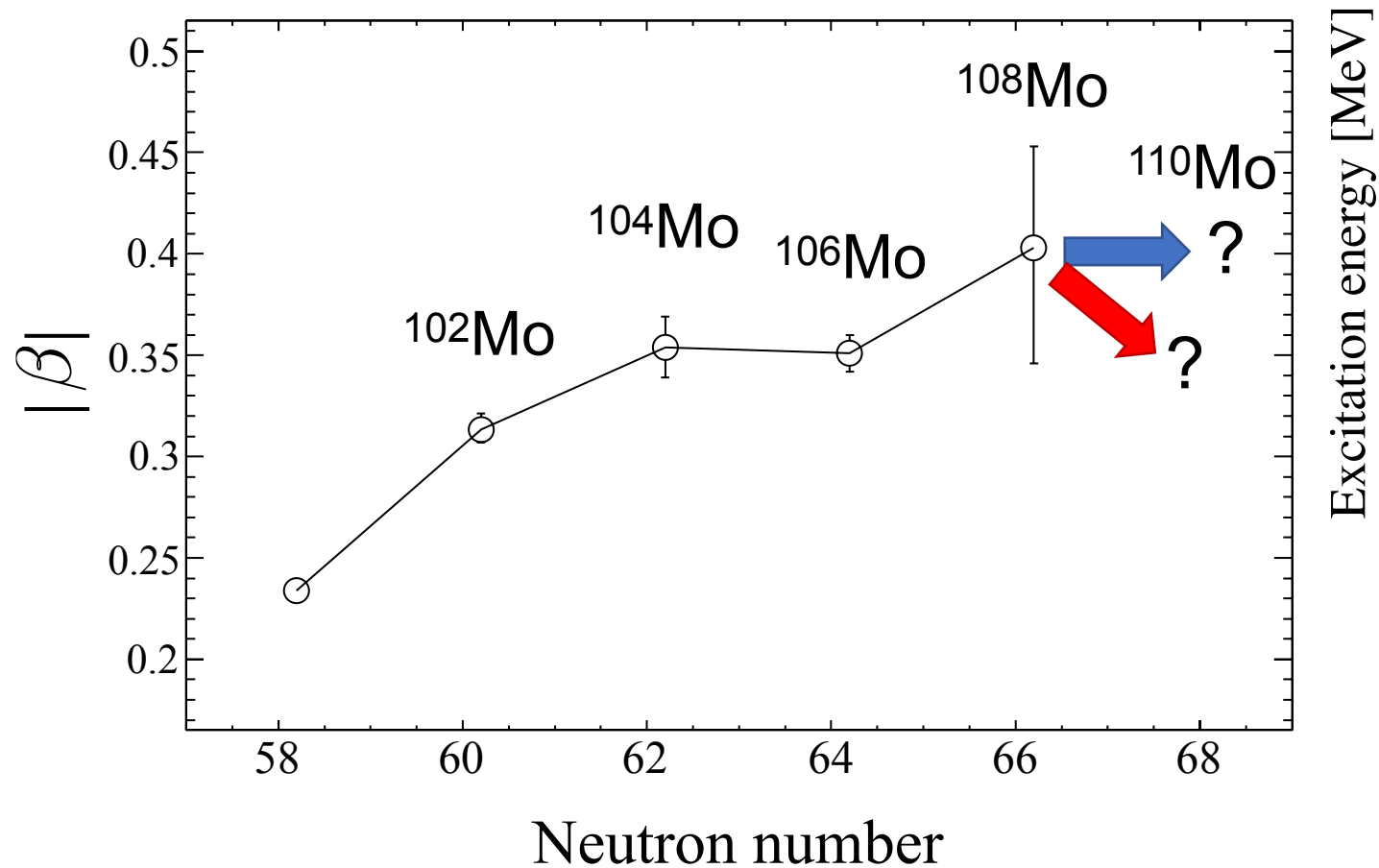


More Information → Better Understanding

Rotor models for $A \sim 110$ nuclei



Evolution of β and $E(2_1^+)$

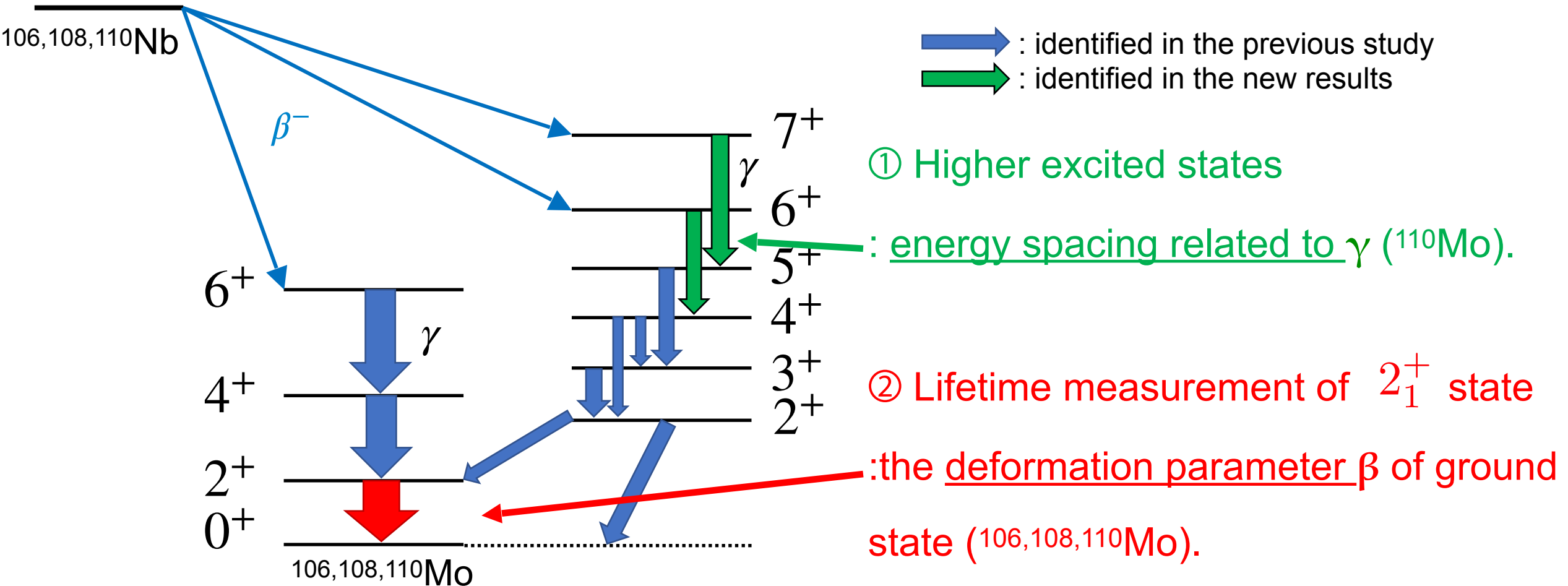


$$\beta \propto \sqrt{\frac{1}{E_\gamma^5}}$$

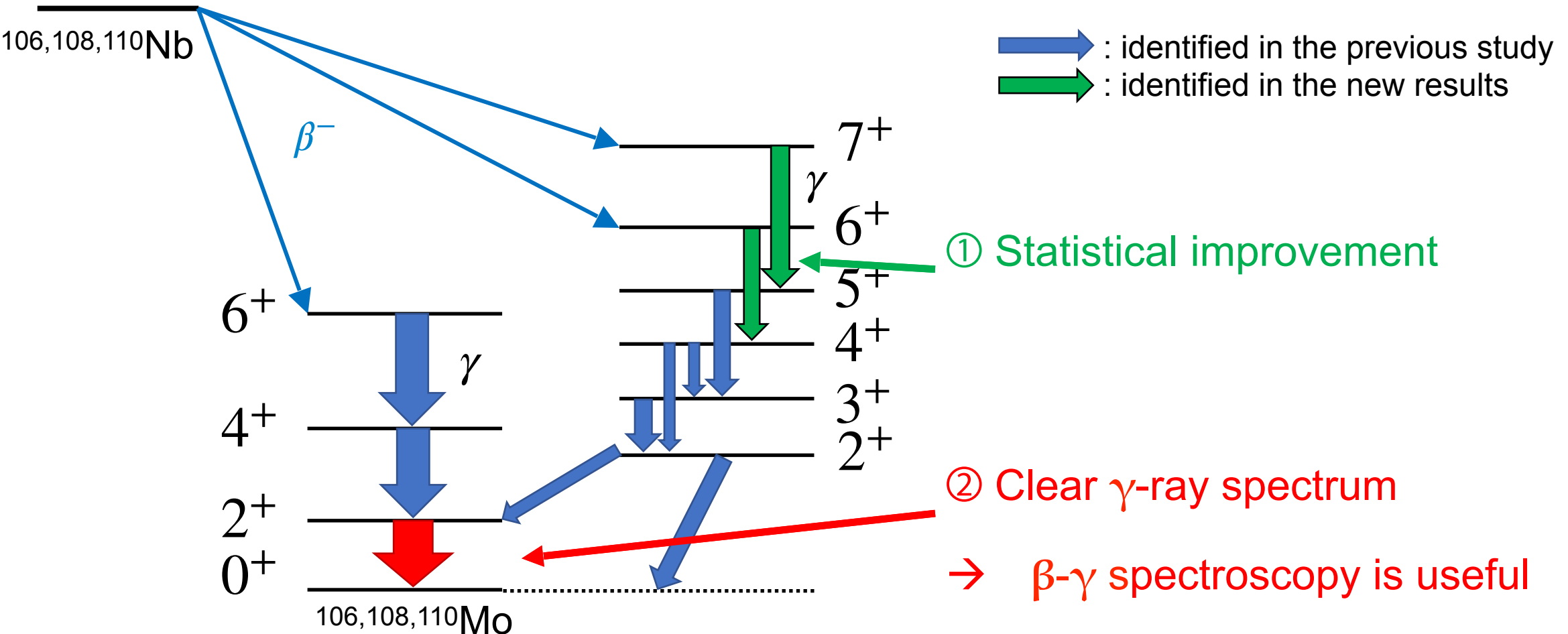
E_γ : γ -ray Energy ($2_1^+ \rightarrow 0_1^+$)

β : deformation parameter

β - γ Decay Spectroscopy

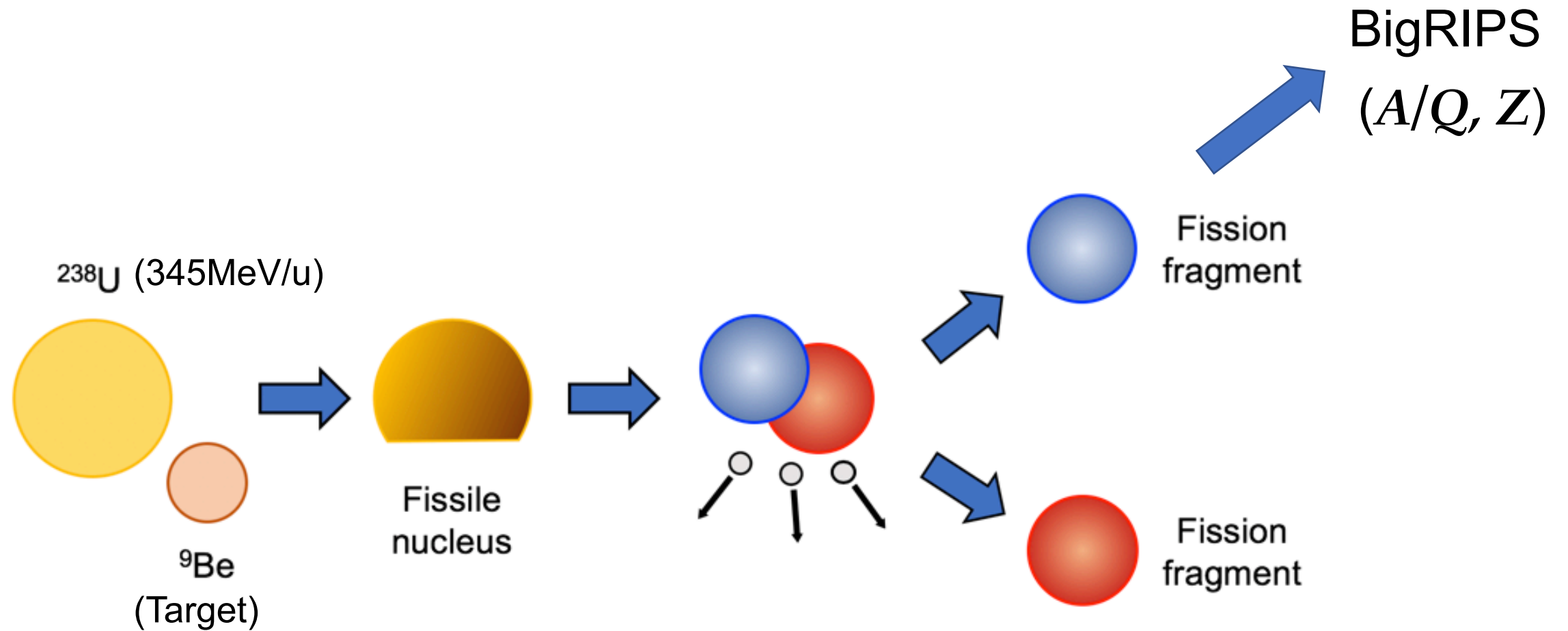


β - γ Decay Spectroscopy

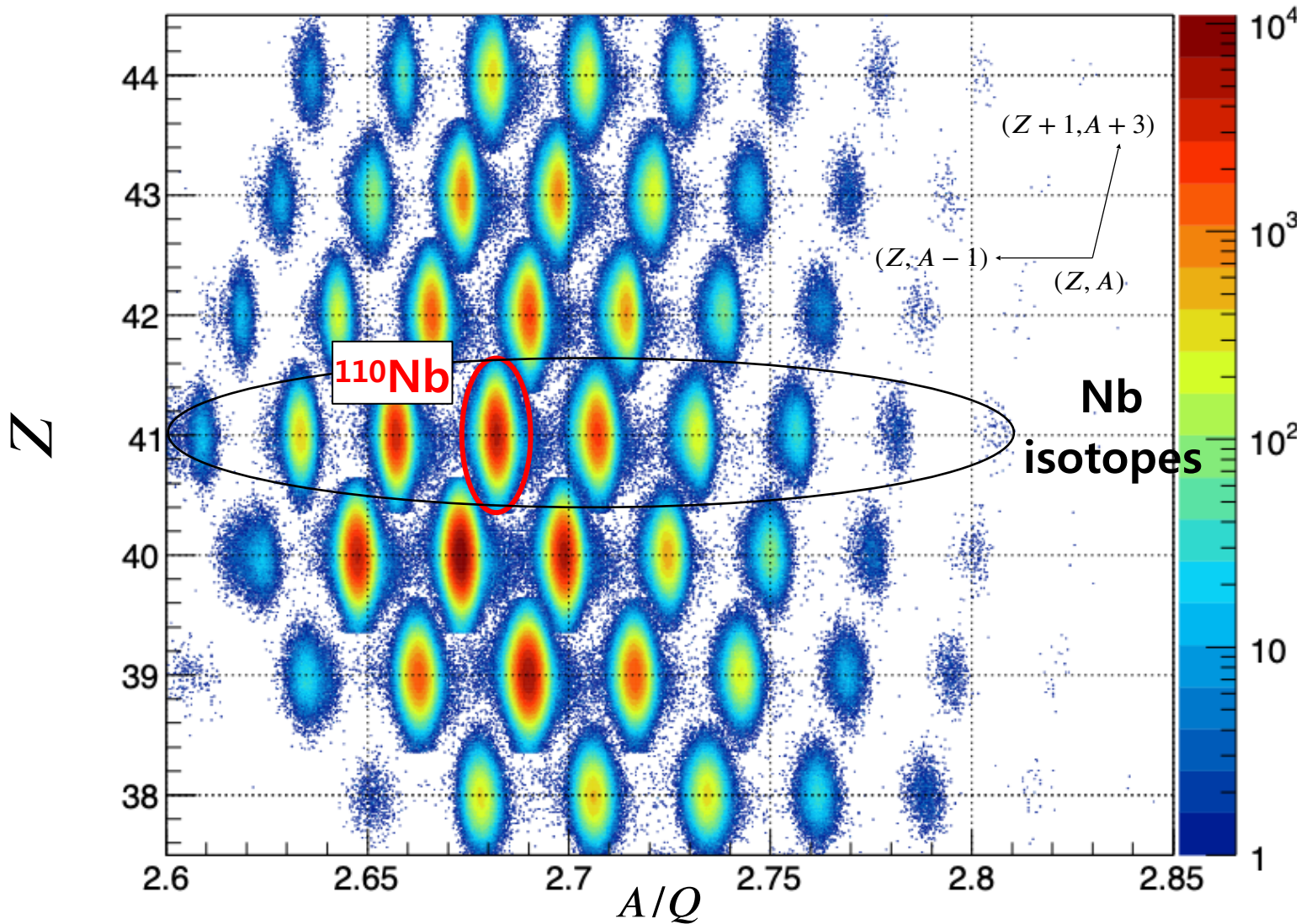


$\rightarrow$ additional information on nuclear structure

Production of RIB at RIKEN



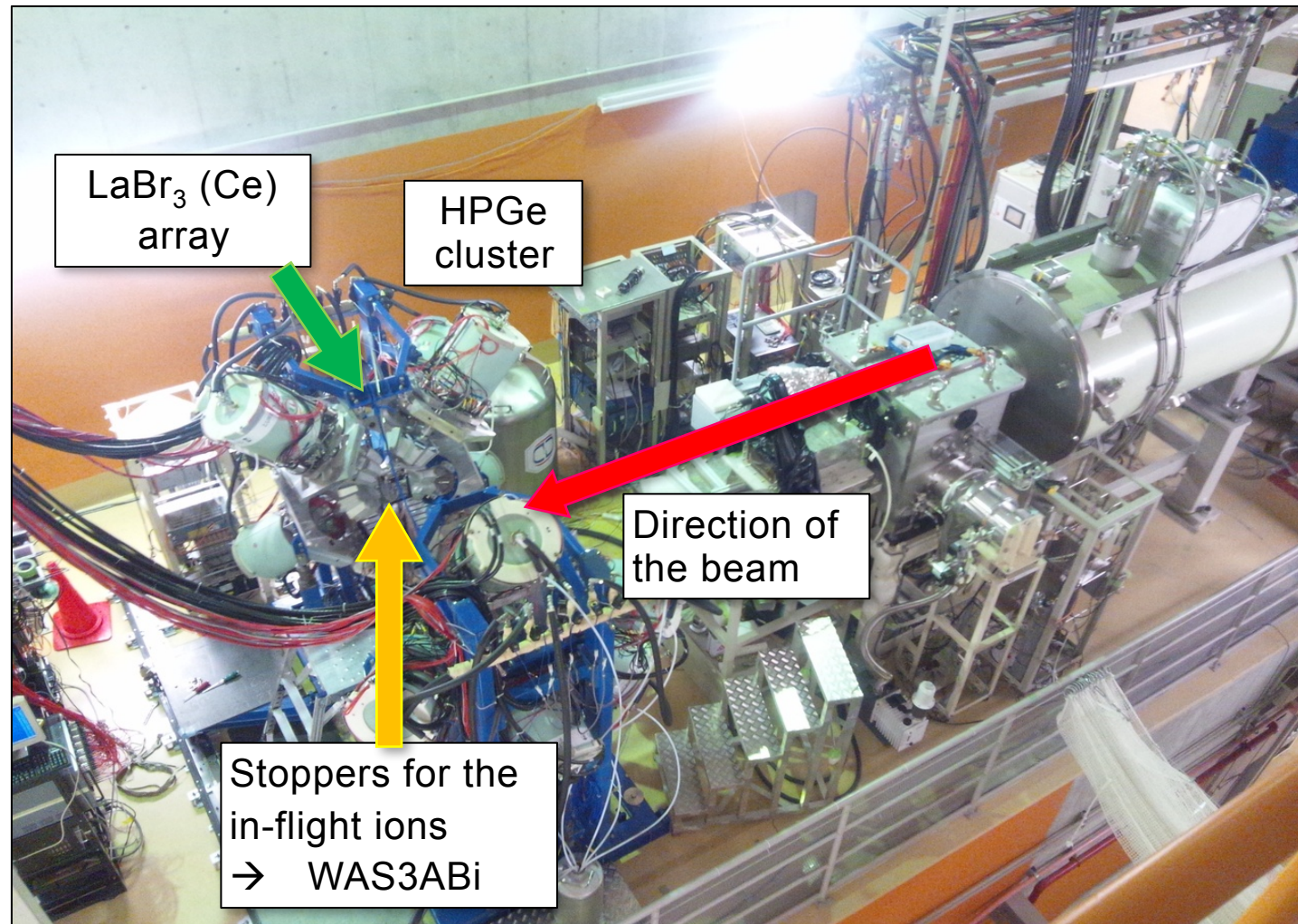
Identification of the Fragments



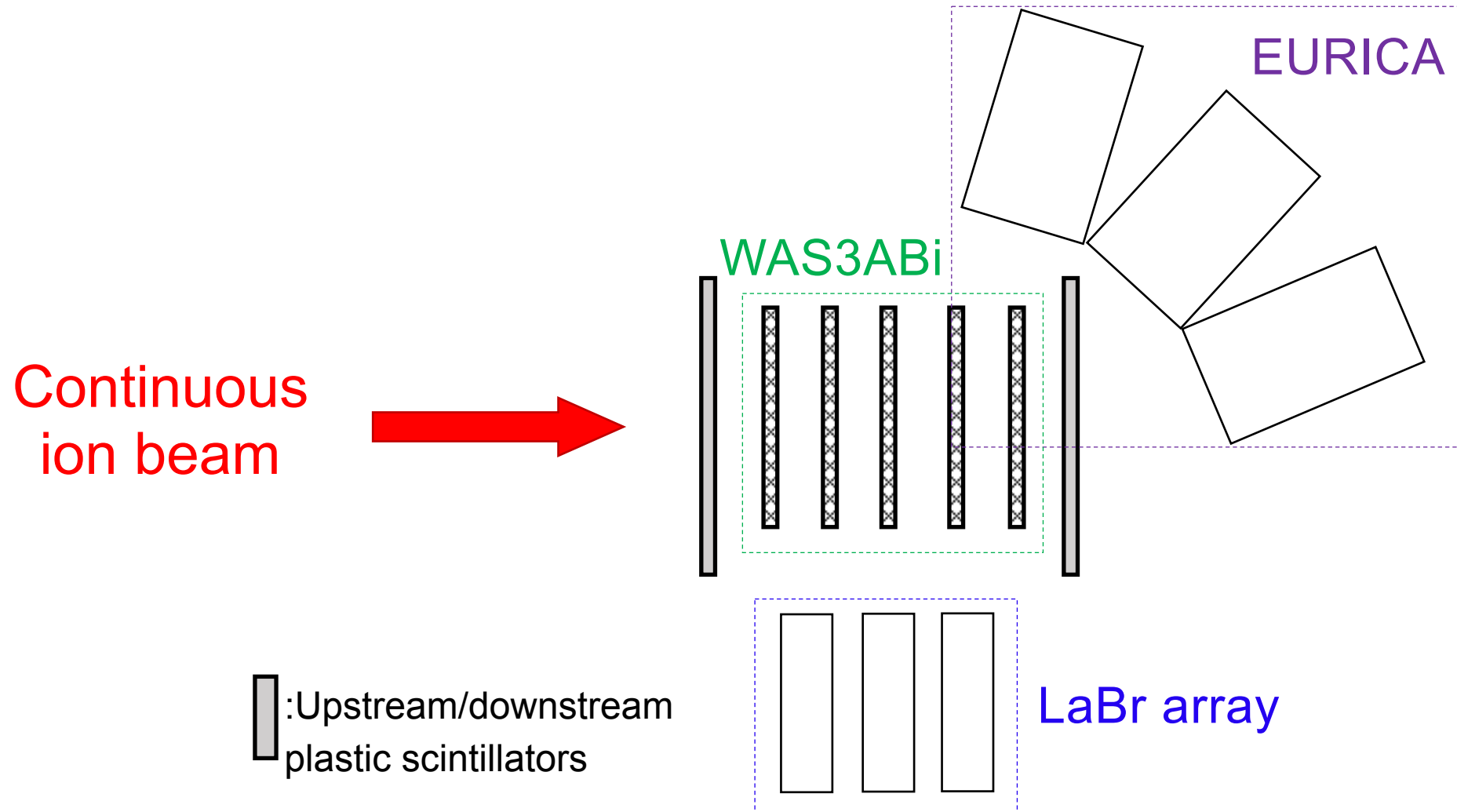
Isotope	The number of implanted ions
^{106}Nb	7.1×10^4
^{108}Nb	1.3×10^5
^{110}Nb	1.9×10^6

Detectors

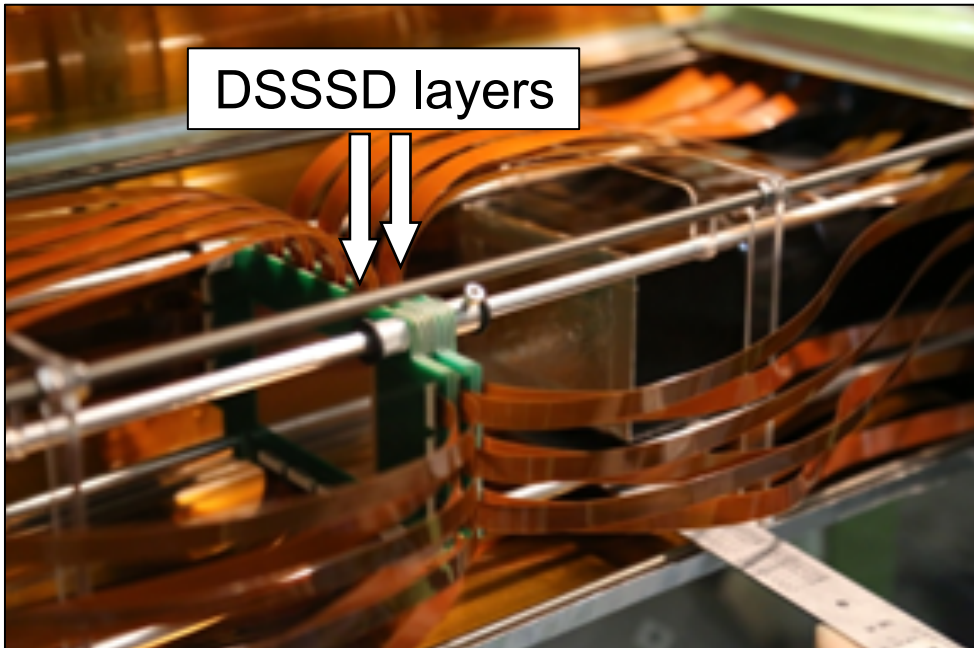
Overview of the experimental area



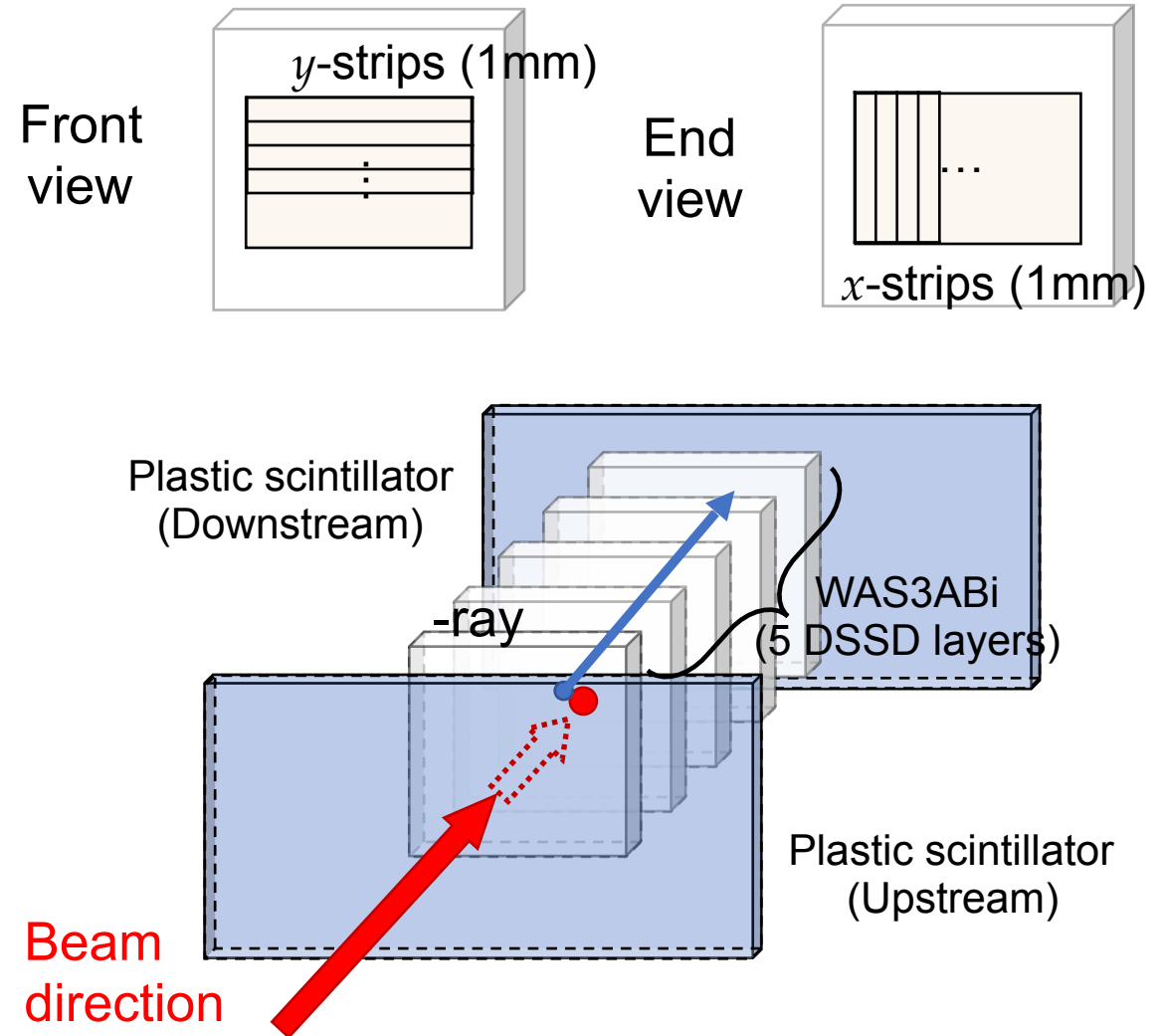
β - γ Decay Spectroscopy at RIBF



Ion Stopping at WAS3ABi

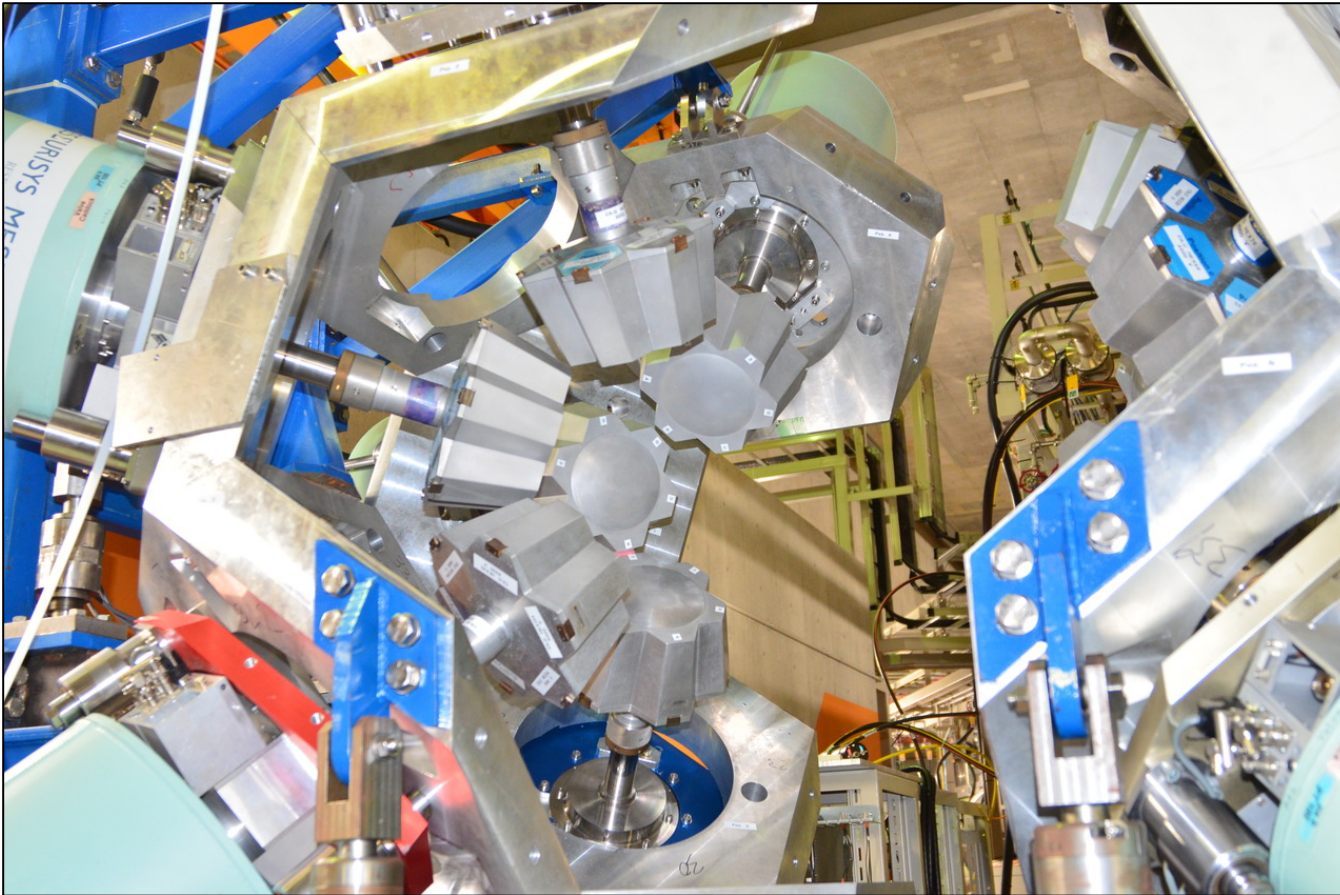


- Five DSSSDs (Double-Sided Silicon Strip Detectors) with 1-mm thickness
- 1-mm width x -strips and y -strips cover 6040 mm active area



Gamma Ray Detection with EURICA*

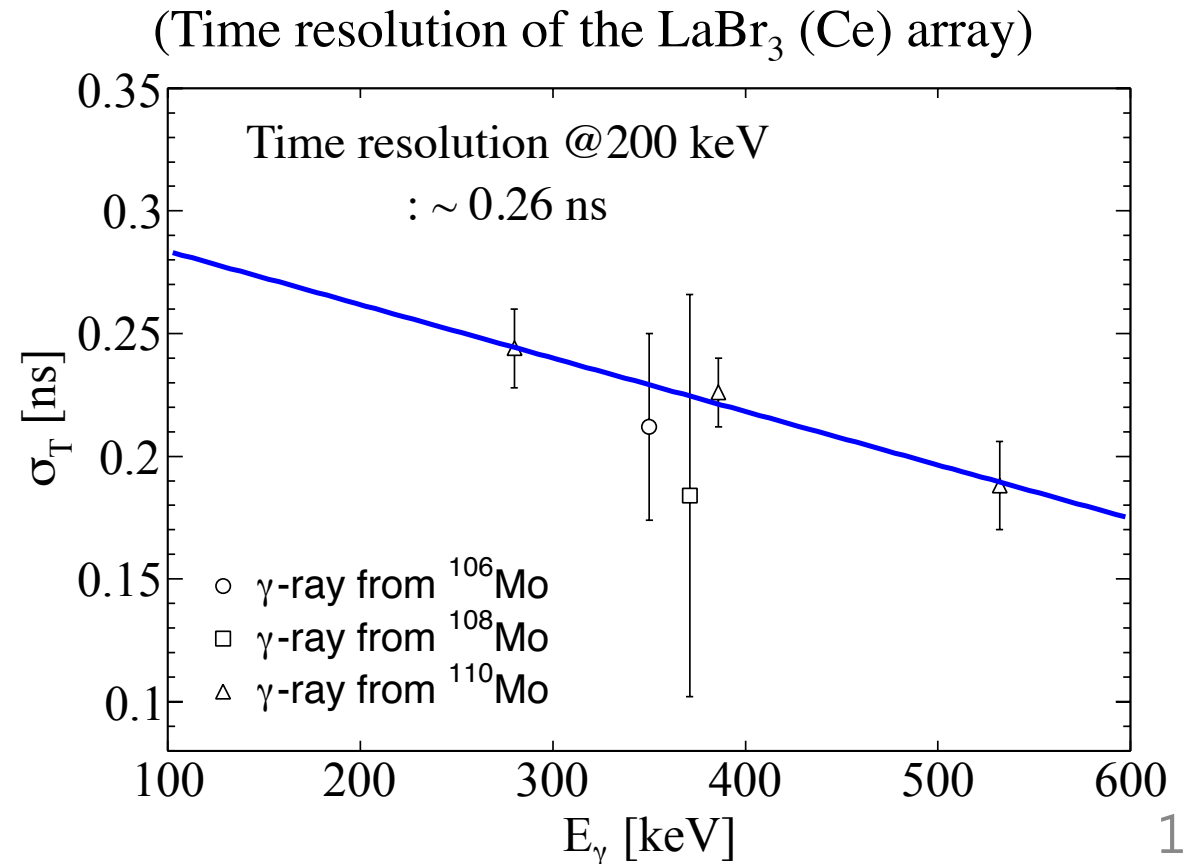
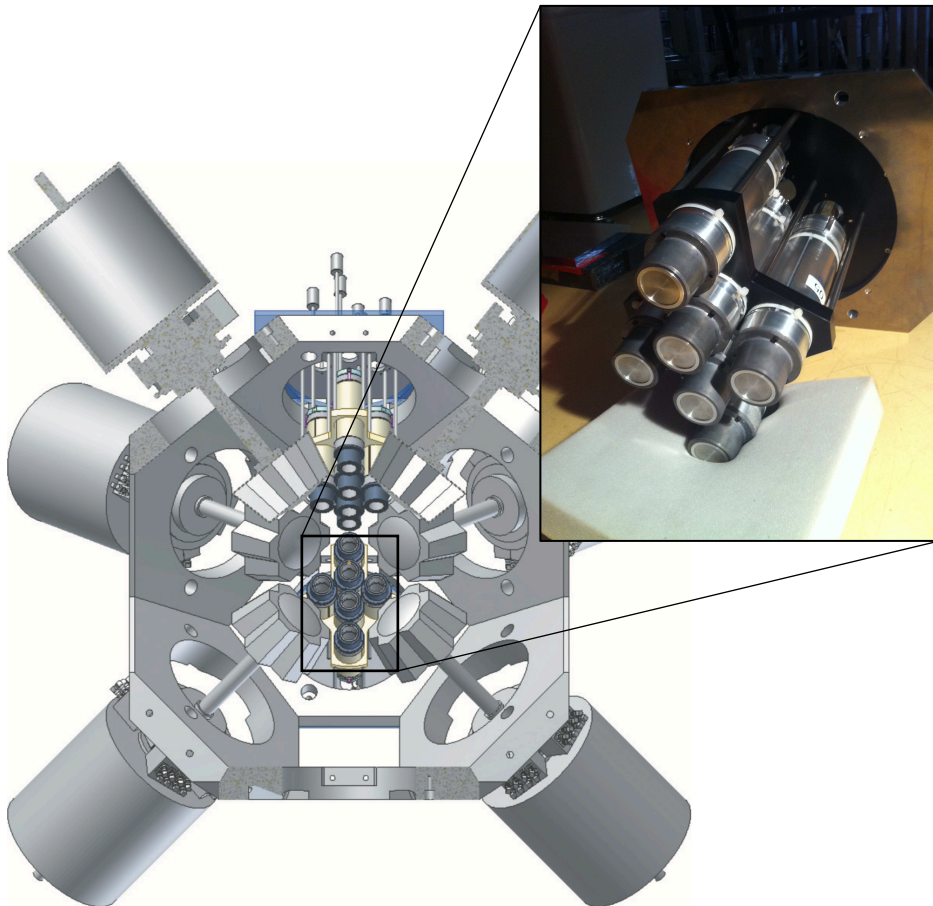
* Euroball-RIKEN Cluster Array



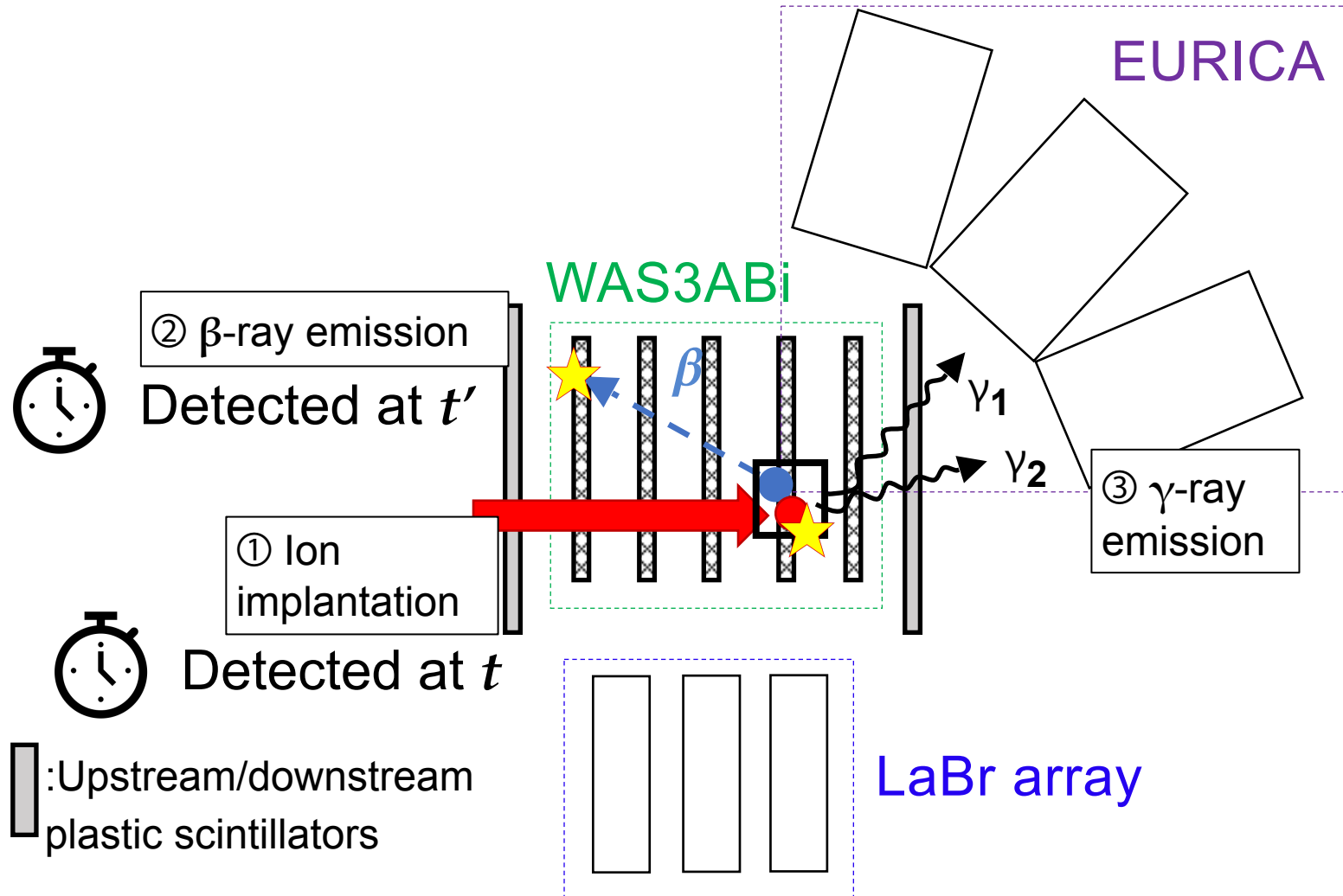
- 84 Ge crystals (7 crystals 12 clusters), designed for $\sim 4\pi$ acceptance
- from Europe (GSI, Germany)

LaBr₃(Ce) Array - Gamma Ray Timing

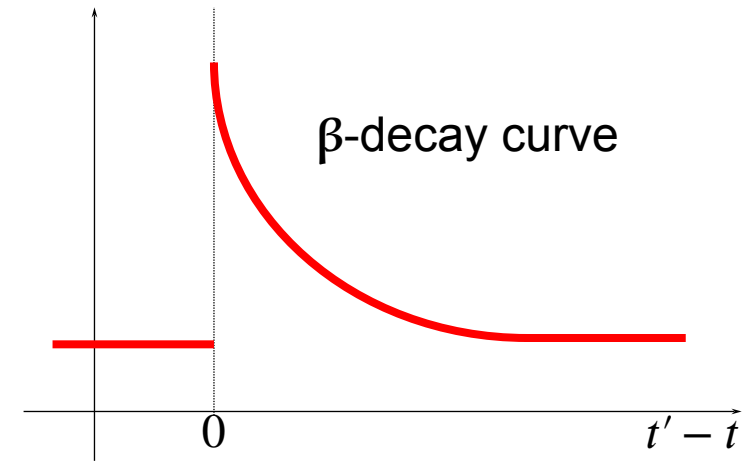
- 18 cerium-doped lanthanum tri-bromide crystals
- from United Kingdom
- **Fast response**



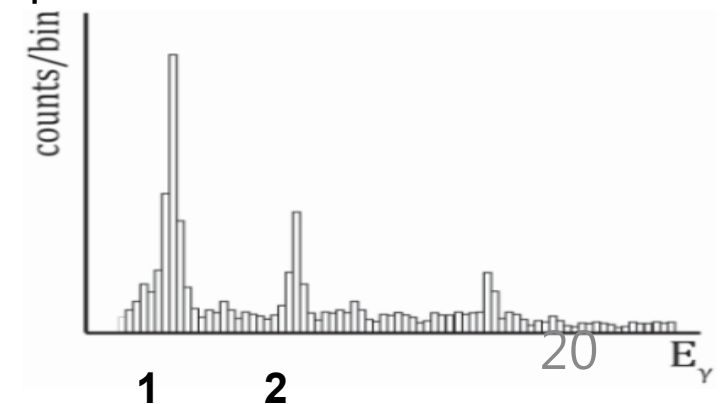
β - γ spectroscopy at RIBF



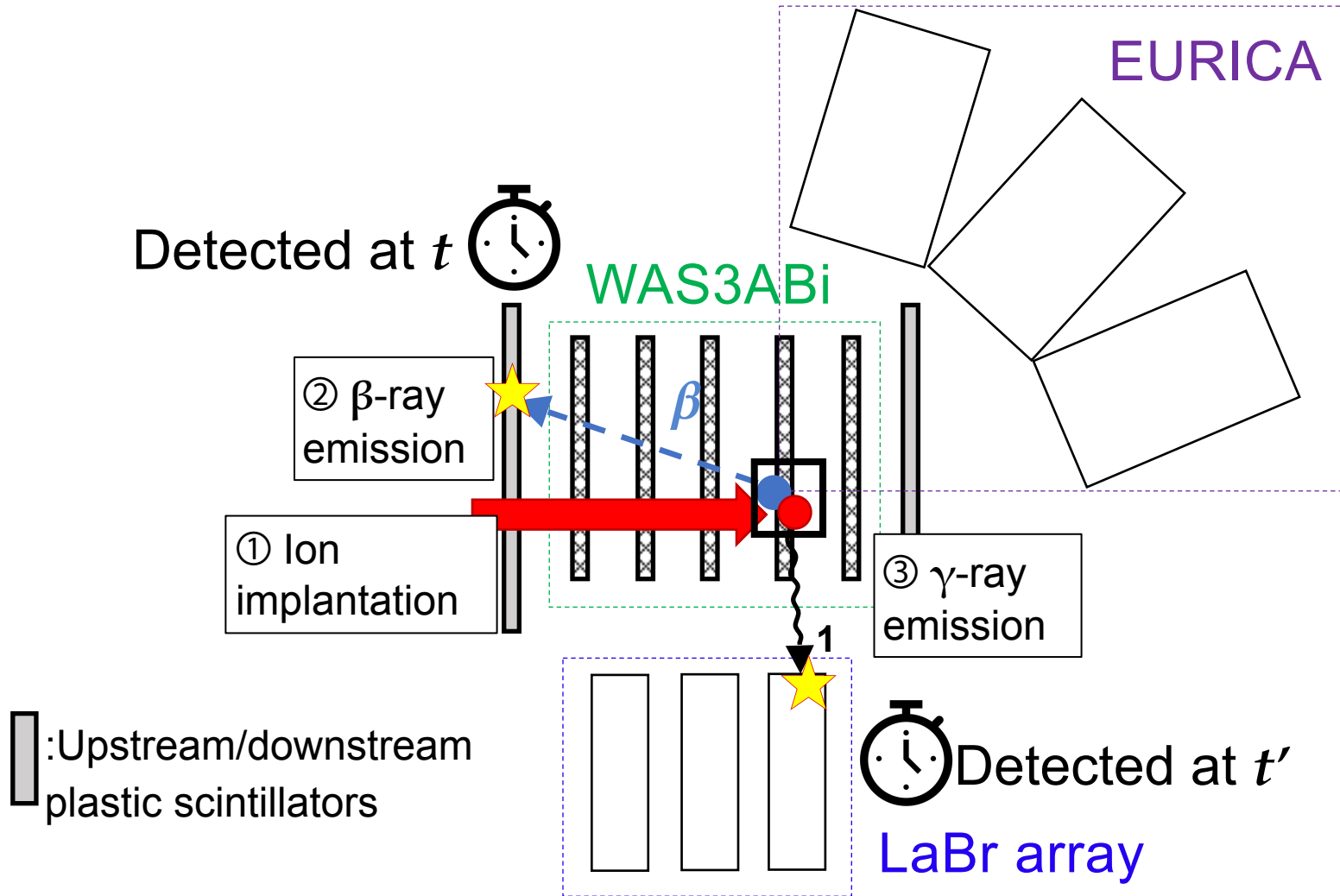
(Example) distribution of time difference



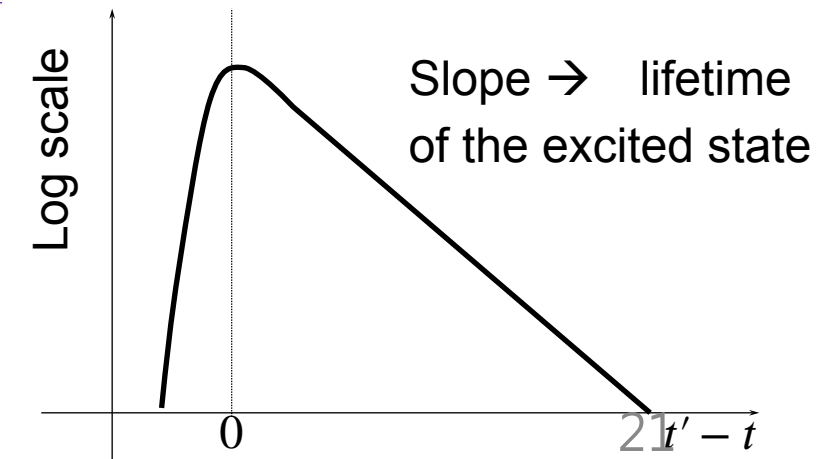
(Example) β -delayed γ -ray spectrum



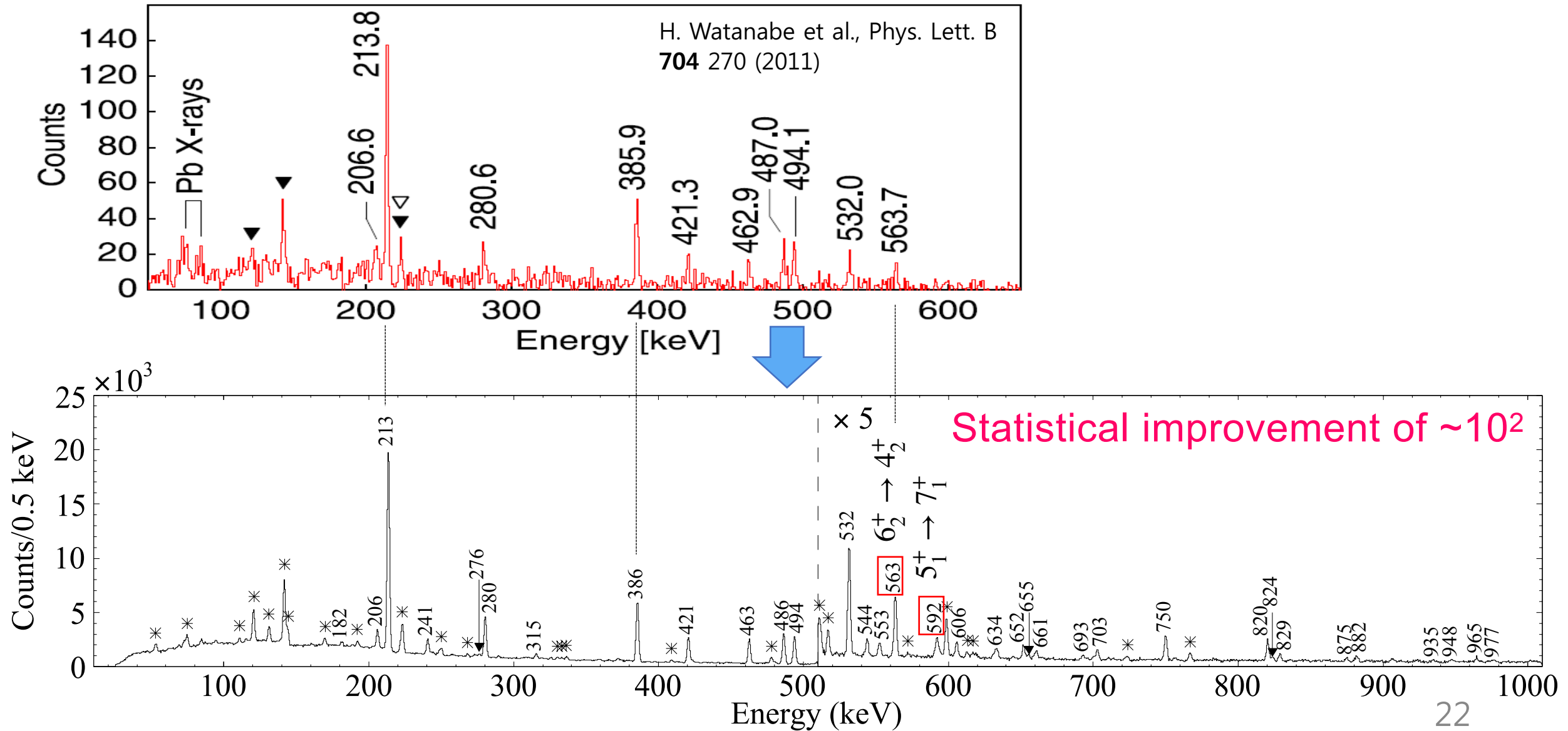
β - γ spectroscopy with LaBr Array



(Example) distribution of time difference $t - t'$

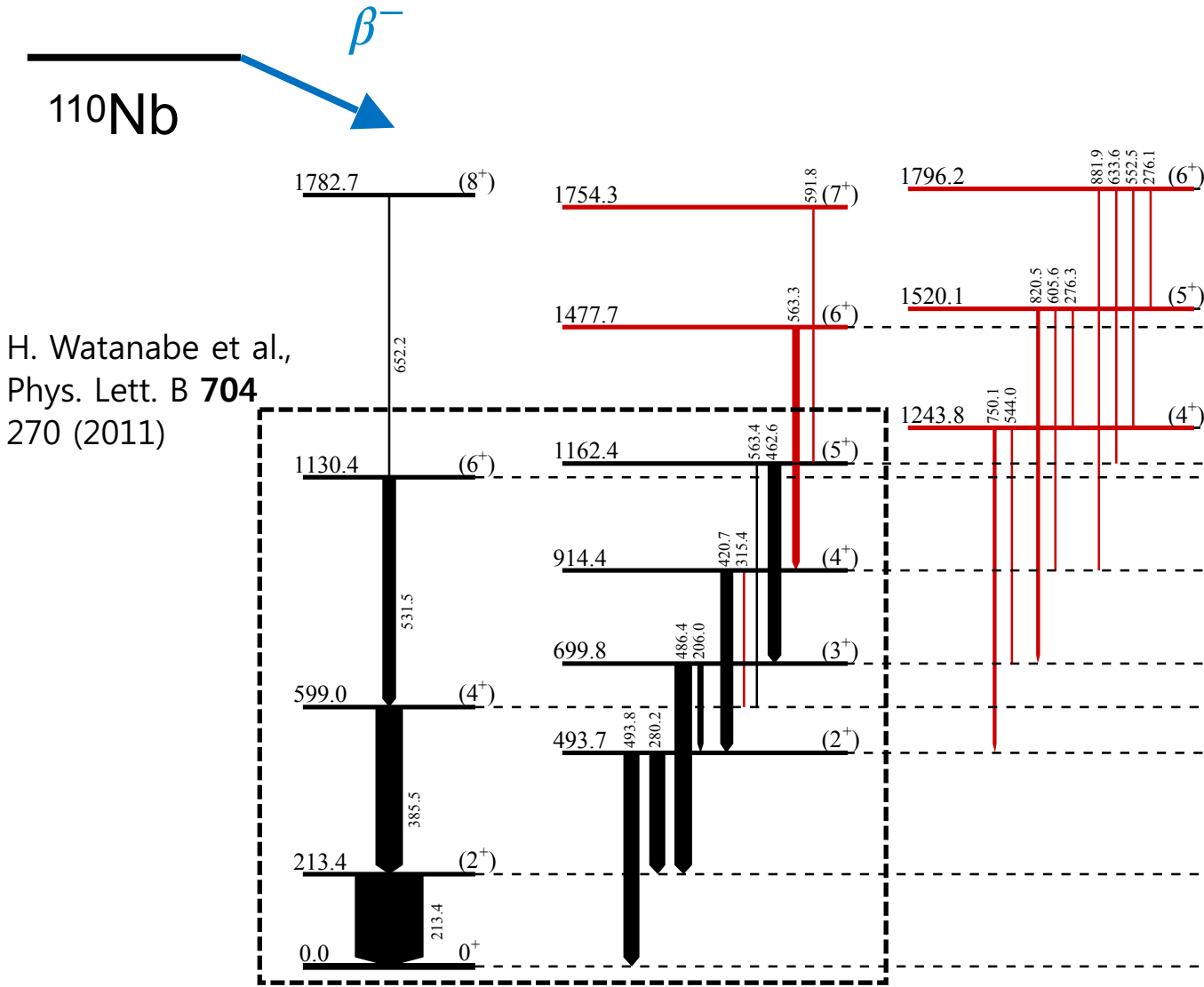


γ -ray spectrum of ^{110}Mo produced from β -decay



β -decay to ^{110}Mo

A partial level scheme of ^{110}Mo



New results

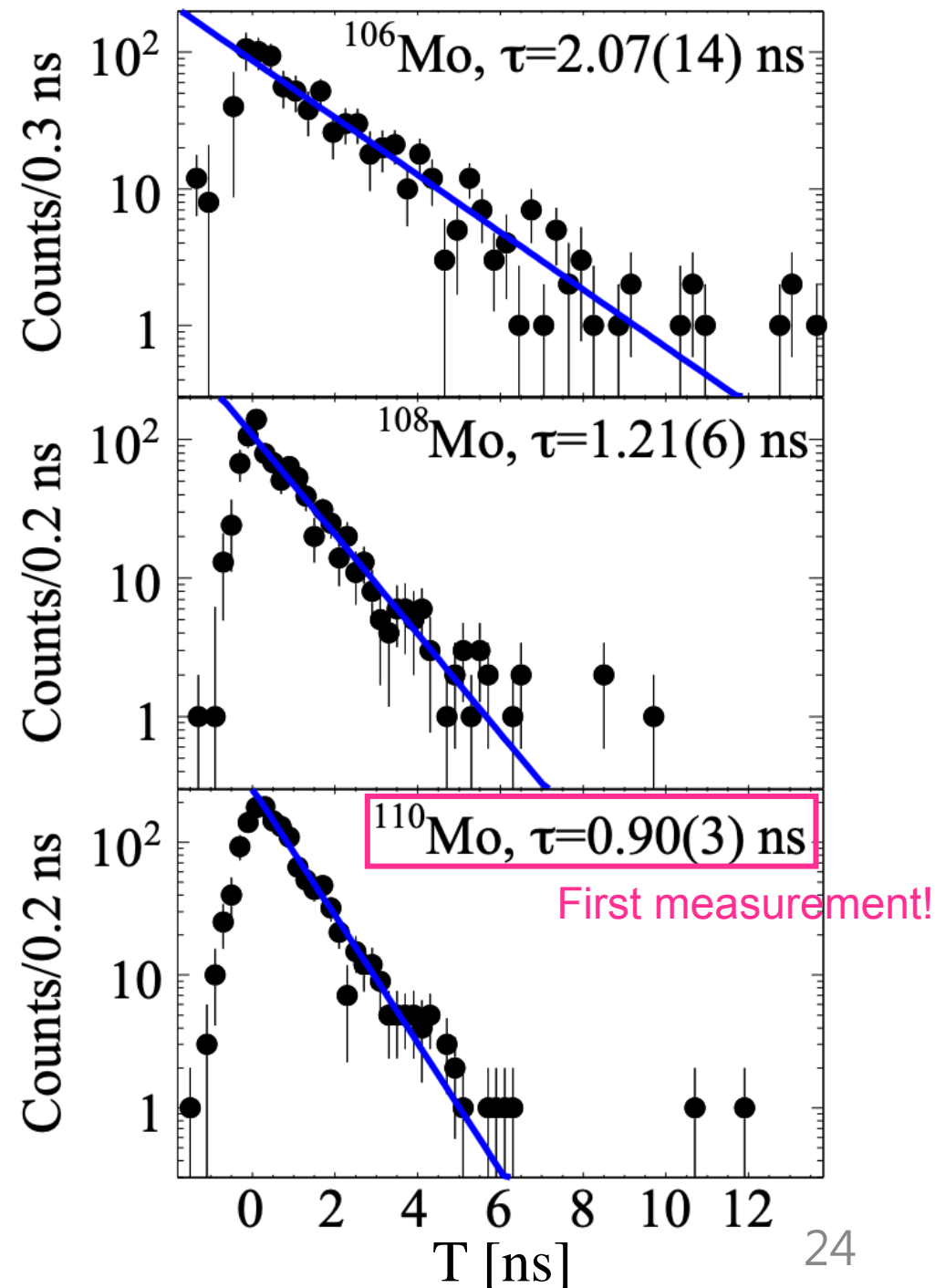
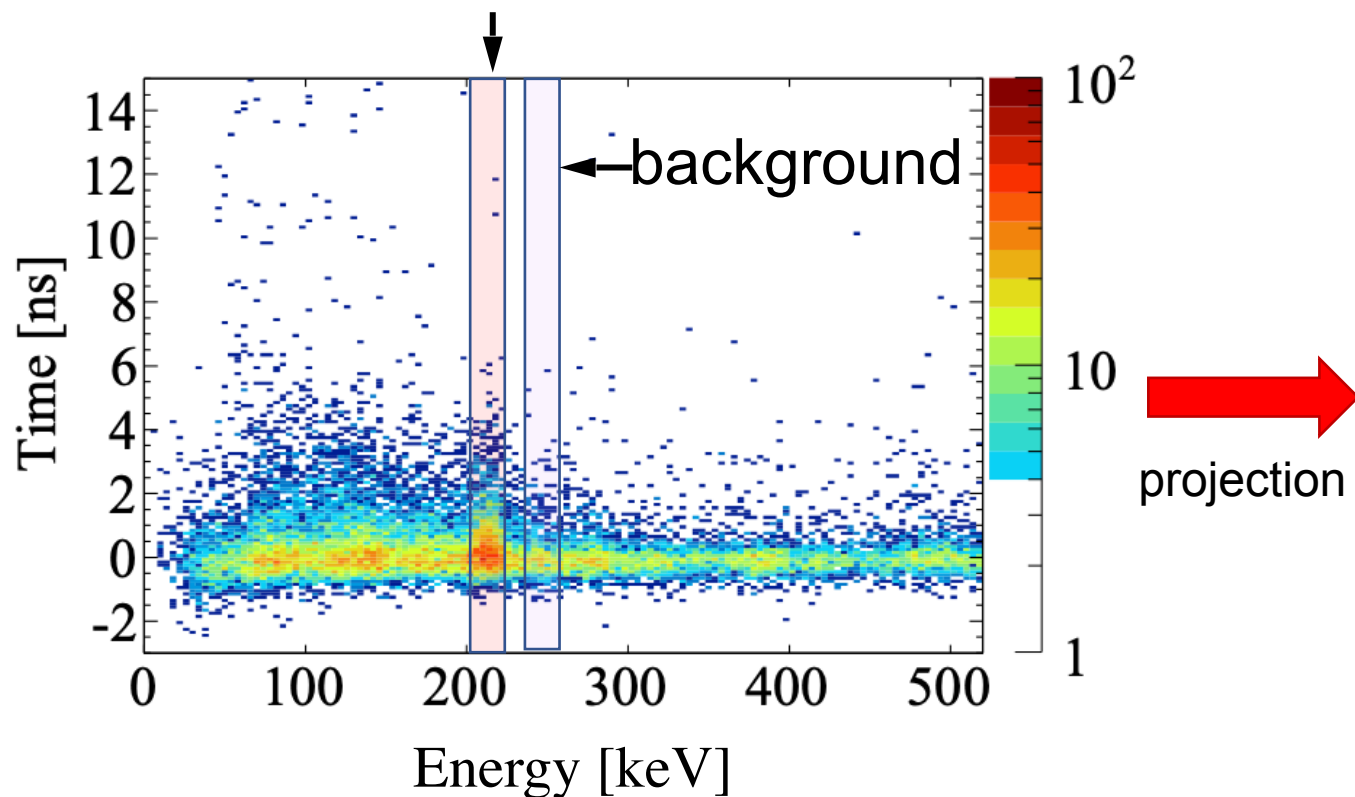
- Observation of the 6^+ , 7^+ states in the second band
 - Observation of the third band ($\gamma\gamma$ -vibration?)
- Kinematic moment of inertia

Red lines: new levels and transitions

Lifetime measurement of 2_1^+ states

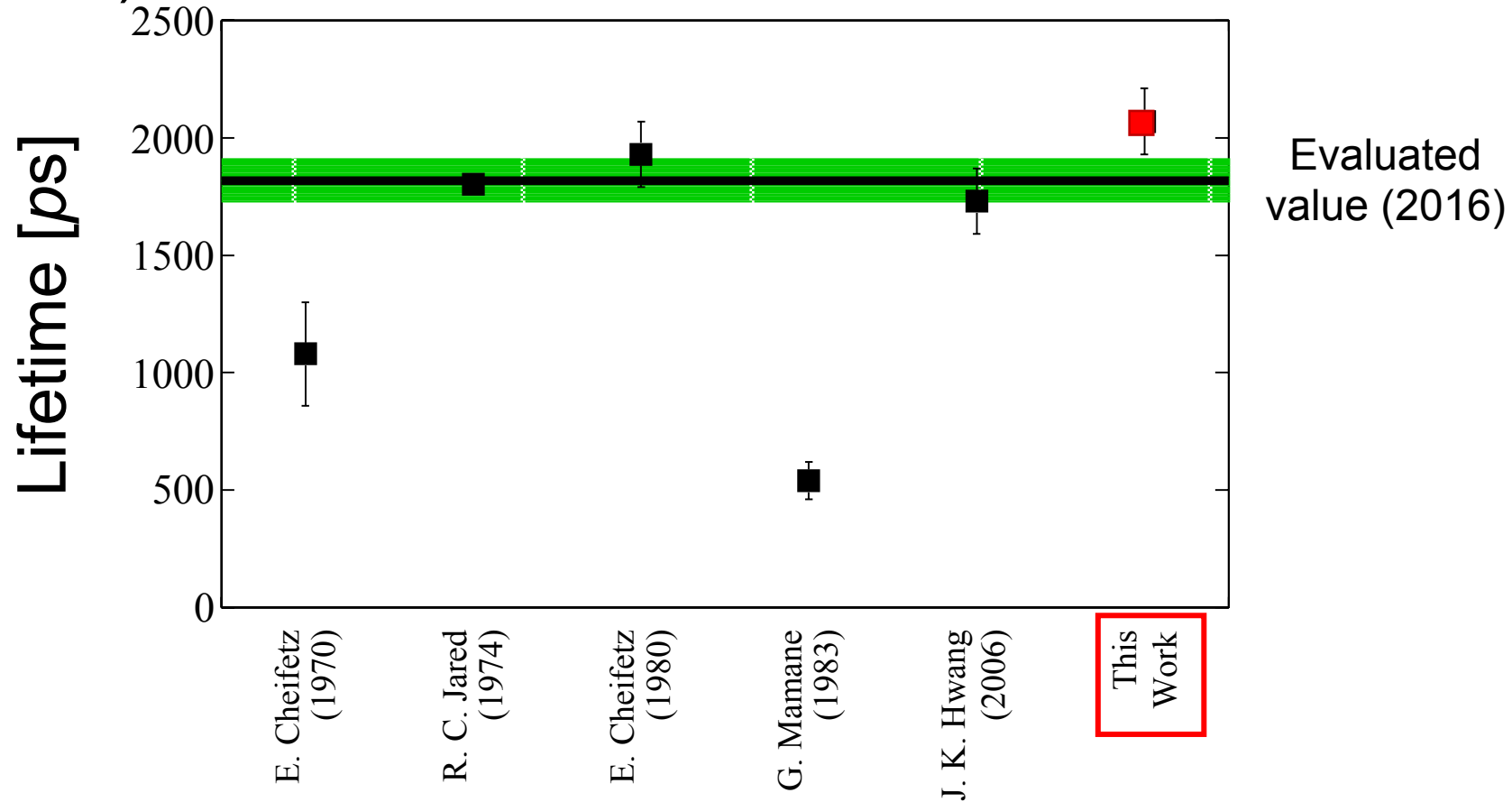
(The energy-time plot of ^{110}Mo)

213.4-keV: $2_1^+ \rightarrow 0_1^+$



Lifetime measurement of 2_1^+ states

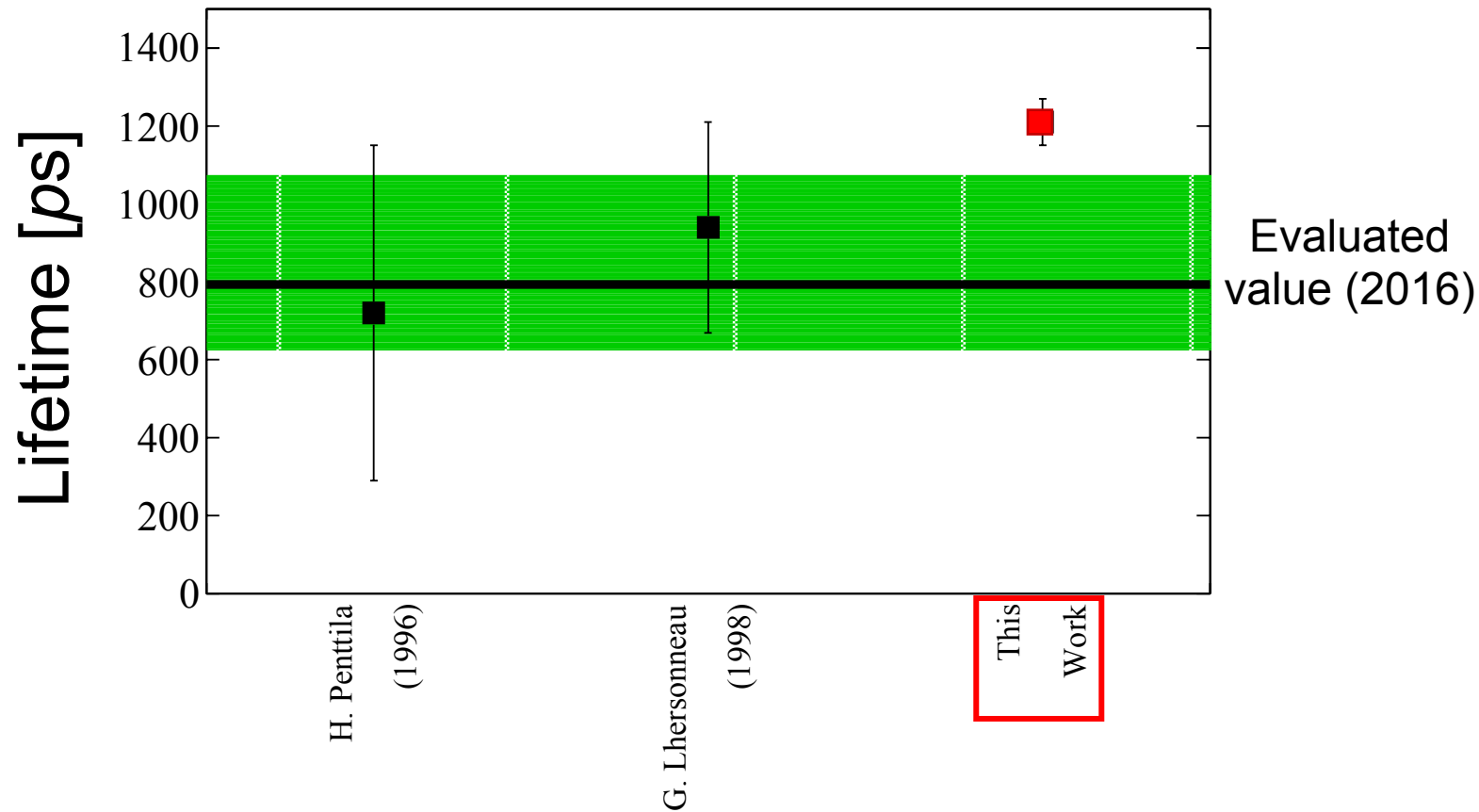
* ^{106}Mo ($N = 64$)



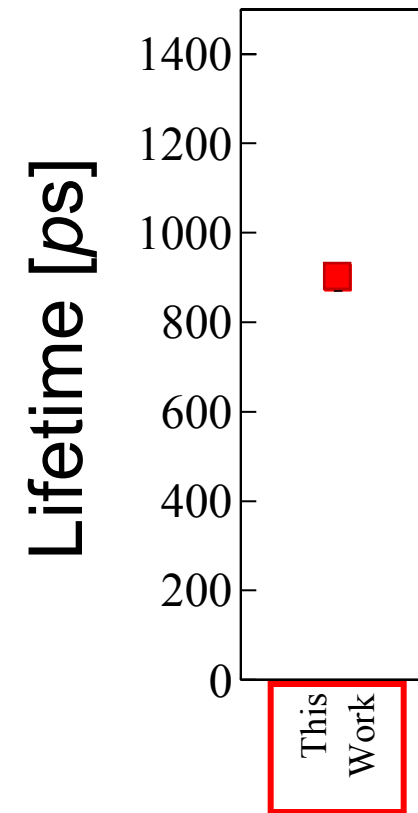
→ First results using LaBr_3 (Ce) detector!

Lifetime measurement of 2_1^+ states

* ^{108}Mo ($N = 66$)

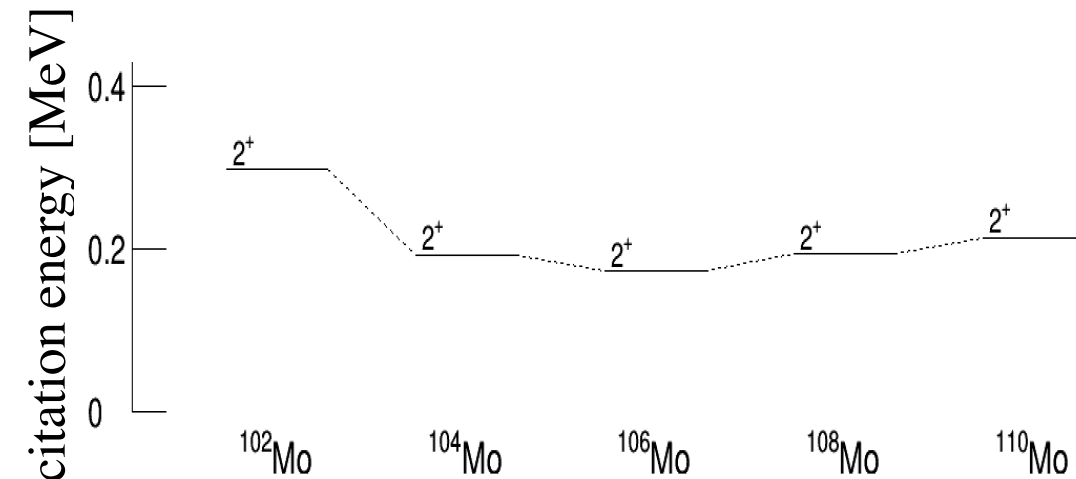
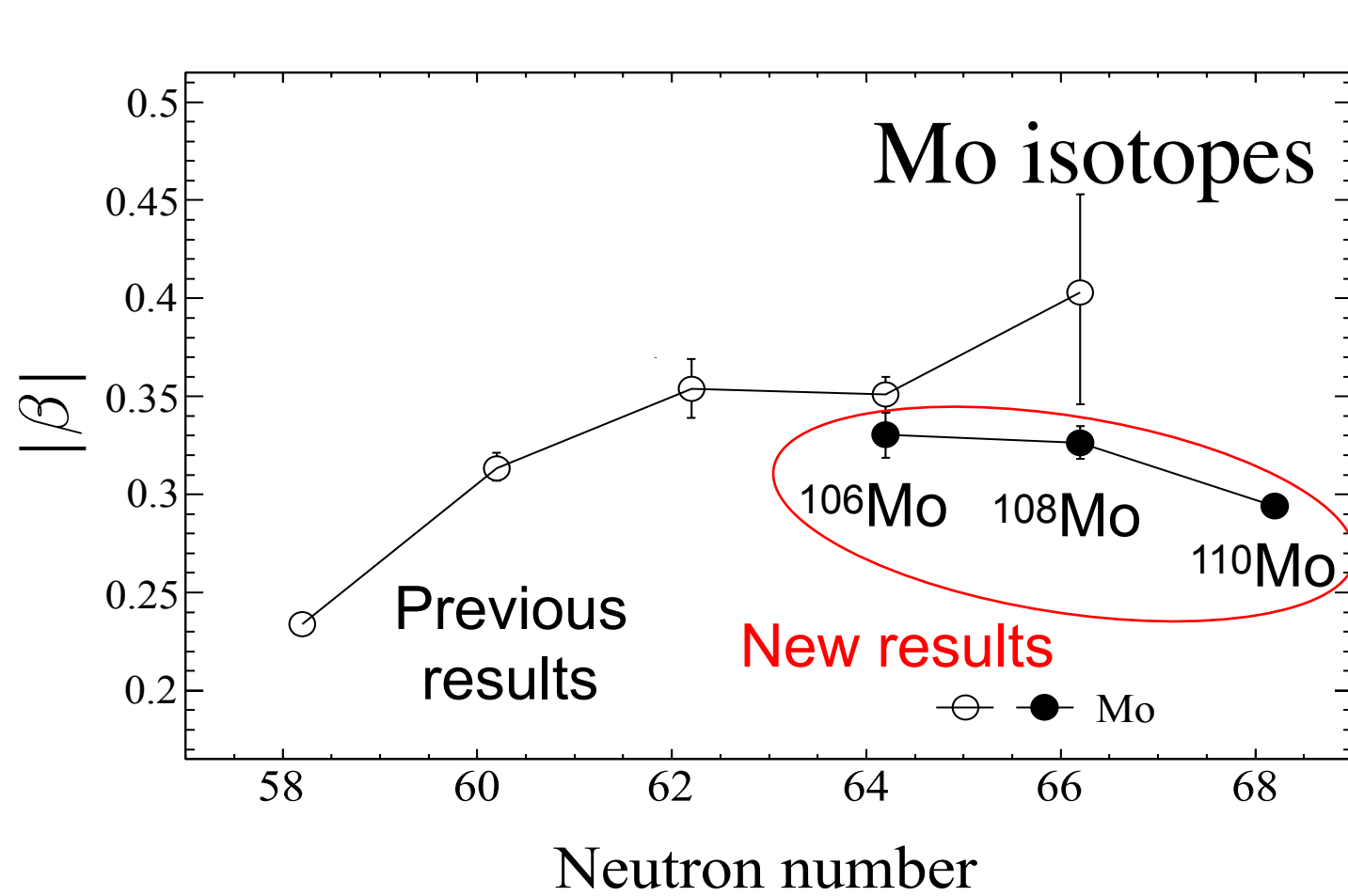


* ^{110}Mo ($N = 68$)



→ First results using LaBr_3 (Ce) detector!

Quadrupole deformation of the ground state



$$\beta = \frac{4\pi}{3ZR_0^2} \frac{1}{\sqrt{\tau(1+\alpha)}} \sqrt{40.81 \times 10^{13} \frac{1}{E_\gamma^5}}$$

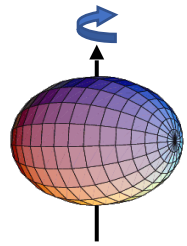
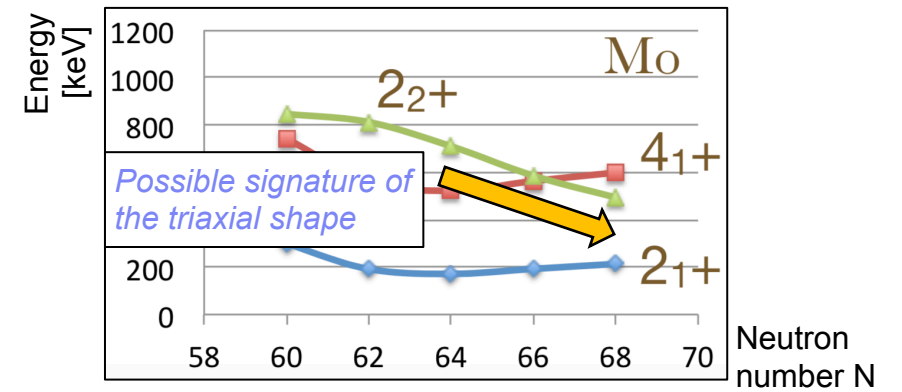
τ : lifetime

α : internal conversion coeff.

R_0 : (fm)

Triaxiality in the second band

Rotor models for $A \sim 110$ nuclei



Axially symmetric rotor with γ -vibration

gr. band

8⁺

6⁺

4⁺

2⁺

0⁺

8⁺

7⁺

6⁺

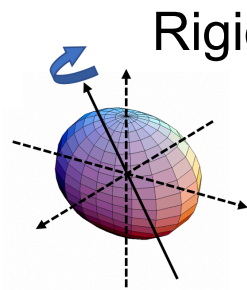
5⁺

4⁺

3⁺

2⁺

γ band
($K = 2$)



Rigid triaxial rotor model

gr. band

8⁺

6⁺

4⁺

2⁺

0⁺

8⁺

7⁺

6⁺

5⁺

4⁺

3⁺

2⁺

second band

γ -unstable rotor model

gr. band

8⁺

6⁺

4⁺

2⁺

0⁺

8⁺

7⁺

6⁺

5⁺

4⁺

3⁺

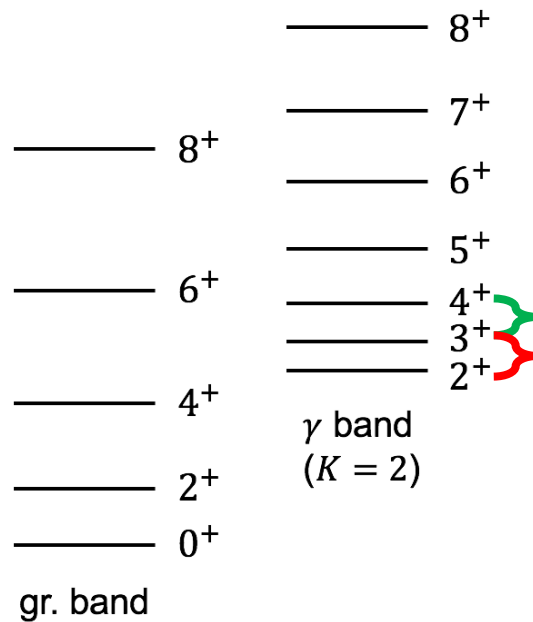
2⁺

second band

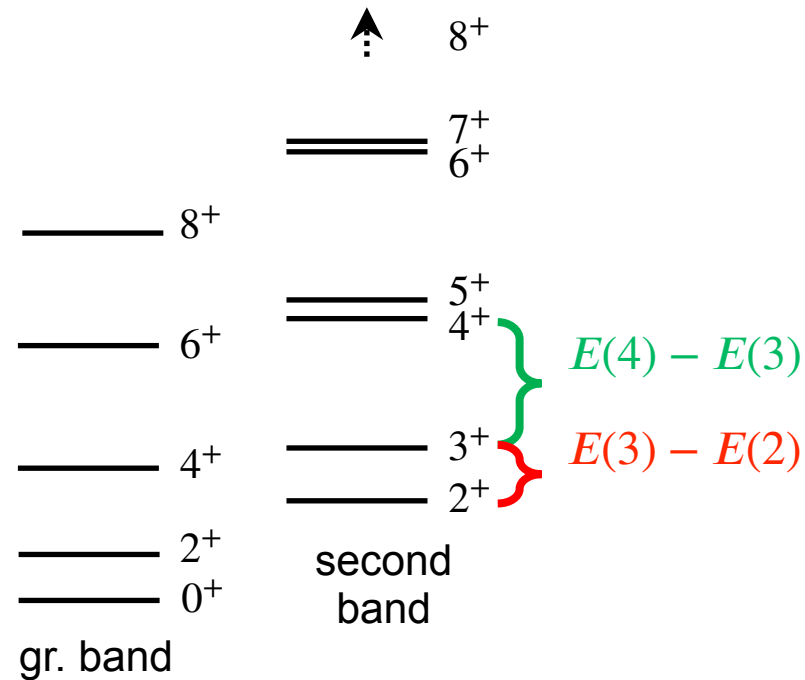
Triaxiality in the second band

Rotor models for $A \sim 110$ nuclei

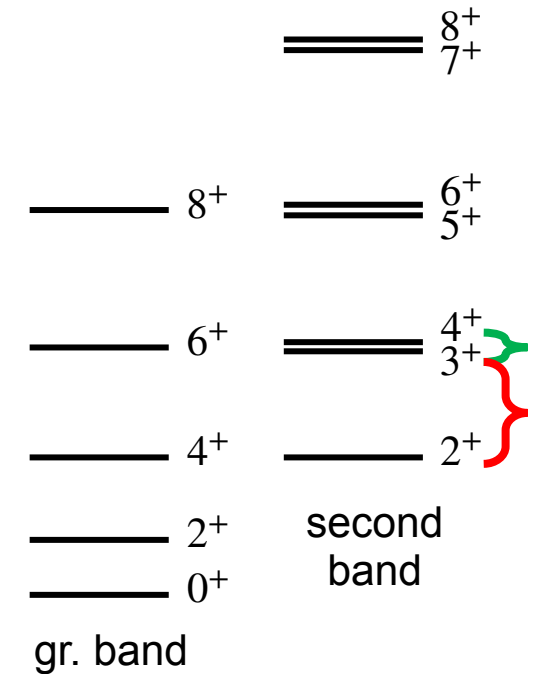
Axially symmetric
rotor with γ -vibration



Rigid triaxial rotor model



γ -unstable rotor model



Level spacing in side band is different for each model

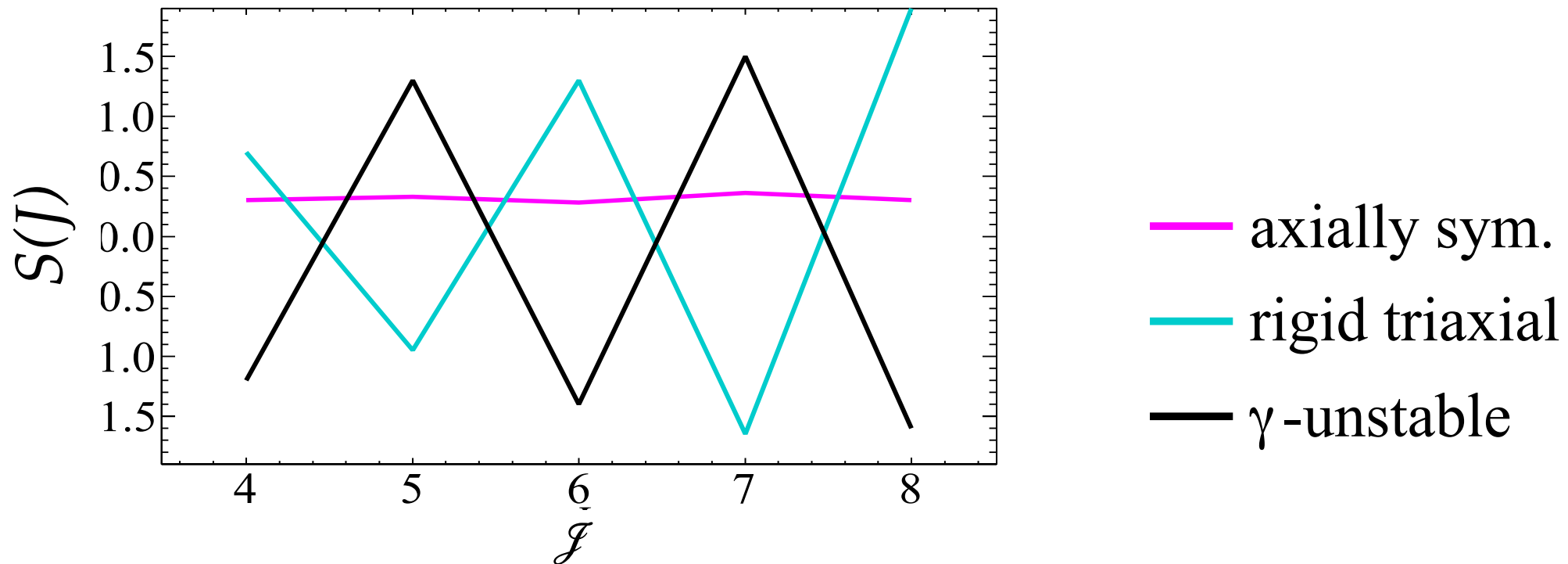
One useful indicator

$$S(\mathcal{J}) \equiv \frac{E_s(J)}{E(2_1^+)} = \frac{[E(J) - E(J-1)] - [E(J-1) - E(J-2)]}{E(2_1^+)}$$

Triaxiality in the second band

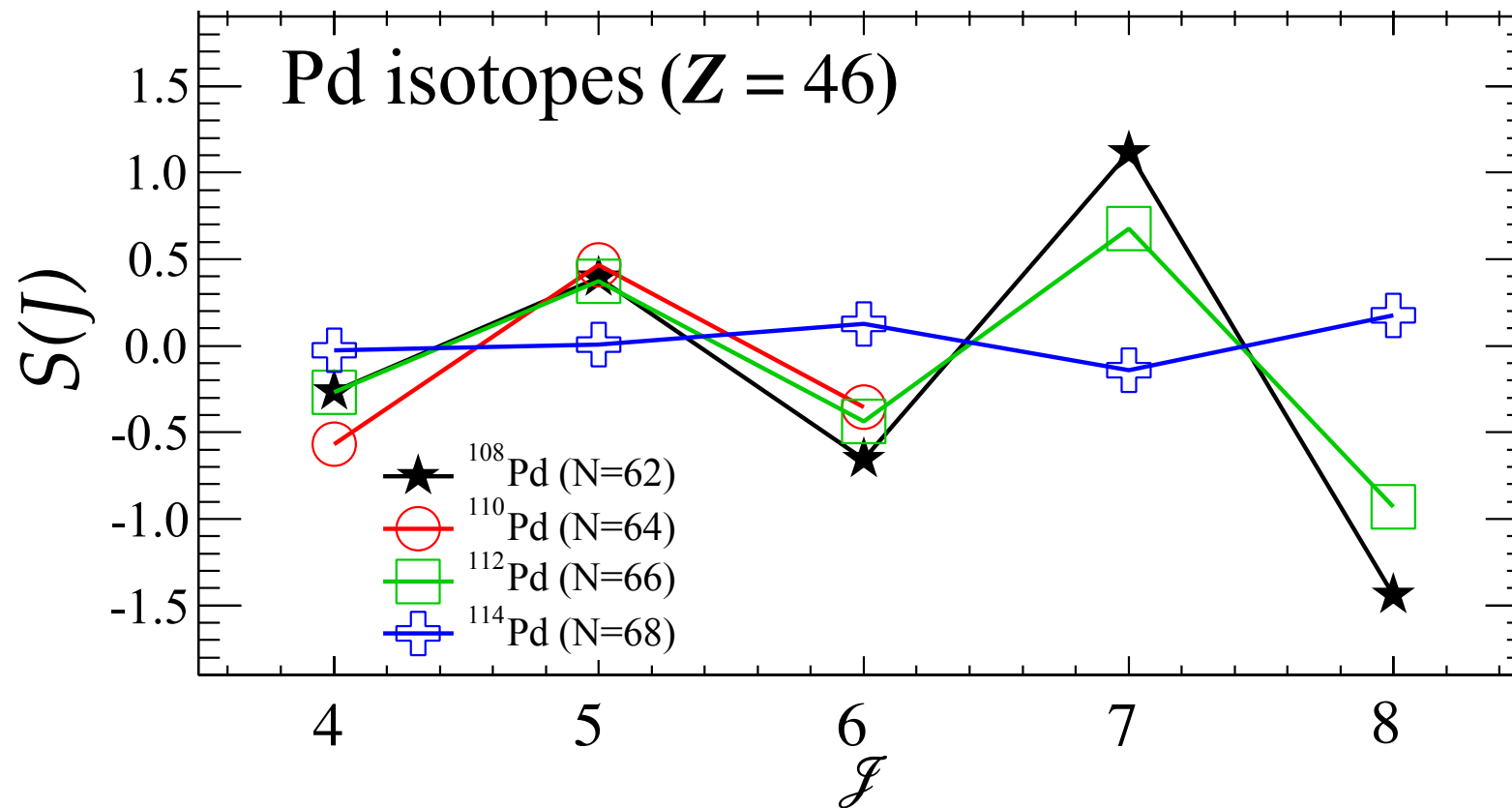
Energy staggering of γ -band

$$S(\mathcal{J}) \equiv \frac{E_s(J)}{E(2_1^+)} = \frac{[E(J) - E(J-1)] - [E(J-1) - E(J-2)]}{E(2_1^+)}$$

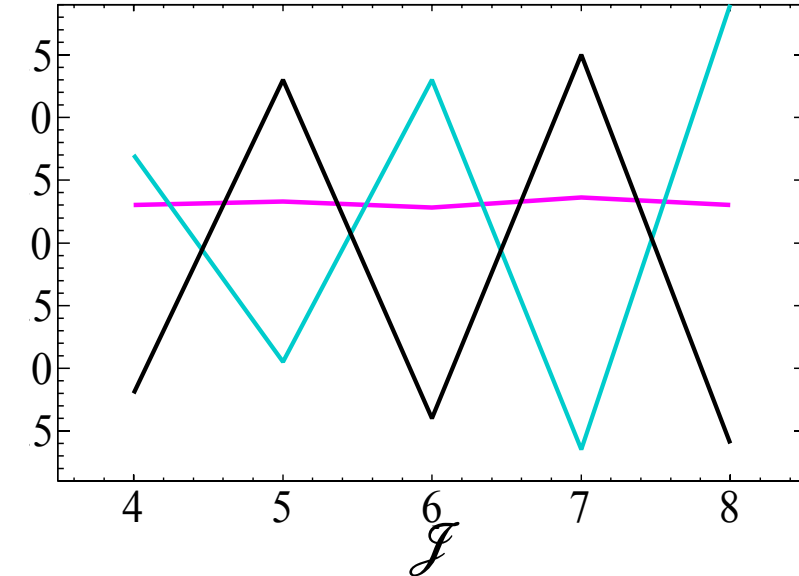


Triaxiality in the second band

$S(J)$ of neutron-rich Mo, Ru, and Pd isotopes



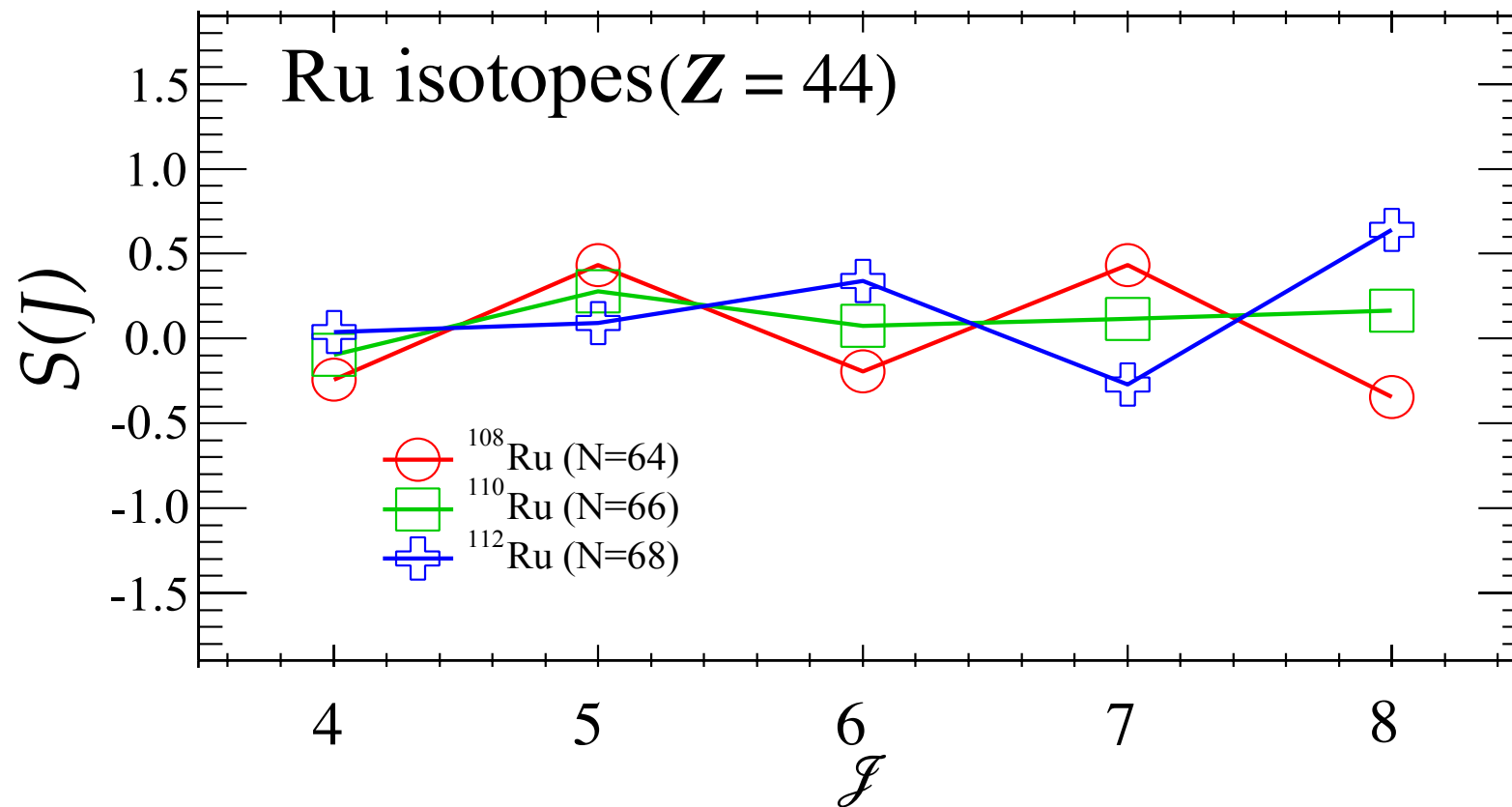
M.A. Caprio, PRC83(2011)064309



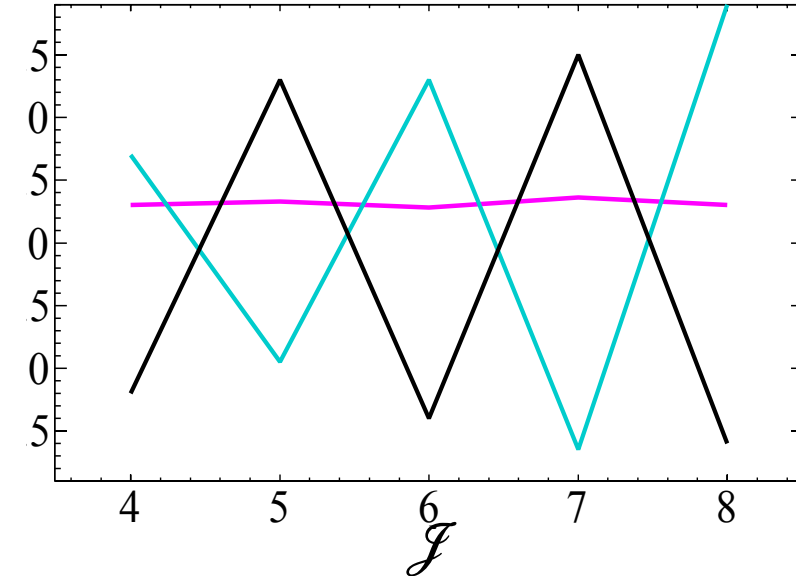
— axially sym.
— rigid triaxial
— γ -unstable

Triaxiality in the second band

$S(J)$ of neutron-rich Mo, Ru, and Pd isotopes

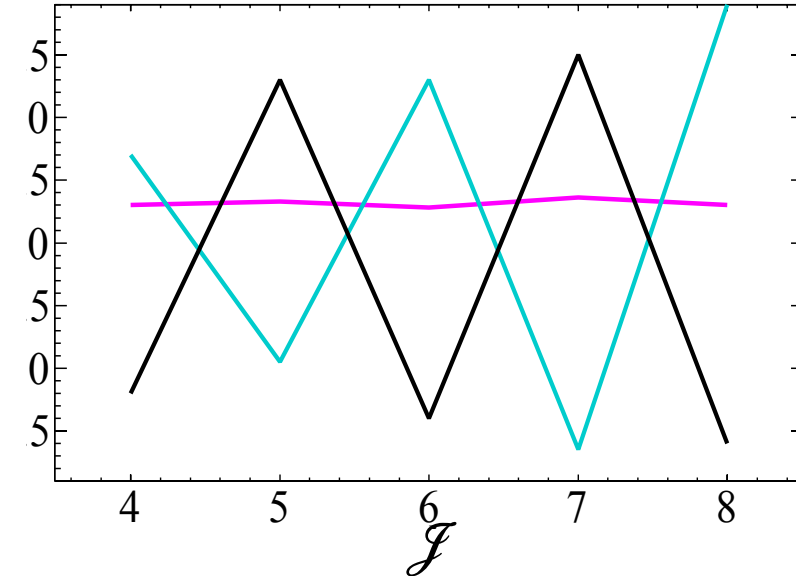
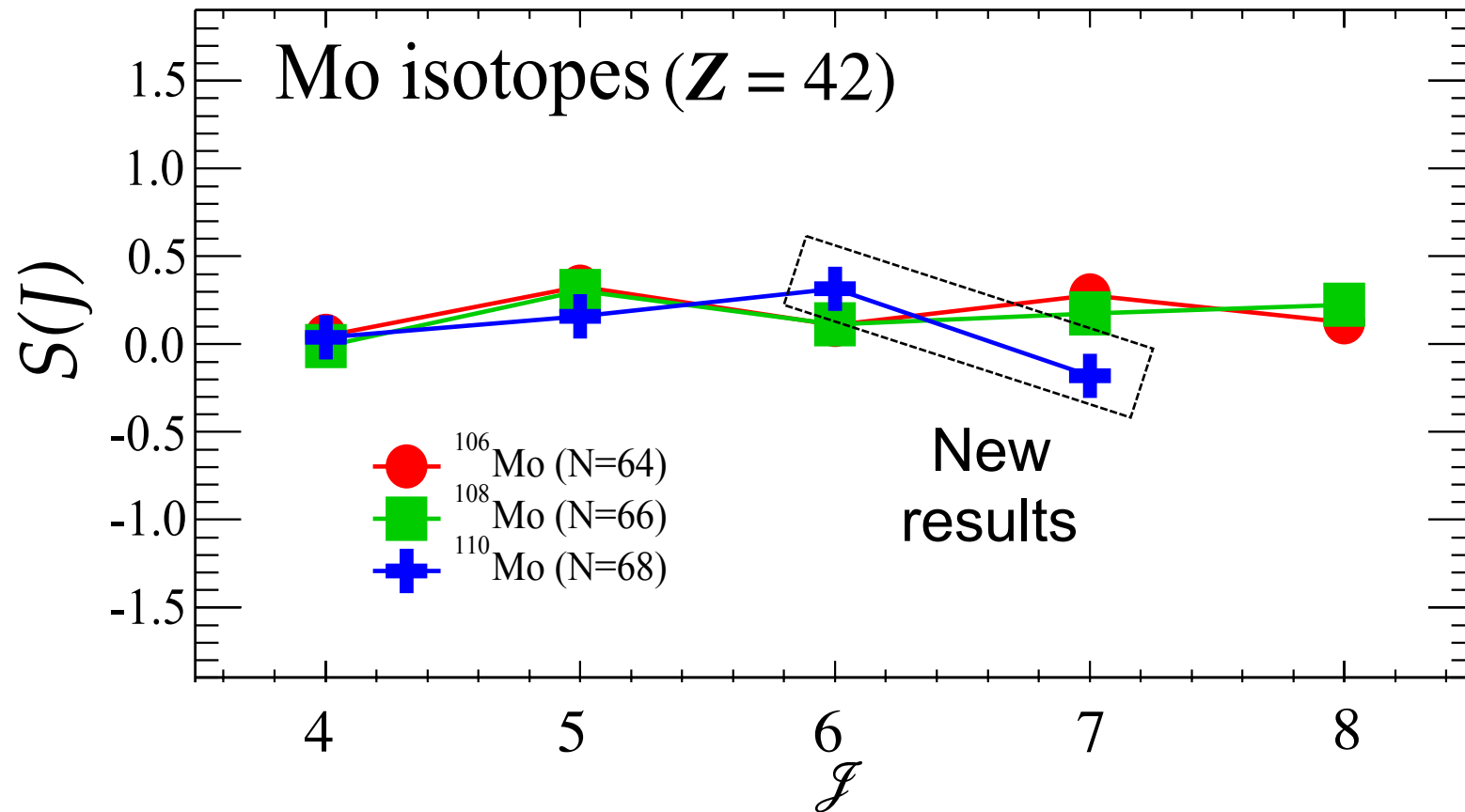


M.A. Caprio, PRC83(2011)064309



Triaxiality in the Second Band

$S(J)$ of neutron-rich Mo, Ru, and Pd isotopes

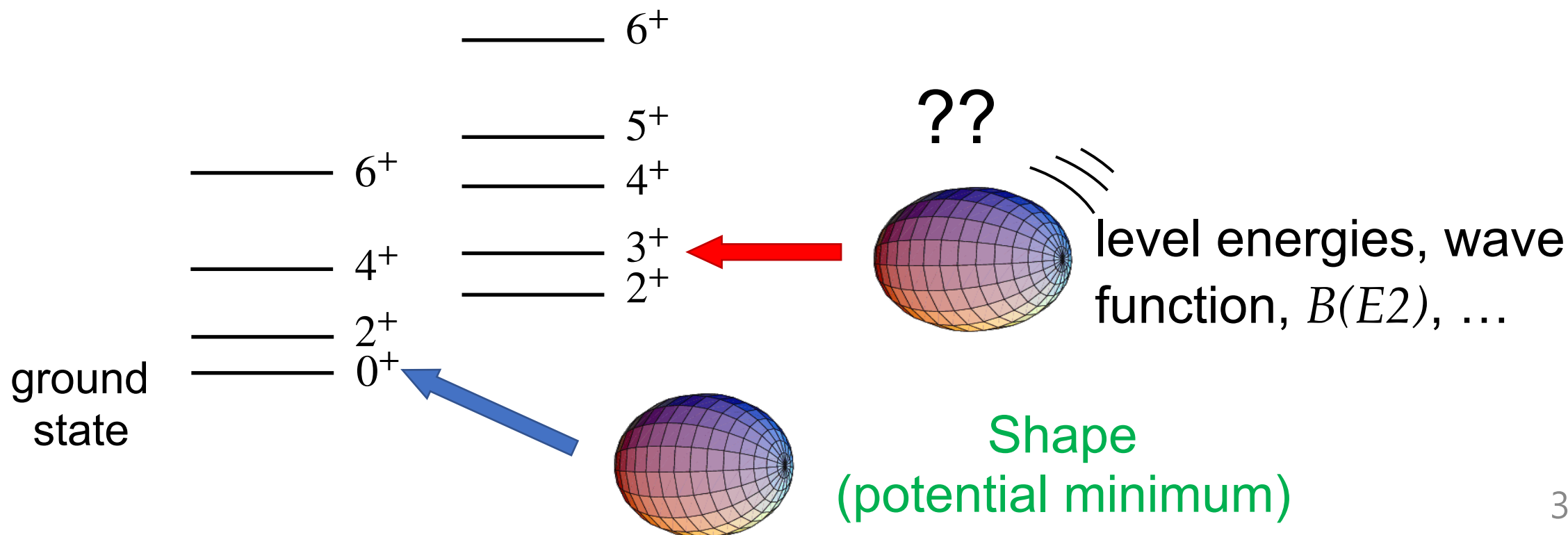


$^{106,108,110}\text{Mo}$: close to the axially symmetric shape

Microscopic 5D Collective Hamiltonian Calculation

Mean field theory is suitable to predict the shape (in body-fixed frame).

Prediction of the observables → Go beyond



Microscopic 5D Collective Hamiltonian Calculation

Beyond mean field (BMF) theory can give information on the observables.

5D-Collective Hamiltonian

$$H = \underbrace{\frac{1}{2} \sum_{k=1}^3 J_k \omega_k^2}_{\text{rotation}} + \underbrace{\frac{1}{2} B_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + B_{\beta\gamma}(\beta, \gamma) \beta \dot{\beta} \dot{\gamma} + \frac{1}{2} B_{\gamma\gamma}(\beta, \gamma) \beta^2 \dot{\gamma}^2}_{\text{vibration}} + \underbrace{V(\beta, \gamma)}_{\text{potential}}$$

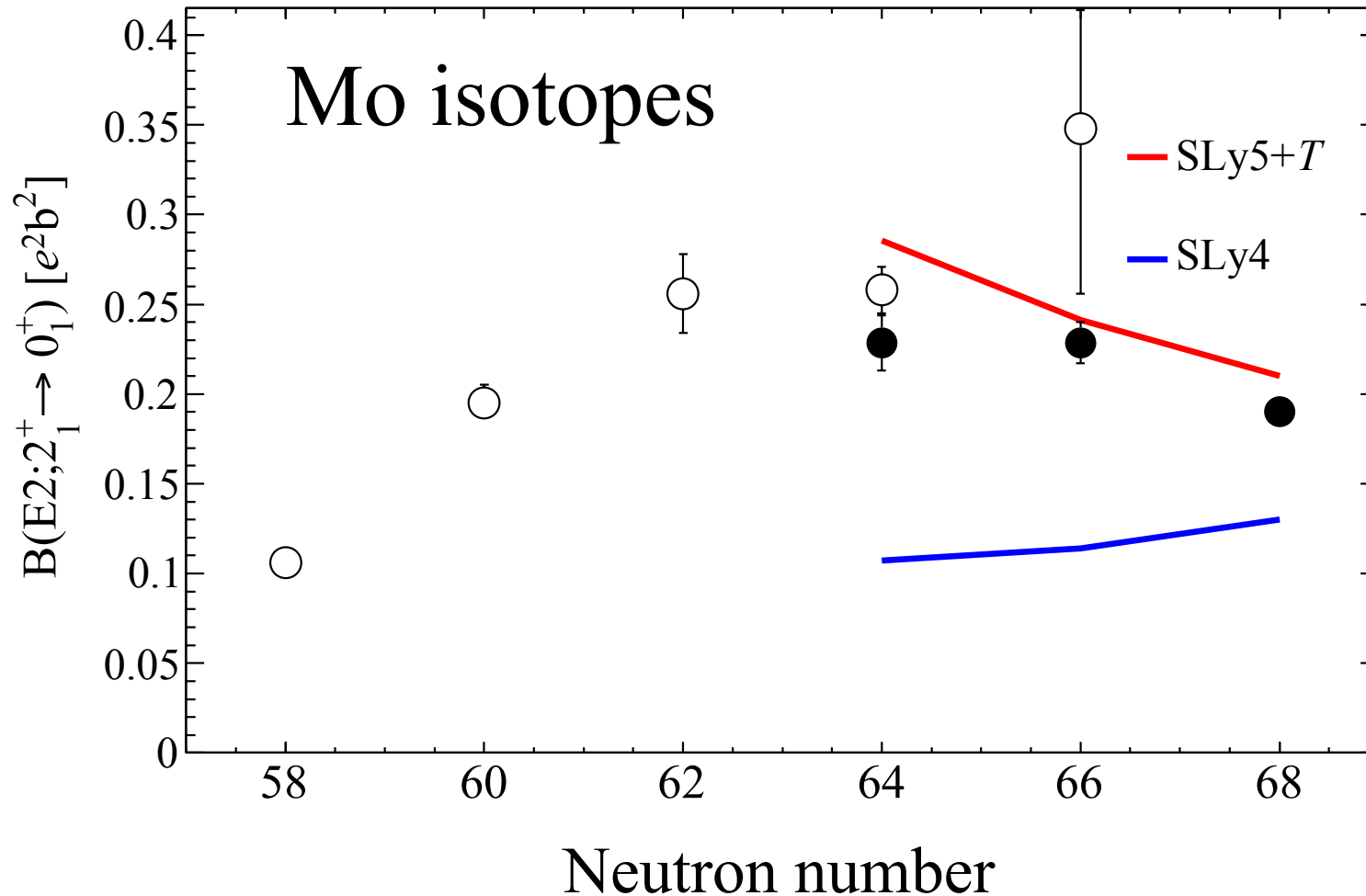
J_k , $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and V : determined through the beyond mean-field approach

Skyrme type interaction SLy4 and SLy5+ T are used and compared.

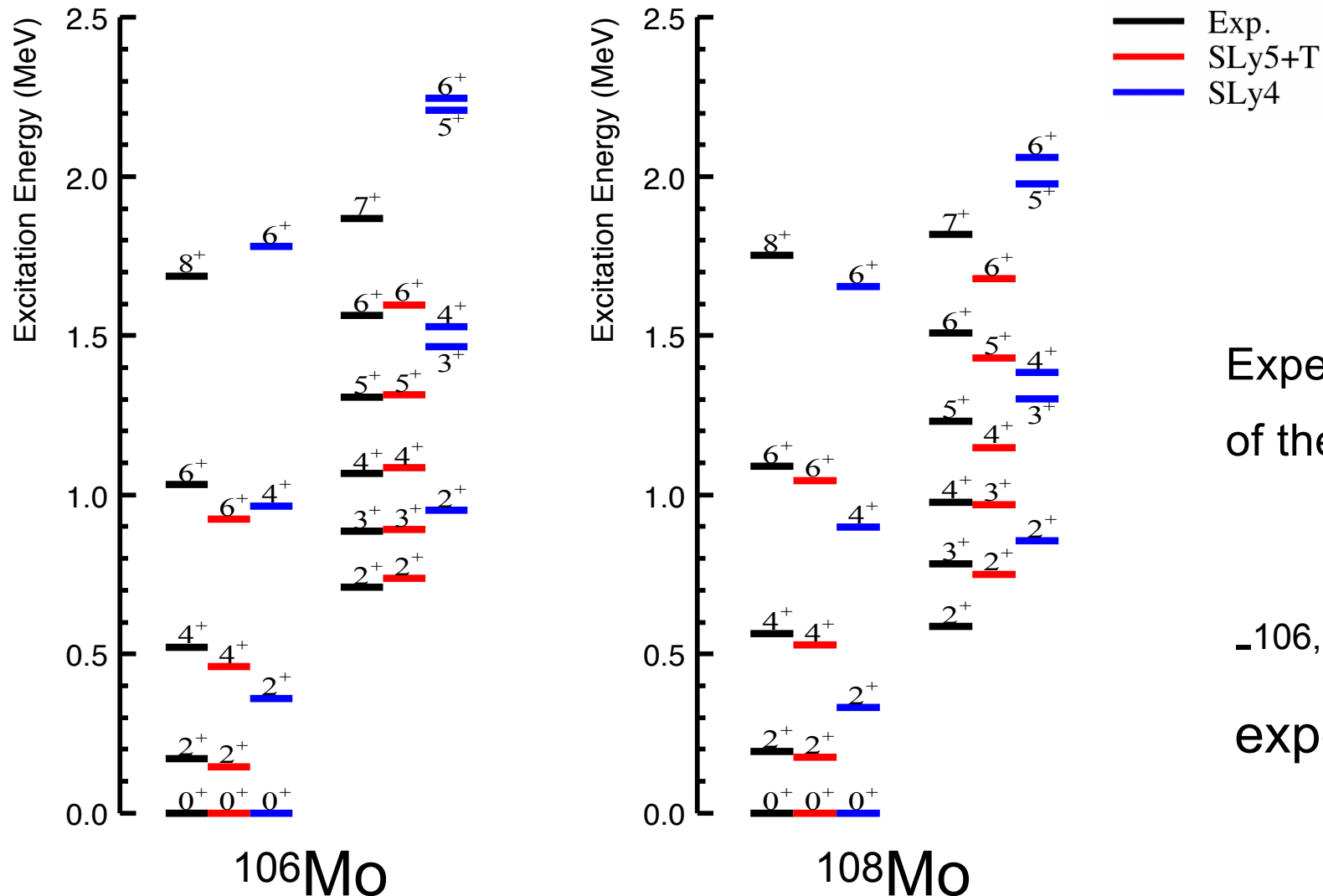
N. Hinohara *et al.*, PRC82(2010)064313, PRC84(2011)061302

K. Sato *et al.*, PRC86(2012)024316

Microscopic 5D Collective Hamiltonian Calculation



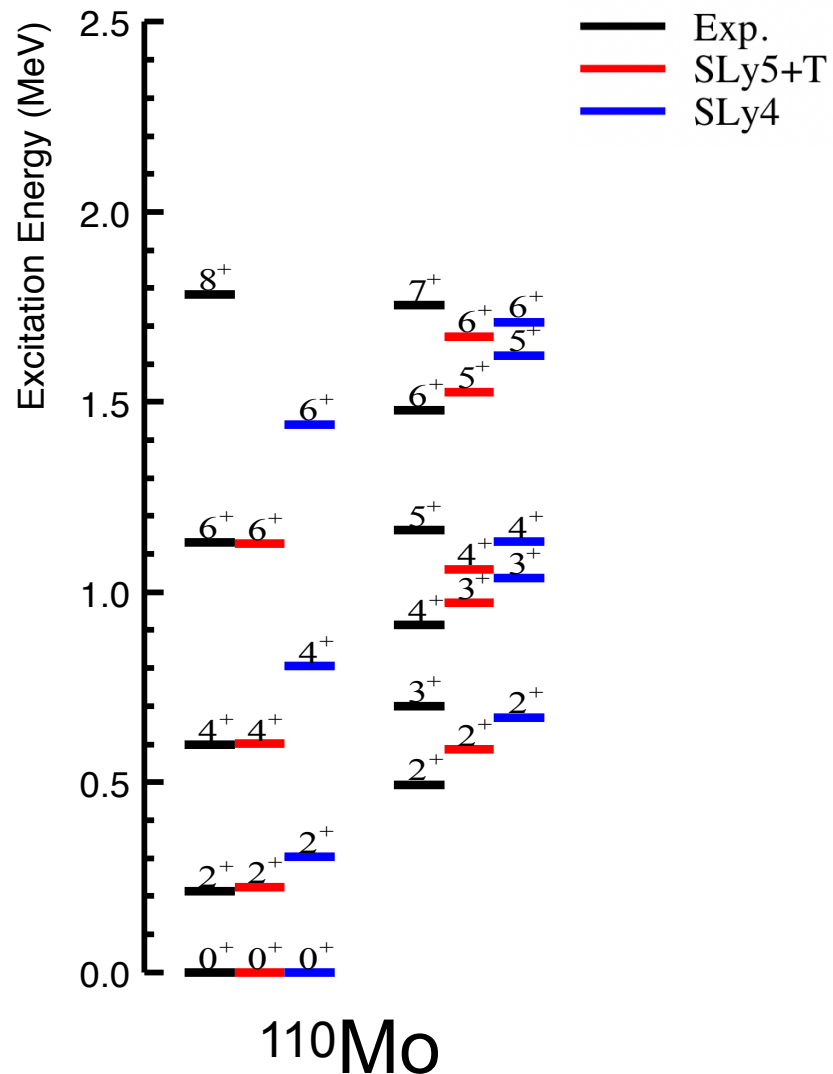
Microscopic 5D Collective Hamiltonian Calculation



Experimental and theoretical energies of the low-lying excited states

$^{106,108}\text{Mo}$: consistent with the experimental results

Microscopic 5D Collective Hamiltonian Calculation



^{-110}Mo

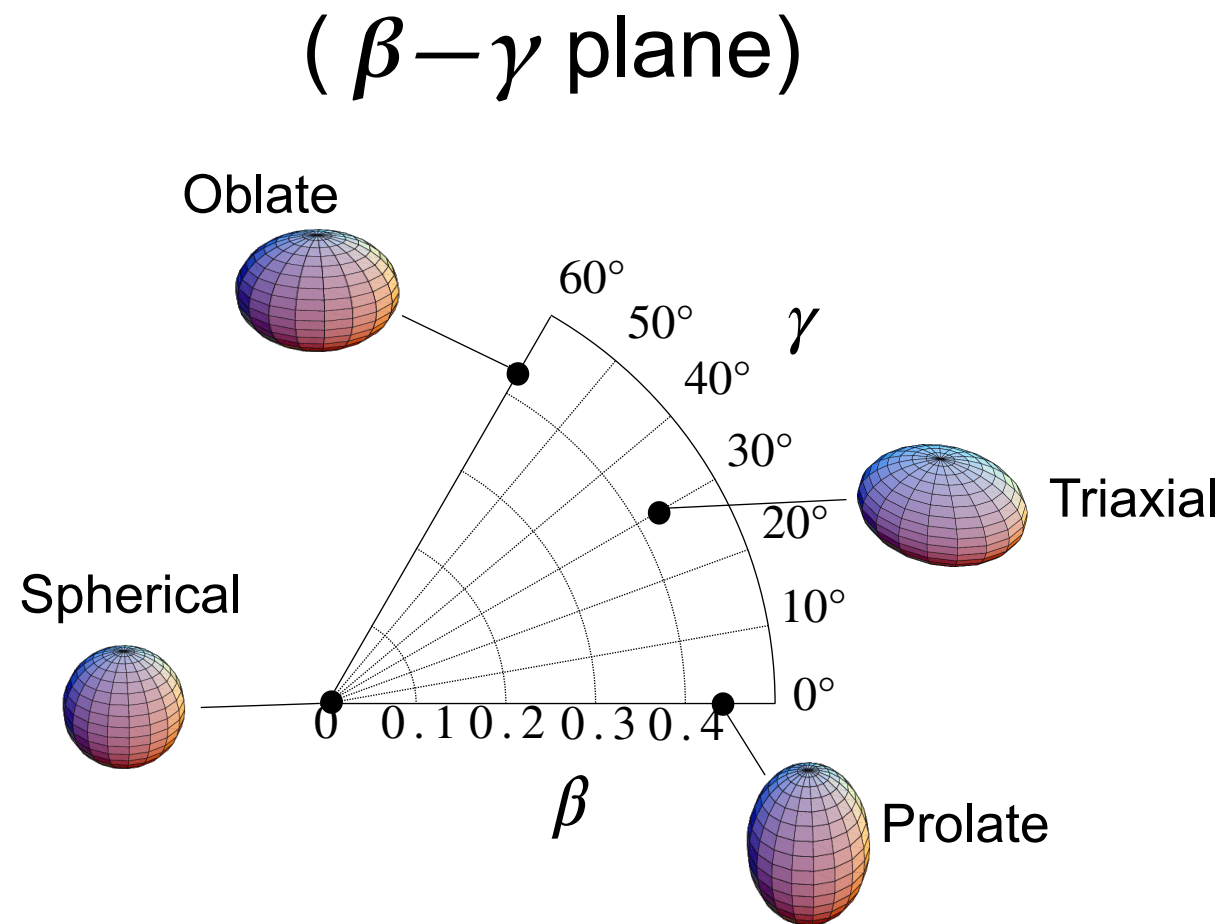
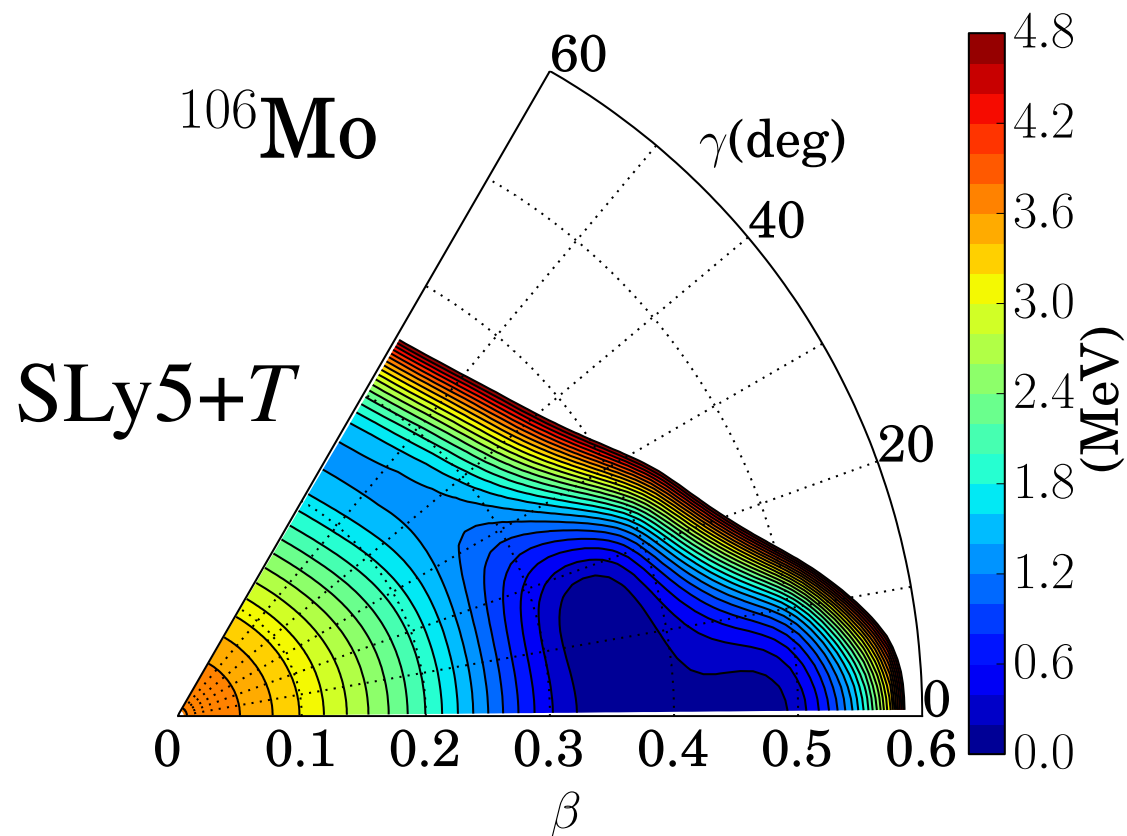
(1) Ground band: good agreement with the experimental results

(2) Side ($K = 2$) band: γ -soft character

→ inconsistent with the experimental results

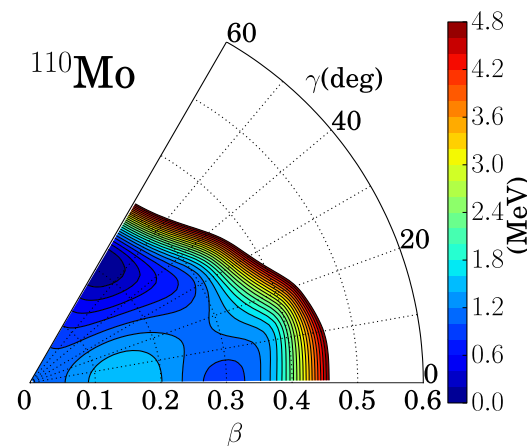
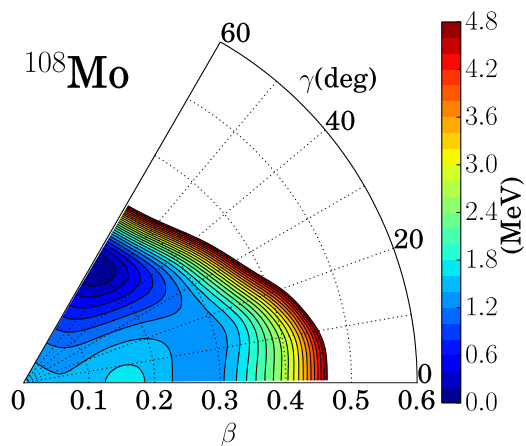
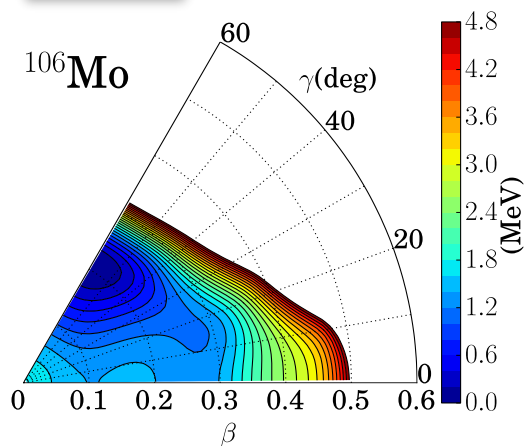
Microscopic 5D Collective Hamiltonian Calculation

Potential Energy Surface (PES)

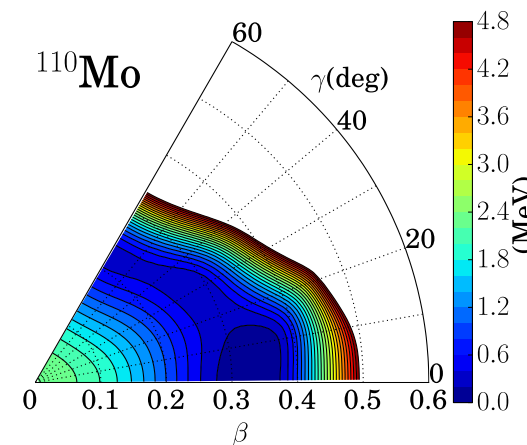
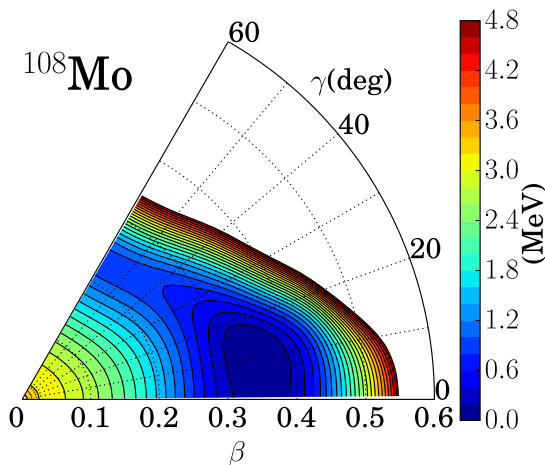
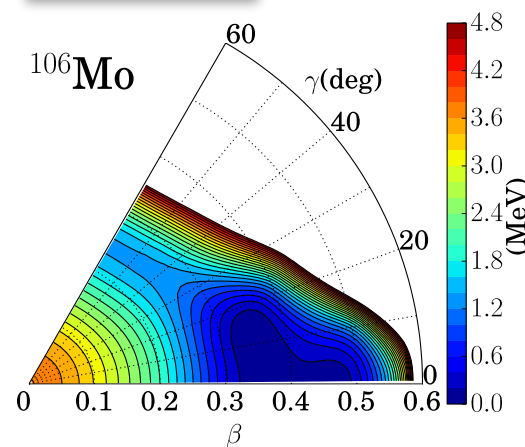


Microscopic 5D Collective Hamiltonian Calculation

SLy4



SLy5+T



$^{106,108}\text{Mo}$: prolate

^{110}Mo : γ -soft, close

to γ -unstable

Summary

The shapes of $^{106,108,110}\text{Mo}$ were studied via the β - γ spectroscopy

- (1) The lifetime measurement of the $E(2_1^+)$ states: the degree of deformation decreases with increasing neutron number.
- (2) The observation of 6^+ , 7^+ states in ^{110}Mo : the energy staggering indicator $S(J)$ for newly observed levels suggests the axially symmetric shape.
- (3) Comparison with BMF calculation: the experimental $B(E2)$ and energy of the low-lying states are consistent with SLy5+T, which indicates prolate ground-state shape.

Acknowledgement: Most of the slides were derived from those of Dr. Jeongsu Ha, who finished his PhD with this experiment.