Equivalent circuit model of MPPC

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GEIGER-APD

R_S : Resistance of the entire APD during a discharge
R_Q :Quenching resistor
C_J : Junction capacitance

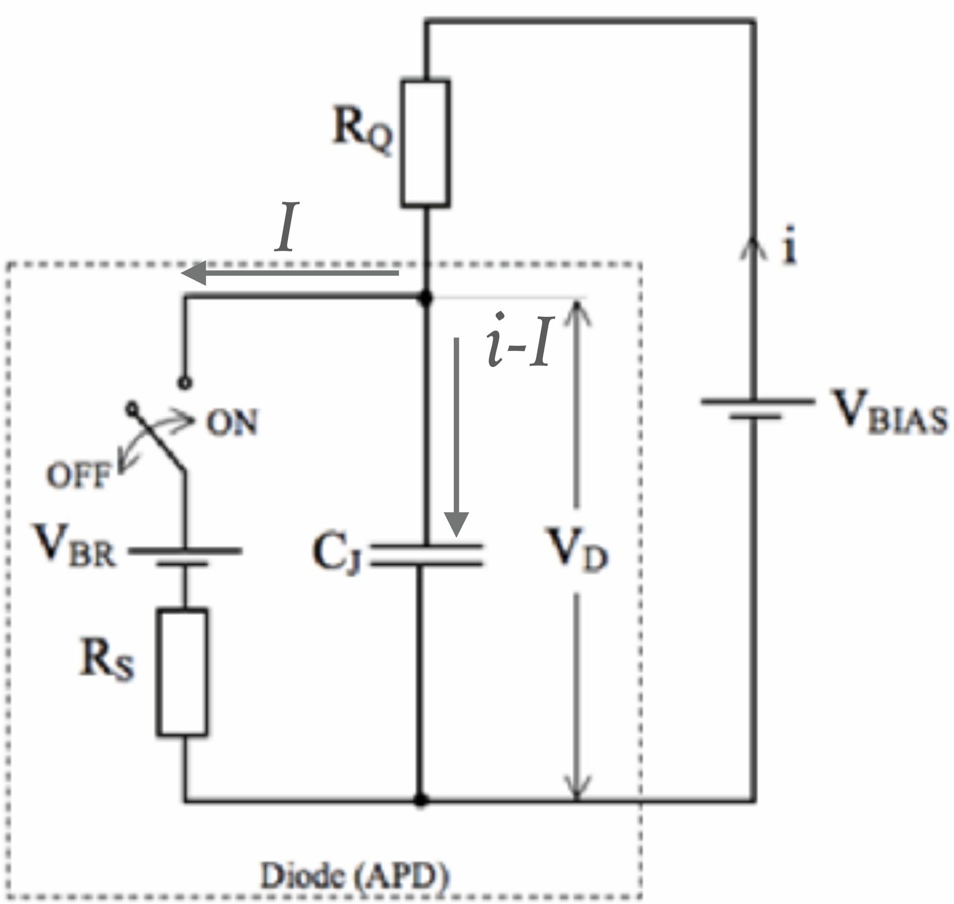
typical values
R_S ~1 k, R_Q~150 k, C_j~0.1 pf

By Kirchhof's current law

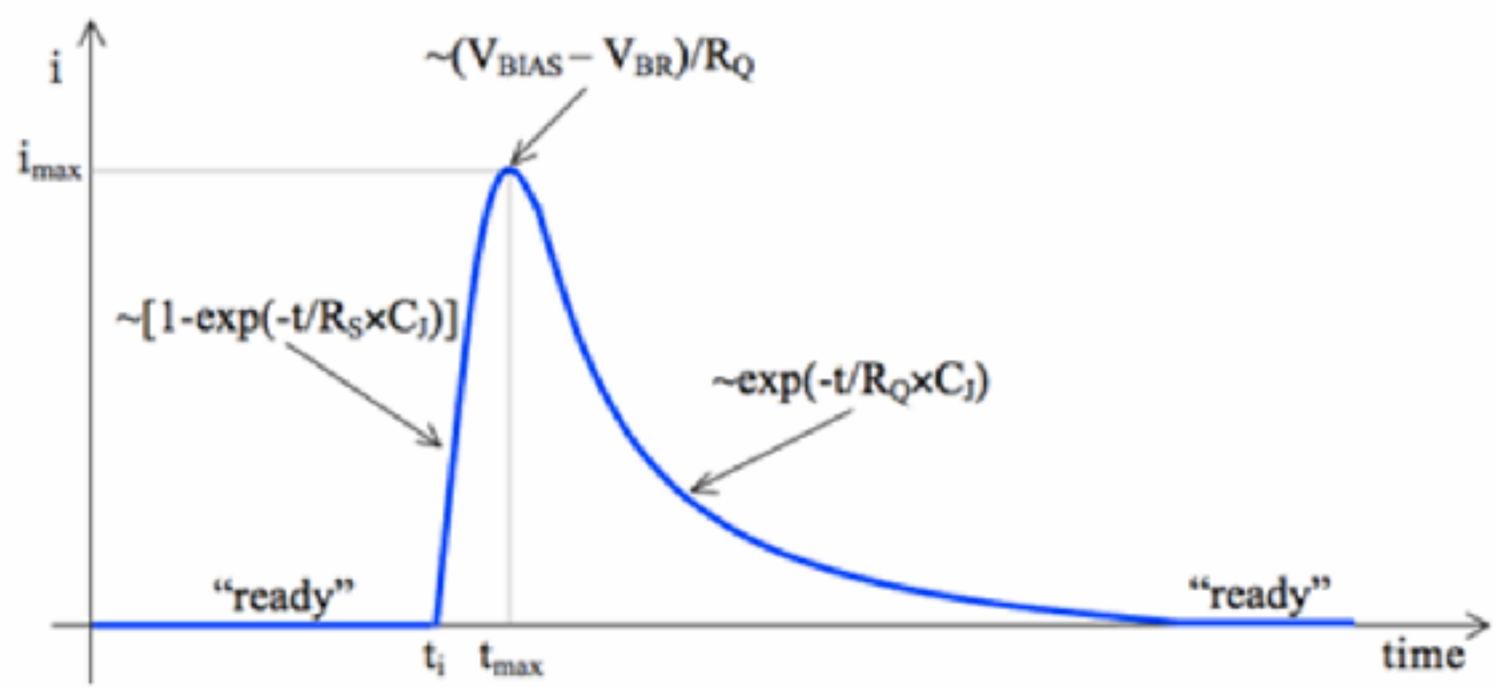
$$V_{BIAS}\text{-}Q/C_j\text{-}iR_Q=0$$
$$V_{BIAS}\text{-}V_{BR}\text{-}(i-I)R_S\text{-}iR_Q=0$$

$$\tau_r = C_j\frac{(R_S R_Q)}{(R_S+R_Q)} \sim C_j R_S \ (\because R_Q \gg R_S)$$
$$i = \frac{V_{BIAS}-V_{BR}}{R_S+R_Q}(1-e^{-t/\tau_r}) \sim \frac{V_{BIAS}-V_{BR}}{R_Q}(1-e^{-t/\tau_r})$$

$$\tau_r = C_j R_Q$$



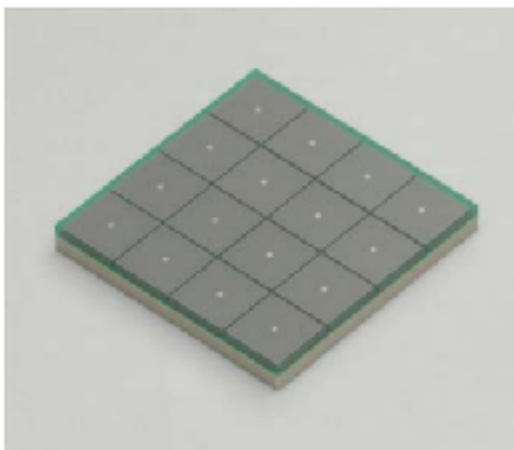
Equivalent circuit of MPPC's single GAPD

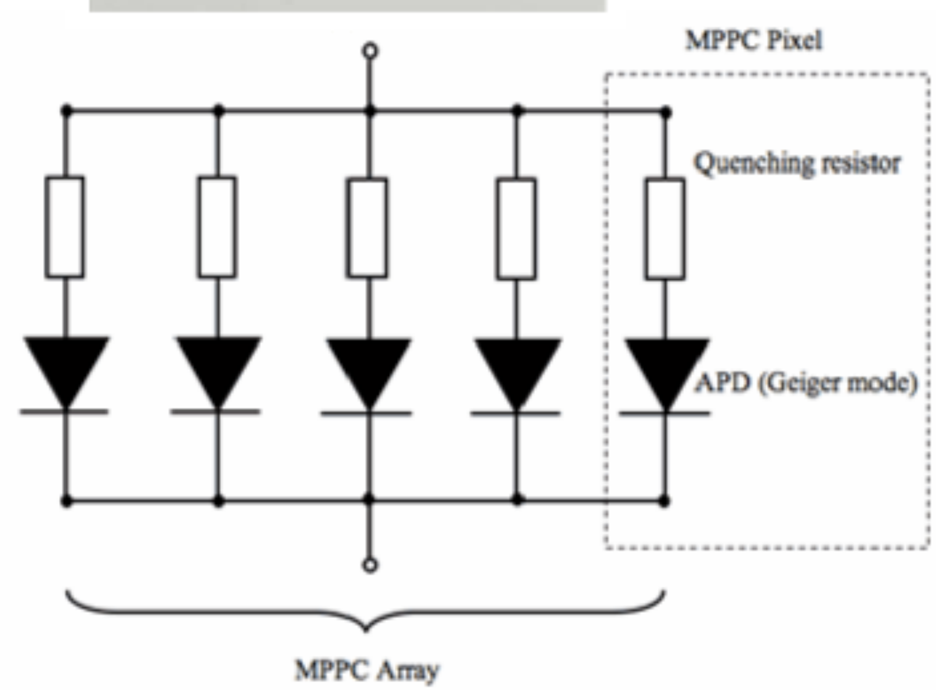


Current flowing through the APD as a function of time

STRUCTURE OF THE MPPC

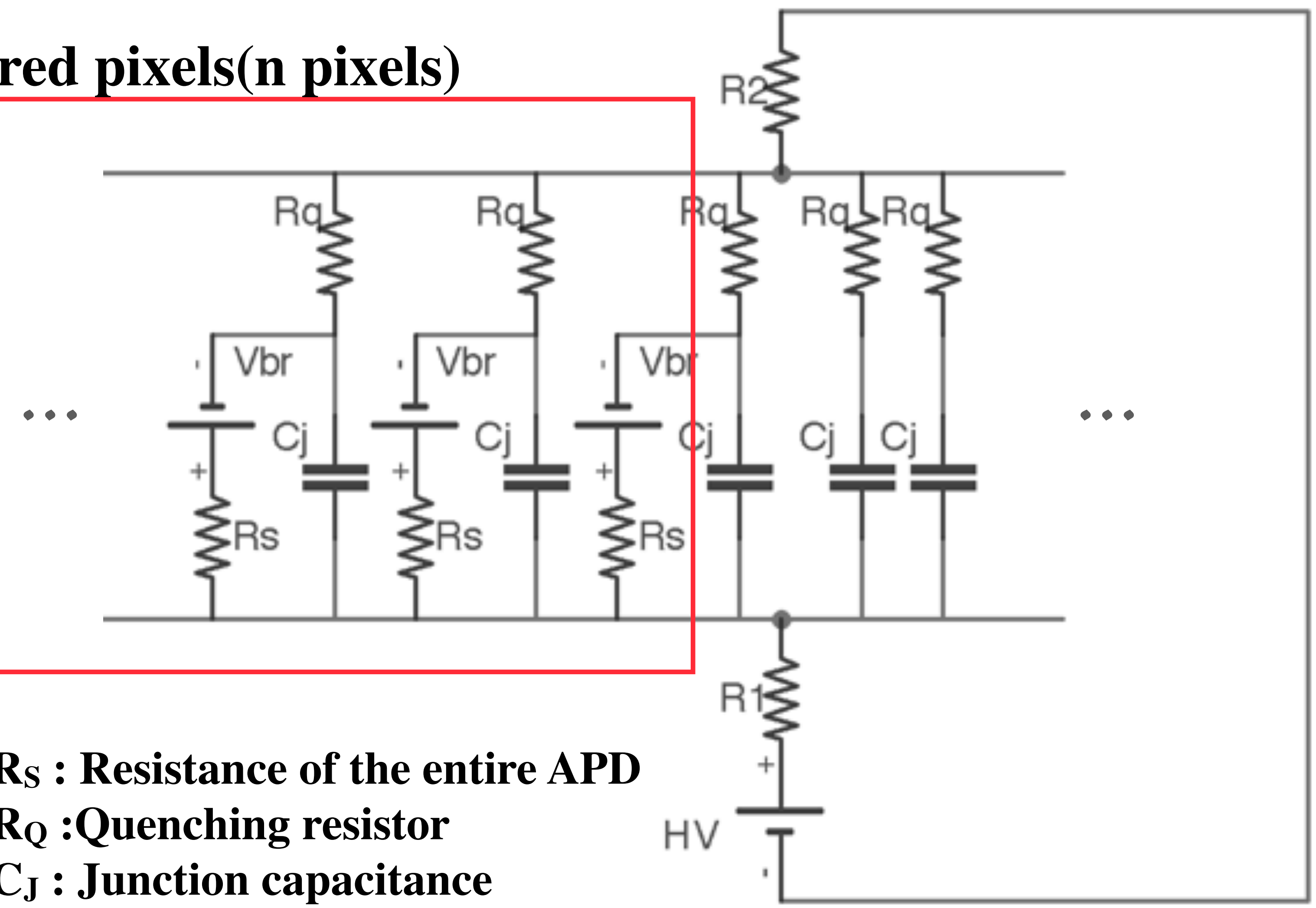
Structure of the MPPC





Equivalent circuit of the MPPC

fired pixels(n pixels)

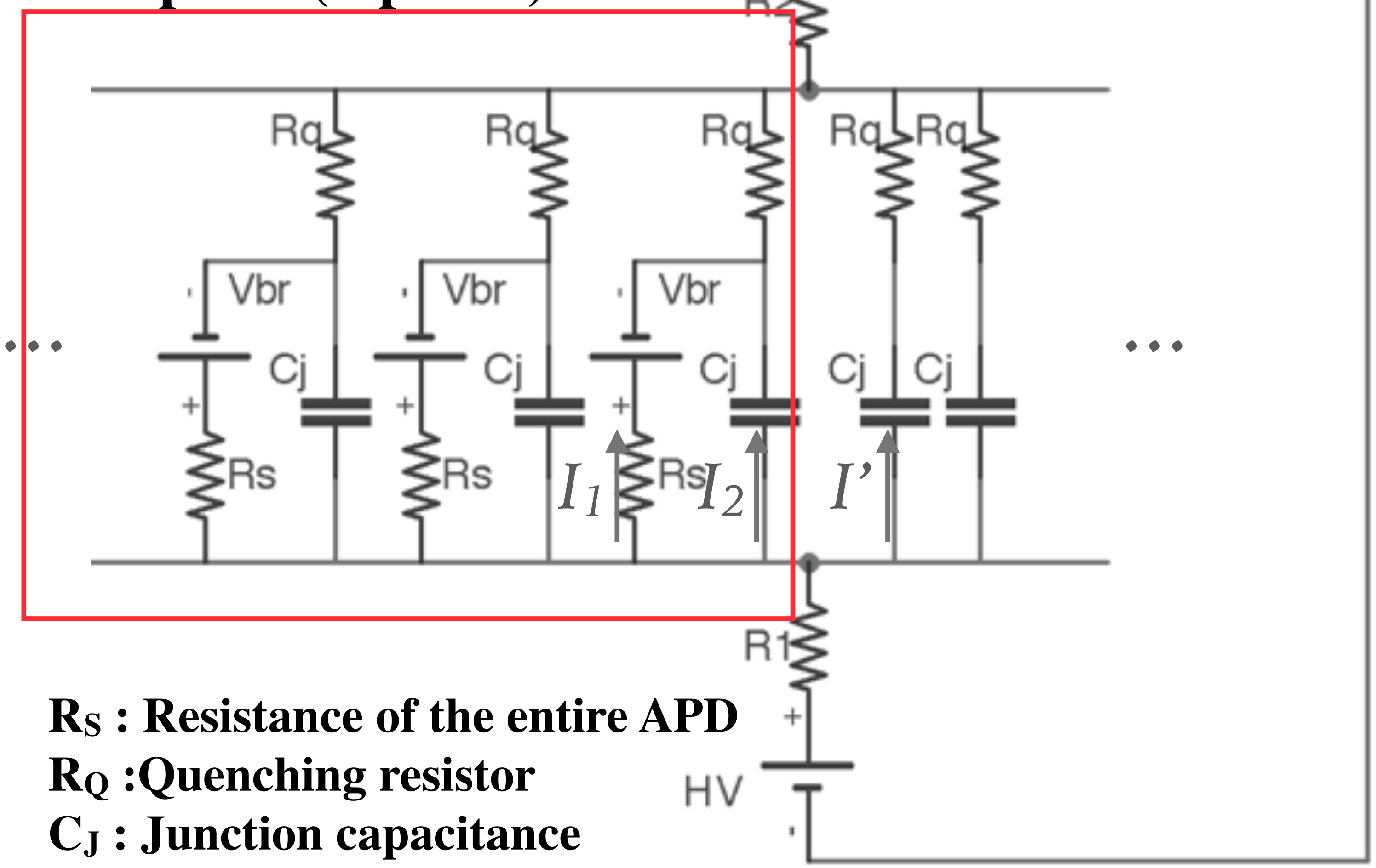


R_S : Resistance of the entire APD
R_Q :Quenching resistor
C_J : Junction capacitance

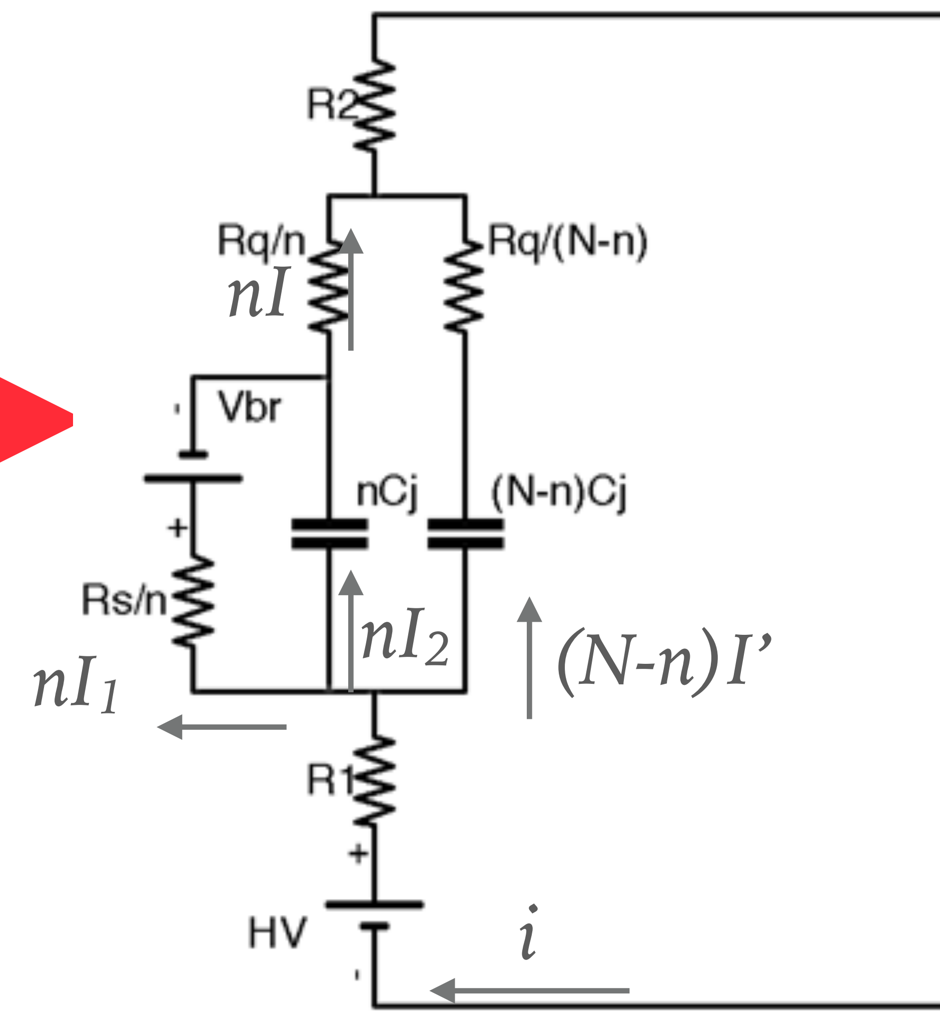
EQUIVALENT CIRCUIT OF THE MPPC

Equivalent circuit of the MPPC

fired pixels(n pixels)



N: total # of pixels of the MPPC
n: # of fired pixels of the MPPC

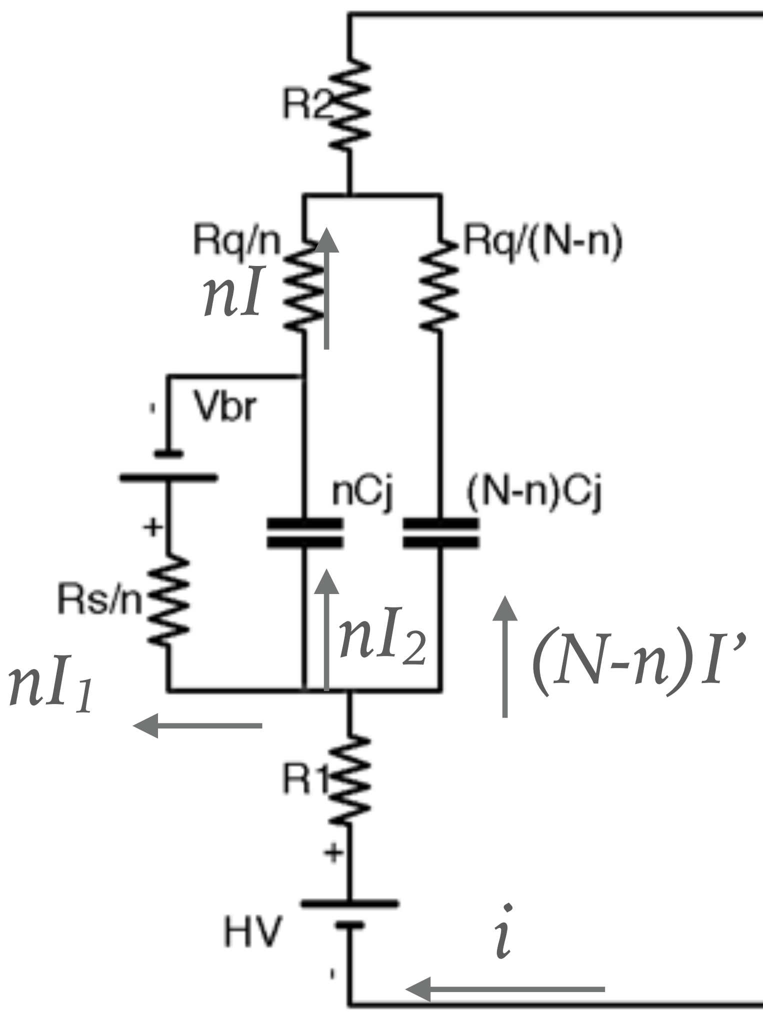


R_S : Resistance of the entire APD
R_Q :Quenching resistor
C_J : Junction capacitance

R1 and R2 : series resistors in circuit (Let R= R1+R2)

EQUIVALENT CIRCUIT OF THE MPPC

Equivalent circuit of MPPC



By Kirchhof's current law

i=nI+(N-n)I', $I = I_1 + I_2$

$$V_{BIAS} - V_{BR} - iR - I_1 R_S - I R_Q = 0$$

$$V_{BIAS} - iR - Q/C_j - I R_Q = 0$$

$$V_{BIAS} - iR - Q'/C_j - I' R_Q = 0$$

Leading edge

$$\tau_r = C_j R_S$$

$$i = \frac{n(V_{BIAS} - V_{BR})}{nR + R_S + R_Q}(1 - e^{-t/\tau_r}) \sim \frac{n(V_{BIAS} - V_{BR})}{R_Q}(1 - e^{-t/\tau_r})$$

$$I = \frac{n(V_{BIAS} - V_{BR})}{nR + R_S + R_Q} \sim \frac{n(V_{BIAS} - V_{BR})}{R_Q}$$

Trailing edge

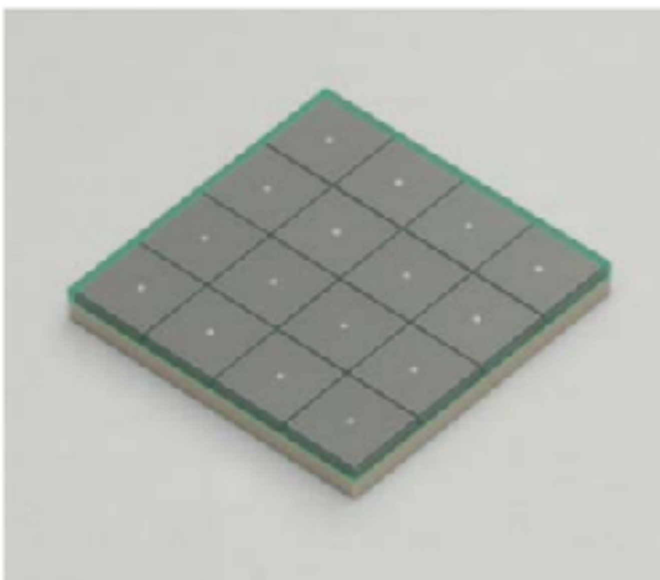
$$\tau_t = C_j(R_Q + NR)$$

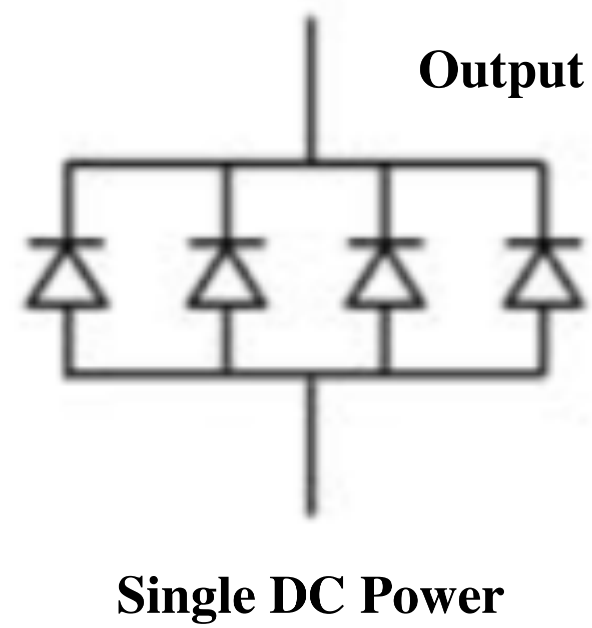
R_S : Resistance of the entire APD
R_Q :Quenching resistor
C_J : Junction capacitance

N: total # of pixels, n: # of fired pixels
R1 and R2 : series resistors in circuit (Let R= R1+R2)

PREVIOUS TEST CONDITION

MPPC has 16 channels

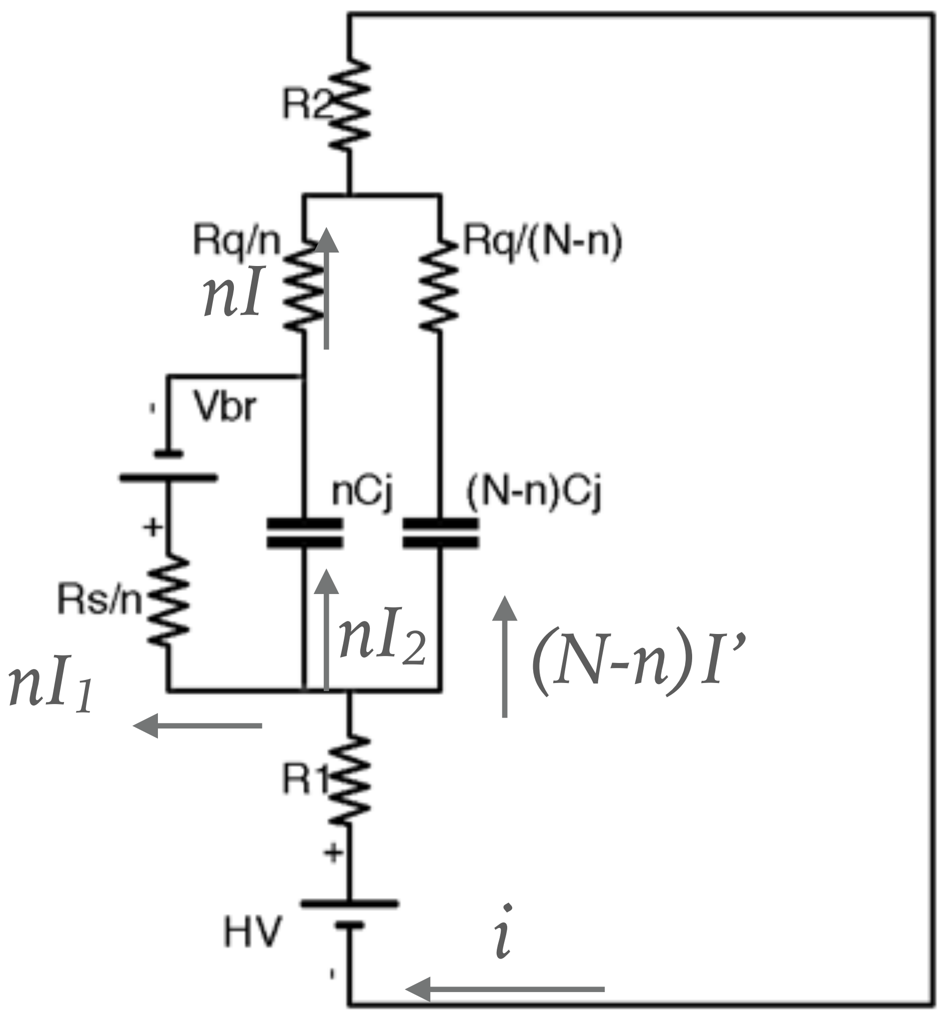




- **Sixteen Channels were connected in parallel**
- **Single DC power applied same voltages for all channels**
- **Current outputs of every channel's were gathered**

MPPCS IN PARALLEL CONNECTION

Equivalent circuit



Assume that number of 'a' MPPCs are connected in parallel
Only one thing is different from single MPPC, N->aN

For the leading edge, all parameters and values still remain same

Leading edge

$$\tau_r = C_j R_S$$

$$i = \frac{n(V_{BIAS}-V_{BR})}{nR+R_S+R_Q}(1-e^{-t/\tau_r}) \sim \frac{n(V_{BIAS}-V_{BR})}{R_Q}(1-e^{-t/\tau_r})$$

$$I = \frac{n(V_{BIAS}-V_{BR})}{nR+R_S+R_Q} \sim \frac{n(V_{BIAS}-V_{BR})}{R_Q}$$

Time constant of fall time has a term promotional to 'a'
So, tailing edge become longer

$$\tau_t = C_j R_Q + \underline{a \times C_j N R}$$

R_S : Resistance of the entire APD
R_Q :Quenching resistor
C_J : Junction capacitance

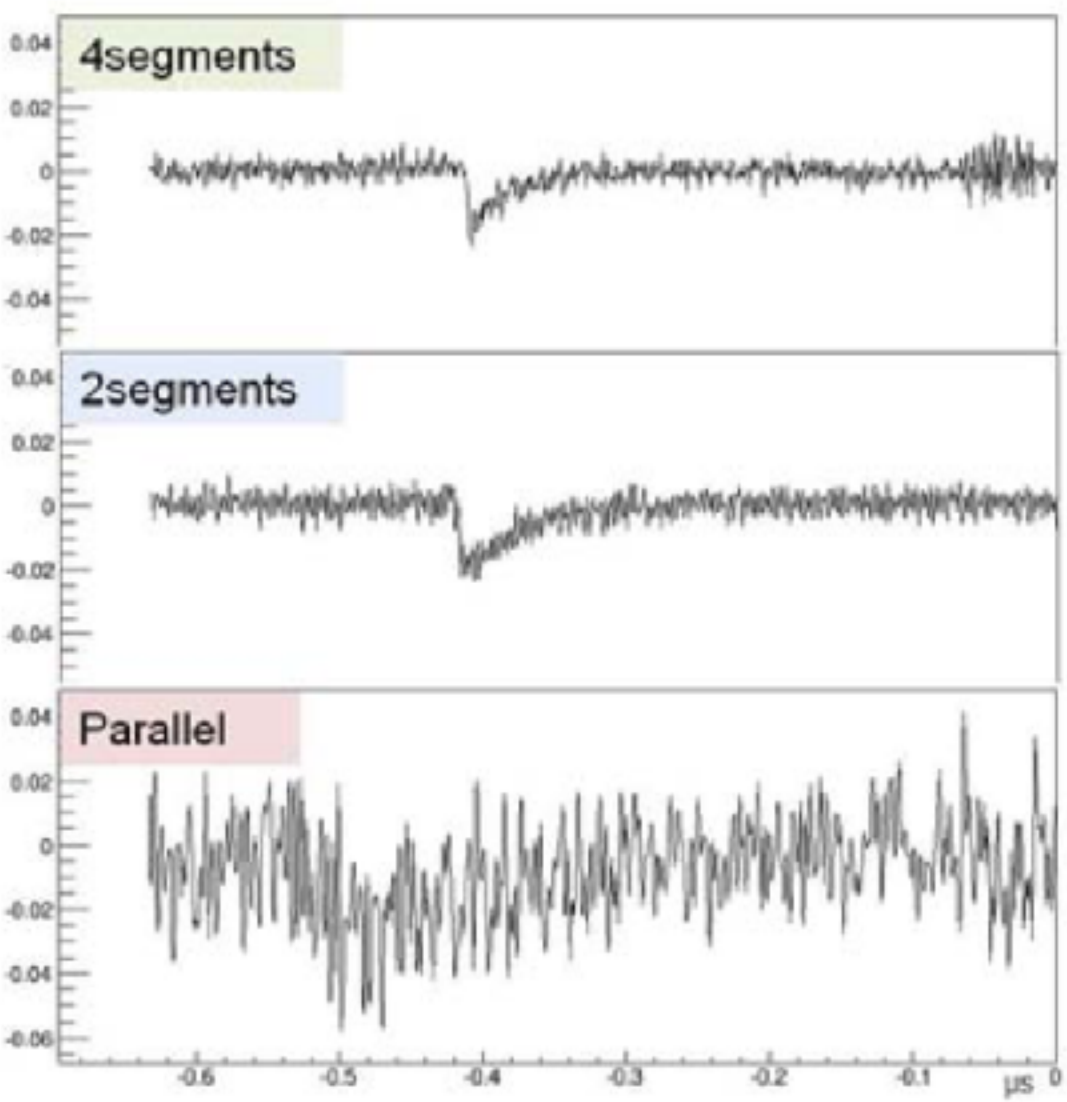
N: total # of pixels, n: # of fired pixels
R1 and R2 : series resistors in circuit (Let R= R1+R2)

REFERENCE OF MULTI-MPPC BIASING

MEG Collaboration's test results

Kaneko, D., Performance of UV-sensitive MPPC for liquid xenon detector in MEG experiment,

DOI:10.1109/NSSMIC.2013.6829484.



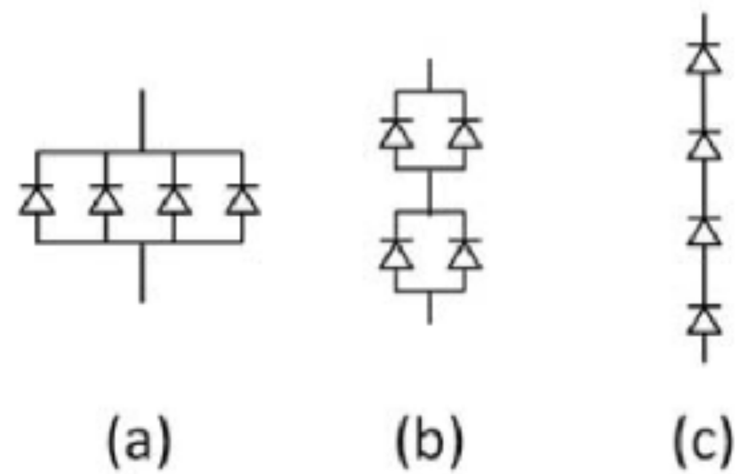
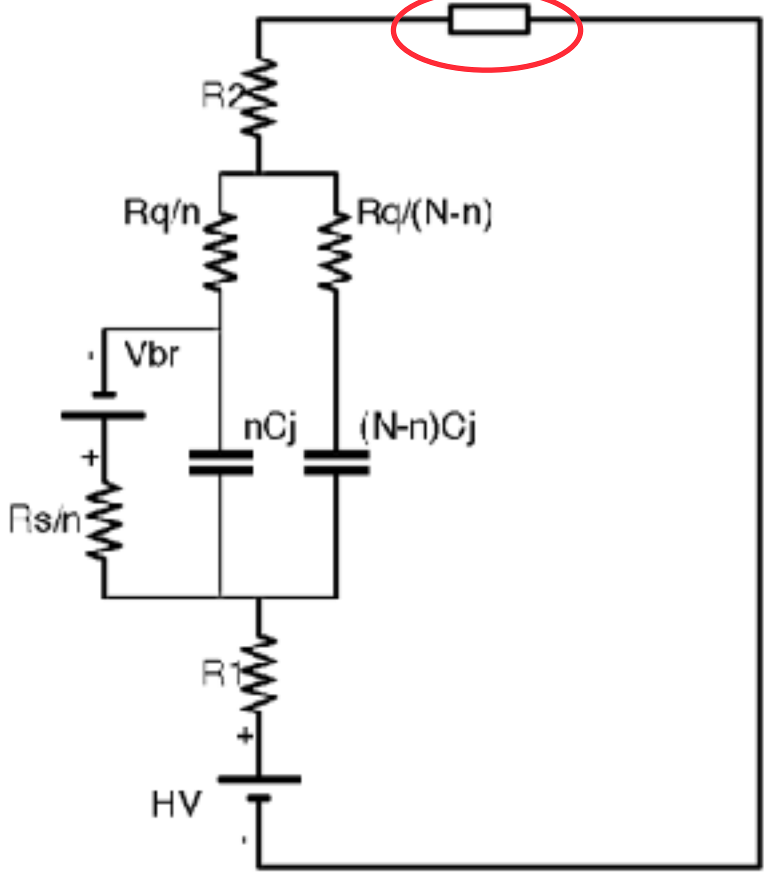


Fig. 10. Connection of 4 MPPCs, 4-parallel, 2-series 2-parallel and 4-series correspond (a), (b) and (c) respectively.

MPPCS IN SERIES CONNECTION

Equivalent circuit

Load of the other MPPCs



Bias voltage should become larger in proportional to the number of MPPCs

Assume the condition that photons are firing only one MPPC
(the other cases can be described as a superposition of above situations)

Because of load of additional MPPCs, it is hard to get exact solutions.
With another assumption dI/dt = 0, rising edge shape remains almost same
But there is a small voltage drop occur through series MPPCs

$$\tau_r \sim C_j R_S$$

For the tailing edge, time constant become small,
So, signal also become shorter

$$\tau_t = C_j R_Q + C_j N R / a$$

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N: total # of pixels, n: # of fired pixels
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