

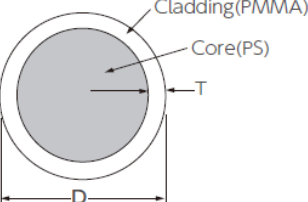
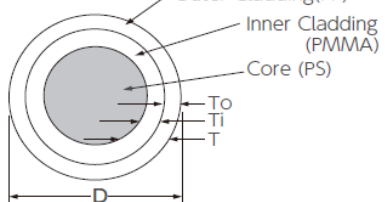
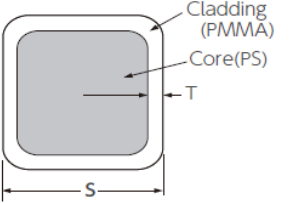
WLS fiber Study

Jae Min Choi

Trapping Efficiency

Trapping Efficiency

Cross-section and Cladding Thickness

	Single Cladding	Multi-Cladding (M)
Round Fiber (D)	 <p>Cladding Thickness¹⁾: T=2% of D Numerical Aperture: NA=0.55 Trapping Efficiency : 3.1%</p>	 <p>Cladding Thickness²⁾: T=2%(To)+2%(Ti) =4% of D Numerical Aperture : NA=0.72 Trapping Efficiency : 5.4%</p>
Square Fiber (SQ)	 <p>Cladding Thickness : T=2% of S Numerical Aperture : NA=0.55 Trapping Efficiency : 4.2%</p>	Not available

1) In some cases, cladding thickness T is 3% of D. 2) In some cases, cladding thickness T is 6% of D. To and Ti are both 3% of D.

<Properties table of Y-11>

Critical Angle for total reflection in WLS fiber

- Refractive Index of core : 1.59
- Refractive index of cladding : 1.49

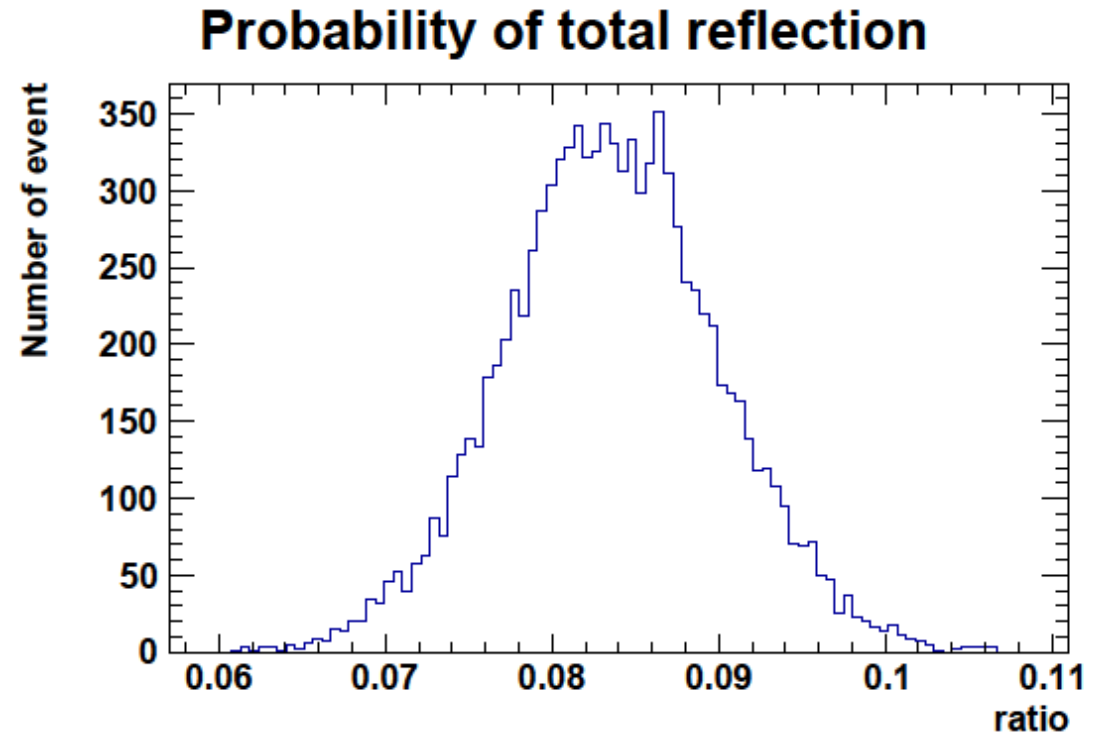
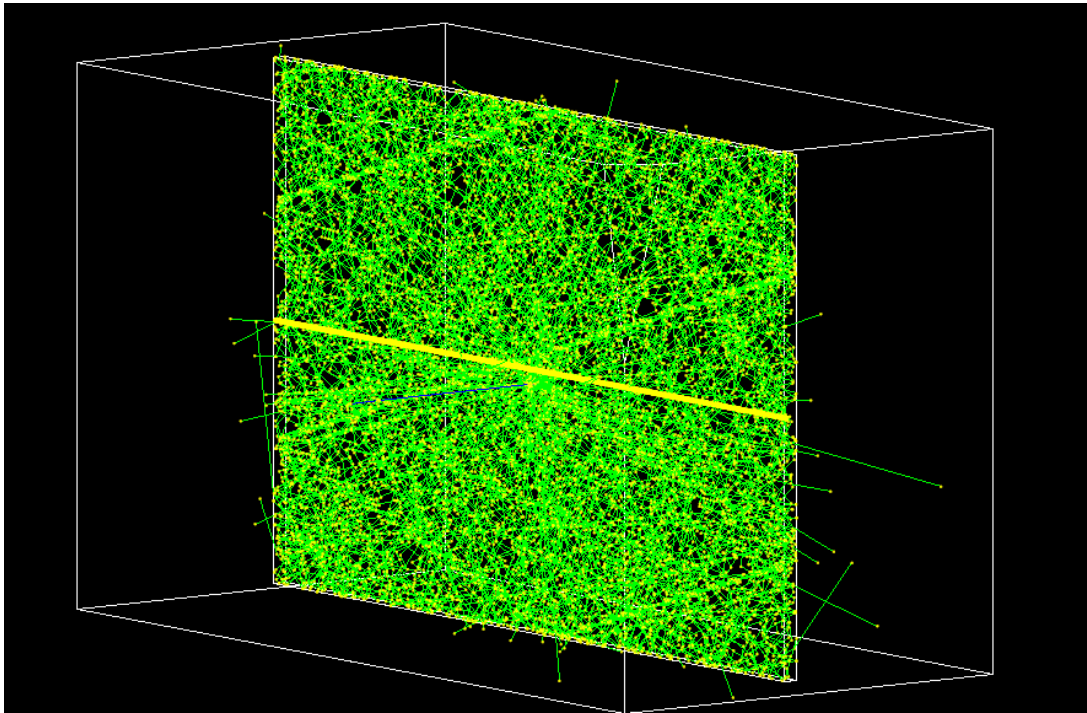
$$\text{critical angle } \theta_c = \sin^{-1}\left(\frac{1.49}{1.59}\right) = 69.57^\circ$$

Assume that there is no attenuation and emission occurs isotropically. Probability for total reflection is

$$P = \frac{2 \times \int_0^{2\pi} \int_0^{20^\circ} \sin\theta d\theta d\phi}{\int d\Omega} = \frac{2 \times 0.06}{2} = 6\%$$

<Calculated results of Trapping Efficiency>

Probability for total reflection



Ratio between the number of WLS process and the number of photon arrived at MPPC

Average of absorption ratio 0.084 is larger than our calculation 0.06.

New Calculation

Equation for total reflection

- As a results

$$\sin \theta = \frac{\cos \psi_c}{1 - \frac{a^2}{R^2} \sin^2 \varphi}$$

Probability for total reflection in (a, 0, 0)

- And by using that

$$P = \frac{1}{2} - \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 \psi_c - \left(\frac{a}{R}\right)^2 \sin^2 \phi}{1 - \left(\frac{a}{R}\right)^2 \sin^2 \phi}} d\phi$$

(this result is for one direction)

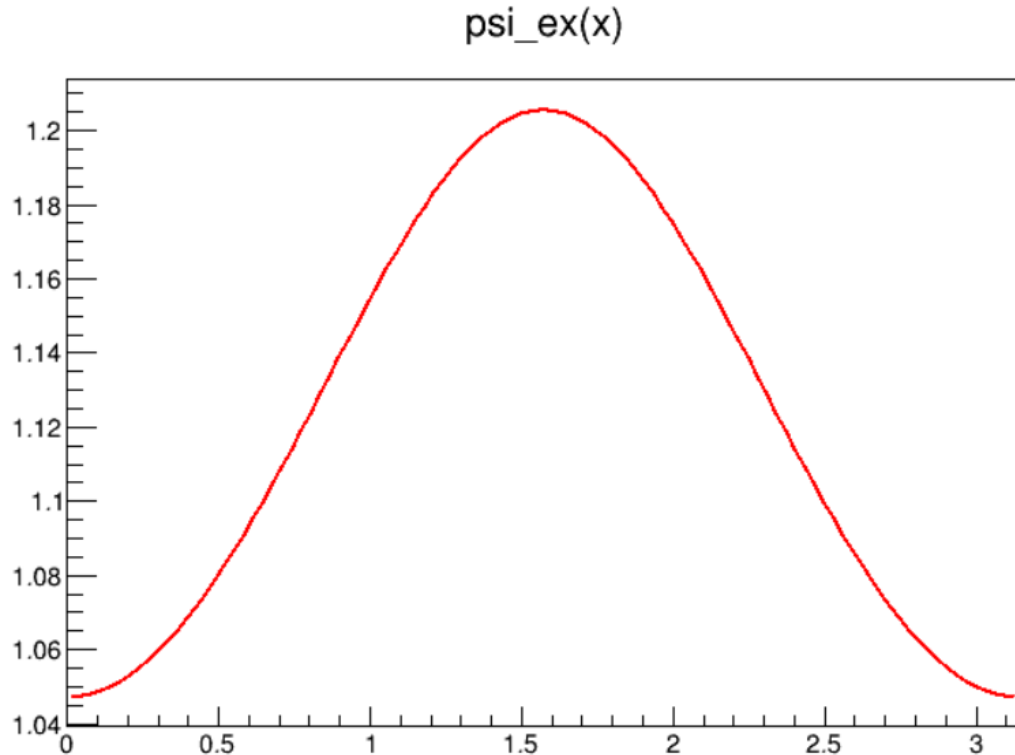
Value of a	Value of $P = \frac{1}{2} - \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 \psi_c - \left(\frac{a}{R}\right)^2 \sin^2 \phi}{1 - \left(\frac{a}{R}\right)^2 \sin^2 \phi}} d\phi$
0.1 R	0.03161
0.2 R	0.03212
0.3 R	0.03302
0.4 R	0.03443
0.5 R	0.03652
0.6 R	0.03959
0.7 R	0.04479
0.8 R	0.05414
0.9 R	0.07899

Range of φ

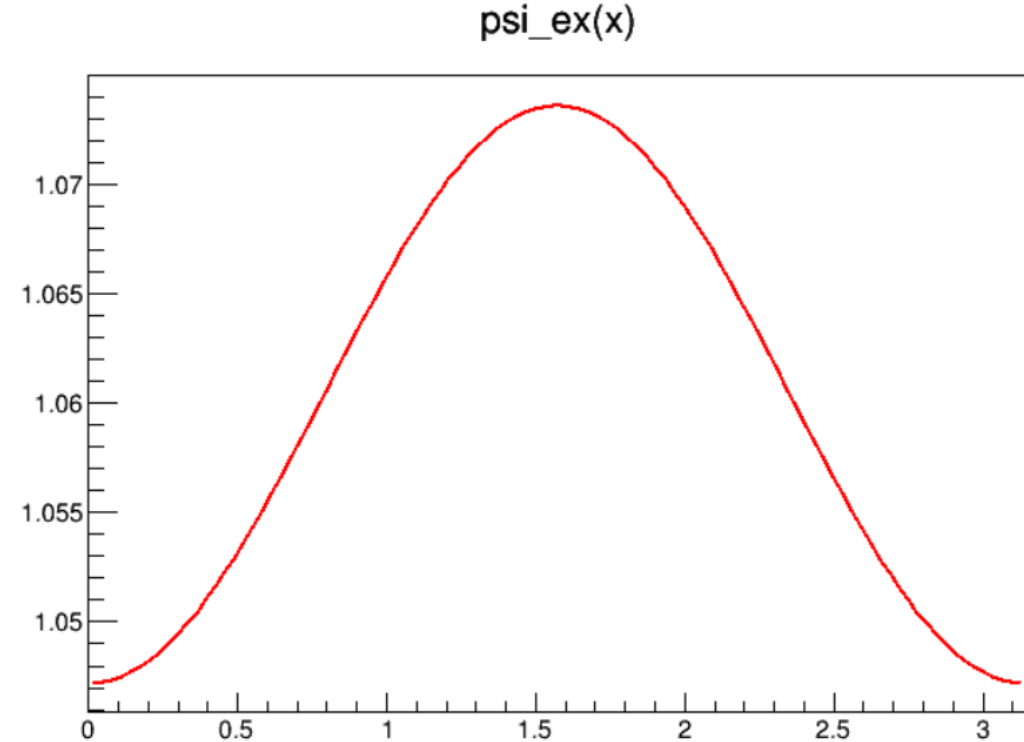
- When $a \rightarrow R$ and $\varphi \rightarrow \pi/2$, the value of integration goes to infinity.
- To avoid this, let's specify the range of φ , that is, setting upper bound of φ as a function of a .

$$-\frac{R}{a} \sin \psi_c \leq \sin \phi \leq \frac{R}{a} \sin \psi_c$$

Angle with normal vector of cylinder



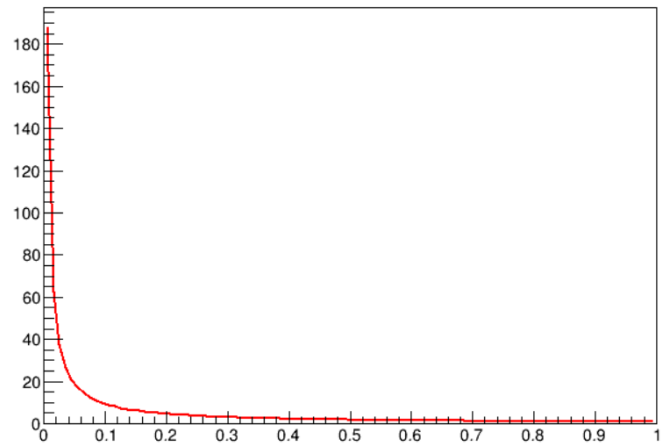
When $a = 0.7R$, $\theta = 30^\circ$



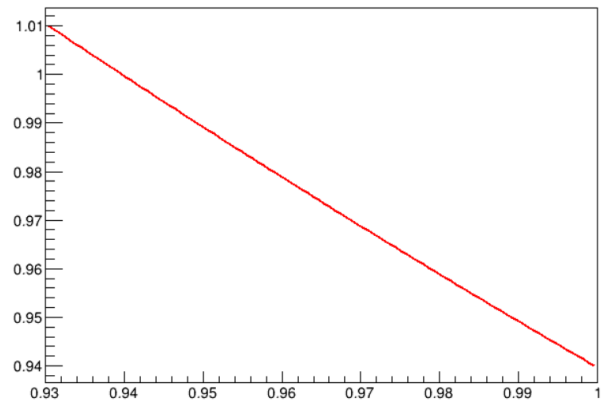
When $a = 0.4R$, $\theta = 30^\circ$

Even if the polar angle is the same, the value at the peak changes depending on the position a . Critical angle is about 1.2217.

phi(x)



phi(x)



Integration

- However, we cannot get results of indefinite integral.

Reflectivity

Fresnel's Equation

- Reflectivity can be calculated by Fresnel's equation.
- If direction of polarization is parallel to incident plane, transmittance and reflectivity is as follows

$$R = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \quad \text{and} \quad T = \alpha\beta \left(\frac{2}{\alpha + \beta}\right)^2$$
$$\left(\alpha = \frac{\cos \theta_t}{\cos \theta_i} \quad \text{and} \quad \beta = \frac{\mu_1 n_2}{\mu_2 n_1}\right)$$

Fresnel's Equation

- If direction of polarization is perpendicular to incident plane, transmittance and reflectivity is as follows

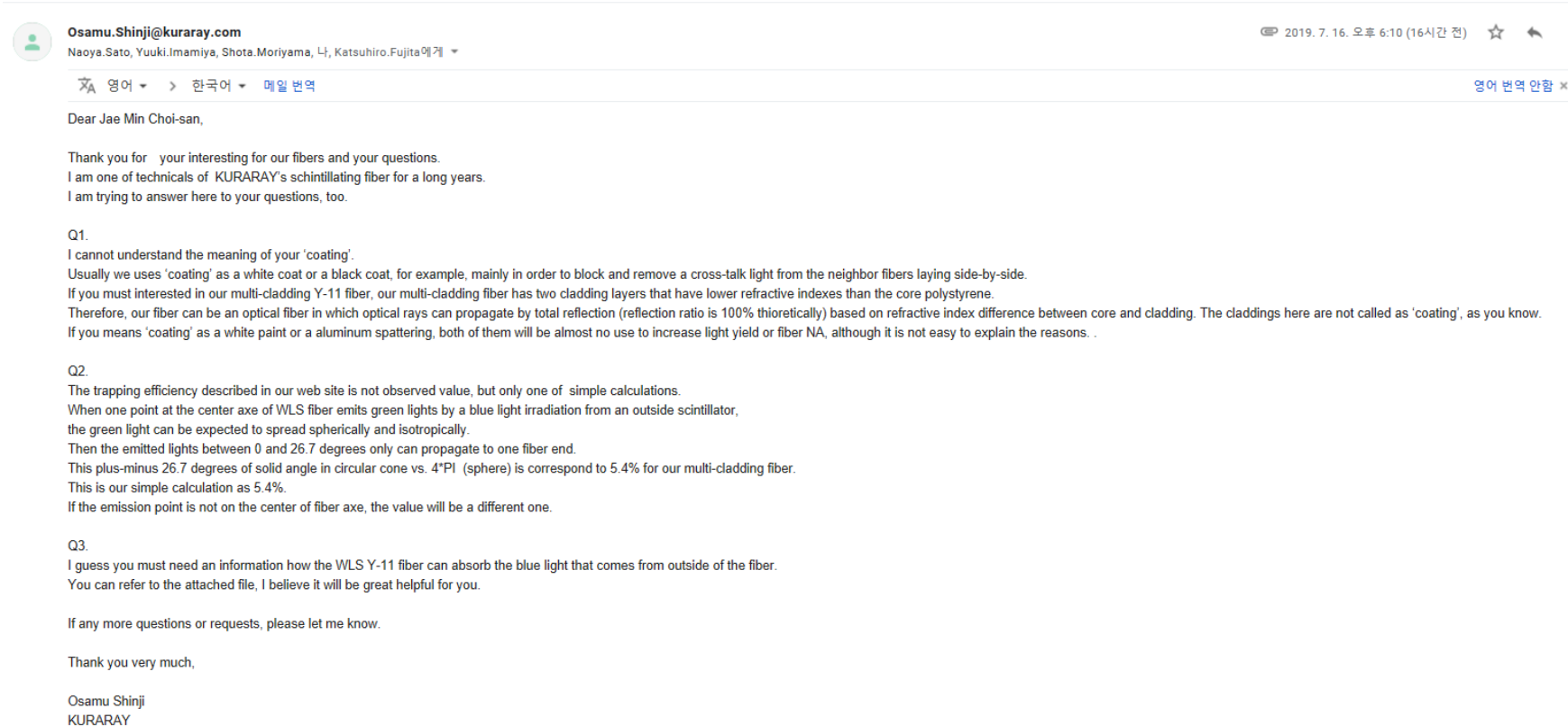
$$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 \quad \text{and} \quad T = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$
$$\left(\alpha = \frac{\cos \theta_t}{\cos \theta_i} \quad \text{and} \quad \beta = \frac{\mu_1 n_2}{\mu_2 n_1}\right)$$

- Anyway, when total internal reflection occurs, α goes to 0, so R is equal to 1.
- That is, reflectivity is equal to 1.

Fresnel's Equation

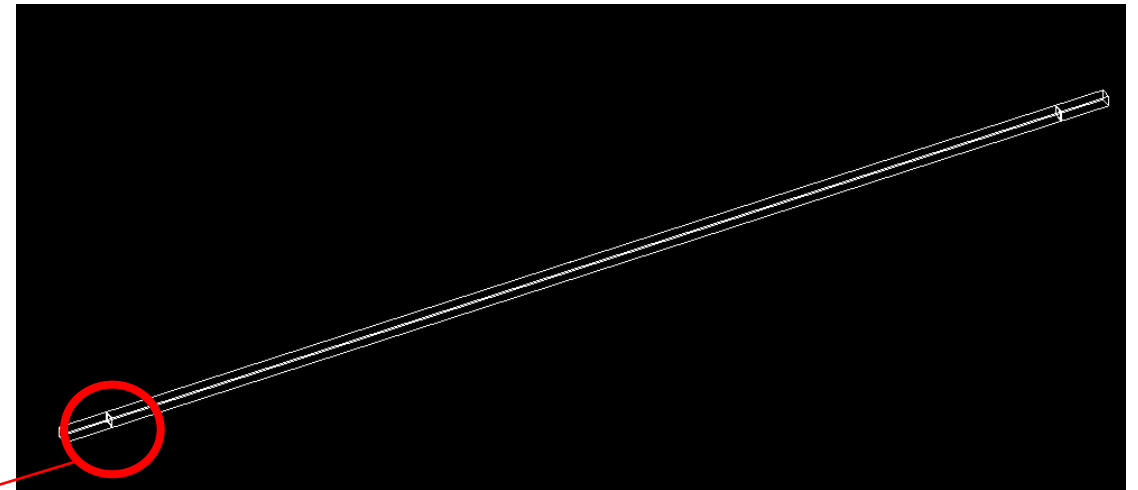
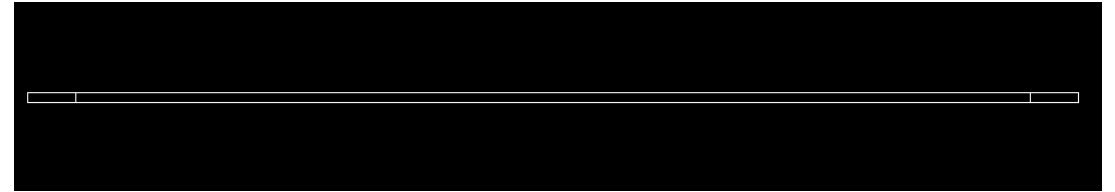
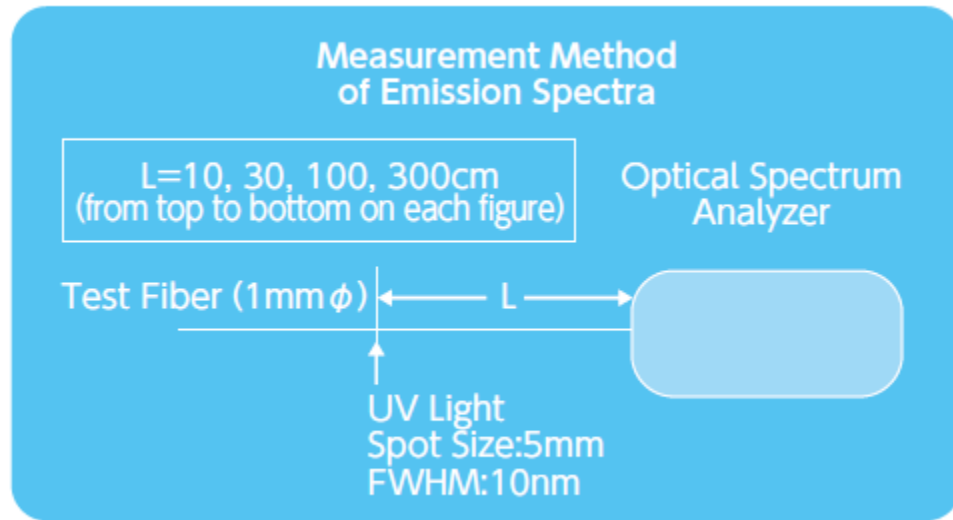
- By previous slide, we can check that Reflectivity is function of incident angle, refractive index of two medium.
- In Geant4, total reflection is realized as `G4OpBoundaryProcess`, and it means that we don't have to set refractivity.
- Reflectivity will be determined automatically.

Mail from Kuraray



- We can confirm this by mail from Kuraray.
- There is no white paint or aluminum spattering for increasing reflectivity.

Design of simulation

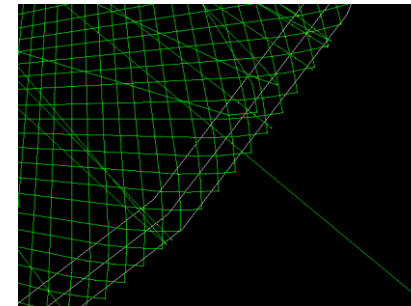
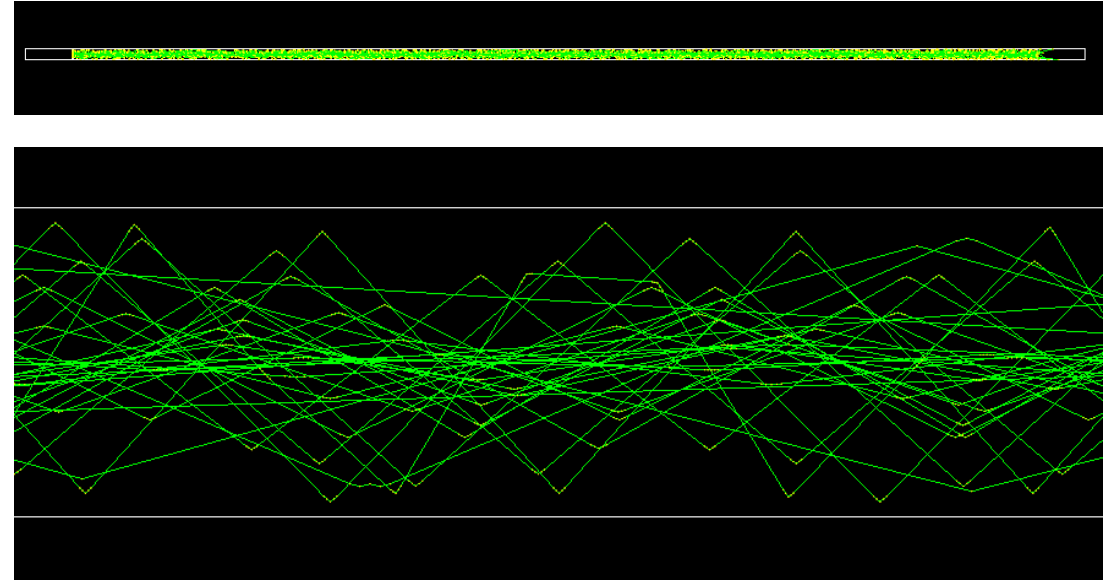
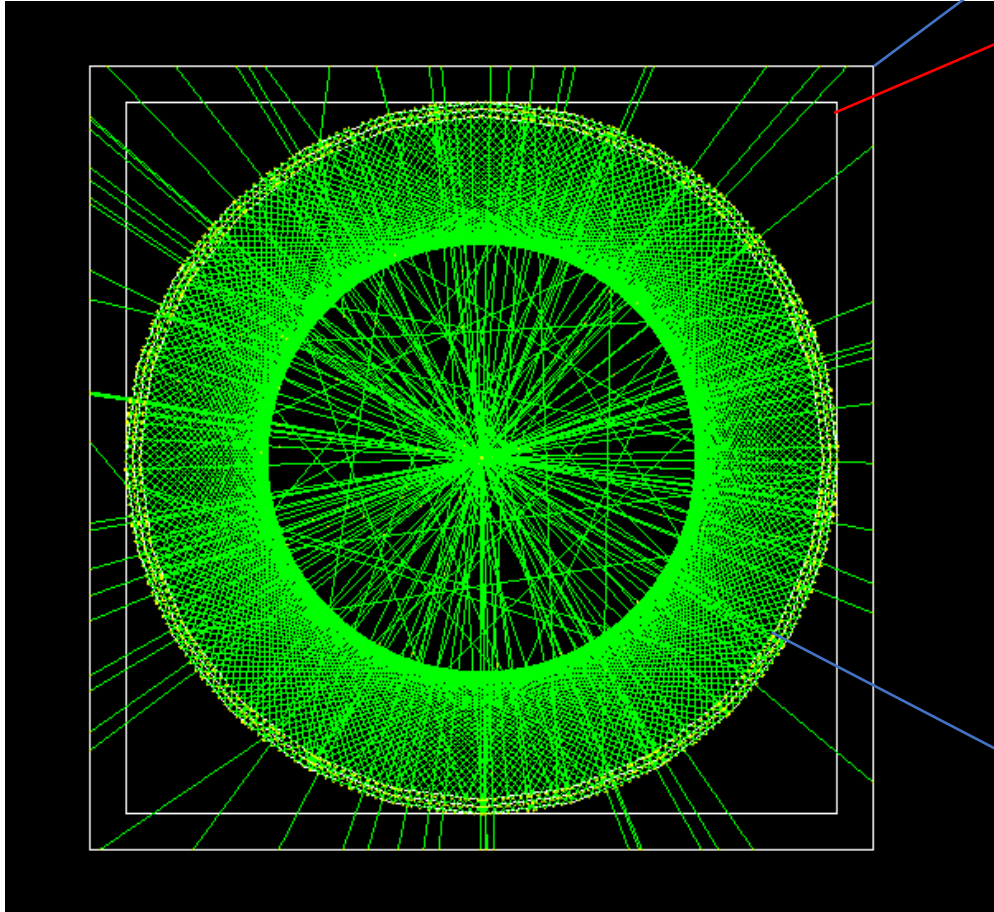


MPPC

Results of Simulation

WorldPV

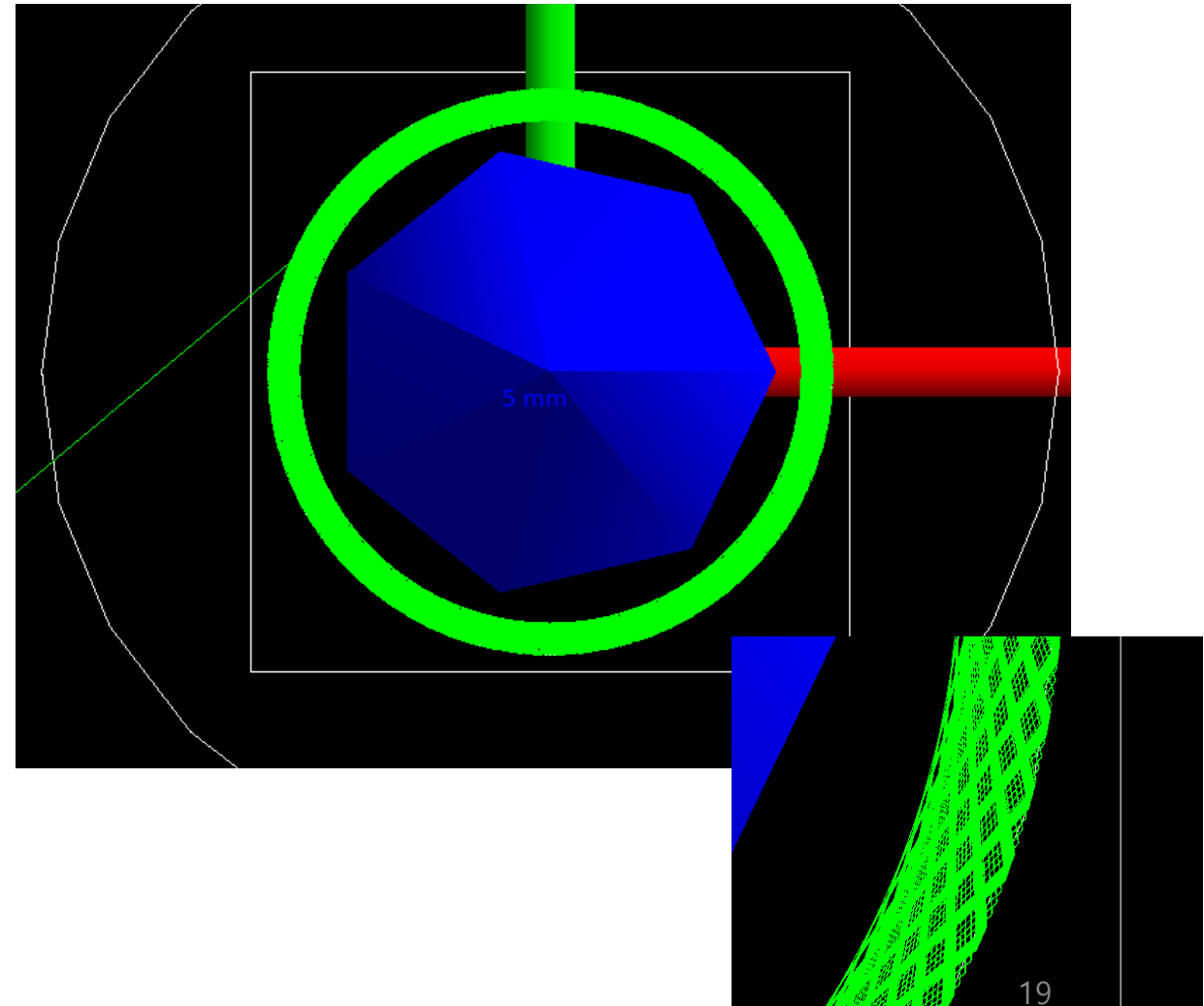
MPPC



From the side near the center of the circle, surface of core, surface of cladding1(PMMA), surface of cladding2(FP)

Visualization of example, WLS

- Compare the visualization results in previous slide, this is reasonable result.
- At least, total reflection work well.



Results of Simulation

- In visualization, photon looks like going out of fiber.
- However, by setting `/tracking/verbose 2`, we confirm that photon is reflected in WLS fiber.
- Some is between 1st and 2nd cladding and other is among core, 1st and 2nd cladding.
- Or going out of world.

```
*****
* G4Track Information: Particle = opticalphoton, Track ID = 2, Parent ID = 0
*****
Step# X(mm) Y(mm) Z(mm) KinE(MeV) dE(MeV) StepLeng TrackLeng NextVolume ProcName
0 0 0 -49.9 2.88e-06 0 0 0 WLS_fiber_corePV initStep
1 0 0 -35.8 2.88e-06 0 14.1 14.1 WLS_fiber_corePV OpWLS
:----- List of 2ndaries - #SpawnInStep= 1(Rest= 0,ALong= 0,Post= 1), #SpawnTotal= 1 -----
: 0 0 -35.8 2.41e-06 opticalphoton
:----- EndOf2ndaries Info -----
*****
* G4Track Information: Particle = opticalphoton, Track ID = 205, Parent ID = 2
*****
Step# X(mm) Y(mm) Z(mm) KinE(MeV) dE(MeV) StepLeng TrackLeng NextVolume ProcName
0 0 0 -35.8 2.41e-06 0 0 0 WLS_fiber_corePV initStep
1 0.187 -0.442 -36.5 2.41e-06 0 0.871 0.871 WLS_fiber_cladding1PV Transportation
2 0.191 -0.451 -36.6 2.41e-06 0 0.022 0.893 WLS_fiber_cladding2PV Transportation
3 0.195 -0.46 -36.6 2.41e-06 0 0.028 0.921 World Transportation
4 0.195 -0.46 -36.6 2.41e-06 0 0 0.921 WLS_fiber_cladding2PV Transportation
5 0.191 -0.451 -36.6 2.41e-06 0 0.028 0.949 WLS_fiber_cladding1PV Transportation
6 0.187 -0.442 -36.6 2.41e-06 0 0.022 0.971 WLS_fiber_corePV Transportation
7 -0.187 0.442 -38.1 2.41e-06 0 1.74 2.71 WLS_fiber_cladding1PV Transportation
8 -0.191 0.451 -38.1 2.41e-06 0 0.022 2.73 WLS_fiber_cladding2PV Transportation
9 -0.195 0.46 -38.1 2.41e-06 0 0.028 2.76 World Transportation
10 -0.195 0.46 -38.1 2.41e-06 0 0 2.76 WLS_fiber_cladding2PV Transportation
11 -0.191 0.451 -38.2 2.41e-06 0 0.028 2.79 WLS_fiber_cladding1PV Transportation
12 -0.187 0.442 -38.2 2.41e-06 0 0.022 2.81 WLS_fiber_corePV Transportation
13 0.187 -0.442 -39.6 2.41e-06 0 1.74 4.55 WLS_fiber_cladding1PV Transportation
14 0.191 -0.451 -39.7 2.41e-06 0 0.022 4.58 WLS_fiber_cladding2PV Transportation
15 0.195 -0.46 -39.7 2.41e-06 0 0.028 4.6 World Transportation
16 0.195 -0.46 -39.7 2.41e-06 0 0 4.6 WLS_fiber_cladding2PV Transportation
17 0.191 -0.451 -39.7 2.41e-06 0 0.028 4.63 WLS_fiber_cladding1PV Transportation
18 0.187 -0.442 -39.7 2.41e-06 0 0.022 4.65 WLS_fiber_corePV Transportation
19 -0.187 0.442 -41.2 2.41e-06 0 1.74 6.39 WLS_fiber_cladding1PV Transportation
20 -0.191 0.451 -41.2 2.41e-06 0 0.022 6.42 WLS_fiber_cladding2PV Transportation
21 -0.195 0.46 -41.2 2.41e-06 0 0.028 6.44 World Transportation
22 -0.195 0.46 -41.2 2.41e-06 0 0 6.44 WLS_fiber_cladding2PV Transportation
23 -0.191 0.451 -41.2 2.41e-06 0 0.028 6.47 WLS_fiber_cladding1PV Transportation
24 -0.187 0.442 -41.3 2.41e-06 0 0.022 6.49 WLS_fiber_corePV Transportation
25 0.187 -0.442 -42.7 2.41e-06 0 1.74 8.24 WLS_fiber_cladding1PV Transportation
26 0.191 -0.451 -42.7 2.41e-06 0 0.022 8.26 WLS_fiber_cladding2PV Transportation
27 0.195 -0.46 -42.8 2.41e-06 0 0.028 8.29 World Transportation
28 0.195 -0.46 -42.8 2.41e-06 0 0 8.29 WLS_fiber_cladding2PV Transportation
29 0.191 -0.451 -42.8 2.41e-06 0 0.028 8.31 WLS_fiber_cladding1PV Transportation
30 0.187 -0.442 -42.8 2.41e-06 0 0.022 8.34 WLS_fiber_corePV Transportation
31 -0.187 0.442 -44.3 2.41e-06 0 1.74 10.1 WLS_fiber_cladding1PV Transportation
32 -0.191 0.451 -44.3 2.41e-06 0 0.022 10.1 WLS_fiber_cladding2PV Transportation
33 -0.195 0.46 -44.3 2.41e-06 0 0.028 10.1 World Transportation
34 -0.195 0.46 -44.3 2.41e-06 0 0 10.1 WLS_fiber_cladding2PV Transportation
35 -0.191 0.451 -44.3 2.41e-06 0 0.028 10.2 WLS_fiber_cladding1PV Transportation
36 -0.187 0.442 -44.4 2.41e-06 0 0.022 10.2 WLS_fiber_corePV Transportation
37 0.187 -0.442 -45.8 2.41e-06 0 1.74 11.9 WLS_fiber_cladding1PV Transportation
38 0.187 -0.442 -45.8 2.41e-06 0 0 11.9 WLS_fiber_corePV Transportation
39 -0.187 0.442 -47.3 2.41e-06 0 1.74 13.7 WLS_fiber_cladding1PV Transportation
40 -0.191 0.451 -47.3 2.41e-06 0 0.022 13.7 WLS_fiber_cladding2PV Transportation
41 -0.195 0.46 -47.3 2.41e-06 0 0.028 13.7 World Transportation
42 -0.195 0.46 -47.3 2.41e-06 0 0 13.7 WLS_fiber_cladding2PV Transportation
43 -0.191 0.451 -47.3 2.41e-06 0 0.028 13.7 WLS_fiber_cladding1PV Transportation
44 -0.187 0.442 -47.4 2.41e-06 0 0.022 13.8 WLS_fiber_corePV Transportation
45 0.187 -0.442 -48.8 2.41e-06 0 1.74 15.5 WLS_fiber_cladding1PV Transportation
46 0.191 -0.451 -48.8 2.41e-06 0 0.022 15.5 WLS_fiber_cladding2PV Transportation
47 0.195 -0.46 -48.8 2.41e-06 0 0.028 15.6 World Transportation
48 0.195 -0.46 -48.8 2.41e-06 0 0 15.6 WLS_fiber_cladding2PV Transportation
49 0.191 -0.451 -48.9 2.41e-06 0 0.028 15.6 WLS_fiber_cladding1PV Transportation
50 0.187 -0.442 -48.9 2.41e-06 0 0.022 15.6 WLS_fiber_corePV Transportation
51 -0.0975 0.23 -50 2.41e-06 0 1.32 16.9 MPPC2PV Transportation
52 -0.111 0.261 -50 2.41e-06 0 0.0604 17 World Transportation
*****
```

Results of Simulation

```

*****
* G4Track Information: Particle = opticalphoton, Track ID = 26, Parent ID = 0
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -49.9  2.88e-06      0          0          0  WLS_fiber_corePV  initStep
1      0      0      -43.3  2.88e-06      0      6.59      6.59  WLS_fiber_corePV  OpWLS
:----- List of 2ndaries - #SpawnInStep= 1(Rest= 0,Along= 0,Post= 1), #SpawnTotal= 1 -----
:      0      0      -43.3  2.39e-06      0          0          0  opticalphoton
:----- EndOf2ndaries Info -----
*****
* G4Track Information: Particle = opticalphoton, Track ID = 180, Parent ID = 26
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -43.3  2.39e-06      0          0          0  WLS_fiber_corePV  initStep
1     -0.449  0.17   -43.9  2.39e-06      0      0.799      0.799  WLS_fiber_cladding1PV  Transportation
2     -0.458  0.174  -44   2.39e-06      0      0.0192     0.818  WLS_fiber_cladding2PV  Transportation
3     -0.468  0.177  -44   2.39e-06      0      0.0224     0.841  World  Transportation
4     -0.468  0.177  -44   2.39e-06      0          0      0.841  WLS_fiber_cladding2PV  Transportation
5     -0.458  0.174  -44   2.39e-06      0      0.0224     0.863  WLS_fiber_cladding1PV  Transportation
6     -0.449  0.17   -44   2.39e-06      0      0.0192     0.882  WLS_fiber_corePV  Transportation
7     0.449   -0.17   -45.3  2.39e-06      0          1.6      2.48  WLS_fiber_cladding1PV  Transportation
8     0.458   -0.174  -45.3  2.39e-06      0      0.0192     2.5   WLS_fiber_cladding2PV  Transportation
9     0.468   -0.177  -45.3  2.39e-06      0      0.0224     2.52  World  Transportation
10    0.468   -0.177  -45.3  2.39e-06      0          0      2.52  WLS_fiber_cladding2PV  Transportation
11    0.458   -0.174  -45.4  2.39e-06      0      0.0224     2.54  WLS_fiber_cladding1PV  Transportation
12    0.449   -0.17   -45.4  2.39e-06      0      0.0192     2.56  WLS_fiber_corePV  Transportation
13    -0.449  0.17   -46.7  2.39e-06      0          1.6      4.16  WLS_fiber_cladding1PV  Transportation
14    -0.458  0.174  -46.7  2.39e-06      0      0.0192     4.18  WLS_fiber_cladding2PV  Transportation
15    -0.468  0.177  -46.7  2.39e-06      0      0.0224     4.2   World  Transportation
16    -0.468  0.177  -46.7  2.39e-06      0          0      4.2   WLS_fiber_cladding2PV  Transportation
17    -0.458  0.174  -46.7  2.39e-06      0      0.0224     4.23  WLS_fiber_cladding1PV  Transportation
18    -0.449  0.17   -46.7  2.39e-06      0      0.0192     4.25  WLS_fiber_corePV  Transportation
19     0.449  -0.17   -48   2.39e-06      0          1.6      5.84  WLS_fiber_cladding1PV  Transportation
20     0.458  -0.174  -48   2.39e-06      0      0.0192     5.86  WLS_fiber_cladding2PV  Transportation
21     0.468  -0.177  -48   2.39e-06      0      0.0224     5.89  World  Transportation
22     0.468  -0.177  -48   2.39e-06      0          0      5.89  WLS_fiber_cladding2PV  Transportation
23     0.458  -0.174  -48.1  2.39e-06      0      0.0224     5.91  WLS_fiber_cladding1PV  Transportation
24     0.449  -0.17   -48.1  2.39e-06      0      0.0192     5.93  WLS_fiber_corePV  Transportation
25    -0.449  0.17   -49.4  2.39e-06      0          1.6      7.53  WLS_fiber_cladding1PV  Transportation
26    -0.449  0.17   -49.4  2.39e-06      0          0      7.53  WLS_fiber_corePV  Transportation
27    0.00652 -0.00247  -50   2.39e-06      0      0.811     8.34  MPPC2PV  Transportation
28    0.0427  -0.0162  -50   2.39e-06      0      0.0632     8.4   World  Transportation

```

- Reflected among core, 1st and 2nd cladding

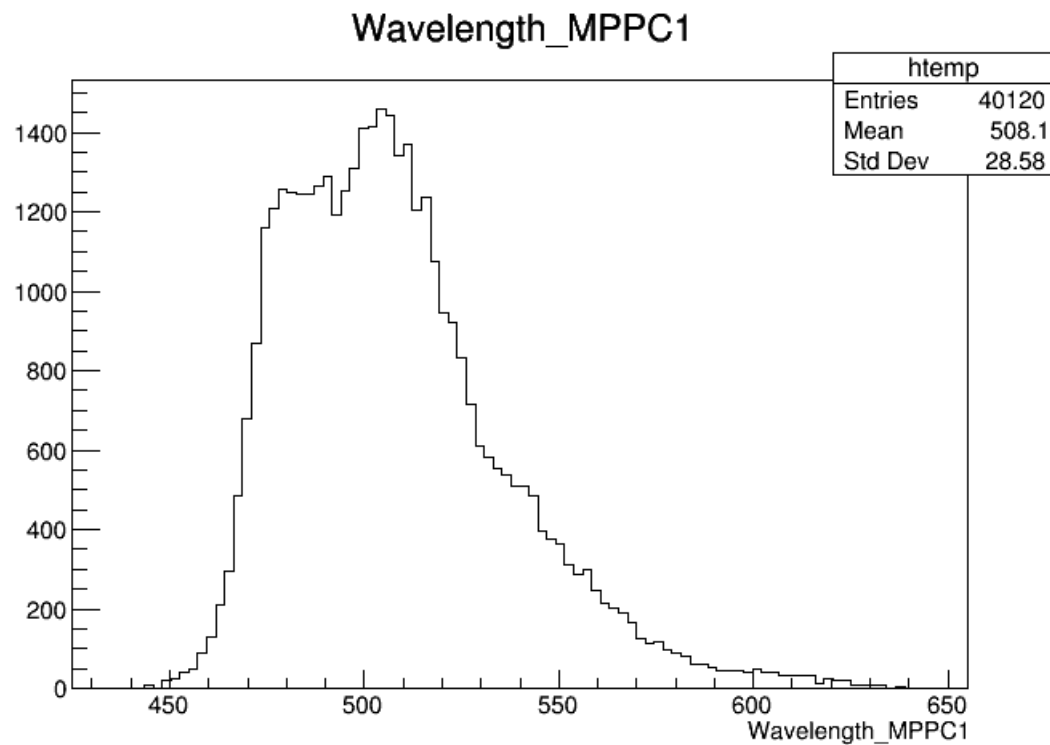
```

*****
* G4Track Information: Particle = opticalphoton, Track ID = 28, Parent ID = 0
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -49.9  2.88e-06      0          0          0  WLS_fiber_corePV  initStep
1      0      0      -43.8  2.88e-06      0      6.13      6.13  WLS_fiber_corePV  OpWLS
:----- List of 2ndaries - #SpawnInStep= 1(Rest= 0,Along= 0,Post= 1), #SpawnTotal= 1 -----
:      0      0      -43.8  2.61e-06      0          0          0  opticalphoton
:----- EndOf2ndaries Info -----
*****
* G4Track Information: Particle = opticalphoton, Track ID = 178, Parent ID = 28
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -43.8  2.61e-06      0          0          0  WLS_fiber_corePV  initStep
1     -0.422  -0.229  -43.8  2.61e-06      0      0.481     0.481  WLS_fiber_cladding1PV  Transportation
2     -0.431  -0.234  -43.8  2.61e-06      0      0.01     0.491  WLS_fiber_cladding2PV  Transportation
3     -0.44  -0.238  -43.8  2.61e-06      0      0.01     0.501  World  Transportation
4     -0.55  -0.298  -43.8  2.61e-06      0      0.126     0.627  OutOfWorld  Transportation
*****
* G4Track Information: Particle = opticalphoton, Track ID = 27, Parent ID = 0
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -49.9  2.88e-06      0          0          0  WLS_fiber_corePV  initStep
1      0      0      -43.3  2.88e-06      0      6.62      6.62  WLS_fiber_corePV  OpWLS
:----- List of 2ndaries - #SpawnInStep= 1(Rest= 0,Along= 0,Post= 1), #SpawnTotal= 1 -----
:      0      0      -43.3  2.56e-06      0          0          0  opticalphoton
:----- EndOf2ndaries Info -----
*****
* G4Track Information: Particle = opticalphoton, Track ID = 179, Parent ID = 27
*****
Step#  X(mm)  Y(mm)  Z(mm) KinE(MeV)  dE(MeV)  StepLeng  TrackLeng  NextVolume  ProcName
0      0      0      -43.3  2.56e-06      0          0          0  WLS_fiber_corePV  initStep
1     0.163  0.451  -43.5  2.56e-06      0      0.541     0.541  WLS_fiber_cladding1PV  Transportation
2     0.167  0.461  -43.5  2.56e-06      0      0.0115     0.553  WLS_fiber_cladding2PV  Transportation
3     0.17  0.47  -43.5  2.56e-06      0      0.0117     0.564  World  Transportation
4     0.199  0.55  -43.6  2.56e-06      0      0.125     0.689  OutOfWorld  Transportation

```

- Going out of World

Result of Simulation



- However, trapping efficiency obtained by simulation, is far larger than we expected.
- Also, the shape of graph differ from we expected.
- There be more problem.

Absorption Length

Experiments for absorption length

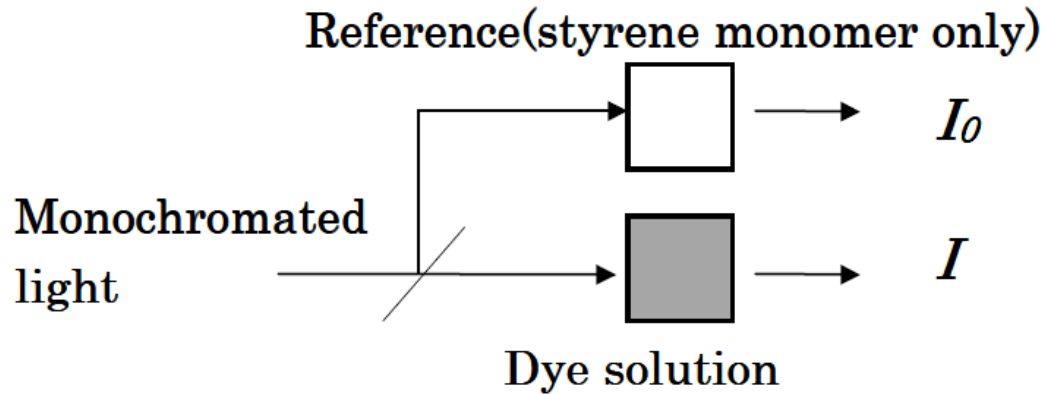
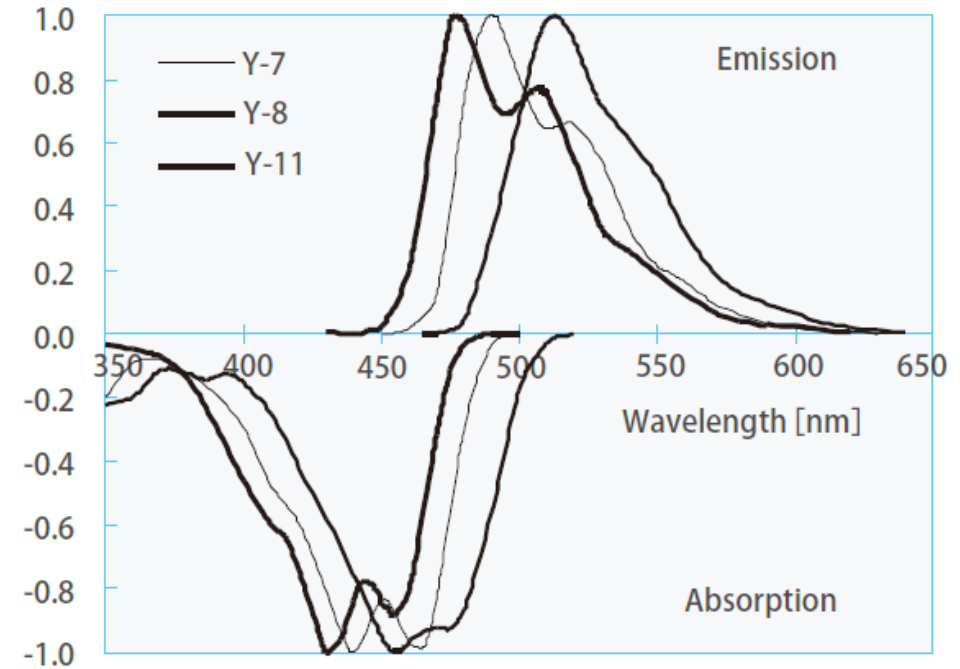


Fig.1 Absorbance measurement
by $d=10\text{mm}$ path cell

Y-7, Y-8, Y-11



Absorbance

- Mathematically, probability of finding a particle at depth x into the material is calculated by Beer-Lambert Law

$$P(x) = e^{-x/\lambda}$$

- And λ is attenuation (absorption) length, and it depends on material and energy.
- Definition of absorbance is as follows.

$$\text{ABS} = k(\lambda)Cd = \log_{10}\left\{\frac{I_0(\lambda)}{I(\lambda)}\right\} \quad \text{when } d = 10 \text{ mm}$$

- For reference, C is equal to 18.2 ppm and k_p (k at peak of absorption) is equal to 0.00638 in Y-11 of Kuraray

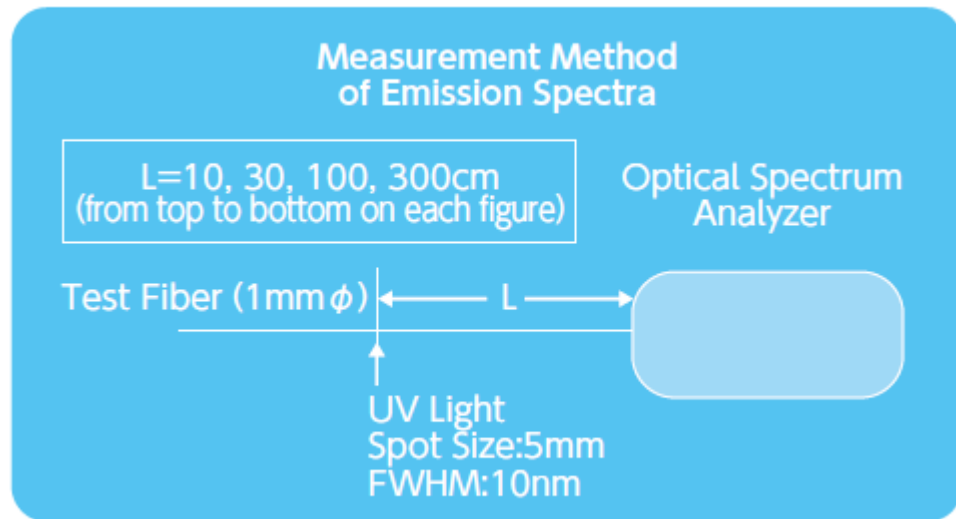
Changing parameter used in Geant4

- If assume that y-axis of absorption spectrum is k, absorption(attenuation) length of Y-11 is as follow.

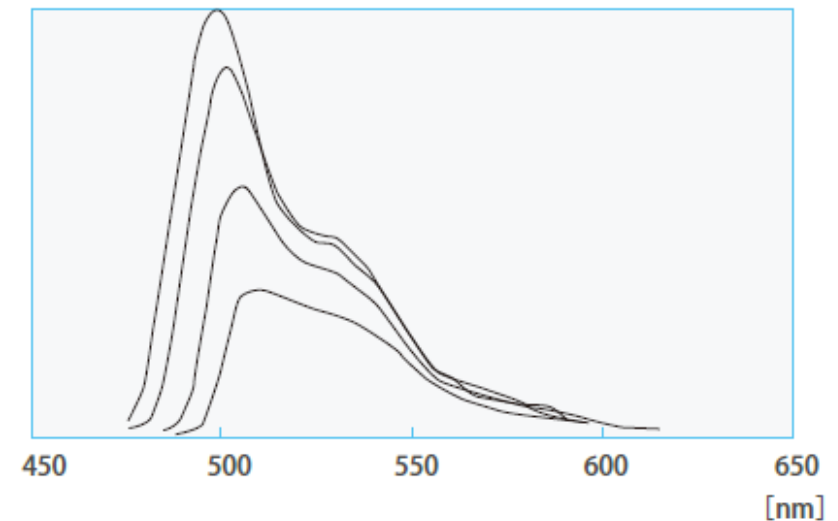
$$\lambda = \frac{1}{kC * \ln 10}$$

- C is concentration of dyne used in Y-11 and k is constant which is function of wavelength.

Comparison of simulation



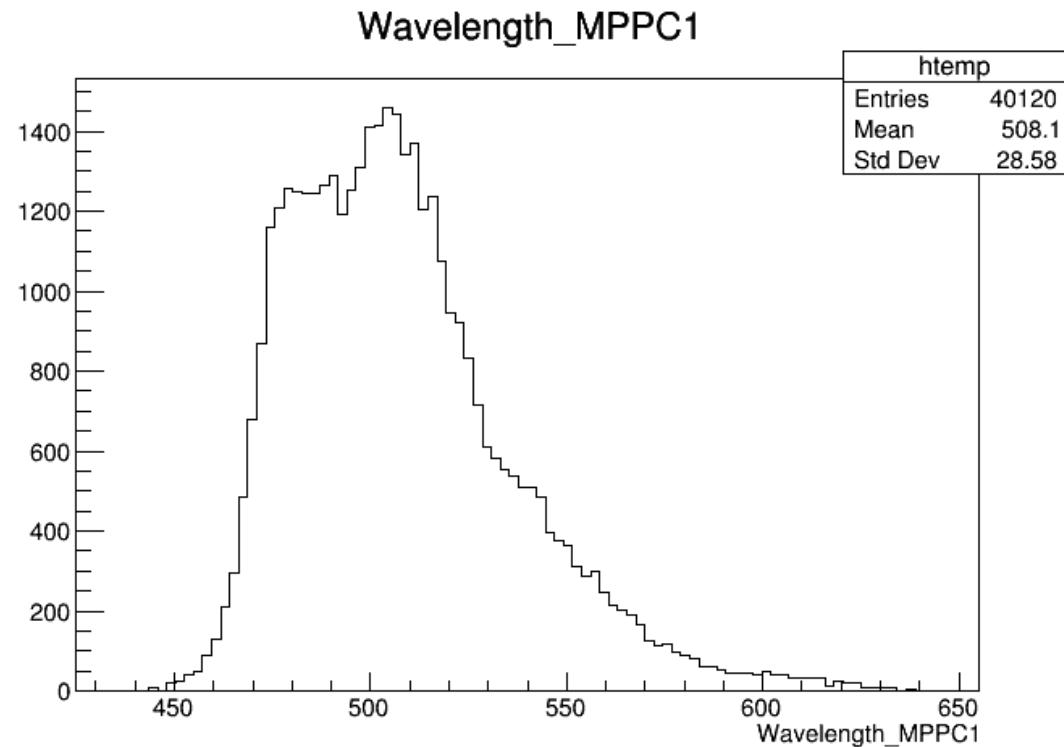
Y-11(200)



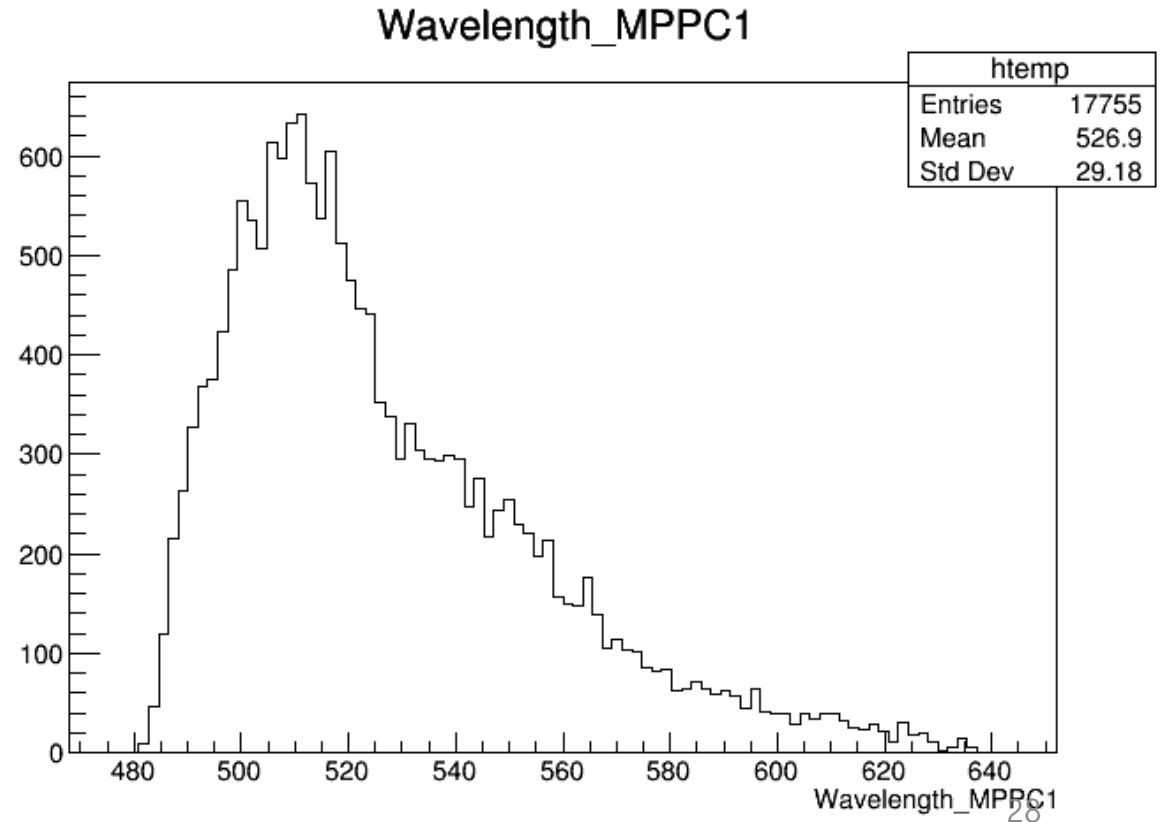
When photon of a wavelength of 430nm shoot in the end of fiber, the emission spectrum should be same as the right.

Comparison with reference

For 10 cm

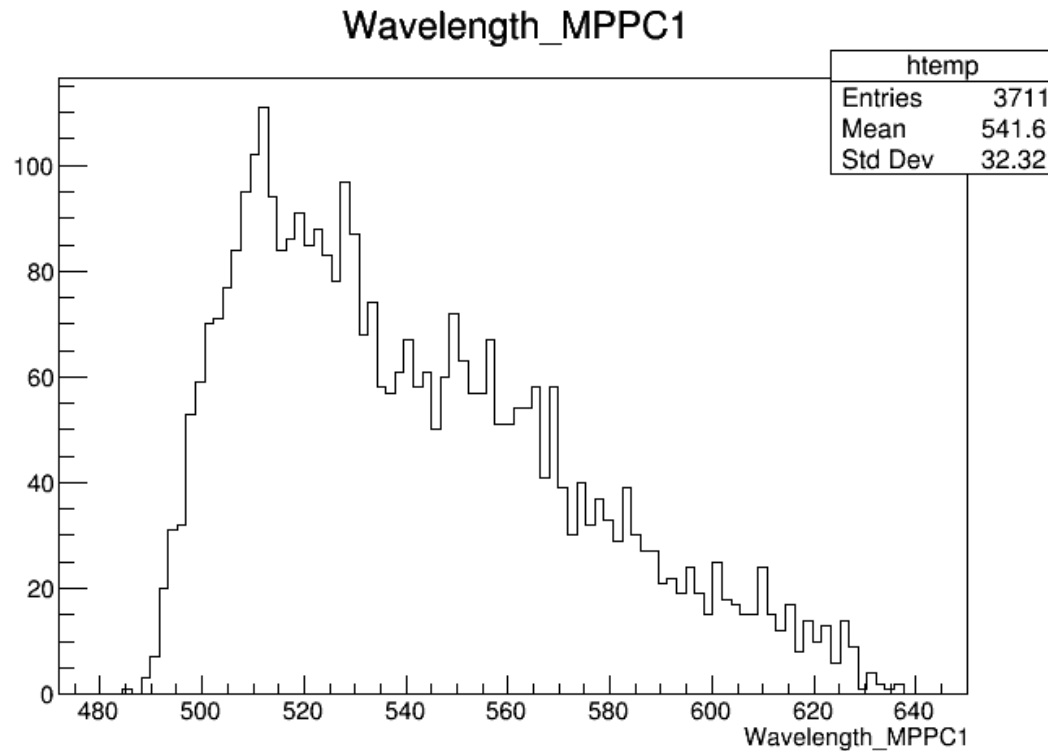


For 30 cm

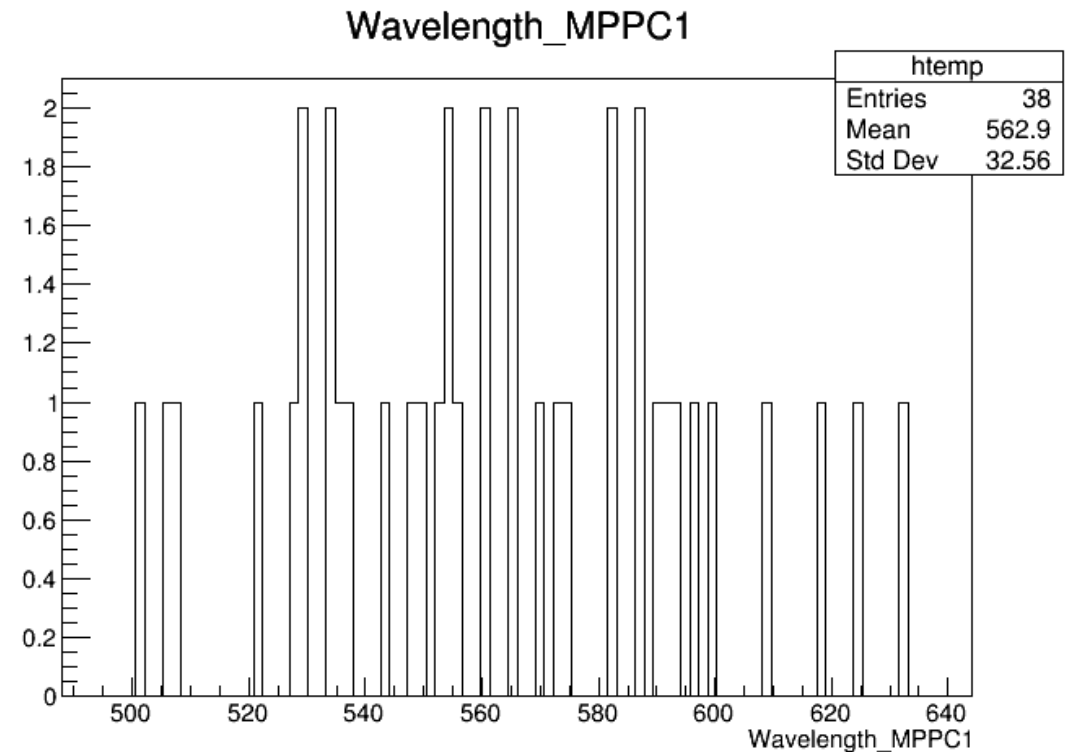


Comparison with reference

For 100 cm



For 300 cm



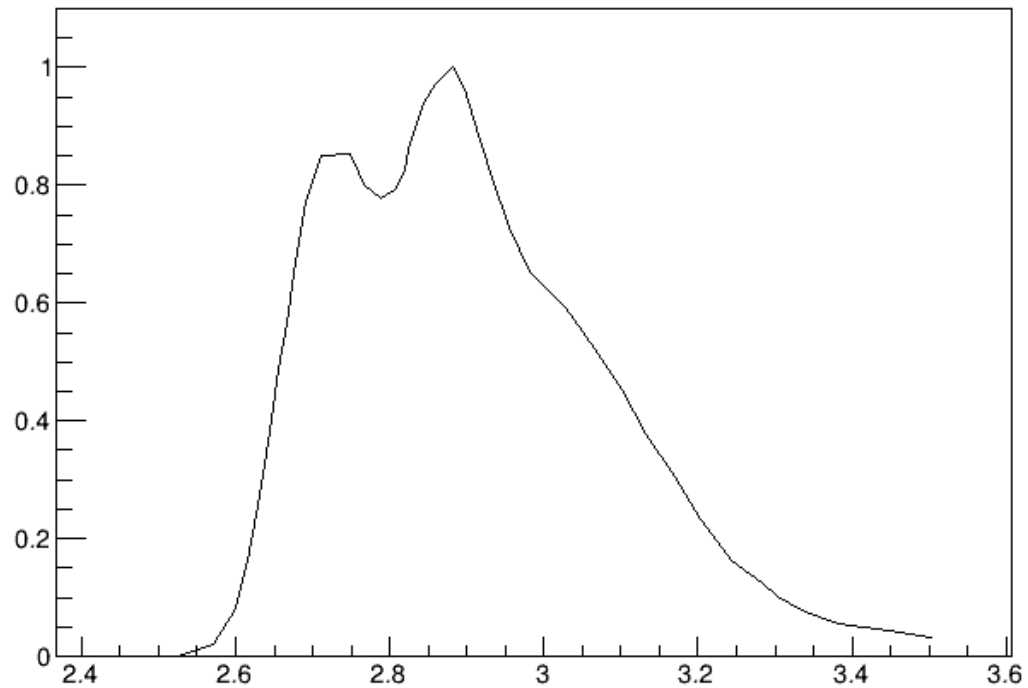
Problem

- However, the shape of graph is not exactly same with reference.
- Also, there is problem of trapping efficiency, that is, the value of probability is much higher than we expected, about 5 times.(in 10cm)
- According to mail from Kuraray, trapping efficiency is calculated only in axis. For this reason, we need to calculate trapping efficiency more detail.

Backup

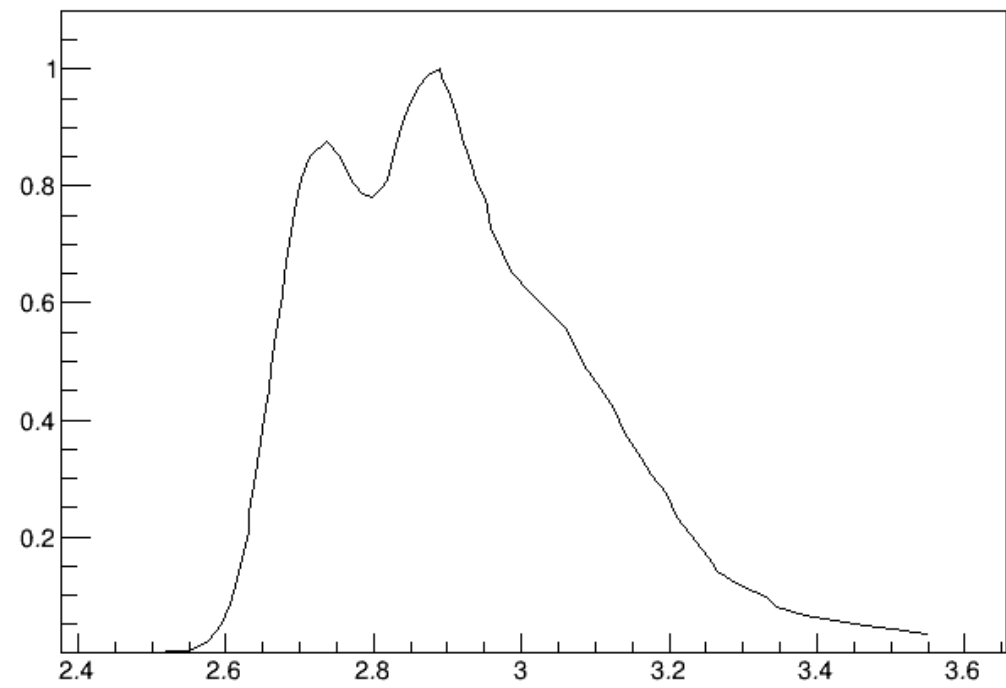
Absorption spectrum used before

Graph



Absorption spectrum used after

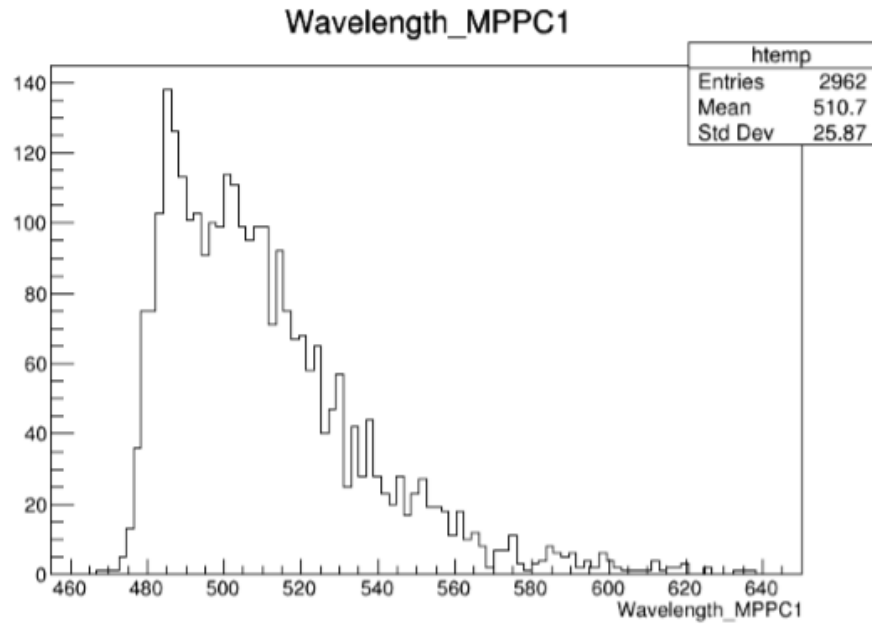
Graph



Backup

Before

For 30 cm



After

For 30 cm

