From homogeneous matter to finite nuclei: KIDS functional

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Rare Isotope Science Project – IBS Daejeon, S.Korea

A rendering of the future RAON complex, under construction in Daejeon

People

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- <u>Tae-Sun Park</u>, **S**KKU
- <u>Yeunhwan Lim</u>, **I**BS (now in Texas)
  - **K**orea
  - **I**BS (that's me and YHL)
  - **D**aegu
  - **S**KKU

- <u>Hana Gil</u>, Kyungpook National University
- <u>Yongseok Oh</u>, Kyungpook National University
- <u>Gilho Ahn</u>, University of Athens, Greece
- <u>Young-Min Kim</u> (UNIST) .....

KIDS sales pitch

- Give KIDS any T=0 EoS you want to test or apply:
  - In terms of $\{\rho_0, E_0, K_0\}, \{J, L, K_{sym}, Q_{sym}\}$, e.g., for your sensitivity studies, or from new constraints
  - In the form of pseudodata from ab initio calculations (χEFT, APR, ...)
  - m_s^*, m_v^* , Q_0, if you wish
- We can reverse-engineer a Skyrme "interaction" which reproduces that EoS
- And can then test it directly in
  - Nuclear structure (Skyrme-Hartree-Fock)
  - Giant/pygmy resonances (RPA)
  - ...

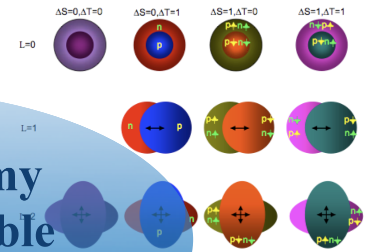
Overview

- **Introduction**
- **KIDS functional:**
  - **Natural Ansatz + Skyrme formalism**
  - Fits to APR: KIDS-ad2 set
  - Successful application to nuclei
  - Effective mass parameters remain free
  - **Proof of principle: From APR straight to nuclei**
- Symmetry energy and KIDS functional
  - Freedom to vary all parameters at will
  - **Relevance of skewness parameter Q_{sym}**
- Summary

My interests and motivation

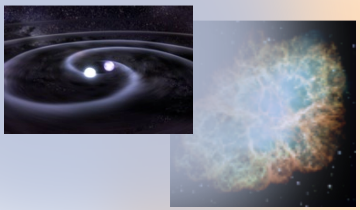
RI facilities

Giant and Pygmy Resonances; stable and exotic nuclei



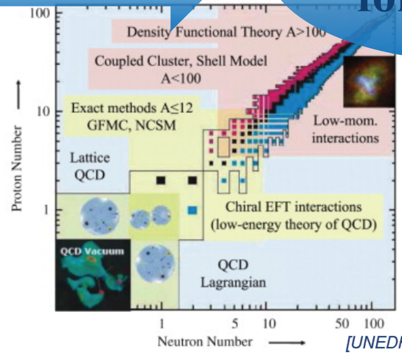
astronomy, HICs

EoS / NM properties



Underlying forces, EDF

pseudodata



[UNEDF]

Skyrme-type interaction and EDF

$$
\begin{aligned}
V_{\rm Skyrme} = \; & t_0(1+x_0P_\sigma)\delta(\mathbf{r}_i-\mathbf{r}_j) \\
& +\tfrac{1}{2}t_1(1+x_1P_\sigma)\{\mathbf{p}_{12}^2\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{12}^2\} \\
& +t_2(1+x_2P_\sigma)\mathbf{p}_{12}\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{12} \\
& +\tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\bar{\mathbf{r}})\delta(\mathbf{r}_i-\mathbf{r}_j) \\
& +it_4\mathbf{p}_{12}\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\mathbf{p}_{12} \,,
\end{aligned}
$$

$$
\begin{aligned}
E_{\rm Skyrme} = \; & 4\pi\int_0^\infty dr\, r^2 \Big\{ \frac{\hbar^2}{2m}\tau + \tfrac{1}{2}t_0(1+\tfrac{1}{2}x_0)\rho^2 - \tfrac{1}{2}t_0(\tfrac{1}{2}+x_0)\sum_q\rho_q^2 \\
& +\tfrac{1}{12}t_3(1+\tfrac{1}{2}x_3)\rho^{\alpha+2} - \tfrac{1}{12}t_3(\tfrac{1}{2}+x_3)\rho^\alpha\sum_q\rho_q^2 \\
& +\tfrac{1}{4}[t_1(1+\tfrac{1}{2}x_1)+t_2(1+\tfrac{1}{2}x_2)]\rho\tau \\
& -\tfrac{1}{4}[t_1(\tfrac{1}{2}+x_1)-t_2(\tfrac{1}{2}+x_2)]\sum_q\rho_q\tau_q \\
& -\tfrac{1}{16}[3t_1(1+\tfrac{1}{2}x_1)-t_2(1+\tfrac{1}{2}x_2)]\rho\nabla^2\rho \\
& +\tfrac{1}{16}[3t_1(\tfrac{1}{2}+x_1)+t_2(\tfrac{1}{2}+x_2)]\sum_q\rho_q\nabla^2\rho_q \\
& -\tfrac{1}{2}t_4[\rho\nabla J+\sum_q\rho_q\nabla J_q]\Big\} \,, \qquad (2.6)
\end{aligned}
$$

"interaction"

↑ HF,RPA,... ↓

energy density

EoS

t_0: attraction

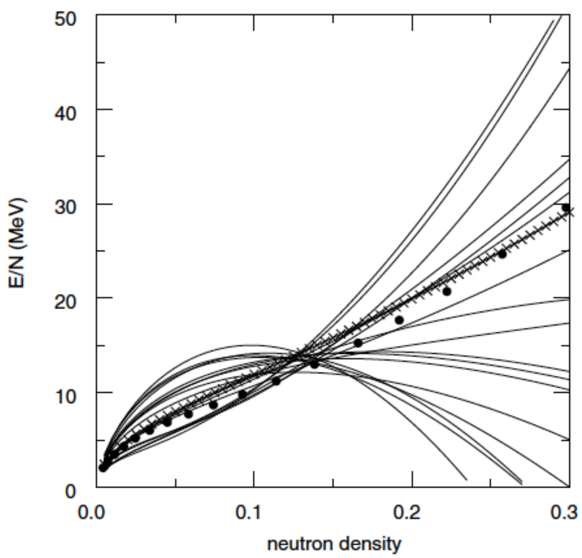
$t_3\rho^\alpha$: repulsion

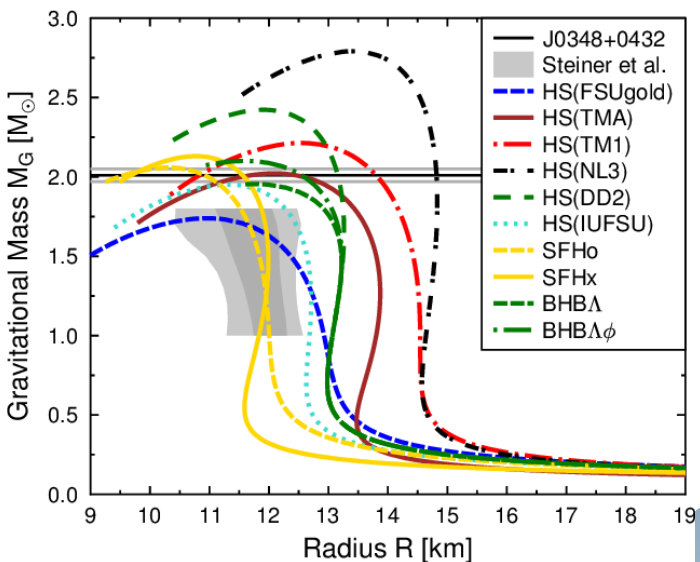
$t_{1,2}$:kinetic

Phenomenological energy-density functionals

- Hundreds of EDF models for nuclei and nuclear matter
  - Typically ~10 parameters fitted to nuclear properties using different data sets and fitting protocols
  - Very different predictions below and above ρ_0
  - Very different predictions at large isospin asymmetries
    - [cf Dutra et al., PRC85(2012)035201]

B.A.Brown, PRL85 (2000)





Phenomenological energy-density functionals

- Only few of the hundreds of EDF models can simultan eously describe nuclear matter and finite nuclei

[M.Dutra et al.,PRC85(2012)035201; P.D.Stevenson et al., AIP Conf.Proc.1529,262]

- Spurious correlations among parameters (e.g., K_0,m*)
- ... while binding energies and radii "prefer" different values for the effective mass

[M.Bender et al., Rev. Mod. Phys. 75,121

Not satisfactory!

- *If an EoS is "realistic", it should be able to correspo nd to nucler properties by definition*

NUCLEAR ENERGY DENSITY FUNCTIONAL FOR KIDS

- Natural Ansatz for energy density – inspired by QMBT / EFT
- Convenient Skyrme formalism for nuclei

KIDS Ansatz: Expansion in $k_F \sim \rho^{1/3}$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- If I have SNM and PNM, namely $c_i(0)$ and $c_i(1)$ (plus the quadratic approximation) I obtain analytically: $\{\rho_0, E_0, K_0, Q_0\}, \{J, L, K_{sym}, Q_{sym}\}$
- And vice versa; or I can fit to SNM/PNM pseudodata
- First, a few words on:
  - Motivation for Ansatz
  - Why 4 terms? Why low order?

for details: PP,Park,Lim,Hyun,Phys. Rev. C 97,014312 (2018)

Ansatz motivation - part I: Brueckner theory

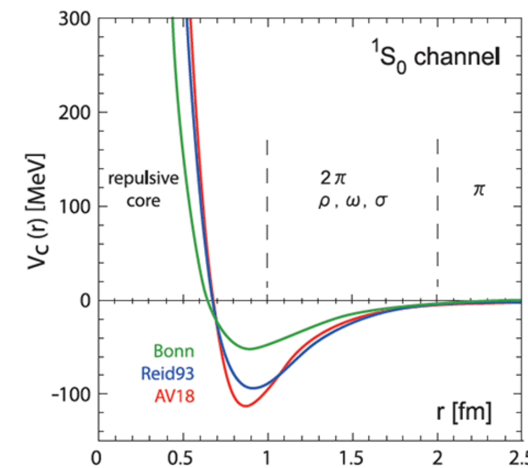
Fetter and Walecka, “Quantum theory of many-particle systems”

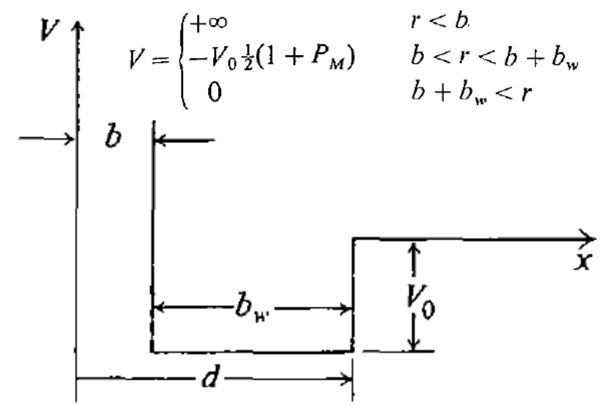
- Realistic potential: strong repulsive core plus attraction at longer range
- Apply Brueckner methodology in the calculation of nuclear matter energy

→ Result: $k_F^2, k_F^3, k_F^4, k_F^5, k_F^6, \ldots ,$ converging

- Even powers: from repulsive part
- Odd powers: from both

→ The Fermi momentum is the relevant variable : **powers of $\rho^{1/3}$**





Ansatz motivation - part II: effective field theory

PP,Park,Lim,Hyun,Phys. Rev. C 97,014312

- Saturation density is low...
  - with respect to (effective) boson exchange range (?)
    - one-pion exchange: vanishing expectation value
    - next boson: rho with m_ρ~775MeV~$4fm^{-1}$
  - Effective Lagrangian in powers of $\mathbf{k_F/m_\rho}$
- Expansion of E/A in powers of k_F
  - ... which means, again, powers of $\rho^{1/3}$
  - The Fermi momentum as the relevant variable
  - k_F^3 and k_F^4 (i.e., coupling~$\rho^{1/3}$) known to be important for obtaining saturation [Kaiser et al.,NPA697(2002)]
- *Dilute Fermi gas: plus logarithmic terms*

H.-W. Hammer, R.J. Furnstahl / Nuclear Physics A 678 (2000) 277–294

Nuclear energy density functional for KIDS

PP,Park,Lim,Hyun,Phys. Rev. C 97,014312

Natural Ansatz for potential energy: powers of $k_F \sim \rho^{1/3}$

But how many powers? Which are relevant?

- Fit to homogeneous matter pseudodata
  - Variational Monte Carlo (APR, FP)
- Statistical analysis of fit quality; naturalness
- Keep only the important terms! No overtraining

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

- SNM: 3 terms suffice in converging hierarchy ($c_3(0)=0$)
- PNM: 4 terms necessary (*different preferences*)

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PP,Park,Lim,Hyun,Phys. Rev. C 97,014312

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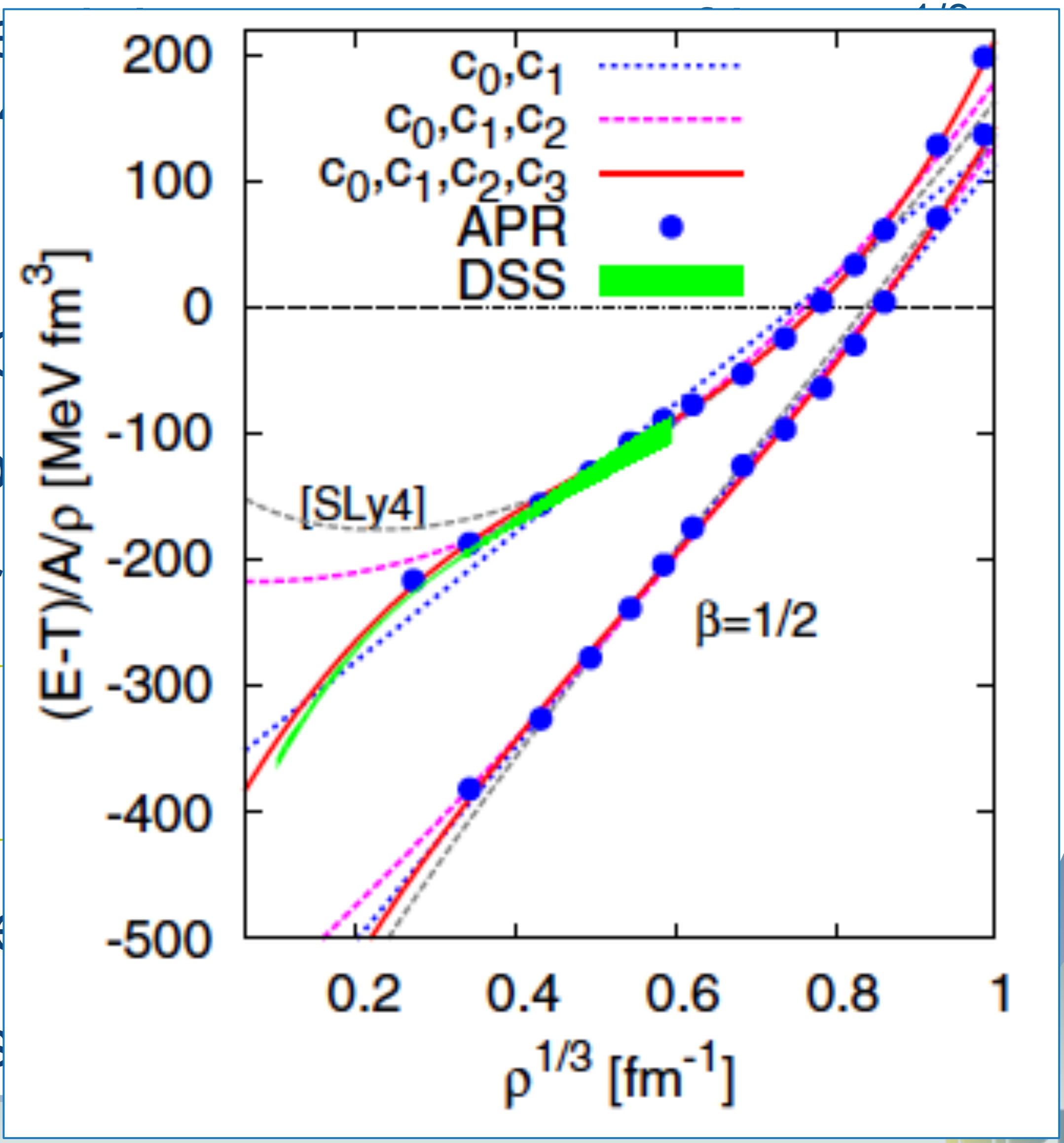

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- ❑ PNM: 4 terms necessary (*different preferences*)

*** for APR, more terms could lead to overfitting*

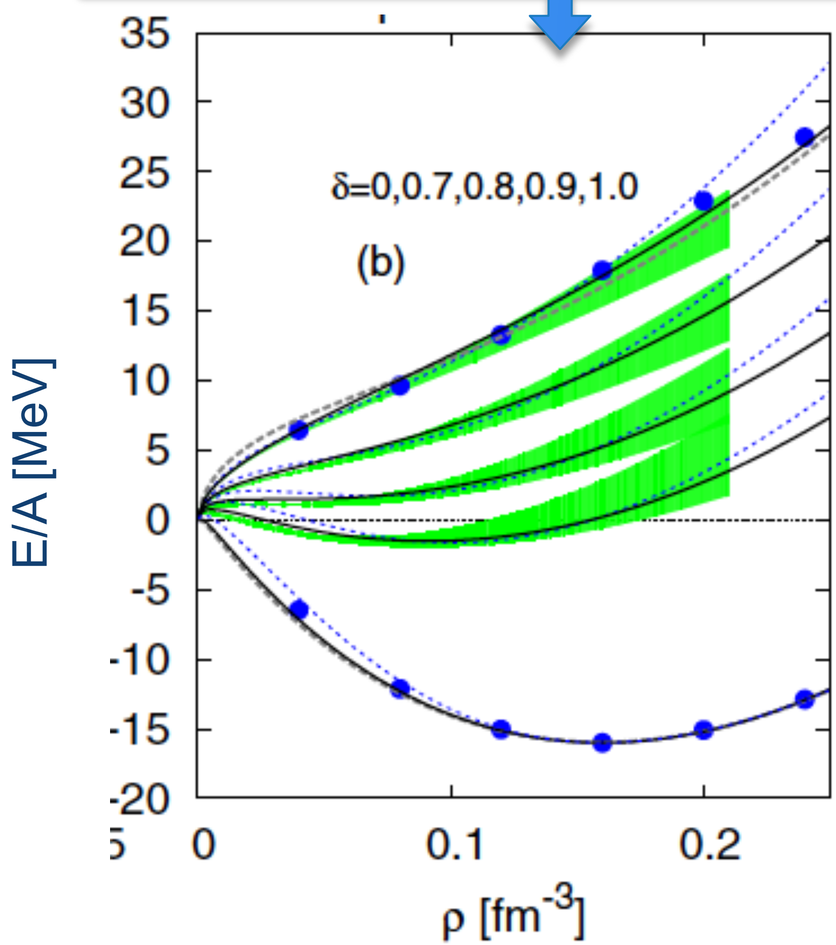
Parameter set KIDS-ad2

- Symmetric nuclear matter:
  - Set ρ_0=0.16 fm -3, E_0=-16MeV, K_0 = 240 MeV
  - Determine $c_{0,1,2}(0)$ (analytical expressions)
  - Leads to Q_0=-373 MeV
- Pure neutron matter:
  - Fit $c_{0,1,2,3}(1)$ to the APR pseudodata for PNM
  - Resulting symmetry-energy parameters:

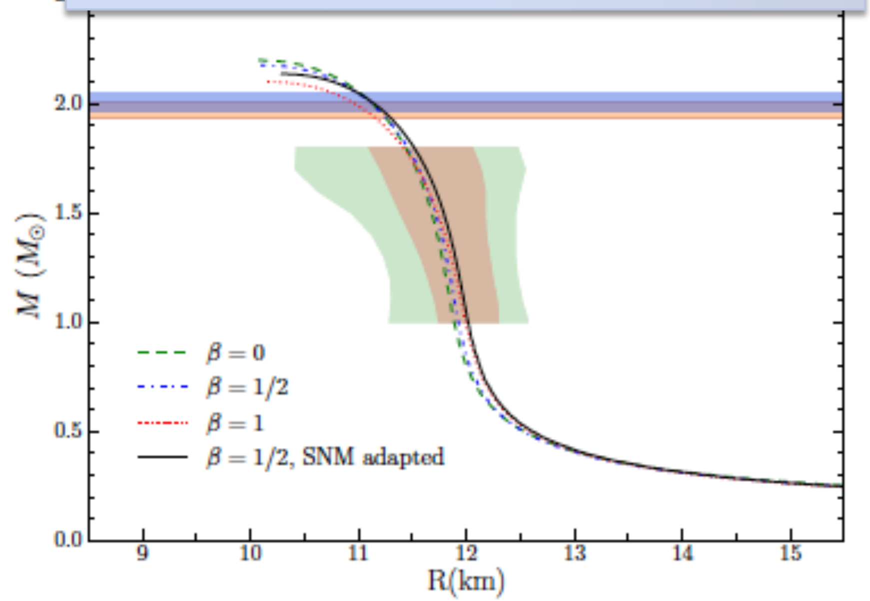
J=33MeV, L=49MeV, K_{sym}=-157MeV, Q_{sym}=586MeV

Interpolations and extrapolations

Calculations with chiral interactions reproduced, although they were not used for fitting

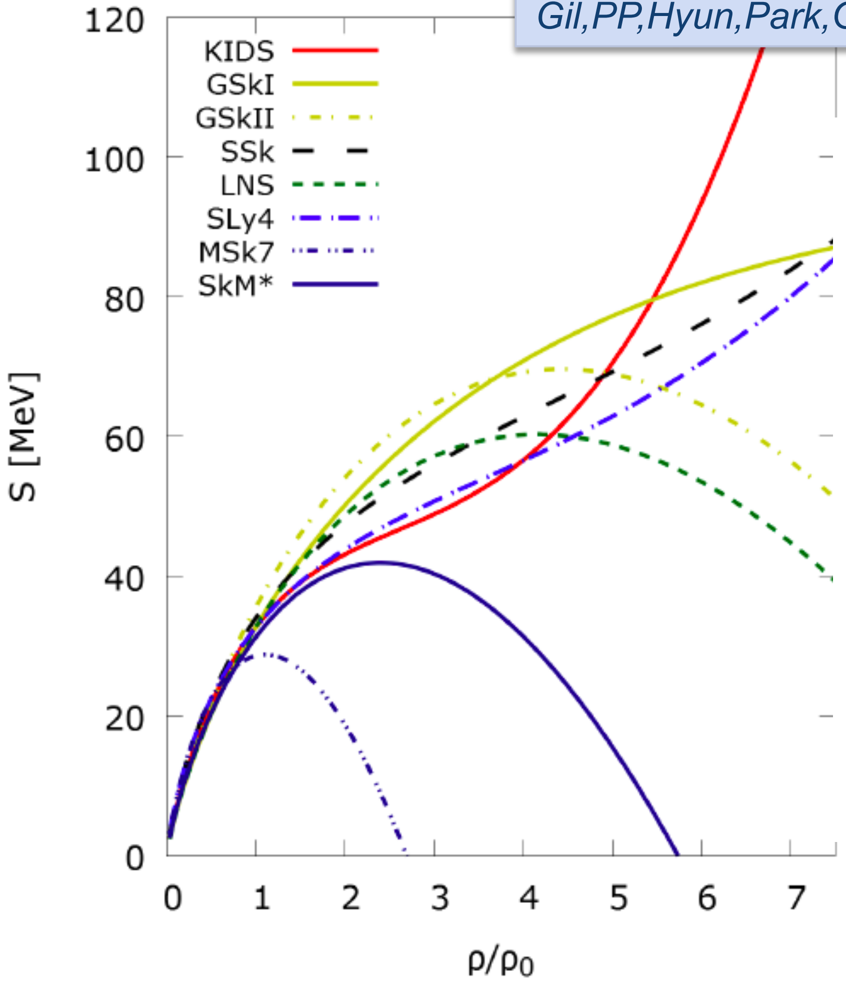


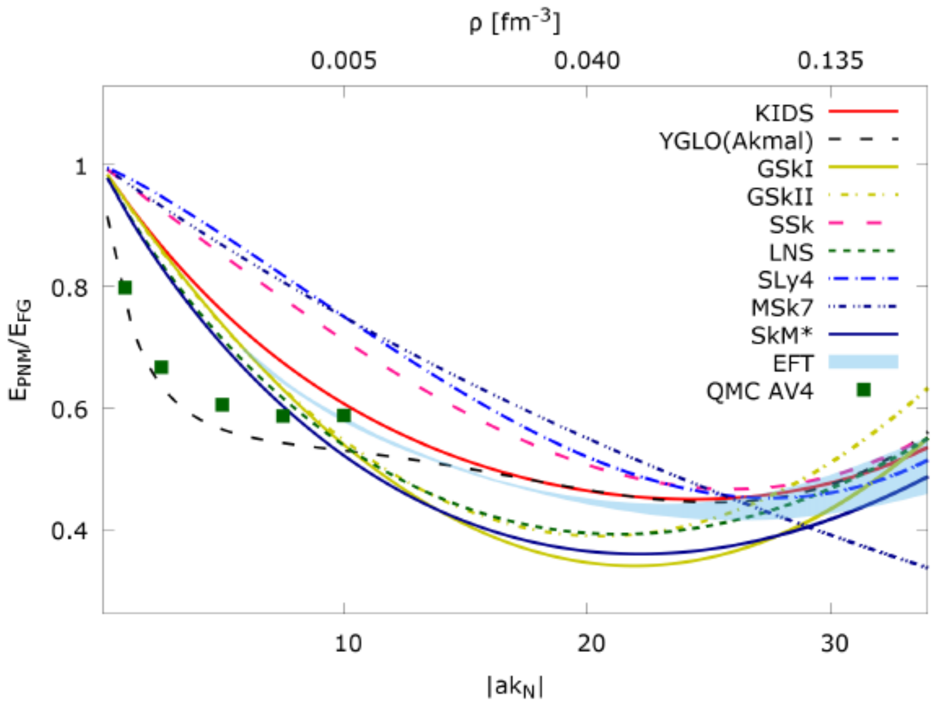
PP,Park,Lim,Hyun,Phys. Rev. C 97,014312



Comparisons with other models

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321





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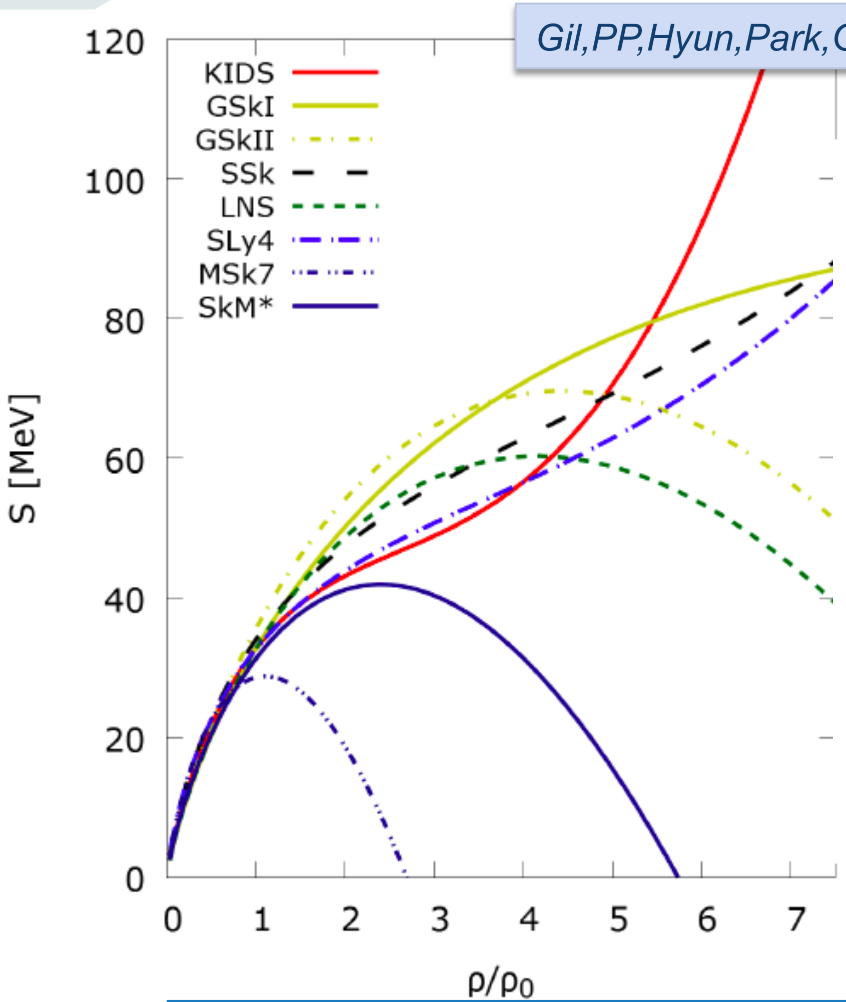
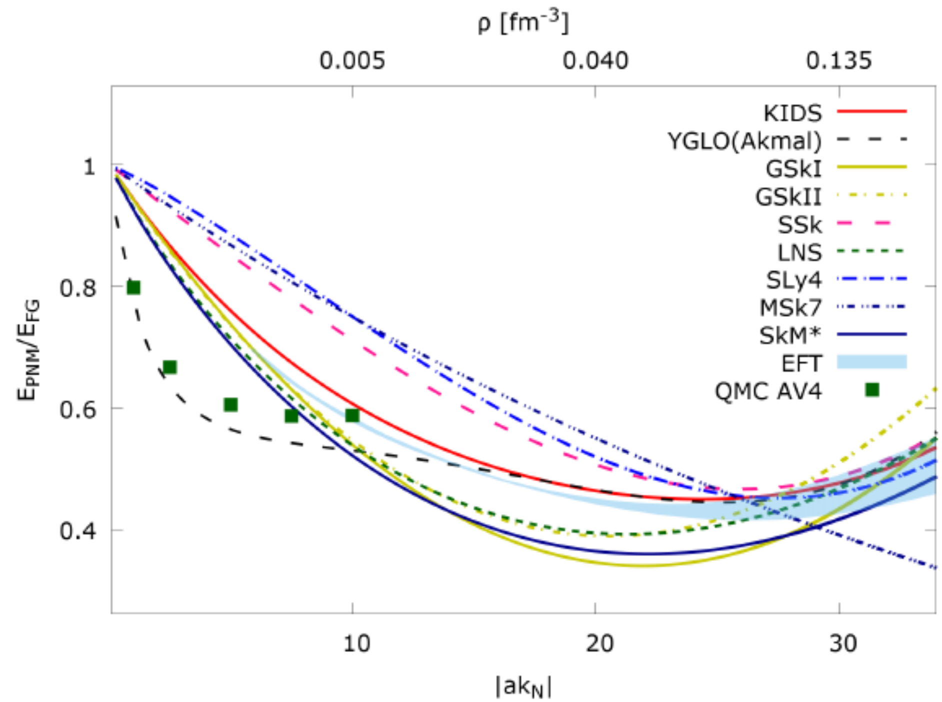




Excellent! Can we now take this straight to nuclei?

PROOF OF PRINCIPLE: APR TAKEN TO NUCLEI

Skyrme parameters by reverse engineering

$$v_{i,j} = (t_0 + y_0 P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2 + \text{h.c.}]$$
$$+(t_2 + y_2 P_\sigma)\boldsymbol{k}' \cdot \delta(\boldsymbol{r}_{ij})\boldsymbol{k} + iW_0\, \boldsymbol{k}' \times \delta(\boldsymbol{r}_{ij})\,\boldsymbol{k} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)$$
$$+\frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n} P_\sigma)\rho^{n/3}\delta(\boldsymbol{r}_{ij}), \qquad (3)$$

⇐ **Minimal Skyrme-type "force"**

$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s,$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right)$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2\,, \quad \theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2\,.$$

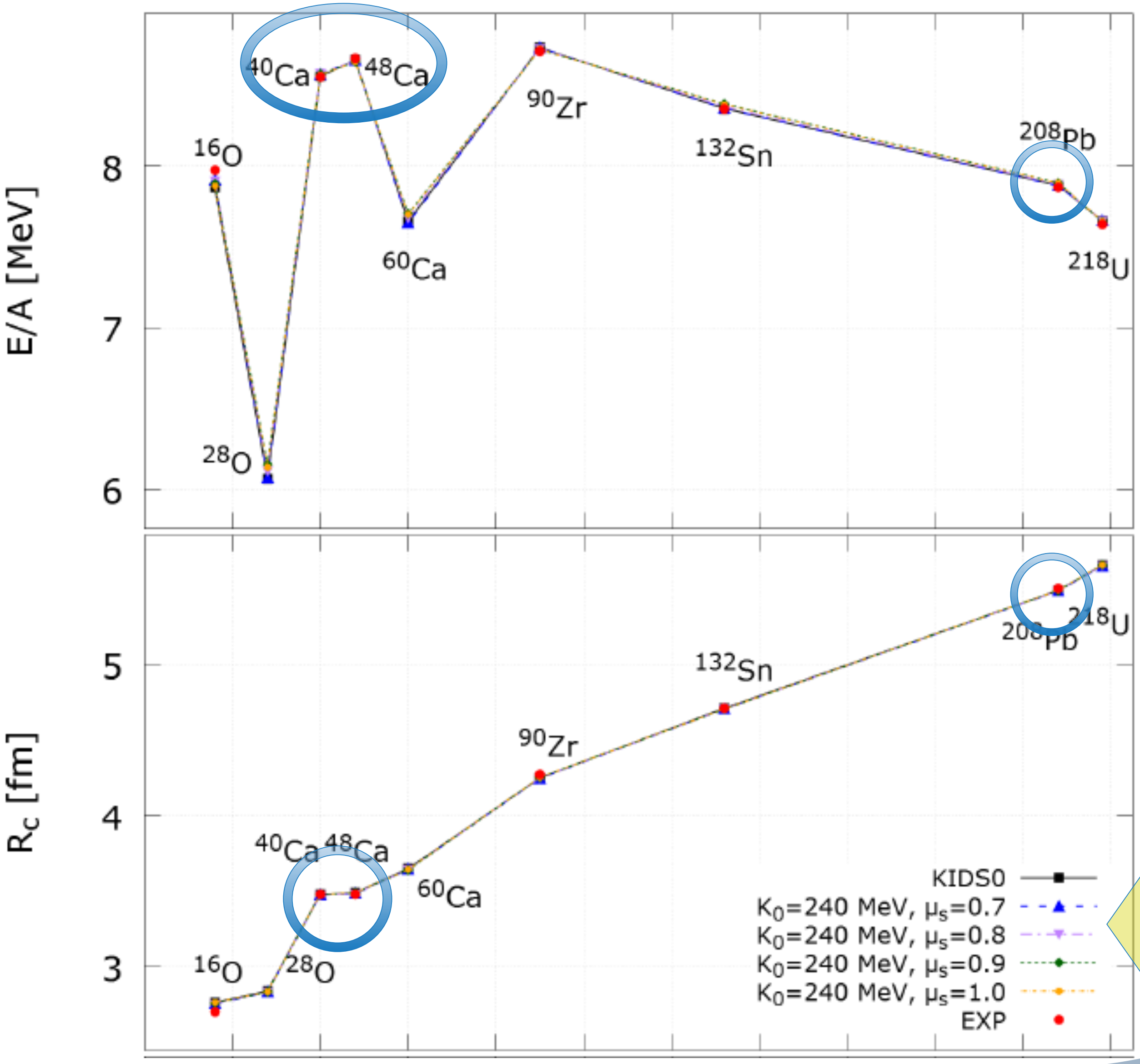
unconstrained from homogenous matter ➔ vary freely
But the total $c_2(0)$, $c_2(1)$ will remain unchanged!

Procedure

For given KIDS functional $c_i(0)$, $c_i(1)$ (i.e., fixed SNM, PNM)

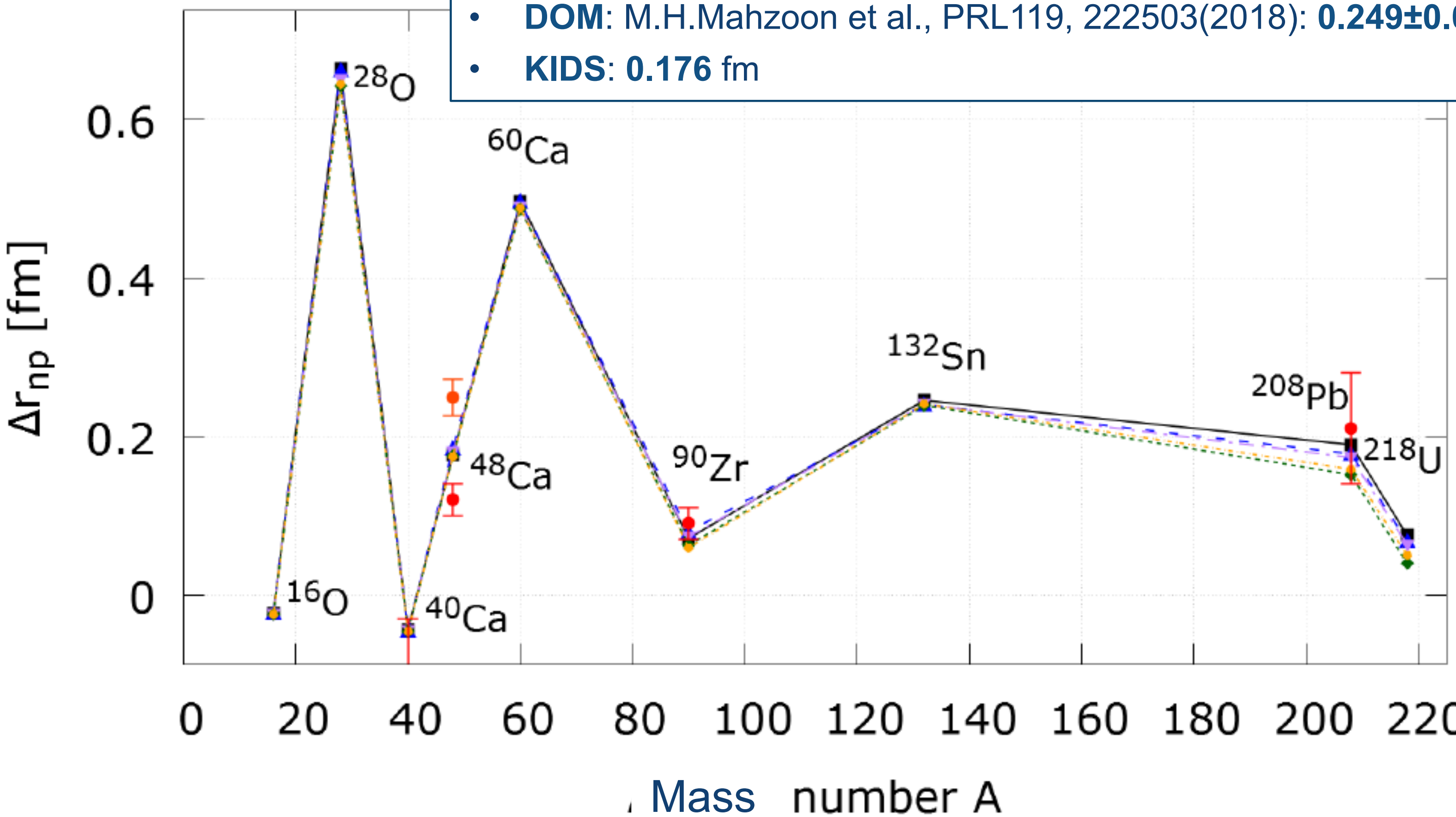
- Chose effective masses (vary at will)
- All t_i, y_i are now known except t_1,t_2,x_1,x_2
- The two combinations θ_s,θ_μ also known (eff. masses)
- **Two independent free parameters plus spin-orbit W_0**
  - Fit only to ^{40}Ca, ^{48}Ca, ^{208}Pb
  - Only bulk properties: E/A, charge radius: 6 data

Binding energy, charge radii



predictions independent of the effective mass assumed

Neutron skin thickness



neutron skin of 48Ca:

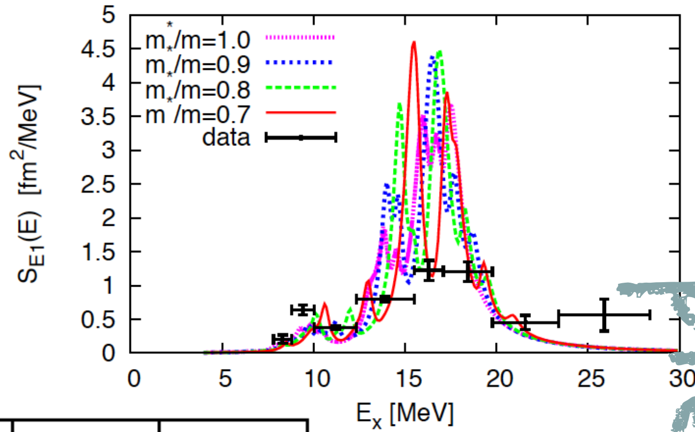
- **CCM**: G.Hagen et al., Nature Phys. 12,186(2016): **0.12-0.15** fm
- **DOM**: M.H.Mahzoon et al., PRL119, 222503(2018): **0.249±0.023** fm
- **KIDS**: **0.176** fm

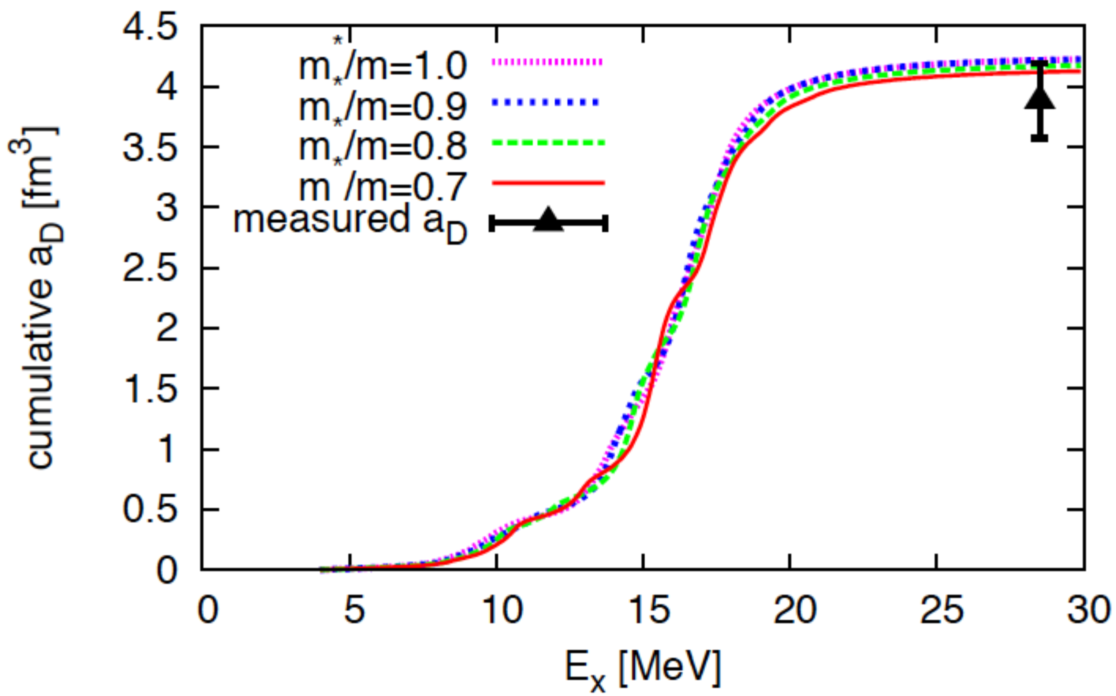
Data: antiprotonic atoms, PREX (^{208}Pb), DOM (^{48}Ca, upper)

Predictions of APR EoS for the neutron skin thickness!

KIDS-ad2: Predictions for ^{68}Ni (not fitted)

- Binding energy per particle:
  - KIDS-ad2: **8.68~8.69** MeV [*]
  - AME2016: **8.68247(4)** MeV
- Dipole polarizability:





preliminary

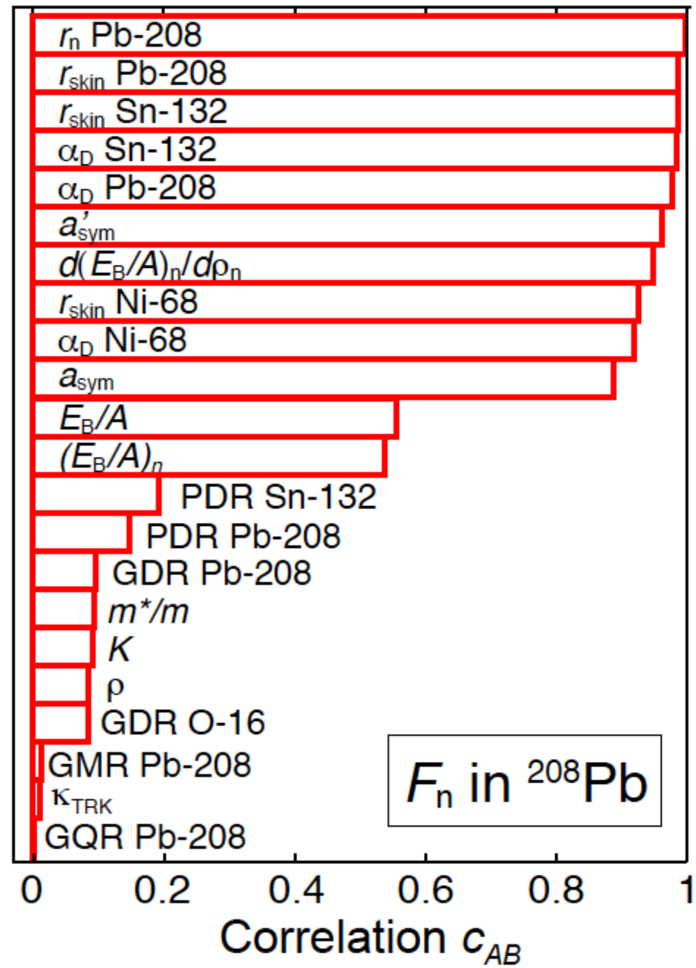
[*] *for m*/m=1.0~0.7:8.68794; 8.68176; 8.68838; 8.68912 MeV*

[**] *a_D measurementT.Aumann and D.Rossi, private communication*

Dipole polarizability a_D

- Theoretical studies within Skyrme and covariant density functional theory:
- a_D correlated with neutron skin thickness
- and with symmery energy and its density dependence

PDR: no correlation



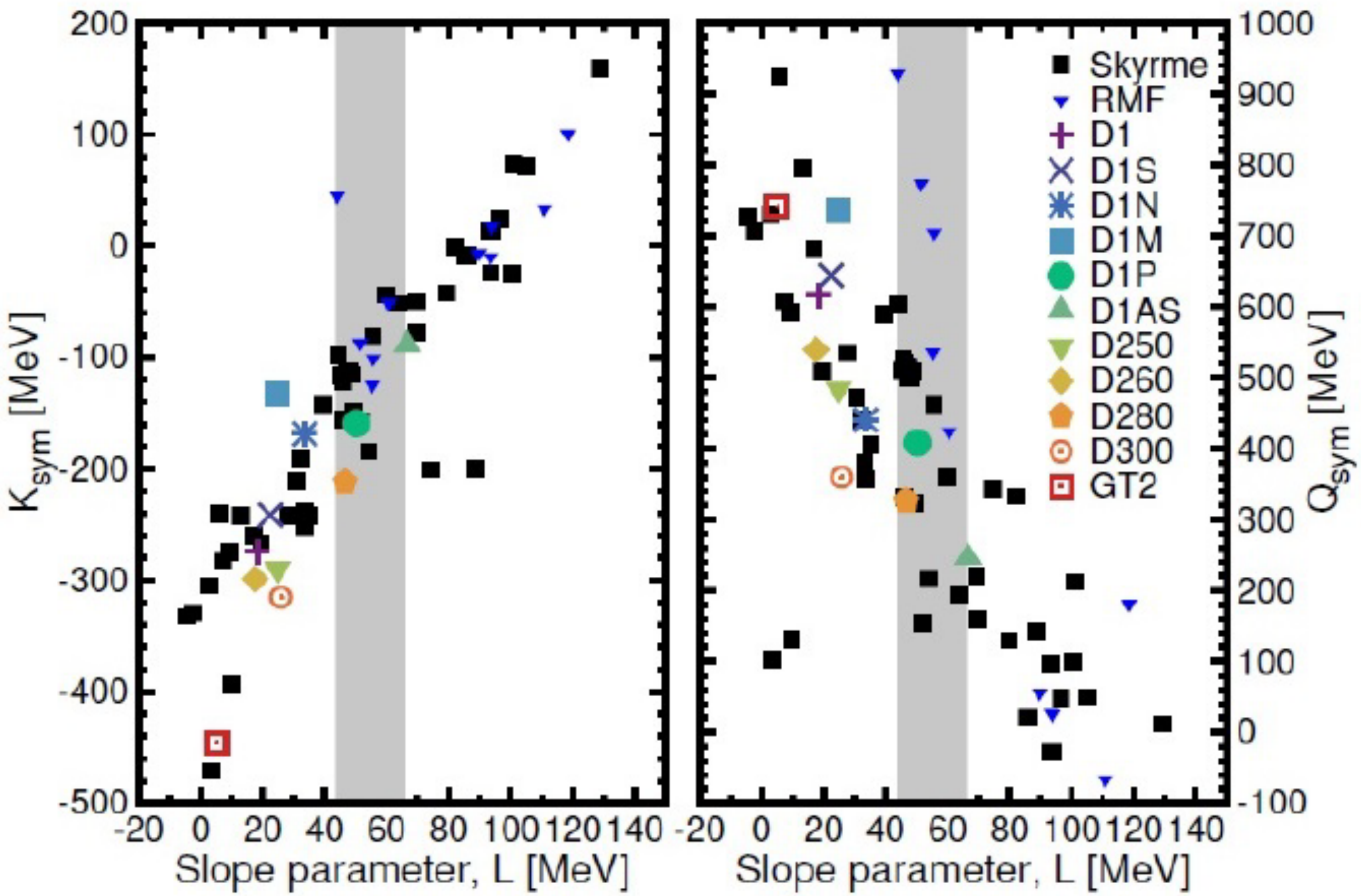
P.-G.Reinhard, W.Nazarewicz, PRC81

Remarks

- For given **immutable** EoS (no refitting), a Skyrme-type functional can easily be reverse-engineered
- Bulk, static properties: practically independent of the effective mass!
  - We can vary EoS parameters and m* independently and examine effect on observables
- *So far I showed results with KIDS-ad2 based on APR.* ***Next: An exploration of symmetry-energy parameters***

EXPLORING THE SYMMETRY ENERGY PARAMETERS

Curvature K_{sym} and skewness Q_{sym}



R. Sellahewa and A. Rios, Phys. Rev. C 90, 054327 (2014)

KIDS: Analytical relations

- Symmetric nuclear matter:
  - $\{\rho_0, E_0, K_0\}$ ➔ 3x3 system ➔ $\{c_i(0); i=0,1,2; c_3(0)=0\}$
  - *Feasible but unnecessary:*
  - *$\{\rho_0, E_0, K_0, Q_0\}$ ➔ 4x4 system ➔ $\{c_i(0); i=0,1,2,3\}$*

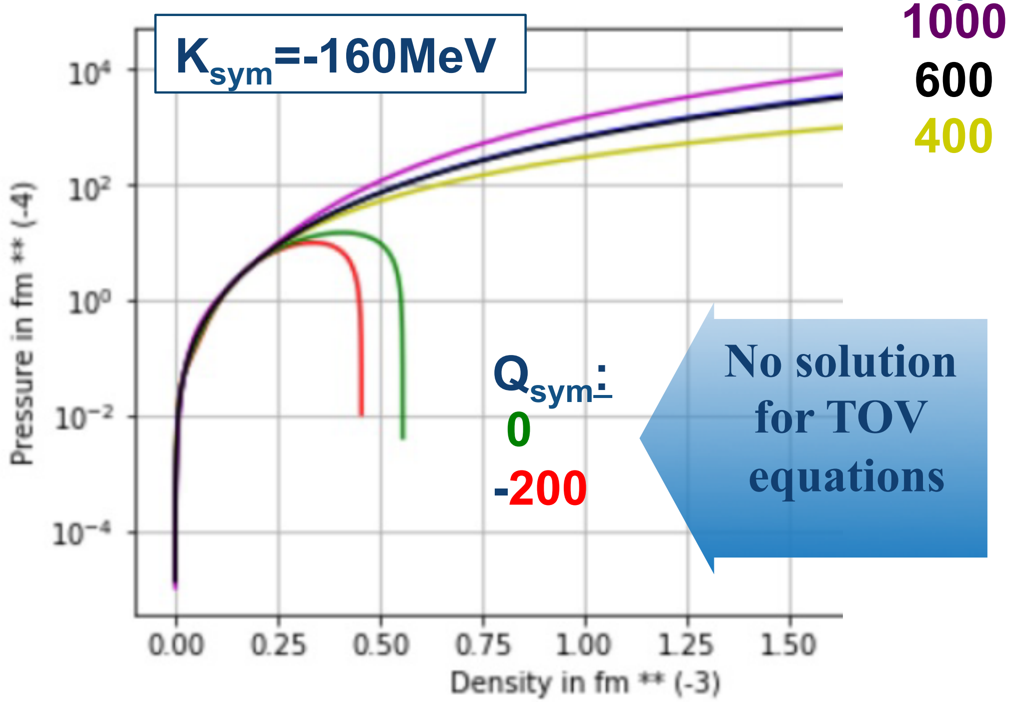
- Symmetry energy:
  - $\{J, L, K_{sym}, Q_{sym}\}$ ➔ 4x4 system ➔ $\{[c_i(1)-c_i(0)]; i=0,1,2,3$

Let us keep SNM, J, L, K_{sym} steady and equal to the KIDS-ad2 values; vary Q_{sym}; and solve for $c_i(1)$

Exploring symmetry energy parameters

- For steady (J,L, K_{sym}) = (33,50,-160) MeV, vary Q_{sym}

	J(MeV)	L(MeV)	K_{sym}(MeV)	Q_{sym}(MeV)
ad – 2	32.76	49.11	-156.69	586.29
Values used	33	50	-160	-200, 0, 400, 600 και 1000

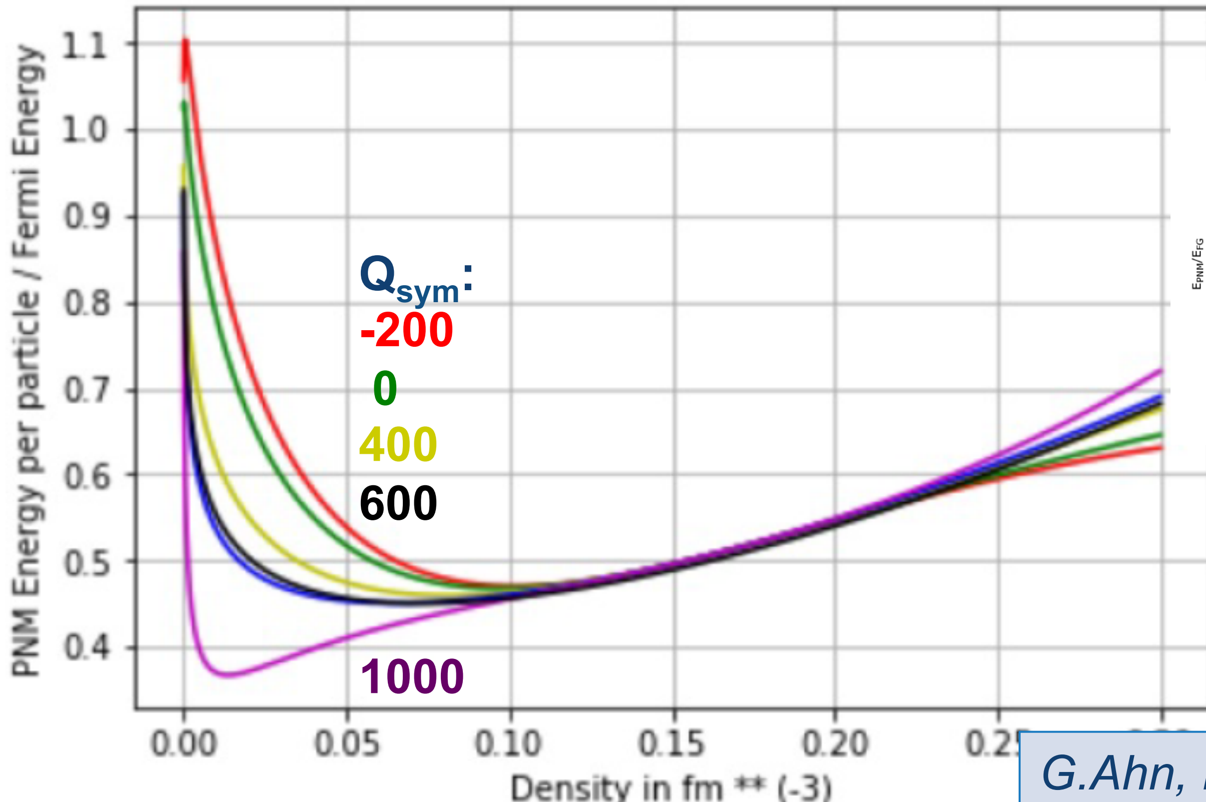


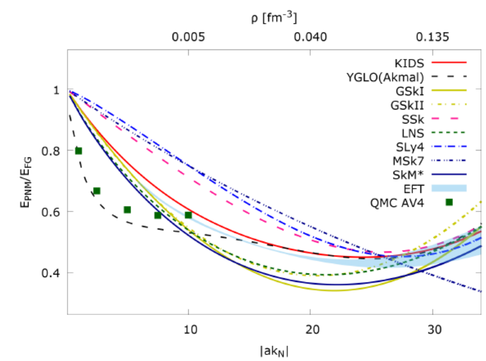
G.Ahn, MSc Thesis (NKUA, 2018)

Exploring symmetry energy parameters

RAON

❖ Dilute neutron matter





G.Ahn, MSc Thesis (NKUA, 2018)

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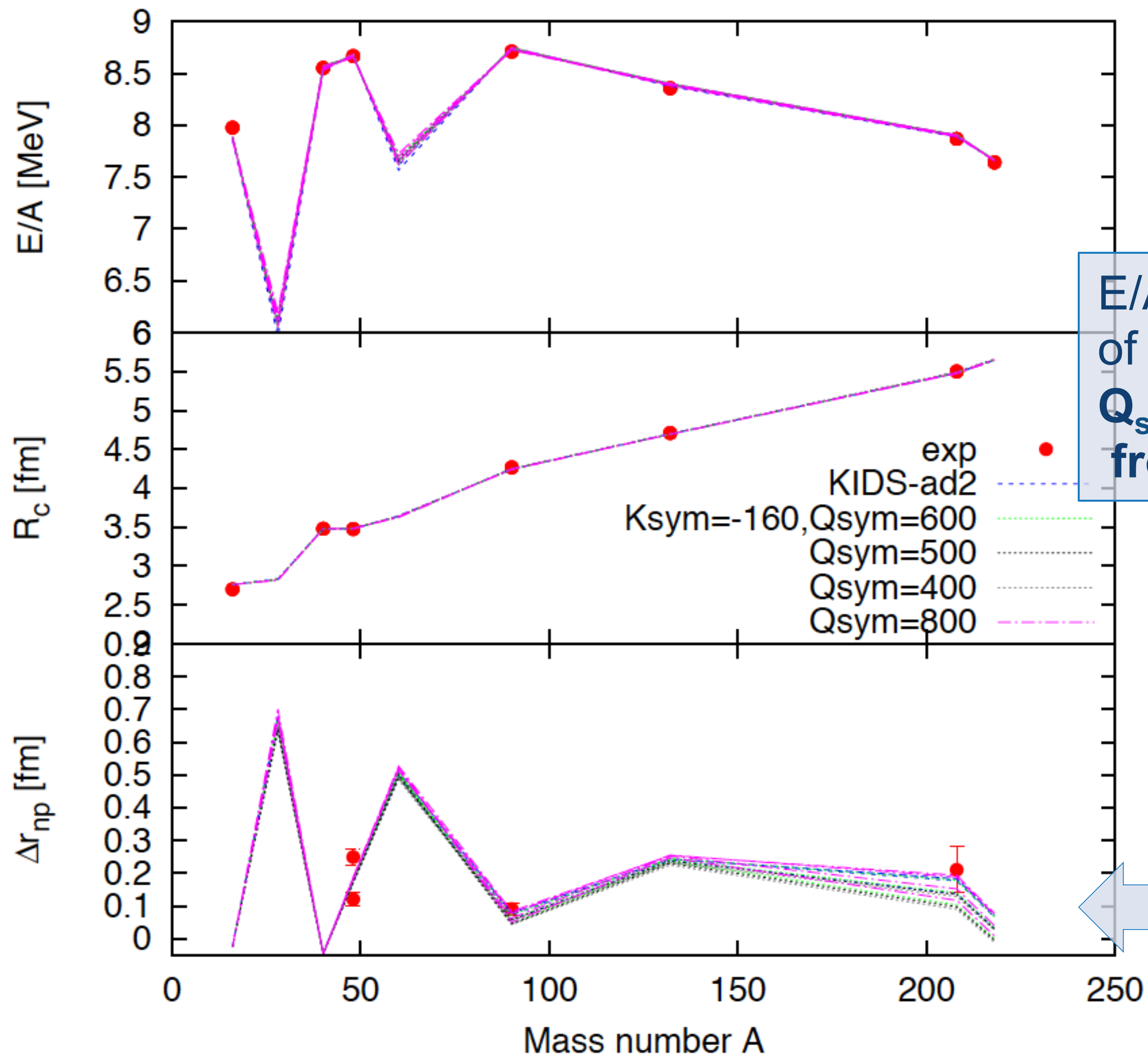

E/A, R_c independent of Q_{sym} ✔

Q_{sym} not constrainable from such data

Neutron skin thickness vs Q_{sym}?

Auras??

Exploring symmetry energy parameters

- For steady (J,L) = (33,50) MeV, vary K_{sym}, Q_{sym}
- Solutions of TOV obtained for the following cases:

		Max Mass($M_\odot$)	$R_{1.4}$(km)	ρ_{max}(fm^{-3})
(-160,600) MeV	(I)	2.05	11.29	0.877
(-160,1000) MeV	(II)	1.92	11.70	0.615
(0,1000) MeV	(III)	1.96	12.07	0.632
(-157,586) MeV	ad − 2	2.06	11.27	0.906

G.Ahn, MSc Thesis (NKUA, 2018)

Summary

- Natural Ansatz + Skyrme formalism: **KIDS functional**
  - 3 terms in expansion sufficient for SNM: $\{\rho_0, E_0, K_0\}$
  - **4 terms necessary for neutron matter and symmetry energy**: $\{J, L, K_{sym}, Q_{sym}\}$
- **From fixed EoS straight to nuclei**
- APR: static, bulk nuclear properties insensitive to
  - Effective-mass parameters
  - High-order parameters of symmetry energy
- **Flexibility to choose parameter values at will** for sensitivity studies **or adjust them to**
  - Dynamical observables (e.g., giant resonances)
  - Ab initio pseudodata (polarized matter, neutron drops...)
  - **Astrophysical and HIC constraints**

Skyrme- type "interaction" by reverse engineering

Thank you!

Thanks to my collaborators and contributors

- Chang Ho Hyun, Hana Gil, TaeSun Park, Yeunhwan Lim
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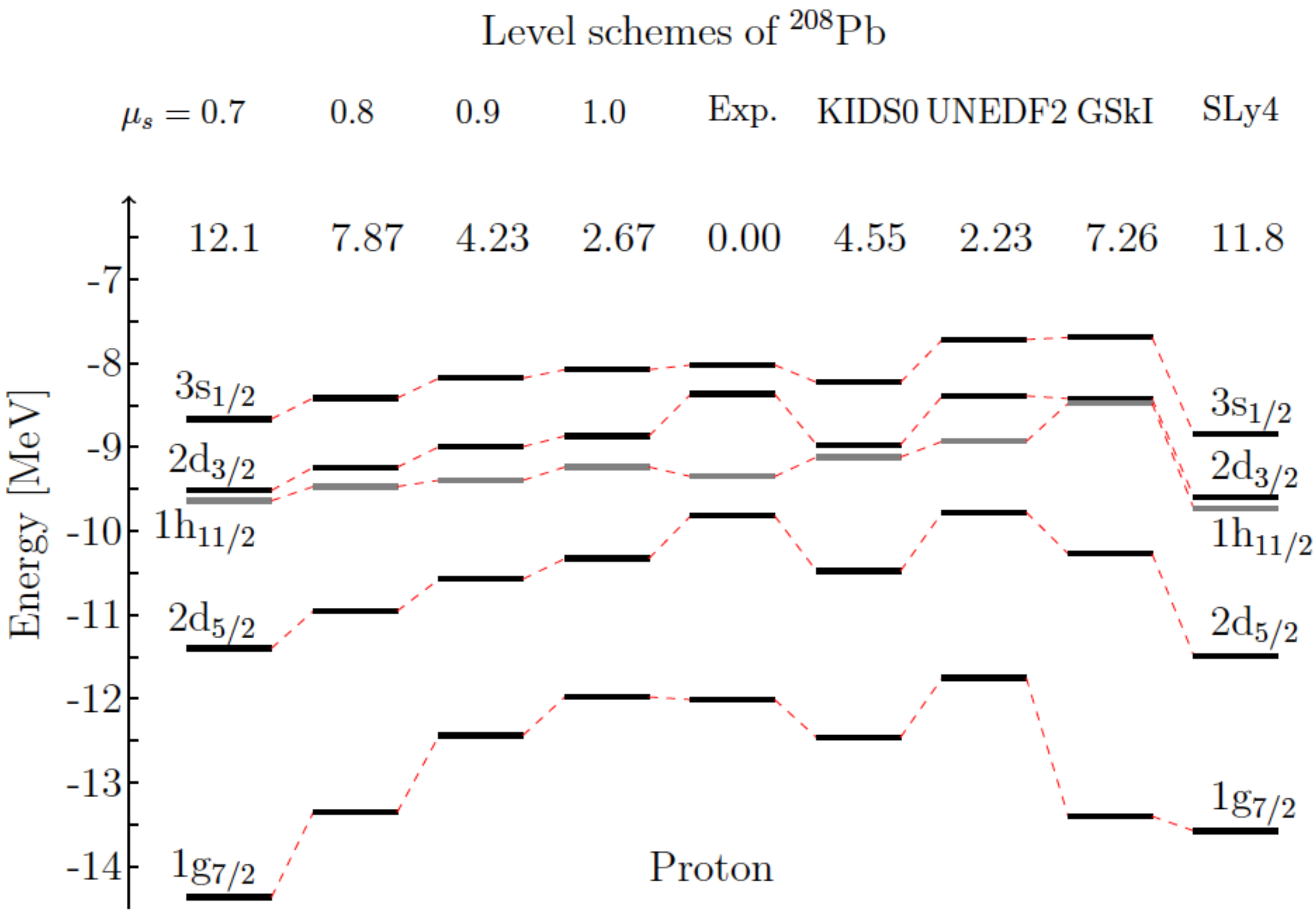
- J.W.Clark
- E.Mavrommatis, G.Ahn
- R.Roth, R.Trippel, J. Wambach, V.Yu.Ponomarev, A.Richter...
  - ... and the experimental groups in Darmstadt, S.Africa, and Cologne...
- H.Hergert

Single-particle levels



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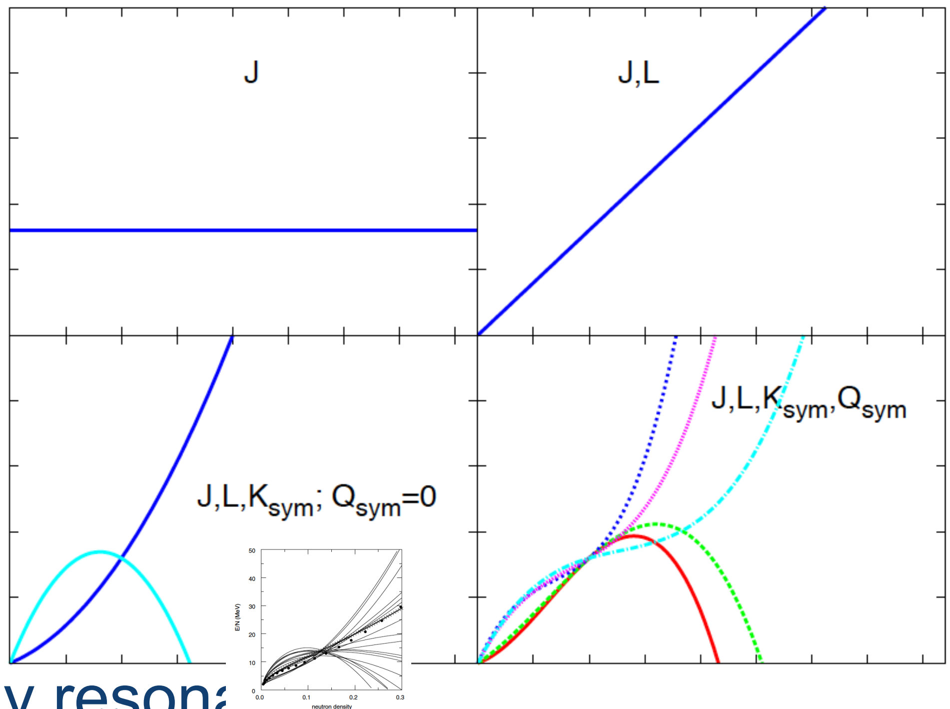
KIDS sales pitch

- Give KIDS any EoS you want to test or apply:
  - In terms of {ρ_0,E_0,K_0},{J,L,K_{sym},Q_{sym}}, e.g., for your sensitivity studies
  - In the form lations (χEFT, APF
  - m_s*, m_v* , G
- KIDS can rev ction" which reprodu
- And can then
  - Nuclear stru
  - Giant/pygmy resona
  - ...



Summary

- For given **immutable** EoS, a Skyrme-type functional can easily be reverse-engineered ★*world first*★
- Bulk, static properties: practically independent of the effective mass!
  - We can vary EoS parameters and m* independently and examine effect on observables
- Prospects abound!
  - Giant and pygmy resonances, polarizability...
  - Higher-order momentum dependencies...
  - An exploration of symmetry-energy parameters underway