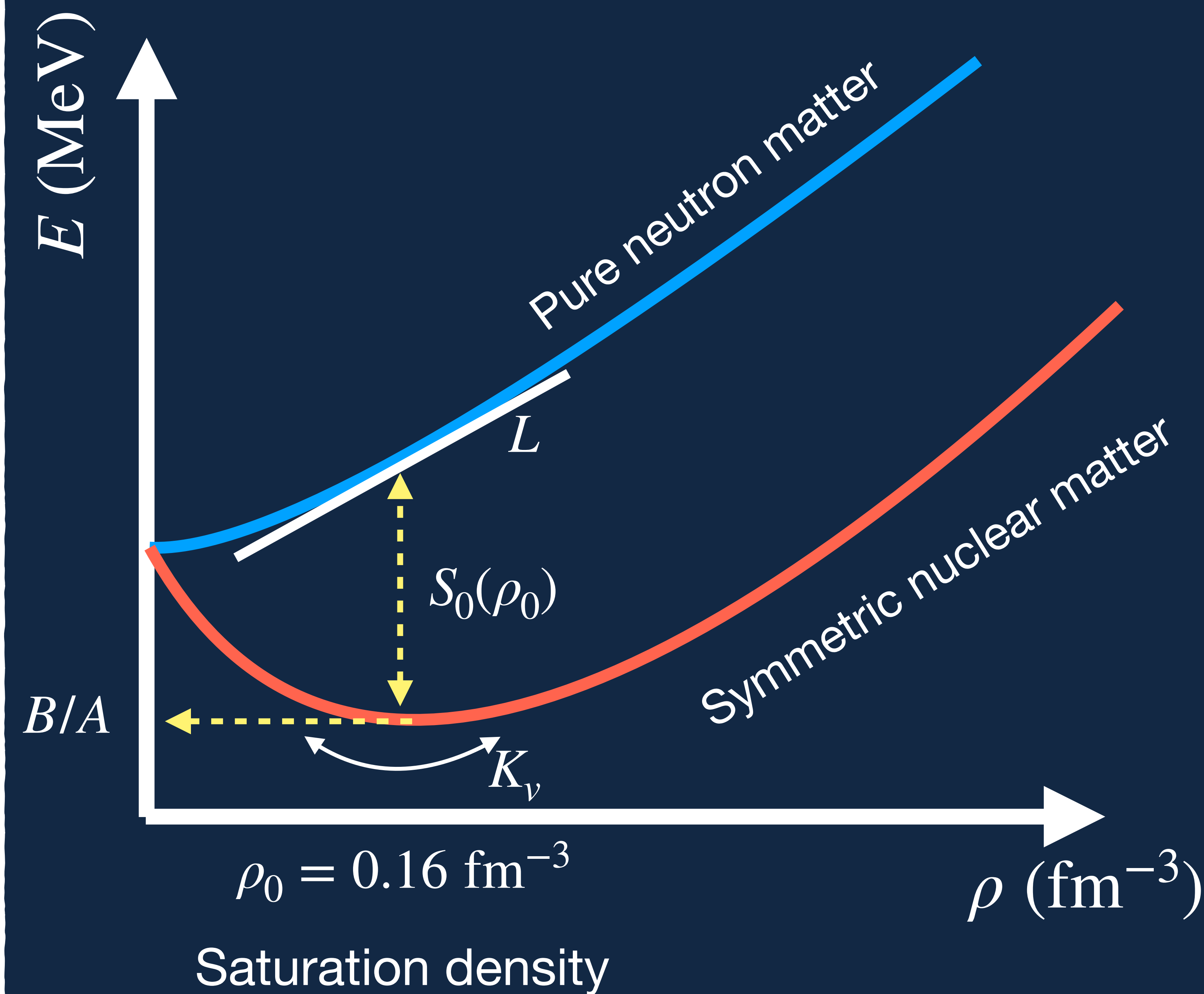
Relations between Symmetry Energy and Tidal Deformability

Soonchul Choi
(Soongsil University)

In collaboration with
Myung-Ki Cheoun (Soongsil University)
Tsuyoshi Miyatsu (Tokyo University of Science)
Youngshin Kwon (Korea Aerospace University)
Kyungsik Kim (Korea Aerospace University)
Hungchong Kim (Korea Aerospace University)

Saturation properties



Binding energy

$B/A \approx -16.0$ MeV

Symmetry energy

$S_0(\rho_0) = 32.5 \pm 0.5$ MeV [1]

Slope parameter

$L =$ 50 – 100 MeV [2]

Incompressibility

$K =$ 180 – 260 MeV [3]

Effective mass

$m^*/m = 0.7 - 0.8$ [4]

[1]PRL 108 052501 (2012) [3]PLB 778 207212 (2018)
[2]PRL 102 122501 (2009) [4]Nucl. Phys. A 950 64109 (2016)

The effect of L and K on neutron stars

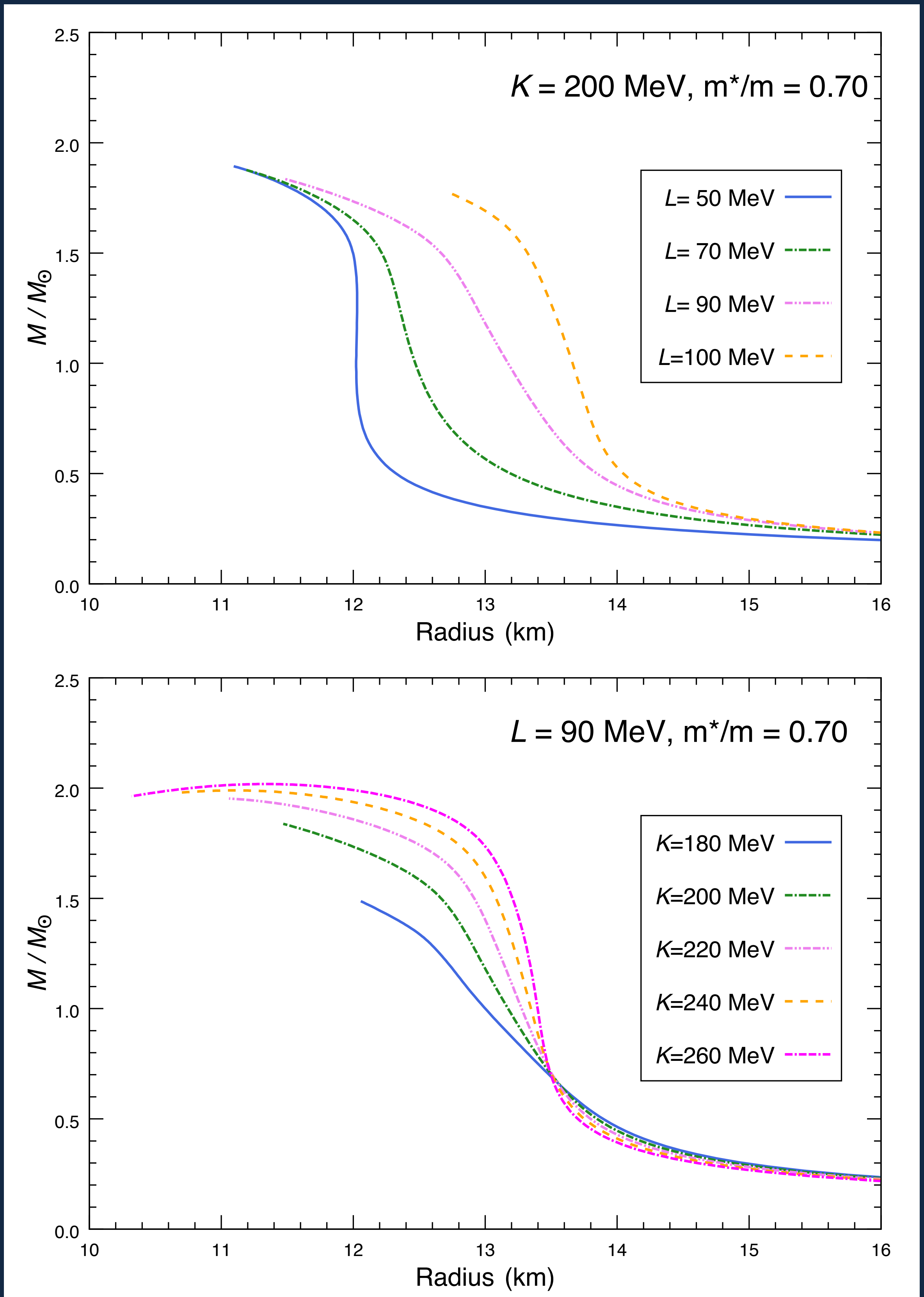
In these range,

$$L = 50 - 100 \text{ MeV} \qquad K = 180 - 260 \text{ MeV}$$

neutron stars can cover wide range of mass and radius.

$$R(1.4M_\odot): \quad 12 - 14 \text{ km}$$

Maximum mass : $1.5 - 2.1M_\odot$



LETTER

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A two-solar-mass neutron star measured using Shapiro delay

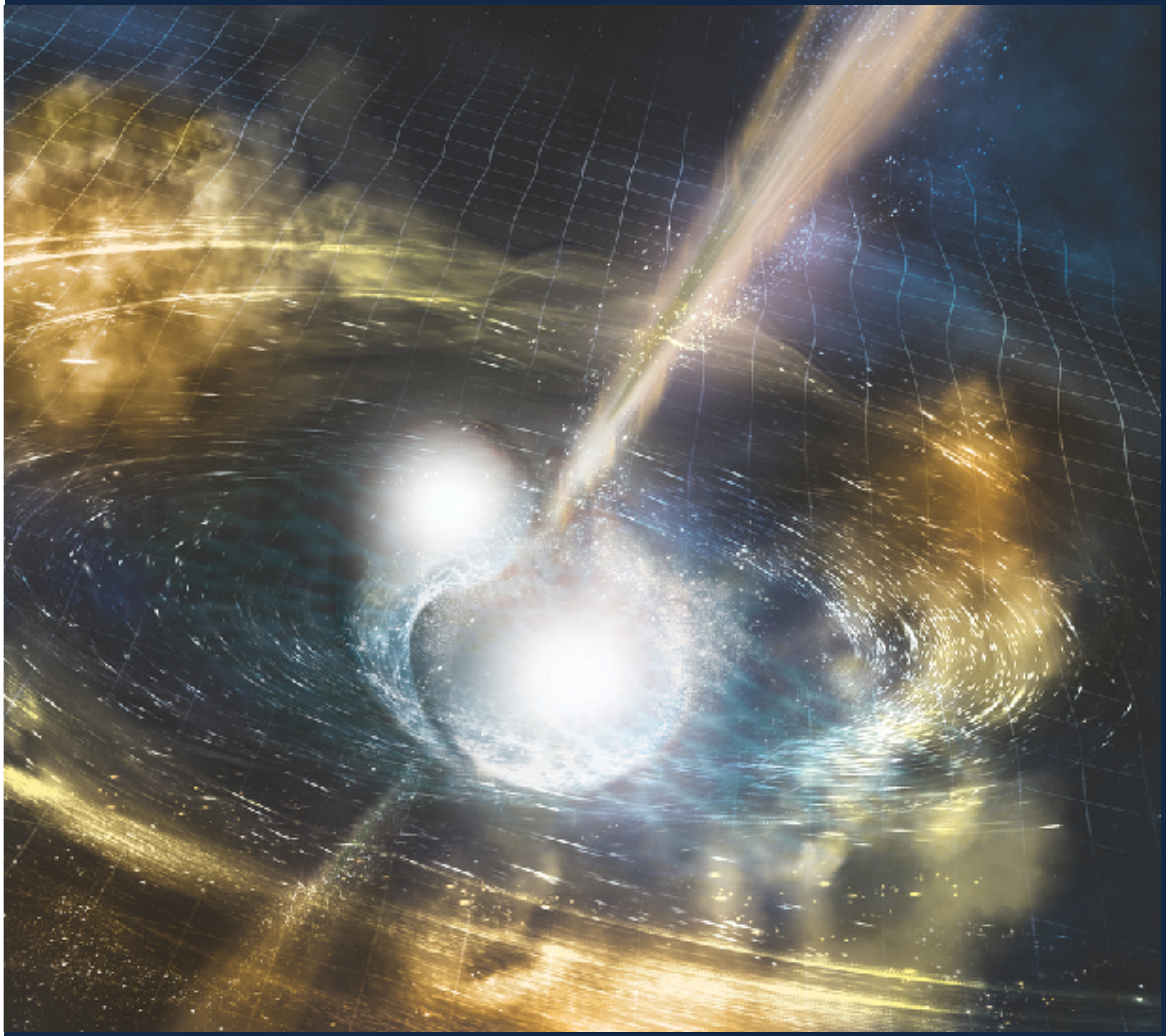
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3]

Neutron stars are composed of the densest form of matter known to exist in our Universe, the composition and properties of which are still theoretically uncertain. Measurements of the masses or radii of these objects can strongly constrain the neutron star matter equation of state and rule out theoretical models of their composition[1,2]. The observed range of neutron star masses, however, has hitherto been too narrow to rule out many predictions of 'exotic' non-nucleonic components[3–6]. The Shapiro delay is a general-relativistic increase in light travel time through the curved space-time near a massive body[7]. For highly inclined (nearly edge-on) binary millisecond radio pulsar systems, this effect allows us to infer the masses of both the neutron star and its binary companion to high precision[8,9]. Here we present radio timing observations of the binary millisecond pulsar J1614-2230[10,11] that show a strong Shapiro delay signature. We calculate the pulsar mass to be $(1.97 \pm 0.04)M_{\odot}$, which rules out almost all currently proposed[2–5] hyperon or boson condensate equations of state ($M_{\odot}$, solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks[12].

Table 1 | Physical parameters for PSR J1614-2230

Parameter	Value
Ecliptic longitude (λ)	245.78827556(5)°
Ecliptic latitude (β)	−1.256744(2)°
Proper motion in λ	9.79(7) mas yr^{-1}
Proper motion in β	−30(3) mas yr^{-1}
Parallax	0.5(6) mas
Pulsar spin period	3.1508076534271(6) ms
Period derivative	$9.6216(9) \times 10^{-21}$ s s^{-1}
Reference epoch (MJD)	53,600
Dispersion measure*	34.4865 pc cm^{-3}
Orbital period	8.6866194196(2) d
Projected semimajor axis	11.2911975(2) light s
First Laplace parameter ($e\sin\omega$)	$1.1(3) \times 10^{-7}$
Second Laplace parameter ($e\cos\omega$)	$-1.29(3) \times 10^{-6}$
Companion mass	0.500(6)$M_{\odot}$
Sine of inclination angle	0.999894(5)
Epoch of ascending node (MJD)	52,331.1701098(3)
Span of timing data (MJD)	52,469–55,330
Number of TOAs†	2,206 (454, 1,752)
Root mean squared TOA residual	1.1 μs
Right ascension (J2000)	16 h 14 min 36.5051(5) s
Declination (J2000)	−22° 30′ 31.081(7)″
Orbital eccentricity (e)	$1.30(4) \times 10^{-6}$
Inclination angle	89.17(2)°
Pulsar mass	1.97(4)$M_{\odot}$
Dispersion-derived distance‡	1.2 kpc
Parallax distance	>0.9 kpc
Surface magnetic field	1.8×10^{8} G
Characteristic age	5.2 Gyr
Spin-down luminosity	1.2×10^{34} erg s^{-1}
Average flux density* at 1.4 GHz	1.2 mJy
Spectral index, 1.1–1.9 GHz	−1.9(1)
Rotation measure	−28.0(3) rad m^{-2}

Tidal deformability



NSF/LIGO/Sonoma State University/A. Simonnet
https://www.ligo.caltech.edu/image/ligo20171016d

Mass

$$m_1 = 1.36 - 1.60\ M_\odot$$
$$m_2 = 1.17 - 1.36\ M_\odot$$

Distance

40^{+8}_{-14} Mpc

Dimensionless
tidal deformability

$$\Lambda(1.4 M_\odot) \leq 800$$

PRL 119 161101 (2017)

Tidal deformability

If we consider a static, spherically symmetric star of mass (m) placed in a time-independent external quadrupole tidal field.

The star will develop in response a quadrupole moment

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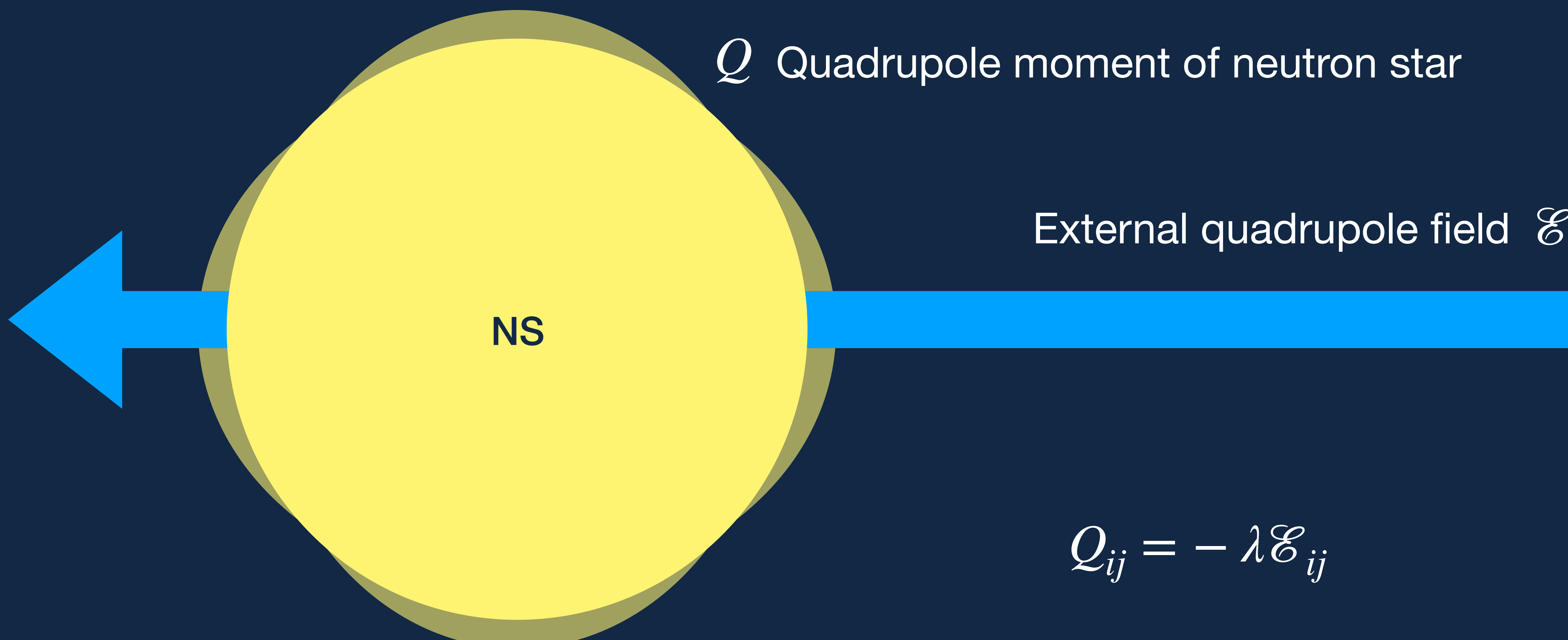


$$Q_{ij} = -\lambda \mathscr{E}_{ij}$$

PRD 81 123016 (2010)

Tidal deformability

λ : a constant which is related to the tidal deformability Λ

$$\Lambda = \frac{5}{2}\lambda\frac{1}{M}\left(\frac{c^2}{G}\right)^5$$

$\Lambda \leq 800$ PRL 119 161101 (2017)

$1.94\ M_\odot \leq M_{\mathrm{NS}} \leq 2.01\ M_\odot$ Nature 467 1081-1083 (2010)

By using these observation data,
we can give the constraint on slope parameter and incompressibility.

Relativistic mean field model

Lagrangian density for static uniform nuclear matter :

$$\mathscr{L} = \sum_B \bar{\psi}_B[i\gamma_\mu\partial^\mu - M_B^*(\sigma_0, \sigma_0^*) - g_{\omega B}\gamma_0\omega_0 - g_{\phi B}\gamma^0\phi^0 - g_{\rho B}\gamma^0\rho_0 I_{3B}]\psi_B$$
$$-\frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{2}m_{\sigma^*}\sigma_0^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_0^2 - U_{NL}(\sigma_0, \omega_0, \rho_0)$$

$B = N, \Lambda, \Sigma^{+,0,-}, \Xi^{0,-}$

The effective baryon mass in matter :

$$M_B^*(\sigma, \sigma^*) = M_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^*$$

The following non-linear term is also introduced in order to reproduce the saturation properties of nuclear matter :

$$U_{\mathrm{NL}}(\sigma, \omega^\mu\rho^\mu) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \Lambda_v g_{\omega B}^2 g_{\rho B}^2(\omega_0^2\rho_0^2)$$

Coupling constants

Coupling constants, $g_{\sigma N}$, $g_{\omega N}$, and $g_{\rho N}$ are determined so as to reproduce the binding energy per nucleon, and symmetry energy at saturation density.

$$B/A = -16.0 \text{ MeV}$$
$$S_0(\rho_0) = 32.5 \text{ MeV}$$
$$\rho_0 = 0.16 \text{fm}^{-3}$$

The SU(3) symmetry in the vector-meson gives the relations of the coupling constants as

$$g_{\omega\Lambda} = g_{\omega\Sigma} = \frac{1}{1+\sqrt{3}z\tan\theta_v} g_{\omega N}$$

$$g_{\rho\Sigma} = 2g_{\rho N}, \quad g_{\rho\Xi} = g_{\rho N} \quad g_{\rho\Lambda} = 0$$

$$g_{\omega\Xi} = \frac{1-\sqrt{3}z\tan\theta_v}{1+\sqrt{3}z\tan\theta_v} g_{\omega N}$$

$$g_{\phi\Lambda} = g_{\phi\Sigma} = \frac{-\tan\theta_v}{1+\sqrt{3}z\tan\theta_v} g_{\omega N}$$

$$g_{\phi\Xi} = -\frac{\sqrt{3}z+\tan\theta_v}{1+\sqrt{3}\tan\theta_v} g_{\omega N}$$

$$g_{\phi N} = \frac{\sqrt{3}z-\tan\theta_v}{1+\sqrt{3}z\tan\theta_v} g_{\omega N}$$

In the present calculation, we refer to the Nijmegen extended-soft-core (ESC) model [1] to fix the mixing angle (θ_v) and z as

$$\theta = 37.50^\circ, \quad z = 0.1949$$

[1]Prog. Theor. Phys. Suppl. 185 14 (2010)

Coupling constants

We need to consider the hyperon coupling. The σ-Y and σ*-Y are determined as follows. The potential for hyperon Y in symmetric nuclear matter is calculated as

$$U_Y^{(N)} = -g_{\sigma Y}\sigma_0 + g_{\omega Y}\omega_0$$

$U_\Lambda^{(N)} = -28$ MeV [1] $U_\Sigma^{(N)} = 30$ MeV [2] $U_\Xi^{(N)} = -18$ MeV [3]

The potential for Y in Y-hyperon matter is written as :

$$U_Y^{(Y)} = -g_{\sigma Y}\sigma_0^{(Y)} - g_{\sigma^* Y}\sigma_0^{*(Y)} + g_{\omega Y}\omega^{(Y)} + g_{\phi Y}\phi_0^{(Y)}$$

$U_\Xi^{(\Xi)} \simeq U_\Lambda^{(\Xi)} \simeq 2U_\Xi^{(\Lambda)} \simeq 2U_\Lambda^{(\Lambda)}$ [3] $U_\Lambda^{(\Lambda)} = -5$ MeV Nagara event

[1]PRC 54 1416 (1996), [2]PRC 88 015802 (2013), [3]PRC 77 025801 (2008)

Non-Linear potential

We add the following NL potential to the Lagrangian density

$$U_{\mathrm{NL}}(\sigma, \omega^\mu \boldsymbol{\rho}^\mu) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 - \Lambda_v g_{\omega B}^2 g_{\rho B}^2 (\omega_0^2 \rho_0^2)$$

Investigated range of incompressibility (K), and the slope parameter (L) at saturation density is

$$K = 180 - 260 \text{ MeV}$$
$$L = 50 - 100 \text{ MeV} .$$

We fix the effective mass ratio as

$$m^*/m = 0.7$$

Result
(Dependence on slope parameter)

- The slope parameter does not affect the maximum mass of neutron star.

- Around the 1.3 solar mass neutron star

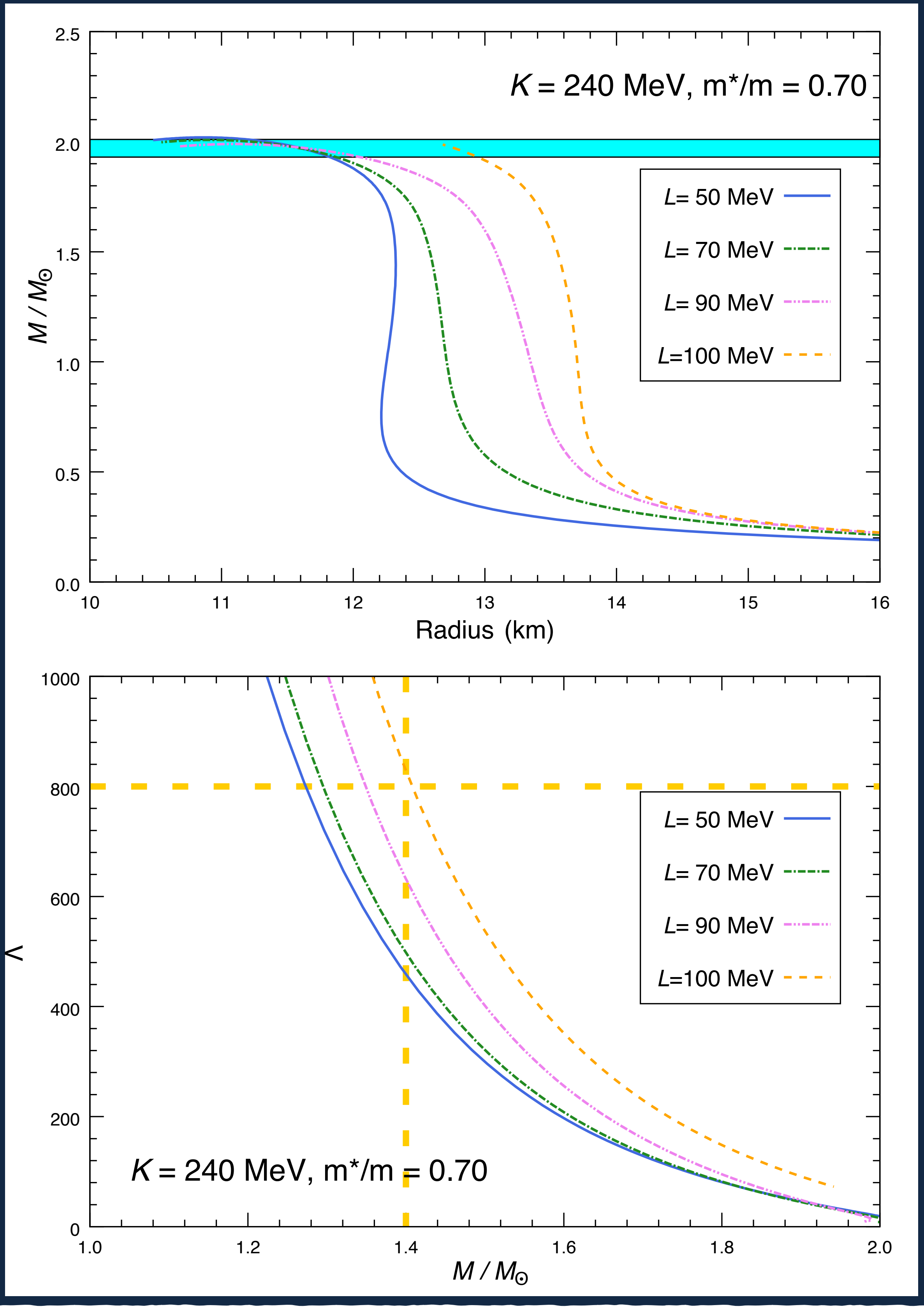
  Slope parameter ⟶ Large

  Radius ⟶ Large

- In the tidal deformability case, the large slope parameter gives the high tidal deformability.

The onset of Lambda particle Kv = 240 MeV

	L = 50 MeV	70 MeV	90 MeV	100 MeV
ρ_Λ	0.450	0.440	0.410	0.380



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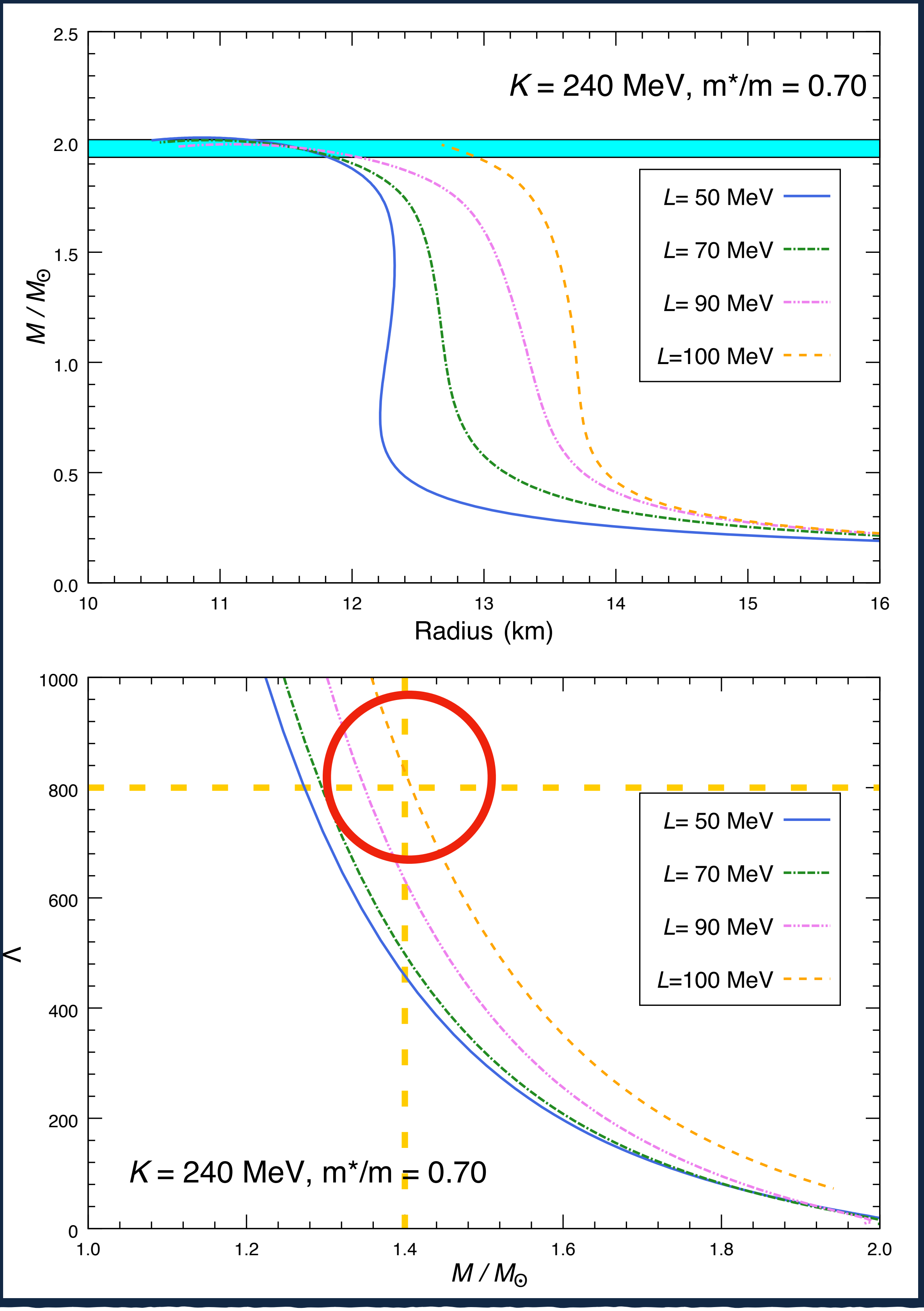


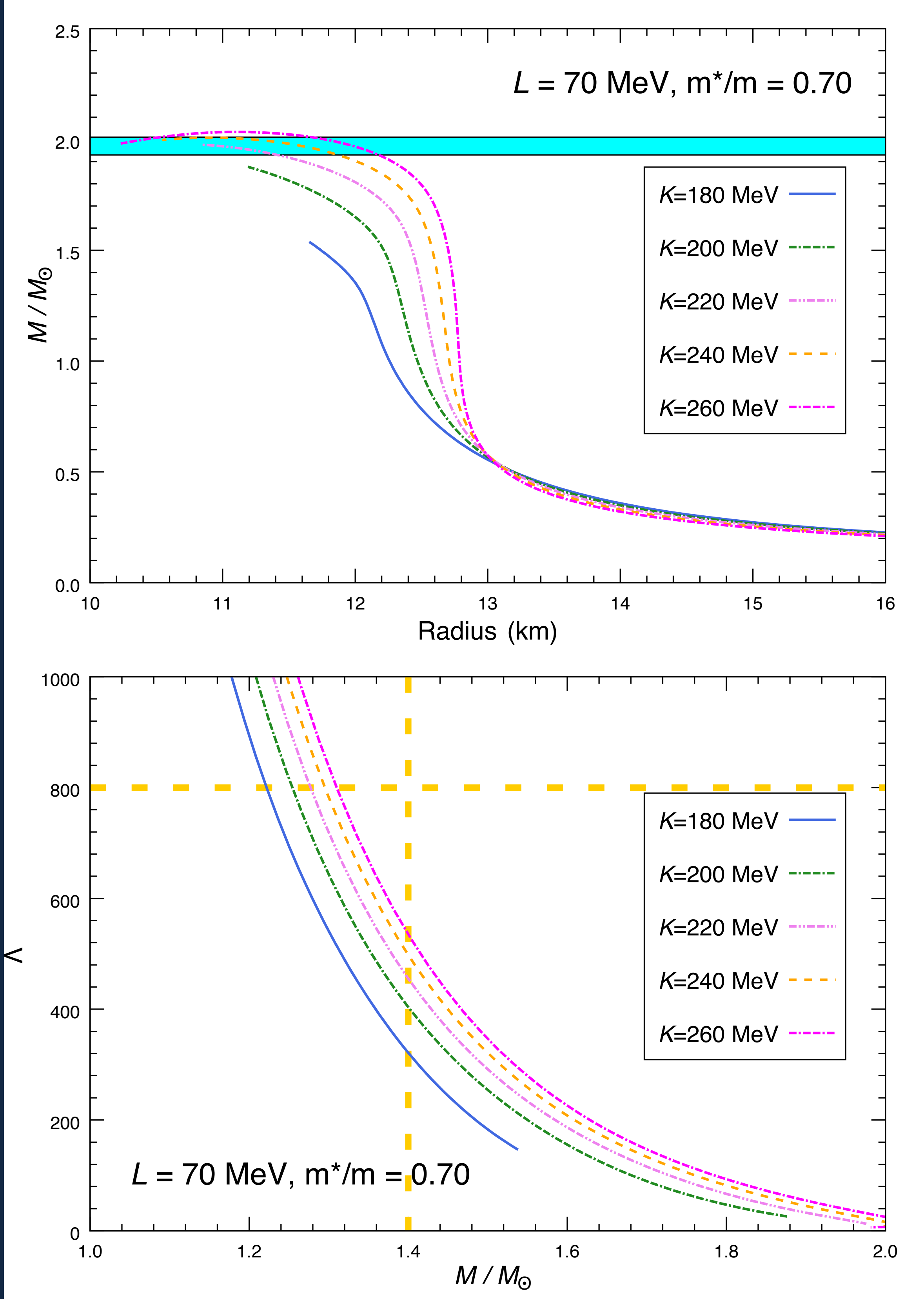
Result
(Dependence on incompressibility)

- The incompressibility affects both maximum mass and radius.

Incompressibility $\longrightarrow$ Large

maximum mass $\longrightarrow$ Large

- The two-solar mass neutron star cannot be described in K = 180 and 200 MeV cases.



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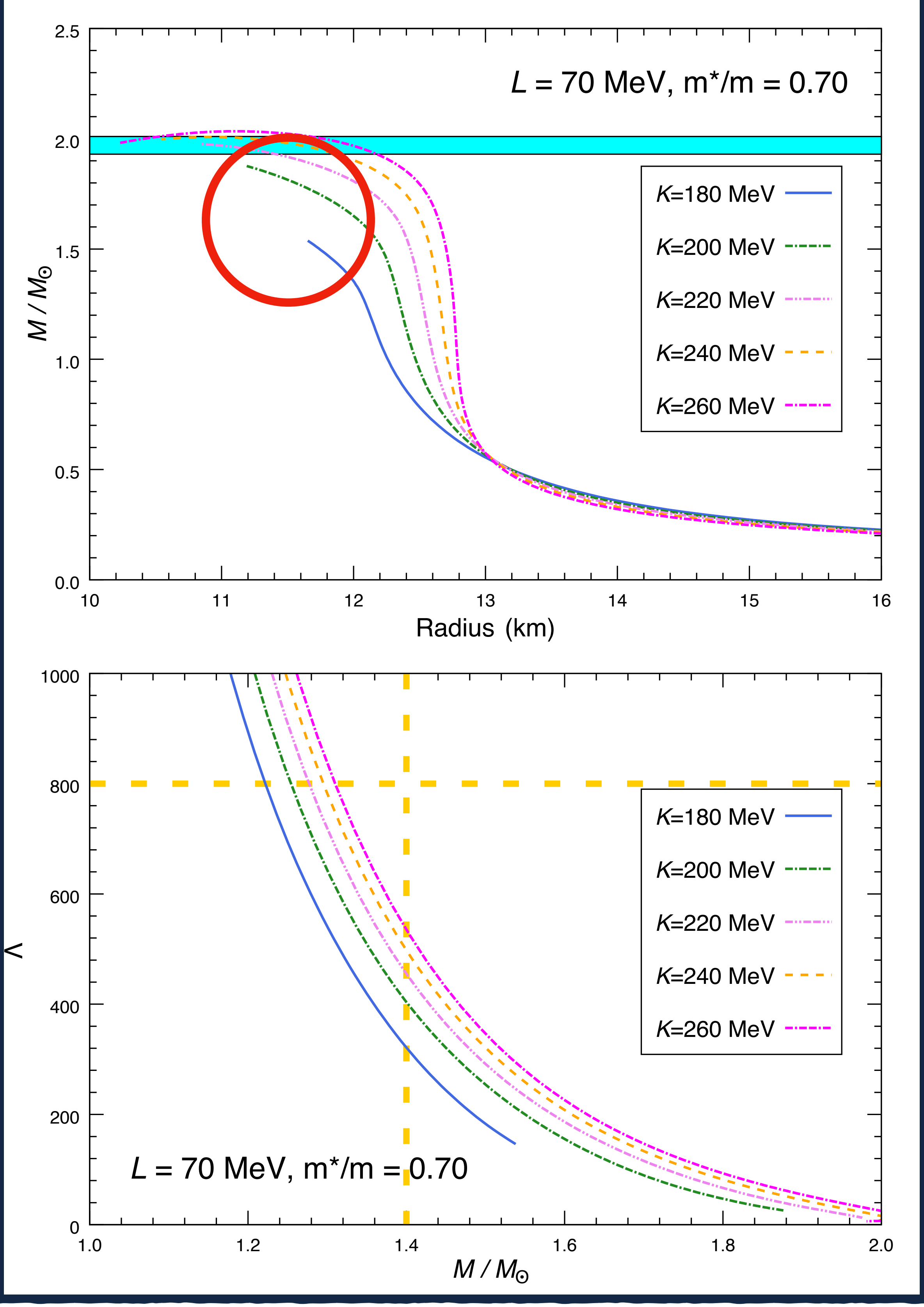

Summary

$m^*/m = 0.7$	$K = 180$ MeV	200 MeV	220 MeV	240 MeV	260 MeV
$L = 50$ MeV	×	×	○	○	○
70 MeV	×	×	○	○	○
90 MeV	×	×	○	○	○
100 MeV	×	×	×	×	×

- We changed the slope parameter and incompressibility by using RMF model with non-linear potential.
- The slope parameter affects the radius and tidal deformability.
- The incompressibility can change the maximum mass of the neutron star.
- Neutron stars prefer the high incompressibility and low slope parameter.
- **We can reduce the ambiguity of slope parameter and incompressibility by using the astronomical observation data**

Thank you for your attention