NuSYM 2018

Gravitational waves and Tidal deformability of Neutron Stars

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in collaboration with

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First detection of GW from a BNS

GW170817

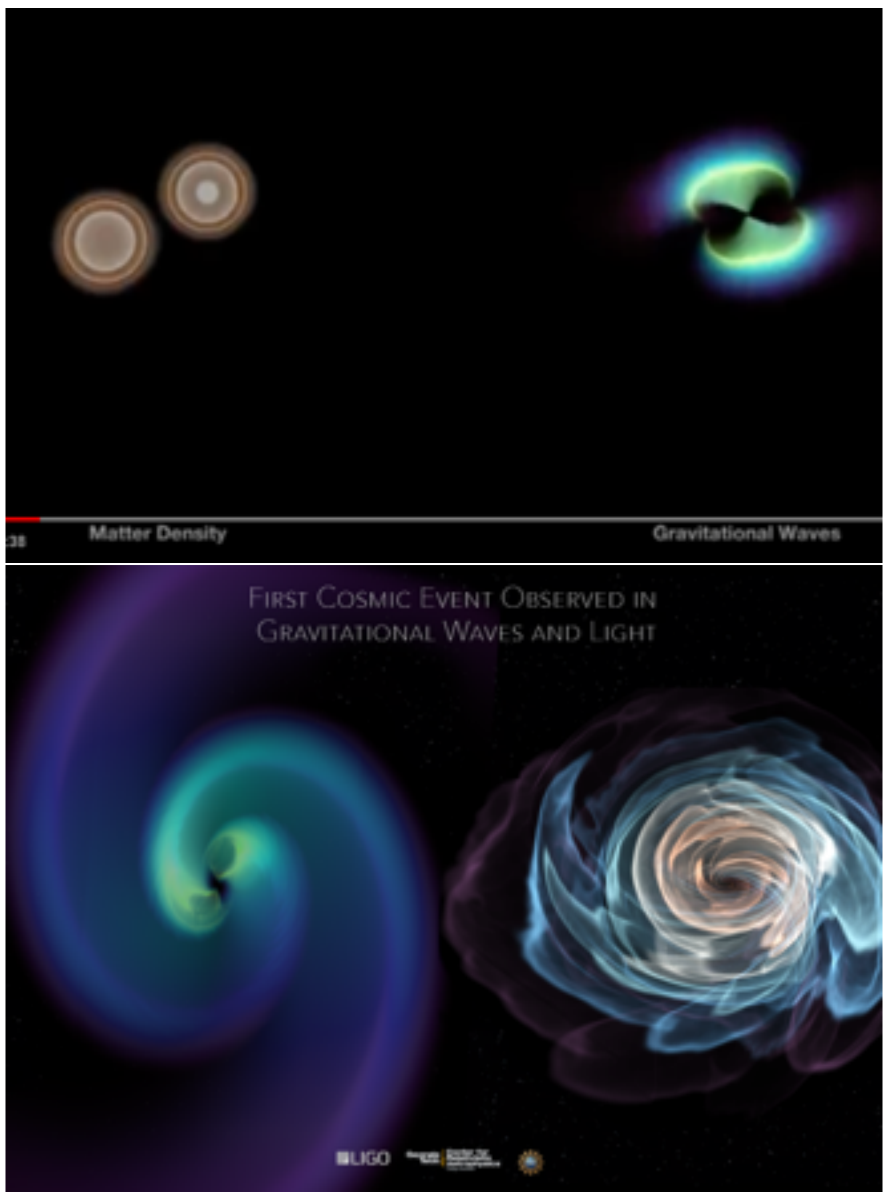
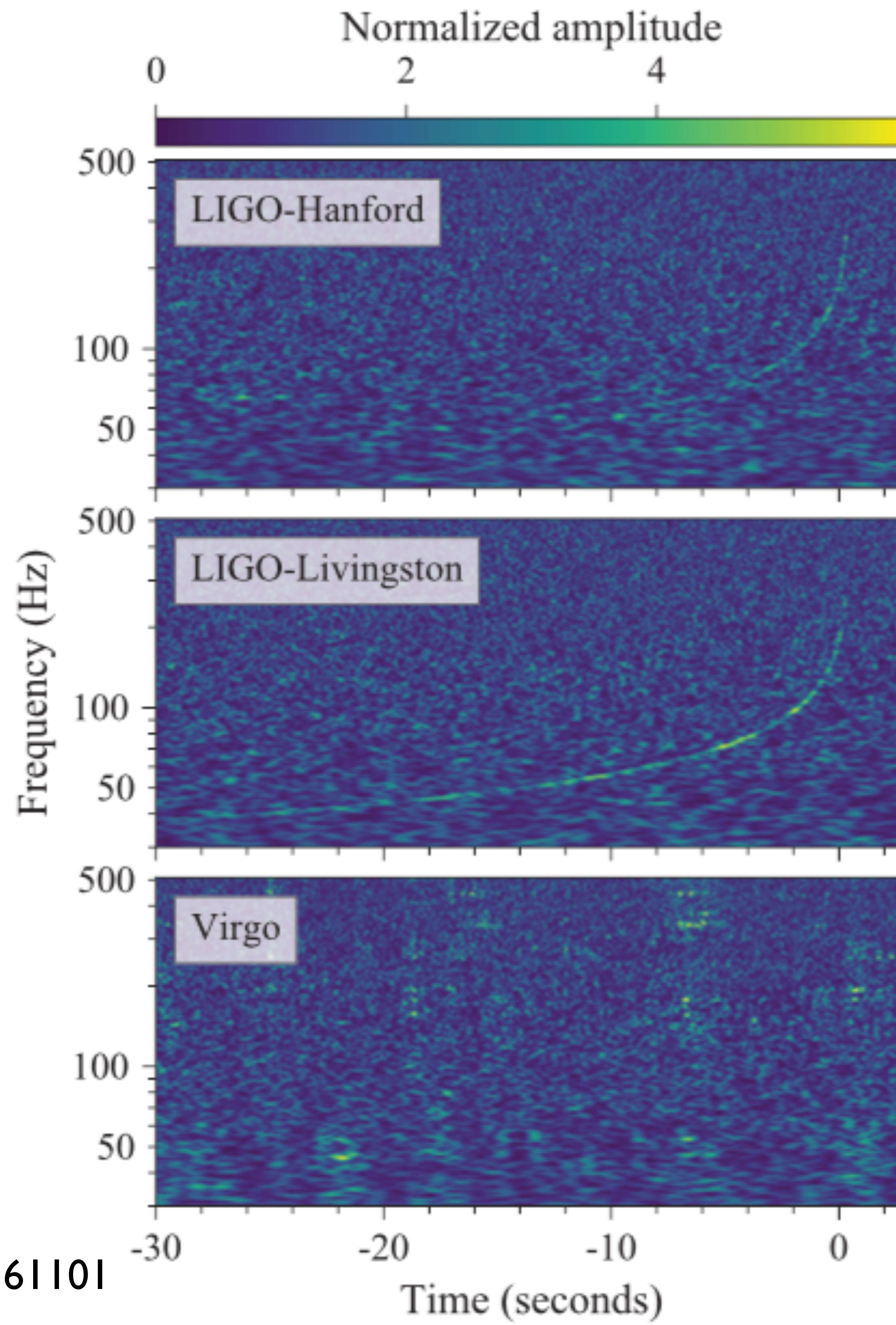


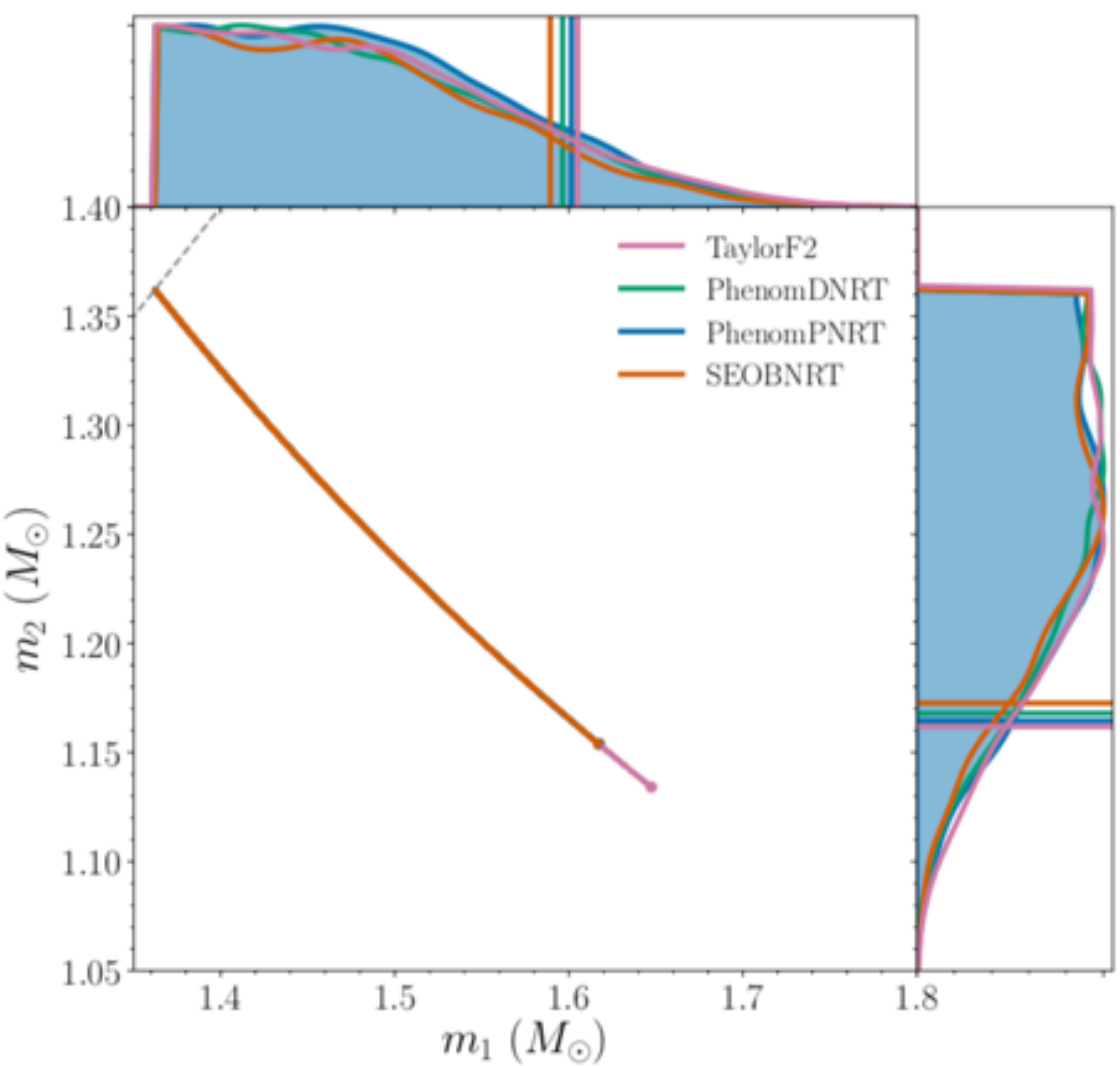
Image credit: Karan Jani/Georgia Tech.



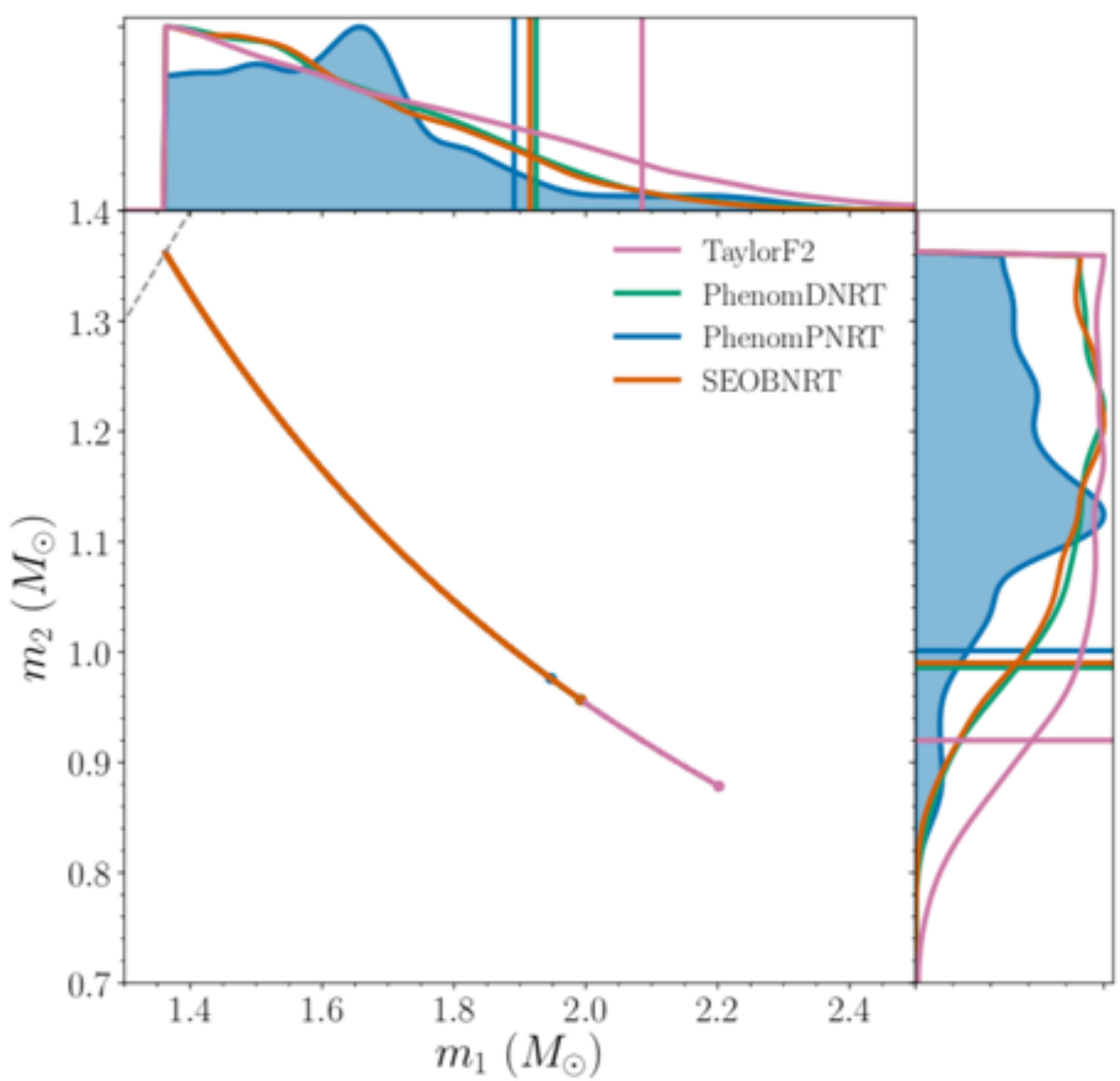
PhysRevLett.119.161101

Parameter Estimation (1)

low spin prior:
m1=1.36~1.60 $M_\odot$
m2=1.16~1.36 $M_\odot$



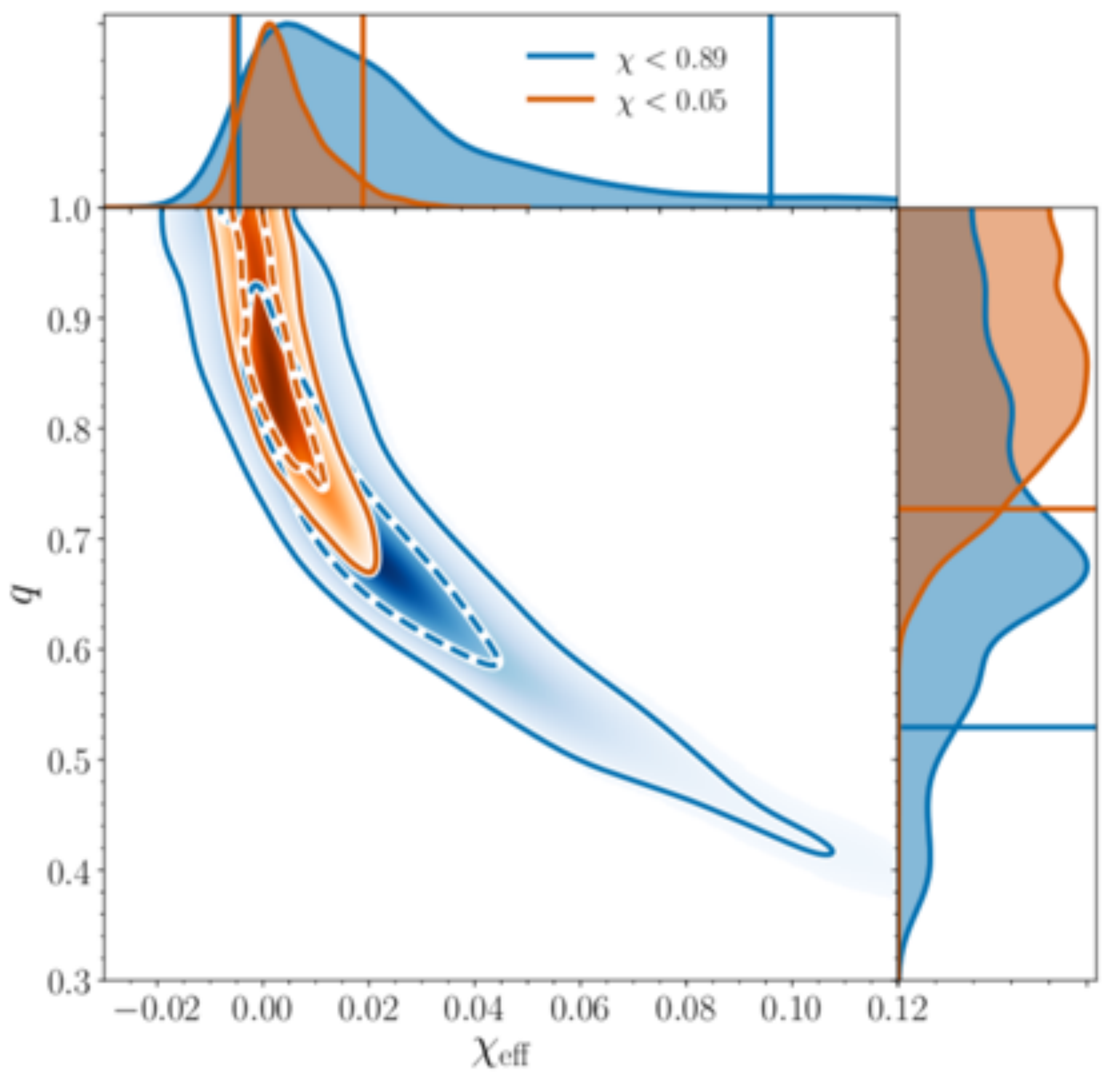
high spin prior:
m1=1.36~1.89 $M_\odot$
m2=1.00~1.36 $M_\odot$



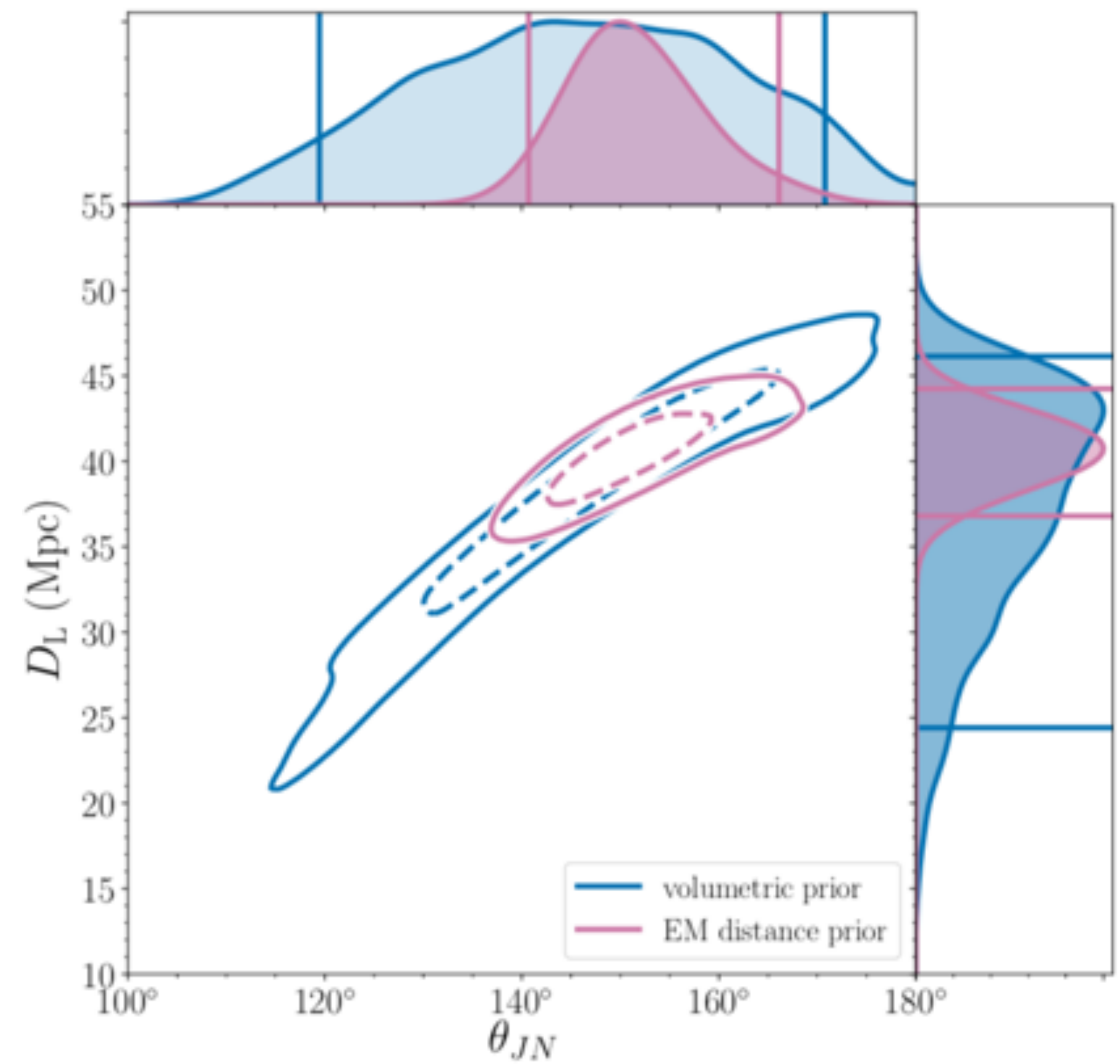
Abbott et al. (LSC and Virgo), arxiv:1805.11579

Parameter Estimation (2)

mass ratio q=
low spin: 0.73~1.0
high spin: 0.53~1.0



D_L ~ 40 Mpc
(consistent with D of NGC4993)



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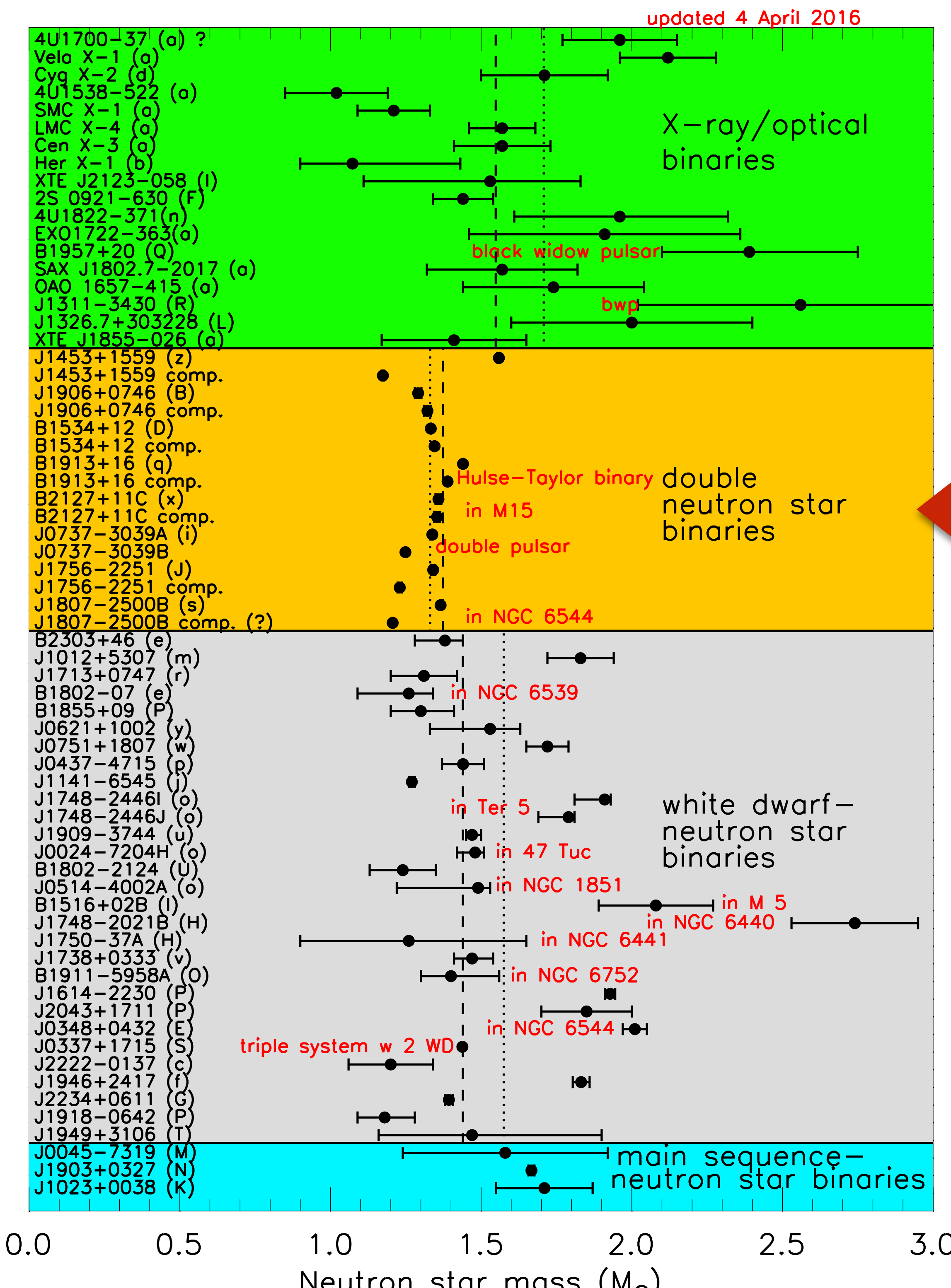
Neutron Star of Known Mass



GW170817: BNS

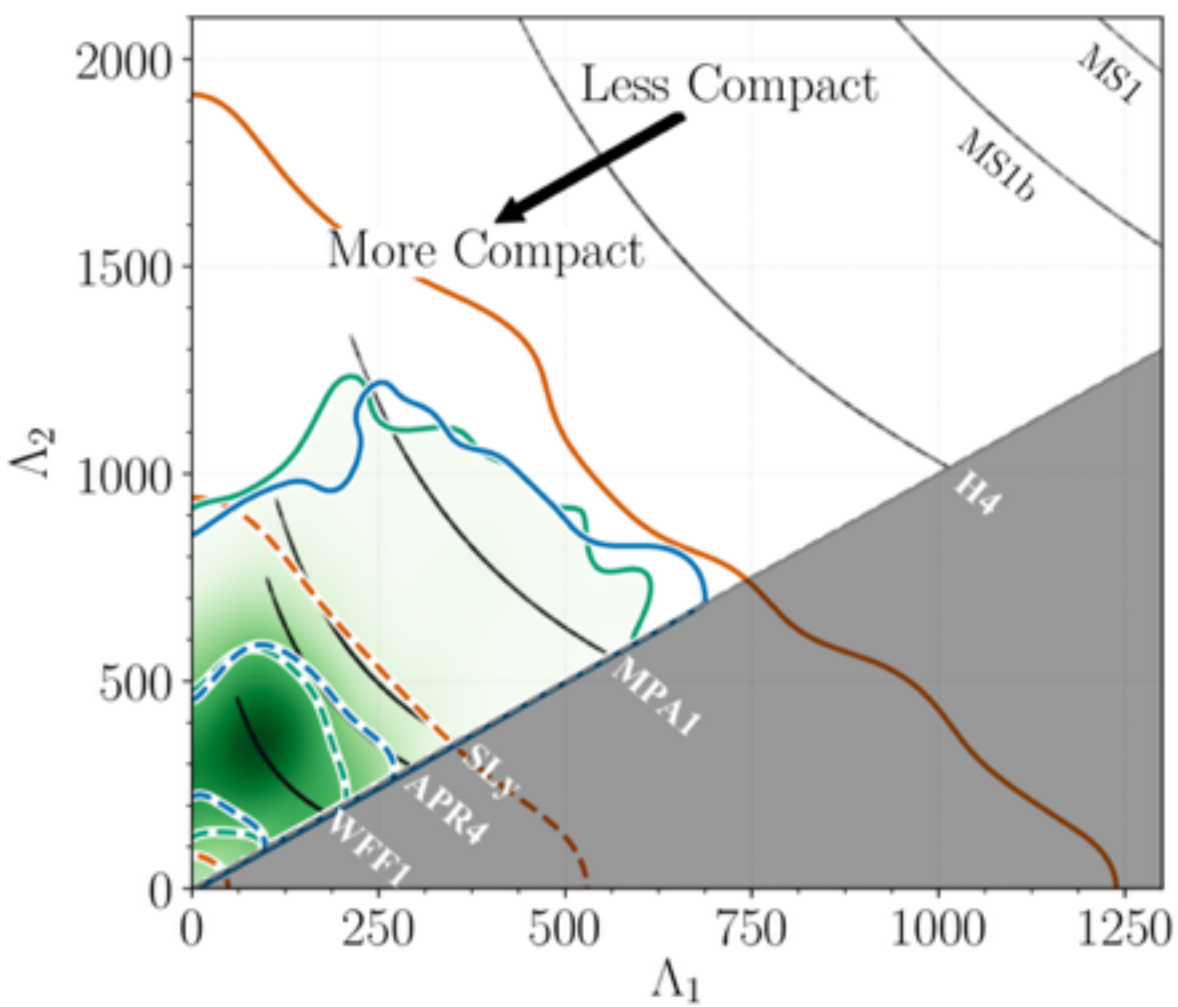
M1: 1.36~1.60 $M_\odot$ (1.36~2.26)

M2: 1.17~1.36 $M_\odot$ (0.86~1.36)

J. Lattimer, Annu.Rev.Nucl.Part.Sci.62,485(2012) and https://stellarcollapse.org by C. Ott

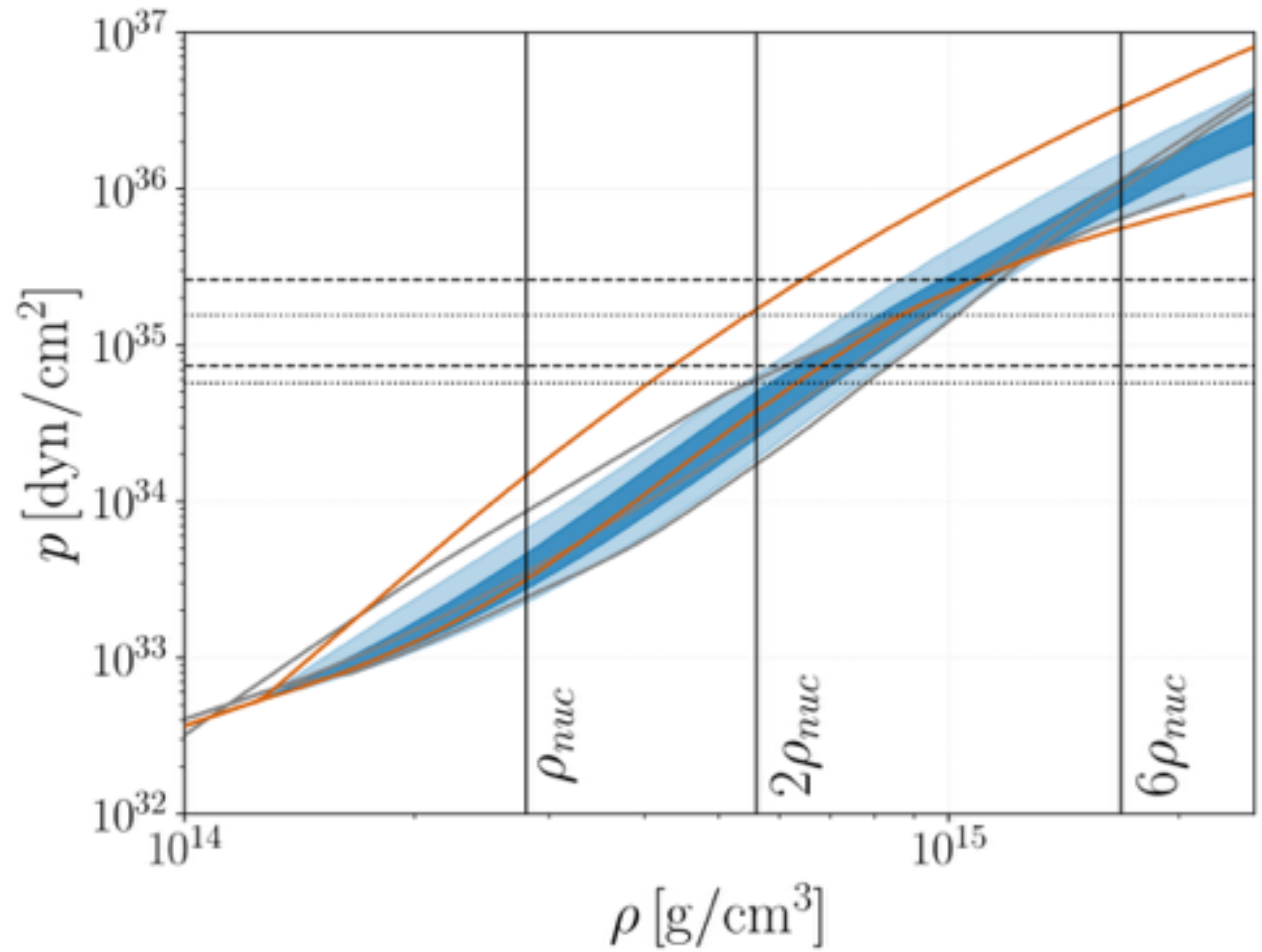
A new constraint by GW Obs. (1)

$\Lambda(1.4M_\odot)=190^{+390}_{-120}$



$P(2\ \rho_{nuc})= 3.5^{+2.7}_{-1.7}\ \times 10^{34}$ dyne/cm^2

$P(6\ \rho_{nuc})= 9.0^{+7.9}_{-2.6}\ \times 10^{35}$ dyne/cm^2



Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PRL accepted)

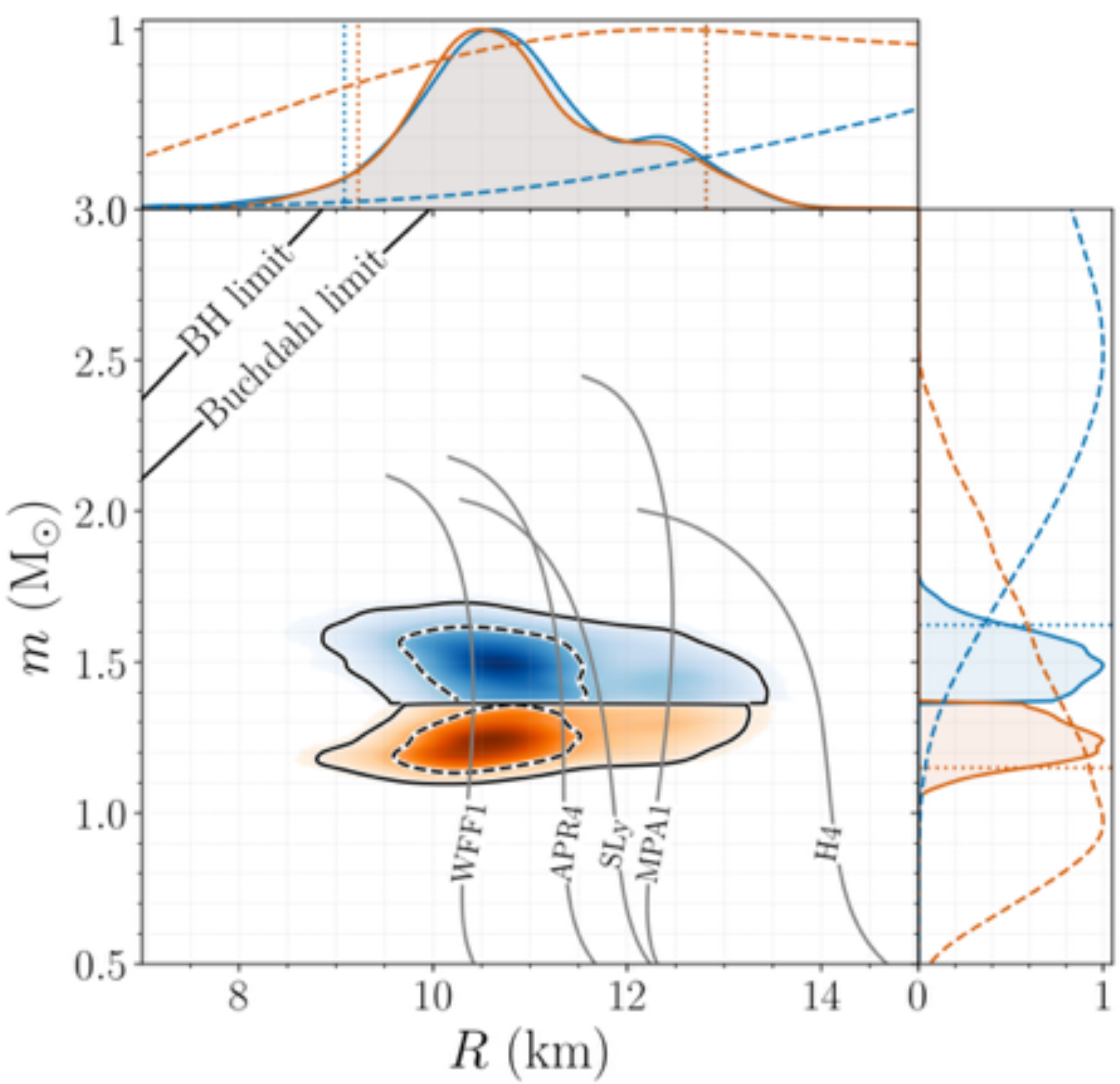
$\rho_{nuc}= 2.8\ \times 10^{14}$ g/cm^3

A new constraint by GW Obs. (2)

EoS insensitive relations (Yagi&Yunes,PR2017)

$R_1 = 10.8^{+2.0}_{-1.7}$ km

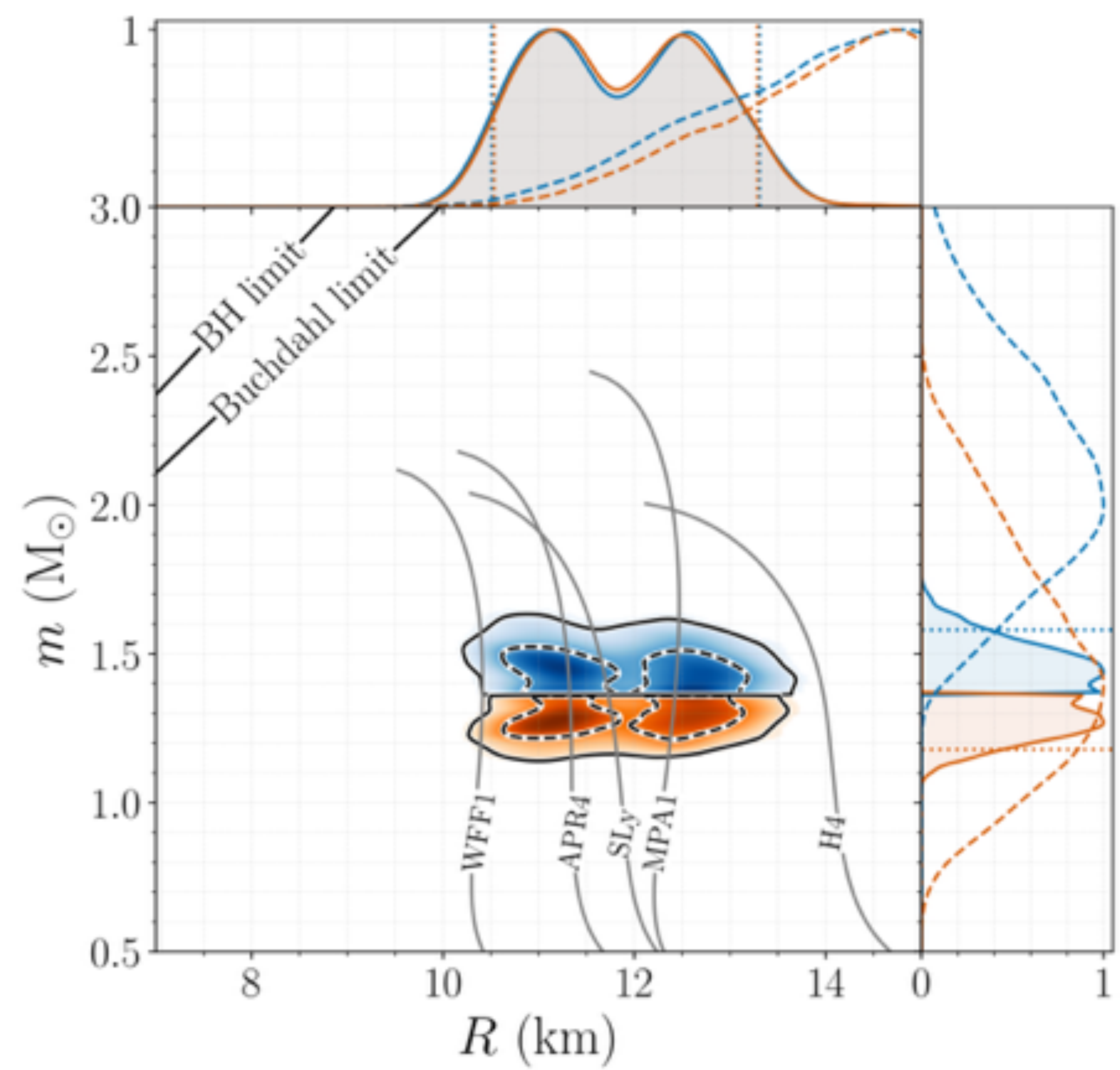
$R_2 = 10.7^{+2.1}_{-1.5}$ km



Parametrized EoS: M_{max} >= 1.97 $M_\odot$

$R_1 = 11.9^{+1.4}_{-1.4}$ km

$R_2 = 11.9^{+1.4}_{-1.4}$ km



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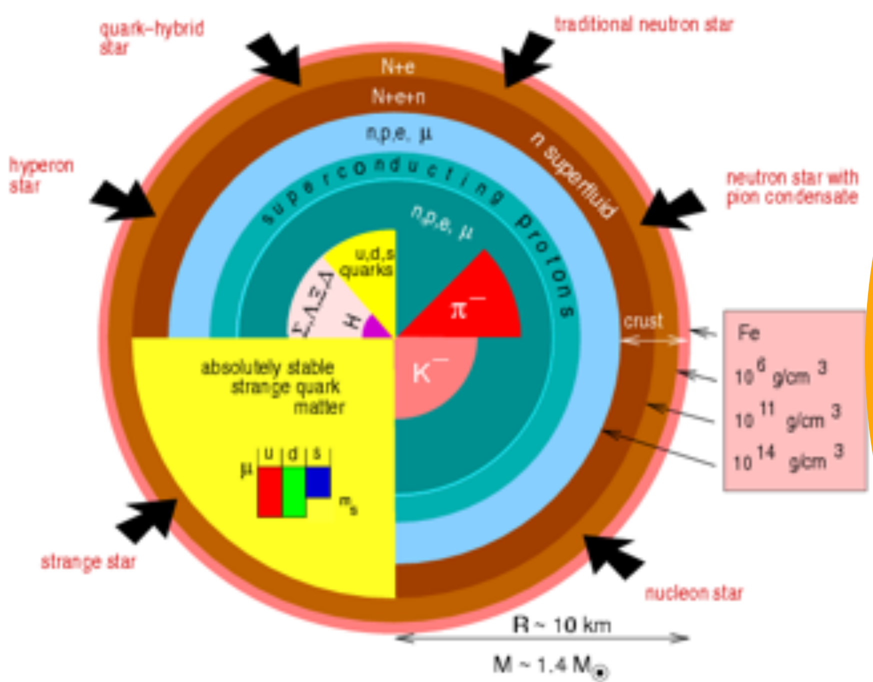
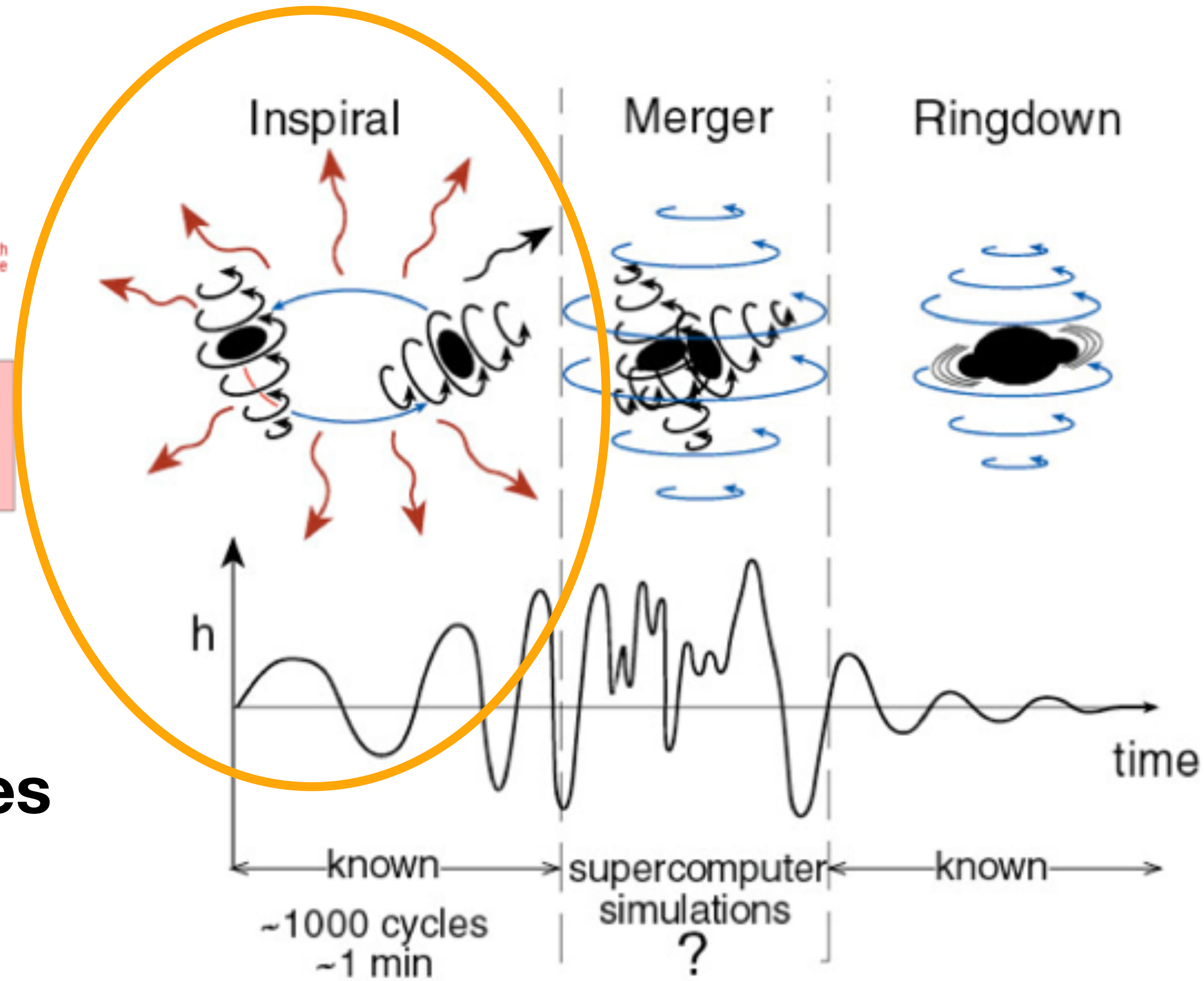
Response of NS to GW during inspiral





perturbative approaches

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ :Tidal deformability
k2 :Tidal Love number

GW waveform in Frequency Domain

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T(f)}$$

M. Favata, PRL.112.101101 (2014)

$$\Psi_T(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{\rm pp}_{\rm 3.5PN} + \Delta\Psi^{\rm spin}_{\rm 3PN} + \Delta\Psi^{\rm ecc.}_{\rm 2PN} + \Delta\Psi^{\rm tidal}_{\rm 6PN} + \Delta\Psi^{\rm tm}_{\rm 6PN}), \quad (1)$$

$$\Delta\Psi^{\rm tidal}_{\rm 6PN} = -\frac{39}{2}\tilde{\Lambda} v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right), \quad (4)$$

$$v = (\pi f M)^{1/3}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \lambda/M^5 \to G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Tidal Love number, k2

$$\frac{dH}{dr} = \beta \qquad \frac{d\beta}{dr} = 2\left(1 - 2\frac{M}{r}\right)^{-1} \times H \left\{ -2\pi \left[5\epsilon + 9P + (d\epsilon/dP)(\epsilon + P)\right] + \frac{3}{r^2} + 2\left(1 - 2\frac{M}{r}\right)^{-1} \left(\frac{M}{r^2} + 4\pi r P\right)^2 \right\} + \frac{2\beta}{r}\left(1 - 2\frac{M}{r}\right)^{-1} \left\{ -1 + \frac{M}{r} + 2\pi r^2 (\epsilon - P) \right\}$$

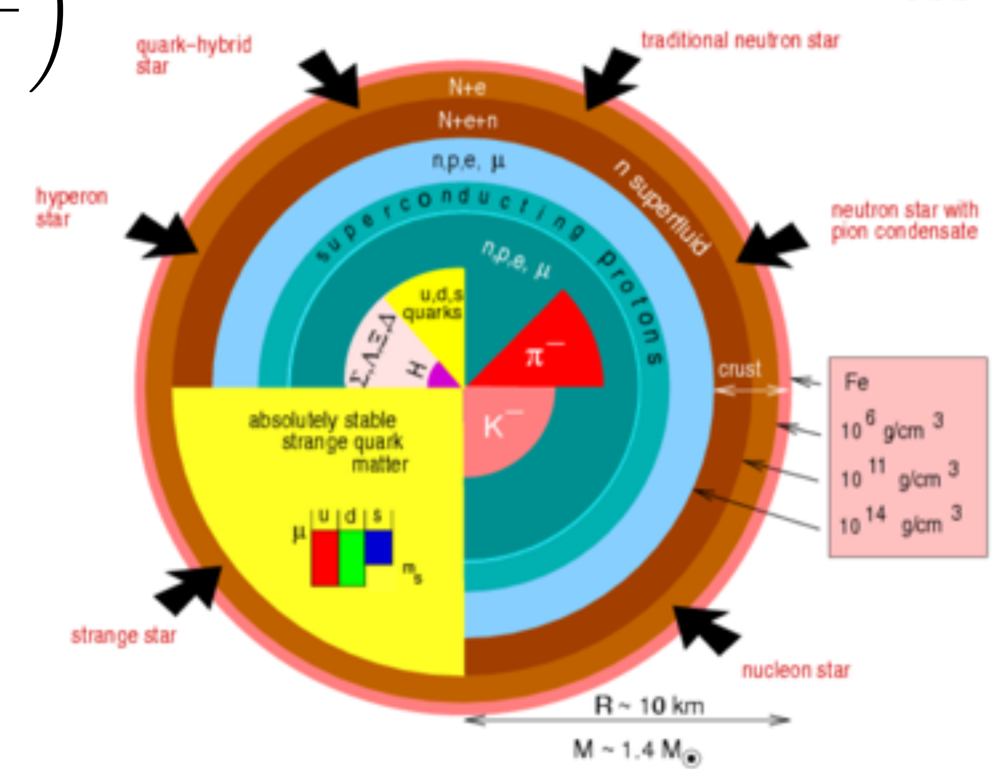
TOV

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{Mc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

Compositions of a NS

F. Weber 2005



k2 (λ,Λ) depends on NS EoS !!

TOV Eq. vs. Diff. Eq. for Tidal deformability

a spherical symmetric star in hydrostatic equilibrium

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\begin{aligned} ds_0^2 &= g^{(0)}_{\alpha\beta}\, dx^\alpha\, dx^\beta \\ &= -e^{\nu(r)}\, dt^2 + e^{\lambda(r)}\, dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \end{aligned}$$

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta + p g^{(0)}_{\alpha\beta},$$

TOV eq.

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{M c^2}\right)\left(1 - \frac{2GM}{r c^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

Mass & Radius

TOV Eq. vs. Diff. Eq. for Tidal deformability

static linearized perturbations due to an external tidal field

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu}$$

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta},$$

T. Hinderer (2008), K. Thorne and A. Campolattaro (1967)

$$h_{\alpha\beta} =$$
$$\mathrm{diag}\left[-e^{\nu(r)}H_0(r), e^{\lambda(r)}H_2(r), r^2K(r), r^2\sin^2\theta K(r)\right]Y_{2m}(\theta,\varphi).$$

$$\delta T^0_0 = -\delta\rho = -(dp/d\rho)^{-1}\delta p \qquad \delta T^i_i = \delta p$$

$$H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\}$$
$$+H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - \nu'^2\right] = 0,$$

k2 or λ

Electric-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Electric-type tidal coefficients

$$G\mu_\ell = \frac{a_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{2k_\ell}{(2\ell-1)!!}R^{2\ell+1}. \quad (48)$$

$$H'' + C_1 H' + C_0 H = 0.$$

$$C_1 = \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') = \frac{2}{r} + e^{\lambda}\left[\frac{2m}{r^2} + 4\pi r(p-e)\right], \quad (28)$$

$$\begin{aligned} C_0 &= e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(e+p)\right] \\ &\quad + \nu'' + (\nu')^2 + \frac{1}{2r}(2 - r\nu')(3\nu' + \lambda') \\ &= e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(5e+9p)\right] \\ &\quad - (\nu')^2, \end{aligned} \quad (29)$$

we calculated the case, l=2

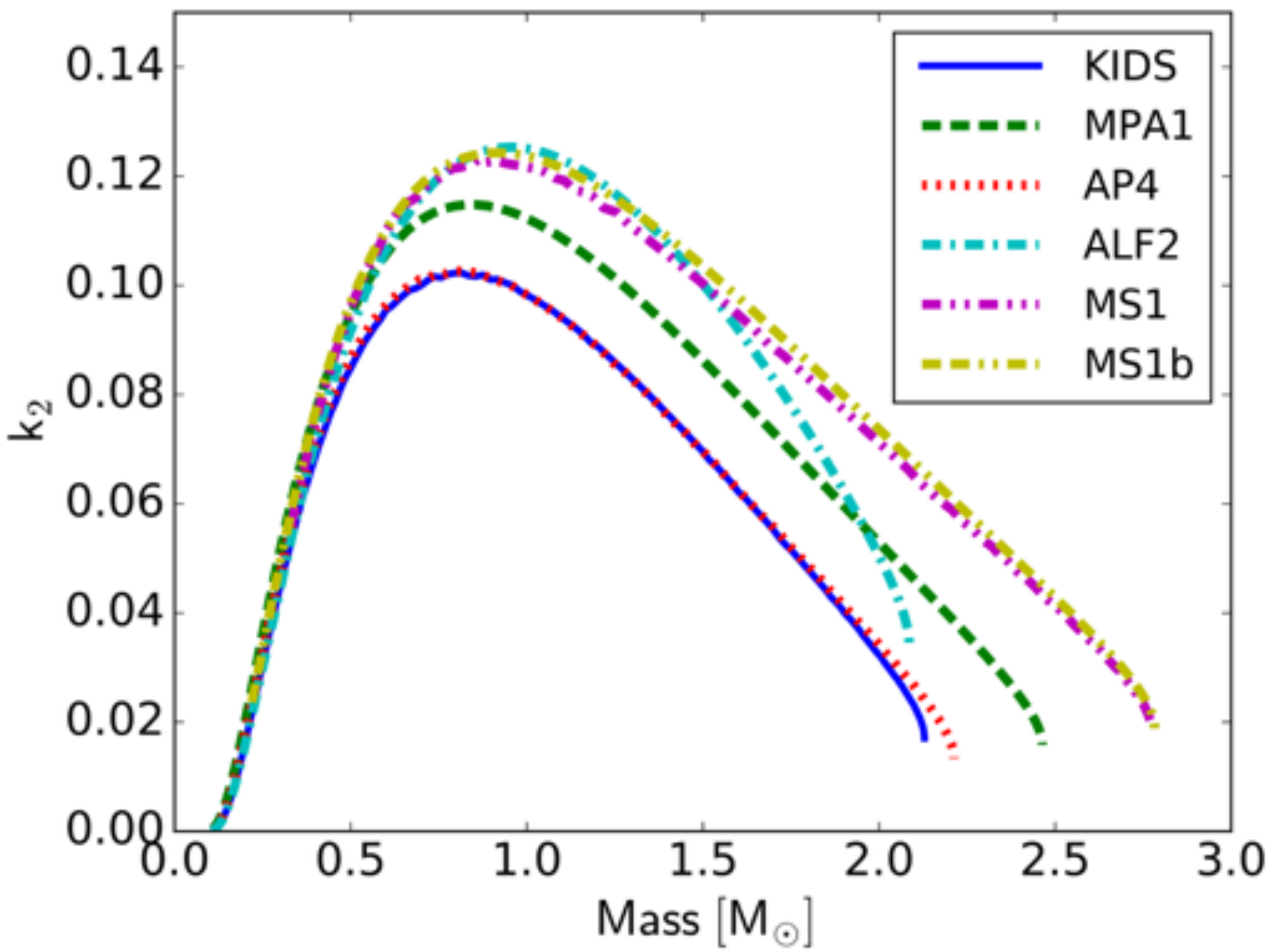
$$\begin{aligned} k_2 = \frac{8}{5}(1-2c)^2c^5[2c(y-1)-y+2]\Big\{&2c(4(y+1)c^4 + (6y-4)c^3 + (26-22y)c^2 + 3(5y-8)c - 3y + 6) \\ &- 3(1-2c)^2(2c(y-1)-y+2)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (50)$$

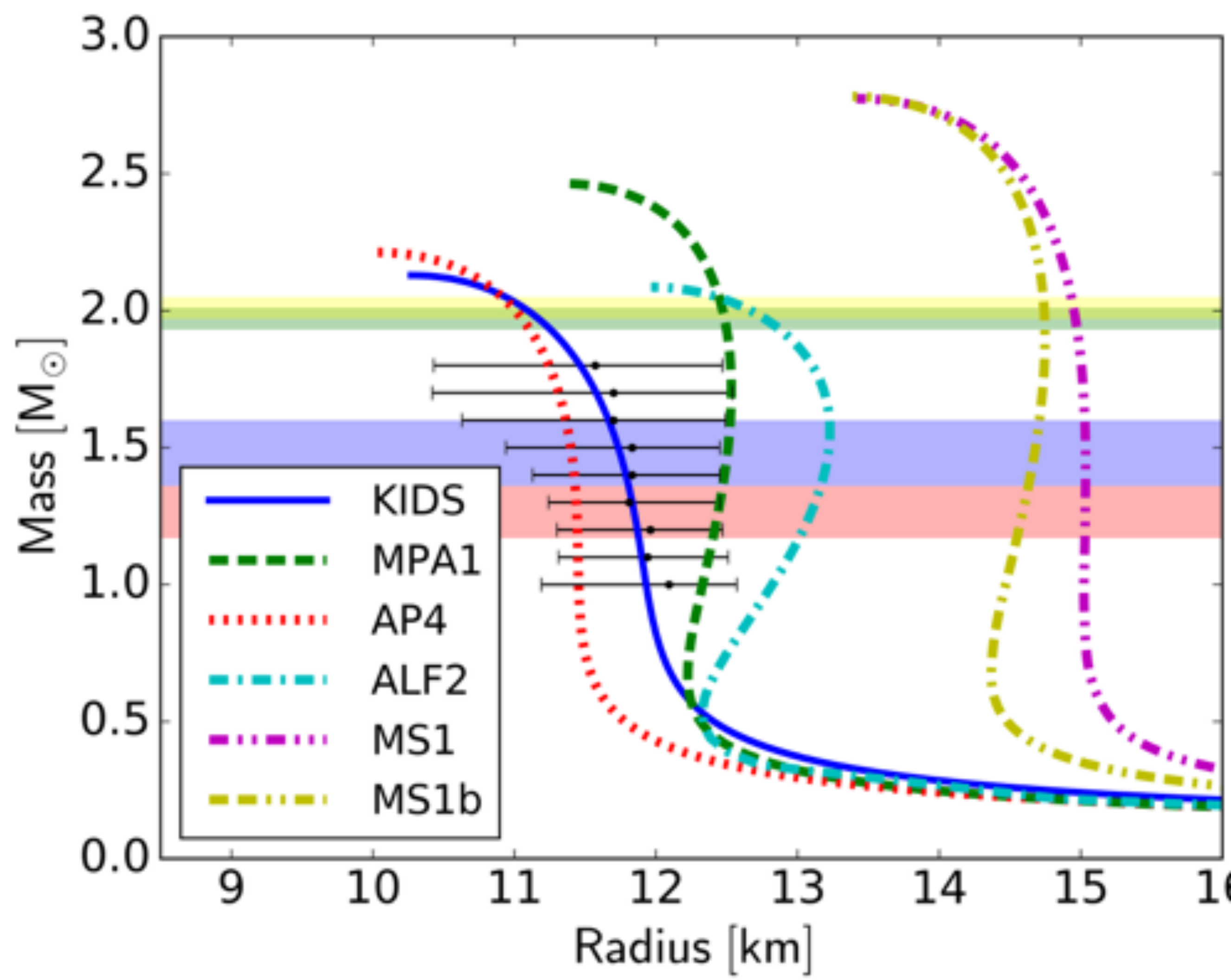
$$y_R = \frac{rH'(r)}{H(r)}\Big|_{r=R}$$

$$\begin{aligned} k_3 = \frac{8}{7}(1-2c)^2c^7[2(y-1)c^2 - 3(y-2)c + y - 3]\Big\{&2c[4(y+1)c^5 + 2(9y-2)c^4 - 20(7y-9)c^3 + 5(37y-72)c^2 \\ &- 45(2y-5)c + 15(y-3)] - 15(1-2c)^2(2(y-1)c^2 - 3(y-2)c + y - 3)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (51)$$

Tidal deformability of a NS

Kim et al., New Physics: Sae Mulli (2018)





GW170817 - Abbott et al. (LSC and Virgo), arxiv:1805.11579
- Mchirp = 1.188 M☉
- low spin prior : $\Lambda = 300^{+500}_{-190}$ (symmetric) / 300^{+420}_{-230} (HPD)
- high spin prior : Λ = 0 ~ 630

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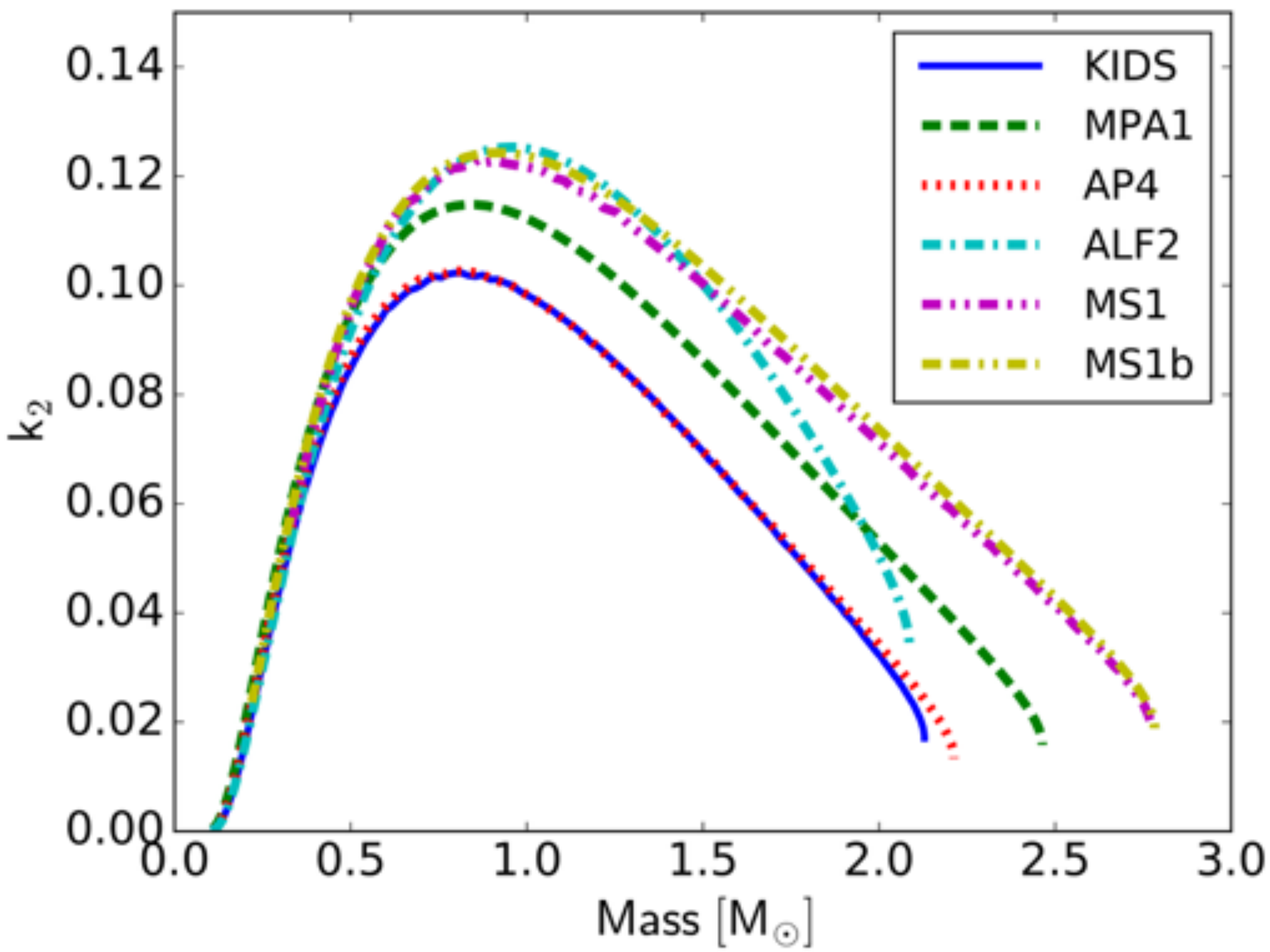

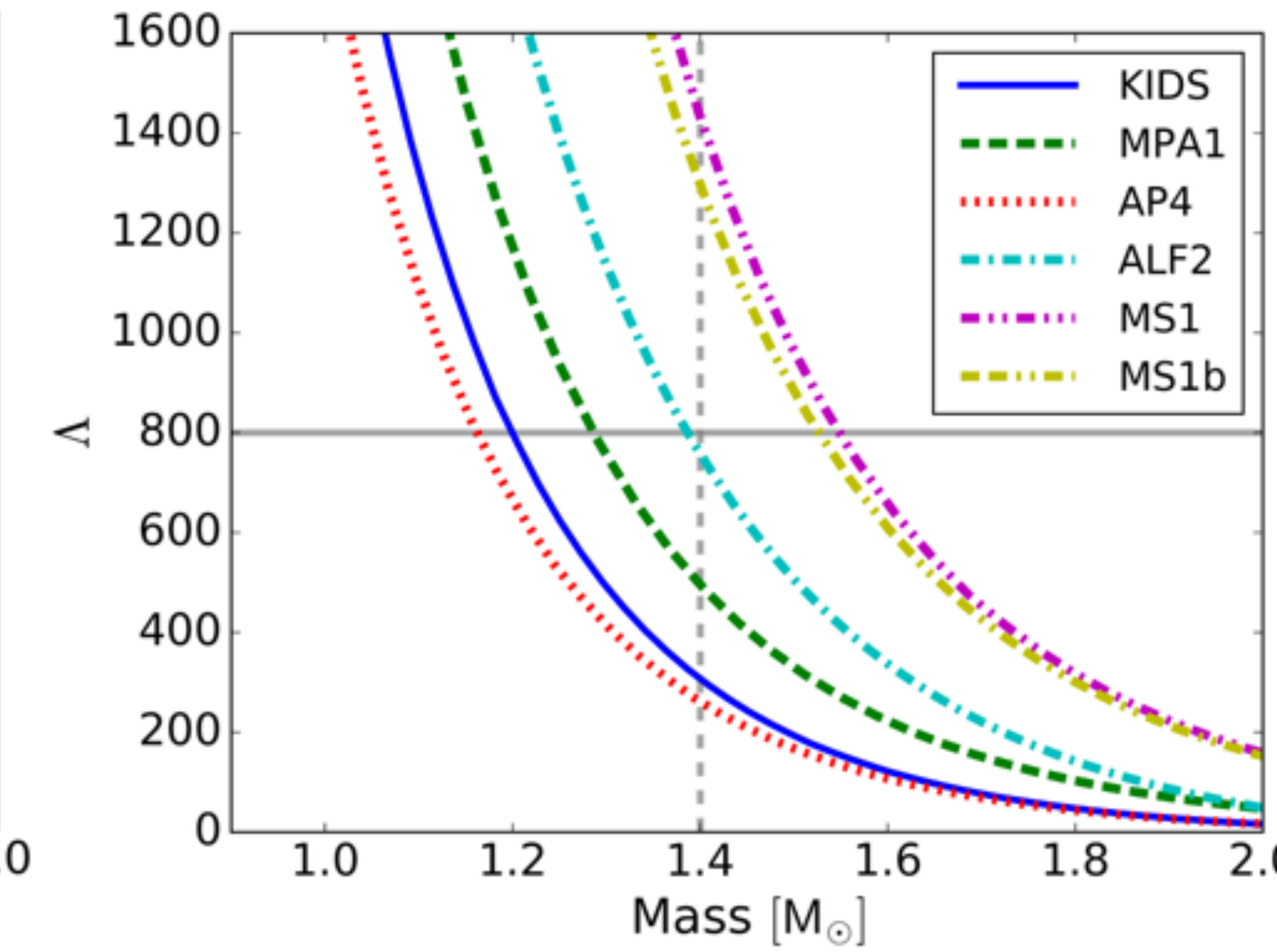

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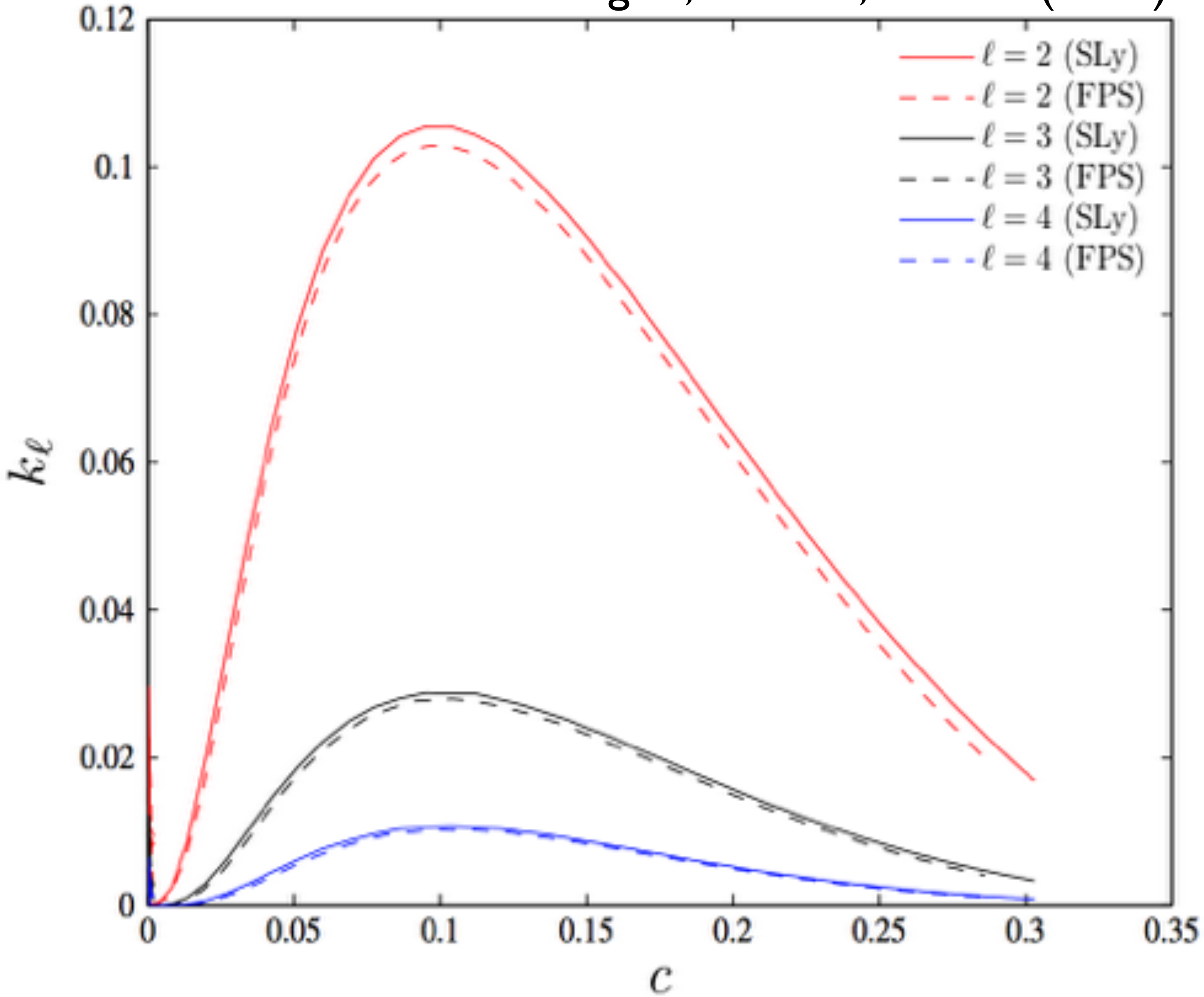
Higher Tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)



$$\Lambda = \lambda/M^5 \to G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

$$x \equiv (M\omega)^{2/3}$$

$$\Delta\Psi_2^{tidal} \sim \lambda_2 \; x^{5/2}$$

$$\Delta\Psi_3^{tidal} \sim \lambda_3 \; x^{9/2}$$

$$|\Delta\Psi_3^{tidal}/\Delta\Psi_2^{tidal}| \sim \mathcal{O}(10^{-3})$$

We hardly expect to observe higher tidal coefficients in the waveform

Accumulated GW phase (1)

the number of wave cycles in frequency domain

$$\Delta N_{\mathrm{cyc},\Psi} = \frac{1}{2\pi}\left[\Psi(f_2) - \Psi(f_1) + (f_1 - f_2)\frac{d\Psi}{df_1}\right], \qquad (7.8)$$

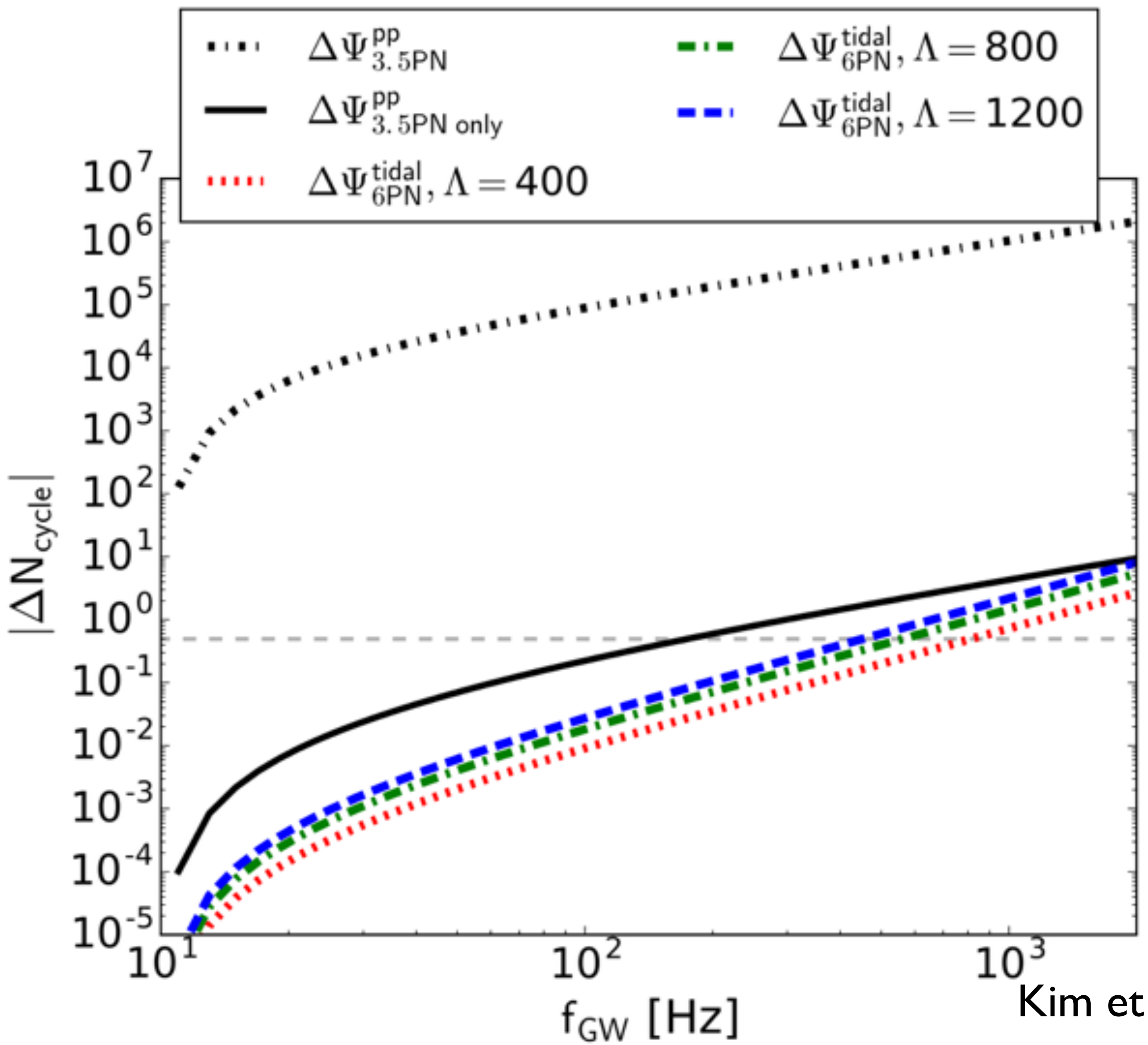
f_l = 10 Hz,
the low frequency cutoff for Advanced LIGO
due to seismic noises

Waveform models:
TaylorT2 for ΔN_{cyc}
TaylorF2(SPA) $\Delta N_{cyc,\Psi}$

Moore et al., PRD.93.124061(2016)

	$1.4M_\odot + 1.4M_\odot$, $f_2 = 1000$ Hz		
PN order	ΔN_{cyc}	$\Delta N_{\mathrm{cyc},\Psi}$	$\Delta N^{\mathrm{norm}}_{\mathrm{useful}}$
0PN(circ)	16 031	986 372	1821
0PN(ecc)	−463	−36 137	−6.37
1PN(circ)	439	21 743	125
1PN(ecc)	−15.8	−1193	−0.332
1.5PN(circ)	−208	−8520	−94.8
1.5PN(ecc)	1.67	103	0.113
2PN(circ)	9.54	294	6.70
2PN(ecc)	−0.215	−15.4	−0.008 17
2.5PN(circ)	−10.6	−218	−10.6
2.5PN(ecc)	0.0443	2.61	0.004 73
3PN(circ)	2.02	18.2	2.80
3PN(ecc)	0.002 00	0.119	−0.000 238
3.5PN(circ)	−0.662	−4.39	−0.977
Total	15 785	962 445	1843

Accumulated GW phase (2)



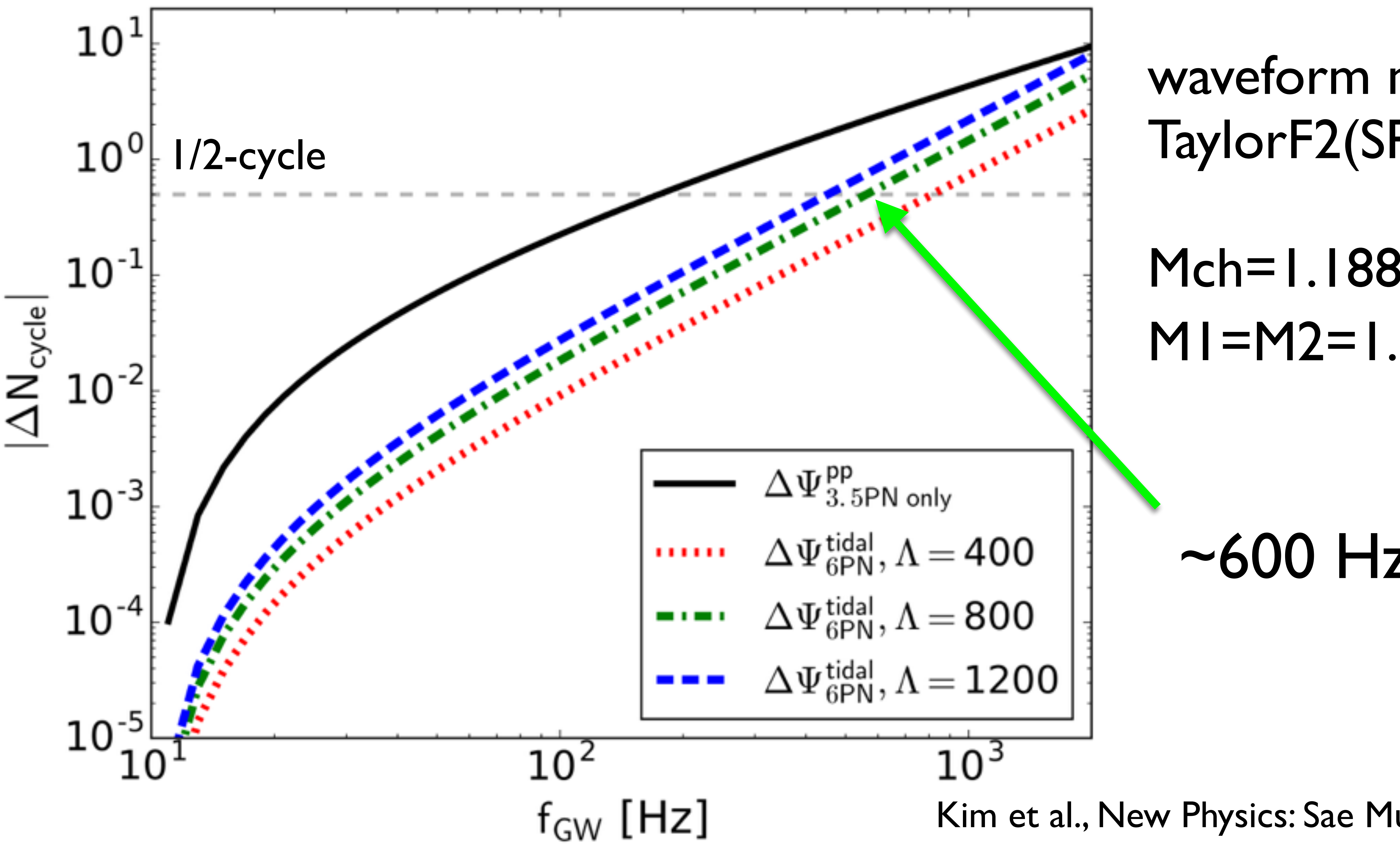
waveform model:
TaylorF2(SPA)

Mch=1.188M$_\odot$

M1=M2=1.365M$_\odot$

Kim et al., New Physics: Sae Mulli (2018)

Accumulated GW phase (2)



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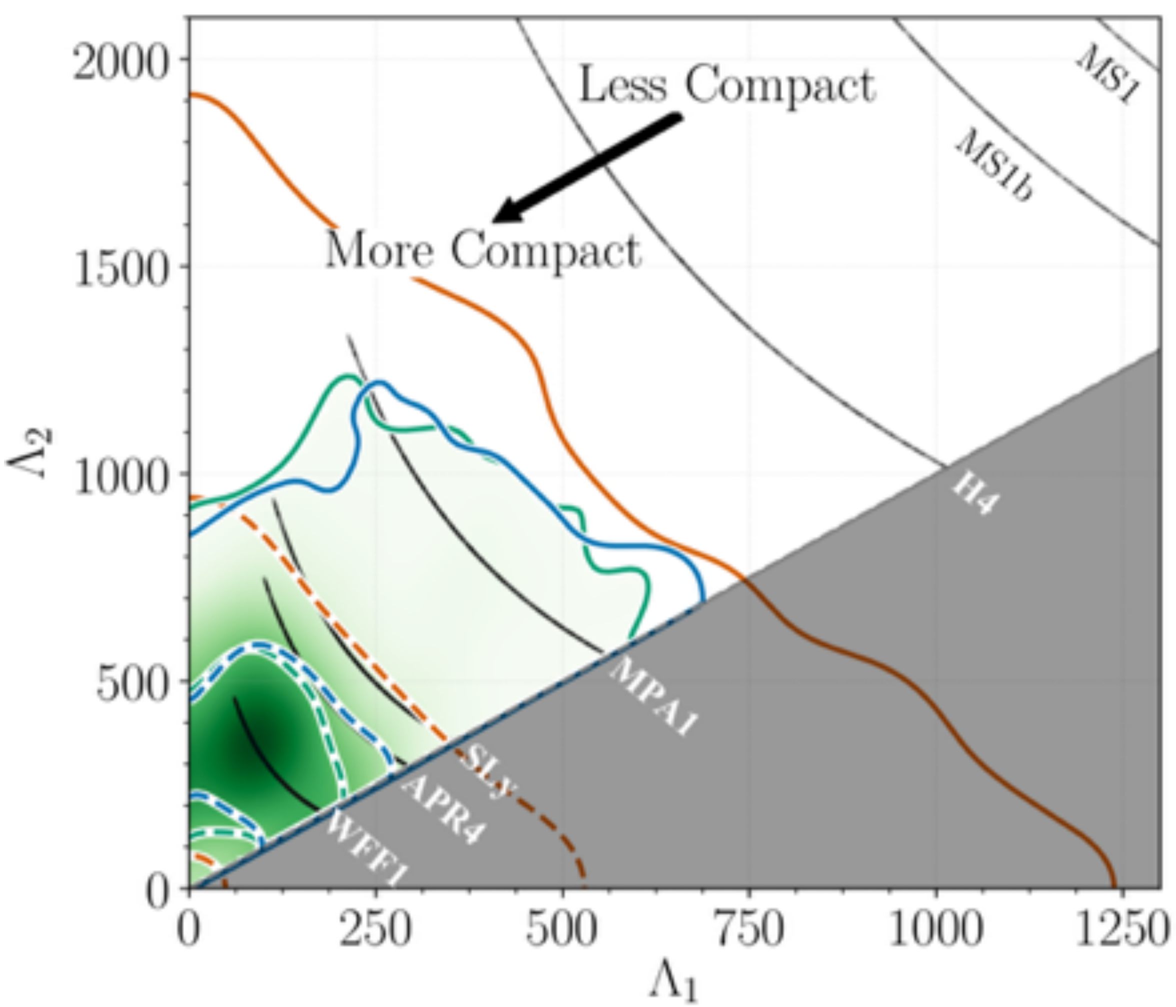
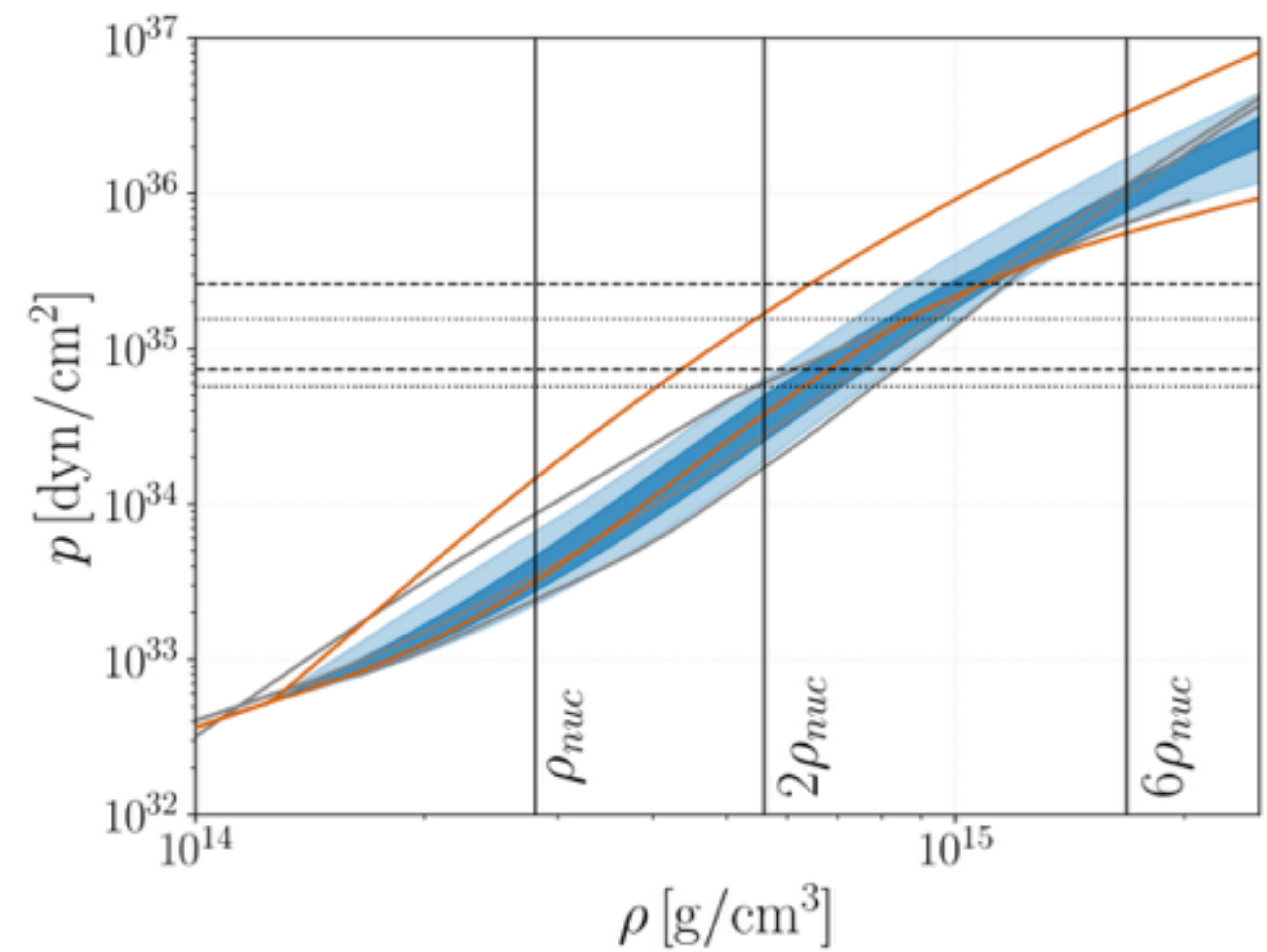
A new constraint by GW Observation

$\Lambda(1.4M_\odot)=190^{+390}_{-120}$



$P(2\ \rho_{nuc}) = 3.5^{+2.7}_{-1.7} \times 10^{34}$ dyn/cm^2

$P(6\ \rho_{nuc}) = 9.0^{+7.9}_{-2.6} \times 10^{35}$ dyn/cm^2

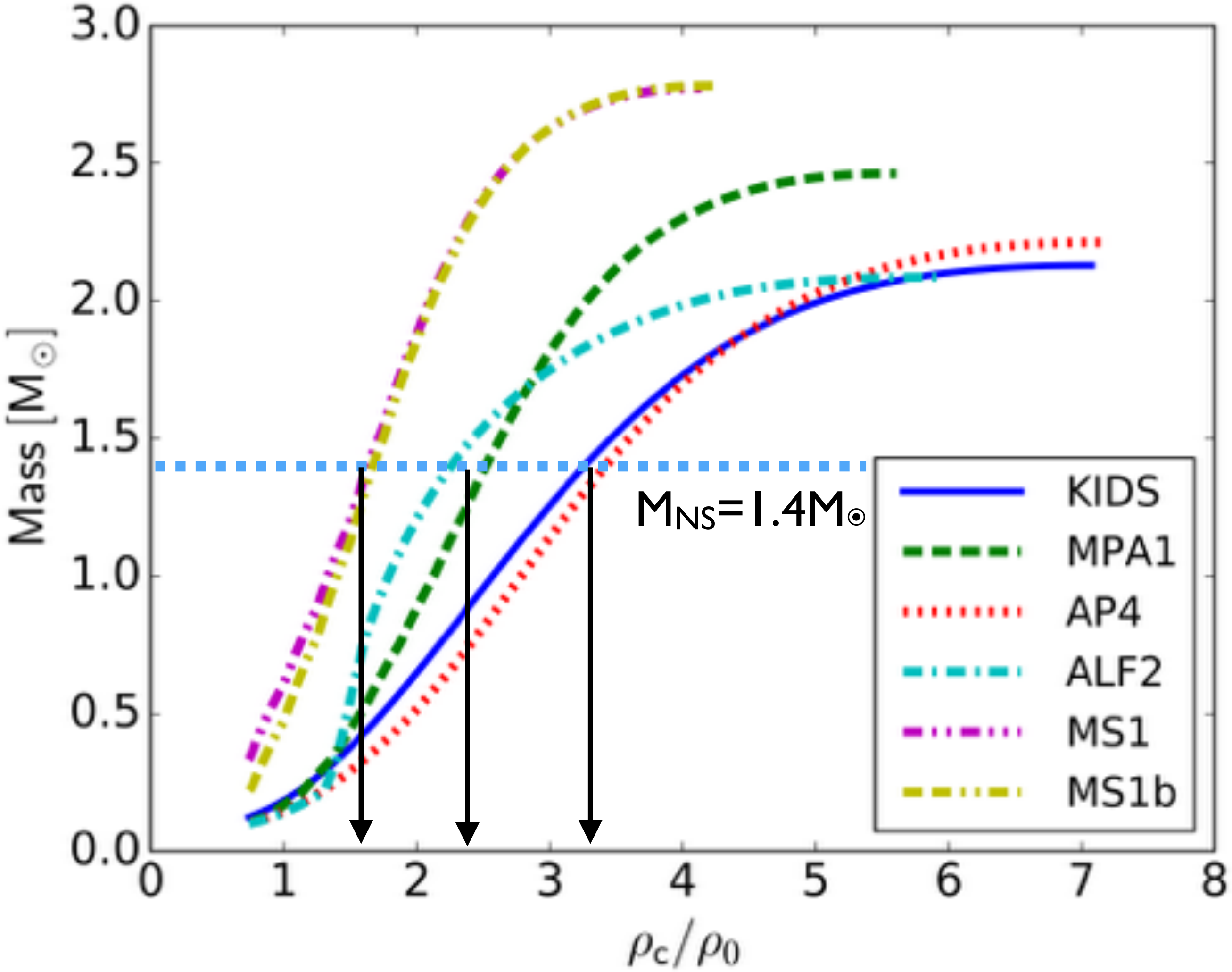


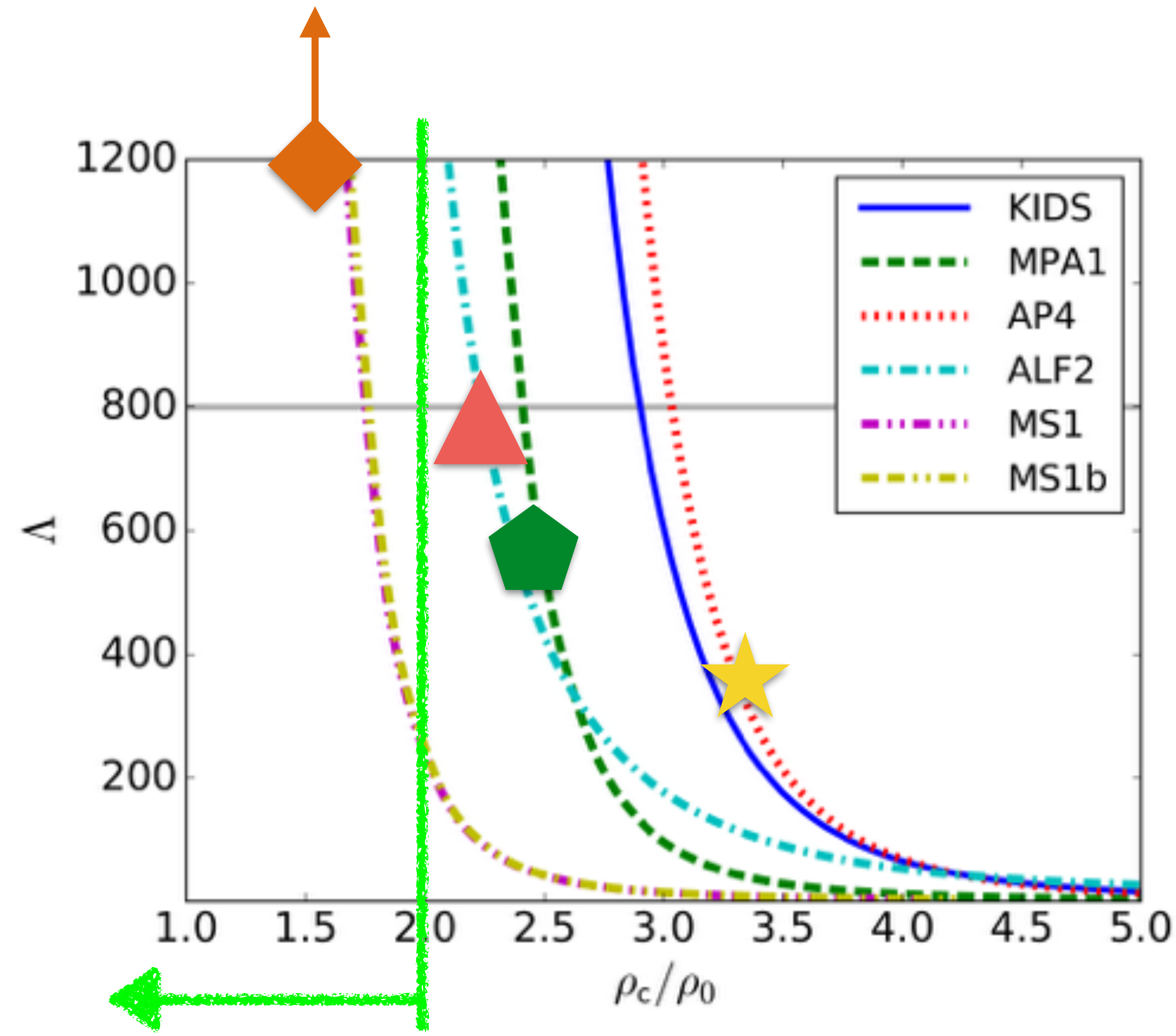
Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PRL accepted)

$\rho_{nuc} = 2.8 \times 10^{14}$ g/cm^3

Central Density at M_{NS}=1.4 $M_\odot$

Kim et al., New Physics: Sae Mulli (2018)





ruled out ?

GW170817 - Abbott et al. (LSC and Virgo), arxiv:1805.11579

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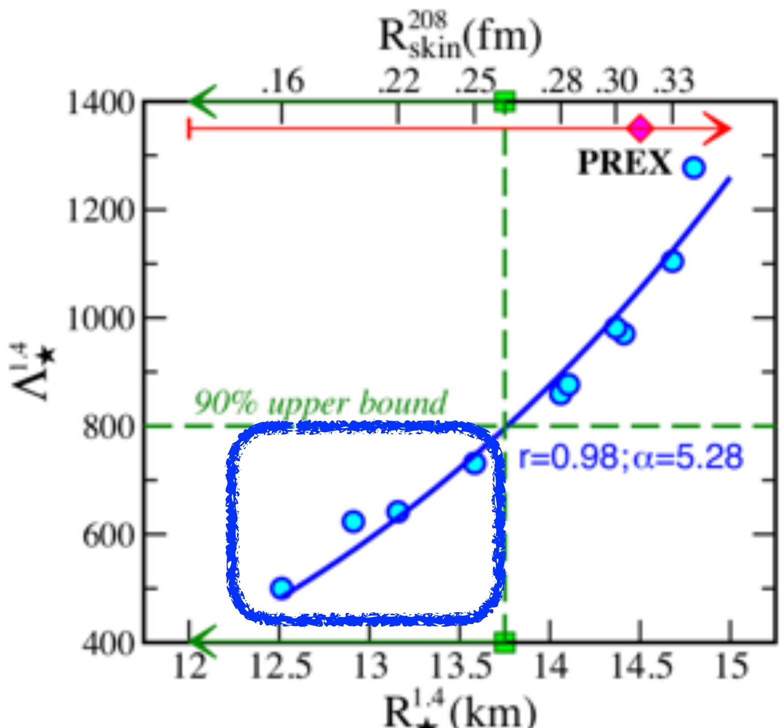
$$\Lambda(1.4M_\odot)=190^{+390}_{-120}$$

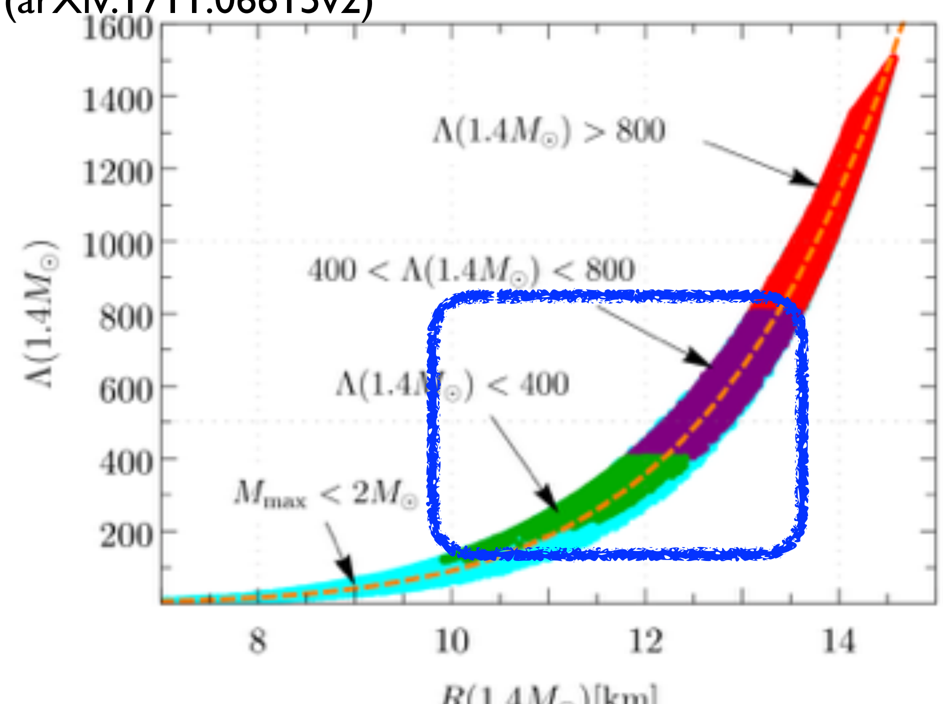
Recent Researches

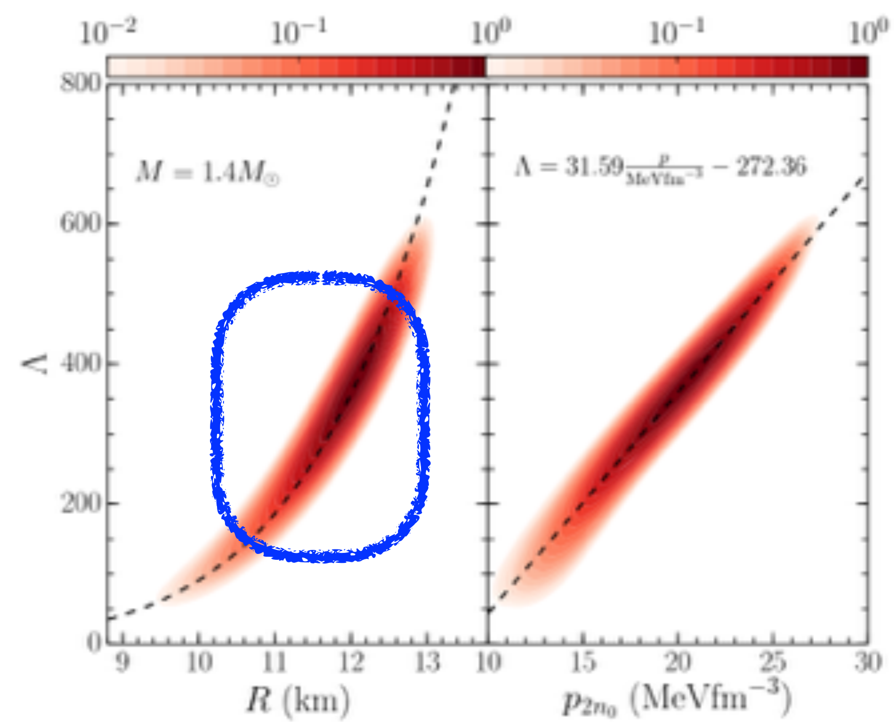


[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, PRL. 120.172702 (arXiv:1711.06615v2)

EOS	R	β	k_2	λ	L
APR	11.55	0.179	0.0721	1.48	62
MDI ($x = 0$)	11.85	0.174	0.0707	1.65	62
MDI ($x = -1$)	13.59	0.152	0.0831	3.85	107
DBHF+Bonn B	12.64	0.163	0.0946	3.06	69
FPS	10.84	0.191	0.0664	1.00	35
SLY4	11.72	0.176	0.0762	1.68	47

[B] P.G. Krastev, and B.-A. Li, arXiv:1801.04620v1



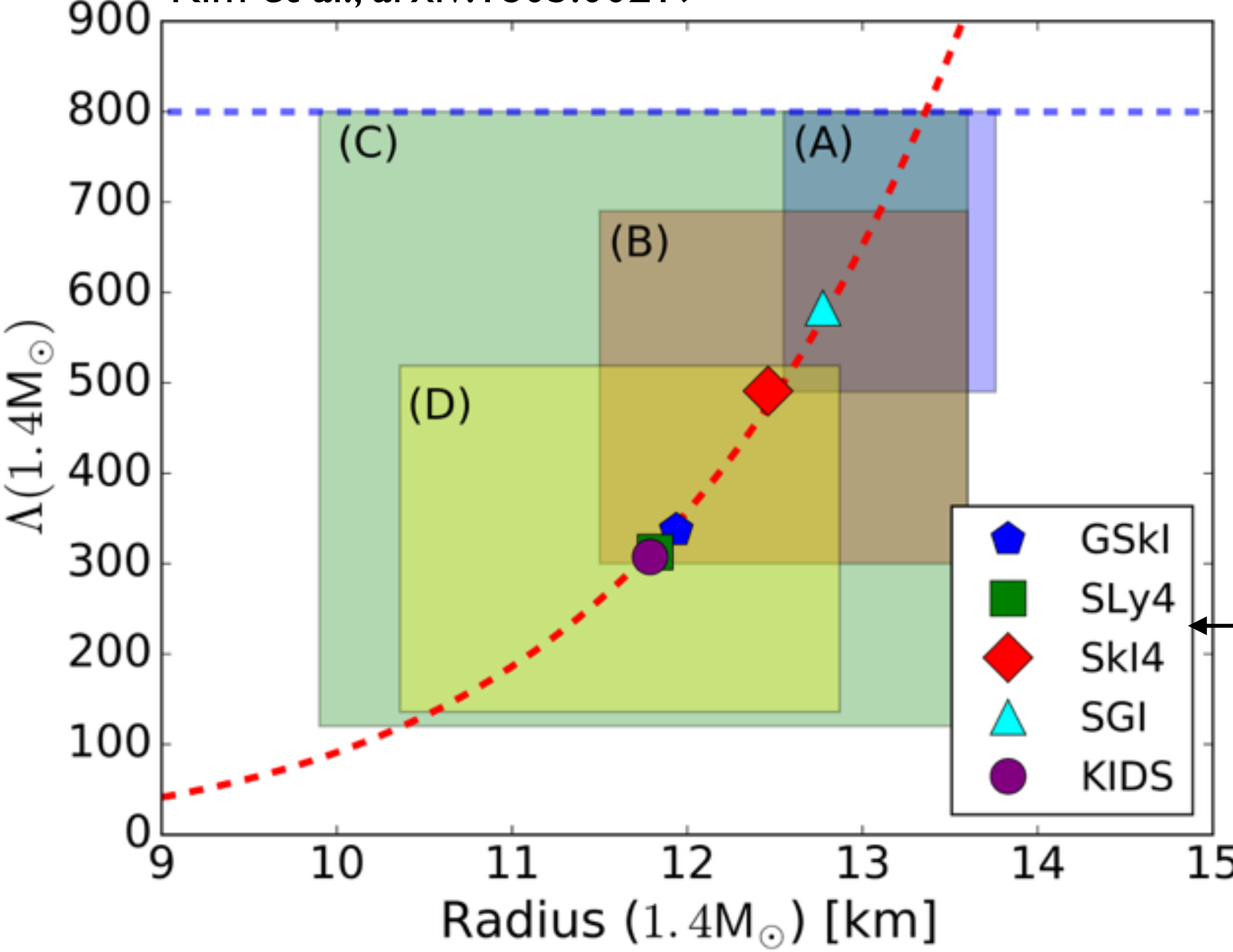


[D] Y.Lim and J. Holt, PRL.121.062701 (arXiv:1803.02803v2)

[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, PRL. 120.172703 (arXiv:1711.02644v2)

Comparison with recent works

Kim et al., arxiv:1805.00219



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[B] P.G. Krastev, and B.-A. Li, arXiv: 1801.04620v1
[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, PRL.120.172703 (arXiv: 1711.02644v2)
[D] Y.Lim and J. Holt, PRL.121.062701 (arXiv:1803.02803v2)

5 Skyrme force models satisfying 11 constraints from Dutra et al. Phys.Rev. C 85, 035201 (2012)

Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6} (R/km)^{7.5}$ (fitting function in [C])

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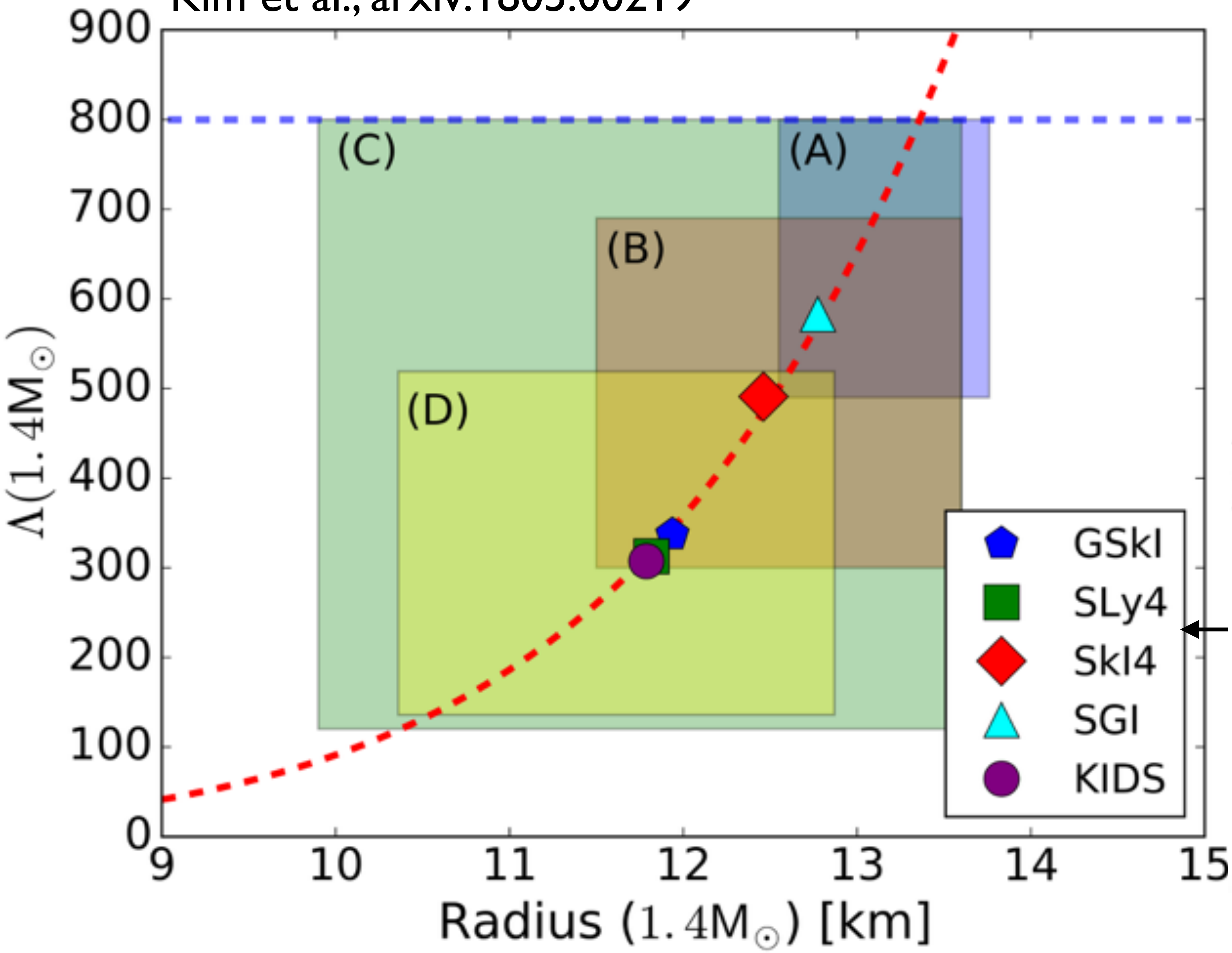

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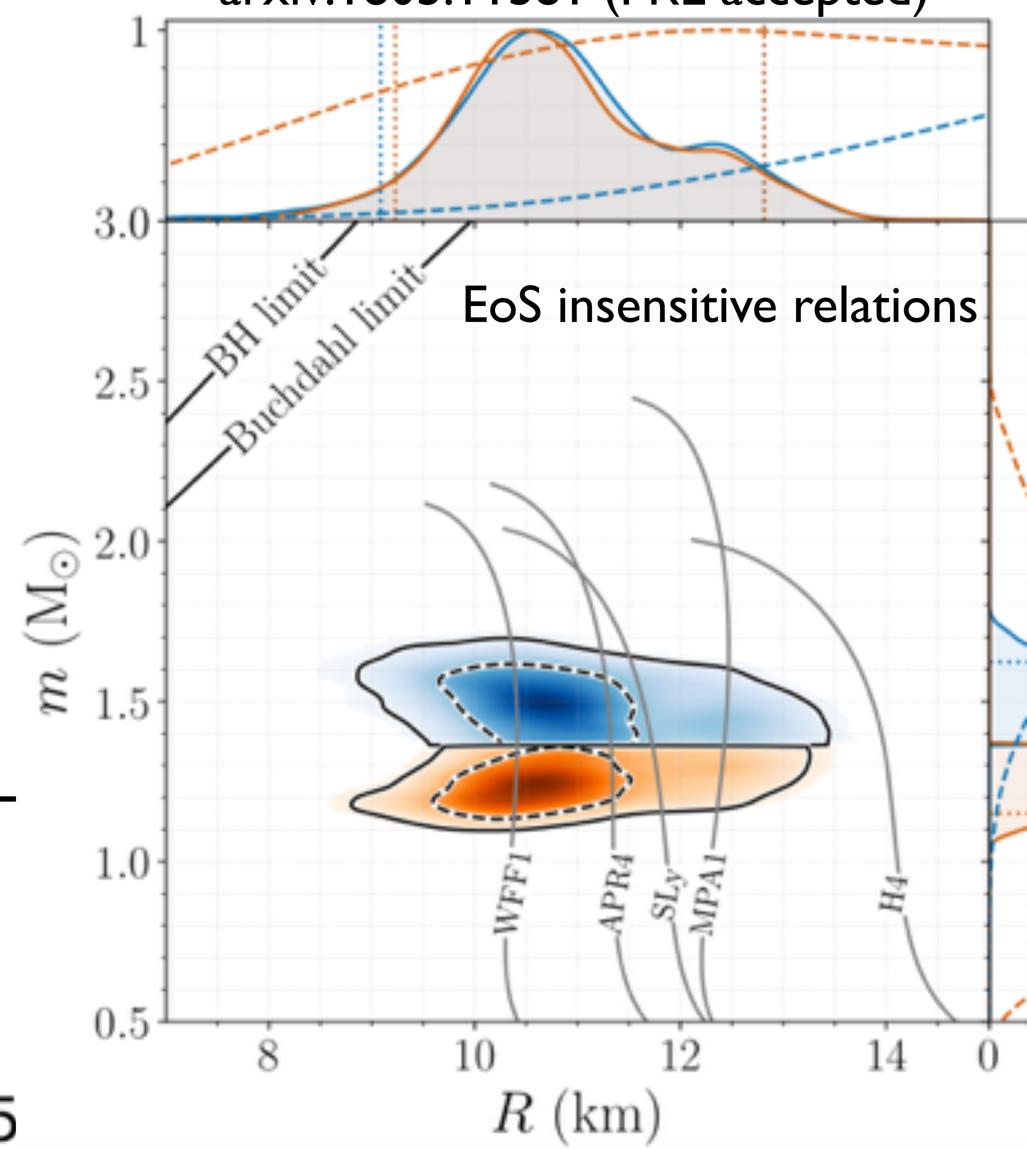

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Prospects of the Observing Runs

"Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO, Advanced Virgo and KAGRA", arXiv:1304.0670v4, LIGO-P1200087-v45, Living Rev. Relativity, 21, 3 (2018)

Epoch			2015 – 2016	2016 – 2017	2018 – 2019	2020+	2024+
Planned run duration			4 months	9 months	12 months	(per year)	(per year)
Expected burst range/Mpc		LIGO	40 – 60	60 – 75	75 – 90	105	105
		Virgo	—	20 – 40	40 – 50	40 – 70	80
		KAGRA	—	—	—	—	100
Expected BNS range/Mpc		LIGO	40 – 80	80 – 120	120 – 170	190	190
		Virgo	—	20 – 65	65 – 85	65 – 115	125
		KAGRA	—	—	—	—	140
Achieved BNS range/Mpc		LIGO	60 – 80	60 – 100	—	—	—
		Virgo	—	25 – 30	—	—	—
		KAGRA					
Estimated BNS detections			0.05 – 1	0.2 – 4.5	1 – 50	4 – 80	11 – 180
Actual BNS detections			0	1	—	—	—
90% CR	% within	5 deg^2	< 1	1 – 5	1 – 4	3 – 7	23 – 30
		20 deg^2	< 1	7 – 14	12 – 21	14 – 22	65 – 73
	median/deg^2		460 – 530	230 – 320	120 – 180	110 – 180	9 – 12
Searched area	% within	5 deg^2	4 – 6	15 – 21	20 – 26	23 – 29	62 – 67
		20 deg^2	14 – 17	33 – 41	42 – 50	44 – 52	87 – 90

We expect to observe more BNS and/or NS-BH

Summary

1. Tidal deformability of a neutron star can be observed by gravitational-wave detection.

    - The most dominant tidal coefficient is l=2 electric-type coefficient $\lambda 2$.

    - The weighted Λ in BNS was estimated by observation of GW170817

2. Tidal deformability is a new constraint on nuclear equation of states provided by GW observation.

    - more compact NS EoS is preferred.

3. In coming GW Obs., NS EoS can be studied more precisely.

4. Further investigation on Λ and NS EoS will be conducted by using Bayesian analyses on GW DA as well as HIC to study Esym.

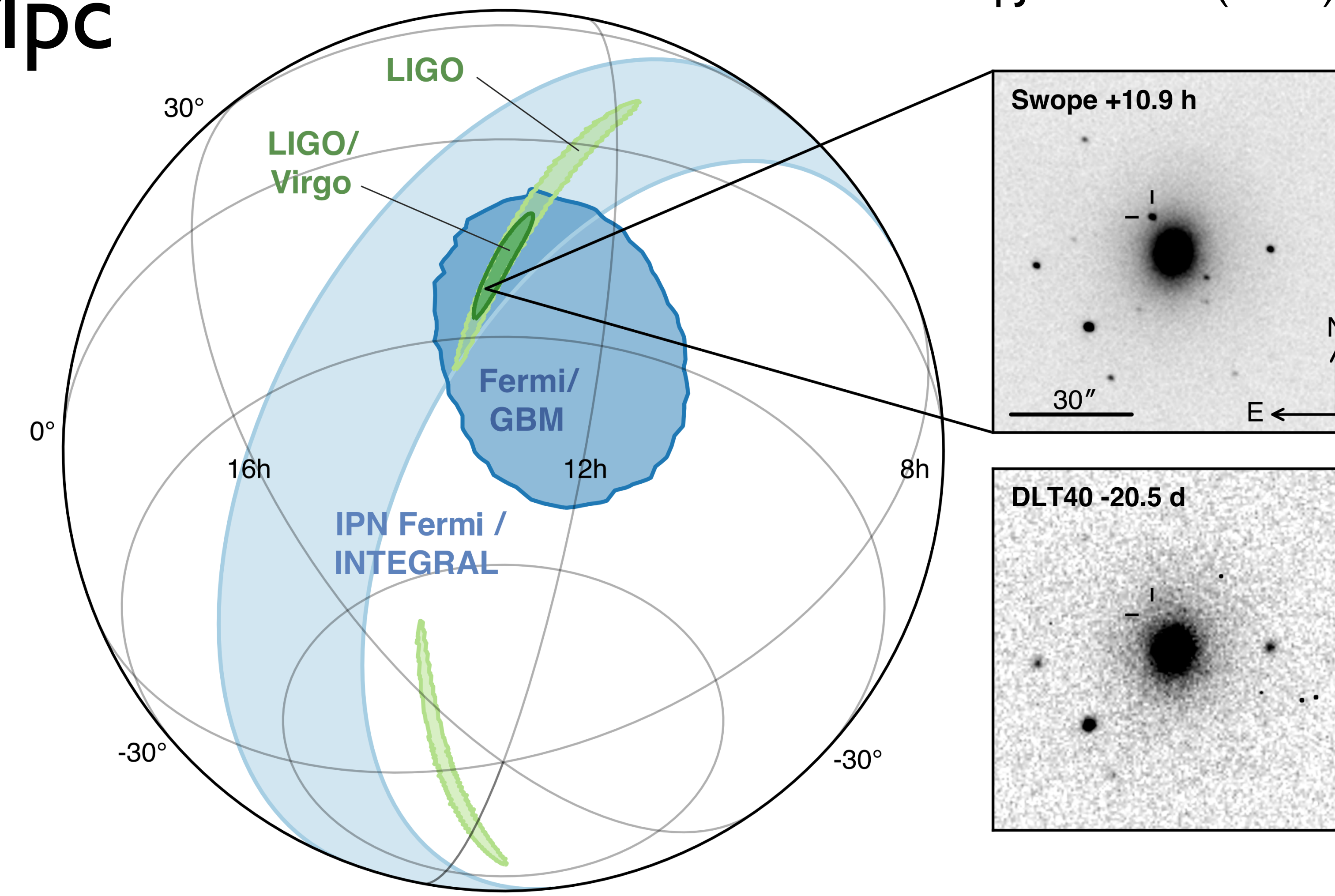
Thank you for your attention.

Extra Slides

Localization

$D_L \sim 40$ Mpc

ApJL.848.L12(2017)



AT2017gfo in NGC4993

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$G\sigma_\ell = \frac{\ell-1}{4(\ell+2)}\frac{b_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{\ell-1}{4(\ell+2)}\frac{j_\ell}{(2\ell-1)!!}R^{2\ell+1},$$

$$G\sigma_2 = \frac{1}{48}j_2R^5 = \frac{1}{48}b_2\left(\frac{GM}{c_0^2}\right)^5,$$

$$\psi'' + \frac{e^\lambda}{r^2}[2m + 4\pi r^3(p-e)]\psi' - e^\lambda\left[\frac{\ell(\ell+1)}{r^2} - \frac{6m}{r^3} + 4\pi(e-p)\right]\psi = 0. \quad (31)$$

$$j_\ell \equiv c^{2\ell+1}b_\ell = -c^{2\ell+1}\frac{\psi_P'(\hat{r}) - c y_{\rm odd}\psi_P(\hat{r})}{\psi_Q'(\hat{r}) - c y_{\rm odd}\psi_Q(\hat{r})}\bigg|_{\hat{r}=1/c}.$$

$$j_2 = \frac{96c^5(2c-1)(y-3)}{5(2c(12(y+1)c^4 + 2(y-3)c^3 + 2(y-3)c^2 + 3(y-3)c - 3y + 9) + 3(2c-1)(y-3)\log(1-2c))}. \quad (73)$$

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

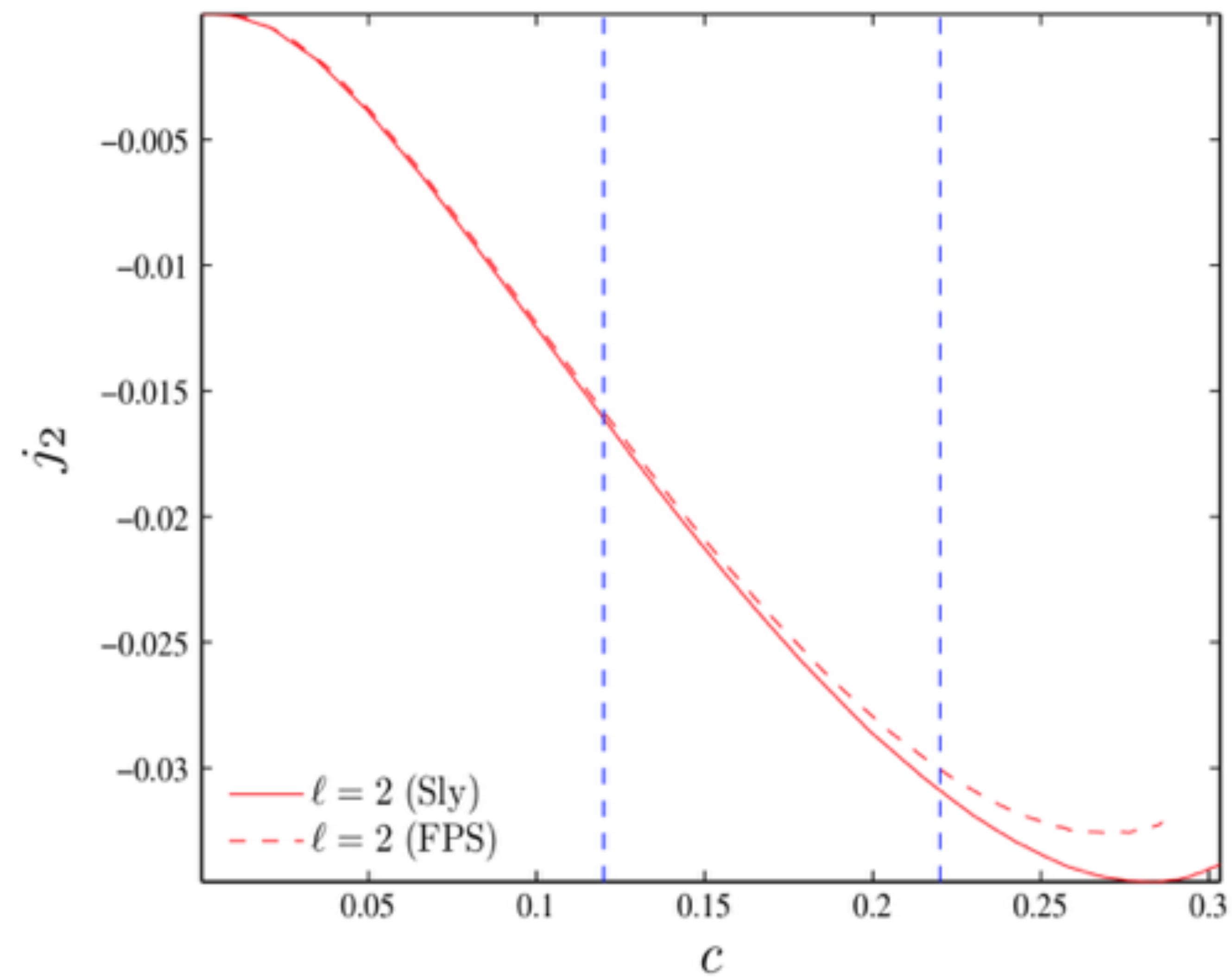
Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$\psi'' + \ldots$$

$$G\sigma_\ell = \frac{\ell - 1}{4(\ell + 2)} \frac{b_\ell}{(2\ell - 1)!!} \left(\frac{GM}{c_0^2}\right)^{2\ell+1}$$
$$= \frac{\ell - 1}{4(\ell + 2)} \frac{j_\ell}{(2\ell - 1)!!} R^{2\ell+1},$$

$$j_\ell \equiv \ldots$$



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Higher order terms in GW phase (1)

Rotational-tidal phasing of the binary neutron star waveform

- arxiv:1805.01882

$$\Psi = \frac{3M}{128\mu} x^{-2.5} \left[1 - \frac{39}{2} \tilde{\Lambda}\, x^5 + \tilde{\Sigma}\, x^6 - \tilde{X}\, x^{6.5} - \tilde{\Lambda}_3\, x^7 + \tilde{\Sigma}_3\, x^8 \right], \qquad (8)$$

coupled to spins

$$\tilde{X} = \frac{1}{21 M^6} c^{12} \Big\{ \chi^{(1)} \left[36(35 + 614q)\hat{\lambda}_2^{(1)} - (7 - 4751q)\hat{\sigma}_2^{(1)} - 2316q\hat{\lambda}_3^{(1)} - 3474q\hat{\sigma}_3^{(1)} \right] + \chi^{(2)} \left[36(35 + 614/q)\hat{\lambda}_2^{(2)} - (7 - 4751/q)\hat{\sigma}_2^{(2)} - 2316\hat{\lambda}_3^{(2)}/q - 3474\hat{\sigma}_3^{(2)}/q \right] \Big\},$$

$$\tilde{\Lambda}_3 = \frac{4000}{9M^7} c^{14} (q\lambda_3^{(1)} + \lambda_3^{(2)}/q),$$

$$\tilde{\Sigma}_3 = \frac{29925}{11M^7} c^{14} (q\sigma_3^{(1)} + \sigma_3^{(2)}/q).$$

Higher order terms in GW phase (2)

Post-Newtonian spin-tidal couplings for compact binaries - arxiv:1805.01487

PN order	λ_2	σ_2	$\lambda_{23,32},\ \sigma_{23,32}$	λ_3	σ_3
5	LO $\propto \Lambda$				
6	NLO $\propto \delta\Lambda$	LO $\propto \Sigma$			
6.5	**NNLO** $\propto \tilde{\Lambda}$	**NLO** $\propto \tilde{\Sigma}$	**LO** $\propto \tilde{\Gamma}$		
7	…	…	…	LO	
8	…	…	…	…	LO

$$\psi(x) = \frac{3}{128\nu x^{5/2}}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\nu\right)x + \left(\frac{113}{3} \times \right.\right.$$
$$\left. \times\ (\eta_1\chi_1 + \eta_2\chi_2) - \frac{38}{3}\nu(\chi_1 + \chi_2)\right)x^{1.5} + O(x^2)$$
$$\left. + \Lambda x^5 + (\delta\Lambda + \Sigma)x^6 + (\tilde{\Lambda} + \tilde{\Sigma} + \tilde{\Gamma})x^{6.5} + O(x^7)\right\}, \tag{15}$$

$$\Lambda = \left(264 - \frac{288}{\eta_1}\right)\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2)\,, \tag{16}$$

$$\delta\Lambda = \left(\frac{4595}{28} - \frac{15895}{28\eta_1} + \frac{5715\eta_1}{14} - \frac{325\eta_1^2}{7}\right)\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2)\,, \tag{17}$$

$$\Sigma = \left(\frac{6920}{7} - \frac{20740}{21\eta_1}\right)\frac{c^8\sigma_2^{(1)}}{M^5} + (1 \leftrightarrow 2)\,.$$

$$\tilde{\Lambda} = \left[\left(\frac{593}{4} - \frac{1105}{8\eta_1} + \frac{567\eta_1}{8} - 81\eta_1^2\right)\chi_2 + \left(-\frac{6607}{8} + \frac{6639\eta_1}{8} - 81\eta_1^2\right)\chi_1\right]\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1 \leftrightarrow 2)\,, \tag{19}$$

$$\tilde{\Sigma} = \left[\left(-\frac{9865}{3} + \frac{4933}{3\eta_1} + 1644\eta_1\right)\chi_2 - \chi_1\right]\frac{c^8\sigma_2^{(1)}}{M^5} + (1 \leftrightarrow 2)\,, \tag{20}$$

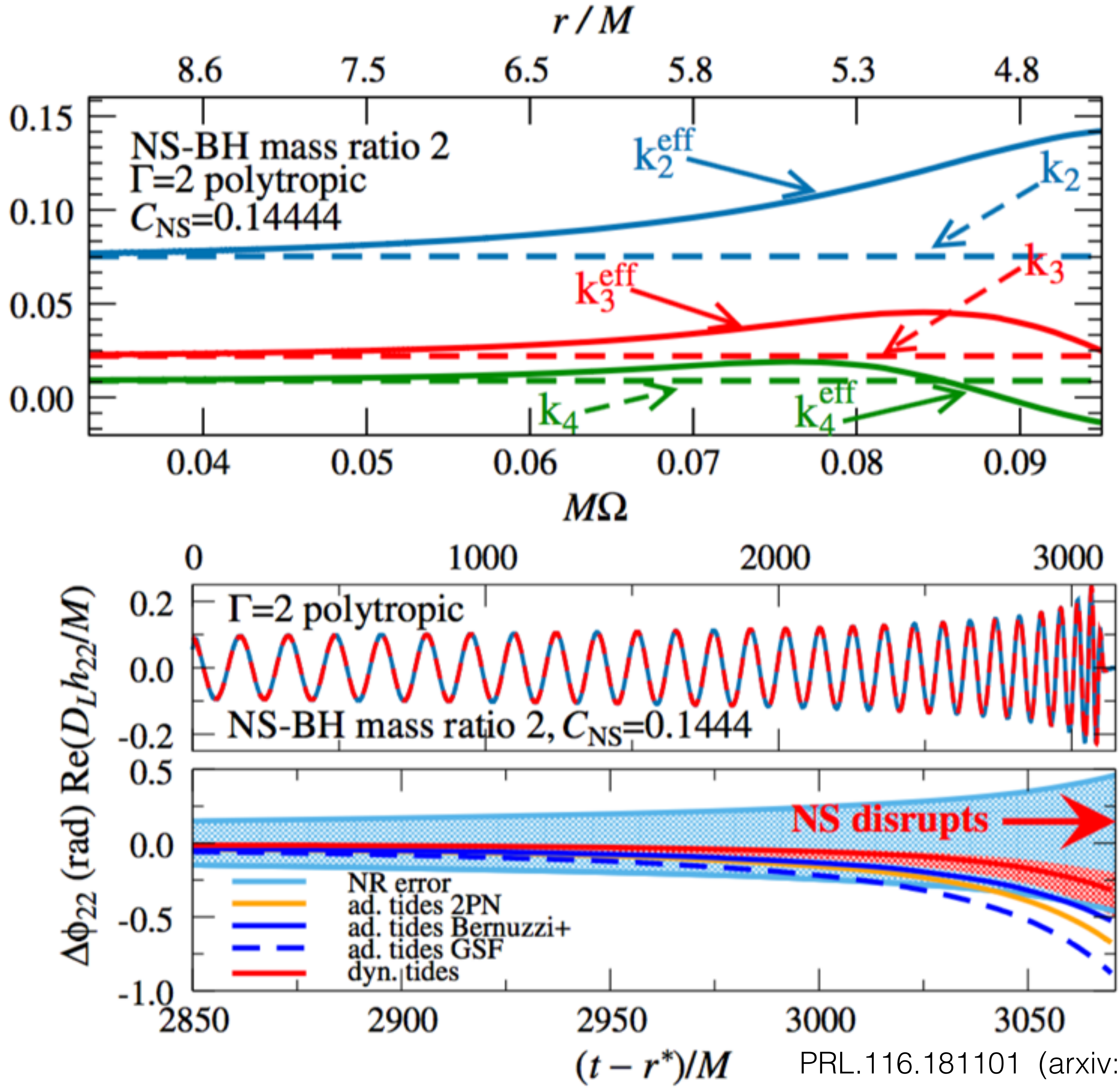
$$\tilde{\Gamma} = \frac{c^{10}\chi_1}{M^4}\left[\left(856\eta_1 - 816\eta_1^2\right)\lambda_{23}^{(1)} - \left(\frac{4993\eta_1}{18} - \frac{2497\eta_1^2}{9}\right)\sigma_{23}^{(1)} - \nu\left(272\lambda_{32}^{(1)} - 204\sigma_{32}^{(1)}\right)\right] + (1 \leftrightarrow 2)\,. \tag{21}$$

Dynamic tide

$$k_\ell^{\rm eff} = k_\ell\left[a_\ell + \frac{b_\ell}{2}\left(\frac{Q^{\rm DT}_{m=\ell}}{Q^{\rm AT}_{m=\ell}} + \frac{Q^{\rm DT}_{m=-\ell}}{Q^{\rm AT}_{m=-\ell}}\right)\right],$$

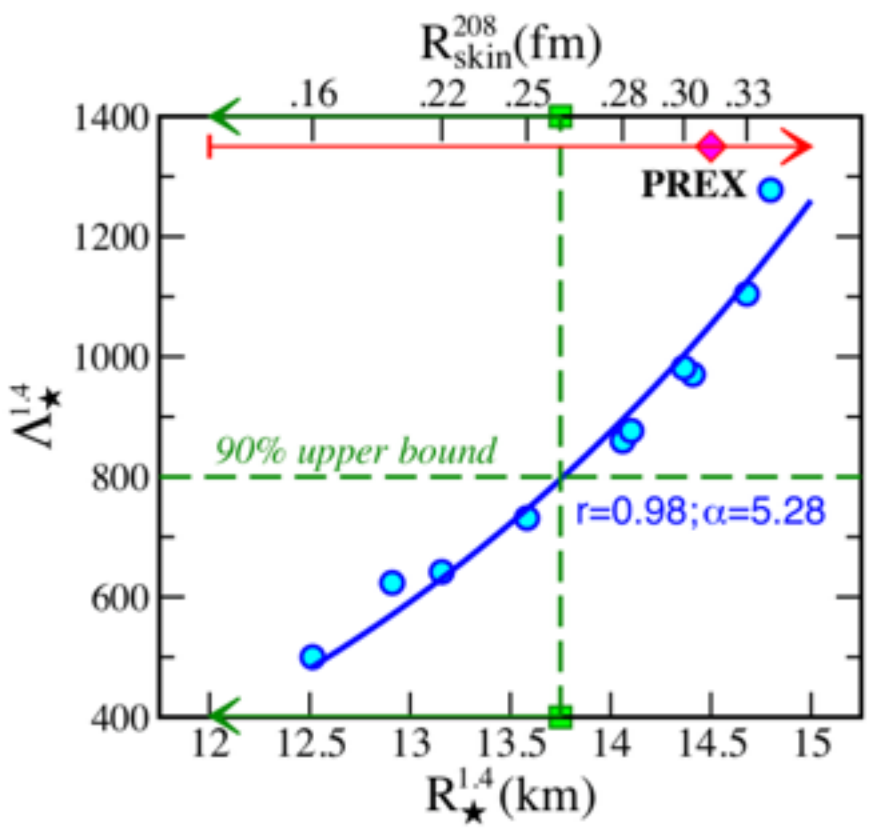
$$\frac{Q_m^{\rm DT}}{Q_m^{\rm AT}} \approx \frac{\omega_f^2}{\omega_f^2-(m\Omega)^2} + \frac{\omega_f^2}{2(m\Omega)^2\epsilon_f\Omega'_f(\phi-\phi_f)} \pm \frac{i\omega_f^2}{(m\Omega)^2\sqrt{\epsilon_f}} e^{\pm i\Omega'_f\epsilon_f(\phi-\phi_f)^2}\int_{-\infty}^{\sqrt{\epsilon_f}(\phi-\phi_f)} e^{\mp i\Omega'_f s^2} ds, \tag{2}$$

PRL.116.181101 (arxiv:1602.00599)



PRL.116.181101 (arxiv:1602.00599)

Recent Researches (1)



EOS	R	β	k_2	λ	L
APR	11.55	0.179	0.0721	1.48	62
MDI ($x = 0$)	11.85	0.174	0.0707	1.65	62
MDI ($x = -1$)	13.59	0.152	0.0831	3.85	107
DBHF+Bonn B	12.64	0.163	0.0946	3.06	69
FPS	10.84	0.191	0.0664	1.00	35
SLY4	11.72	0.176	0.0762	1.68	47

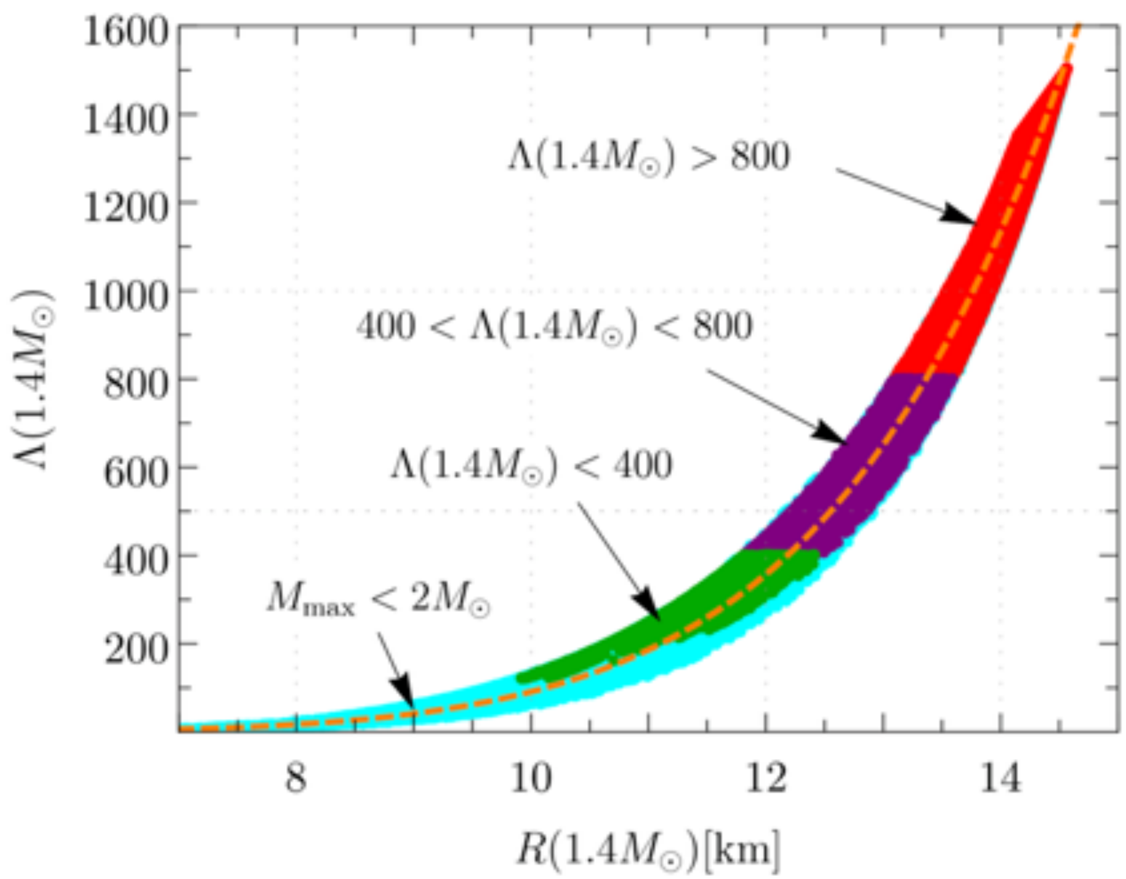
[B] P.G. Krastev, and B.-A. Li, arXiv:1801.04620v1

[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, arXiv:1711.06615v2

- RMF models
- Correlating neutron skin of ^{208}Pb , Λ(1.4M☉) and R(1.4M☉)
- 490 < Λ(1.4M☉) < 800
- 12.55 km < R(1.4M☉) < 13.76 km

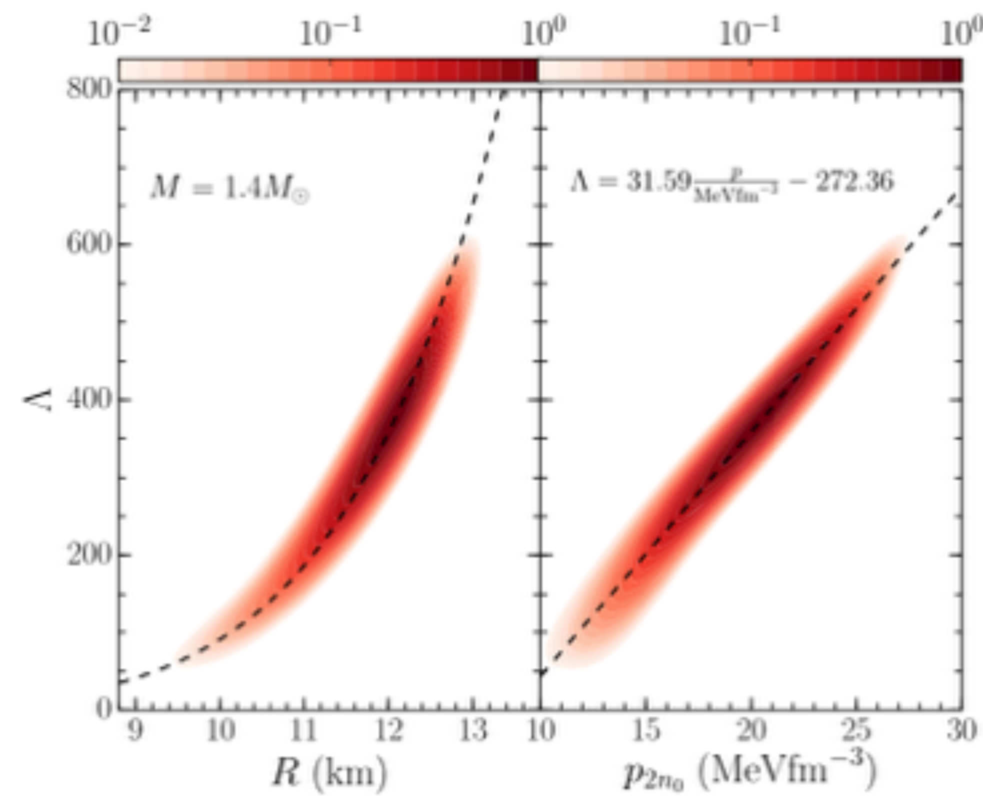
- MDI EoS
  - SNM part and symmetry energy constrained by heavy-ion reaction data up to 4.5 ρ0 and 1.2 ρ0, respectively
- 341 < Λ(1.4M☉) < 782
- 11.5 km < R(1.4M☉) < 13.6 km

Recent Researches (2)



[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, arXiv:1711.02644v2

- Low density EoS from EFT
- High density EoS from pQCD
- Polytropic interpolation in the intermediate density
- $120 < \Lambda(1.4M_\odot) < 800$
- 9.9 km $< R(1.4M_\odot) < 13.6$ km
- $\Lambda(1.4M_\odot) = 2.88 * 10^{-6} (R/km)^{7.5}$



[D] Y. Lim and J. Holt, arXiv:1803.02803

- Prediction with uncertainties inherent EFT
- ~ 73000 energy density functionals
- $350 < \Lambda(1.4M_\odot) < 540$
- 11.65 km $< R(1.4M_\odot) < 12.84$ km

Constraints on Nuclear EoS

- Nuclear data: hundreds of models (Skyrme force, RMF, …)
- Neutron star maximum mass

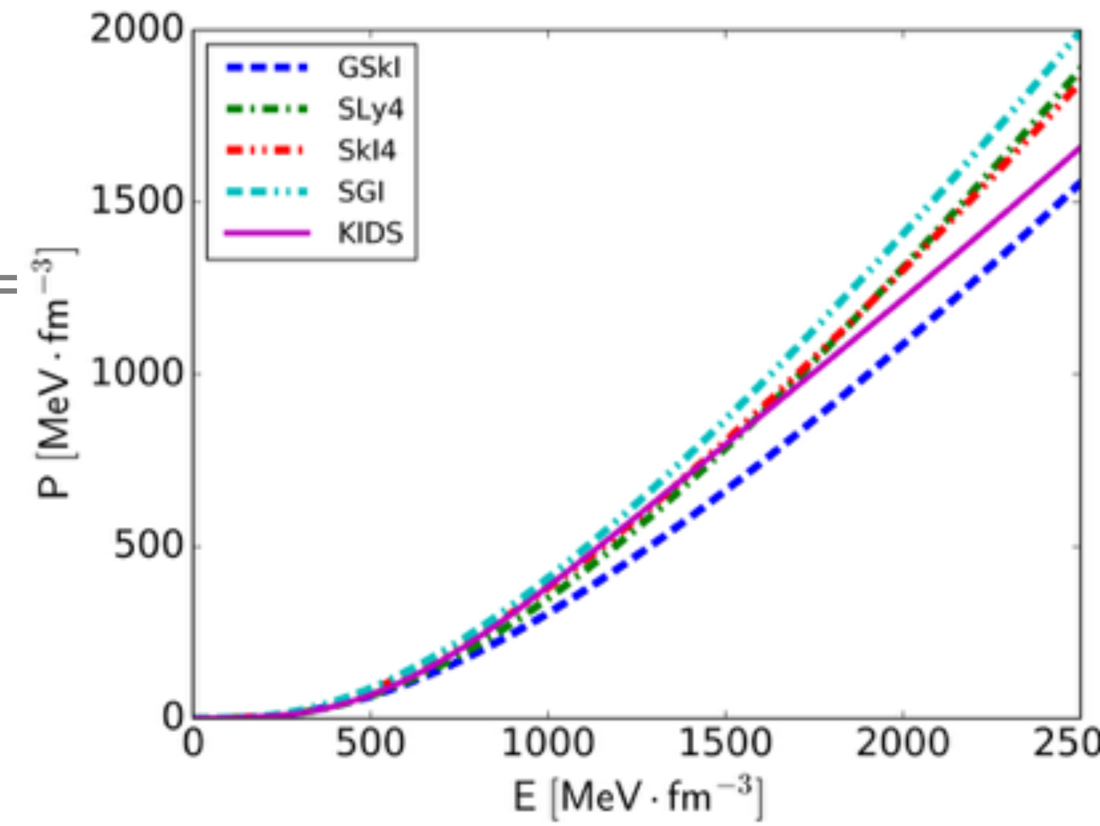
  1.97 ± 0.04 M⊙ [Nature 467, 1081 (2010)]

  2.01 ± 0.04 M⊙ [Science 340, 448 (2013)]
- 11 experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

Selected EoSs

- Skyrme force models
- Basically fitted to properties of well-known nuclei
- Good saturation properties
- Mmax more than 2Msun



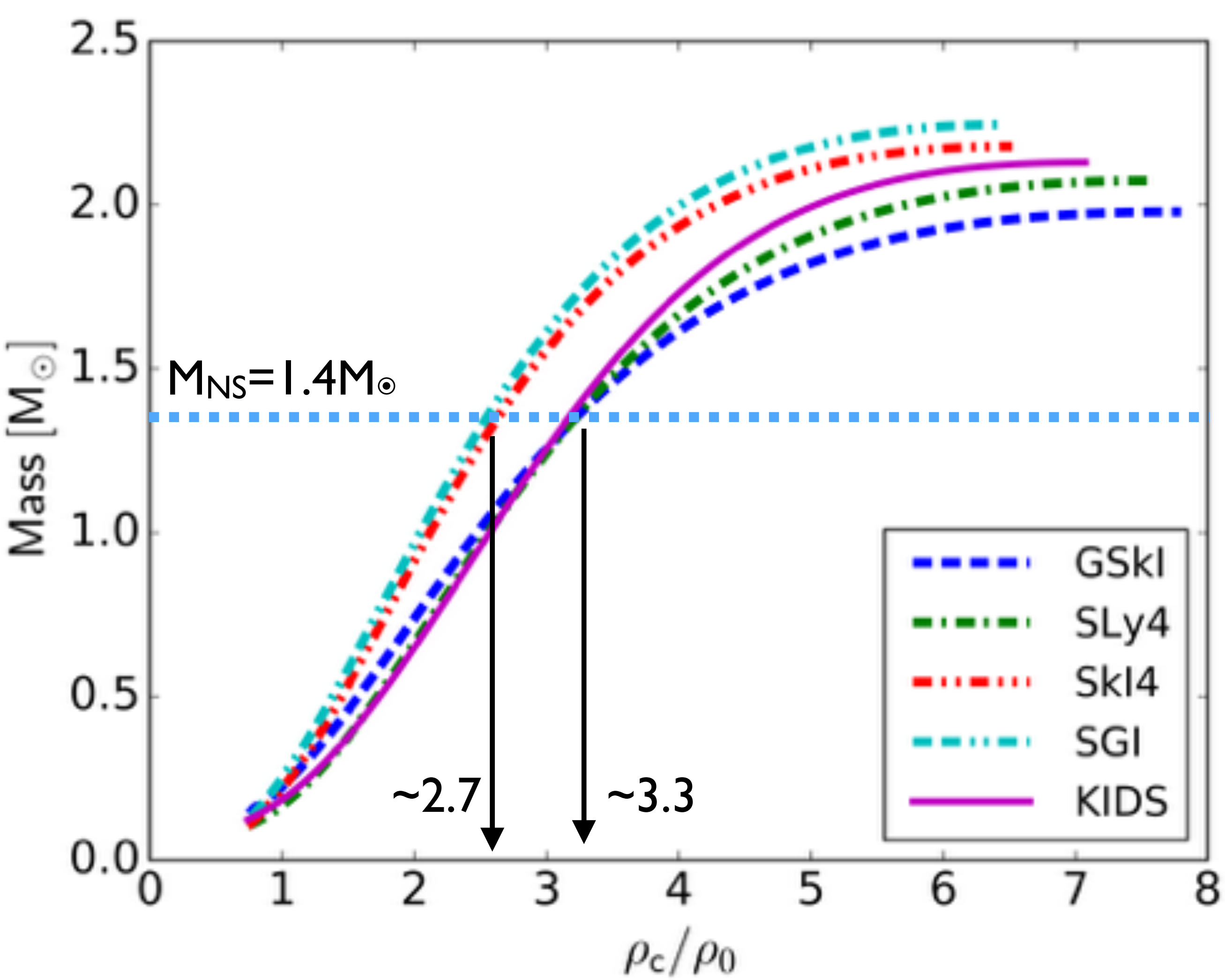
Model	ρ_0	E_0	K_0	$-Q_0$	J	L	$-K_\tau$	$M_{\max}$
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	$200 \sim 260$	$200 \sim 1200$	$30 \sim 35$	$40 \sim 76$	$372 \sim 760$	$\geq 1.93 \sim 2.05$
CSkP	-	-	$202.0 \sim 240.3$	$362.5 \sim 425.6$	$30.0 \sim 35.5$	$48.6 \sim 67.1$	$360.1 \sim 407.1$	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

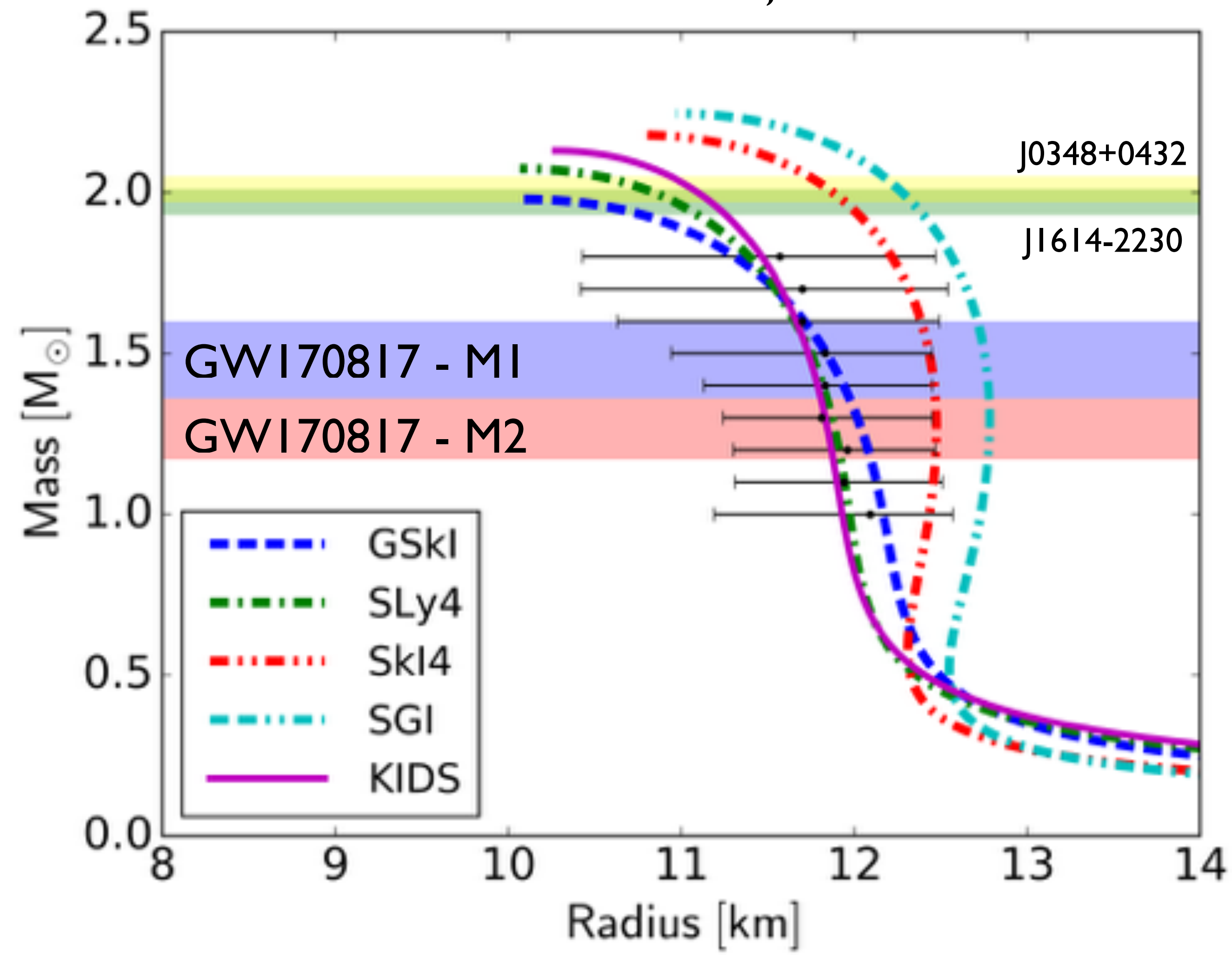
Kim et al., arxiv:1805.00219

KIDS (Korea: IBS-Daegu-Sungkyunkwan): A new systematic expansion scheme for nuclear EDF [Phys. Rev. C 97, 014312 (2018)]

Mass-Radius relations

Kim et al., arxiv:1805.00219

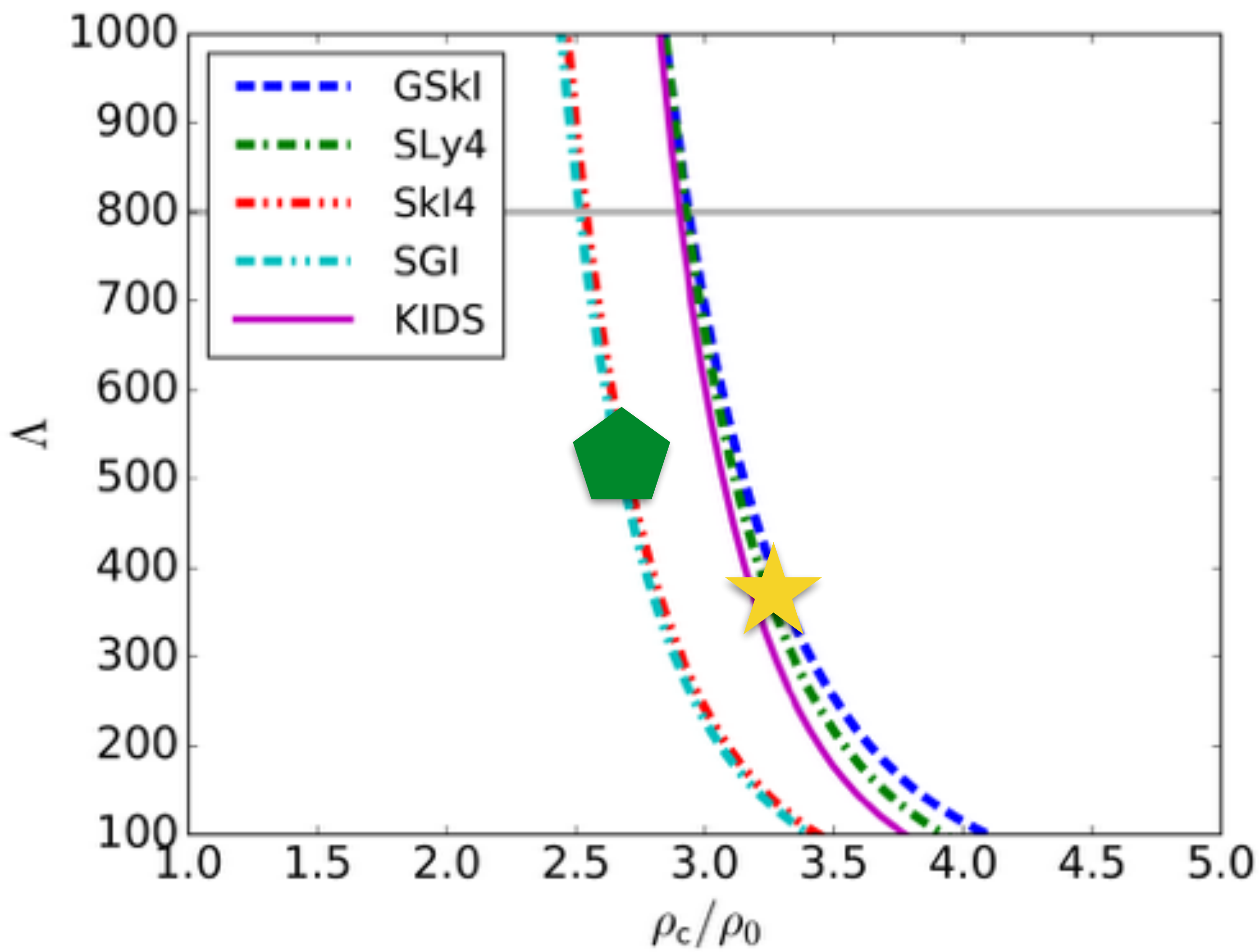


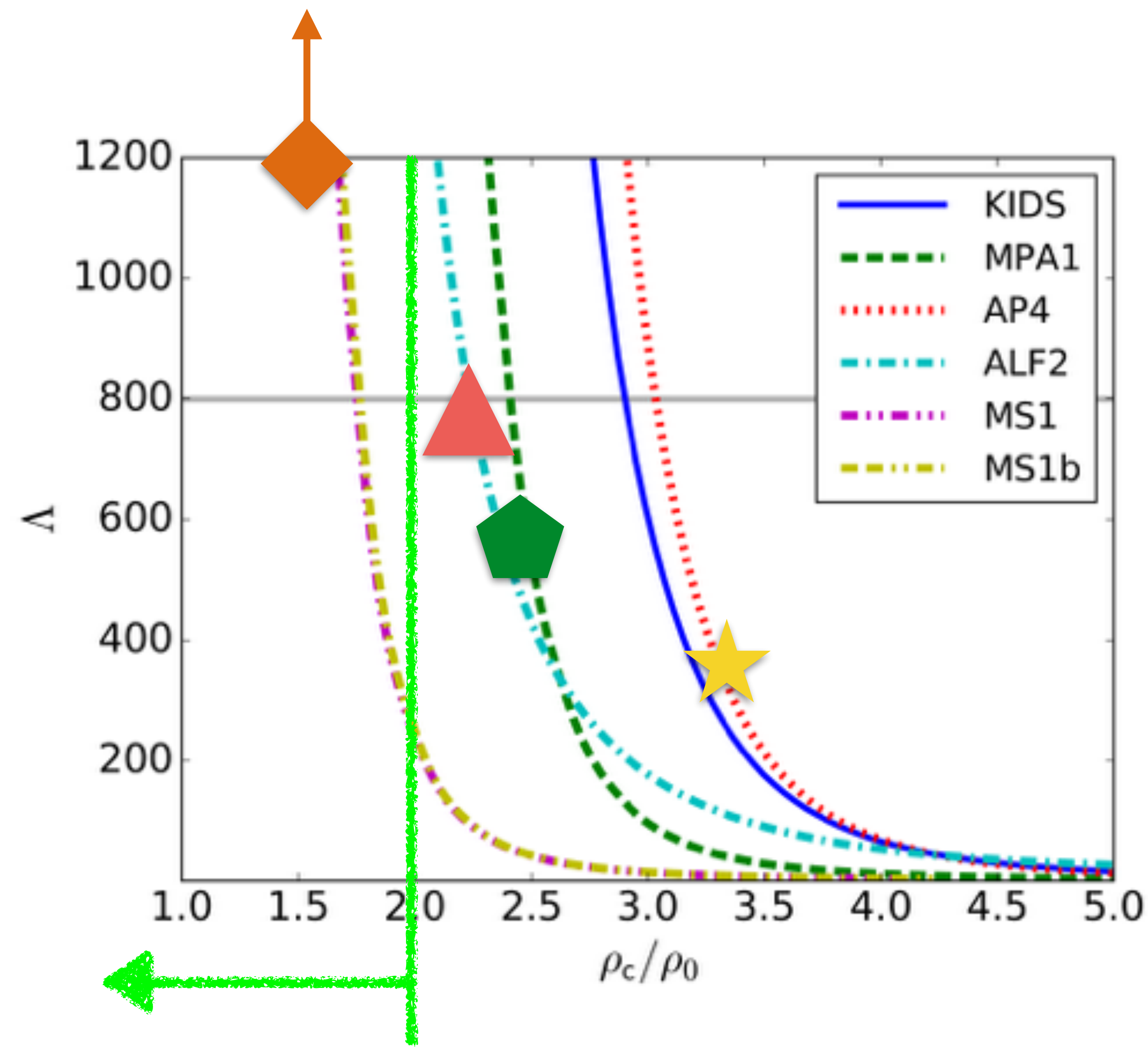


GW170817 - Abbott et al. (LSC and Virgo), arxiv:1805.11579
- Mchirp = 1.188 M⊙
- low spin prior : M_1=1.36~1.60 M⊙ , M_2=1.16~1.36 M⊙
- high spin prior : M_1=1.36~1.89 M⊙, M_2=1.00~1.36 M⊙

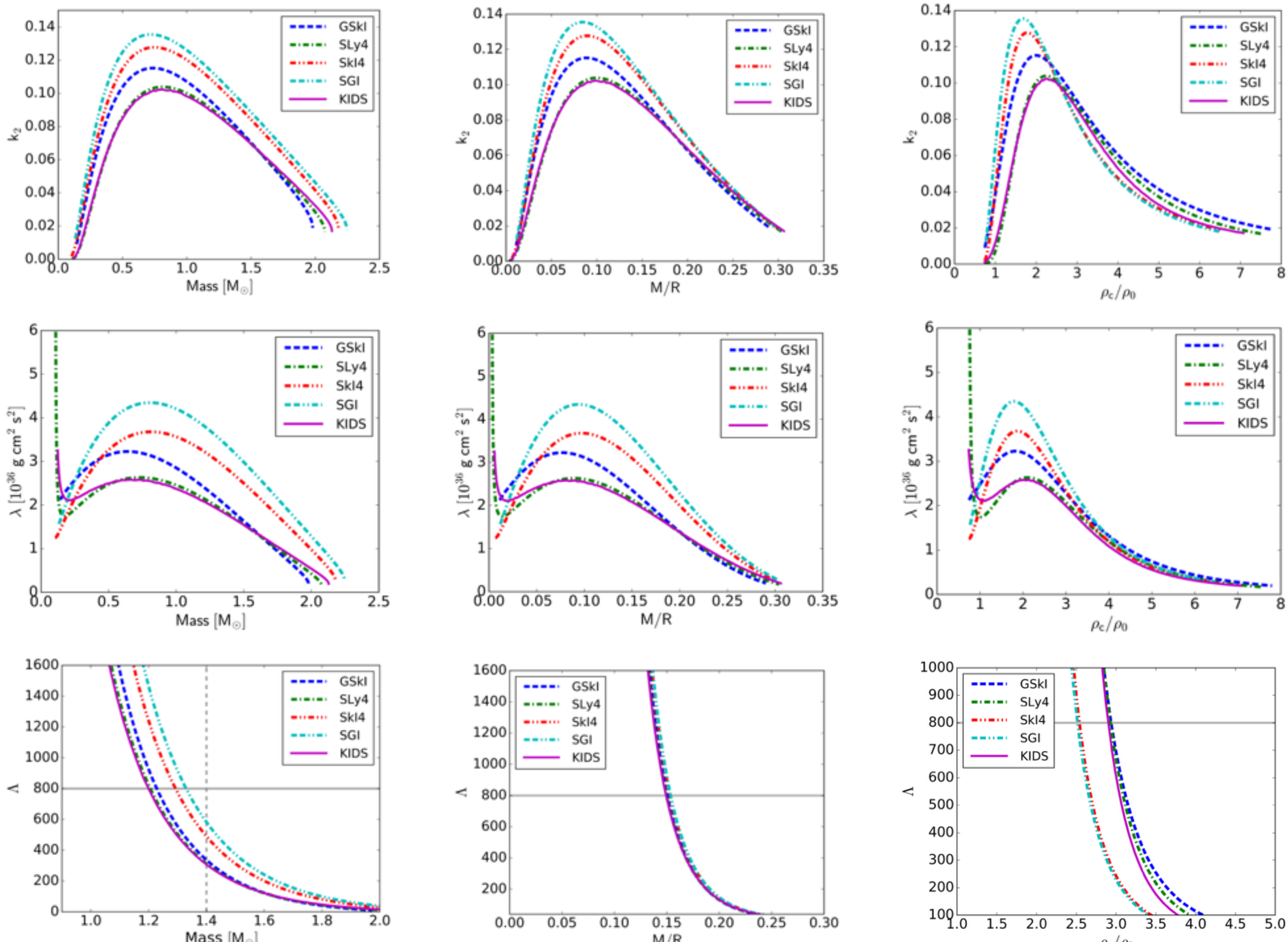
Central Density at M_{NS}=1.4 $M_\odot$

5 Skyrme force models

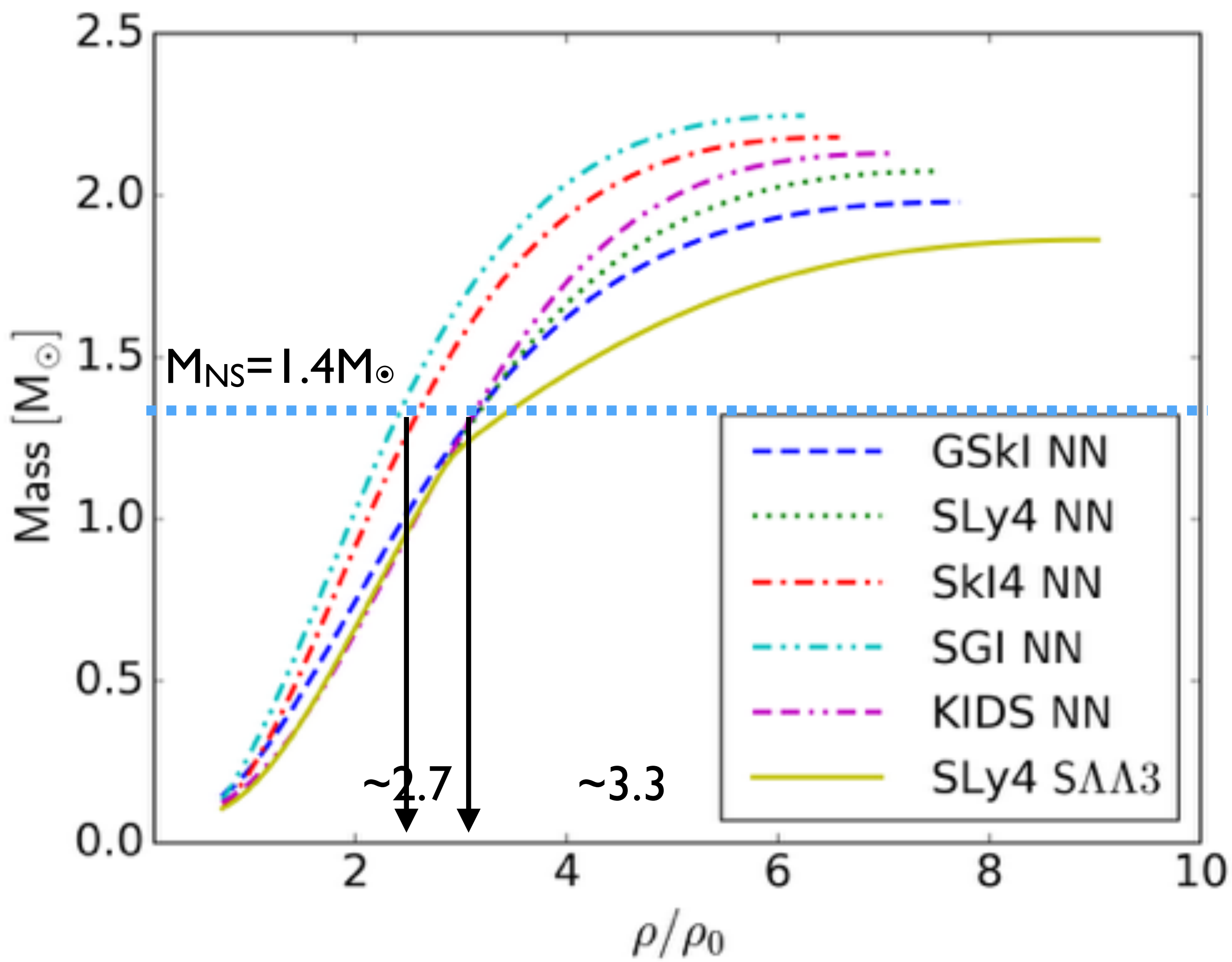


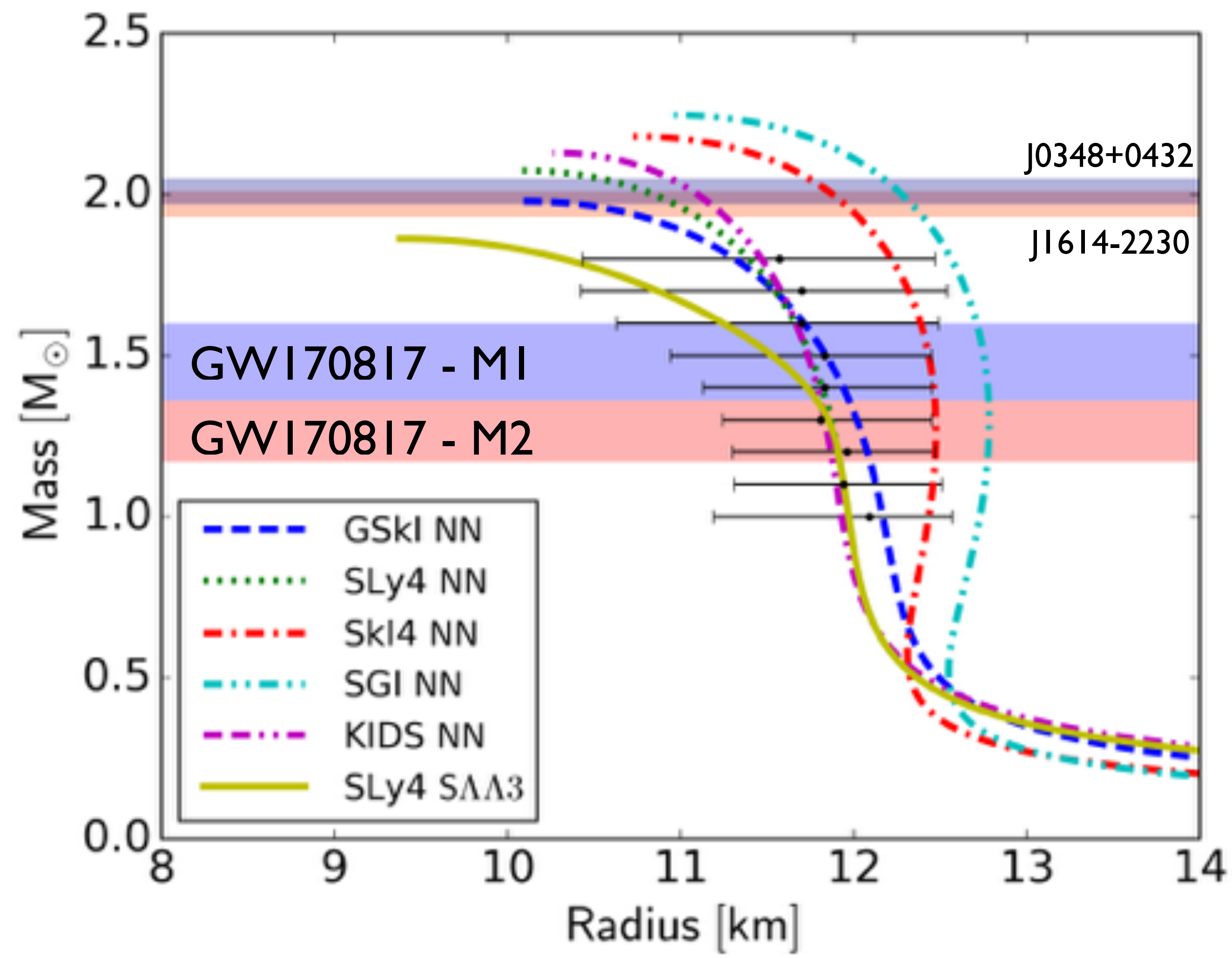


ruled out ?


GSkI
SLy4
SkI4
SGI
KIDS
k_2
Mass $[M_\odot]$
M/R
ρ_c/ρ_0
$\lambda\ [10^{36}$ g cm^2 s$^2]$
Λ

Mass-Radius relations





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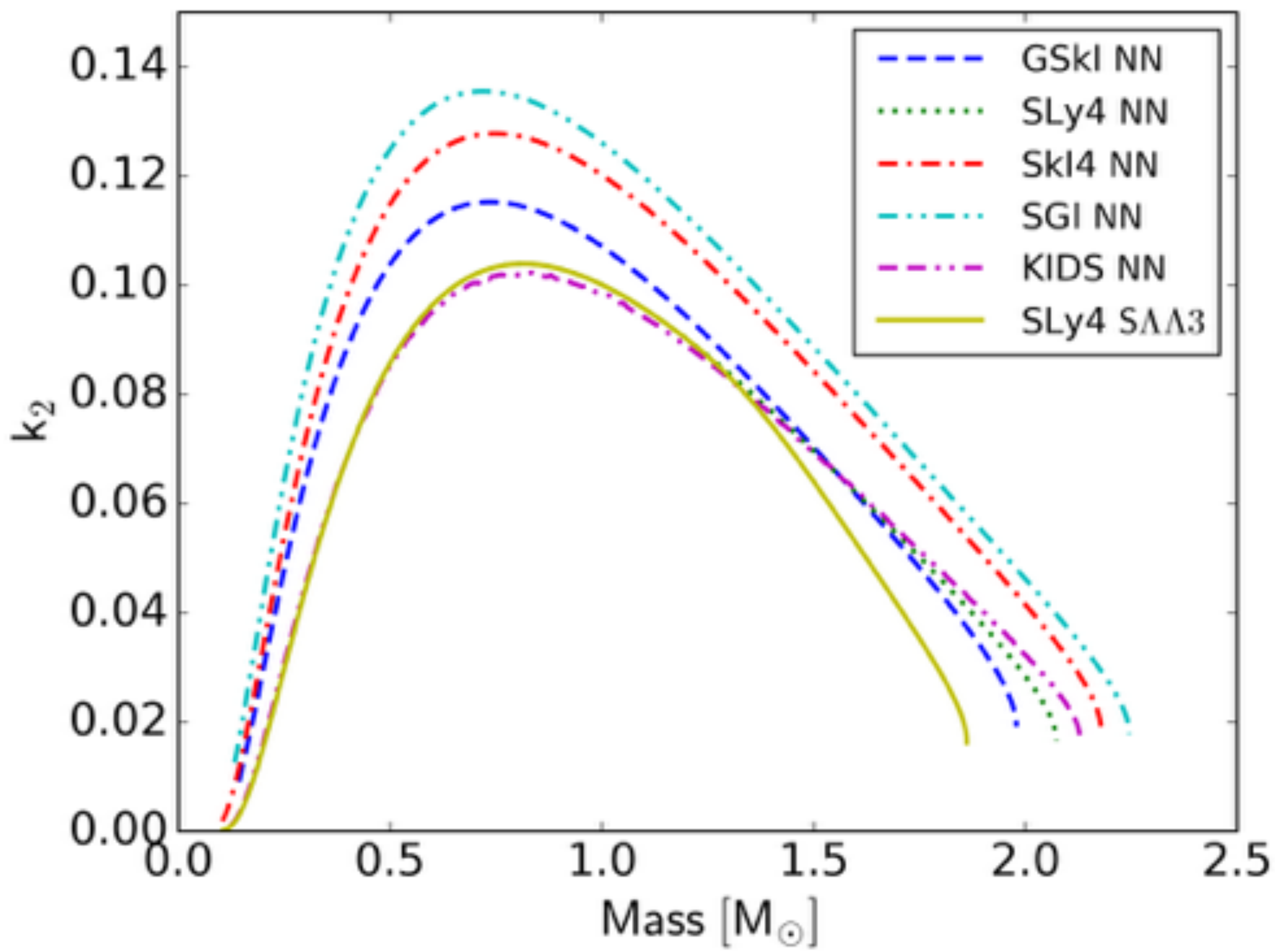
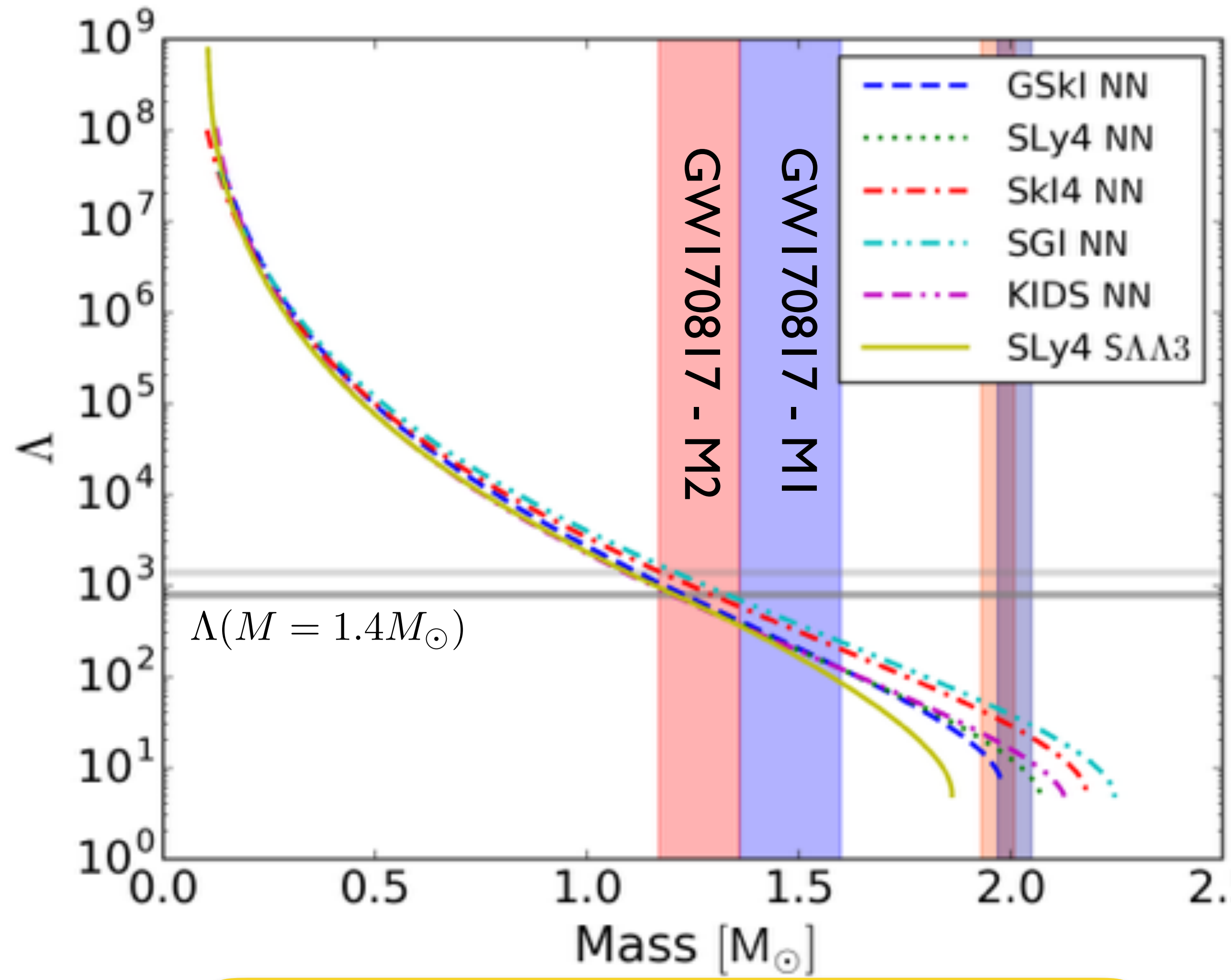
- high spin prior : M1 = 1.36 ~ 2.26 M⊙ , M2 = 0.86 ~ 1.36 M⊙

Tidal deformability of a NS

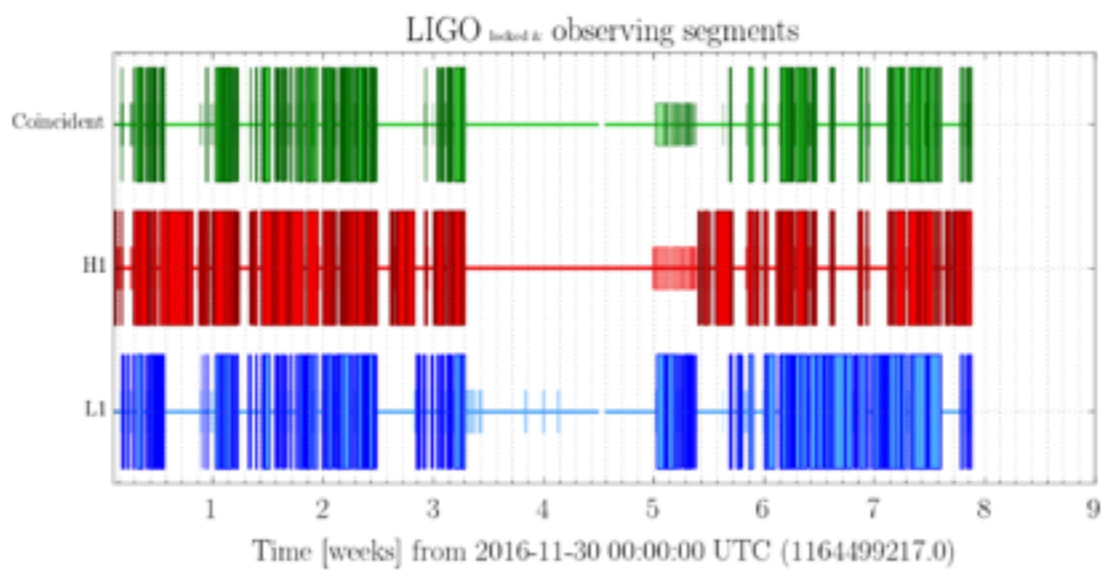




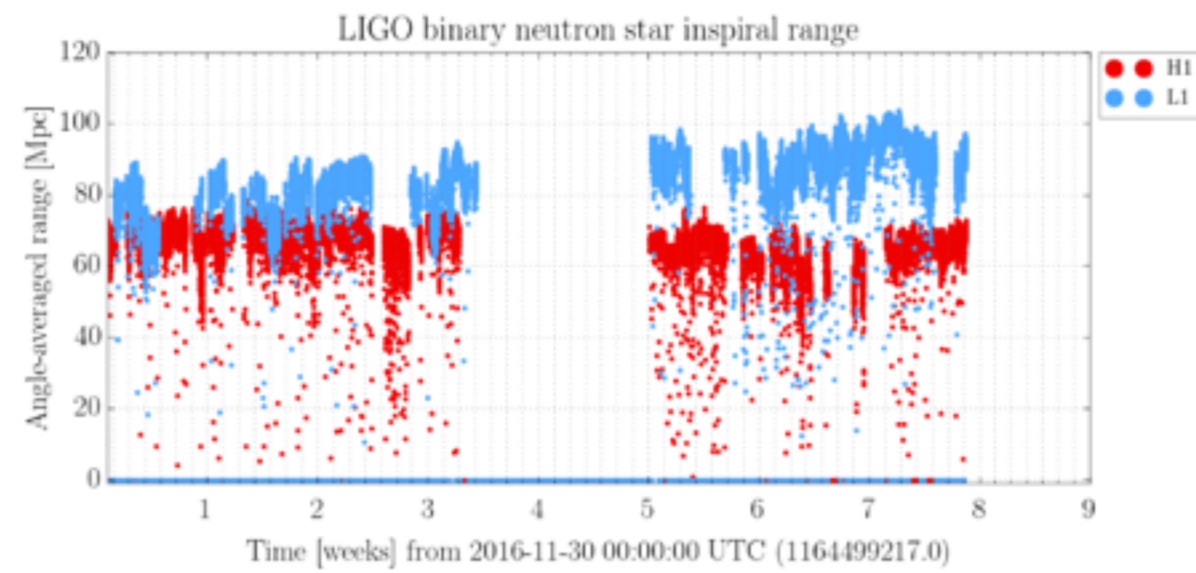
GW170817, Mchirp = 1.188 M⊙
- low spin prior : Lambda(1.4 M⊙) < 800
- high spin prior : Lambda(1.4 M⊙) < 1400

Kim et al. in preparation

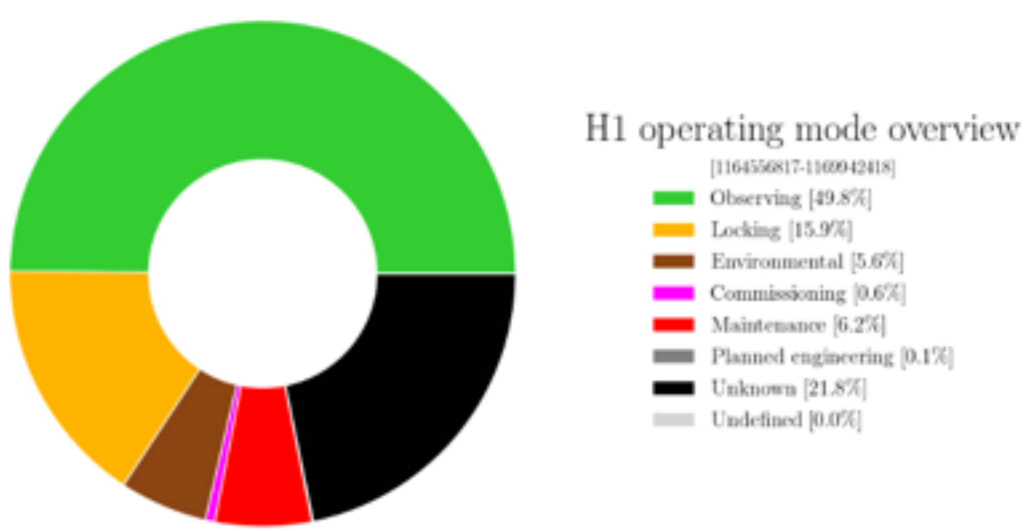
O2 Summary



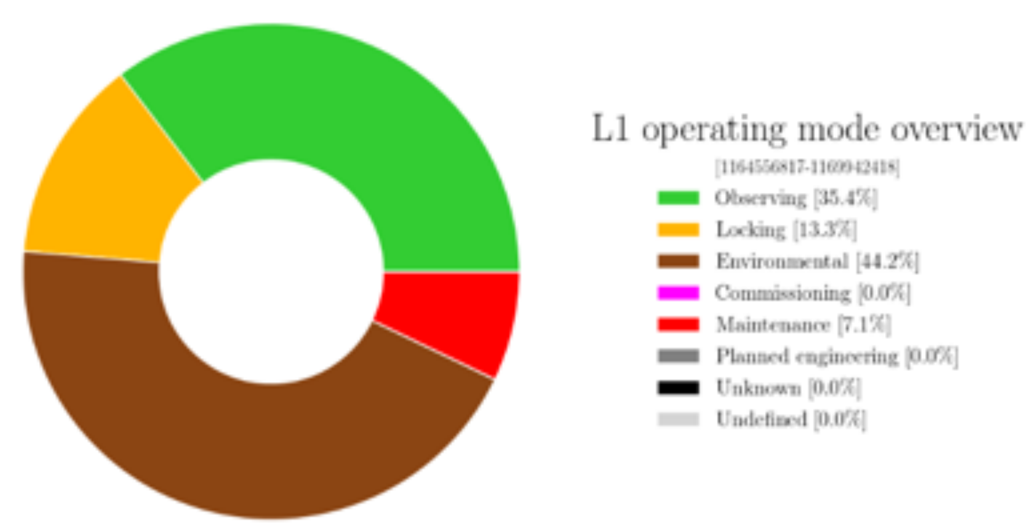
LIGO observing segments
Coincident
H1
L1
1 2 3 4 5 6 7 8 9
Time [weeks] from 2016-11-30 00:00:00 UTC (1164499217.0)



LIGO binary neutron star inspiral range
H1
L1
Angle-averaged range [Mpc]
0 20 40 60 80 100 120
1 2 3 4 5 6 7 8 9
Time [weeks] from 2016-11-30 00:00:00 UTC (1164499217.0)



H1 operating mode overview
Observing [49.8%]
Locking [15.9%]
Environmental [5.6%]
Commissioning [0.6%]
Maintenance [6.2%]
Planned engineering [0.1%]
Unknown [21.8%]
Undefined [0.0%]



L1 operating mode overview
Observing [35.4%]
Locking [13.3%]
Environmental [44.2%]
Commissioning [0.0%]
Maintenance [7.1%]
Planned engineering [0.0%]
Unknown [0.0%]
Undefined [0.0%]