

# Relevant studies on isospin splitting of nucleon effective mass

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## Based on:

**Phys. Rev. C 91, 014611 (2015);**

**Phys. Rev. C 91, 037601 (2015);**

**Phys. Rev. C 91, 047601 (2015);**

**Phys. Rev. C 95, 034324 (2017);**

**Prog. Part. Nucl. Phys. 99, 29 (2018);**

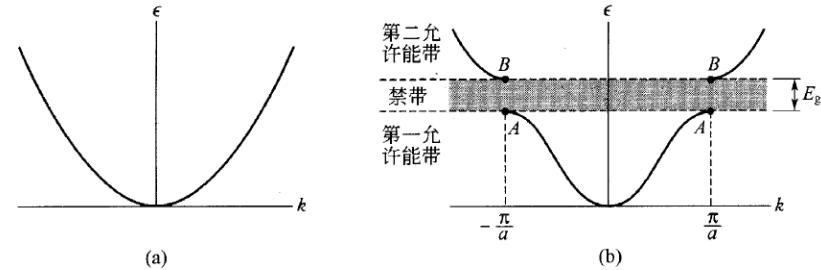
**arXiv: 1807.01849 [nucl-th]**

# Nucleon effective mass

## Electron effective mass:

dispersion relation different from free electrons near the energy gap

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 \epsilon}{dk^2}$$



## Nucleon effective mass:

in-medium interaction lowers the nucleon mass

**P-mass:** 
$$\frac{\tilde{m}_\tau^*}{m} = \left[ 1 + \frac{m}{p} \frac{\partial U_\tau(p, \epsilon_\tau(p))}{\partial p} \right]^{-1}$$
  
 $\tau = n, p$

**E-mass:** 
$$\frac{\bar{m}_\tau^*}{m} = 1 - \frac{\partial U_\tau(p, \epsilon_\tau(p))}{\partial \epsilon_\tau}$$

**Dirac mass:** 
$$m_{Dirac, \tau}^* = m + \Sigma_\tau^s$$
       $\Sigma_\tau^s$ : scalar self-energy

**Skyrme-Hartree-Fock:** non-relativistic, momentum-dependent potential

**Relativistic mean-field:** relativistic, meson exchange

Comparison between **non-relativistic mass** with **relativistic mass**

## Lorentz effective mass:

$$m_{Lorentz, \tau}^* = m \left( 1 - \frac{dU_{SEP, \tau}}{dE_\tau} \right) = (E_\tau - \Sigma_\tau^0) \left( 1 - \frac{d\Sigma_\tau^0}{dE_\tau} \right) - (m + \Sigma_\tau^s) \frac{d\Sigma_\tau^s}{dE_\tau} + m - E_\tau$$

M. Jaminon and C. Mahaux, PRC (1989); B.A. Li, L.W. Chen, and C.M. Ko, Phys. Rep. (2008); Z.X. Li, Nucl. Phys. Rev. (2014)

# Neutron-proton effective mass splitting

## Isospin dynamics in nuclear reactions

$$\frac{d\vec{p}}{dt} = -\nabla U_\tau$$

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} + \nabla_p U_\tau = \frac{\vec{p}}{m_\tau^*}$$

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \delta = \frac{\rho_n - \rho_p}{\rho}$$

Symmetry energy/potential

Effective mass  $\frac{m_\tau^*}{m} = \left[ 1 + \frac{m}{p} \frac{dU_\tau(p)}{dp} \right]^{-1}, \tau = n, p$   
 (non-relativistic p-mass)

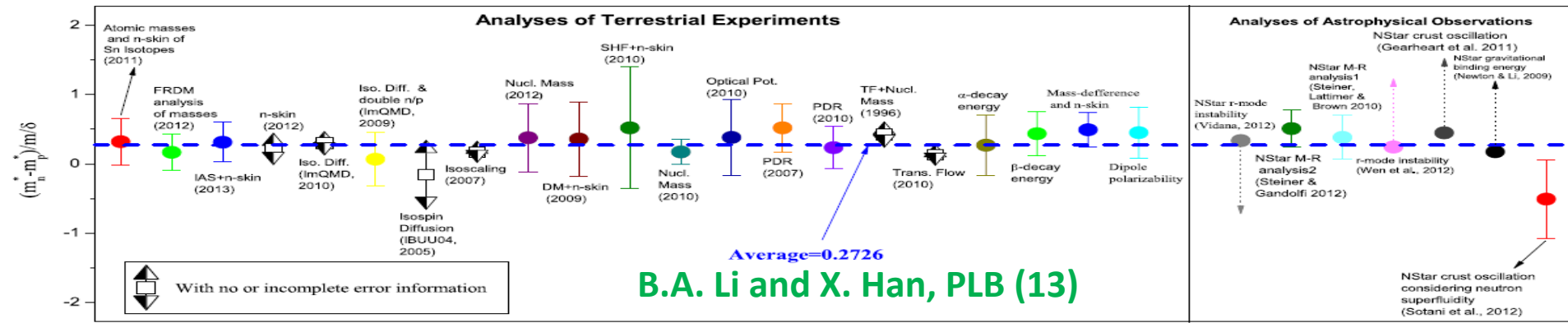
Isoscalar effective mass:  $m_s^* \approx m_{n(p)}^* (\delta = 0)$  Isovector effective mass:  $m_n^* - m_p^* \approx \frac{2m_s^*}{m_v^*} (m_s^* - m_v^*) \delta$

## Hughenoltz–Van Hove theorem

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2 k_F^2}{2m_0^*} + \frac{1}{2} U_{\text{sym}}(\rho, k_F)$$

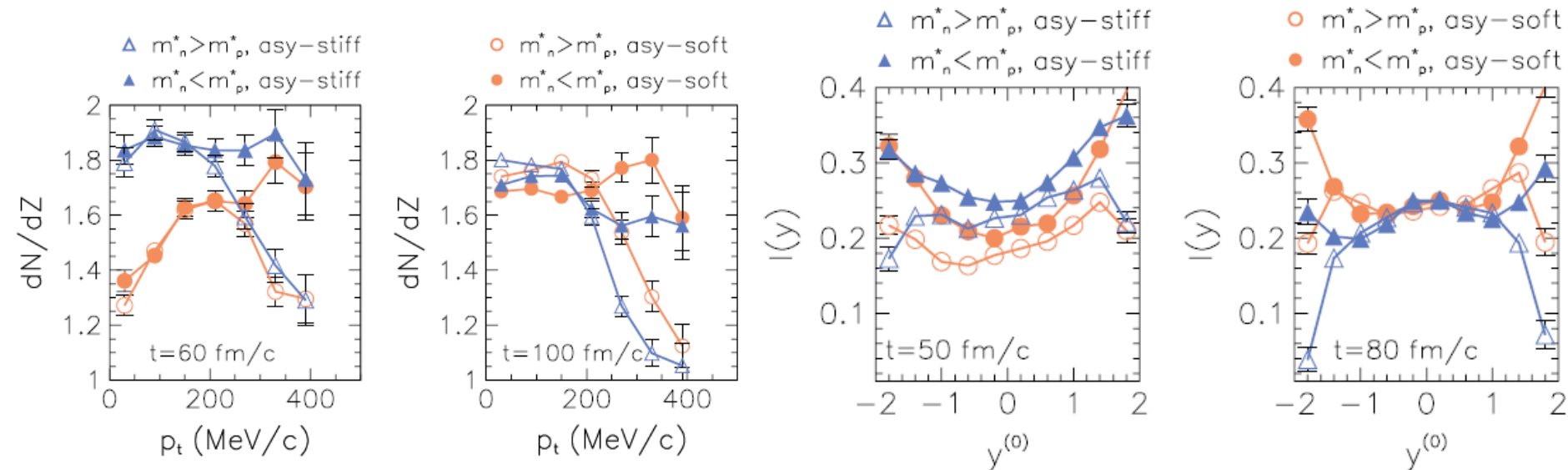
$$L(\rho) \approx \frac{2}{3} \frac{\hbar^2 k_F^2}{2m_0^*} + \frac{3}{2} U_{\text{sym}}(\rho, k_F) + \left. \frac{\partial U_{\text{sym}}}{\partial k} \right|_{k_F} k_F$$

C. Xu, B.A. Li, and L.W. Chen, PRC (10); R. Chen, B.J. Cai, L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC (12)



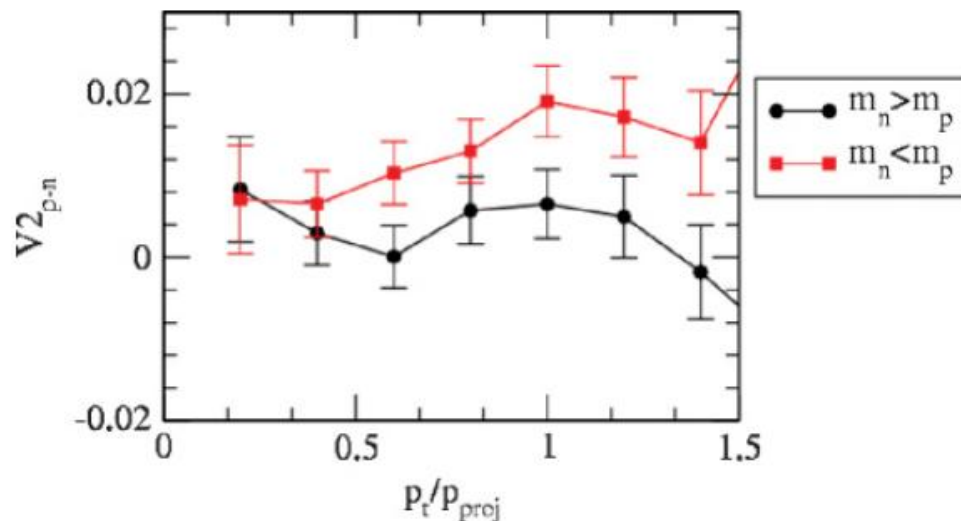
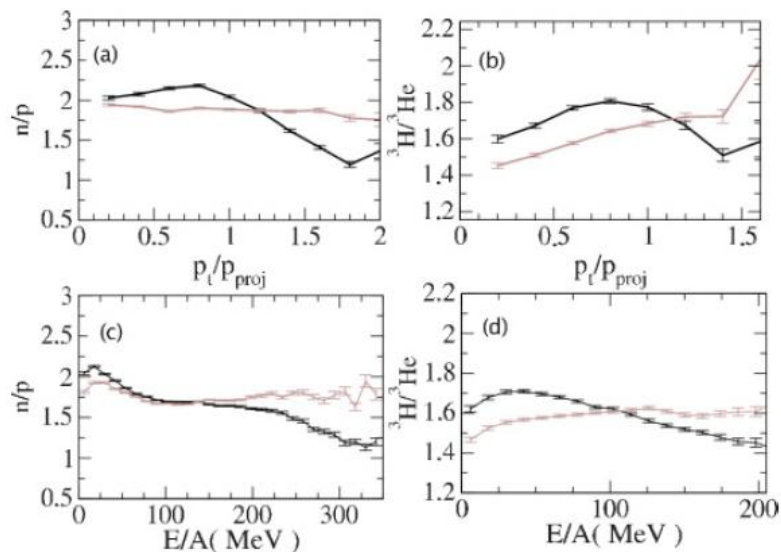
# np effective mass and HIC dynamics I

Fast emitted particles (BNV) (Rizzo, Colonna, Di Toro, PRC (05))  $^{132}\text{Sn}+^{124}\text{Sn}@100\text{AMeV}$



np ratio and  $v_2$  splitting (SMF) (Giorando, Colonna, Di Toro, Greco, and Rizzo, PRC (10))

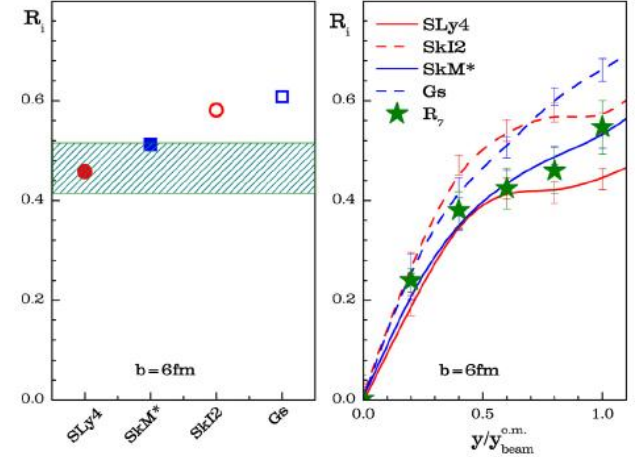
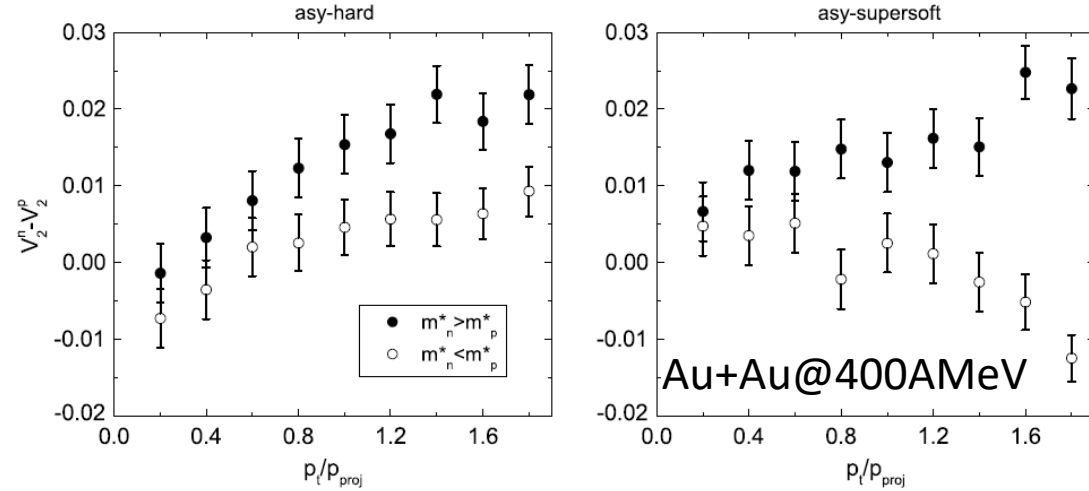
Au+Au@400AMeV



# np effective mass and HIC dynamics II

np flow splitting (LQMD) (Z.Q. Feng, NPA (11))

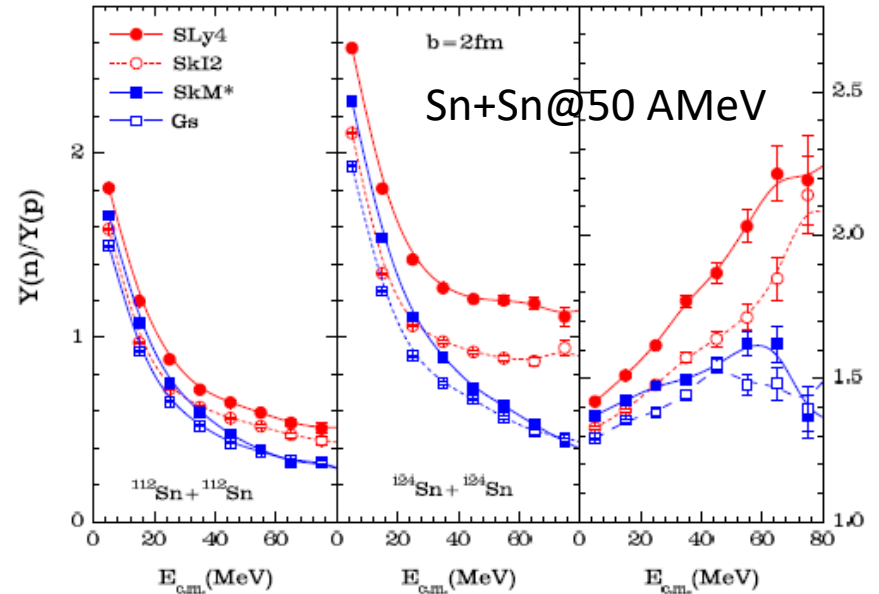
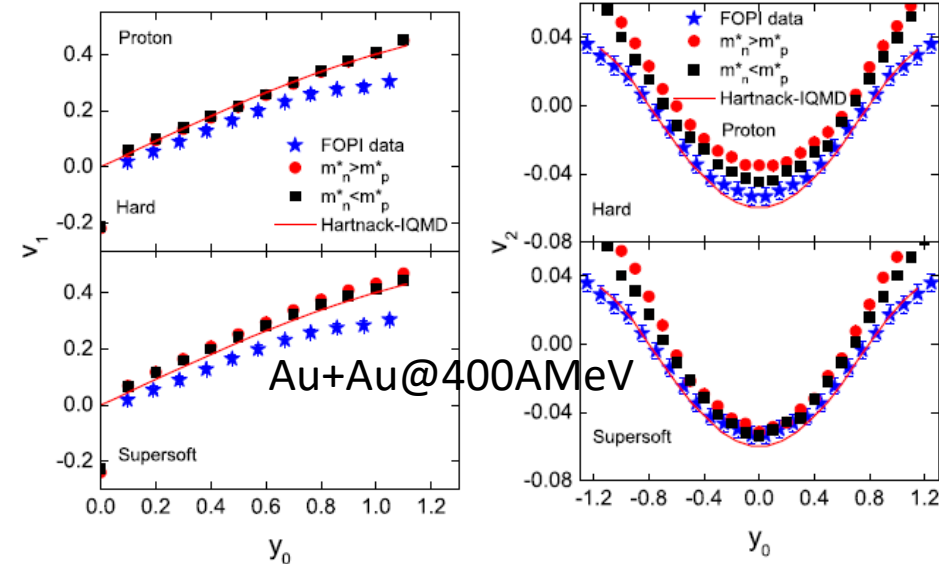
Isospin diffusion  $R_i = \frac{2X - X_{aa} - X_{bb}}{X_{aa} - X_{bb}}$



np flow splitting (IQMD-BNU) (W.J. Xie and F.S. Zhang, PLB (14))

n/p ratio (ImQMD)

(Y.X. Zhang, M.B. Tsang, Z.X. Li, and H. Liu, PLB (14))



# An improved momentum-dependent interaction (ImMDI)

## Effective NN potential:

$$v(\vec{r}_1, \vec{r}_2) = \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^y \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2) \\ + (W + G P_\sigma - H P_\tau - M P_\sigma P_\tau) \frac{e^{-\mu |\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$$

$$A_l(x, y) = A_{l0} + y + x \frac{2B}{\sigma + 1},$$

$$A_u(x, y) = A_{u0} - y - x \frac{2B}{\sigma + 1},$$

## Potential energy density:

$$V(\rho, \delta) = \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_l}{2\rho_0} (\rho_n^2 + \rho_p^2) + \frac{B}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} \\ \times (1 - x\delta^2) + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'}$$

$$C_l(y, z) = C_{l0} - 2(y - 2z) \frac{p_{f0}^2}{\Lambda^2 \ln [(4p_{f0}^2 + \Lambda^2)/\Lambda^2]},$$

$$C_u(y, z) = C_{u0} + 2(y - 2z) \frac{p_{f0}^2}{\Lambda^2 \ln [(4p_{f0}^2 + \Lambda^2)/\Lambda^2]},$$

$$\times \iint d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}.$$

0, we choose the following empirical values:  $\rho_0 = 0.16 \text{ fm}^{-3}$ ,  $E_0(\rho_0) = -16 \text{ MeV}$ ,  $K_0 = 230 \text{ MeV}$ ,  $m_s^* = 0.7m$ ,  $E_{\text{sym}}(\rho_0) = 32.5 \text{ MeV}$ , and  $U_{0, \infty} = 75 \text{ MeV}$ , which lead to  $A_{l0} = A_{u0} = -66.963 \text{ MeV}$ ,  $B = 141.963 \text{ MeV}$ ,  $C_{l0} = -60.4860 \text{ MeV}$ ,  $C_{u0} = -99.7017 \text{ MeV}$ ,  $\Lambda = 2.42401 p_{f0}$ , and  $\sigma = 1.26521$ .

## Mean-field potential:

$$U_\tau(\rho, \delta, \vec{p}) = A_u \frac{\rho_{-\tau}}{\rho_0} + A_l \frac{\rho_\tau}{\rho_0} \\ + B \left( \frac{\rho}{\rho_0} \right)^\sigma (1 - x\delta^2) - 4\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho_{-\tau} \\ + \frac{2C_l}{\rho_0} \int d^3 p' \frac{f_\tau(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\ + \frac{2C_u}{\rho_0} \int d^3 p' \frac{f_{-\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}.$$

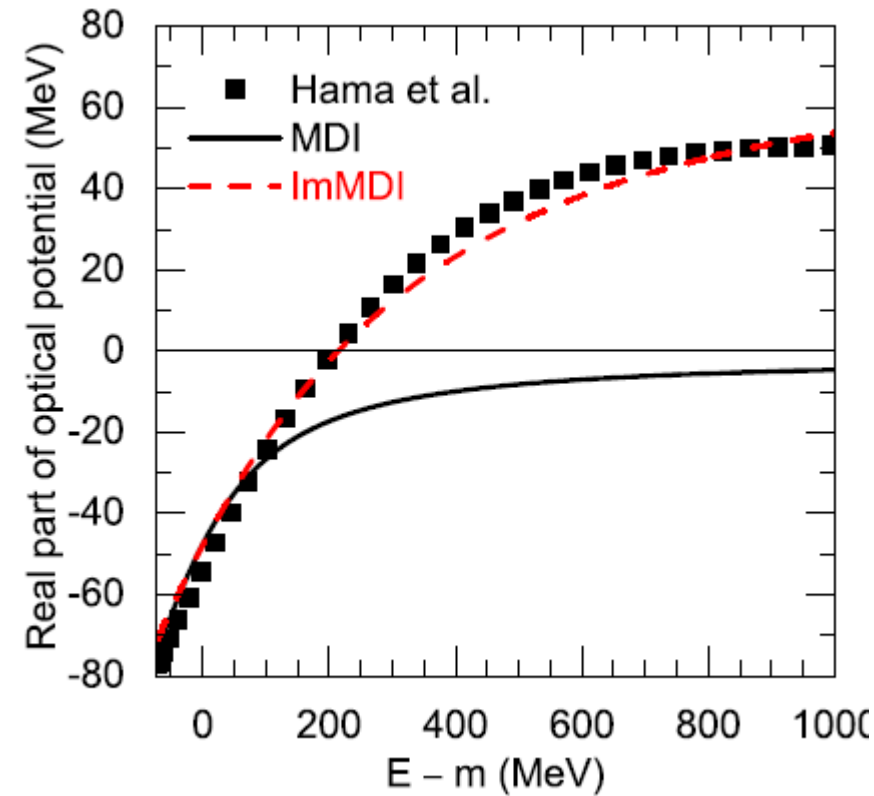
For nuclear matter

$$f_\tau(\vec{r}, \vec{p}) \sim \frac{1}{\exp \left[ \left( \frac{p^2}{2m} + U_\tau(\vec{p}) - \mu_\tau \right) / T \right] + 1}$$

**Relevant parameters: x, y, z**

JX, L.W. Chen, and B.A. Li, PRC 91, 014611 (2015)

## Isoscalar part

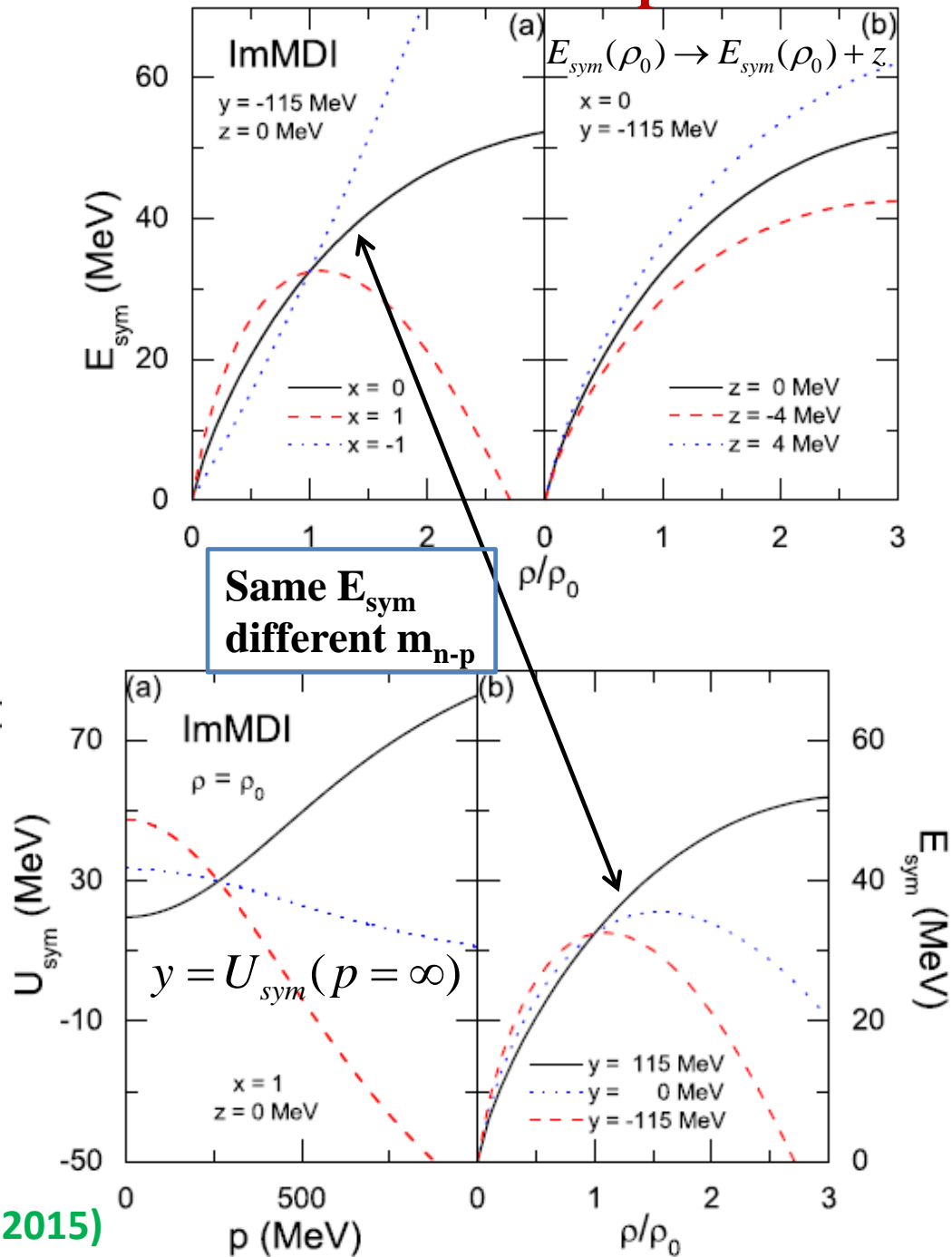


More reliable at higher energies  
More flexible in studying isospin effects

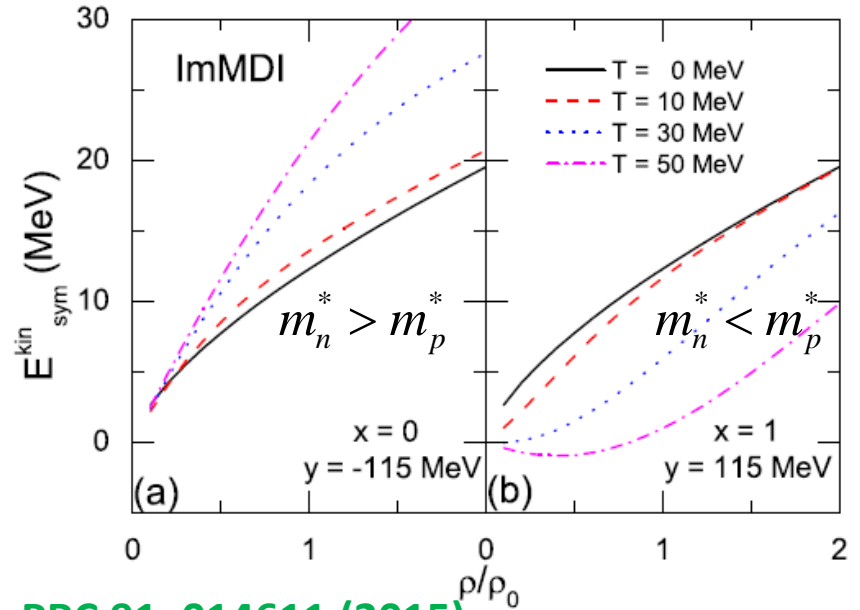
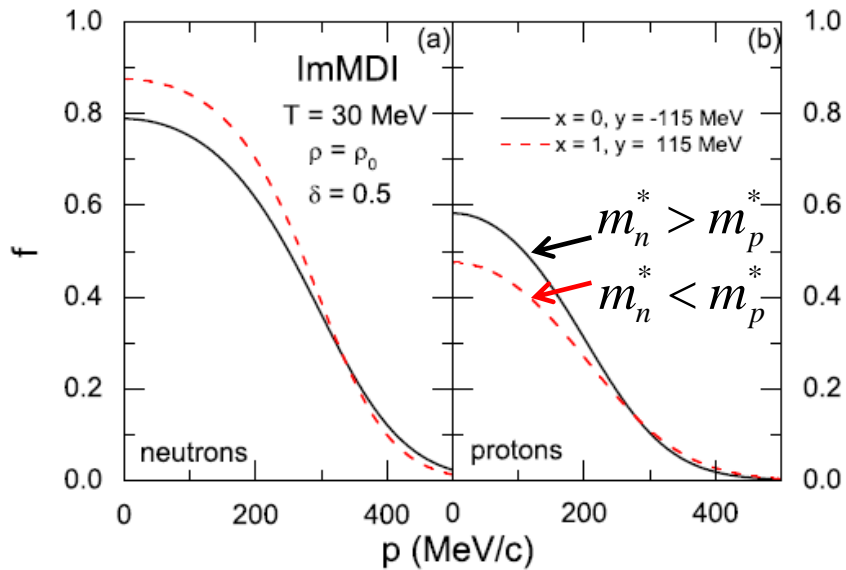
$$U_{n/p} \approx U_0 \pm U_{sym} \delta$$

$$\frac{m_{\tau}^*}{m} = \left( 1 + \frac{m}{p} \frac{dU_{\tau}}{dp} \right)^{-1}$$

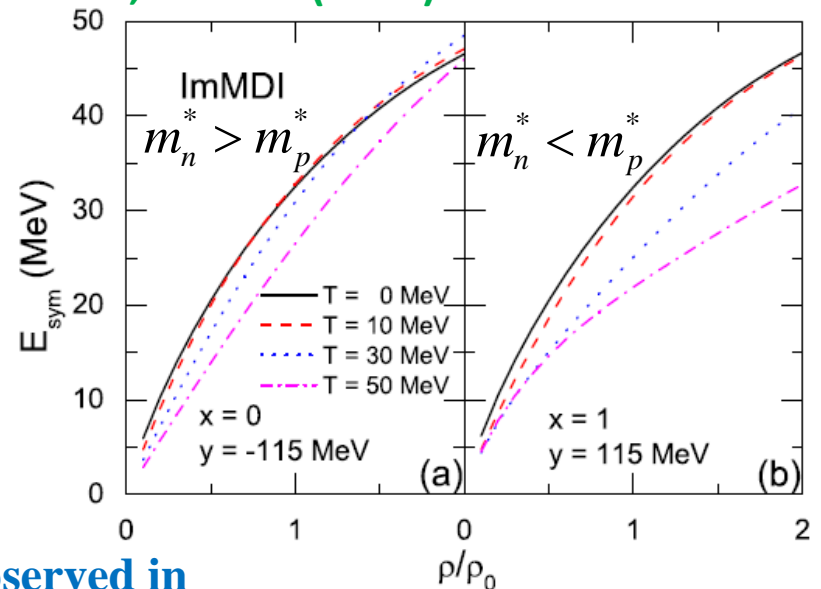
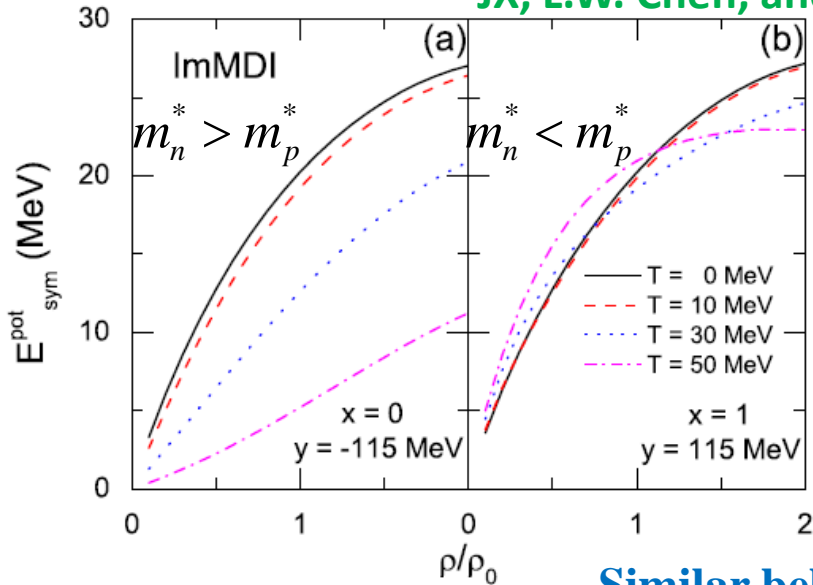
## Isvector part



# np effective mass splitting and nuclear thermodynamics I



JX, L.W. Chen, and B.A. Li, PRC 91, 014611 (2015)

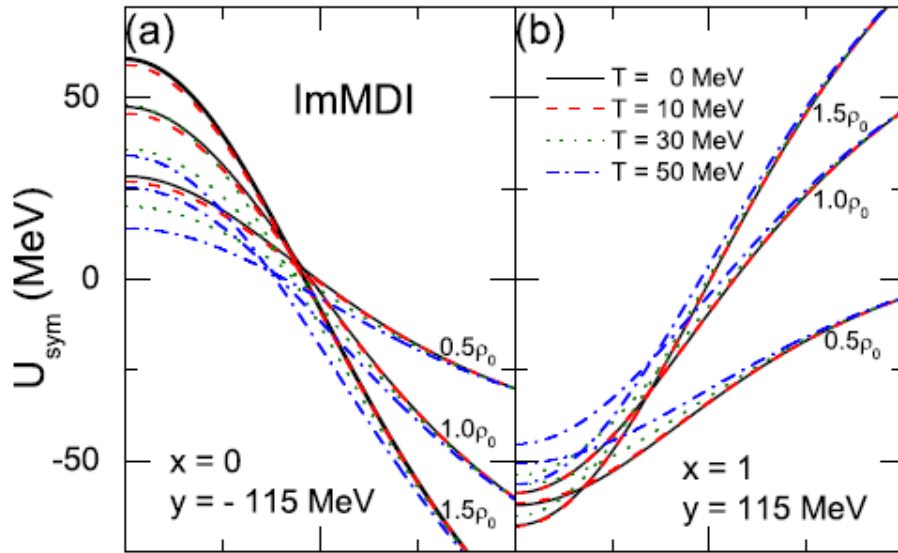


Similar behavior observed in

L. Ou, Z.X. Li, Y.X. Zhang, and H. Liu, PLB (11)

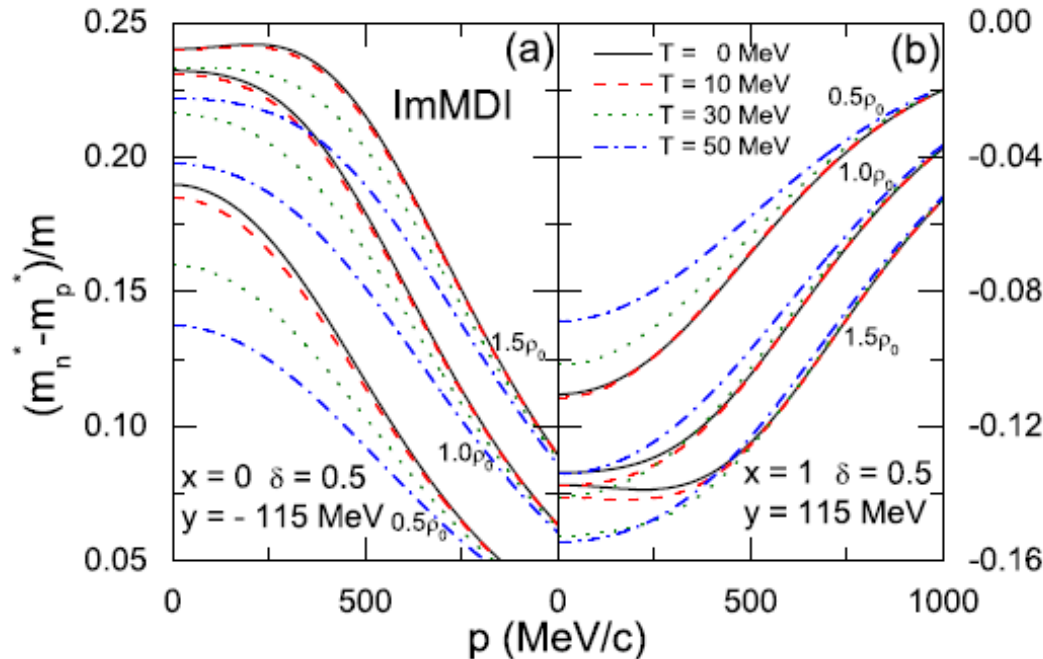
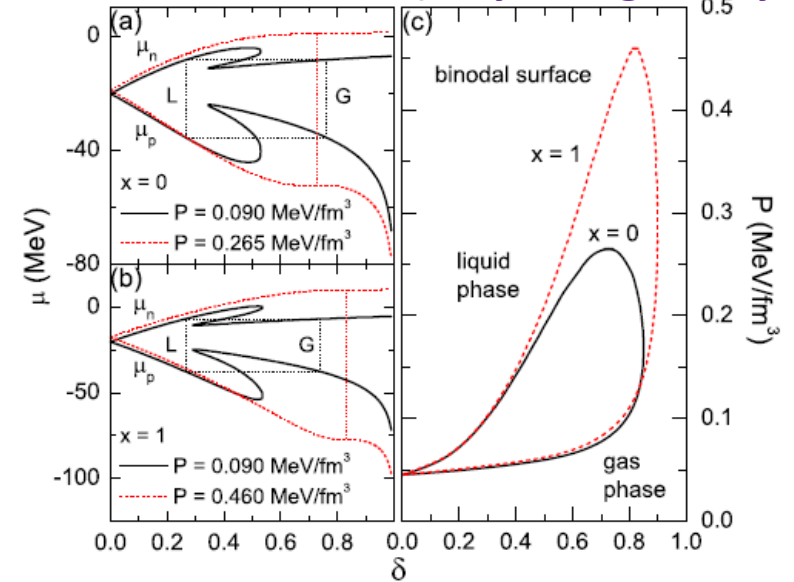


# np effective mass splitting and nuclear thermodynamics II

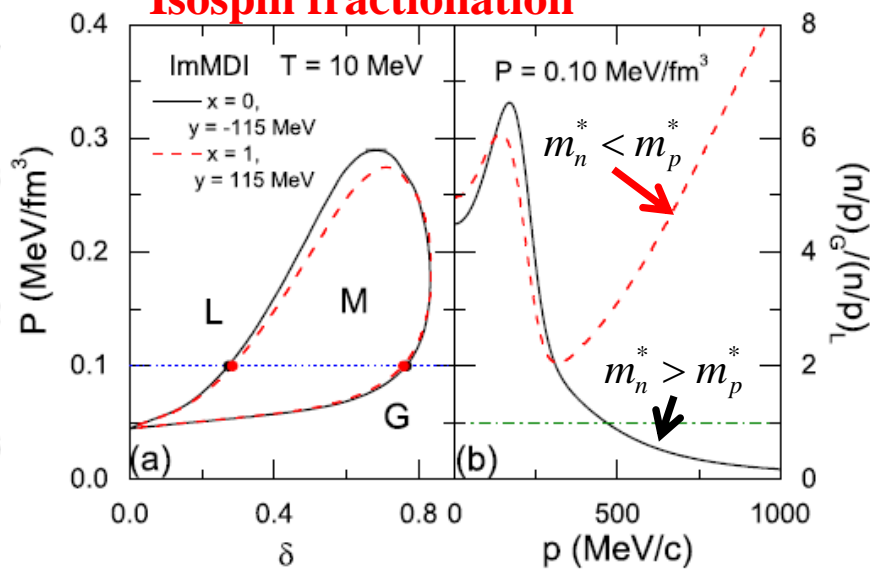


## Nuclear liquid-gas phase transition

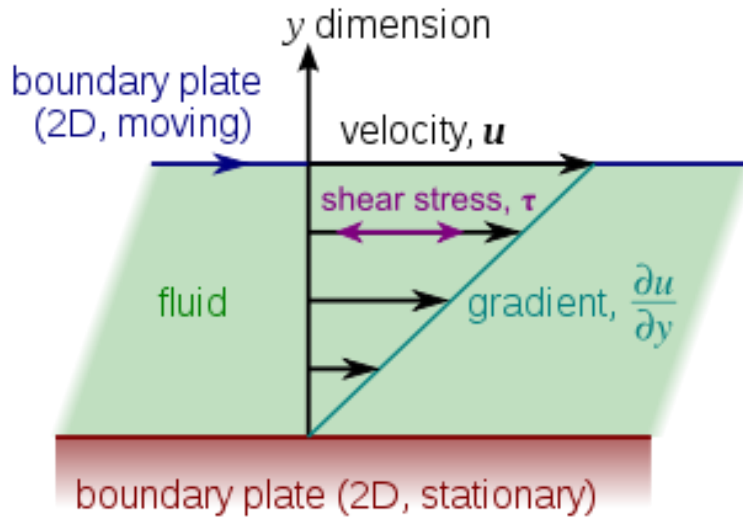
### Gibbs construction (only change x-Esym)



## Isospin fractionation



# Shear viscosity



$$\tau = \frac{F}{A} = \eta \frac{\partial u}{\partial y} \quad \eta \propto \frac{\langle p \rangle}{\sigma}$$

**Strong interaction**  $\rightarrow$  **Small  $\eta$**

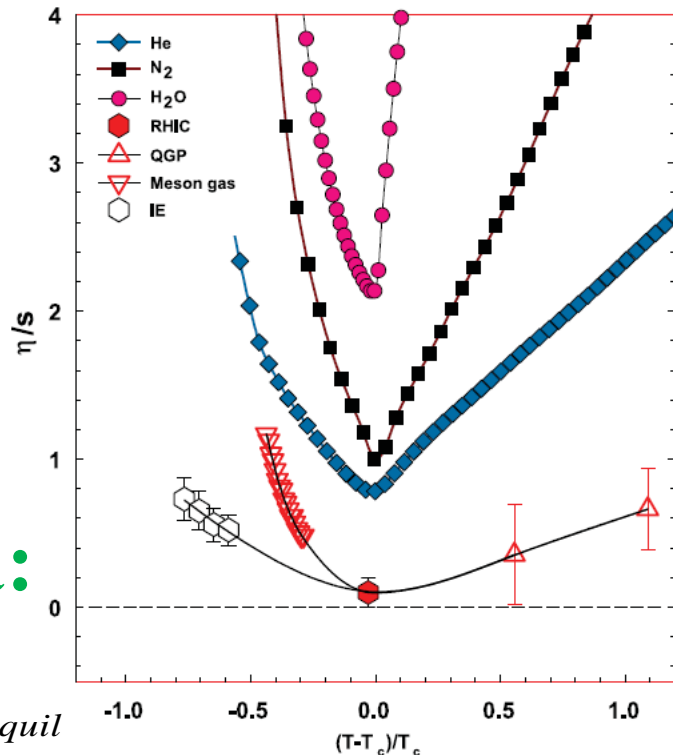
**Viscous hydrodynamics:**  $\frac{\eta}{s}$

**Ideal fluid:**  $\eta = 0$

**Ads/CFT:**  $\frac{\eta}{s} \geq \frac{1}{4\pi}$

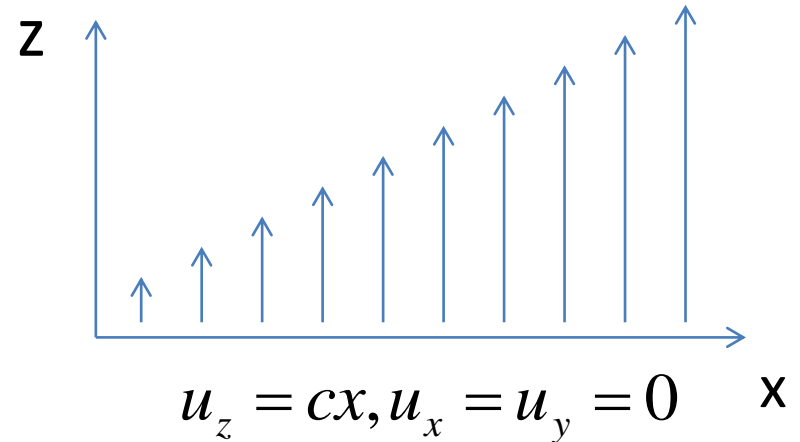
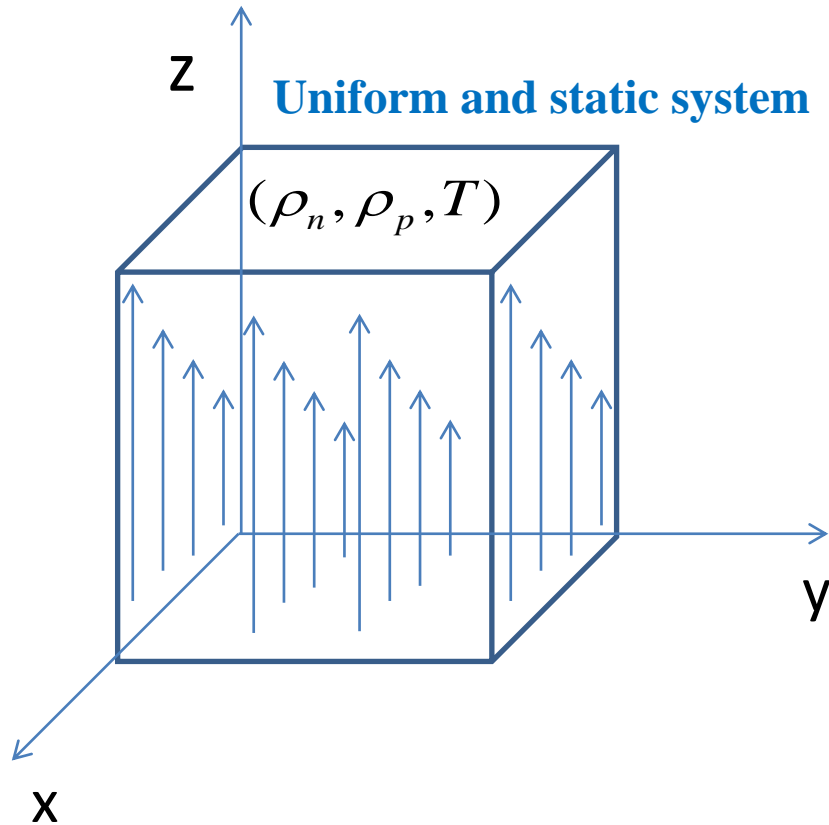
**Green-Kubo's formula:**

$$\eta = \frac{1}{T} \int d^3 r \int_0^\infty dt \langle \pi^{ij}(\vec{0}, 0) \pi^{ij}(\vec{r}, t) \rangle_{equil}$$



R. Lacey *et al.*,  
PRL, 2007

# Shear viscosity from a relaxation time approach



**Shear viscosity:**

$$\eta = \sum_{\tau} -\frac{d}{(2\pi)^3} \int \tau_{\tau}(p) \frac{p_z^2 p_x^2}{p m_{\tau}^*} \frac{dn_{\tau}}{dp} dp_x dp_y dp_z$$

$\tau = n, p$

**Relaxation time:**  $\frac{1}{\tau_{\tau}(p)} = \frac{1}{\tau_{\tau}^{\text{same}}(p)} + \frac{1}{\tau_{\tau}^{\text{diff}}(p)}$

$$n_{\tau}^*(\vec{p}) = \left\{ \exp \left[ \left( \frac{p^2}{2m} + U_{\tau}(\vec{p}) - \mu_{\tau} \right) / T \right] + 1 \right\}^{-1}$$

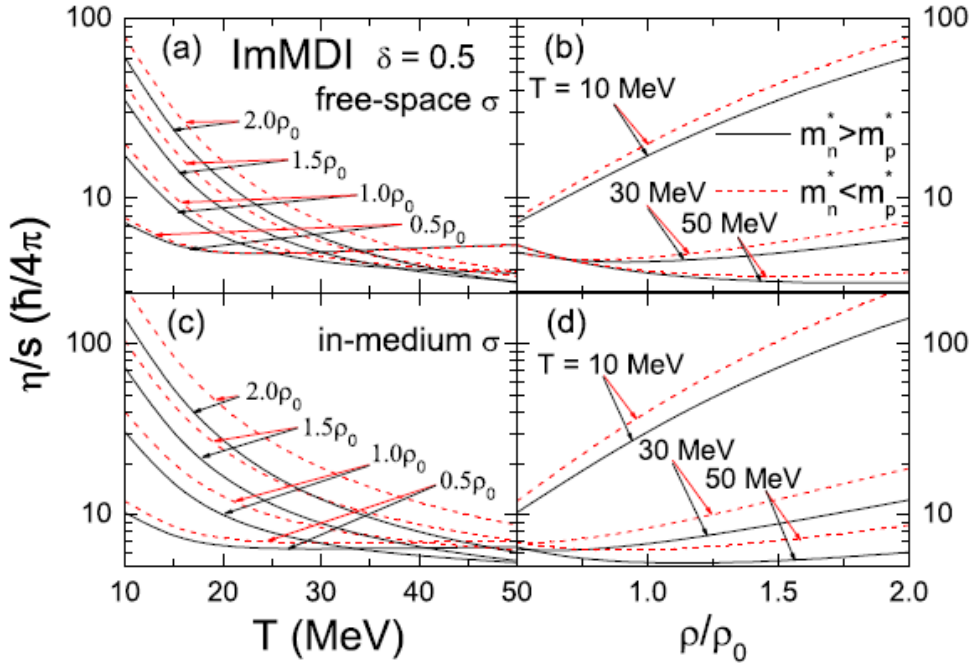
From linearizing isospin-dependent BUU equation

near Fermi surface

$$\sigma_{NN}^{\text{medium}} = \sigma_{NN} \left( \frac{\mu_{NN}^*}{\mu_{NN}} \right)^2$$

# np effective mass splitting and nuclear transport properties

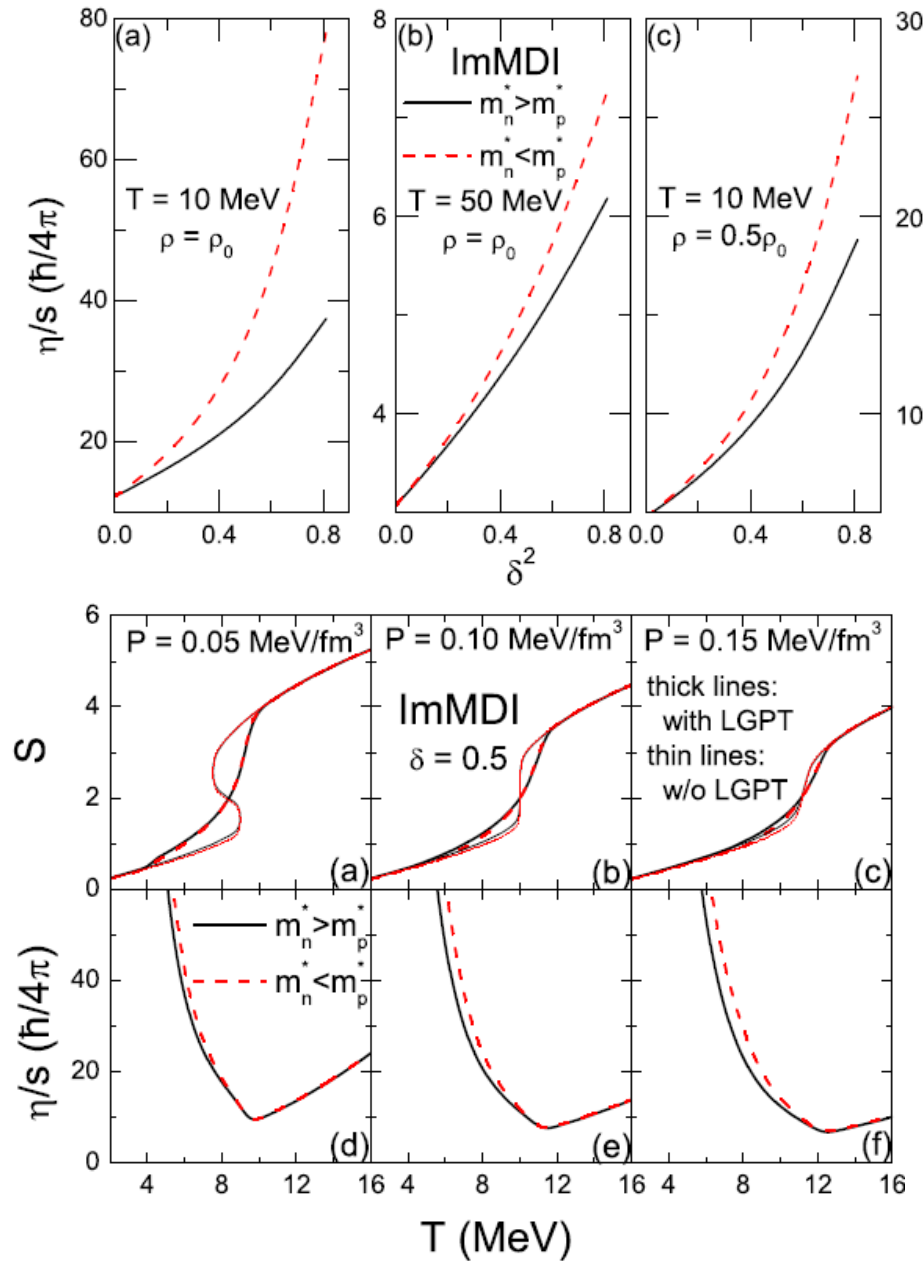
Effects are robust even with  
free-space NN scattering cross section



- 1) Flux between flow layers;
- 2) Effective mass scaling on  $\sigma$

Effects remain even with naive MFP method

$$\tau_{\tau}^{same}(p_1) = \frac{1}{\langle \sigma_{\tau,\tau}^{tr} \rho_{\tau} v_{rel} \rangle} \quad \tau_{\tau}^{diff}(p_1) = \frac{1}{\langle \sigma_{\tau,-\tau}^{tr} \rho_{-\tau} v_{rel} \rangle}$$



# Isovector giant dipole resonance

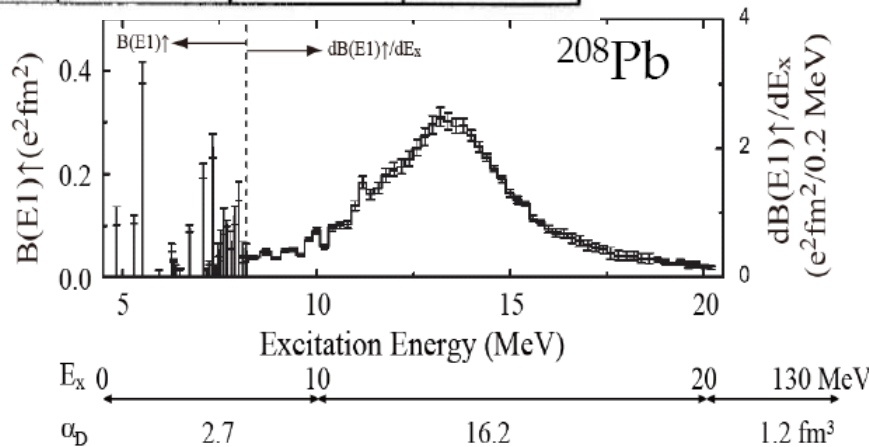
## Symmetry energy as a restoring force

Harmonic oscillator  $\omega \propto \sqrt{\frac{k}{m}}$

Constrain the symmetry energy and the np effective mass splitting using the exp data of  $^{208}\text{Pb}$  giant resonance

With random-phase approximation:  
Z. Zhang and L.W. Chen, PRC (2017)

With IBUU transport model:  
Hai-Yun Kong, JX\*, et al.,  
Phys. Rev. C 95, 034324 (2017)



Electric dipole polarizability Total:  $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$

Photon absorption measurement  $E_{.1} = 13.46 \text{ MeV}$

Subtract quasideuteron excitation  
 $\alpha_D = 19.6 \pm 0.6 \text{ fm}^3$

# Extract $m_s^*$ from ISGQR

Operator of isoscalar giant quadrupole resonance (ISGQR) :

$$\hat{Q} = \sum_{i=1}^A r_i^2 Y_{20}(\hat{r}_i) = \sum_{i=1}^A \sqrt{\frac{5}{16\pi}} (3z_i^2 - r_i^2)$$

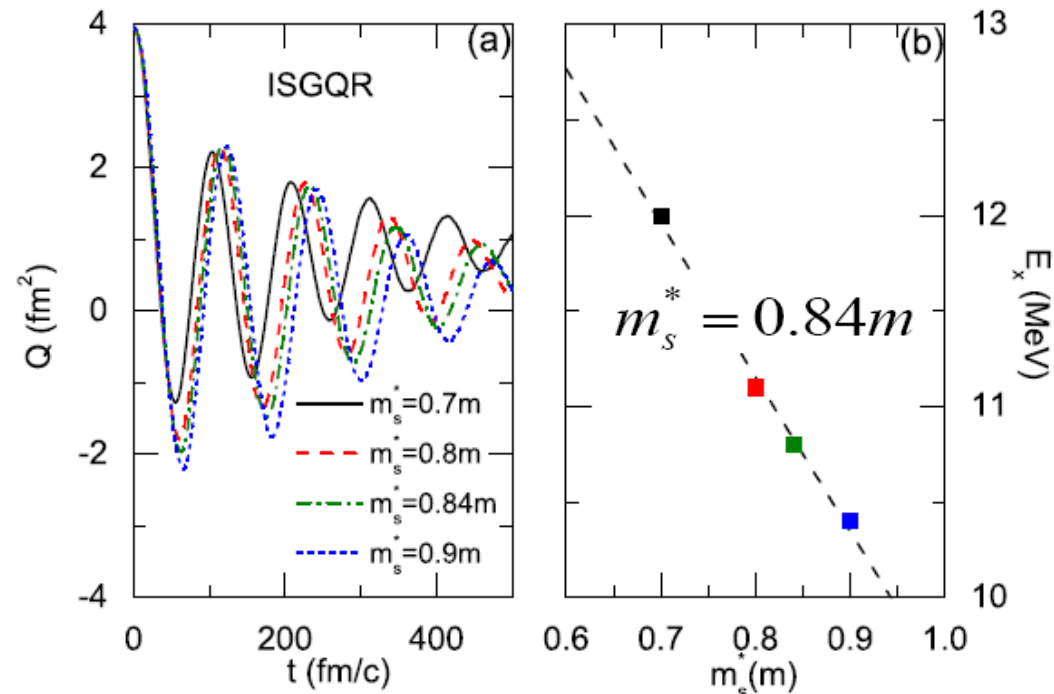
From  $\alpha$ - $^{208}\text{Pb}$  scattering data

$$E_x = 10.9 \pm 0.1 \text{ MeV}$$

Initial excitation of ISGQR  
(based on scaling relation):

$$\begin{cases} x \rightarrow x/\lambda \\ y \rightarrow y/\lambda \\ z \rightarrow \lambda^2 z \end{cases} \begin{cases} p_x \rightarrow \lambda p_x \\ p_y \rightarrow \lambda p_y \\ p_z \rightarrow p_z/\lambda^2 \end{cases}$$

$$\lambda = 1.1$$



Hai-Yun Kong, JX\*, et al.,  
Phys. Rev. C 95, 034324 (2017)

# Extract $L$ and $m_v^*$ from IVGDR

Operator of isovector giant dipole resonance (IVGDR):

$$\hat{D} = \frac{NZ}{A} \hat{X},$$

Initial excitation of IVGDR:

$$p_i \rightarrow \begin{cases} p_i - \eta \frac{N}{A} & (\text{protons}) \\ p_i + \eta \frac{N}{A} & (\text{neutrons}) \end{cases}$$

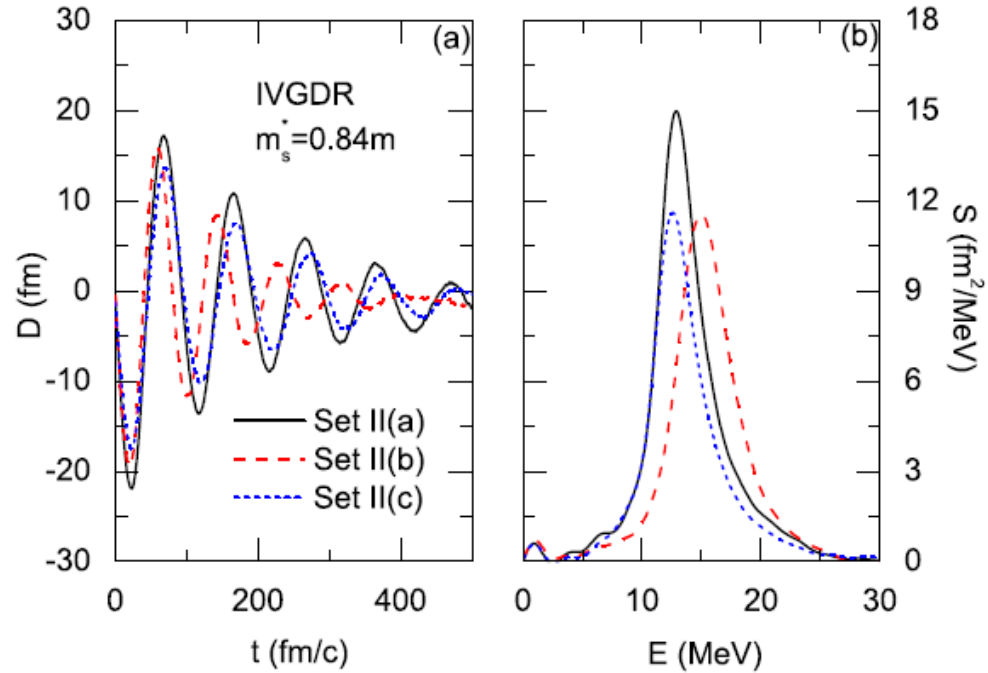
Strength function:

$$S(E) = \frac{-Im[\tilde{D}(\omega)]}{\pi\eta}$$

$$\tilde{D}(\omega) = \int_{t_0}^{t_{max}} D(t) e^{i\omega t} dt$$

Electronic dipole polarizability:

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_f}{\omega^2} d\omega = \int_0^\infty E^{-1} S(E) dE \quad (m_n^* - m_p^*)/m = (0.216 \pm 0.114)\delta$$



(a), (b), and (c) correspond to different values of  $L$ ,  $m_v^*$

Extracted slope parameter of symmetry energy:

$$L = 53.85 \pm 10.29 \text{ (MeV)}$$

Extracted np effective mass splitting:

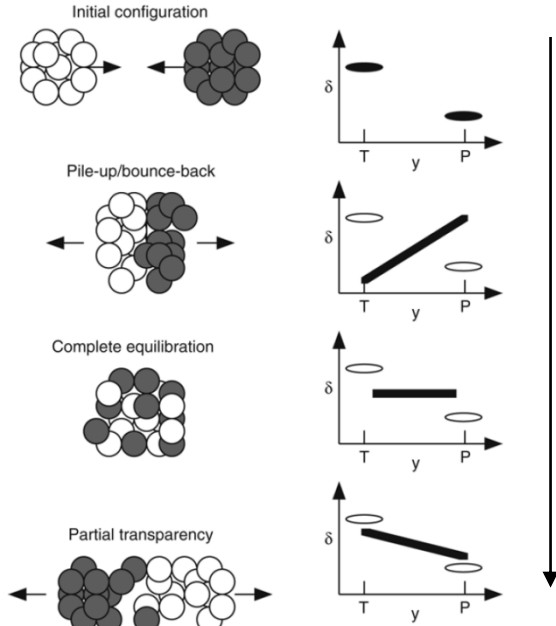
$$(m_n^* - m_p^*)/m = (0.216 \pm 0.114)\delta$$

# Isospin transport in HIC

The isovector current:  $\vec{j}_n - \vec{j}_p = (D_n^\rho - D_p^\rho)\nabla\rho - (D_n^I - D_p^I)\nabla\delta$ .

Isospin drift

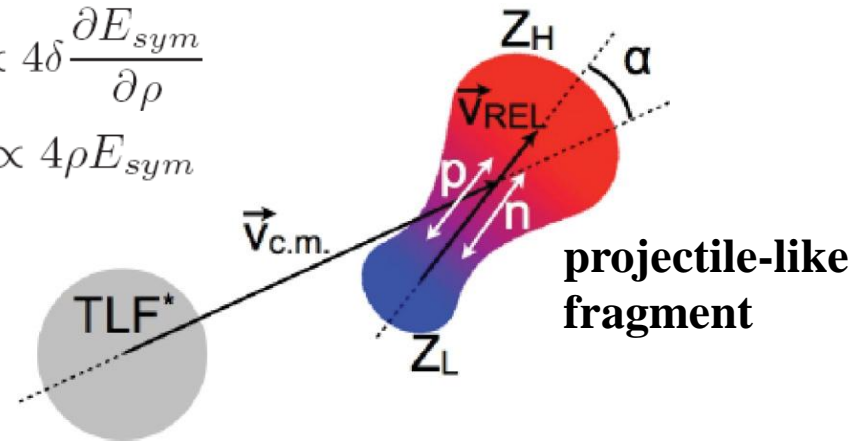
Isospin diffusion



$$D_n^\rho - D_p^\rho \propto 4\delta \frac{\partial E_{sym}}{\partial \rho}$$

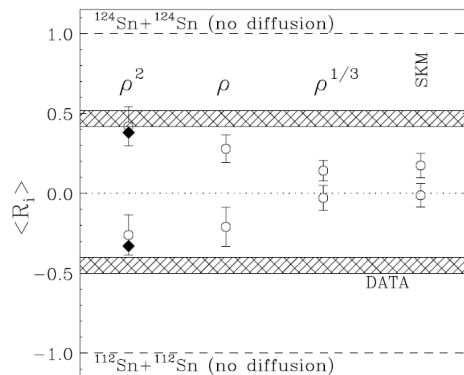
$$D_n^I - D_p^I \propto 4\rho E_{sym}$$

time

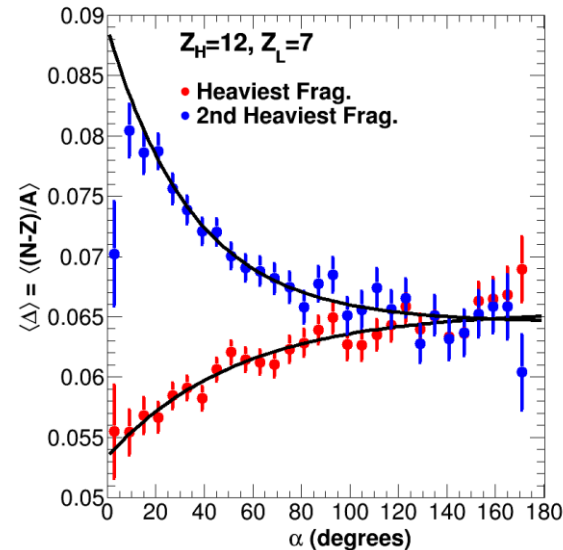


Hudan et al., Phys. Rev. C. 86, 921603(R) (2012)

B.A. Li, L.W. Chen, and C.M. Ko,  
Phys. Rep. 464, 113 (2008)



M. B. Tsang et al., Phys. Rev. Lett. 92, 062701 (2004)



Jedele et al., Phys. Rev. Lett. 118, 062501 (2017).



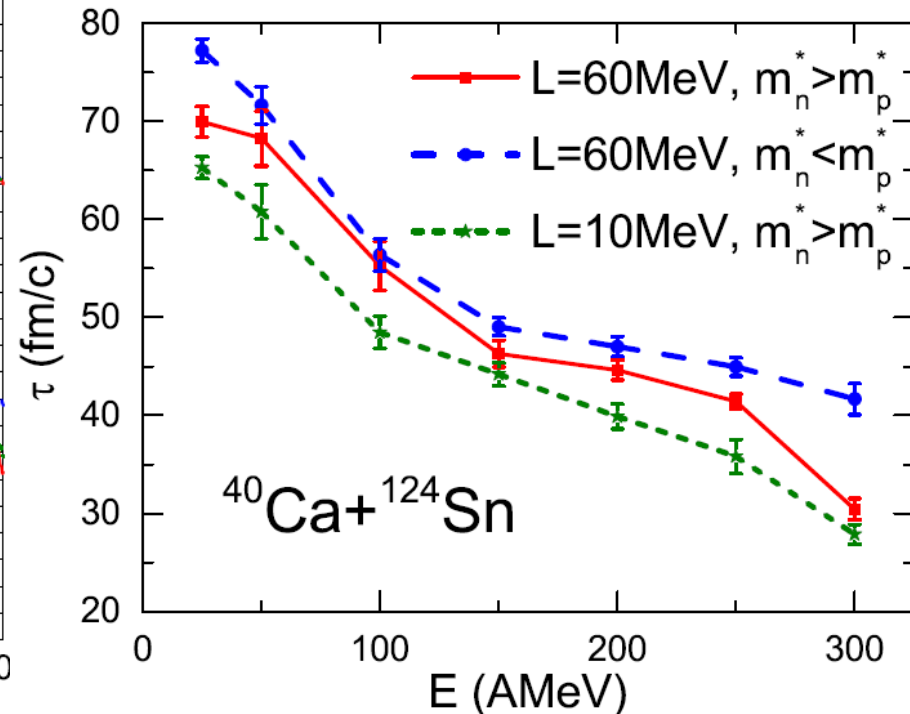
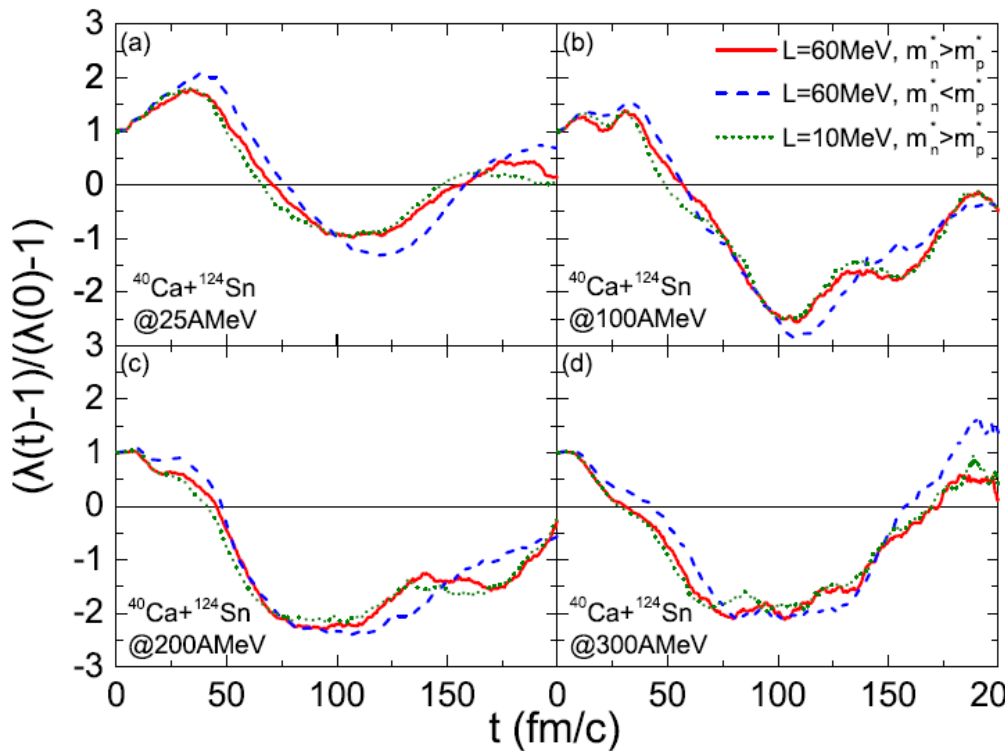
# Isospin transport between projectile and target

$^{40}\text{Ca}+^{124}\text{Sn}@b=1\text{fm}$

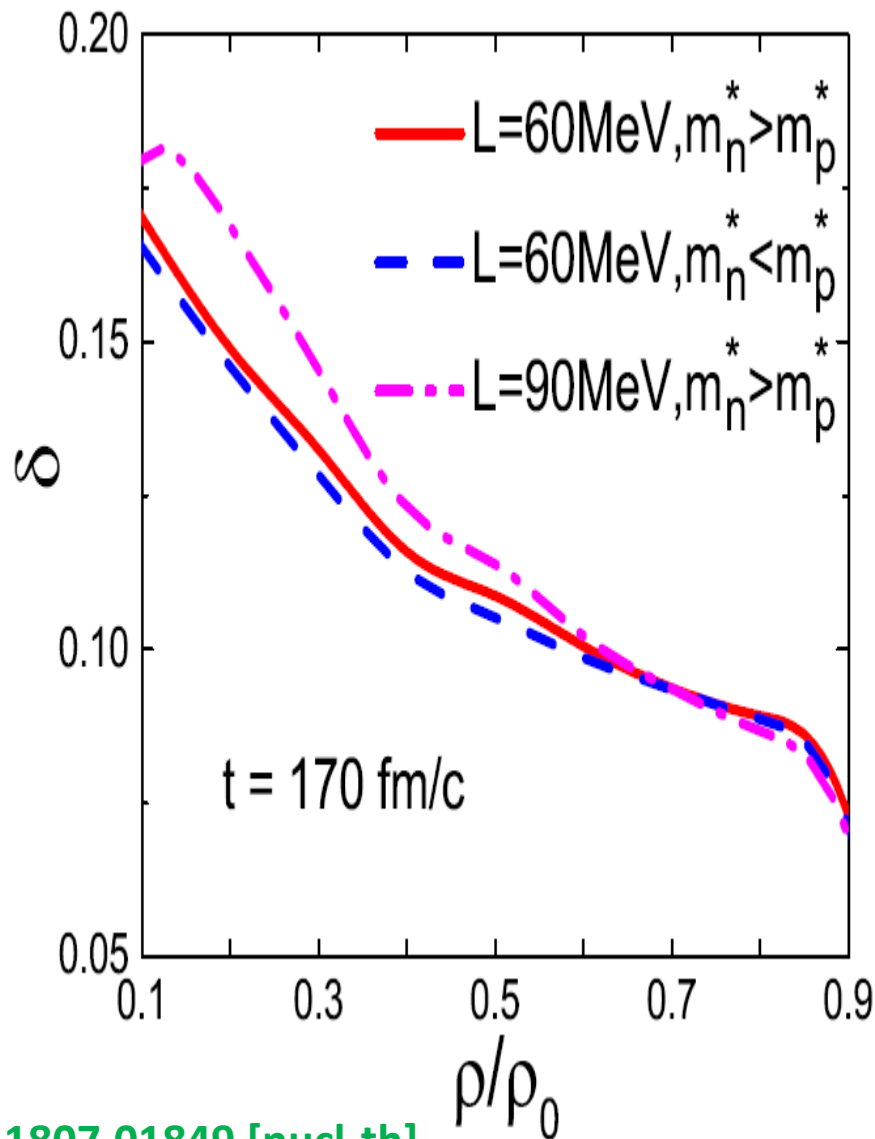
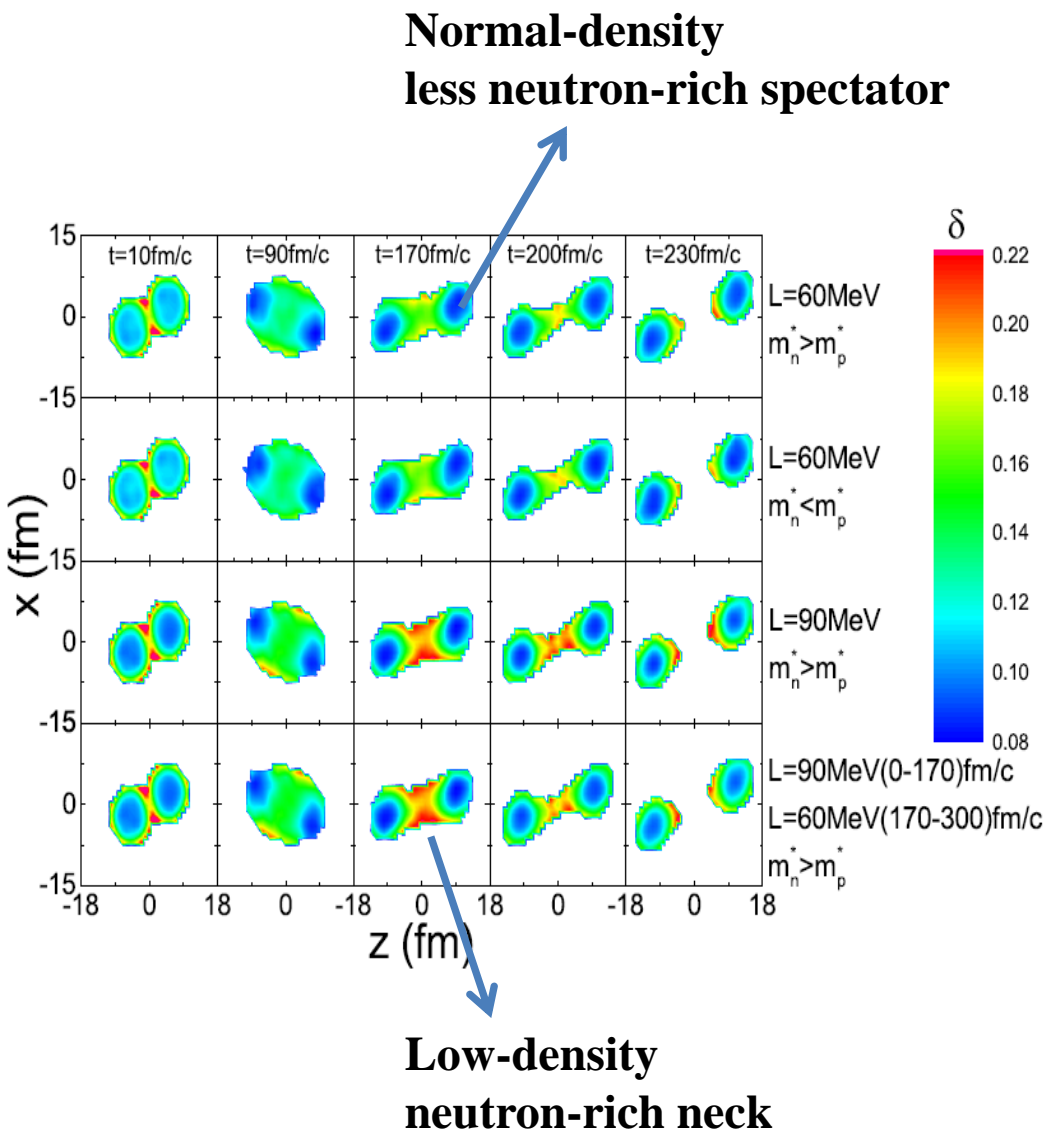
$$\lambda(t) \equiv \frac{(n/p)_{y>0}}{(n/p)_{y<0}}$$

characterizing isospin stopping/equilibrium

Isospin relaxation time  $\tau$  is defined when the isospin equilibration meter  $[\lambda(t)-1]/[\lambda(0)-1]$  first crosses 0.



# Isospin transport between neck and spectator



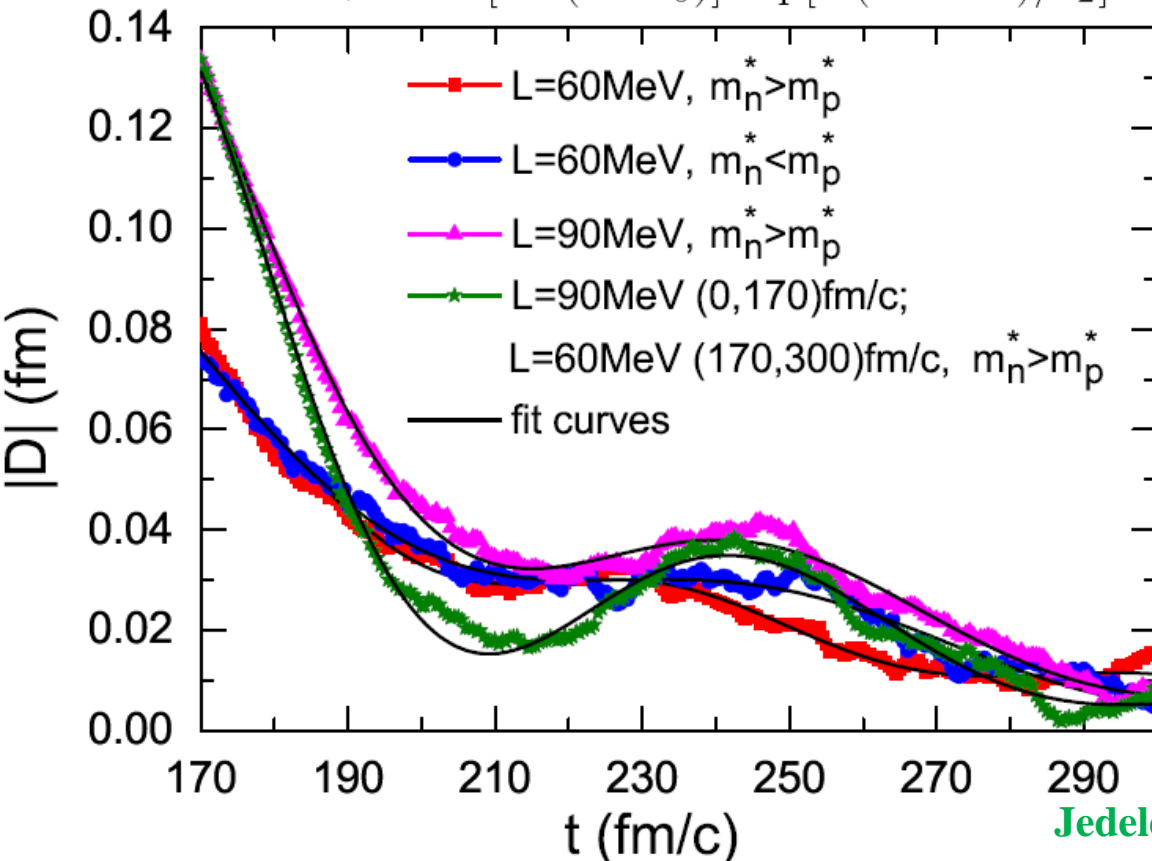
# Isospin transport between neck and spectator

Isvector dipole moment for neck-spectator matter

$$\vec{D}(t) \equiv \vec{R}_Z(t) - \vec{R}_N(t)$$

Fit with

$$|\vec{D}(t)| = a \exp[-(t - 170)/\tau_1] + b \cos[\omega \cdot (t - t_0)] \exp[-(t - 170)/\tau_2]$$



Isospin relaxation time:

Upper envelope of  $|\mathbf{D}|$  drops to  $1/e$  of its initial value

$$\vec{j}_n - \vec{j}_p = (D_n^\rho - D_p^\rho) \nabla \rho - (D_n^I - D_p^I) \nabla \delta$$

Different initial  $\nabla \delta$  from different L

$L(\text{MeV})$	$m_n^* - m_p^*(m)$	$\tau(\text{fm}/c)$
60	$0.426\delta$	$68.00 \pm 1.22$
60	$-0.251\delta$	$73.52 \pm 3.45$
90	$0.426\delta$	$60.02 \pm 1.05$
(90,60)	$0.426\delta$	$56.30 \pm 0.15$

TAMU's experiment:

$$\tau = 100 \pm_{66.7}^{233.3} \text{fm}/c$$

Jedele et al., Phys. Rev. Lett. 118, 062501 (2017)

# Final remarks

- The effect of np effective mass splitting is as important as  $E_{\text{sym}}$  in isospin dynamics of HIC and non-negligible for properties of asymmetric nuclear matter.**

The neutron–proton effective mass splitting  $m_{n-p}(\rho_0)$  in neutron-rich matter of isospin asymmetry  $\delta$  at saturation density.

Approach	$m_{n-p}(\rho_0)$	Reference
Optical model Analyses of nucleon–nucleus scattering data	$(0.41 \pm 0.15)\delta$	[41] X.H. Li et al.
Universal nuclear energy density functional	$0.637\delta$	[92] M. Kortelainen et al.
ISGQR, IVGDR & dipole polarizability of $^{208}\text{Pb}$ using SHF+RPA	$(0.27 \pm 0.15)\delta$	[42] Z. Zhang and L.W. Chen
ISGQR, IVGDR & dipole polarizability of $^{208}\text{Pb}$ using IBUU	$(0.216 \pm 0.114)\delta$	[106] K.Y. Kong et al.
General analyses of symmetry energy using HVH theorem	$(0.27 \pm 0.25)\delta$	[67] B.A. Li and X. Han
Chiral effective field theory	$(0.309 \pm 0.227)\delta$	[128,135] Jeremy Holt et al.
BCPM energy functional	$0.2\delta$	[141] M. Baldo et al.
General analyses of energy density functional	$(0.17 \pm 0.24)\delta$	[143] C. Mondal et al.

## Acknowledge

### Collaborators:

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# Thank you!

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# Backup 1

## MDI: from NN interaction to energy density

### Effective two-body NN interaction:

$$v(\vec{r}_1, \vec{r}_2) = \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2) \\ + (W + B P_\sigma - H P_\tau - M P_\sigma P_\tau) \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$$



Hartree-Fock framework

### Potential energy density:

$$H(\rho, \delta) = \frac{A_1}{2\rho_0} \rho^2 + \frac{A_2}{2\rho_0} \rho^2 \delta^2 + \frac{B}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} (1 - x\delta^2) \\ + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \int \int d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}$$

$$t_3 = \frac{16B}{(\sigma + 1)\rho_0^\sigma},$$

$$x_3 = \frac{3x - 1}{2},$$

$$\alpha = \sigma - 1,$$

$$\mu = \Lambda,$$

$$W = \frac{\Lambda^2}{3\pi\rho_0} (A_1 - A_2 + C_l - C_u),$$

$$B = \frac{\Lambda^2}{6\pi\rho_0} (-A_1 + A_2 - 4C_l + 4C_u)$$

$$H = \frac{\Lambda^2}{3\pi\rho_0} (-2A_2 - C_u),$$

$$M = \frac{\Lambda^2}{3\pi\rho_0} (A_2 + 2C_u).$$

JX and C.M. Ko, PRC 82, 044311 (2010)

# Backup 2

The cross section for the scattering of two nucleons in vacuum, from momentum states  $\mathbf{k}_1$  and  $\mathbf{k}_2$  to states  $\mathbf{k}_3$  and  $\mathbf{k}_4$  is given by

$$\frac{d\sigma}{d\Omega} = \frac{L^3}{v_{\text{rel}}} \frac{2\pi}{\hbar} |t|^2 D_f, \quad (2.1)$$

where  $L^3$  is the normalization volume,  $v_{\text{rel}}$  the relative velocity,

$$v_{\text{rel}} = \hbar |\mathbf{k}_1 - \mathbf{k}_2| / m,$$

and the density of final states

$$D_f = L^3 m |\mathbf{k}_3 - \mathbf{k}_4| / 32\pi^3 \hbar^2.$$

$$\frac{1}{\hbar} \frac{de(k, \rho)}{dk} = \frac{\hbar k}{m} + \frac{1}{\hbar} \frac{d}{dk} U(k, \rho) \equiv \frac{\hbar k}{m^*(k, \rho)}$$

$$D'_f = D_f \frac{m^*[\sqrt{\frac{1}{2}(k_3^2 + k_4^2)}, \rho]}{m}$$

the present context. Using  $t' \approx t$  we obtain

$$\frac{d\sigma'}{d\Omega} = \frac{v_{\text{rel}}}{v'_{\text{rel}}} \frac{D'_f}{D_f} \frac{d\sigma}{d\Omega} \quad (2.8)$$

$$= \frac{|\mathbf{k}_1 - \mathbf{k}_2|}{m} \left[ \left| \frac{\mathbf{k}_1}{m^*(k_1, \rho)} - \frac{\mathbf{k}_2}{m^*(k_2, \rho)} \right| \right]^{-1} \\ \times \frac{m^*[\sqrt{(k_3^2 + k_4^2)/2}, \rho]}{m} \frac{d\sigma}{d\Omega}. \quad (2.9)$$

Effective mass scaling of NN cross section

Pandharipande and Peiper  
PRC (1992)

# Asymmetric nuclear matter: The role of the isovector scalar channel

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(Received 5 October 2001; published 19 March 2002)

## Langrangian $\sigma, \omega, \rho, \delta$

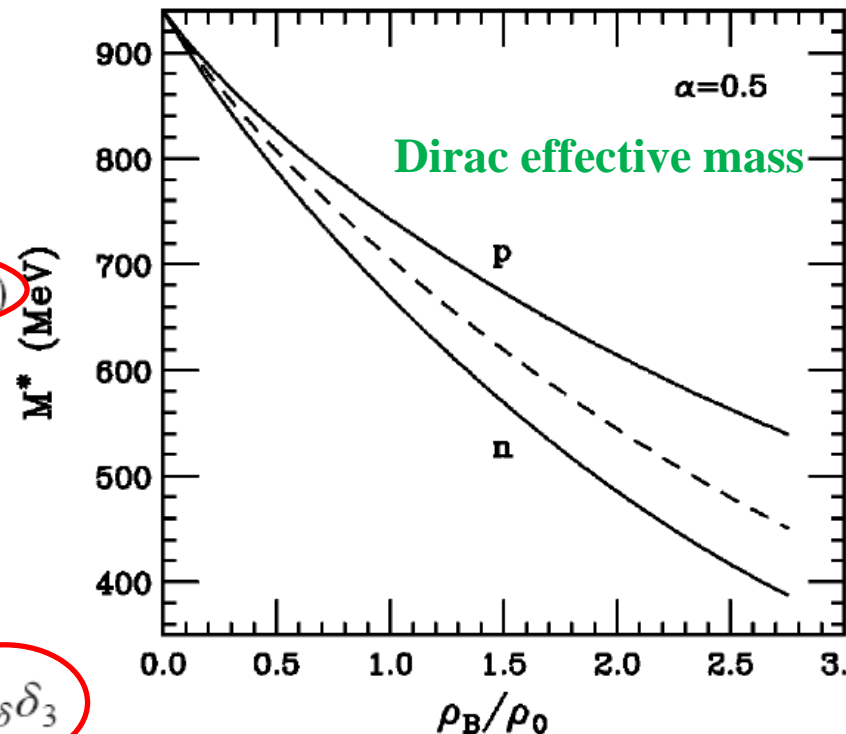
$$\begin{aligned} \mathcal{L} = & \bar{\psi} [i\gamma_\mu \partial^\mu - (M_N - g_\sigma \phi - g_\delta \vec{\tau} \cdot \vec{\delta}) - g_\omega \gamma_\mu \omega^\mu \\ & - g_\rho \gamma^\mu \vec{\tau} \cdot \vec{b}_\mu] \psi + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_\sigma^2 \phi^2) - U(\phi) \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \vec{b}_\mu \cdot \vec{b}^\mu + \frac{1}{2} (\partial_\mu \vec{\delta} \cdot \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}. \end{aligned}$$

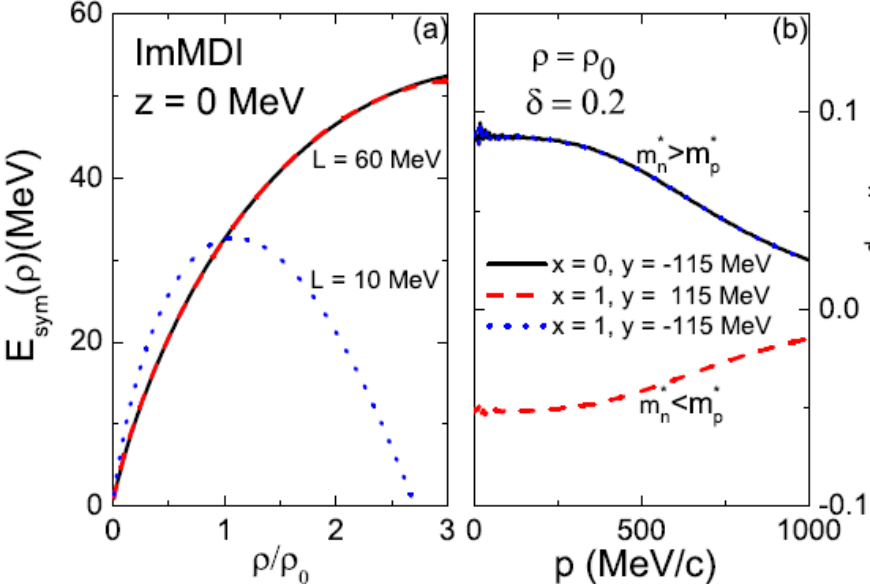
## Field equation

$$m_\delta^2 \delta_3 = g_\delta \bar{\psi} \tau_3 \psi = g_\delta \rho_{S3}$$

## Nucleon effective mass

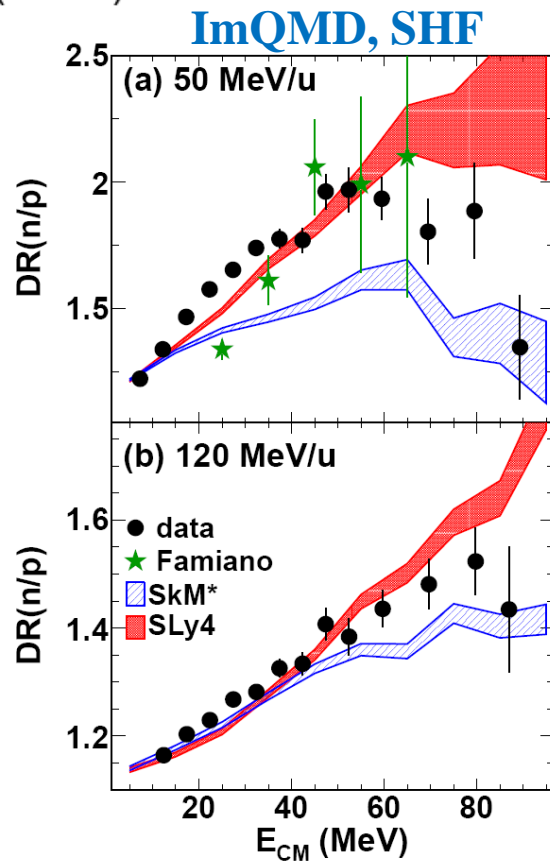
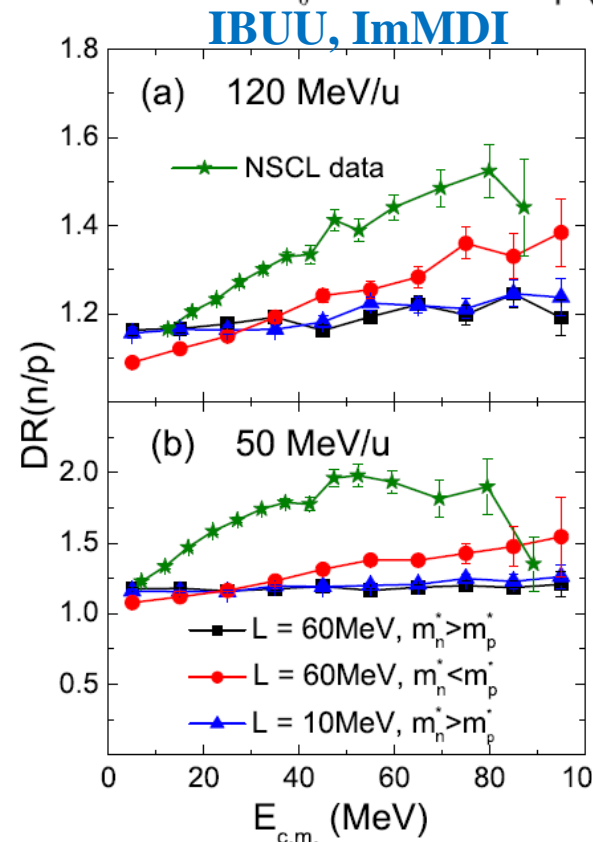
$$M_i^* = M_N - g_\sigma \phi \mp g_\delta \delta_3$$





Skyrme	$S_0$ (MeV)	L (MeV)	$m_n^*/m_n$	$m_p^*/m_p$
SLy4	32	46	0.68	0.71
SkM*	30	46	0.82	0.76

D.D.S. Coupland et al., arXiv:1406.4546



$$DR(n/p) = \frac{[Y(n)/Y(p)]_{124\text{Sn}+124\text{Sn}}}{[Y(n)/Y(p)]_{112\text{Sn}+112\text{Sn}}}$$

Still below the NSCL/MSU data  
no matter how the symmetry energy  
and effective mass splitting is adjusted.

H.Y. Kong, Y. Xia, JX\*,  
L.W. Chen, B.A. Li, and Y.G. Ma  
PRC 91, 047601 (2015)



# How to explain DR(n/p) data?

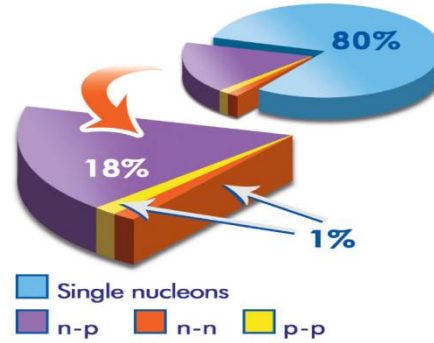
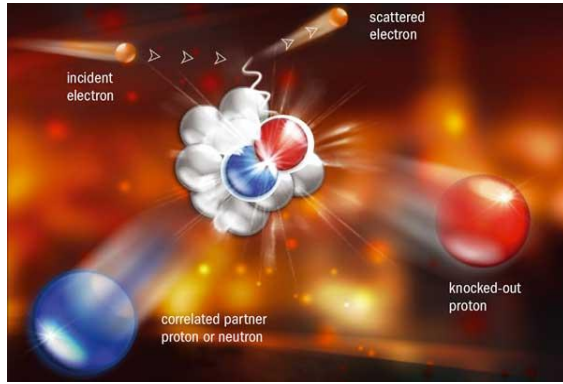
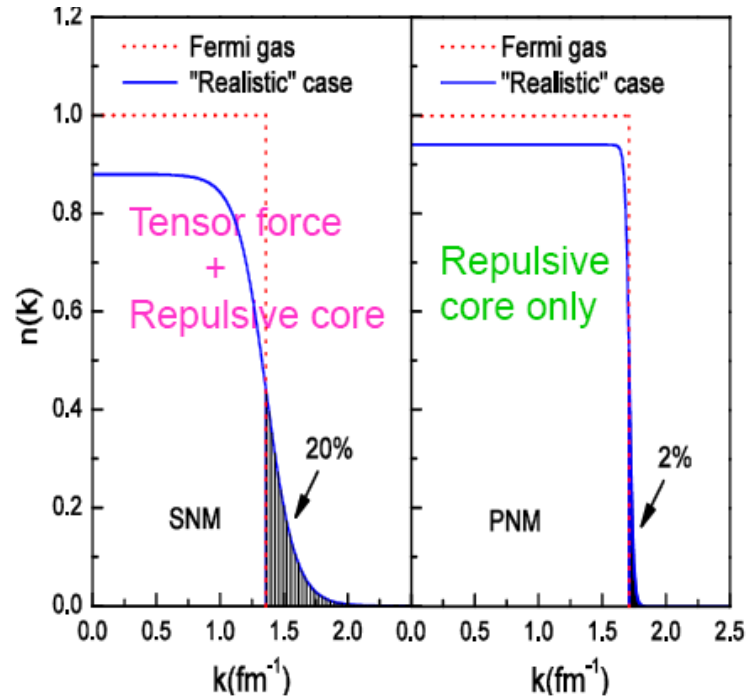


Figure 3: The average fraction of nuclei in the various initial state configurations of  $^{12}\text{C}$ .

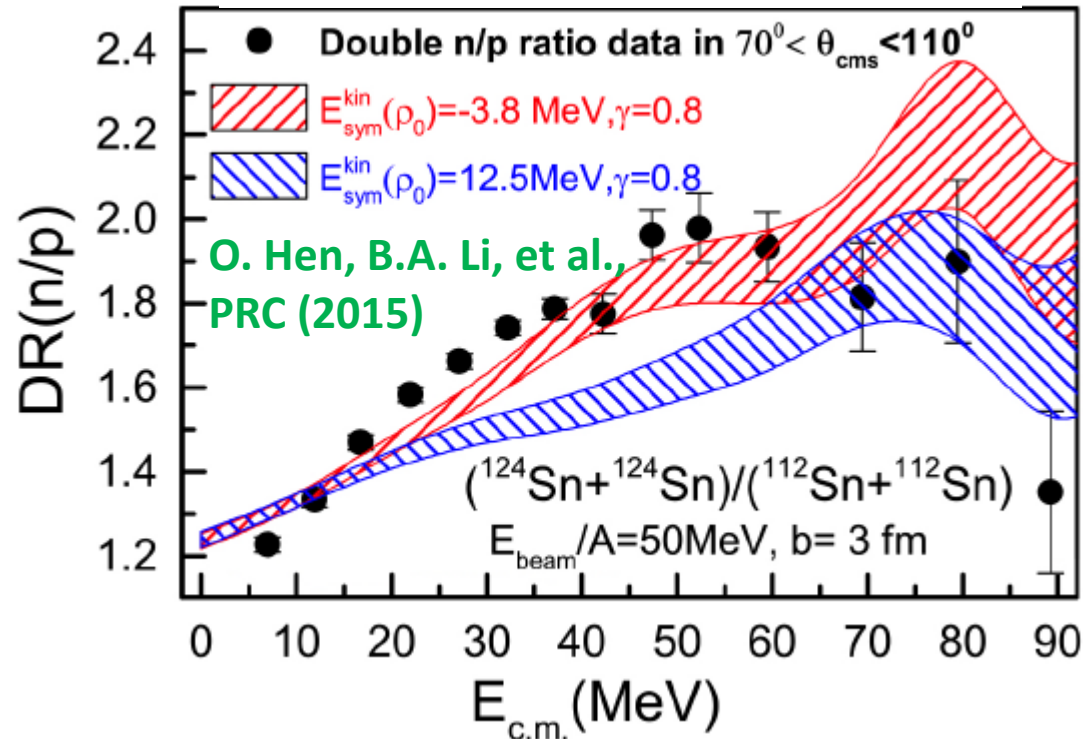
$$E_{kin} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk$$

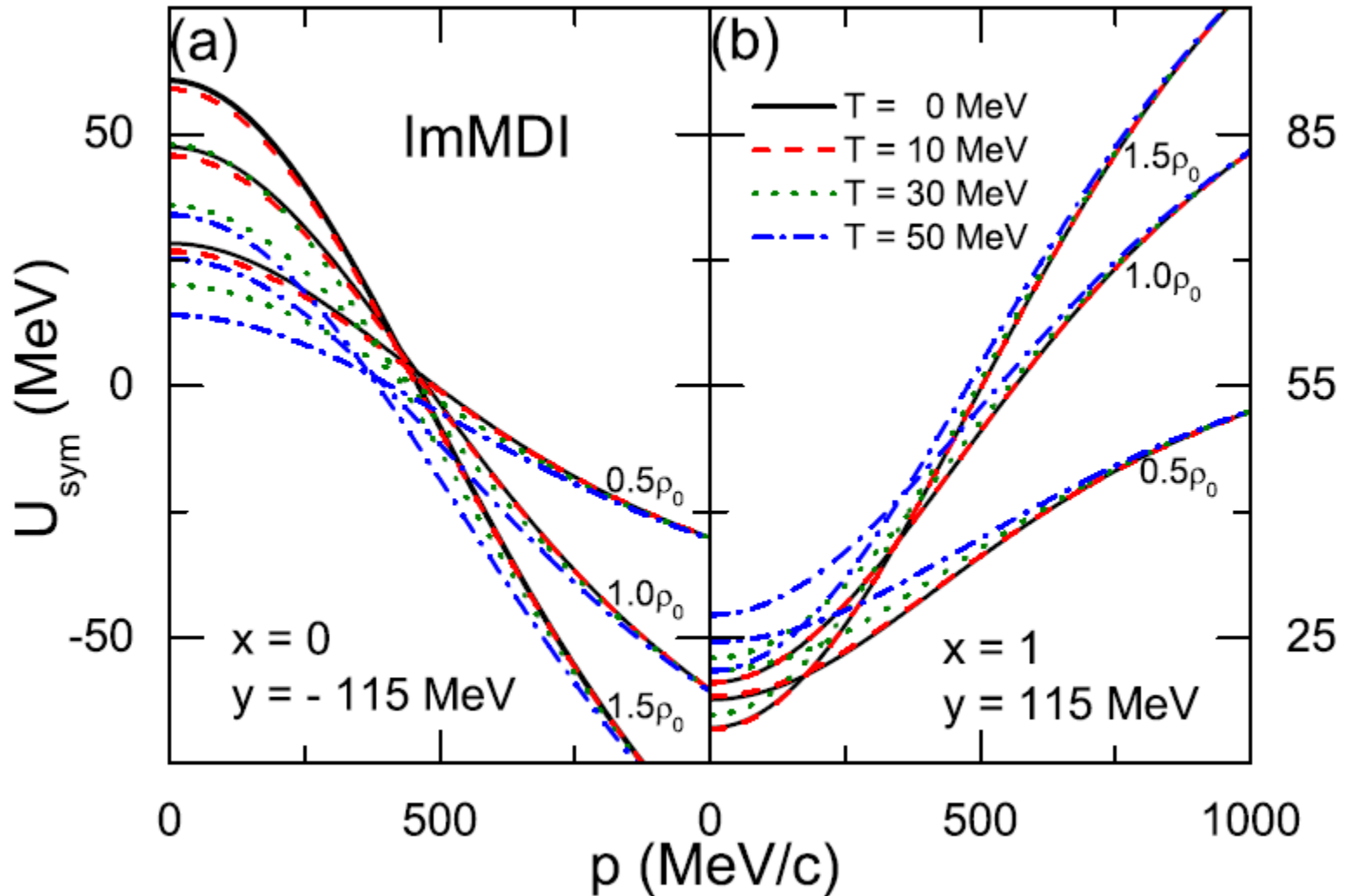
$$E_{sym}^{kin} = E_{PNM}^{kin} - E_{SNM}^{kin} < 0$$



$$E_{sym}^{pot}(\rho) = [E_{sym}(\rho_0) - \eta E_{sym}^{kin}(\rho_0)]_{\text{FG}} (\rho/\rho_0)^\gamma$$

$$V_{sym}^{n/p}(\rho, \delta) = [E_{sym}(\rho_0) - \eta E_{sym}^{kin}(\rho_0)]_{\text{FG}} (\rho/\rho_0)^\gamma \times [\pm 2\delta + (\gamma - 1)\delta^2], \quad ($$





**( $x=0, y=-115$  MeV) and ( $x=1, y=115$  MeV) have almost the same density dependence of  $E_{\text{sym}}$ , but  $U_{\text{sym}}$  at lower momenta is different, and the difference increases with increasing density.**