

Relativistic covariance corrections to transport models and symmetry energy constraints

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Motivation

Previous studies:
TuQMD

pionic observables
elliptic flow

D.C, PRC 95, 014601 (2017)
D.C. EPJA 54, 40 (2018)

Conclusions of these studies:

- pions: large dependence on the pion optical potential
- elliptic flow: need to perform experimental measurements at different energies for n/p and n/H(or ch) elliptic flow ratios

Eventually:

- describe kaon production to constrain the symmetry energy above $2 \rho_0$

Known facts:

- probed density and relativistic effects increase with projectile energy

However:

TuQMD – relativistic kinematics

- non-relativistic dynamics

Expected magnitude of correction : $\sim \gamma = 1.1$ @ 400 MeV/u

Gogny inspired potential (MDI2)

momentum dependent potential **MDI2**

$$\frac{E}{N}(\rho, \beta, x, y) = \frac{1}{2} A_1 u + \frac{1}{2} A_2(x, y) u \beta^2 + \frac{B u^\sigma}{\sigma + 1} (1 - x \beta^2) + \frac{D u^2}{3} (1 - y \beta^2)$$

$$+ \frac{1}{u \rho_0^2} \sum_{\tau, \tau'} C_{\tau\tau'} \int \int d^3 p d^3 p' \frac{f_\tau(p, p') f_{\tau'}(p, p')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}$$

$$A_2(x, y) = A_2^0 + \frac{2x B}{\sigma + 1} \bar{u}^{\sigma-1} + \frac{2y D}{3} \bar{u} \quad u = \frac{\rho}{\rho_0}$$

Fit:

U_∞, K, J_0, m^* -isoscalar

$S(\tilde{u}), L, K_{\text{sym}}, \delta m_{\text{isv}}$ -isovector

A_l^0, x, y

$C_l - C_u$
 δ_{n-p}^*

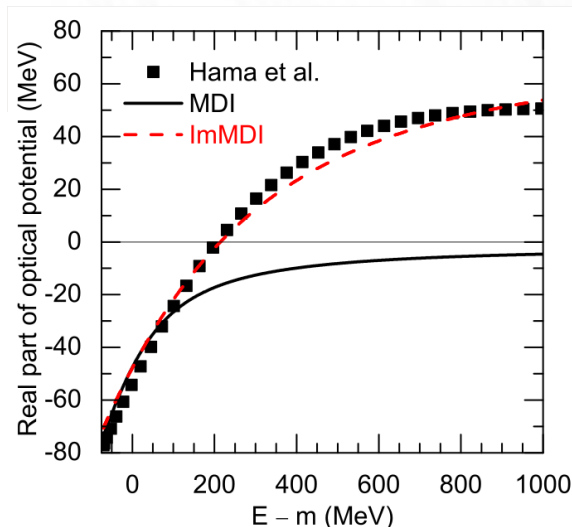
momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)

(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))

used previously to test model dependence: flow ratio PRC 88, 44912 (2013)

pion multiplicity ratio PLB 753, 166 (2016)

independent part: extra term (vary L vs. K_{sym} and also J_0 vs. K independently)

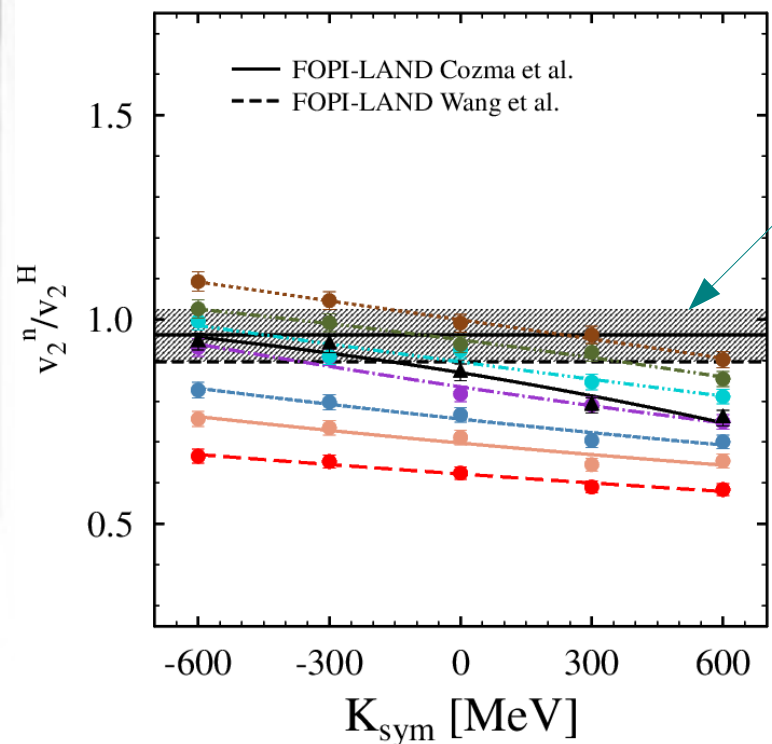


from J. Xu et al. PRC 91, 014611 (2015)

Input		Parameters	
ρ_0 [fm ⁻³]	0.16	A [MeV]	708.001
E_B [MeV]	-16.0	C_l [MeV]	-13.183
m_s^*/m	0.70	C_u [MeV]	-140.405
$\delta_{n-p}^* (\rho_0, \beta = 0.5)$	0.165	B [MeV]	137.305
K_0 [MeV]	245.0	σ	1.2516
J_0 [MeV]	-350.0	$\tilde{A}_l$ [MeV]	-130.495
$\tilde{\rho}$ [fm ⁻³]	0.10	$\tilde{A}_u$ [MeV]	-8.828
$S(\tilde{\rho})$ [MeV]	25.4	D [MeV]	7.357

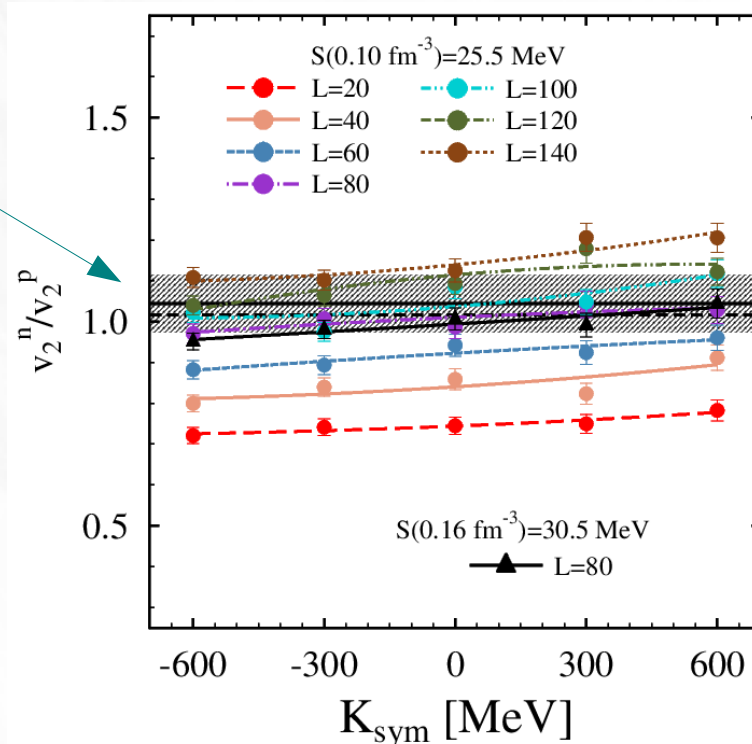
Sensitivity to K_{sym} (and S_0)

3 dimensional parameter space: heavy-ion observables $\rightarrow$ 1 dim constraint
 +nuclear structure $\rightarrow$ determine values



$S(0.10 \text{ fm}^{-3}) = 25.5 \text{ MeV}$

FOPI-LAND



B.A. Brown, PRL111, 232502 (2013)

n/p and n/H (n/ch) elliptic flow ratios probe on average different densities

$1.4-1.5 \rho_0$

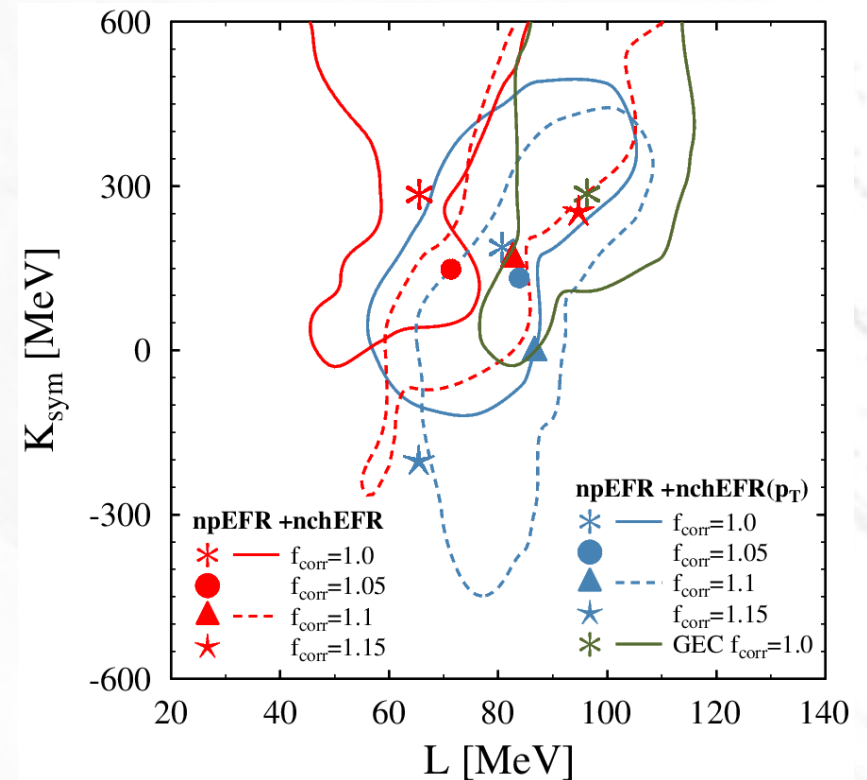
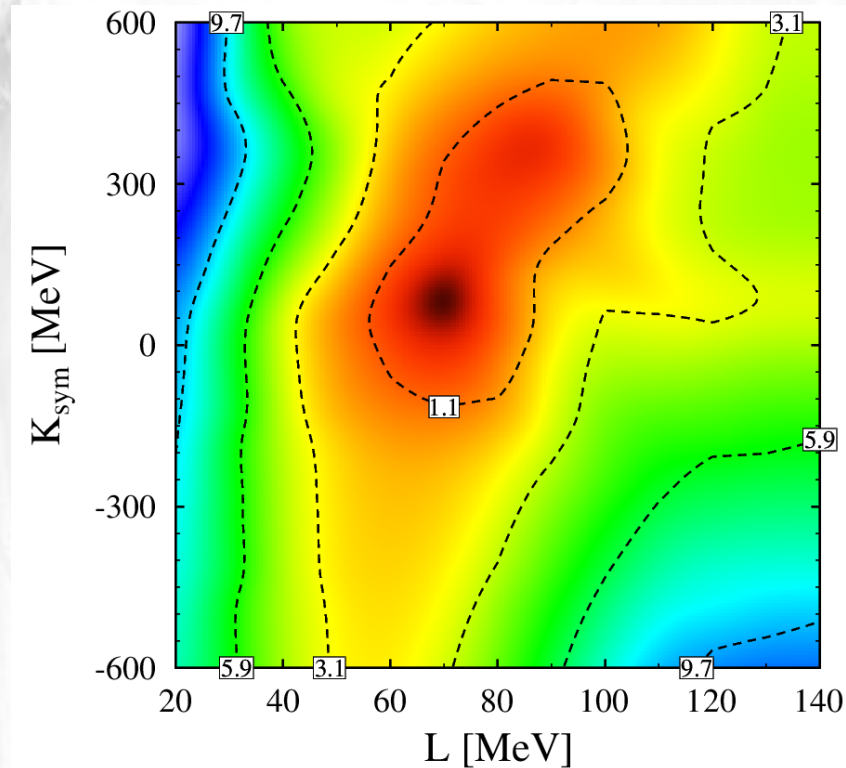
$1.0-1.1 \rho_0$

P. Russotto et al, PRC 94, 034608 (2016)

Constraints for L and K_{sym}

MDI2

n/p [FOPI/LAND] + n/ch (p_T)[ASYEOS] elliptic flow ratios



neutron-proton v_2^n/v_2^p + neutron-charged part. v_2^n/v_2^{ch}

$$L = 85 \pm 22(\text{exp}) \pm 20(\text{th}) \pm 12(\text{sys}) \text{ MeV}$$

$$K_{\text{sym}} = 96 \pm 315(\text{exp}) \pm 170(\text{th}) \pm 166(\text{sys}) \text{ MeV}$$

D.C. EPJA 54, 40 (2018)

Equations of Motions

RQMD Hamiltonian:

$$H \approx \sum_{i=1}^{i=N} u_i (p_i^2 - m_i^2 - 2m_i V_i)$$

$$u_i = \frac{1}{2p_i^0}, p_i^0 = \sqrt{\vec{p}_i^2 + m_i^2 + 2m_i V_i}$$

particular time fixation
compatible with particle
production

T. Maruyama et al., Prog. Theor. Phys. 96, 263 (1996)

Equations of Motion:

$$\frac{d\vec{r}_i}{d\tau} \approx \frac{-\partial H}{\partial \vec{p}_i} = \frac{\vec{p}_i}{p_i^0} + \sum_{j=1}^N \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{p}_i}$$

$$\frac{d\vec{p}_i}{d\tau} \approx \frac{\partial H}{\partial \vec{r}_i} = - \sum_{j=1}^N \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{r}_i}$$

approximation:

$$p_i^0 = \sqrt{(\vec{p}_i^2 + m_i^2)}$$

covariant expressions for distance
and momentum difference

$$\vec{r}_{ij}^2 \rightarrow -q_{ij}^2 = -q_{ij}^2 + \frac{(q_{ij} \cdot P_{ij})^2}{P_{ij}^2}$$

$$\vec{p}_{ij}^2 \rightarrow -p_{ij}^2 = -p_{ij}^2 + \frac{(p_{ij} \cdot P_{ij})^2}{P_{ij}^2}$$

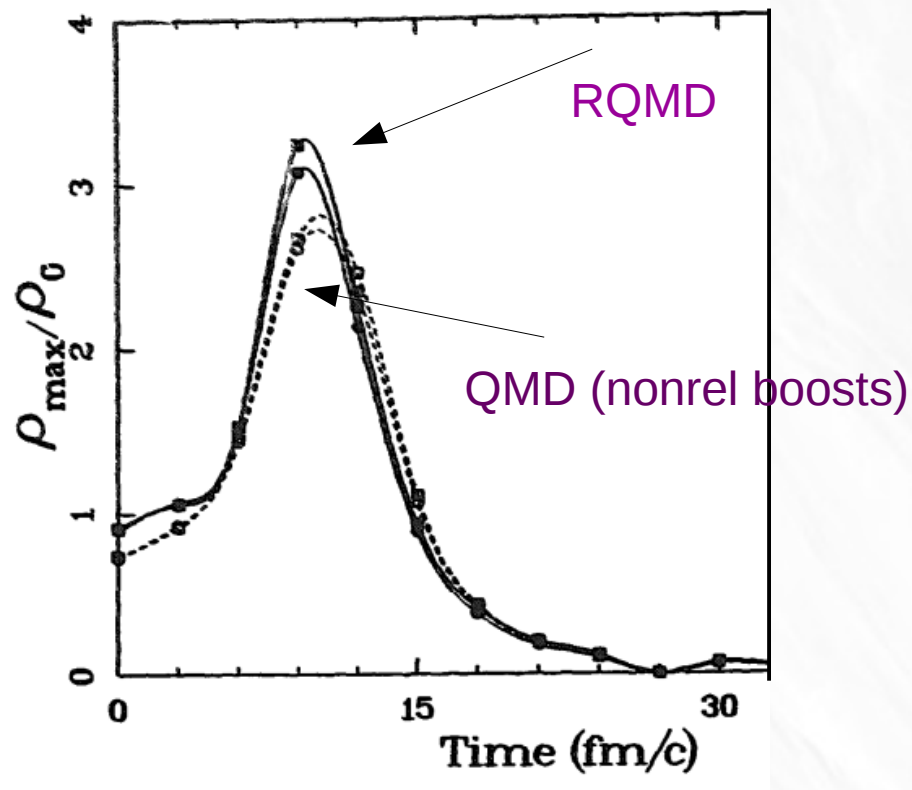
Non-relativistic limit: V_i identified with the non-relativistic potential

Pauli blocking: surface correction terms evaluated in the rest frame of particle i , rather than in computational frame

Additionally: algorithm to select initial configurations (nuclei) with improved stability

Consequences of Relativistic Dynamics

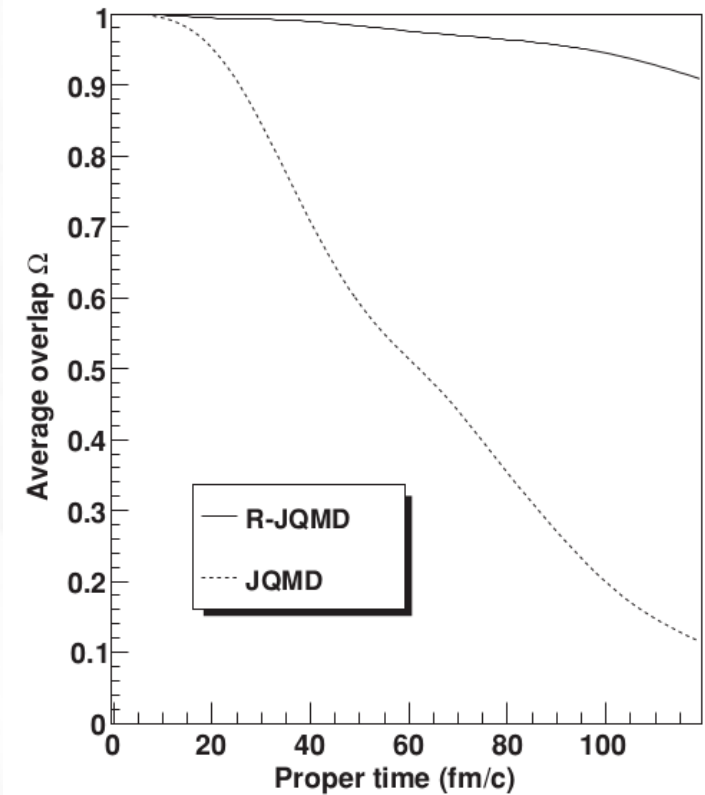
maximum density reached



$^{12}\text{C}+^{12}\text{C}$ 800 MeV/u $b=0.0$ fm

T. Maruyama et al., NPA 534, 720 (1991)

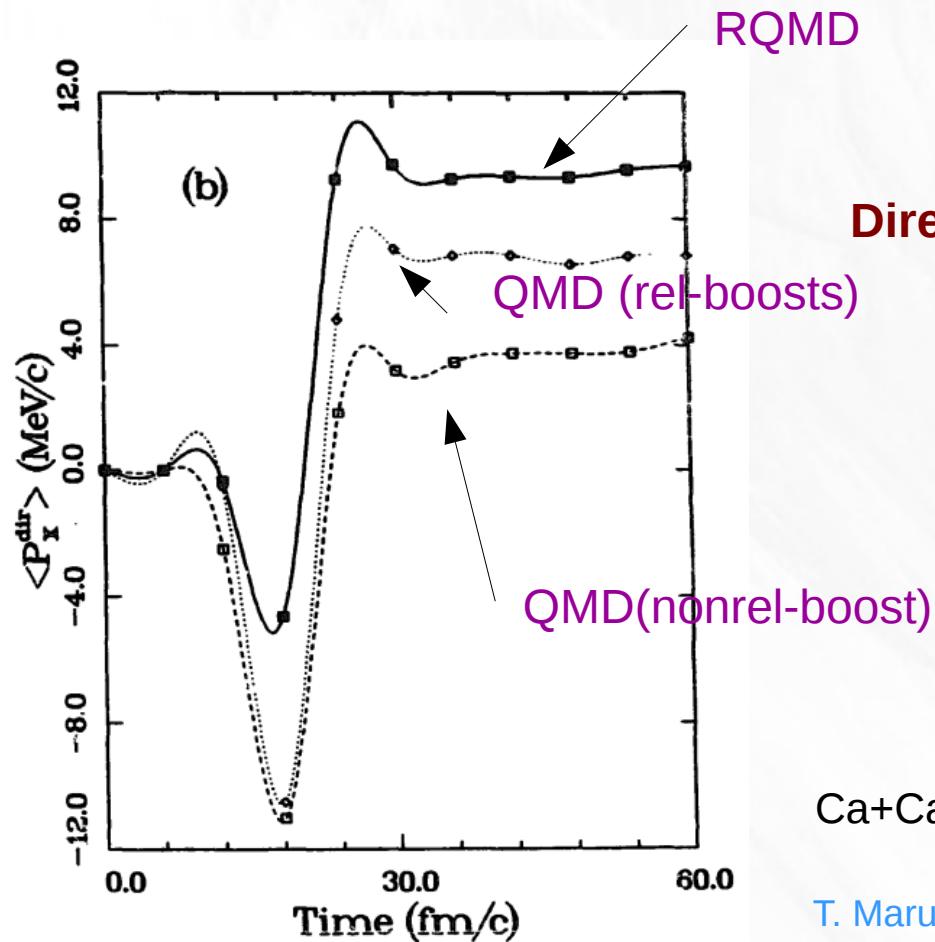
stability of nucleus's wavefunction



^{40}Ca , $\gamma=1.24$

D. Mancusi et al., PRC 79, 014614 (2009)

Consequences of Relativistic Dynamics



Directed transverse momentum

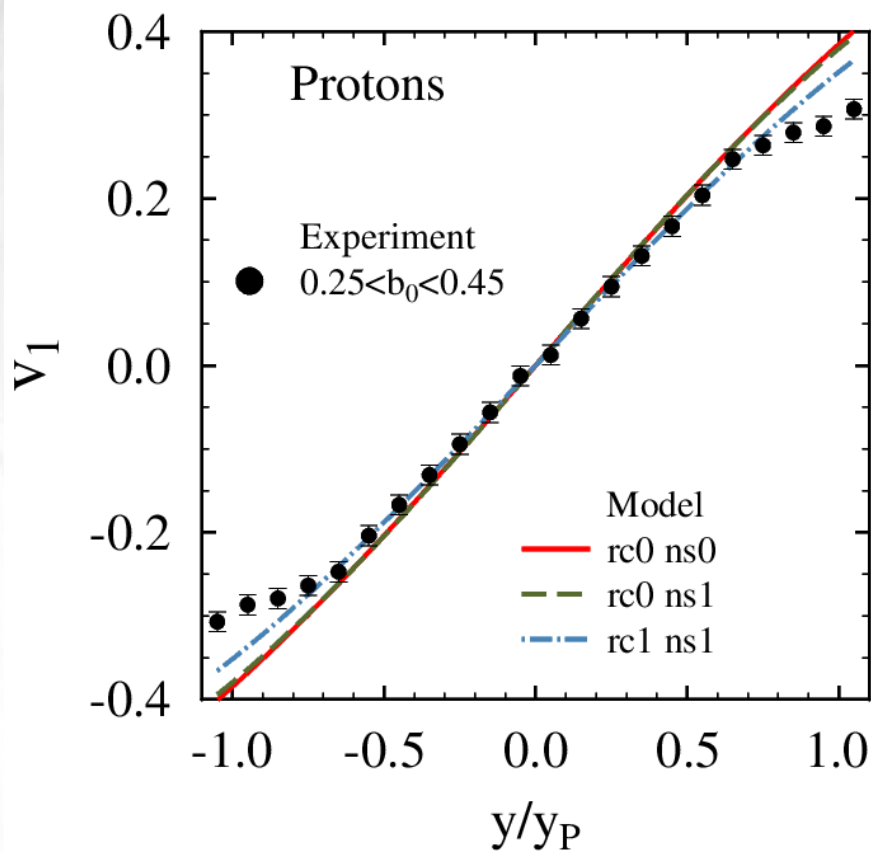
$$\langle p_x^{dir} \rangle = \frac{1}{N} \sum_{i=1}^N \text{sign}(y^{(i)}) p_x^{(i)}$$

Ca+Ca @ 1050 MeV/u, $b=3.9$ fm

T. Maruyama et al., NPA 534, 720 (1991)

Directed Flow

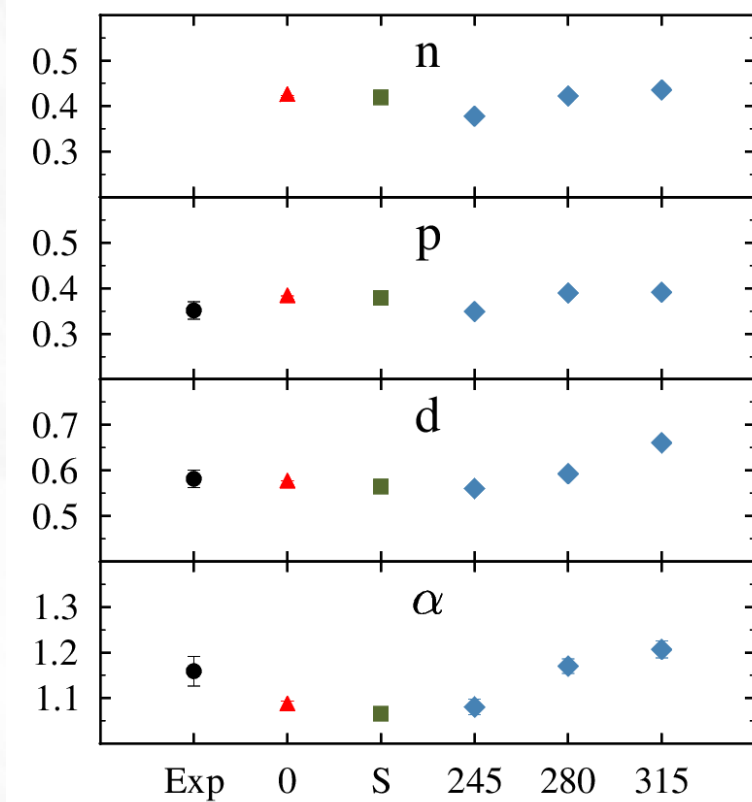
Au+Au @ 400 MeV/u
 $0.25 < b_0 < 0.45$; $u_{t0} > 0.4$



Experimental data: [W. Reisdorf et al. \(FOPI\), NPA 876, 1 \(2012\)](#)

$$v_1 = v_{11} y_0 + v_{13} y_0^3 + \dots$$

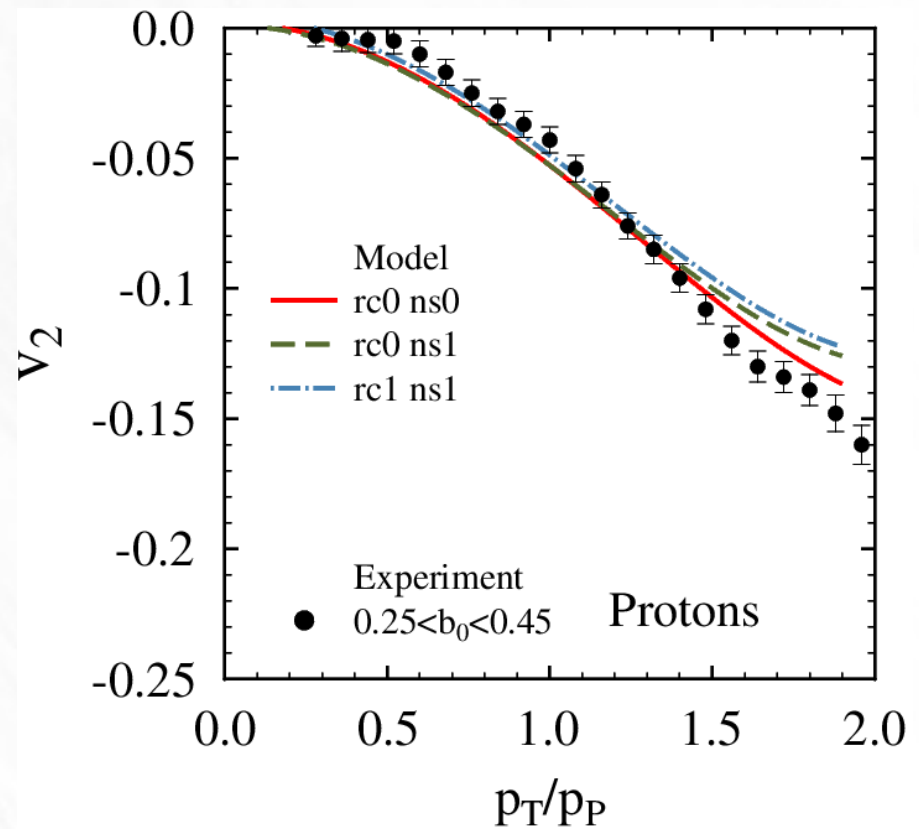
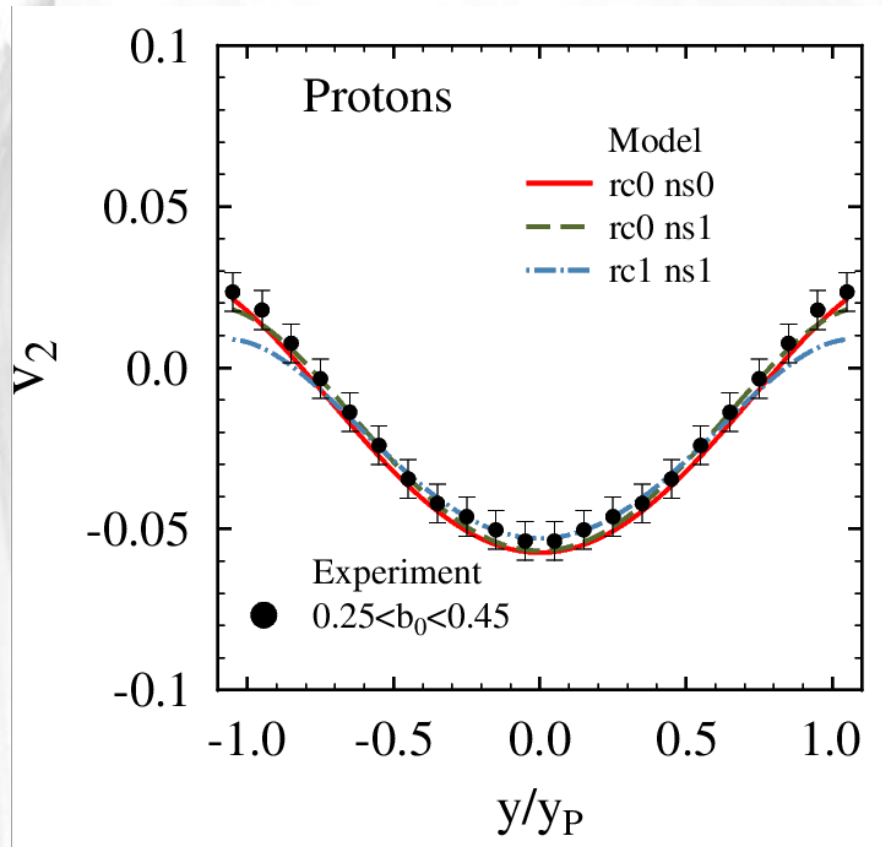
v_{11}



Model: asyEoS $L=70$ MeV
 $K_{\text{sym}} = 0$ MeV

Elliptic Flow

Au+Au @ 400 MeV/u
 $0.25 < b_0 < 0.45$; $u_{t0} > 0.4$



Experimental data: [W. Reisdorf et al. \(FOPI\), NPA 876, 1 \(2012\)](#)

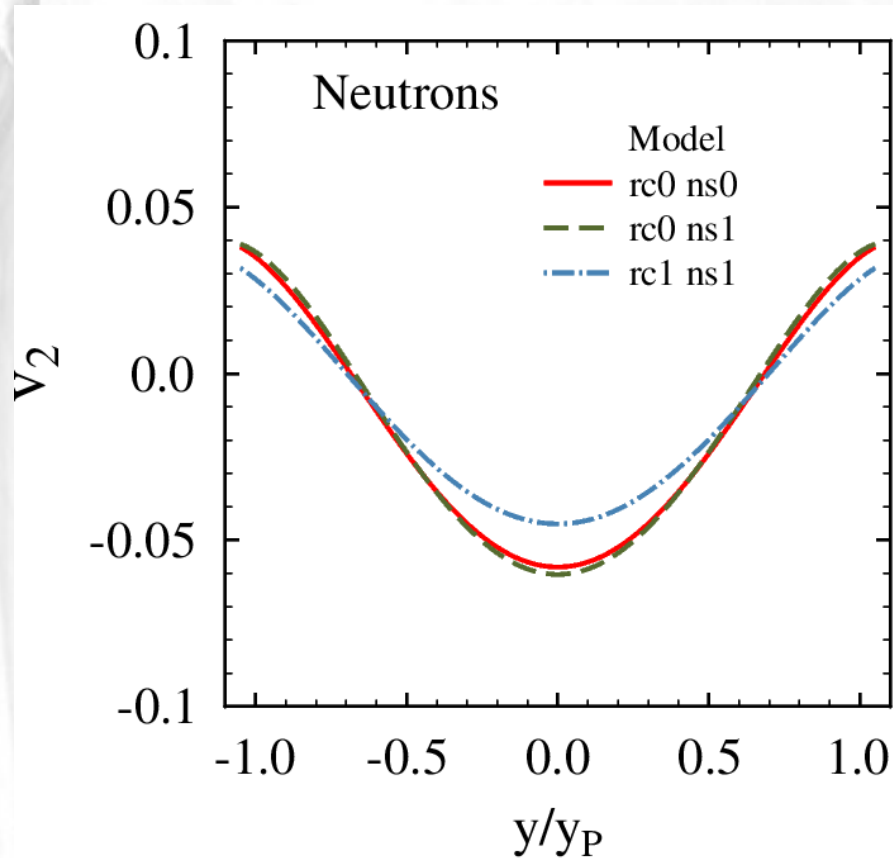
rc – relativistic covariance
 ns - nucleus stability

Elliptic Flow

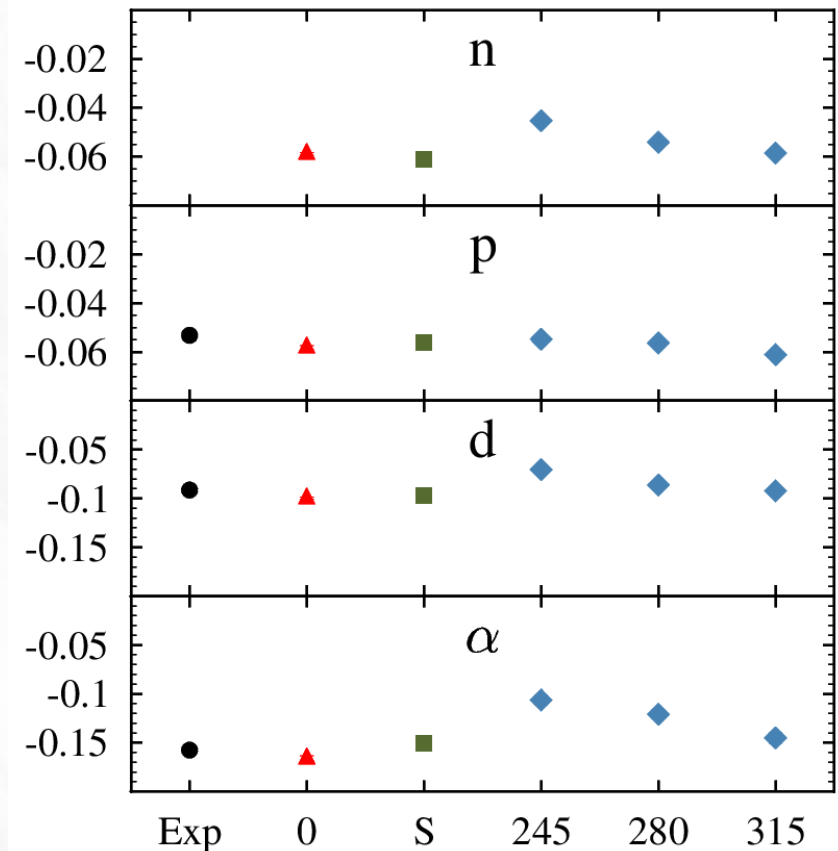
Au+Au @ 400 MeV/u
 $0.25 < b_0 < 0.45$; $u_{t0} > 0.4$

$$v_2 = v_{20} + v_{22} y_0^2 + \dots$$

v_{20}



Experimental data: [W. Reisdorf et al. \(FOPI\), NPA 876, 1 \(2012\)](#)

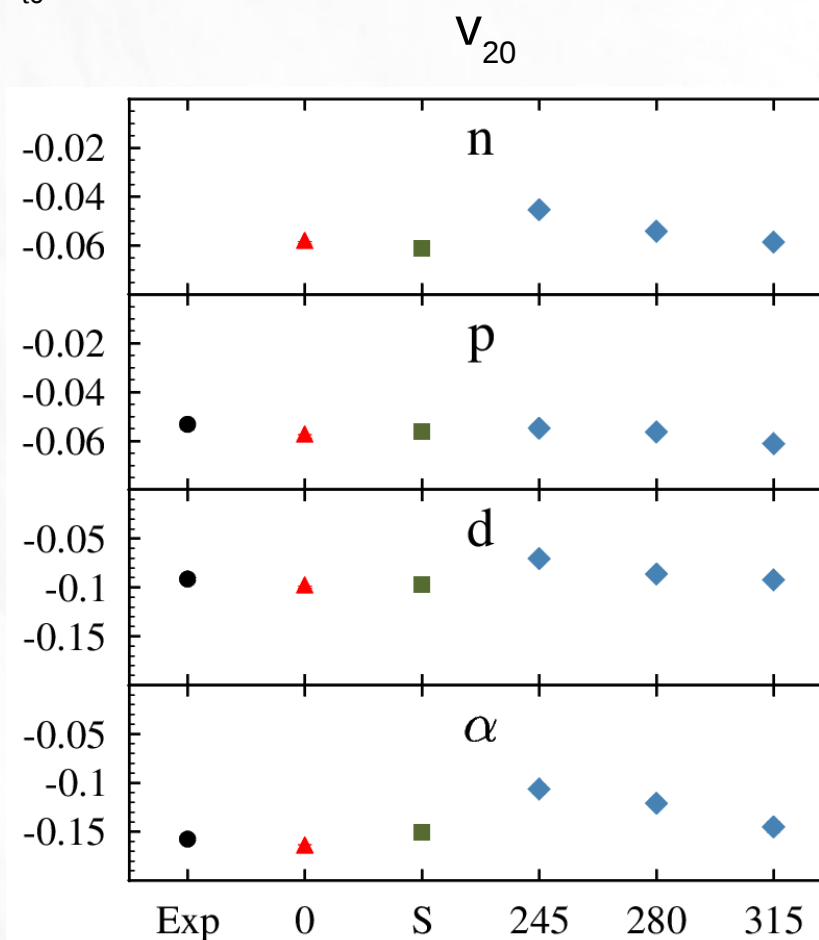
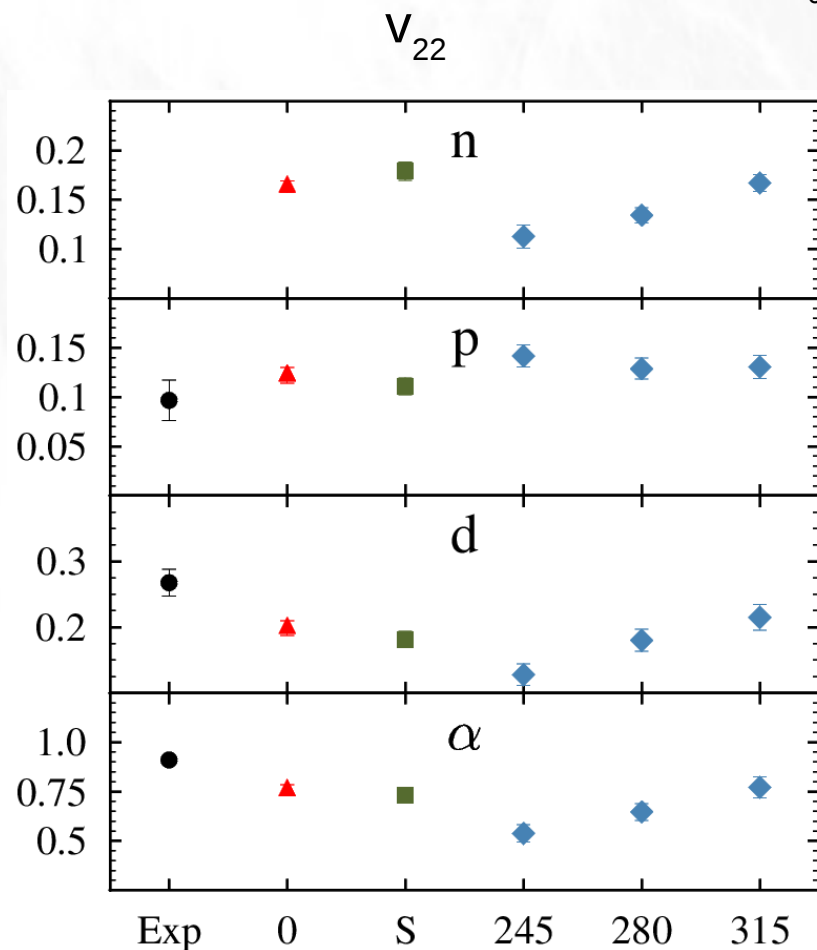


rc – relativistic covariance
 ns - nucleus stability

Elliptic Flow

Au+Au @ 400 MeV/u
 $0.25 < b_0 < 0.45$; $u_{t0} > 0.4$

$$v_2 = v_{20} + v_{22} y_0^2 + \dots$$



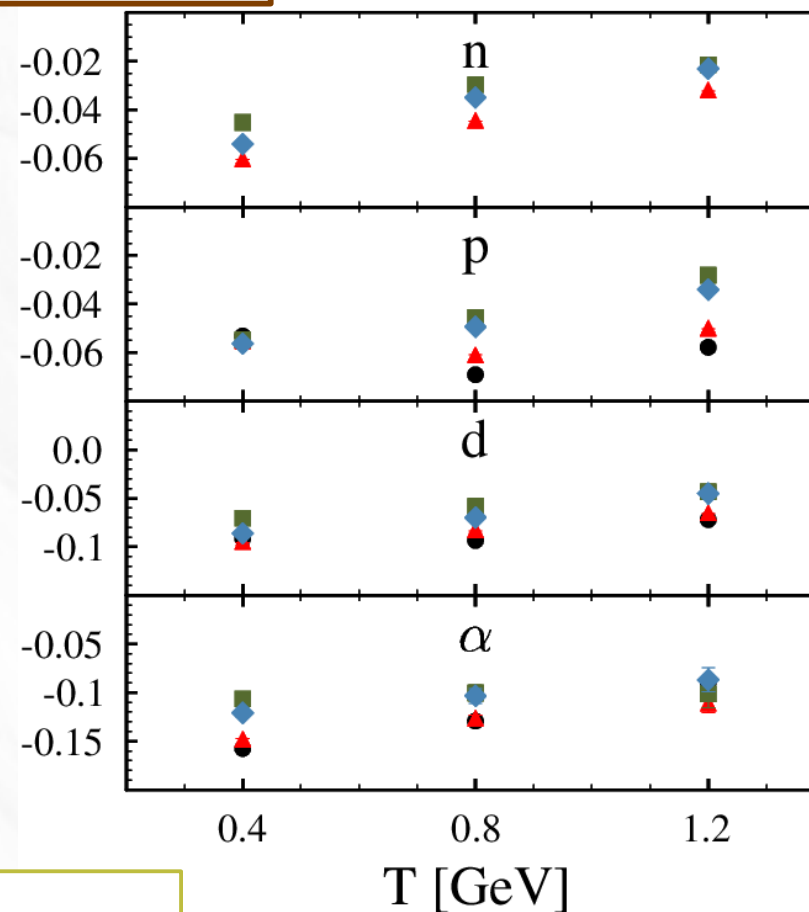
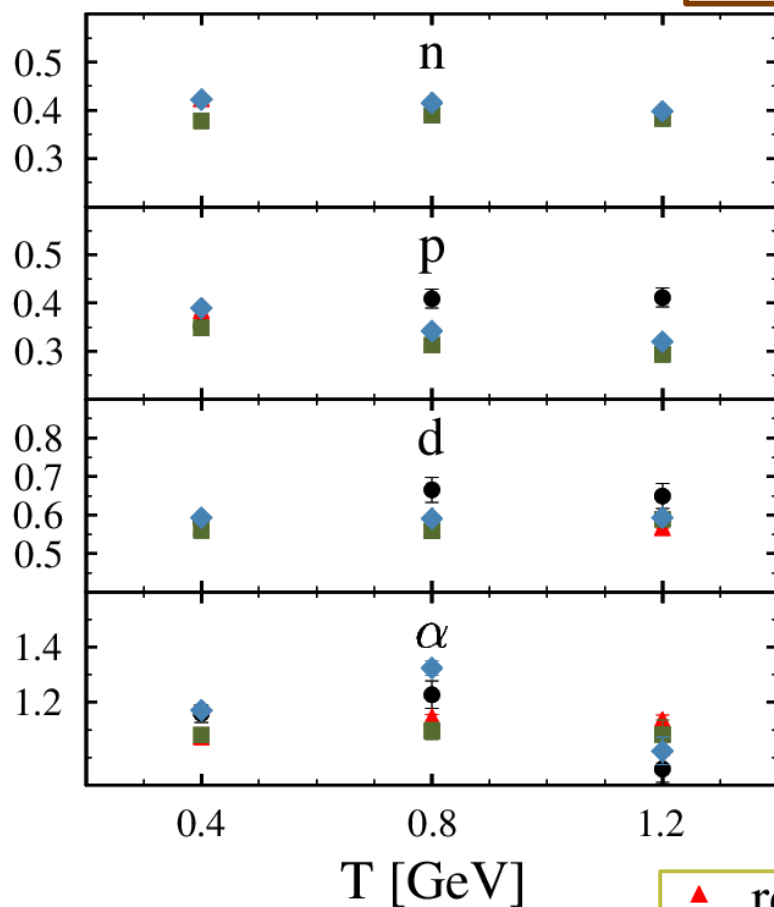
Experimental data: [W. Reisdorf et al. \(FOPI\), NPA 876, 1 \(2012\)](#)

Energy Dependence

V_{11}

FOPI
 $0.25 < b_0 < 0.45; u_{t0} > 0.4$

V_{20}

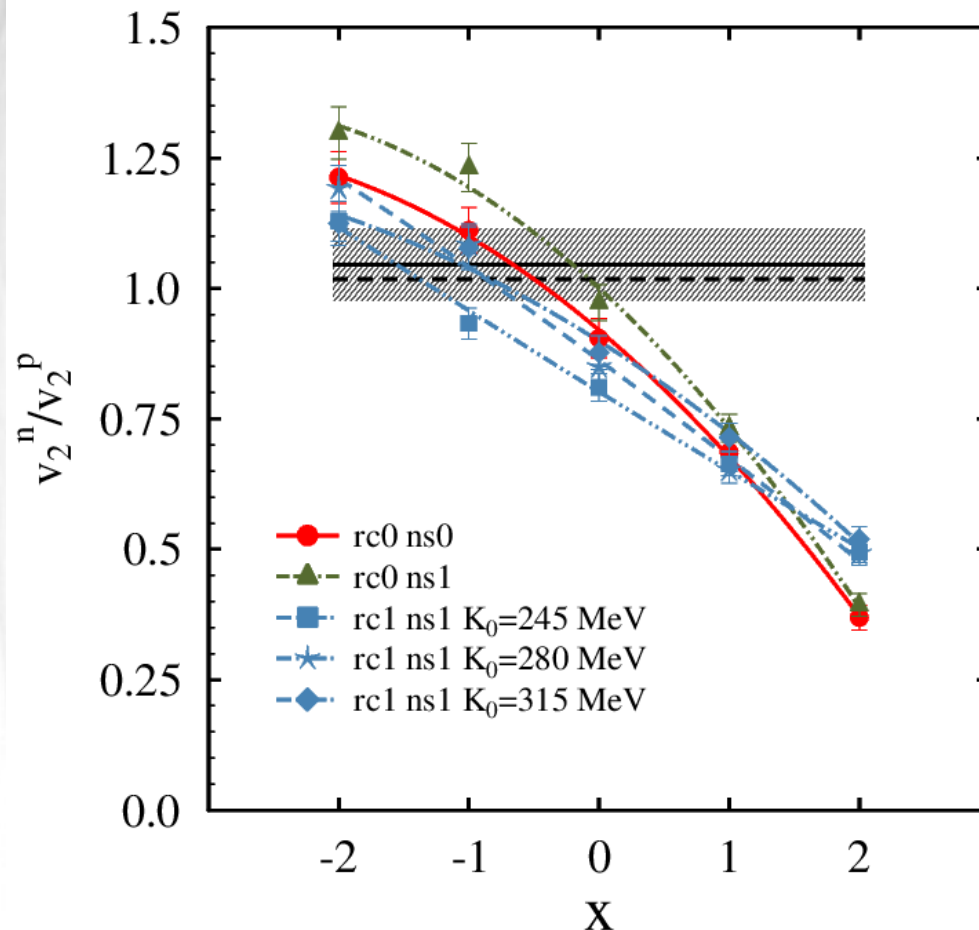


- ▲ rc0 ns1
- rc1 ns1 K=245
- ◆ rc1 ns1 K=280

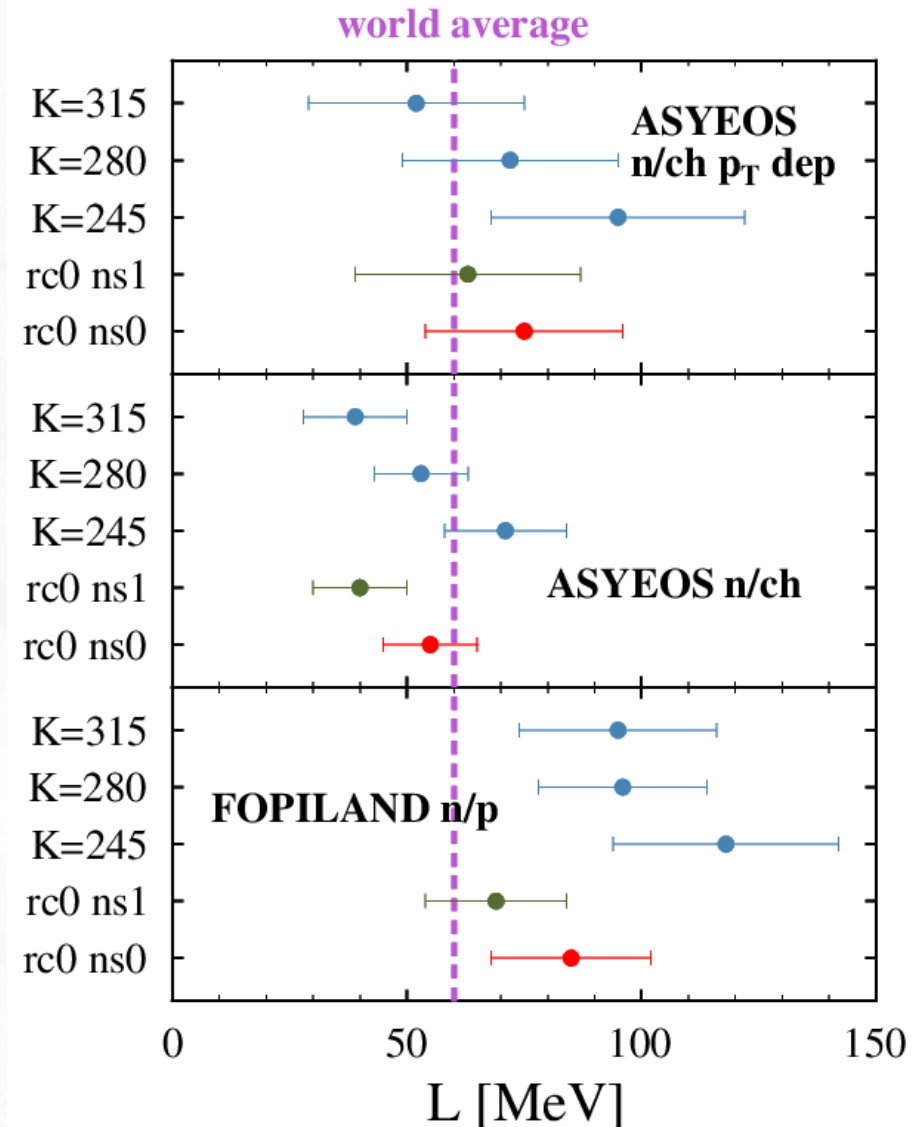
Symmetry Energy Constraint

1 free parameter (cMDI2)

FOPILAND n/p elliptic flow ratio



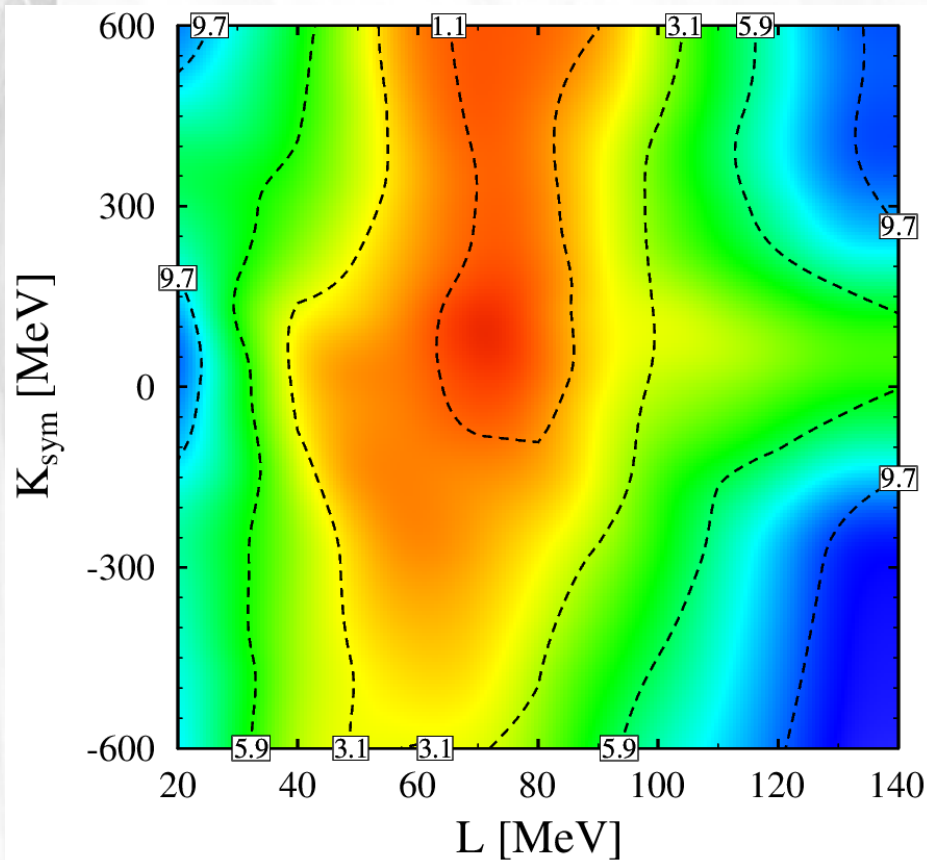
increased sensitivity to compressibility by a factor ~ 3 !!



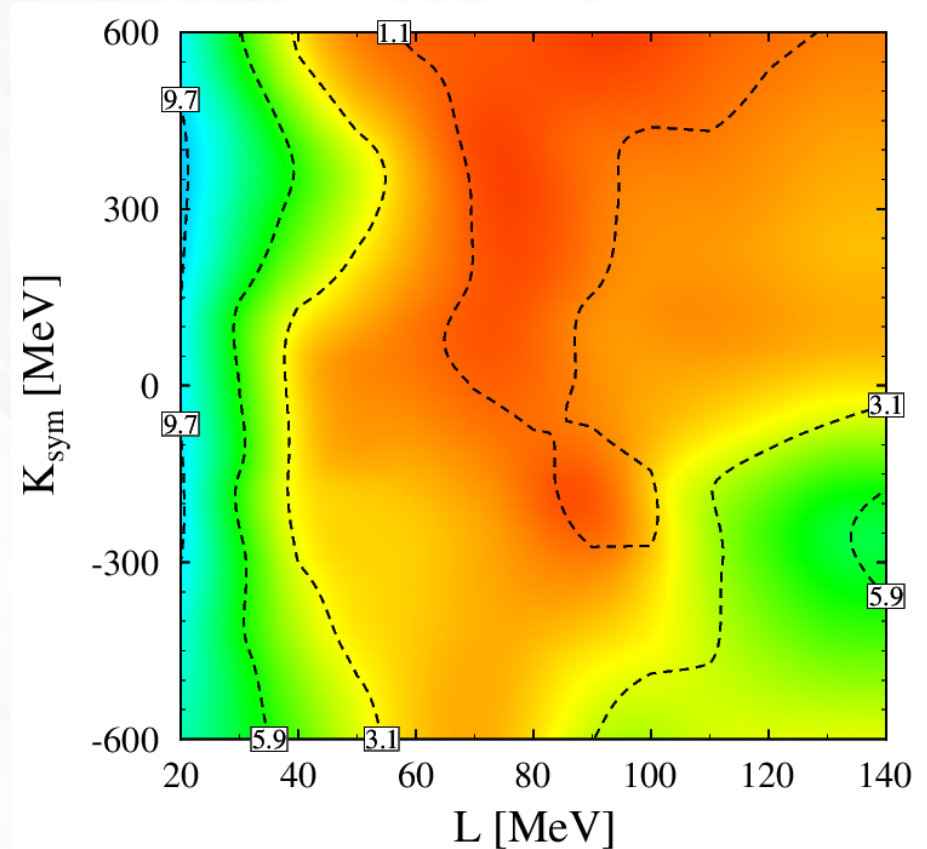
Symmetry Energy Constraint

2 free parameters (full MDI2) → extract both L and K_{sym}

non-relativistic dynamics ($K=245$ MeV)



relativistic corrections included ($K=280$ MeV)



Relativistic corrections → less sensitivity in the stiffer region, allowing for a stiffer SE above saturation

Conclusions

First results with a QMD transport model that accounts for relativistic dynamics effects in the meanfield propagation.

Failure to include these effects → spurious density dependence of symmetry energy

Relativistic dynamics → sizable impact on flows, increasing with impact energy (directed, elliptic flow)
→ need to readjust model parameters
→ qualitative approach: tune compressibility modulus to reproduce magnitude of flows
→ reasonable description at 400 MeV/u; insufficient in the region 1 GeV/u.

Symmetry Energy Constraints: compatible with the non-relativistic dynamics model

However: less sensitivity towards stiffer asy-EoS
non-negligible sensitivity to the compressibility modulus