Symmetry energy parameters and nuclear structure in the KIDS framework

KIDS = Korea: IBS-Daegu-SKKU

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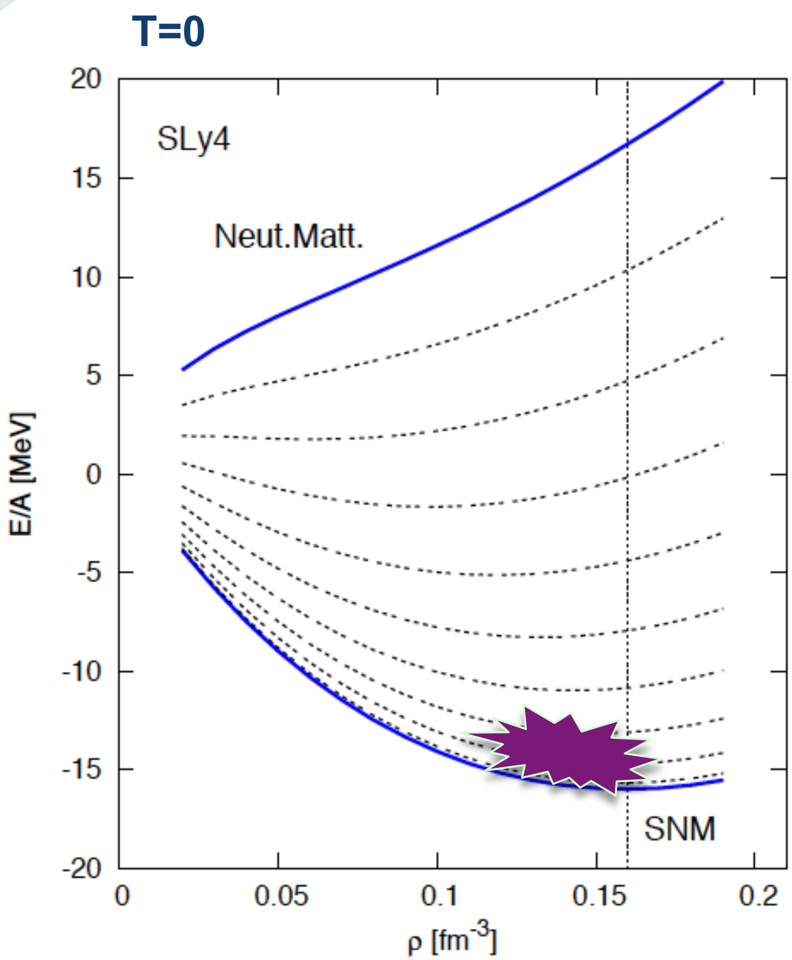
Overview

- Introduction
  - Symmetry energy parameters
  - Issues: effective mass; correlations between parameters
- KIDS EoS: Nuclear matter
  - Physical motivation
  - Validation: interpolation and extrapolation
- KIDS EDF: Nuclei
  - Effective nucleon mass decoupled from bulk static properties
  - Correlations of observables with parameters explored freely
- Conclusion, prospects

SYMMETRY ENERGY PARAMETERS

Equation of state



Characterization at saturation density by convention

- Symmetric matter:
  - Density ρ_0
  - Binding energy E_0
  - Incompressibility (curvature) K_0
- Asymmetric matter:
  - Symmetry energy J
  - Slope L
  - Curvature K_{sym}
  - Skewness Q_{sym}
  - ...

Symmetry energy

❖ Two definitions

- Derivative:

$$S_{\text{der}}(\rho) = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\mathcal{E}(\rho,\delta)\Big|_{\delta=0}$$

- Difference:

$$S_{\text{diff}}(\rho) = \mathcal{E}(\rho,1) - (\rho,0)$$

❖ *For the purposes of this talk, and for simplicity, the distinction is not important:*

$$S(\rho) : S_{\text{diff}}(\rho) \approx S_{\text{der}}(\rho)$$

Symmetry energy parameters

❖ Expansions around the saturation density:

$$S(\rho) = J + Lx + \frac{1}{2}K_{\mathrm{sym}}x^2 + \frac{1}{6}Q_{\mathrm{sym}}x^3 + \frac{1}{24}R_{\mathrm{sym}}x^4 + O(x^5),$$

❖ where $x = (\rho - \rho_0)/(3\rho_0)$ and we have defined the slope, curvature, skewness, and kyrtosis parameters:

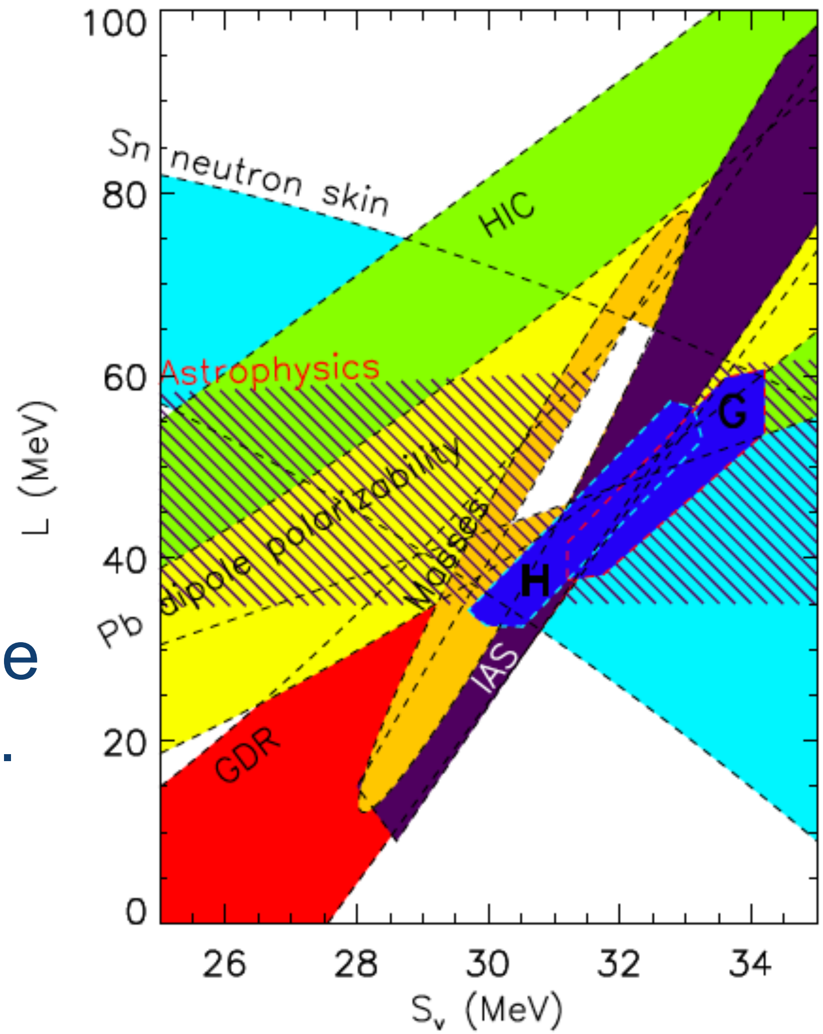
$$L \equiv 3\rho_0 \frac{d}{d\rho} S(\rho)\Big|_{\rho=\rho_0}, \qquad Q_{\mathrm{sym}} \equiv 27\rho_0^3 \frac{d^3}{d\rho^3} S(\rho)\Big|_{\rho=\rho_0},$$

$$K_{\mathrm{sym}} \equiv 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{S(\rho)}{\rho}\Big|_{\rho=\rho_0} \qquad R_{\mathrm{sym}} \equiv 81\rho_0^4 \frac{d^4}{d\rho^4} S(\rho)\Big|_{\rho=\rho_0}.$$

❖ *High-order parameters Ksym, Qsym less constrained; active in both high and low densities*

There is one EoS

- Only one point on this graph corresponds to nature
  - Only one point in the multiparameter space $\{\rho_0, e_0, K_0, \ldots, J, L, K_{sym}, Q_{sym}, ..\}$ corresponds to nature
- Nuclear properties shall be compatible with that point.
  - Microscopic calculations for nuclei: Relying on EDFT



Roca-Maza&Paar,PPNP101(2018)96; originally from Lattimer&Steiner, EPJA50(2014)40

Obstacles

- Analyses are model dependent
- Nuclear DFT:
  - Only few of the hundreds of EDF models can simultaneously describe nuclear matter and finite nuclei

Dutra et al.,PRC85(12)035201
Stevenson et al., AIP Conf.Proc.1529,262

  - ... while binding energies and radii “prefer” different values for the effective mass

Bender et al., Rev. Mod. Phys. 75,121

- Assumptions about the effective mass affect extrapolations (in basic Skyrme models)
- Artificial correlations among parameters

Effective mass

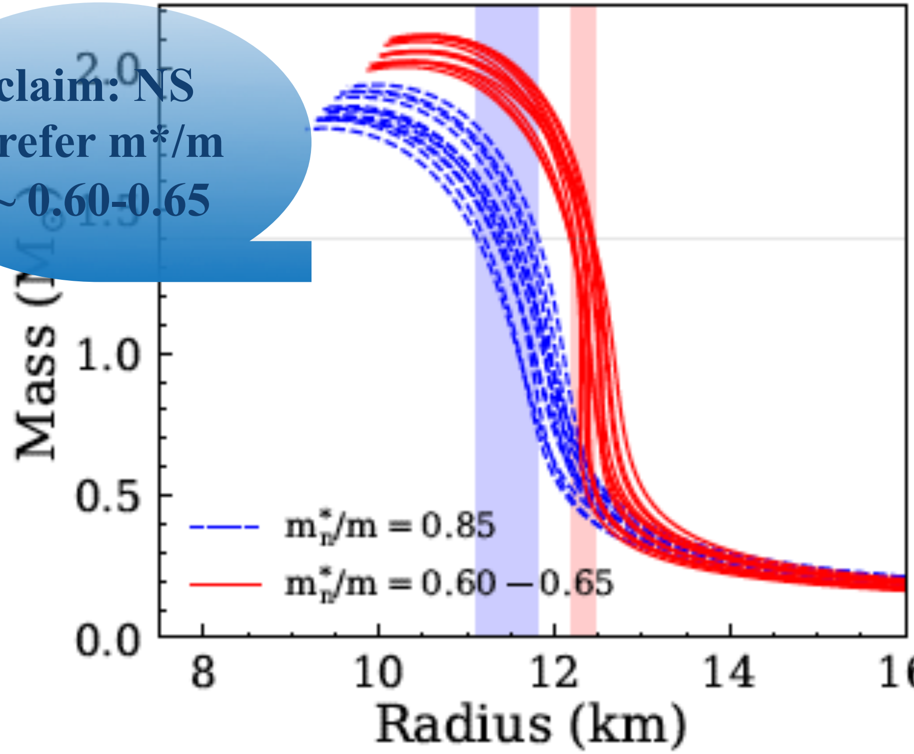
Effective mass

❖ Traditional Skyrme functionals

$$\mathcal{E}(\rho) = f\rho^{2/3} + a_0\rho + a_\gamma\rho^{1+\gamma} + a_2\rho^{5/3}$$

$$\frac{m*}{m} = \frac{f}{f + a_2\rho}$$

claim: NS prefer m*/m ~ 0.60-0.65



Tsang et al, arXiv:1908.11842

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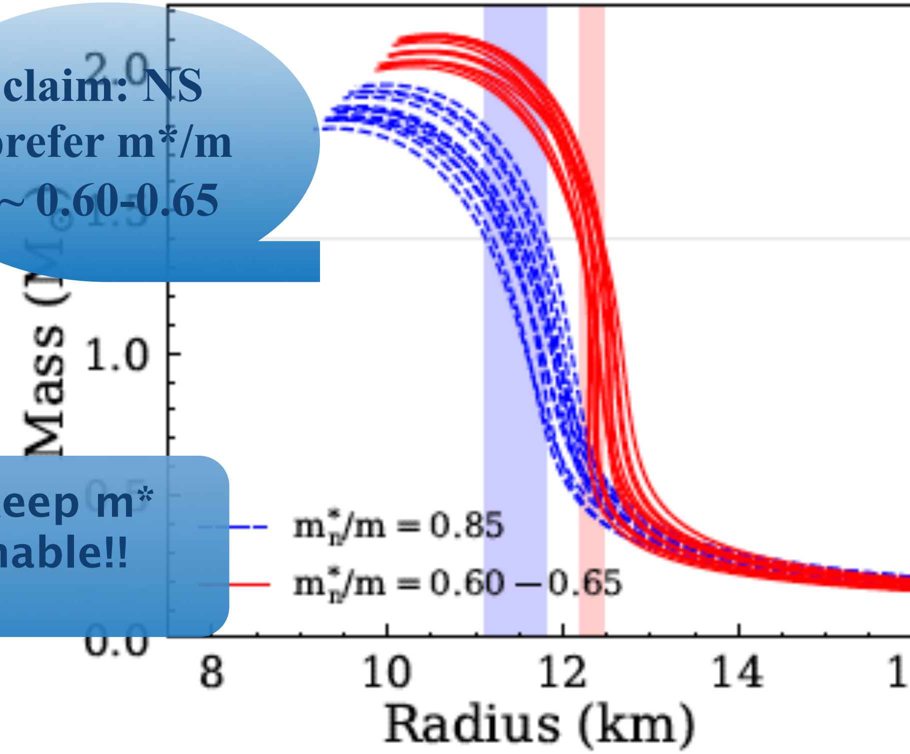
Neutron stars want higher a_2!

Please keep m* reasonable!!



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Effective mass

❖ Traditional Skyrme functionals

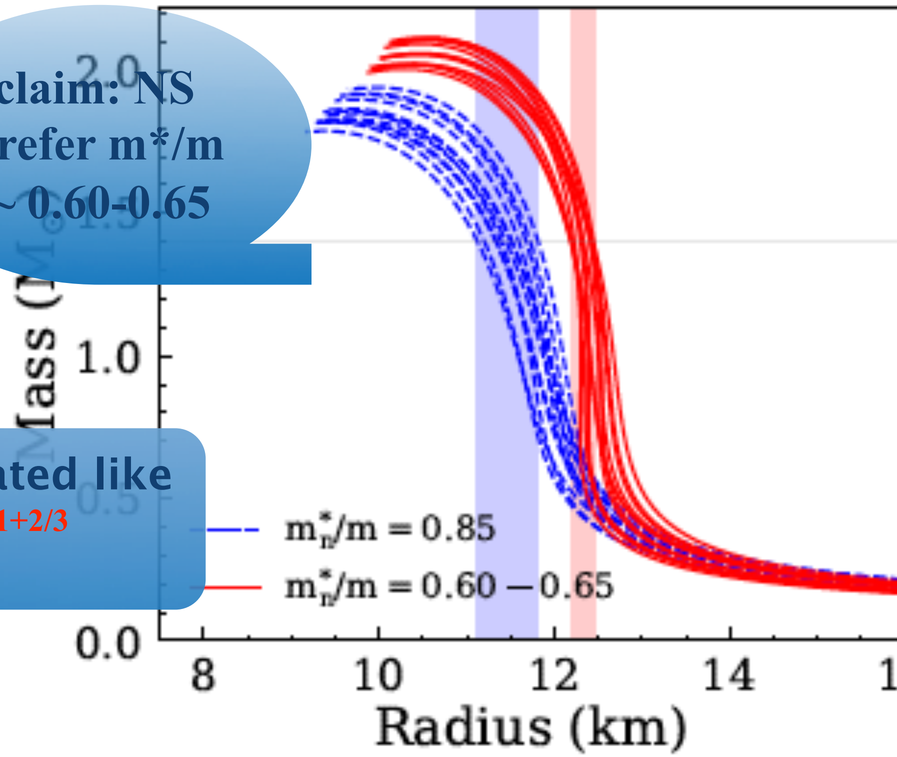
$$\mathcal{E}(\rho) = f\rho^{2/3} + a_0\rho + a_\gamma\rho^{1+\gamma} + a_2\rho^{5/3}$$

$$\frac{m*}{m} = \frac{f}{f + k a_2 \rho}$$

Only part of a_2 in the denominator

The rest treated like $(1-k)a_2\rho^{1+2/3}$

claim: NS prefer m*/m ~ 0.60-0.65



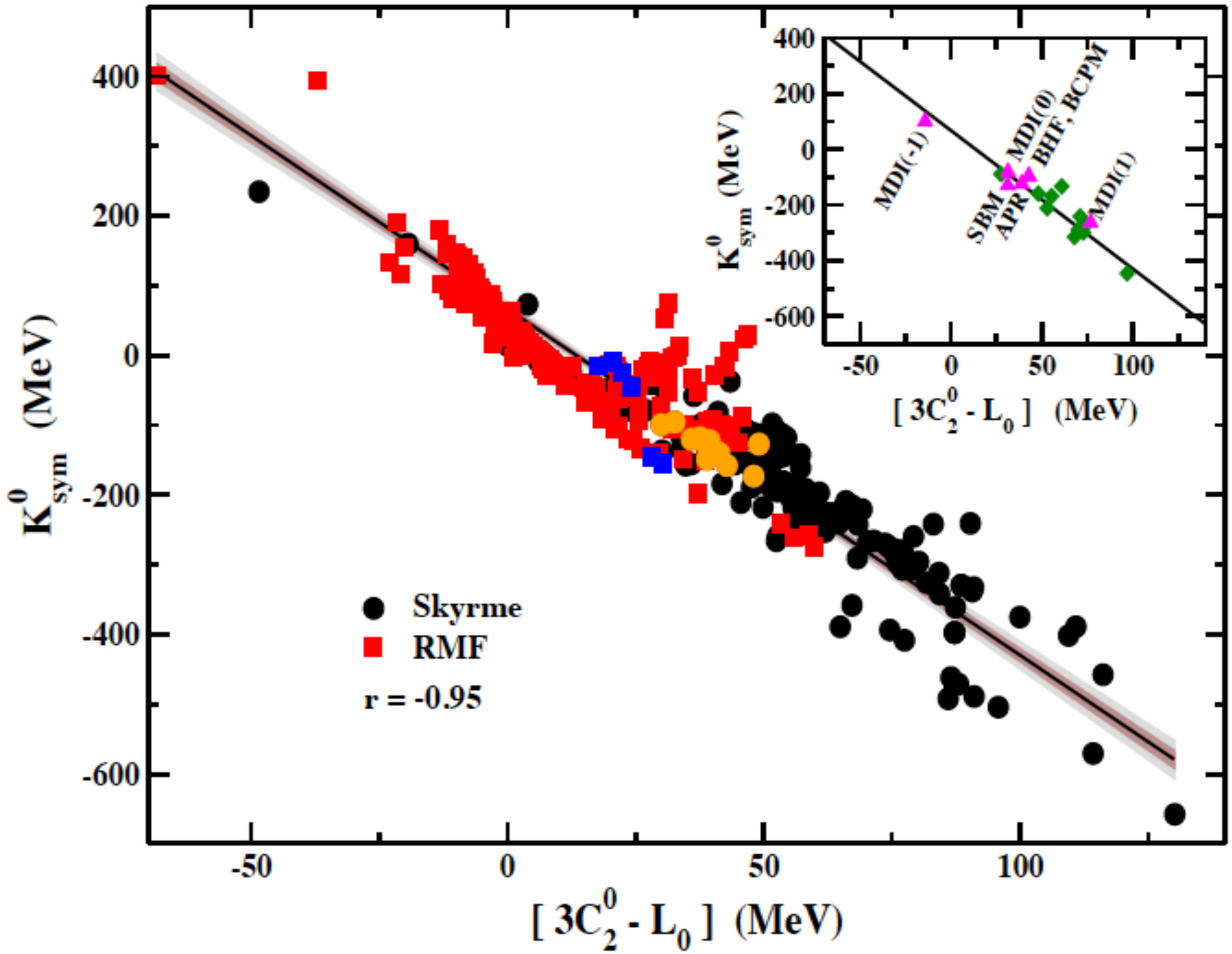
… almost like that… **KIDS**

Correlations between symmetry energy parameters

Correlations between symmetry energy parameters

❖ Interdependence?

Mondal et al., PRC96(2017)021302



Standard Skyrme: an underdetermined EoS

- SNM:
  - 4 Skyrme parameters...

$$\mathcal{E}(\rho,0) = f\rho^{2/3} + a_0\rho + a_\gamma\rho^{1+\gamma} + a_2\rho^{5/3}$$

  - for 4 EoS parameters

$$\mathcal{E}_0, \rho_0, K_0, \ldots \qquad \mu_s \equiv m^*/m$$

- Symmetry energy:
  - **3** additional Skyrme parameters...

$$S(\rho) = g\rho^{2/3} + b_0\rho + b_\gamma\rho^{1+\gamma} + b_2\rho^{5/3}$$

  - For **4+** EoS parameters

$$J, L, K_{\text{sym}}, Q_{\text{sym}} \ldots \qquad \mu_v \equiv m^*_{\text{IV}}/m$$

- **Trivial relation:**

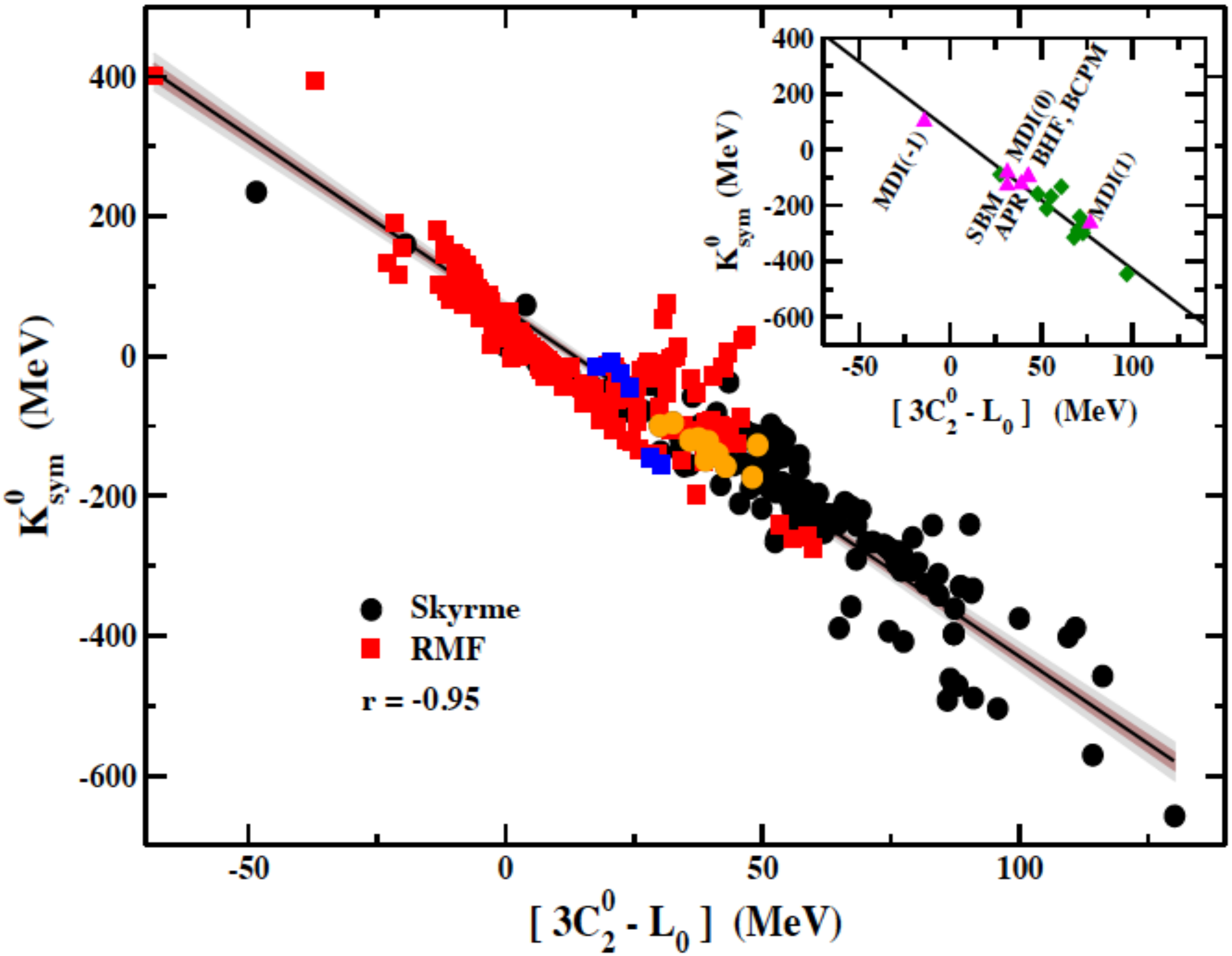
$$K_{\text{sym}} = 3(1+\gamma)(L-3J) + (1+3\gamma)g\rho_0^{2/3} + 2(2-3\gamma)b_2\rho_0^{5/3}$$

***(f and g are shorthand for the usual kinetic energy constants)*

Symmetry energy parameters

❖ Interdependence?

Mondal et al., PRC96(2017)021302

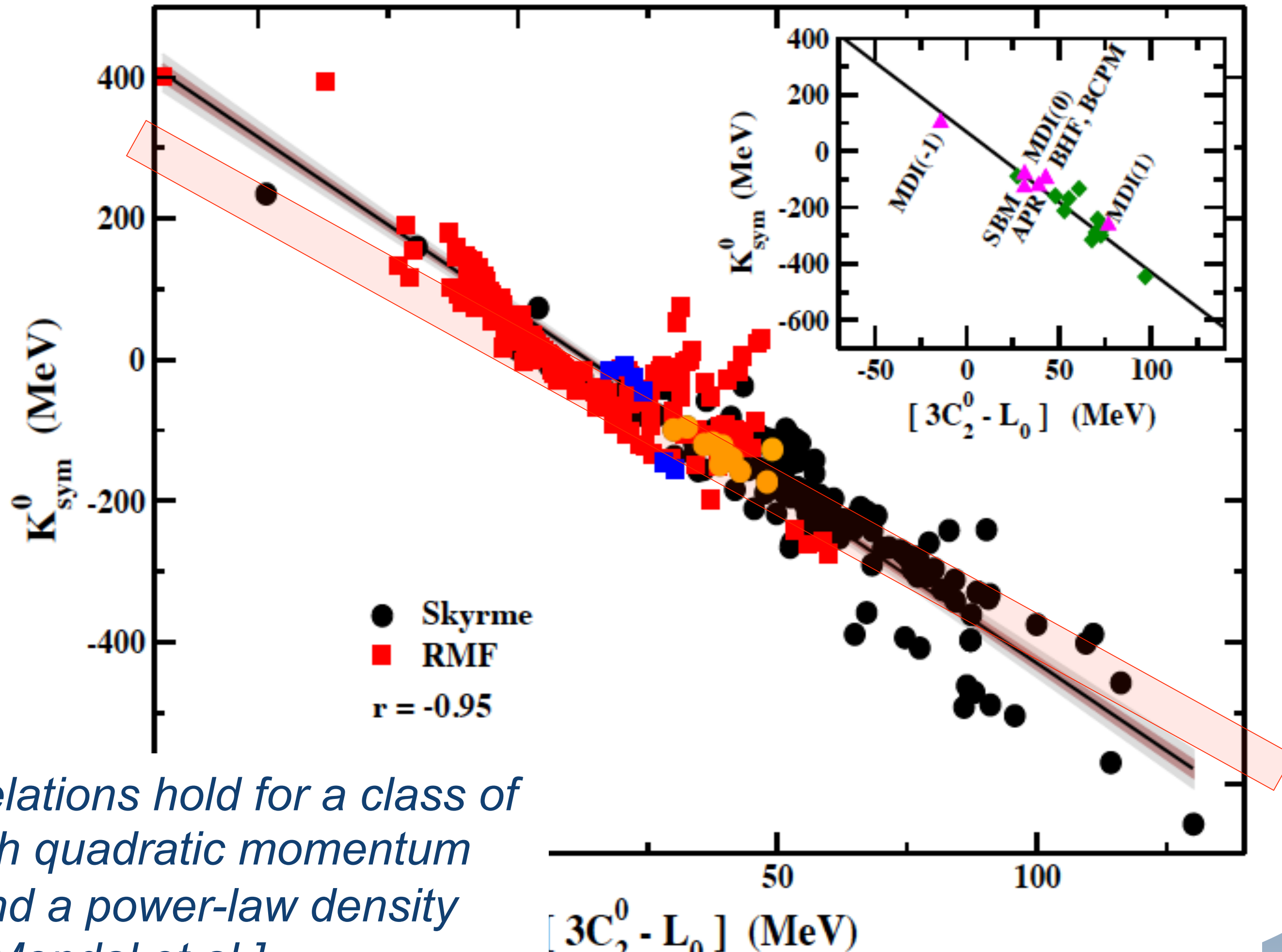


Symmetry energy parameters

❖ Interdependence?

for L≈40-60MeV
and γ=1/3

Mondal et al., PRC96(2017)021302



Indeed, "The relations hold for a class of interactions with quadratic momentum dependence and a power-law density dependence" [Mondal et al.]

Standard Skyrme: an underdetermined EoS

❖ SNM:

- 4 Skyrme parameters...

$$\mathcal{E}(\rho,0) = f\rho^{2/3} + a_0\rho + a_\gamma\rho^{1+\gamma} + a_2\rho^{5/3}$$

- for 4 EoS parameters

$$\mathcal{E}_0, \rho_0, K_0, \ldots$$

$$\mu_s \equiv m^*/m$$

very restrictive

❖ Symmetry energy:

- **3** additional Skyrme parameters...

$$S(\rho) = g\rho^{2/3} + b_0\rho + b_\gamma\rho^{1+\gamma} + b_2\rho^{5/3}$$

- For **4+** EoS parameters

$$J, L, K_{\rm sym}, Q_{\rm sym} \ldots$$

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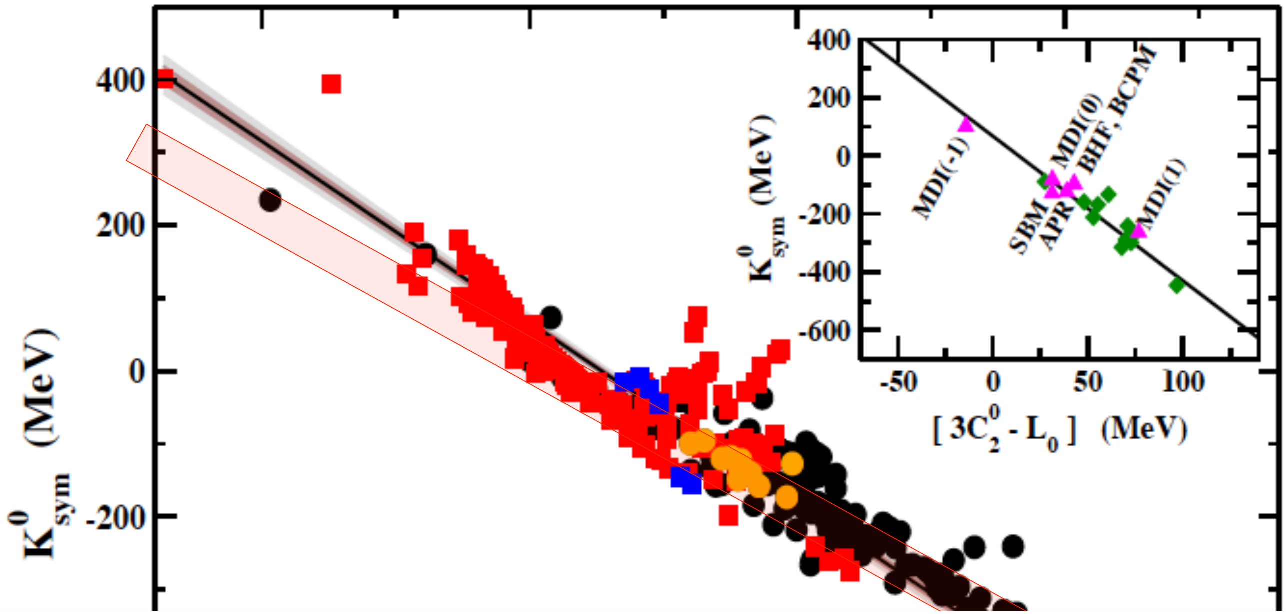
Symmetry energy parameters

RAON

for L≈40-60MeV and γ=1/3

❖ Interdependence?

et al., PRC96(2017)021302



We need to examine all consequential parameters independently

Must go beyond Skyrme

"The relations hold for a class of interactions with quadratic momentum dependence and a power-law density dependence"

KIDS FRAMEWORK

1) Homogeneous matter : EOS
2) Finite nuclei: EDF

KIDS FRAMEWORK

1) **Homogeneous matter : EOS**
2) Finite nuclei: EDF

KIDS Ansatz: Expansion in $k_F \sim \rho^{1/3}$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- If I have SNM and PNM, namely $c_i(0)$ and $c_i(1)$ (plus the quadratic approximation) I obtain analytically: $\{\rho_0, E_0, K_0, Q_0\}, \{J, L, K_{sym}, Q_{sym}\}$
- And vice versa;
- Ansatz motivated by basic QMBT and EFT

for details: PP,Park,Lim,Hyun,Phys. Rev. C 97,014312 (2018)

- Why 4 terms? Why low order?

KIDS Ansatz: Expansion in $k_F \sim \rho^{1/3}$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- If I have SNM and PNM, namely

$$c_i(\delta) = \alpha_i + \beta_i\delta^2$$

the quadratic approximation) I ob

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$$\begin{bmatrix} K_0 + 2\mathcal{T}_0 \\ \vdots \\ Q_{\rm sym} - 8\mathcal{T}_{\rm sym\,0} \\ \cdot \end{bmatrix} = \mathbb{A} \begin{bmatrix} \alpha_0\rho_0 \\ \vdots \\ \beta_3\rho_0^2 \\ \cdot \end{bmatrix}$$

3)

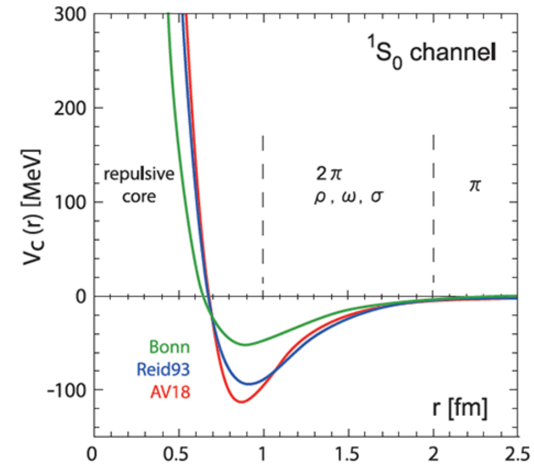
Fetter and Walecka, "Quantum theory of many-particle systems"

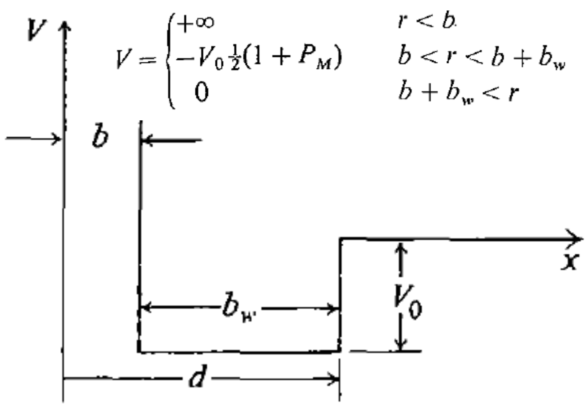
- ❖ Realistic potential: strong repulsive core plus attraction at longer range
- ❖ Apply Brueckner methodology in the calculation of nuclear matter energy

➔ Result: $k_F^2, k_F^3, k_F^4, k_F^5, k_F^6, \ldots,$ converging

- ◆ Even powers: from repulsive part
- ◆ Odd powers: from both

➔ The Fermi momentum is the relevant variable : **powers of ρ$^{1/3}$**





Guessing the analytical form - part II: EFT

- Saturation density is low...
  - with respect to (effective) boson exchange range (?)
    - one-pion exchange: vanishing expectation value
    - next boson: rho with m_ρ~775MeV~4fm^{-1}
  - Powers of $\mathbf{k_F/m_\rho}$
- Expansion of E/A in powers of k_F
  - ... which means, again, powers of $\rho^{1/3}$
  - The Fermi momentum as the relevant variable
  - k_F^3 and k_F^4 (i.e., coupling~$\rho^{1/3}$) known to be important for obtaining saturation [Kaiser et al.,NPA697(2002)]
- *Dilute Fermi gas: plus logarithmic terms*

H.-W. Hammer, R.J. Furnstahl / Nuclear Physics A 678 (2000) 277–294

KIDS Ansatz: Expansion in $k_F \sim \rho^{1/3}$

But how many powers? Which are relevant?

- Fit to homogeneous matter pseudodata
  - Variational Monte Carlo (APR, FP)
- Statistical analysis of fit quality; naturalness
- Keep only the important terms! No overtraining

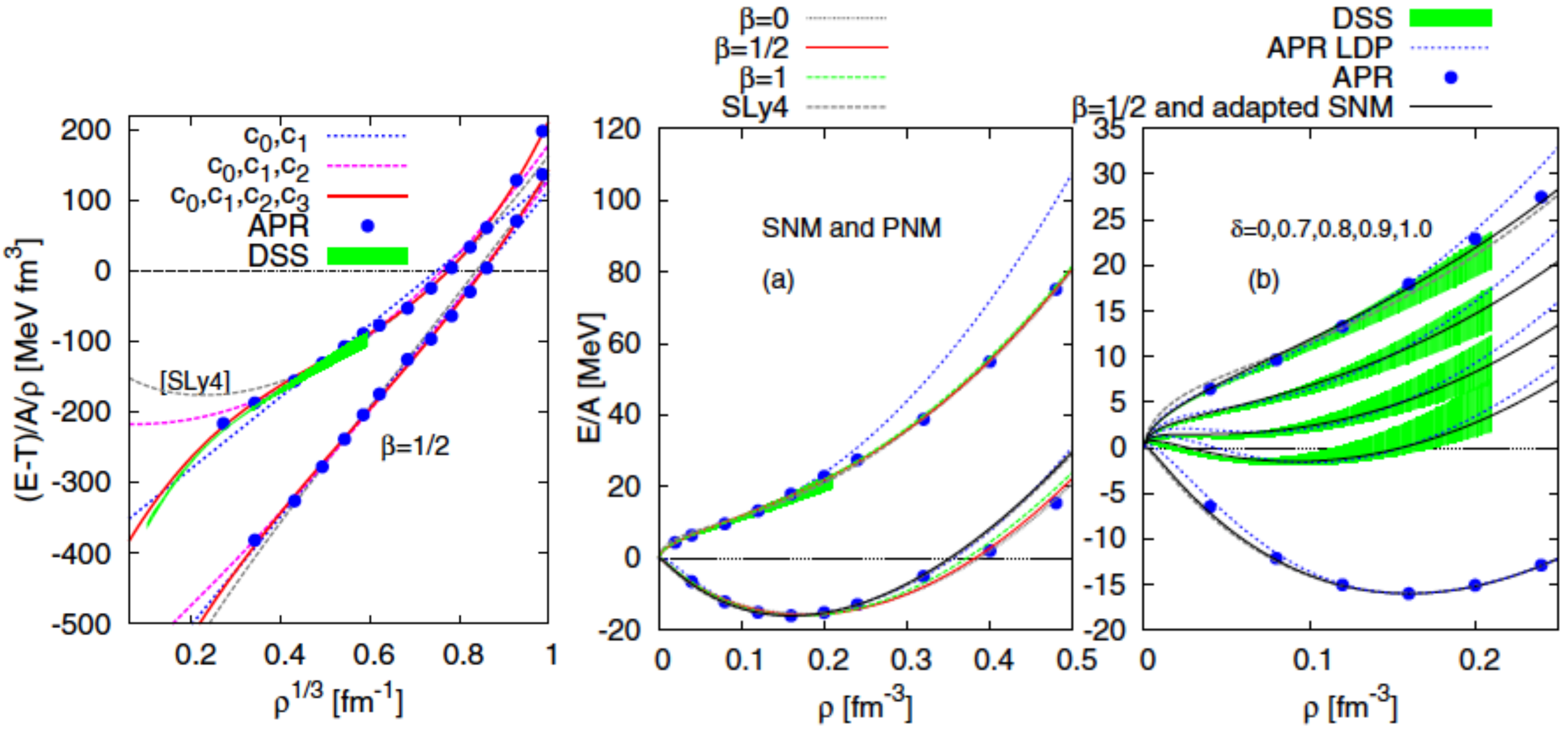
$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- SNM: 3 terms suffice in converging hierarchy ($c_3(0)=0$)
- PNM: 4 terms necessary (*different preferences*)

See PP,Park,Lim,Hyun,Phys. Rev. C 97,014312 (2018), Gil,Kim,Hyun,PP,Oh,Phys. Rev. C 100,014312 (2019)

Fits to APR pseudodata

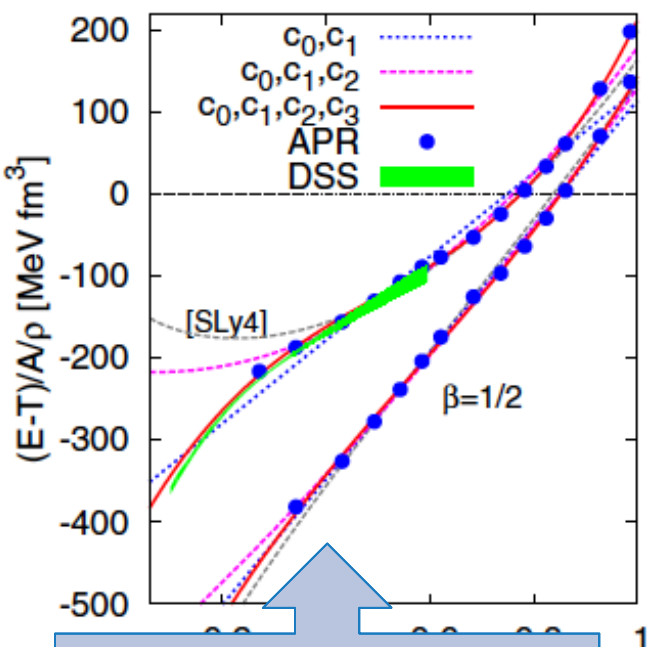
PP,Park,Lim,Hyun,Phys.Rev.C97,014312(2018)



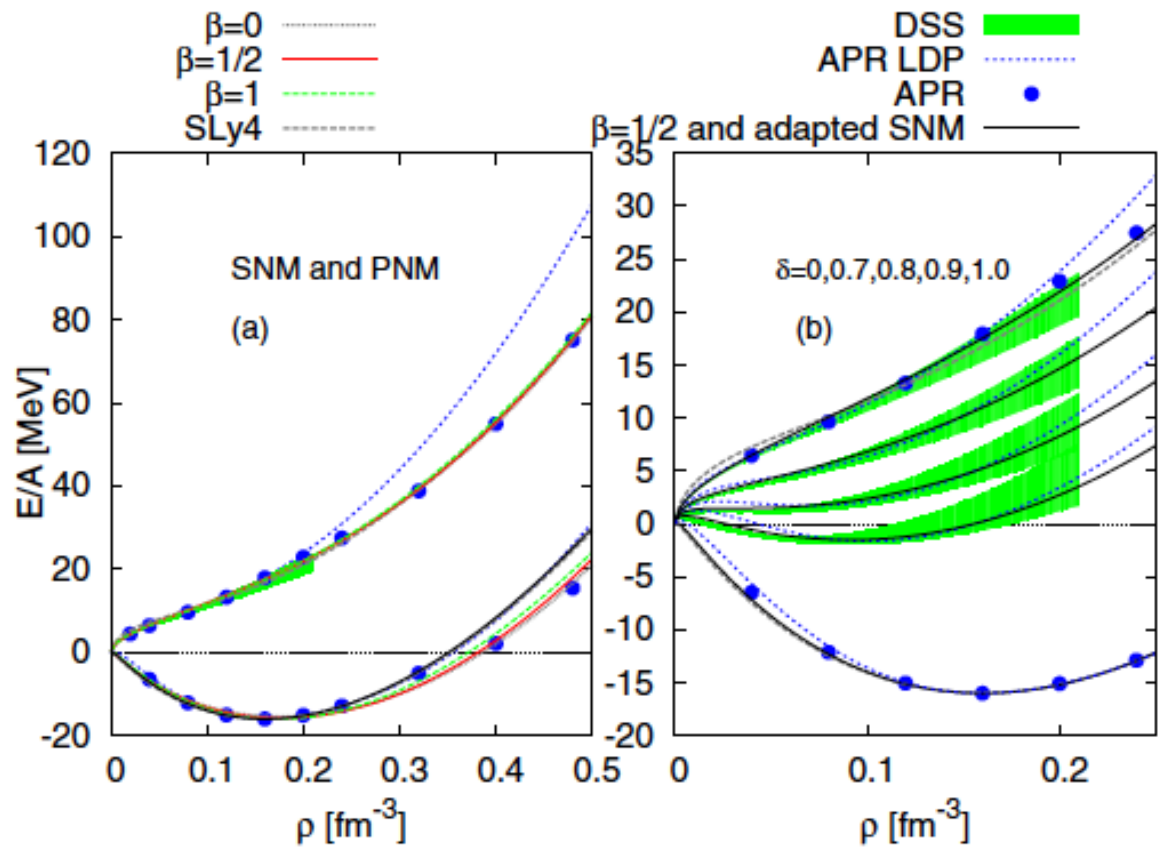
[*] β: controls the weight we put on the lower-density data

Fits to APR pseudodata

PP,Park,Lim,Hyun,Phys.Rev.C97,014312(2018)



Symmetric matter: practically linear in $\rho^{1/3}$



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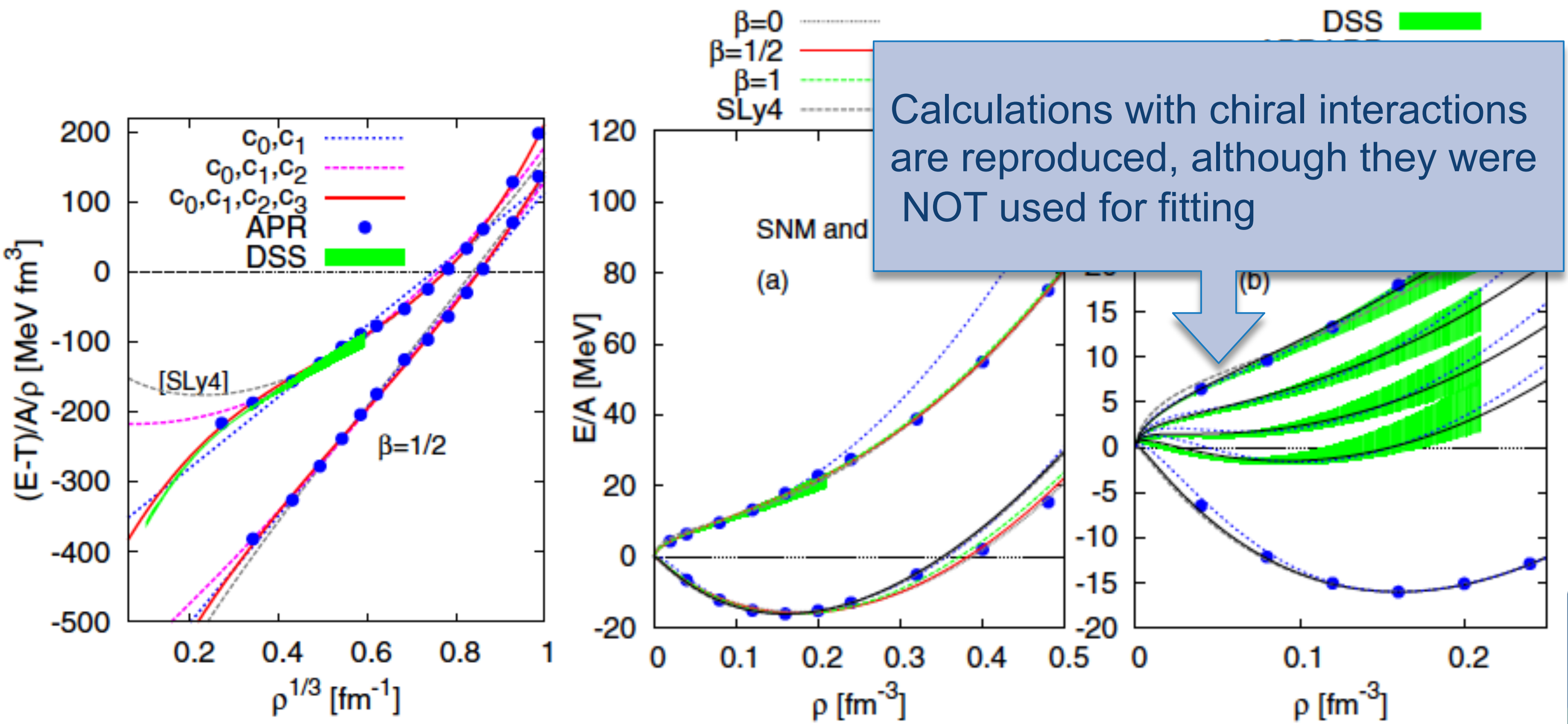
PP,Park,Lim,Hyun,Phys.Rev.C97,014312(2018)



Calculations with chiral interactions are reproduced, although they were NOT used for fitting

[*] β: controls the weight we put on the lower-density data

Parameter set KIDS-ad2

- Symmetric nuclear matter:
  - Set ρ_0=0.16 fm^{-3}, E_0=-16MeV, K_0 = 240 MeV
  - Determine $c_{0,1,2}(0)$ (analytical expressions)
  - Leads to Q_0=-373 MeV
- Pure neutron matter:
  - Fit $c_{0,1,2,3}(1)$ to the APR pseudodata for PNM
  - Resulting symmetry-energy parameters:

J=33MeV, L=49MeV, K_{sym}=-157MeV, Q_{sym}=586MeV

Parameter set KIDS-ad2

- Symmetric nuclear matter:
  - Set ρ_0=0.16 fm^{-3}, E_0=-16MeV, K_0 = 240 MeV
  - Determine $c_{0,1,2}(0)$
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  - Resulting symmetr

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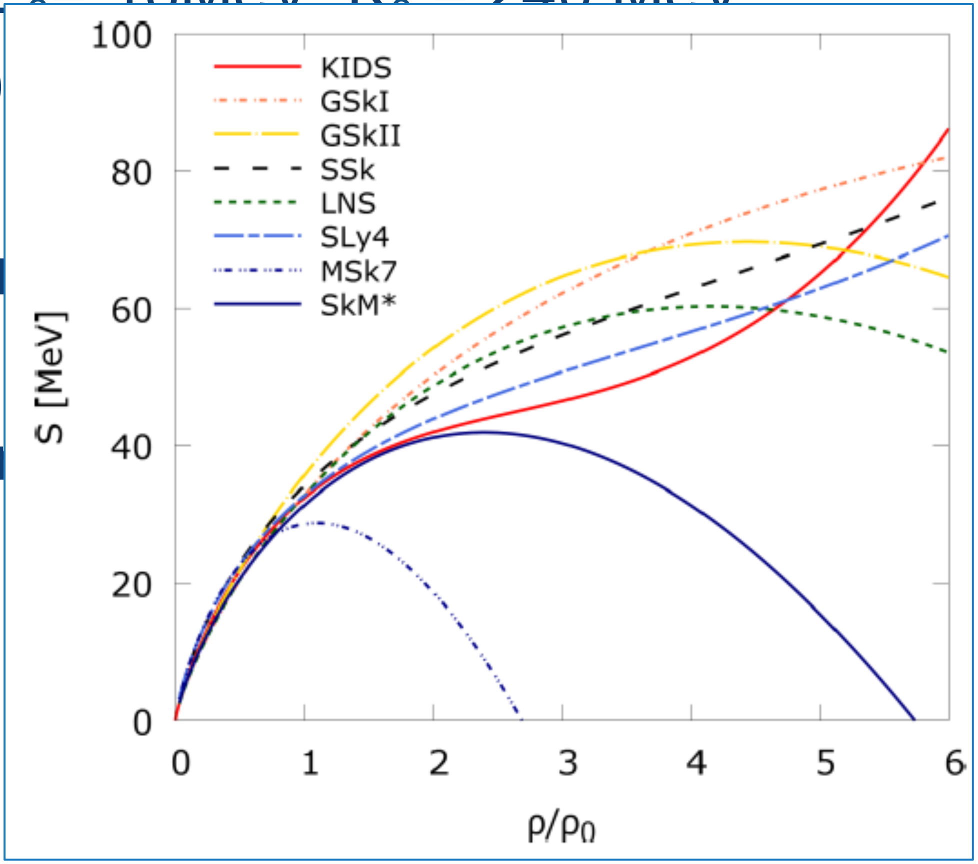
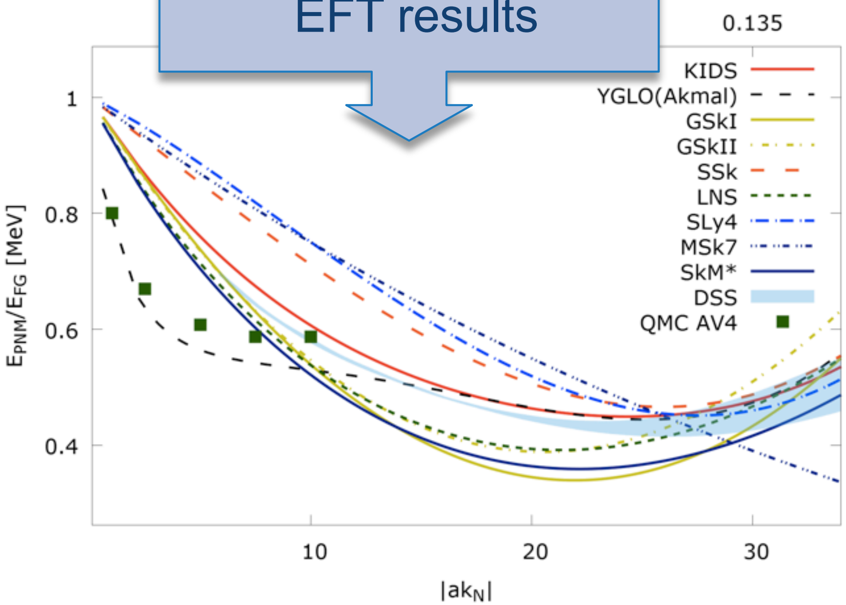


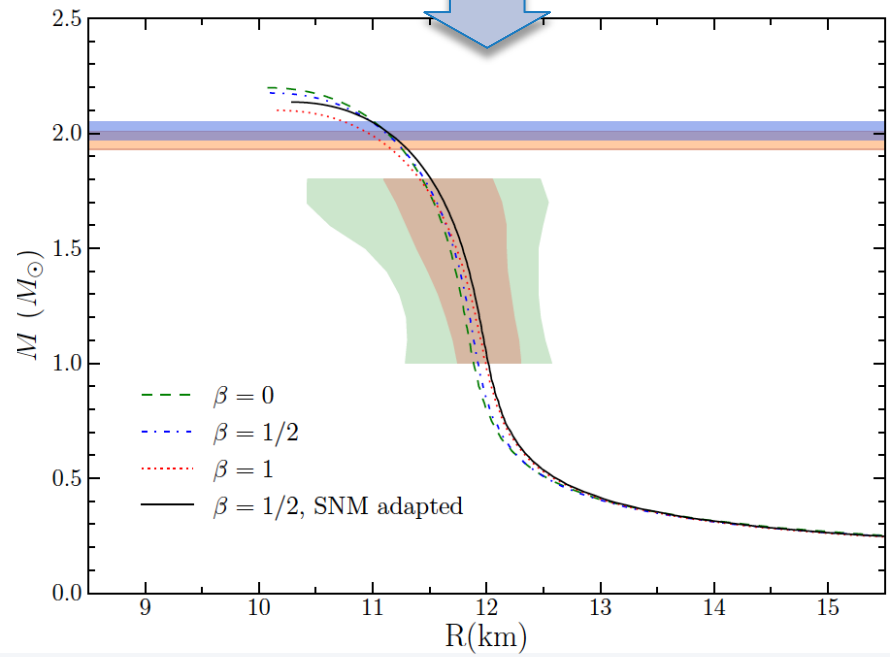
Gil,PP,Hyun,Oh,PRC99,064319(2019)

Extrapolations to dilute and dense matter

Follows closely chiral EFT results



Mass-Radius relation of NS: Compatible with observational data



We are now free to vary EoS parameters and examine systematiclly effects on NS, etc.
... but also nuclei

KIDS FRAMEWORK

1) Homogeneous matter : EOS
2) **Finite nuclei: EDF**

Ansatz

powers of Fermi momentum ~ $\rho^{1/3}$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

$$\begin{aligned} & t_0(1+x_0P_\sigma)\delta(\mathbf{r}_i-\mathbf{r}_j) \\ & + \frac{1}{2}t_1(1+x_1P_\sigma)[\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{k}^2 + \mathbf{k}'^2\delta((\mathbf{r}_i-\mathbf{r}_j)] \\ & + t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{k} \\ & + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha\delta(\mathbf{r}_i-\mathbf{r}_j) \\ & + iW_0\,\mathbf{k}'\times\delta(\mathbf{r}_i-\mathbf{r}_j)\,\mathbf{k}\cdot(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j), \qquad (3) \end{aligned}$$

kinetic energy: $\mathcal{T} = \mathcal{T}_p + \mathcal{T}_n\,;\ \mathcal{T}_{p,n} = \frac{3}{5}\frac{\hbar^2}{2m_{p,n}}x_{p,n}^{5/3}(3\pi^2\rho)^{2/3}\,;\ x_{p,n} \equiv \rho_{p,n}/\rho$

asymmetry:

$$c_k(\delta) = \alpha_k + \delta^2\beta_k$$

$$\delta = (\rho_n - \rho_p)/\rho$$

correspondence with Skyrme

Nuclear potential	Order	KIDS parameter	Skyrme parameter
$\mathcal{E}_0$	k_F^3	$c_0(\delta)$	(t_0, x_0)
$\mathcal{E}_1$	k_F^4	$c_1(\delta)$	$(t_3, x_3), \alpha = 1/3$
$\mathcal{E}_2$	k_F^5	$c_2(\delta)$	$(t_1, x_1), (t_2, x_2), (t_3', x_3'), \alpha' = 2/3$
$\mathcal{E}_3$	k_F^6	$c_3(\delta)$	$(t_3'', x_3''), \alpha'' = 1$

Skyrme-like parameters by reverse engineering

$$v_{i,j} = (t_0 + y_0 P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2 + \text{h.c.}]$$
$$+(t_2 + y_2 P_\sigma)\boldsymbol{k}' \cdot \delta(\boldsymbol{r}_{ij})\boldsymbol{k} + iW_0\, \boldsymbol{k}' \times \delta(\boldsymbol{r}_{ij})\, \boldsymbol{k} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)$$
$$+\frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n} P_\sigma)\rho^{n/3}\delta(\boldsymbol{r}_{ij}), \qquad (3)$$

Minimal Skyrme-type "force"

$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s,$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right)$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2\,, \quad \theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2\,.$$

unconstrained from homogenous matter ➔ vary freely

But the total $c_2(0)$, $c_2(1)$ will remain unchanged!

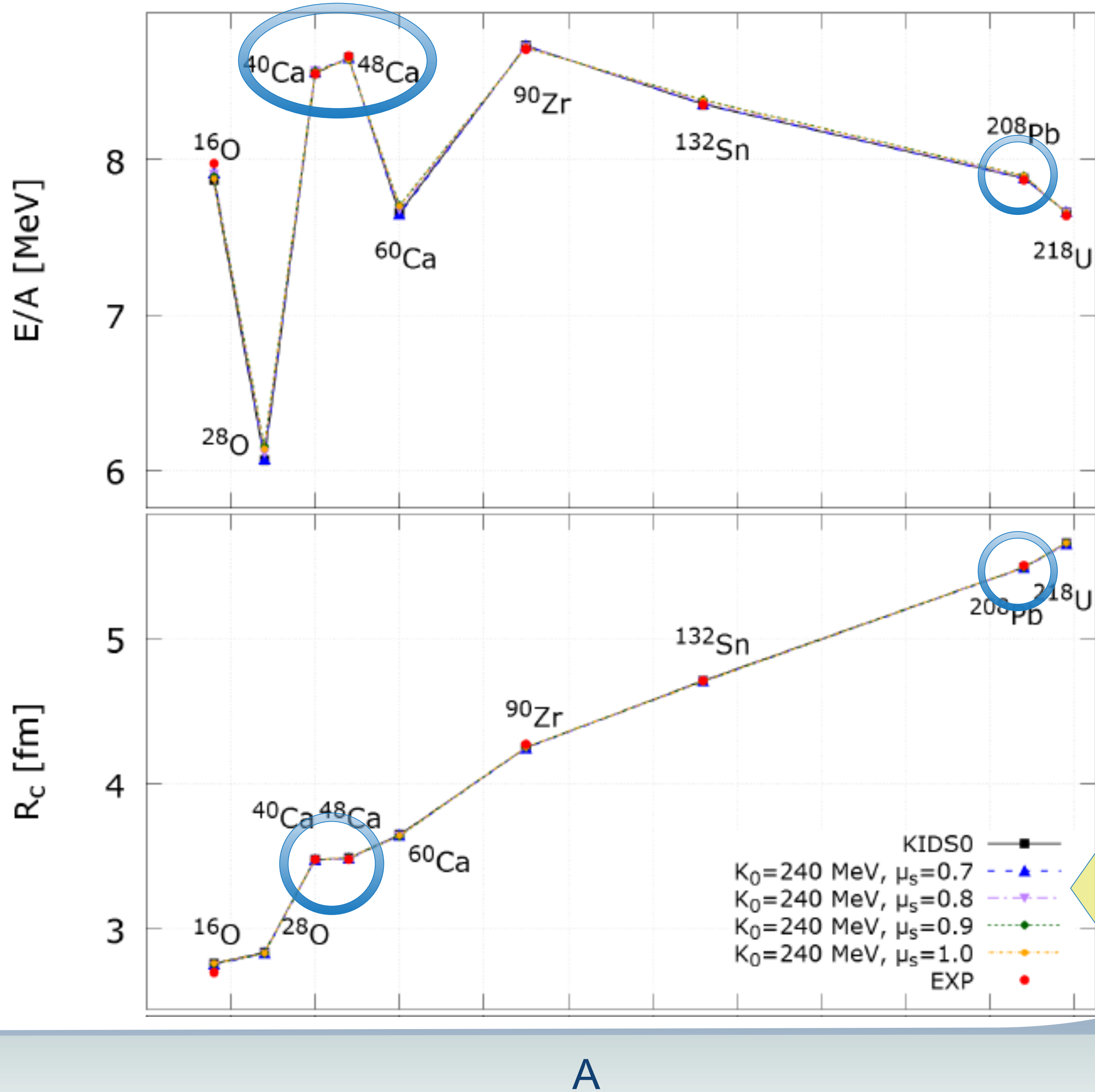
Procedure

For given KIDS functional $c_i(0)$, $c_i(1)$ (i.e., fixed SNM, PNM)

- Chose effective masses (vary at will)
- All t_i, y_i are now known except t_1, t_2, x_1, x_2
- The two combinations θ_s, θ_μ also known (eff. masses)
- **Two independent free parameters plus spin-orbit W_0**
  - Fit only to ^{40}Ca, ^{48}Ca, ^{208}Pb
  - Only bulk properties: E/A, charge radius: 6 data

Proof of concept with KIDS-ad2
~Taking APR to nuclei~

Binding energy, charge radii

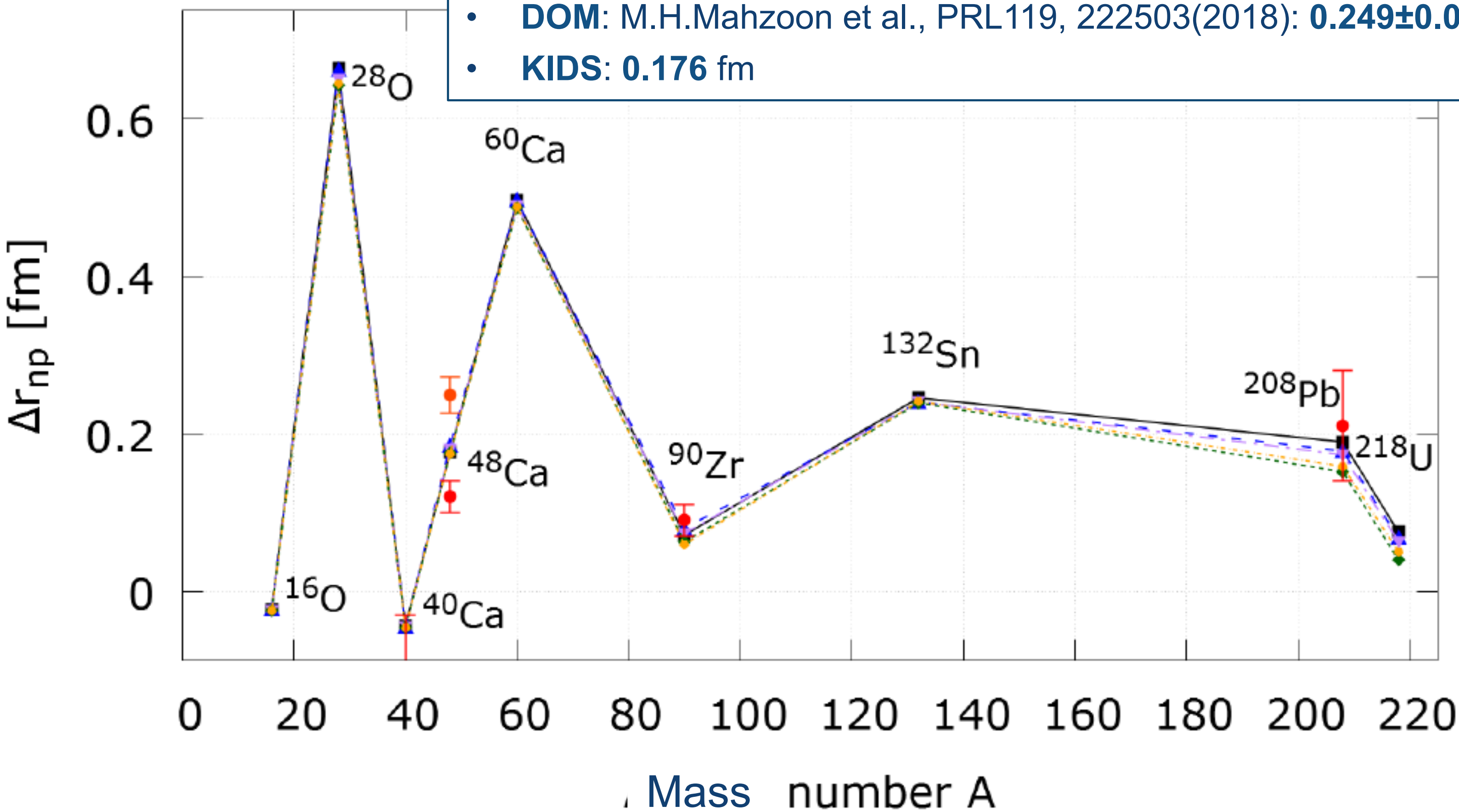


predictions independent of the effective mass assumed

Neutron skin thickness

neutron skin of 48Ca:

- **CCM**: G.Hagen et al., Nature Phys. 12,186(2016): **0.12-0.15** fm
- **DOM**: M.H.Mahzoon et al., PRL119, 222503(2018): **0.249±0.023** fm
- **KIDS**: **0.176** fm



Data: antiprotonic atoms, PREX (^{208}Pb), DOM (^{48}Ca, upper)

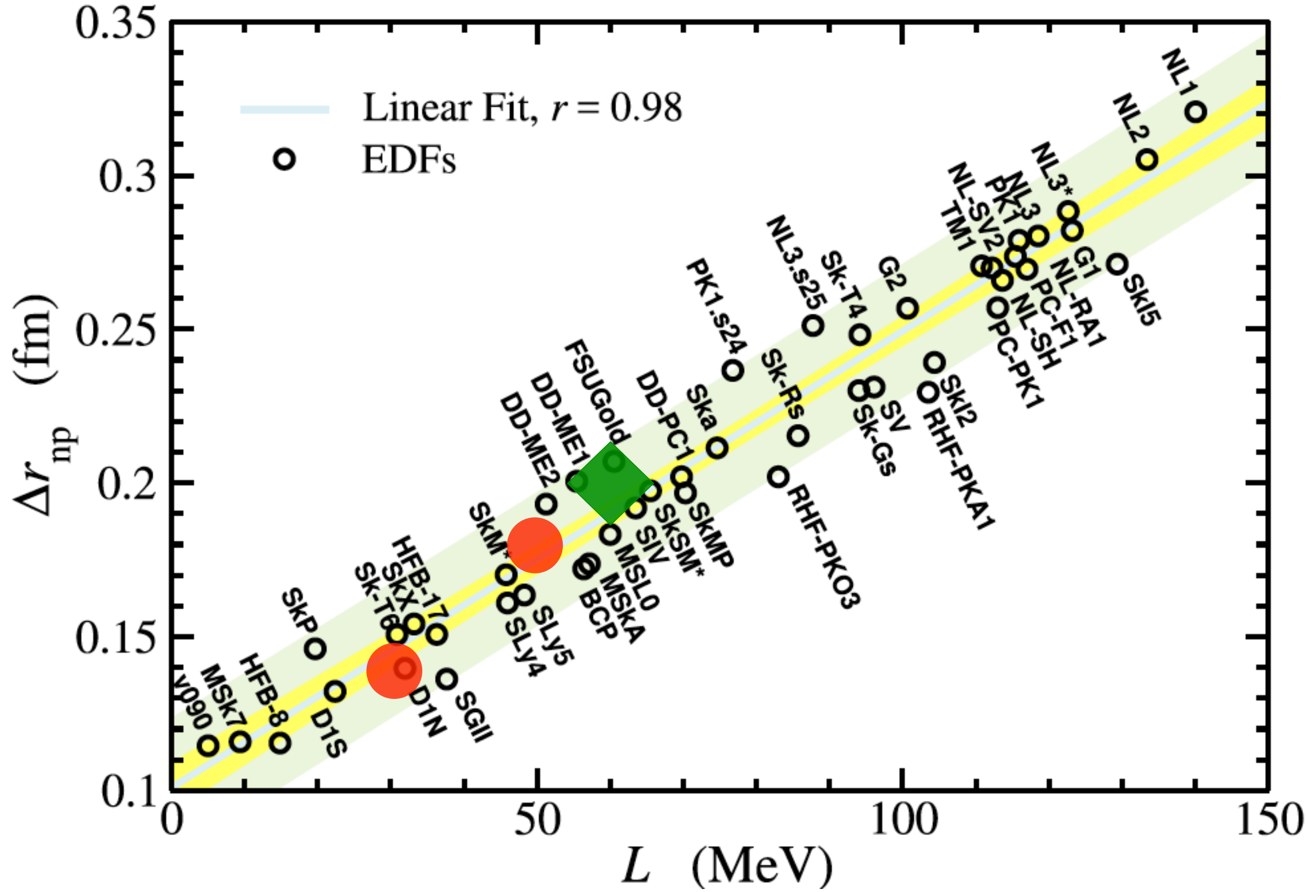
Predictions of APR EoS for the neutron skin thickness!

L and neutron skin thickness

RAON

[Roca-Maza et al., PRL106,252501(2013)]

❖ Correlation based on DFT models

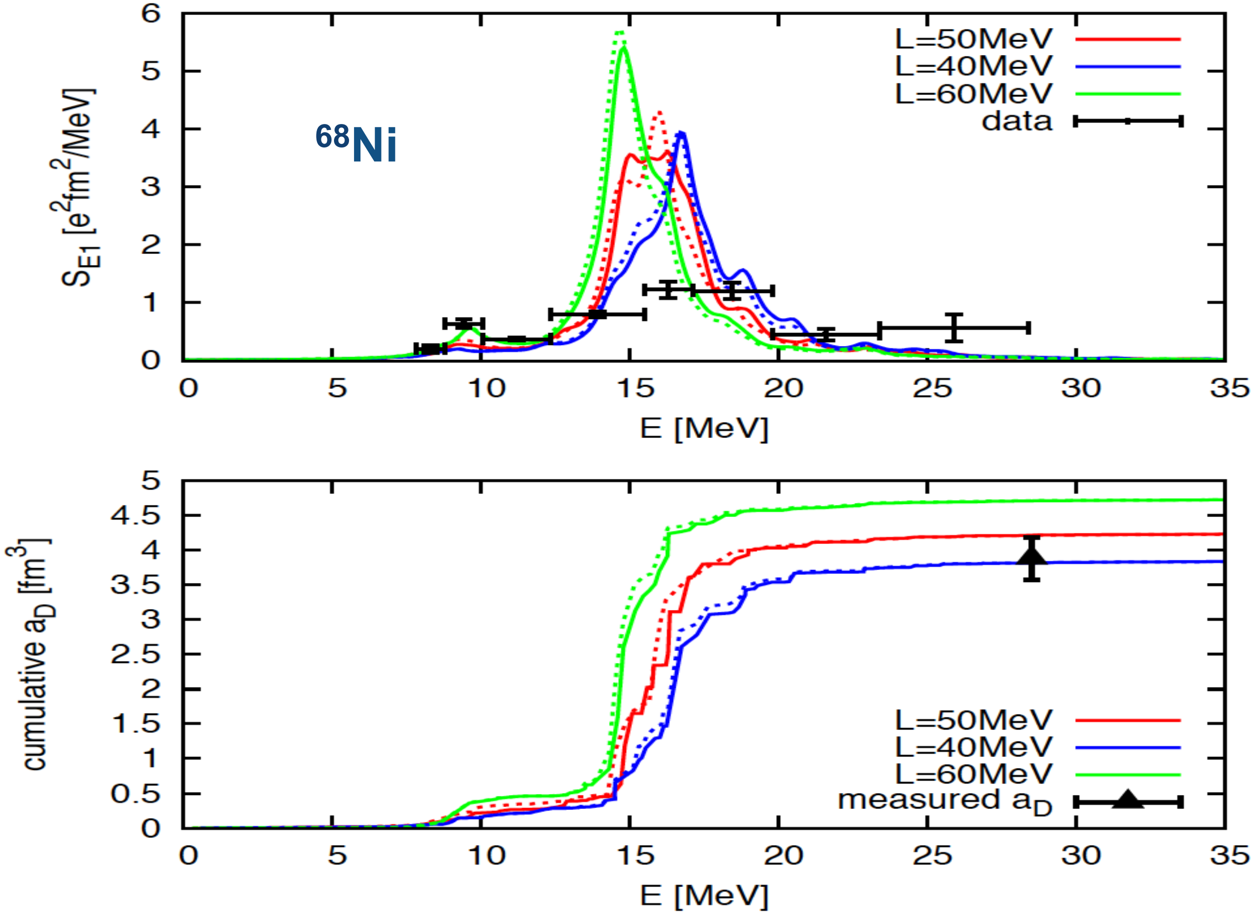


❖ KIDS points ●: Varying L only, around the baseline value; Diamond ◆ corresponds to QMC pseudodata

[Carlson et al., RMP87,1067(2015)]

L and electric dipole polarizability

Data: Rossi et al., PRL111,232503(2013)



Full lines: κ=0.20
Dotted: κ=0.15

❖ Enhancement factor (isovector m*) decouples from static polarizability

INTERRELATIONS OF EOS PARAMETERS?

In progress

Varying L, Ksym, etc.

- Begin with baseline KIDS-ad2 parameters
- Vary one or more EoS parameters at will: L, K0, Ksym, etc -> Corresponding EDF
- Fit to 6 data (energy and charge radius of ^{40}Ca, ^{48}Ca, ^{208}Pb)

$$\chi^2_{\mathrm{fit}} = \sum_{d=1}^{6} \left| \frac{O_i^{\mathrm{calc}} - O_i^{\mathrm{exp}}}{O_i^{\mathrm{exp}}} \right|^2$$

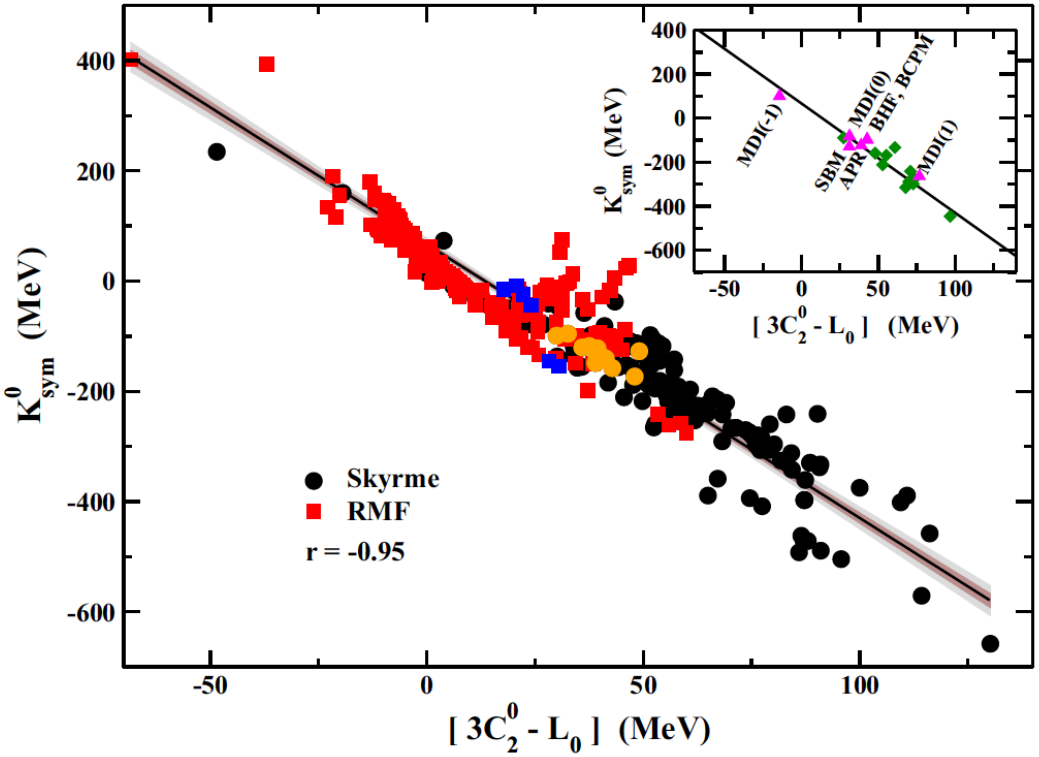
- Obtain predictions for the nuclei ^{16}O, ^{28}O, ^{60}Ca, ^{90}Zr, ^{100}Sn, ^{132}Sn, ^{208}Pb, ^{218}U
- Deviations from known data give a “prediction” error

$$\chi^2_{\mathrm{pred}} = \sum_{d>6} \left| \frac{O_i^{\mathrm{calc}} - O_i^{\mathrm{exp}}}{O_i^{\mathrm{exp}}} \right|^2$$

K_{sym} vs (3J-L)

❖ Reported linear fit: $K_{sym} - 4.97(L\text{-}3J) \approx$ 67 ± 2 MeV

Mondal et al., PRC96(2017)021302

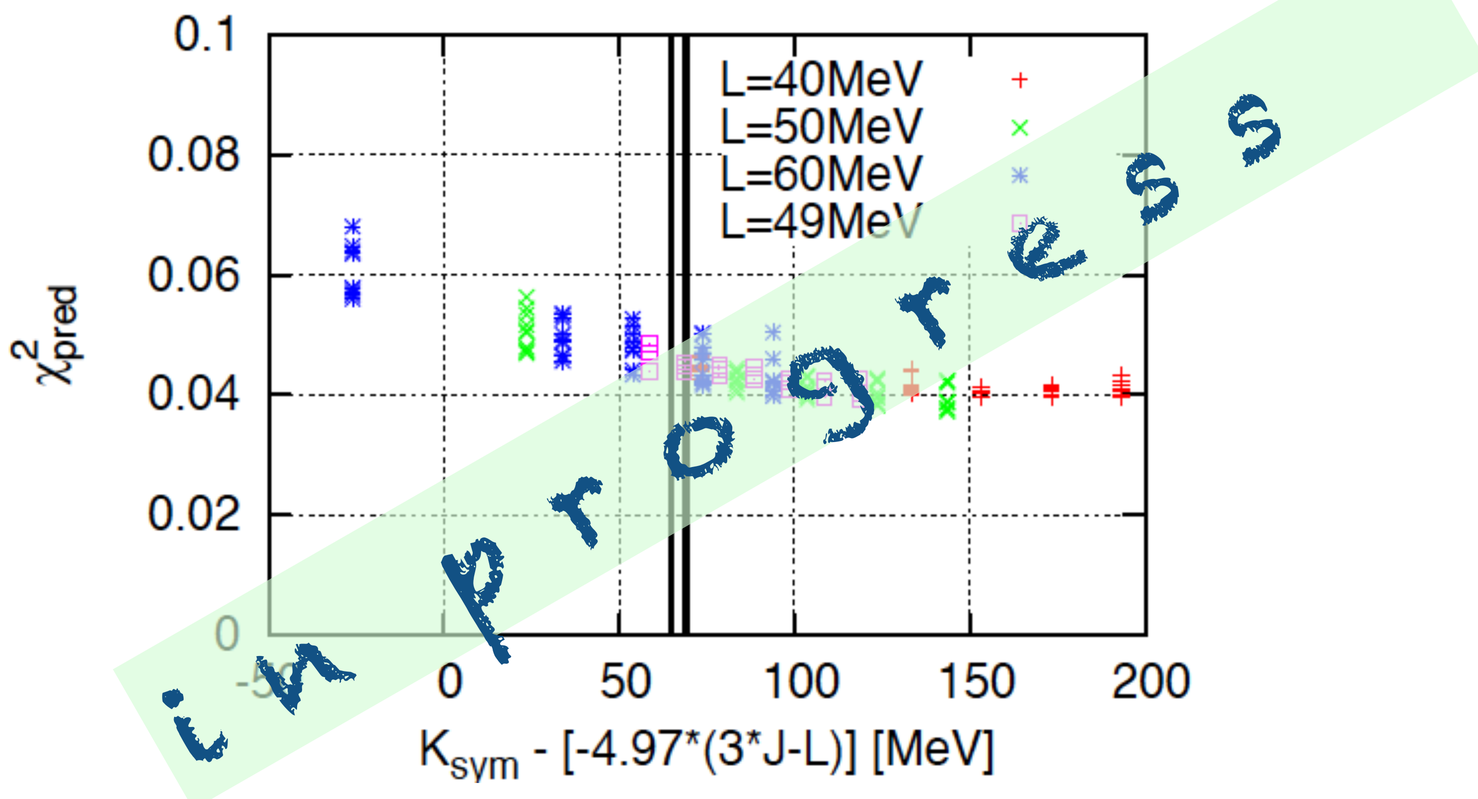


❖ If this is physical, fits for various (L,J,K_{sym}) combinations should favor the above value

K_{sym} vs (3J-L)

RAON

❖ Reported linear fit: K_{sym} – 4.97(L-3J) ≈ 67 ± 2 MeV

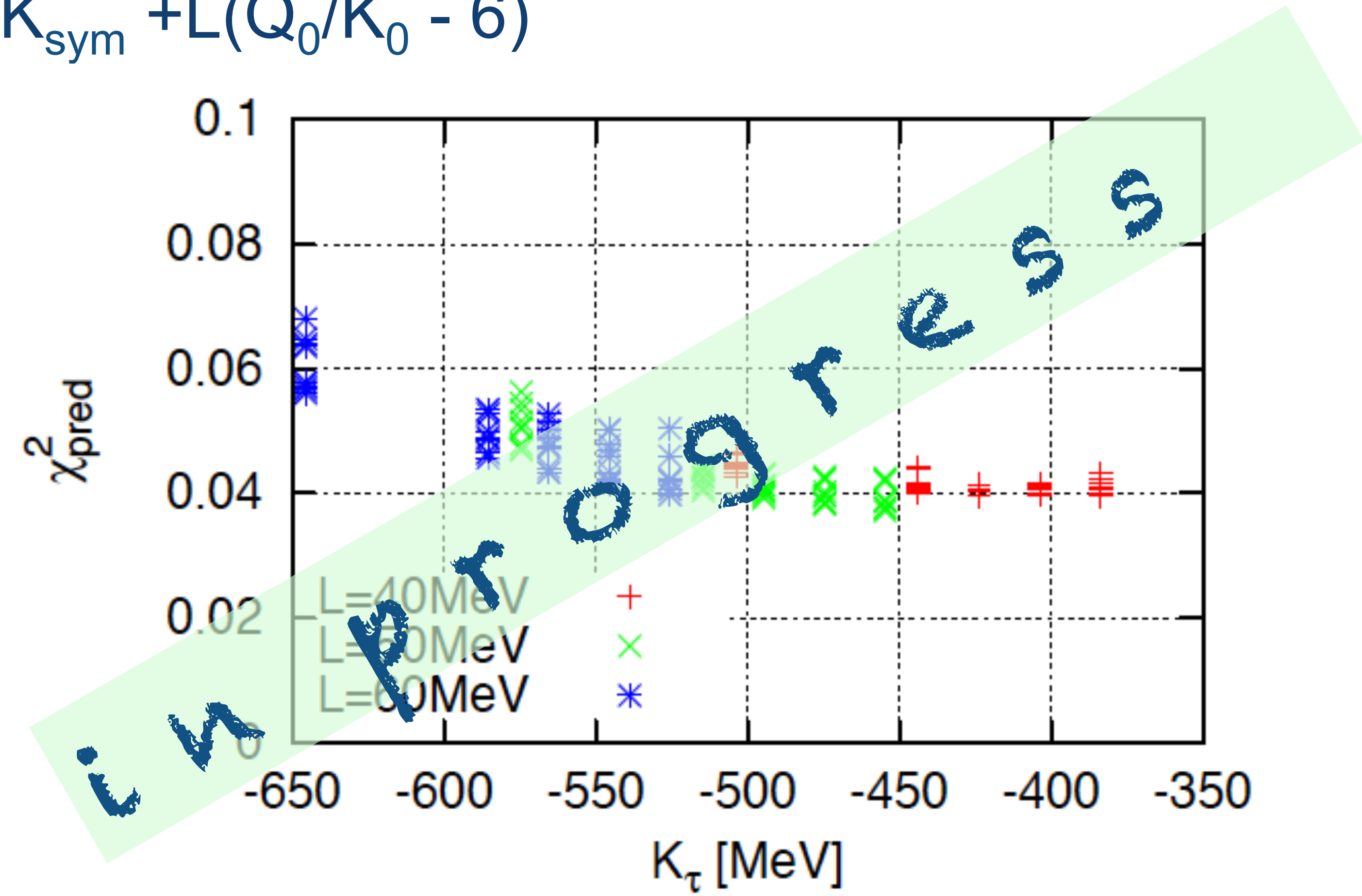


❖ No favored value for the examined combinations: Flat trend

K_0:240MeV; -K_{sym}:140-220MeV; J=33MeV

Liquid drop parameter K_τ

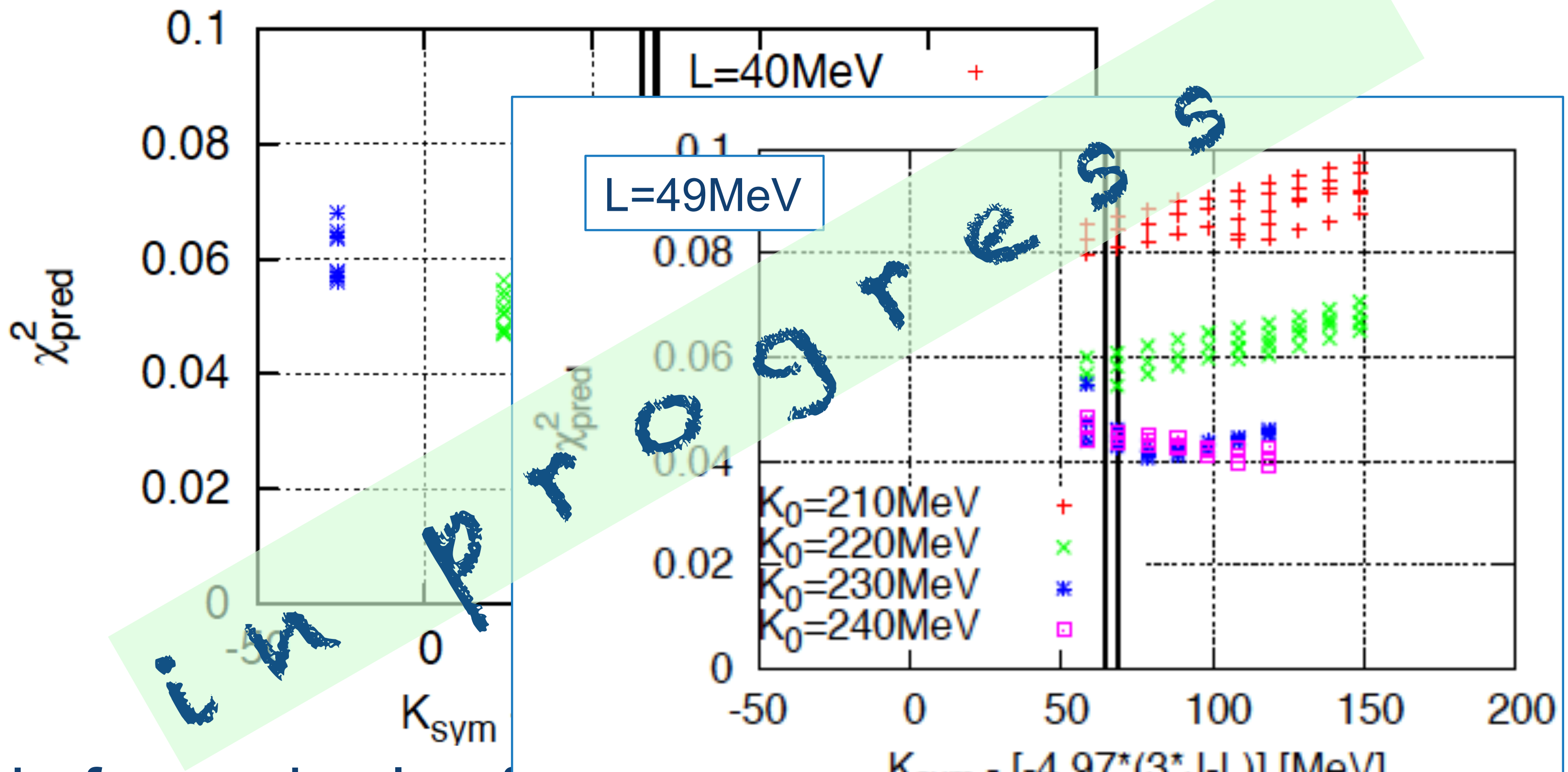
❖ $K_\tau = K_{sym} + L(Q_0/K_0 - 6)$



K_0:240MeV; $-K_{sym}$:140-220MeV; J=33MeV

K_{sym} vs (3J-L)

❖ Reported linear fit: K_{sym} – 4.97(L-3J) ≈ <u>67 ± 2 MeV</u>



❖ No favored value for the examined combinations: Flat trend

K_0:240MeV; -K_{sym}:140-220MeV; J=33MeV

Conclusion-Prospects

- KIDS functional: direct connection of all EoS parameters of interest with nuclei.
  - Study their effects on observables
  - Avoid overfitting as well as artificial correlations
  - Effective mass decouples from bulk properties
  - In progress: K_{sym}, K_{τ} vs L and J
- Eventually, there is only one EoS. Plan: to determine it in the KIDS framework by **multiparametric fits** (->*bootstrapping*)
- **Exotic nuclei to be included**

Thank you!

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