

Stiff Symmetry Energy from Thick Isovector Aura

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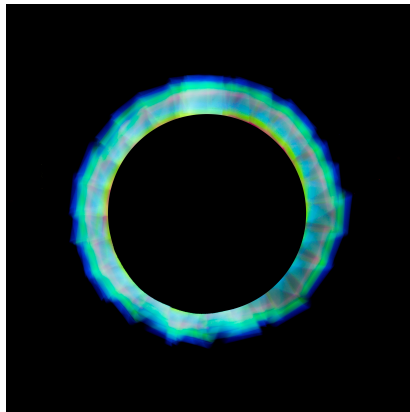


Aura

Historically Kirlian Photography



Living Being



Nucleus



Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth F , yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \dots$$

An isovector quantity G changes sign. Example:
 $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \dots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \dots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in n - p space. Isospin $\vec{T} = \sum_{i=1}^A \vec{\tau}_i$ SU(2)



Charge Symmetry & Charge Invariance

Charge symmetry:
 $n \leftrightarrow p$ invariance

Charge invariance:
symmetry under
rotations in
 n - p space

Isospin doublets

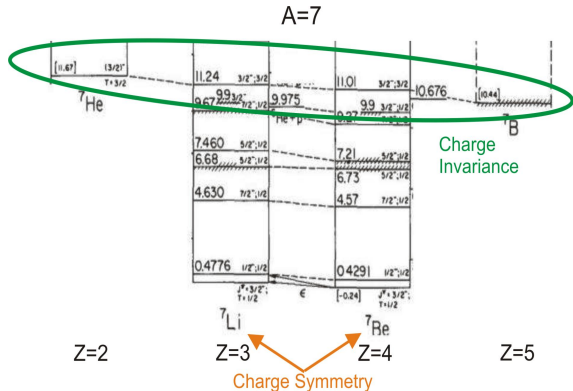
$$p : (\tau, \tau_z) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$n : (\tau, \tau_z) = \left(\frac{1}{2}, -\frac{1}{2}\right)$$

Net isospin

$$\vec{T} = \sum_{i=1}^A \vec{\tau}_i$$

Isobars: Nuclei with the same A



$$T = \frac{3}{2}, \dots$$

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Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$



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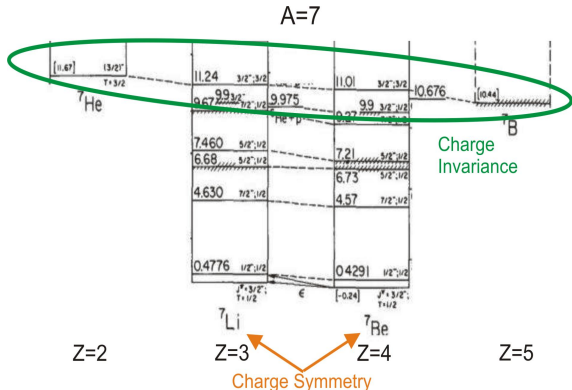
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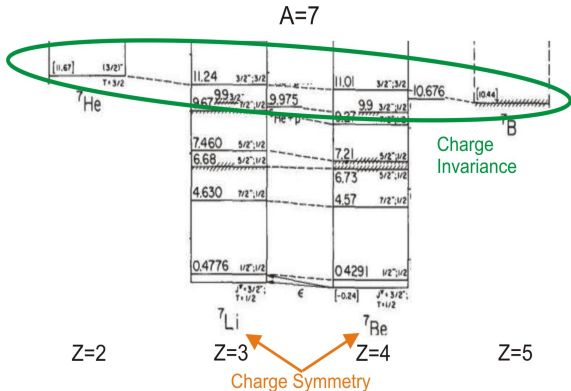
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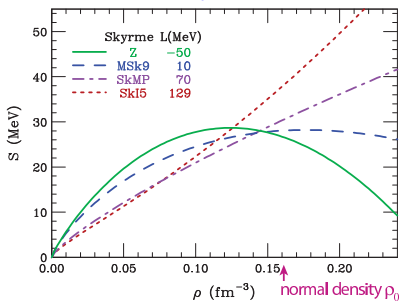
Examples: Nuclear Energy, Densities

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter

(a)symmetry energy

$$\rho = \rho_n + \rho_p$$



Net $\rho = \rho_n + \rho_p$ isoscalar

Difference $\rho_n - \rho_p$ isovector

$\rho_a = \frac{A}{N-Z} (\rho_n - \rho_p)$ isoscalar

$$\rho_{n,p}(r) = \frac{1}{2} \left[\rho(r) \pm \frac{N-Z}{A} \rho_a(r) \right]$$

Energy min in Thomas-Fermi:

$$\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$$

low $S \Leftrightarrow$ high ρ_a

$$S(\rho) = S(\rho_0) + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \dots$$

Unknown: $S(\rho_0)$? L ?



Stiffness of EOS & Mass & Radius of n -Star

$$\frac{E}{A} = \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2$$

$$S \simeq a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0}$$

In neutron matter:

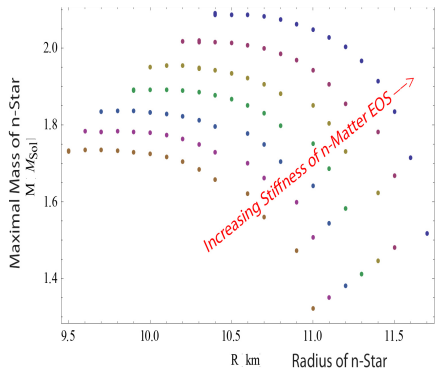
$$\rho_p \approx 0 \text{ \& } \rho_n \approx \rho.$$

$$\text{Then, } \frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho)$$

Pressure:

$$P = \rho^2 \frac{d}{d\rho} \frac{E}{A} \simeq \rho^2 \frac{dS}{d\rho} \simeq \frac{L}{3\rho_0} \rho^2$$

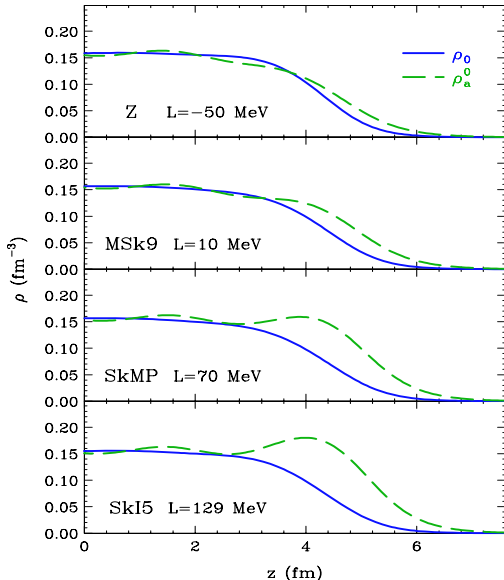
Stiffer symmetry energy correlates with
larger max mass of neutron star & larger radii



Schematic Calculation by Stephen Portillo (Harvard U)



Relation between ρ , ρ_a & $S(\rho)$

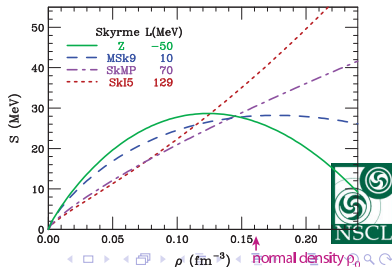


Results f/different Skyrme
ints in half- ∞ matter

PD&Lee NPA818(09)36

Isoscalar ($\rho = \rho_n + \rho_p$; blue) &
isovector ($\rho_a \propto \rho_n - \rho_p$; green)
densities displaced
relative to each other.

As $S(\rho)$ changes, $\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$,
so does displacement.



Probing Independently 2 Densities

Jefferson Lab

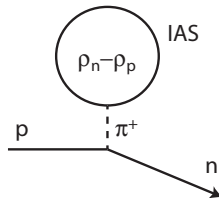
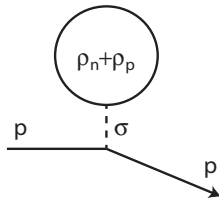
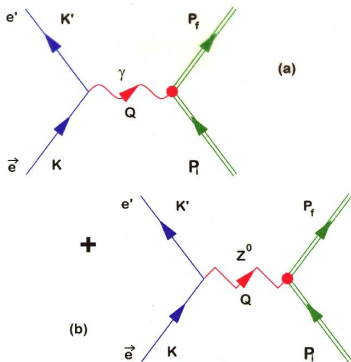
Direct: $\sim p$

Interference: $\sim n$

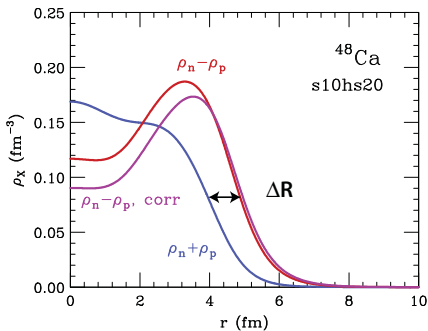
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[after Dao Tien Kho]

elastic: $\sim p + n$

charge exchange: $\sim n - p$



Expectations on Isovector Aura?



Much Larger Than Neutron Skin!

Surface radius $R \simeq \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2}$

rms neutron skin

$$\langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2}$$

$$\simeq 2 \frac{N-Z}{A} \left[\langle r^2 \rangle_{\rho_n - \rho_p}^{1/2} - \langle r^2 \rangle_{\rho_n + \rho_p}^{1/2} \right]$$

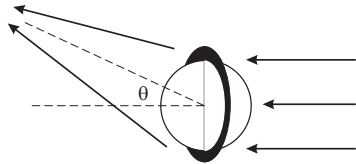
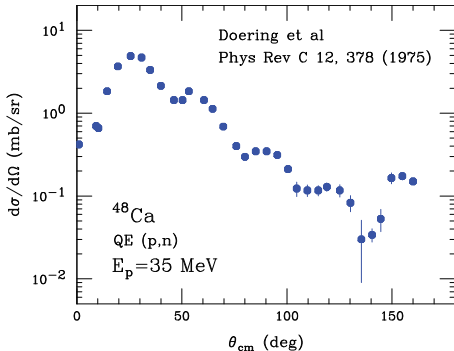
rms isovector aura

Estimated $\Delta R \sim 3 \left(\langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \right)$ for $^{48}\text{Ca}/^{208}\text{Pb}$!

Even before consideration of Coulomb effects that further enhances difference!



Direct Reaction Primer



DWBA:

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr \psi_f^* U_1 \psi_i \right|^2$$

Lane Potential

$$U = U_0 + \frac{4\tau T}{A} U_1$$

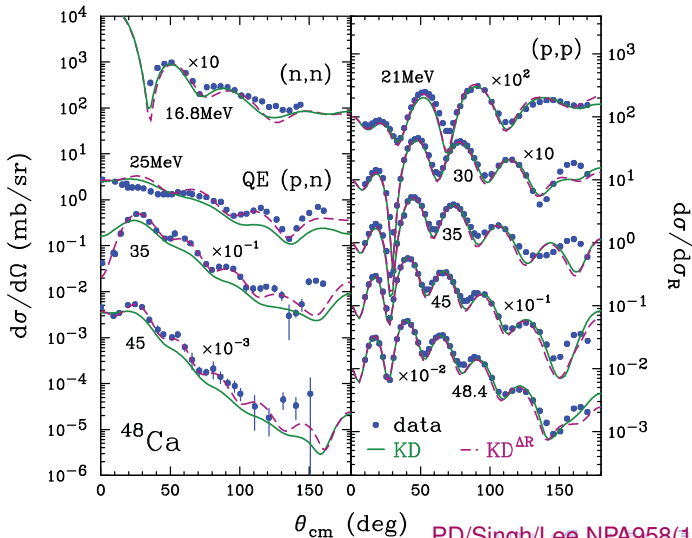
$$U_0 \propto \rho \quad U_1 \propto \rho_n - \rho_p$$

It is common to assume the same geometry for U_0 & U_1 , implicitly ρ & ρ_a , e.g. [Koning&Delaroche NPA713\(03\)231](#)



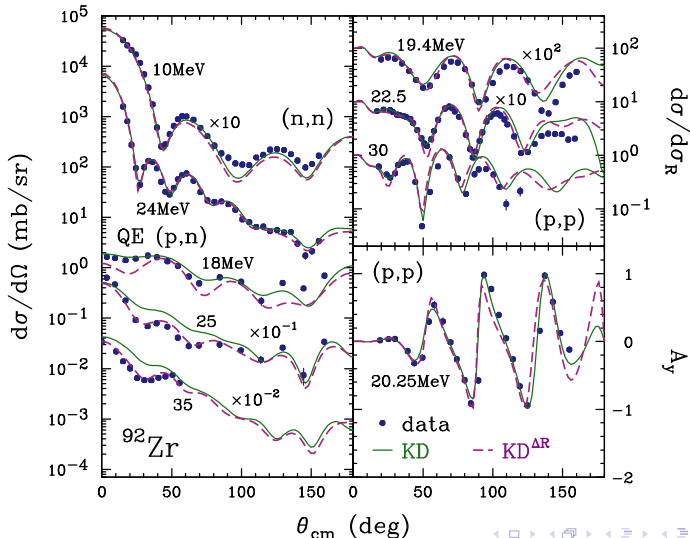
Simultaneous Fits to Elastic & Charge-Change: ^{48}Ca

Different radii for densities/potentials: $R_a = R + \Delta R$



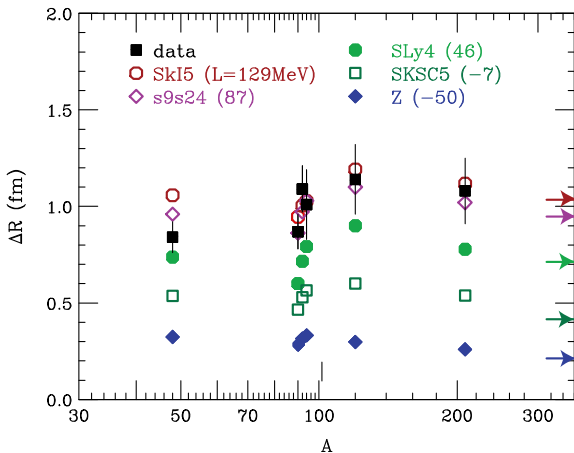
Simultaneous Fits to Elastic & Charge-Change: ^{92}Zr

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Thickness of Isovector Aura

6 targets analyzed, differential cross section + analyzing power

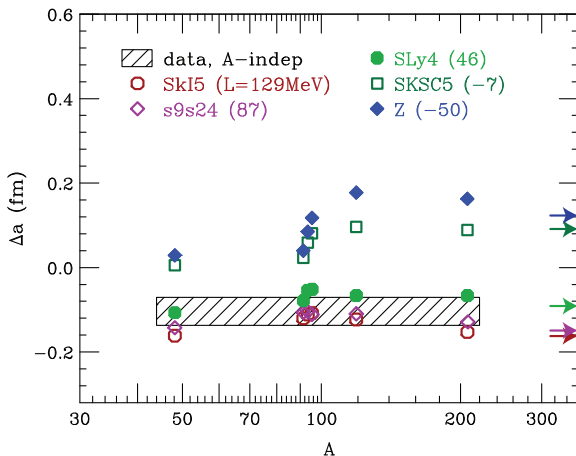


Colored: Skyrme predictions. Arrows: half-infinite matter

Thick ~ 0.9 fm isovector aura! \sim Independent of A .



Difference in Surface Diffuseness



Colored: Skyrme predictions. Arrows: half-infinite matter

Sharper isovector surface than isoscalar!



Bayesian Inference

Probability density in parameter space $p(x)$ updated as experimental data on observables E , value \bar{E} with error σ_E , get incorporated

Probability p is updated iteratively, starting with prior p_{prior}
 $p(a|b)$ - conditional probability

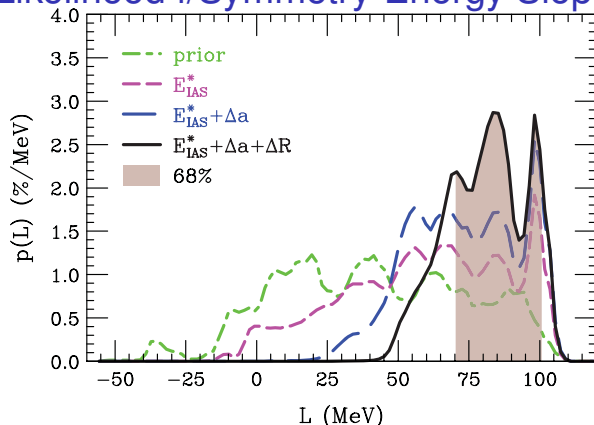
$$p(x|\bar{E}) \propto p_{\text{prior}}(x) \int dE e^{-\frac{(E-\bar{E})^2}{2\sigma_E^2}} p(E|x)$$

For large number of incorporated data, p becomes independent of p_{prior}

In here, p_{prior} and $p(E|x)$ are constructed from all Skyrme ints in literature, and their linear interpolations. p_{prior} is made uniform in plane of symmetry-energy parameters (L, S_0)



Likelihood f/Symmetry-Energy Slope

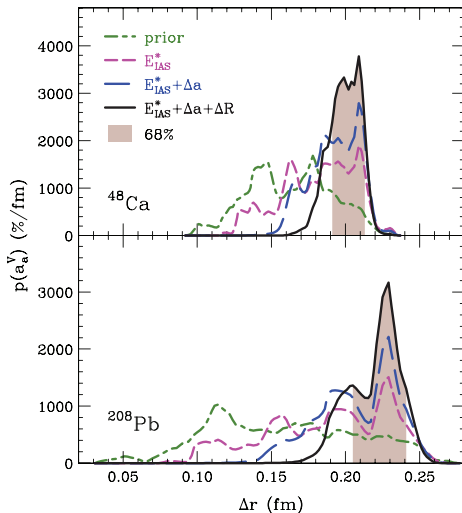


E_{IAS}^* - from excitations to isobaric analog states
in PD&Lee NPA922(14)1

Oscillations in prior of no significance
- represent availability of Skyrme parametrizations



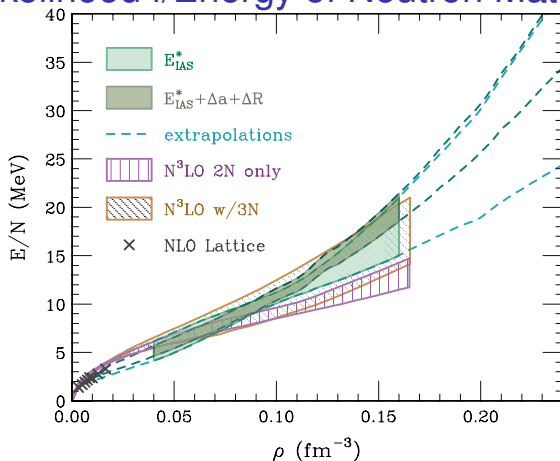
Likelihood f /Neutron-Skin Values



Sizeable n -Skins



Likelihood f /Energy of Neutron Matter

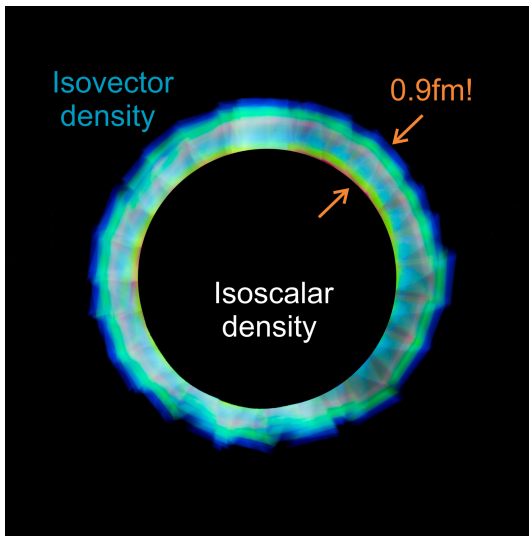


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Some oscillations due to prior



Isvector Aura



Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density
- For large A , displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions
- Such an analysis produces thick isovector aura
 $\Delta R \sim 0.9 \text{ fm!}$
- Symmetry & neutron energies are stiff!
 $L = (70 - 100) \text{ MeV}$, $S(\rho_0) = (33.5 - 36.5) \text{ MeV}$ at 68% level

PD, Lee & Singh NPA818(09)36, 922(14)1, 958(17)147 + in progress
NSF PHY-1403906 & DOE DE-SC0019209



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