Nuclear symmetry energy from finite nuclei to neutron stars

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- Nuclear Symmetry energy
  : Energy difference between PNM and SNM

$$
\begin{aligned}
S_2^{(a)}(n) &= E_N(n) - E_S(n)\,, \\
S_2^{(b)}(n) &= \frac{1}{8}\,\frac{\partial^2 E(n,x)}{\partial x^2}\bigg|_{x=1/2}
\end{aligned}
\tag{1}
$$

Energy per baryon of asymmetric nuclear matter

$$
E(n,x) = \frac{1}{2m\,n}(\tau_n + \tau_p) + (1-2x)^2\,V_n(n) + (1-(1-2x)^2)\,V_s(n) \tag{2}
$$

$$
\begin{aligned}
S_2^{(a)}(n) - S_2^{(b)}(n) &\simeq \frac{1}{2m}\frac{3}{5}(3\pi^2 n)^{2/3}\left[1 - \frac{1}{2^{2/3}} - \frac{5}{9\cdot 2^{2/3}}\right] \\
&= 0.70\left(\frac{n}{n_0}\right)^{2/3}\ \text{MeV}
\end{aligned}
\tag{3}
$$

$$S_2(n) = S_v + L\left(\frac{n-n_0}{3n_0}\right) + \frac{1}{2}K_{\text{sym}}\left(\frac{n-n_0}{3n_0}\right)^2 + \cdots \quad (4)$$

- Neutron star cooling
- r-process
- neutron skin thickness
- direct Urca Process
- Nuclear mass
- Mass and radius of neutron stars

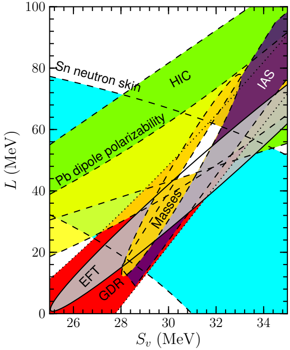


Figure : Allowed area for S_v and L, Lattimer and Lim, ApJ. 771, 51 (2013)

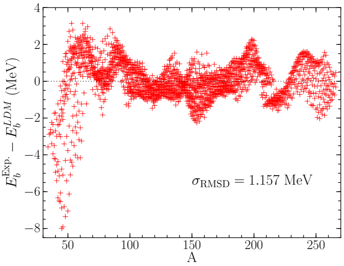
- Symmetry energy in nucleus

$$
\begin{aligned}
E(Z,A) = -BA + E_S A^{2/3} + \frac{S_v}{1+\frac{S_s}{S_v}A^{-1/3}} A I^2 \\
+ E_C \frac{Z^2}{A^{1/3}} + E_{ex}\frac{Z^{4/3}}{A^{1/3}} + E_{diff}\frac{Z^2}{A} + E_{\mathrm{pair}} + E_{\mathrm{shell}},
\end{aligned} \tag{5}
$$

Table : Liquid drop model parameter which minimize the root mean square deviation in total binding energy

B (MeV)	E_S (MeV)	S_v (MeV)	S_s/S_v	Δ (MeV)	RMSD (MeV)	-
15.890	20.219	28.951	1.937	12.196	2.529	w/o shell
15.886	19.777	31.524	2.522	11.257	1.157	w/ shell

Bulk ans Surface part : Isospin asymmetry



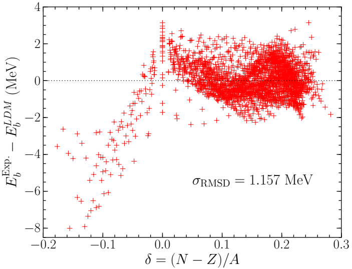


Figure : Binding energy difference in LDM approach

- LDM is fast and accurate to get the total binding energy
  : The most accurate mass model $\sigma_{RMSD} \sim 0.5\,\mathrm{MeV}$

Table : Root Mean Square Deviation in total binding energy from mass models

	FRDM	FRDM12	TF	HFB21	D1M	DZ
σ_{RMSD} (MeV)	0.654	0.579	0.649	0.572	0.789	0.394

- Light and Proton rich : Large deviation (HFB or Few body)

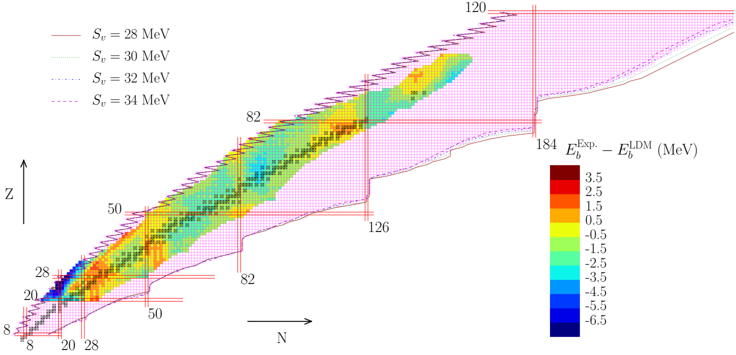


Figure : Neutron dripline and nuclear symmetry energy

- Neutron dripline : $\mu_n > 0$, $S_n < 0$, $S_{2n} < 0$
- As S_v increases, the neutron dripline moves to left
- Proton drip lines don't change

TF approx

- TF assumes the local density is made of plane waves.

$$n_t(r) = \begin{cases} n_{t_i}\left[1-\left(\frac{r}{R_t}\right)^{\alpha_t}\right]^3 & \text{if} \quad r < R_t; \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

We find n_{t_i}, α_t, R_t for a given N, Z minimizing the total binding energy,

$$E(N,Z) = \int 4\pi r^2 \mathcal{E}[n_n(r), n_p(r), \nabla n_n(r), \nabla n_p(r)]\, dr \tag{7}$$

where

$$\mathcal{E} = \mathcal{E}_B + \mathcal{E}_C + \mathcal{E}_{ex} + \mathcal{E}_{grad}\,, \tag{8}$$

where $\mathcal{E}_C$ is the Coulomb energy density, $\mathcal{E}_C$ is the exchange Coulomb energy density, and $\mathcal{E}_{grad}$ is the density gradient energy density which is given as

$$\mathcal{E}_{grad} = \frac{1}{2}\left[Q_{nn}(\nabla n_n)^2 + 2Q_{np}\nabla n_n \nabla n_p + Q_{pp}(\nabla n_p)^2\right]. \tag{9}$$

Density gradient interaction

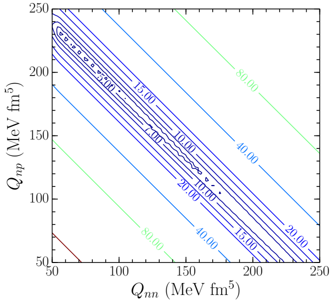


Figure : σ_{RMSD} as a function of Q_{nn} and Q_{np}

- 2336 Nuclei to fit Q_{nn} and Q_{np}

- Gradient interaction in TF vs Surface energy in LDM
  Q_{nn}, Q_{np} vs σ_0, σ_δ,
  $\sigma(x) = \sigma_0 - (1-2x)^2\sigma_\delta$

$$4\pi R^2\sigma(A,Z) = \frac{1}{2}\int 4\pi r^2 dr \times \left[Q_{nn}(\nabla n_n)^2 + 2Q_{np}\nabla n_n \nabla n_p + Q_{pp}(\nabla n_p)^2\right], \tag{10}$$

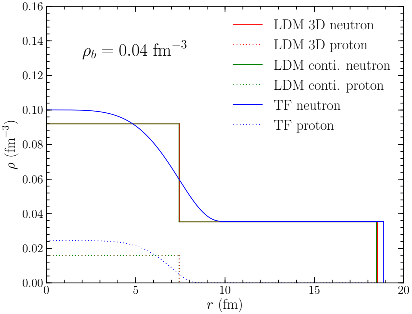


Figure : Density profile at the inner crust of neutron stars

Thus, the planar approximation of semi-infinite nuclear matter gives the correct magnitude of the surface tension where the surface tension is given as

$$\sigma(\delta_L) = \int_{-\infty}^{+\infty} \left[\mathcal{H} - \mu_n n_n - \mu_p n_p\right] , \tag{11}$$

where δ_L represents the isospin asymmetry of dense nuclear matter, $\mathcal{H}$ is the nuclear matter energy density, $\mu_n(\mu_p)$ is the chemical potential of neutron (proton). The density profile of the semi-infinity nuclear matter have the stationary solution, $\delta \int_{-\infty}^{+\infty} \left[\mathcal{H} - \mu_n n_n - \mu_p n_p\right] = 0$. We assume the density profile of semi-infinite nuclear matter follows,

$$\begin{aligned} n_n(z) &= \frac{n_{n_i} - n_{n_o}}{1 + \exp\left(\frac{z - z_n}{a_n}\right)} + n_{n_o} , \\ n_p(z) &= \frac{n_{p_i} - n_{p_o}}{1 + \exp\left(\frac{z - z_p}{a_p}\right)} + n_{p_o} \end{aligned} \tag{12}$$

where $n_{n_{i,o}}$ and $n_{p_{i,o}}$ are dense and dilute nuclear matter densities obtained from the thermodynamic equations for existence,

$$\mu_{n_i} = \mu_{n_o} , \quad \mu_{p_i} = \mu_{p_o} , \quad p_i = p_o . \tag{13}$$

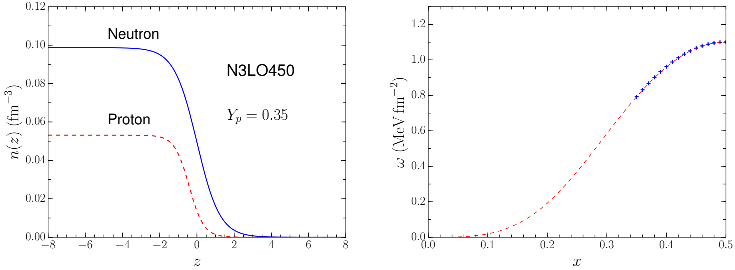


Figure : Left panel : density profile, Right panel : surface tension as a function of proton fraction x

$$\sigma(x) = \sigma_0 \frac{2 \cdot 2^{\alpha} + q}{x^{-\alpha} + q + (1-x)^{-\alpha}}, \tag{14}$$

where $\sigma_0 = \sigma(x = 1/2)$. α and q are fitting parameters for surface tension.

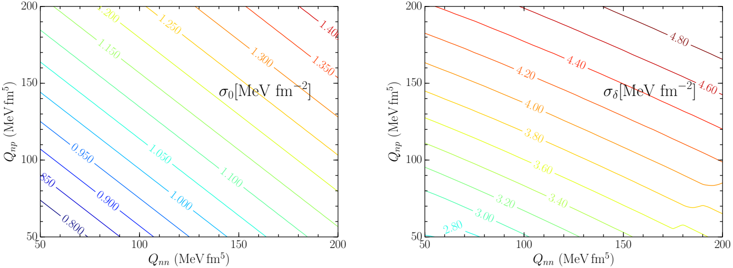


Figure : Gradient coefficients(Q_{nn}, Q_{np}) and surface tension parameters (σ_0, σ_δ)

$$\begin{aligned} \sigma_0 &= \sigma_1 + aQ_{nn} + bQ_{np} , \\ \sigma_\delta &= \sigma_2 + cQ_{nn} + dQ_{np} . \end{aligned} \tag{15}$$

Eq. (15) is the linear approximation for our numerical calculation.

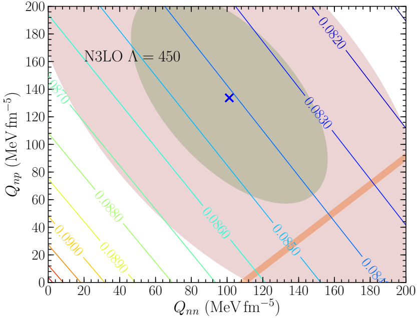


Figure : Contour plot of the core-crust transition density of neutron stars from thermodynamic instability

Thermodynamic instabilities : The matter is stable when the mathematical is maintained,

$$v_0 + 2(4\pi e^2\beta)^{1/2} - \beta k_{TF}^2 > 0\,, \qquad (16)$$

where

$$v_0 = \frac{\partial\mu_p}{\partial\rho_p} - \frac{(\partial\mu_p/\partial\rho_n)^2}{\partial\mu_n/\partial\rho_n}\,, \qquad (17)$$

$$\beta = 2(Q_{pp}+2Q_{np}\zeta+Q_{nn}\zeta^2)\ , \zeta = -\frac{\partial\mu_p/\partial\rho_n}{\partial\mu_n/\partial\rho_n}\,, \qquad (18)$$

and k_{TF} is the Thomas-Fermi wave number,

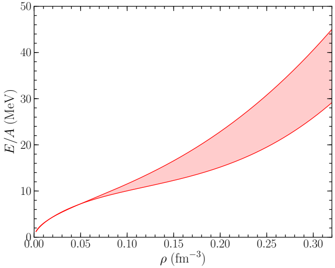
$$k_{TF}^2 = \frac{4e^2}{\pi\hbar c}k_e^2\,, \quad k_e = (3\pi^2\rho_p)^{1/3}\,. \qquad (19)$$

$$\rho_t \simeq \rho_{t_1} + \alpha Q_{nn} + \beta Q_{np} \rightarrow \rho_t \simeq \rho_{t_2} + \gamma\sigma_0 + \delta\sigma_\delta \quad (20)$$

Table : Numerical values for parameters in Eq. (20)

ρ_{t_2} (fm^{-3})	$9.680 \times 10^{-2} \pm 5.211 \times 10^{-4}$
γ (MeV^{-1} fm^{-1})	$-1.998 \times 10^{-2} \pm 4.191 \times 10^{-4}$
δ (MeV^{-1} fm^{-1})	$2.522 \times 10^{-3} \pm 1.063 \times 10^{-4}$

Neutron Skin and $R_{1.4M_\odot}$



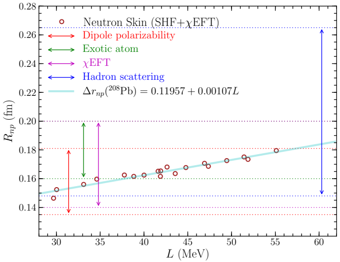


Figure : Left panel : Neutron matter E/A band, Holt and Kaiser, Phys. Rev. C 95, 034326 (2017). Right Panel : Neutron Skin and L (X. Roca-Maza *et al.* Phys. Rev. Lett. 106)

$$\frac{R_{1.4M_\odot}}{\mathrm{km}} \simeq (9.54 \pm 0.49) \left[\frac{p(n_0, x=0)}{\mathrm{MeV\,fm^{-3}}} \right]^{1/4} . \tag{21}$$

$$p(n_0, L) \simeq \frac{L}{3} n_0 . \tag{22}$$

$$\frac{R_{1.4M_\odot}}{\mathrm{km}} = (11.99 \pm 0.06) \left[\frac{\Delta r_{np}\,\mathrm{fm}^{-1} - 0.12}{0.05} \right]^{1/4} . \tag{23}$$

Energy density functional

- Bulk matter energy density $\mathcal{E}(n,x)$

$$\mathcal{E}(n,x) = \frac{1}{2m}\tau_n + \frac{1}{2m}\tau_p + (1-2x)^2 f_n(n) + \left[1-(1-2x)^2\right] f_s(n)\,, \tag{24}$$

where n is the nucleon number density, τ_n and τ_p are the neutron and proton kinetic energy densities, x is the proton fraction,

$$f_s(n) = \sum_{i=0}^{3} a_i n^{(2+i/3)}\,, \quad f_n(n) = \sum_{i=0}^{3} b_i n^{(2+i/3)} \tag{25}$$

- Neutron matter from EFT (N2LO450, N2LO500, N3LO414, N3LO450, N3LO500)
- Symmetric matter from Finite nuclei properties (205 Skyrme force models)
- Or Bayesian (EFT$\rightarrow$ prior, Skyrme $\rightarrow$ Likelihood)

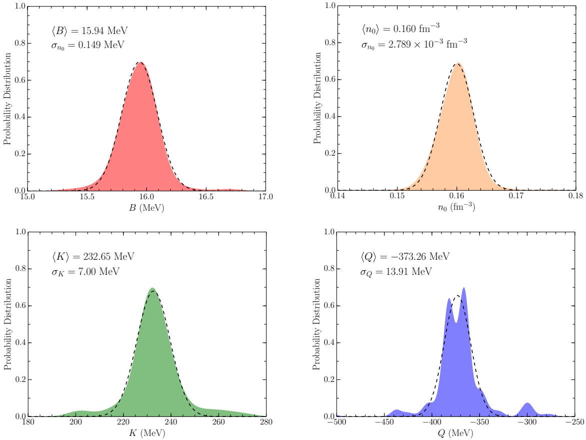


Figure : Saturation properties from Skyrme force models, Dutra *et al.*, Phys. Rev. C 85, 035201 (2012).

- Neutron Star Mass and Radius : TOV equations

$$\frac{dp}{dr} = - e^{2\chi}(\epsilon + p)\left(\frac{m}{r^2} + 4\pi r p\right),$$
$$\frac{dm}{dr} = 4\pi r^2 \epsilon.$$

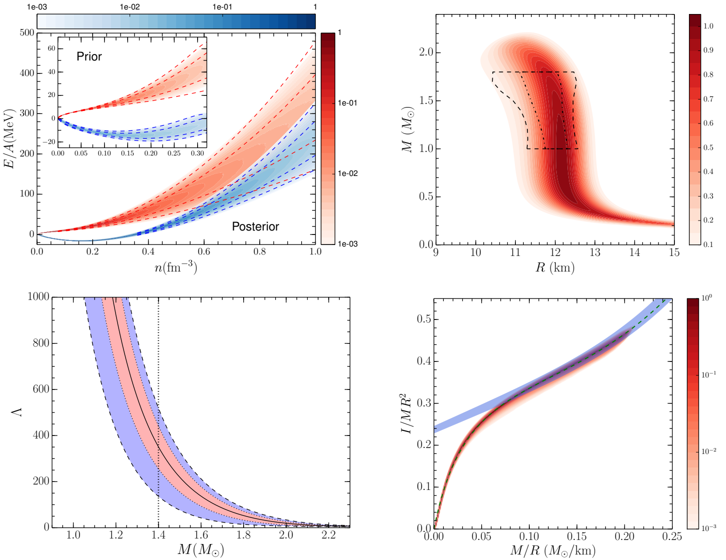
- Tidal deformability : Quadrapole moment (Q_{ij}), Gravitational field

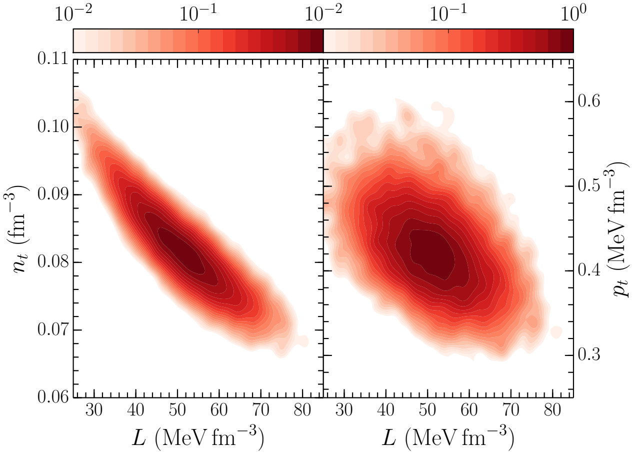
$$Q_{ij} = \int d^3x \delta\rho(x)\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right), \quad \mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\rm ext}}{\partial x^i \partial x^j}. \tag{26}$$

$$Q_{ij} = -\lambda \mathcal{E}_{ij}, \quad \Lambda = G\lambda\left(\frac{c^2}{GM}\right)^5. \tag{27}$$

- Moment of Inertia

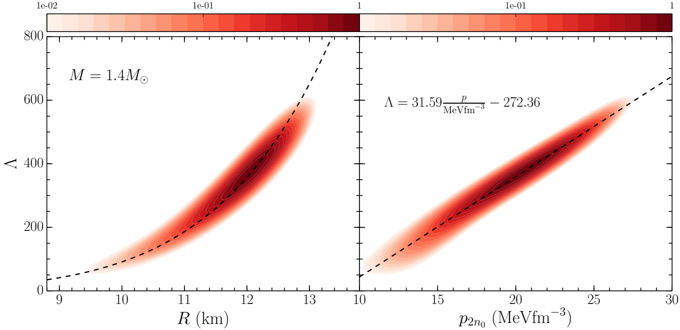
$$I = \frac{8\pi}{3}\int_0^R r^4(\varepsilon + p)e^{(\lambda-\nu)/2}\,\frac{\bar{\omega}}{\Omega}\,dr \tag{28}$$


Prior
Posterior
E/A(MeV)
n(fm^-3)
M (M⊙)
R (km)
Λ
M(M⊙)
I/MR^2
M/R (M⊙/km)

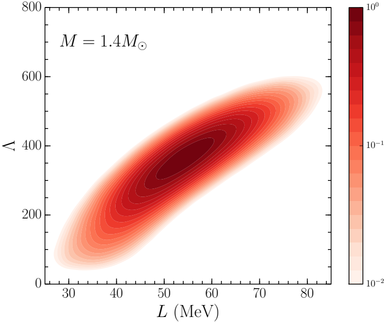


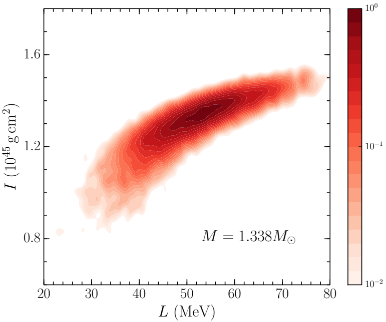
10^{-2}
10^{-1}
10^{-2}
10^{-1}
10^{0}
0.11
0.10
0.09
0.08
0.07
0.06
n_t (fm^{-3})
0.6
0.5
0.4
0.3
p_t (MeV fm^{-3})
30 40 50 60 70 80
30 40 50 60 70 80
L (MeV fm^{-3})
L (MeV fm^{-3})

Figure


1e-02
1e-01
1
1e-01
1
$M = 1.4M_\odot$
$\Lambda = 31.59\frac{p}{\mathrm{MeVfm^{-3}}} - 272.36$
Λ
0
200
400
600
800
9
10
11
12
13
10
15
20
25
30
R (km)
p_{2n_0} ($\mathrm{MeVfm^{-3}}$)

Figure


$M = 1.4M_\odot$
Λ
L (MeV)


$M = 1.338M_\odot$
I (10^{45} g cm^2)
L (MeV)

Summary

- Symmetry energy vs neutron drip lines
- Surface tension vs density gradient interaction
- Surface tension vs neutron star's core-crust transition density
- Neutron skin vs $R_{1.4M_\odot}$
- Symmetry energy density slope (L), macroscopic quantities, microscopic quantites
- Nuclear matter pressure at $2n_0$