Supernova equation of state and symmetry energy at subnuclear densities

H. Togashi (RIKEN)

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H. Suzuki, K. Sumiyoshi, E. Hiyama

Outline

1 : Introduction

2 : Supernova EOS with realistic nuclear forces

3 : Systematic study for non-uniform matter

4 : Summary

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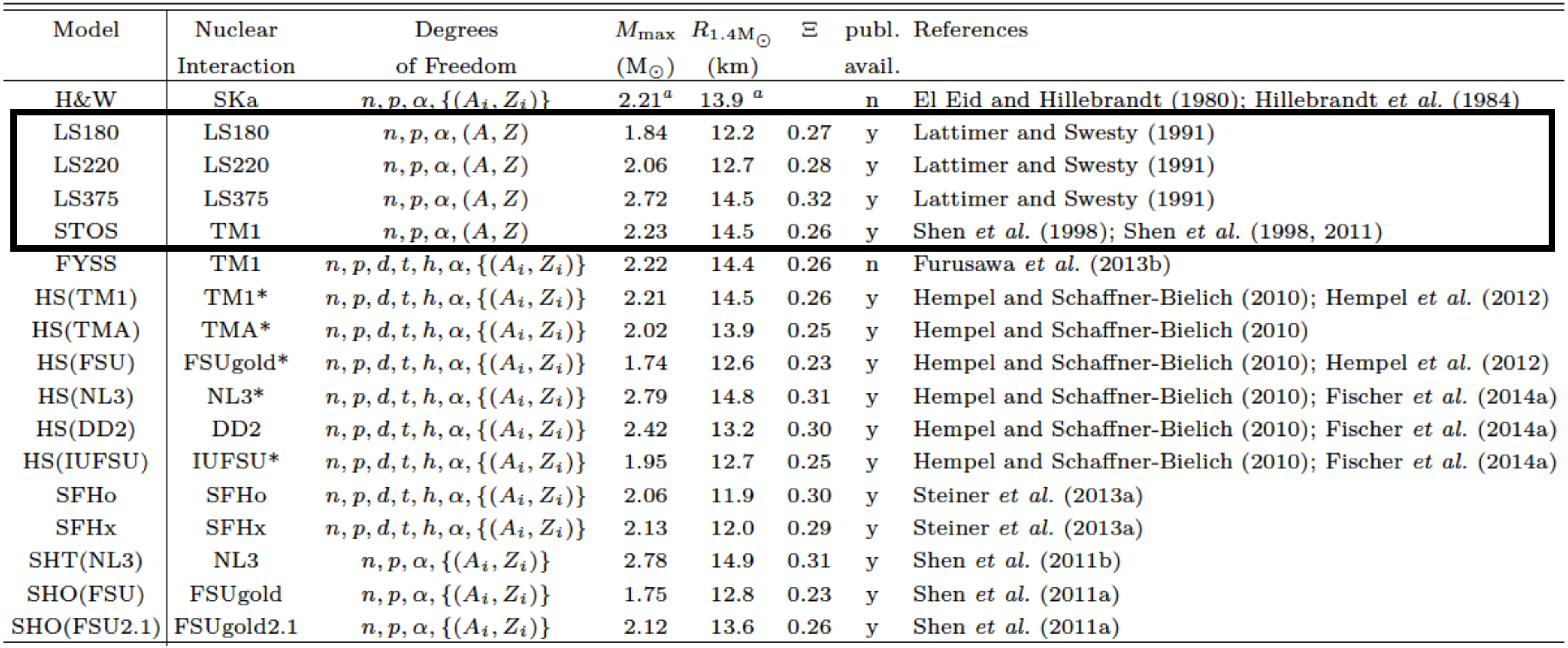
1. Introduction

Supernova equation of state (SN-EOS)



Model	Nuclear Interaction	Degrees of Freedom	$M_{\max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21^a	$13.9\ ^a$		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180	$n, p, \alpha, (A, Z)$	1.84	12.2	0.27	y	Lattimer and Swesty (1991)
LS220	LS220	$n, p, \alpha, (A, Z)$	2.06	12.7	0.28	y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.79	14.8	0.31	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.42	13.2	0.30	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
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SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)

(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

- Lattimer-Swesty EOS : *Skyrme + Compressible liquid drop model*

(NPA 535 (1991) 331)

- Shen EOS : *Relativistic Mean Field Theory + Thomas-Fermi model*

(PTP 100 (1998) 1013)

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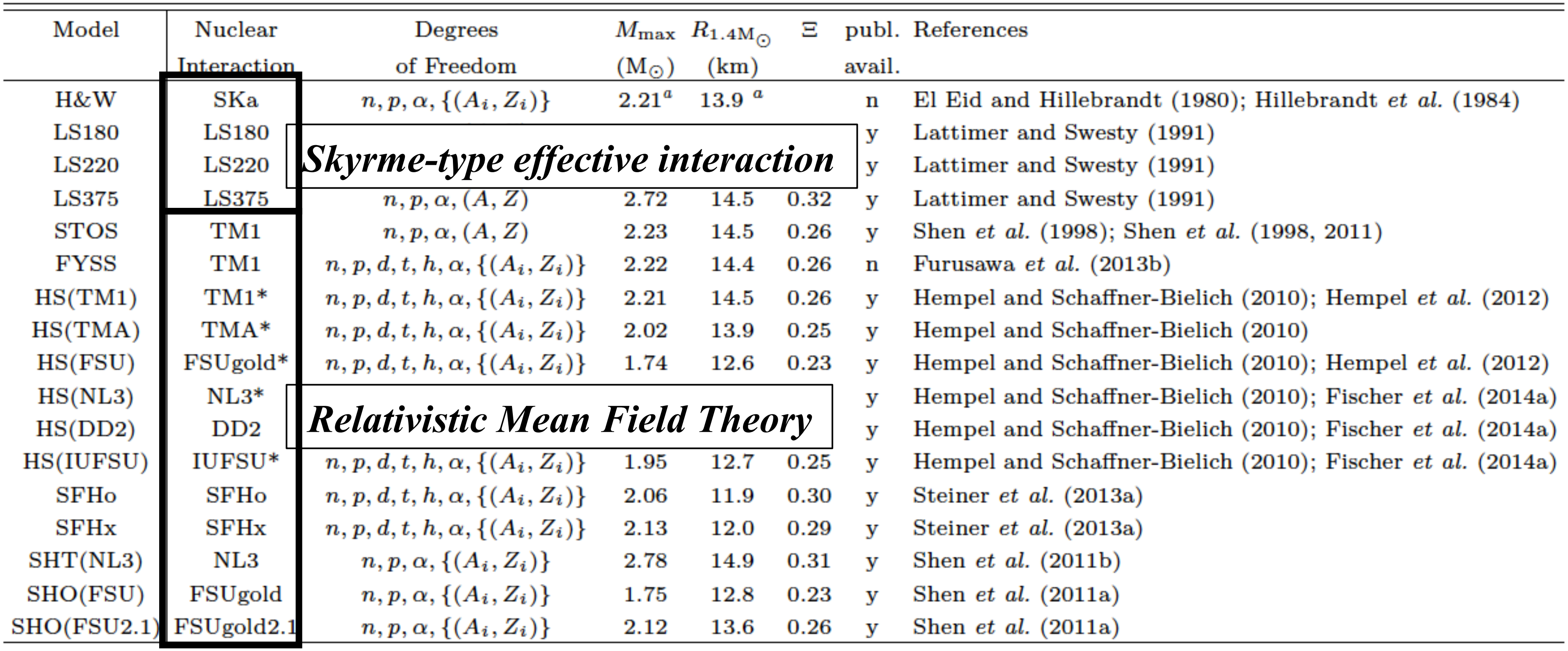

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Skyrme-type effective interaction

Relativistic Mean Field Theory

(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

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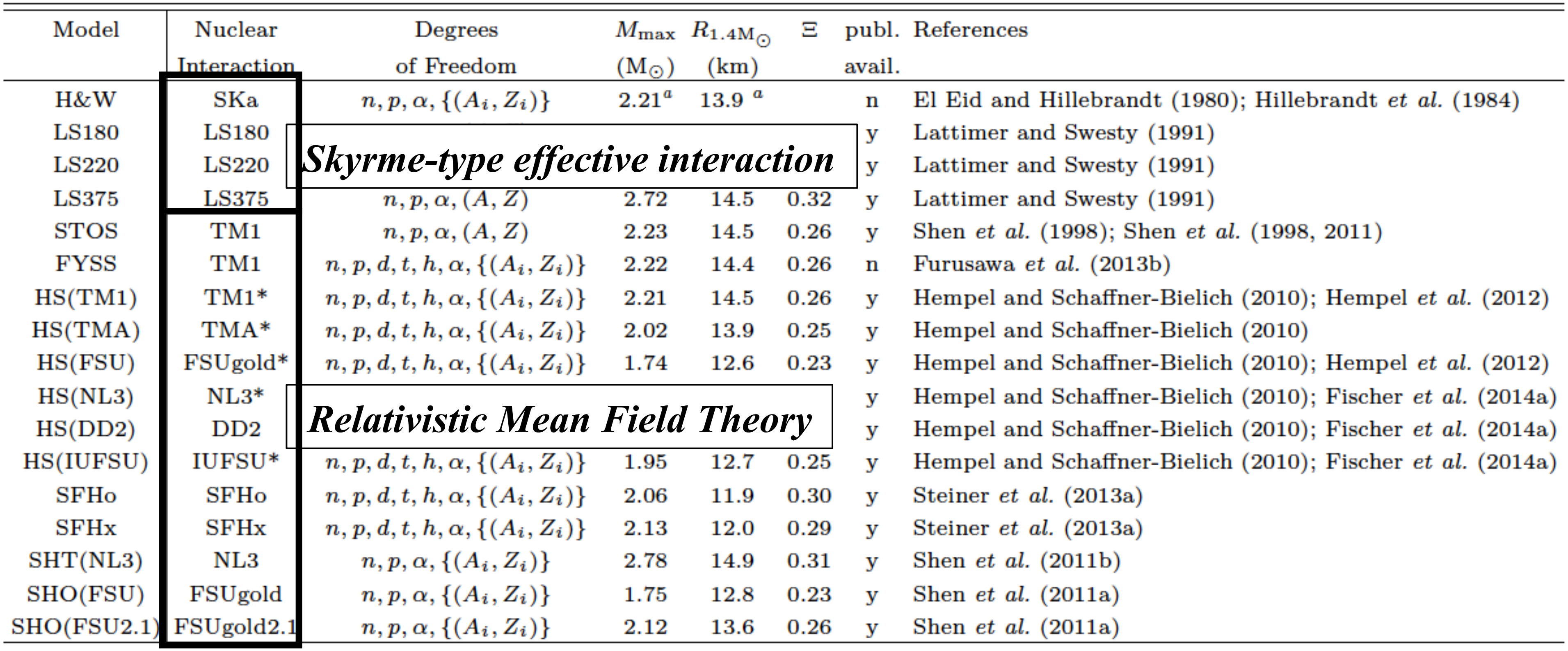

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Skyrme-type effective interaction

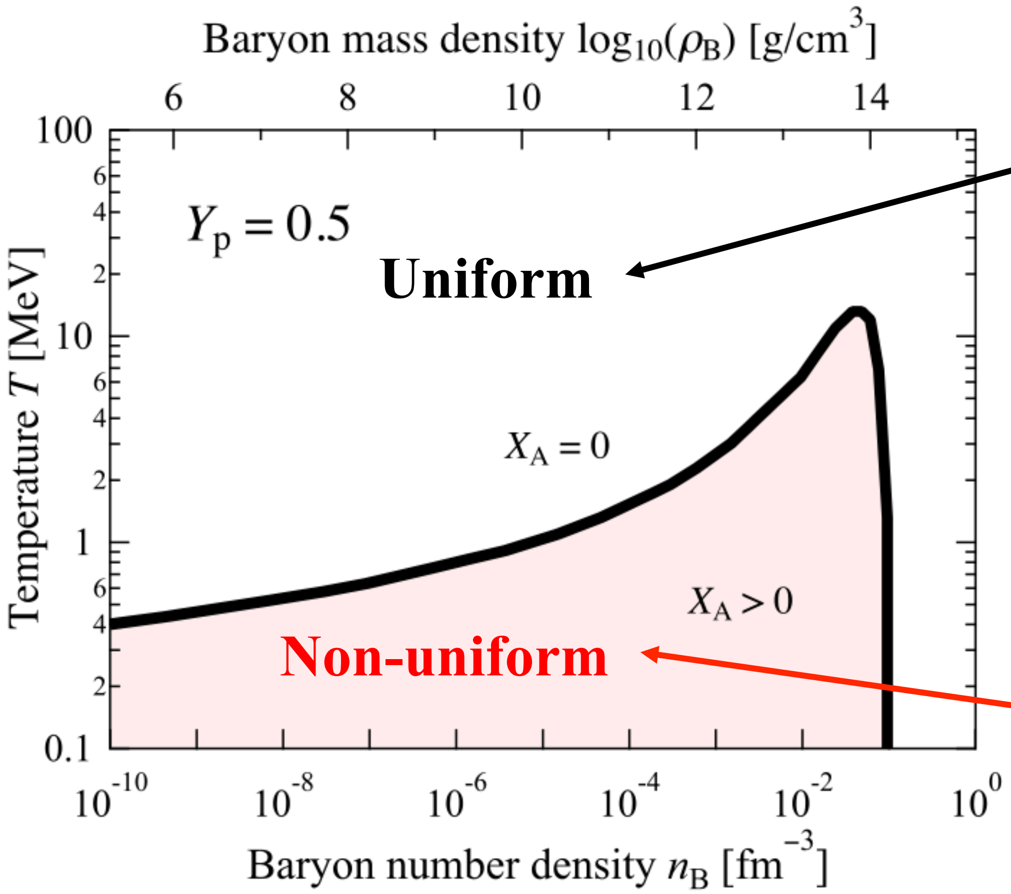
Relativistic Mean Field Theory

(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

TNTYST | AV18+UIX n, p, α, (*A*, *Z*) 2.21 11.5 0.32 y Togashi *et al.* (2017)

We have constructed a new SN-EOS with the variational method starting from bare nuclear forces.

Our procedure to construct a SN EOS table



1: **Cluster variational method with AV18 + UIX potentials**

2: **Thomas-Fermi calculation for non-uniform matter**

- Temperature T : $0 \leq T \leq 100$ MeV
- Density ρ : $10^{5.1} \leq \rho_B \leq 10^{16.0}$g/cm^3
- Proton fraction Y_p : $0 \leq Y_p \leq 0.65$

2. Supernova EOS with realistic nuclear forces

Nuclear Hamiltonian

$$H = -\sum_{i=1}^{N}\frac{\hbar^{2}}{2m}\nabla^{2} + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk}$$

Argonne v18 (AV18) two-body potential

Urbana IX (UIX) three-body potential

Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

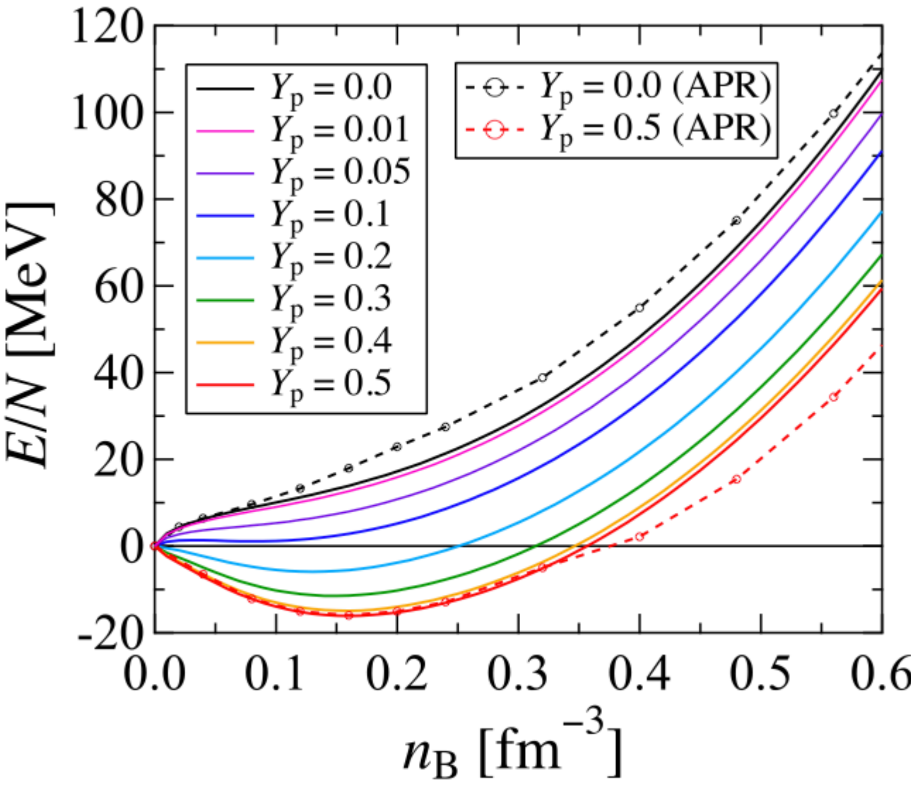
f_{ij}: Correlation function

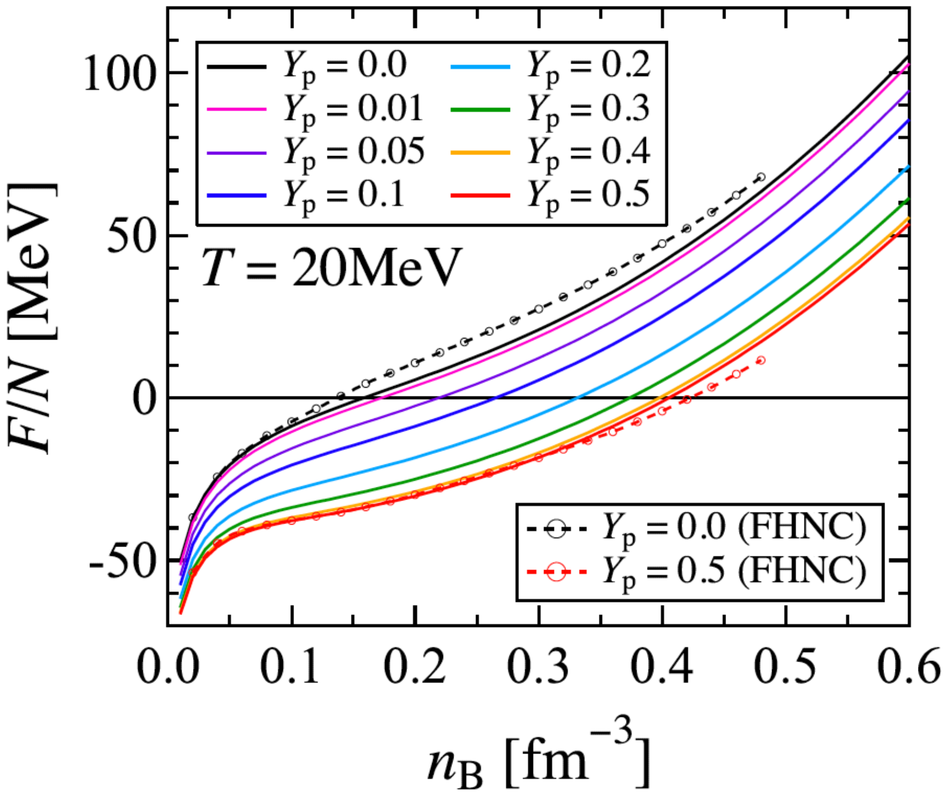
Φ_{F}: Fermi-gas wave function

- The expectation value of the Hamiltonian is calculated in ***the two-body cluster approximation***.
- The prescription by Schmidt and Pandharipande is employed to obtain the free energy ***at finite temperature***.

(Phys. Lett. 87B(1979) 11, PRC 75(2007) 035802)

Nuclear EOS for uniform matter





n_0[fm^{-3}]	E_0[MeV]	K [MeV]	E_{sym}[MeV]
0.16	−16.1	245	30.0

Our EOS : HT and M. Takano, NPA 902 (2013) 53

APR : A. **A**kmal, V. R. **P**andharipande, D. G. **R**avenhall, PRC 58 (1998) 1804

FHNC : A. Mukherjee, PRC 79(2009) 045811

Nuclear EOS for non-uniform matter

*We use the **Thomas-Fermi method** by Shen et al.*

(PTP 100 (1998) 1013, APJS 197(2011) 20)

Free energy of a Wigner-Seitz cell

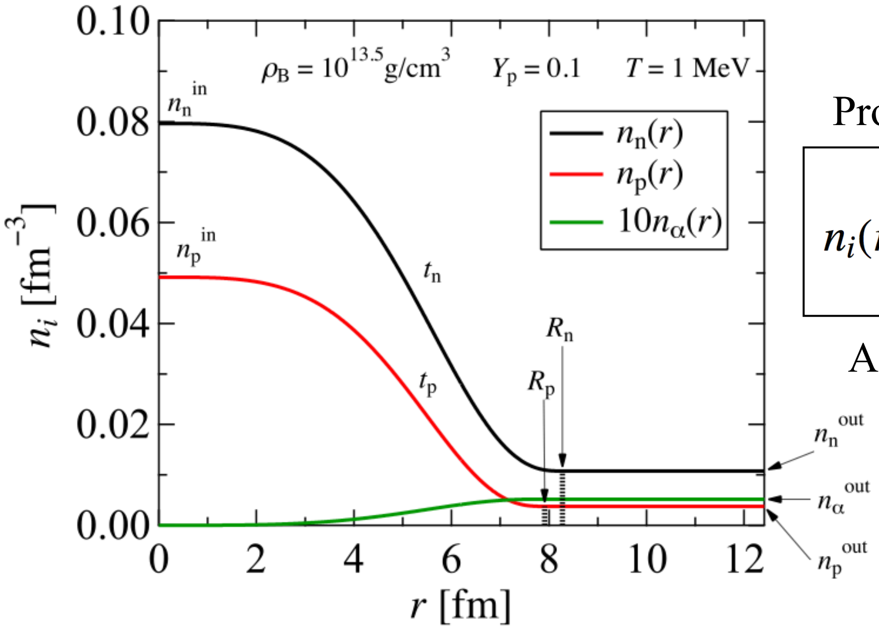
Bulk energy **Gradient energy**

$$F = \int d\boldsymbol{r} \underline{f(n_\mathrm{p}(r), n_\mathrm{n}(r), n_\alpha(r))} + F_0 \int d\boldsymbol{r} |\nabla(n_\mathrm{p}(r) + n_\mathrm{n}(r))|^2$$

$$+ \frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_\mathrm{p}(r) + 2n_\alpha(r) - n_\mathrm{e}][n_\mathrm{p}(r') + 2n_\alpha(r') - n_\mathrm{e}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_\mathrm{bcc} \frac{(Ze)^2}{a}$$

Coulomb energy

Free energy density of uniform matter: $f = f_\mathrm{N} + f_\alpha$



Particle number density distributions

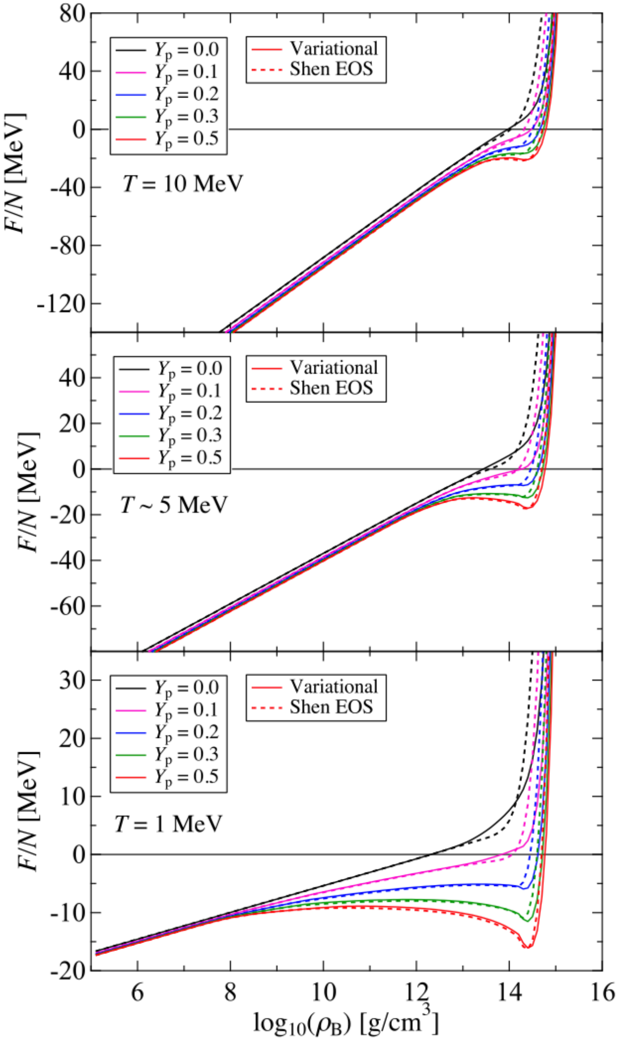
Protons and neutrons (i = p, n)

$$n_i(r) = \begin{cases} (n_i^\mathrm{in} - n_i^\mathrm{out})[1 - (r/R_i)^{t_i}]^3 + n_i^\mathrm{out} & (0 \le r \le R_i) \\ n_i^\mathrm{out} & (R_i \le r \le R_\mathrm{cell}) \end{cases}$$

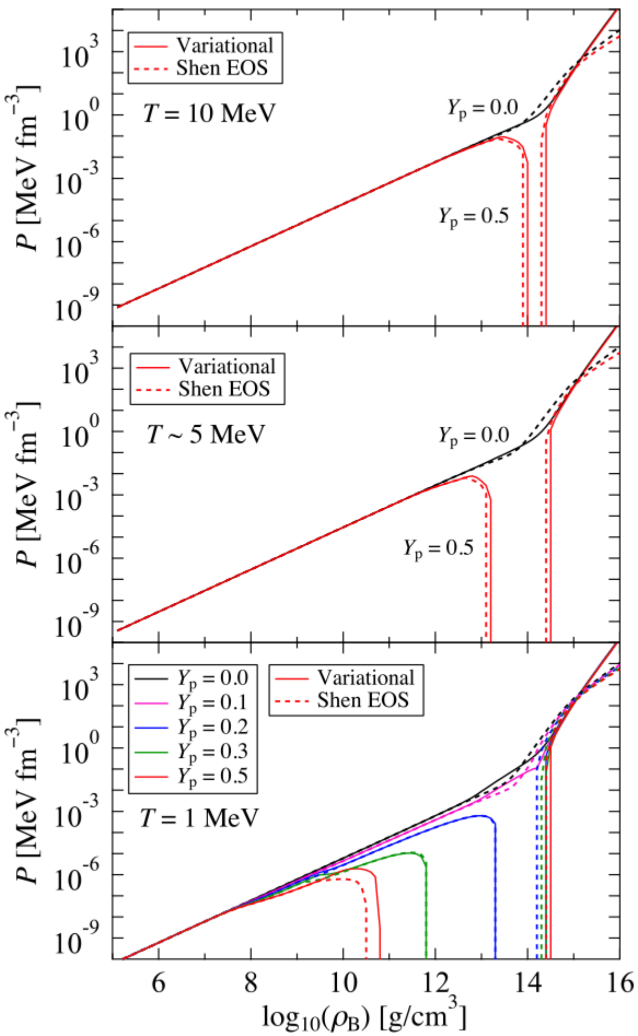
Alpha-particles

$$n_\alpha(r) = \begin{cases} -n_\alpha^\mathrm{out}[1 - (r/R_\mathrm{p})^{t_\mathrm{p}}]^3 + n_\alpha^\mathrm{out} & (0 \le r \le R_\mathrm{p}) \\ n_\alpha^\mathrm{out} & (R_\mathrm{p} \le r \le R_\mathrm{cell}) \end{cases}$$

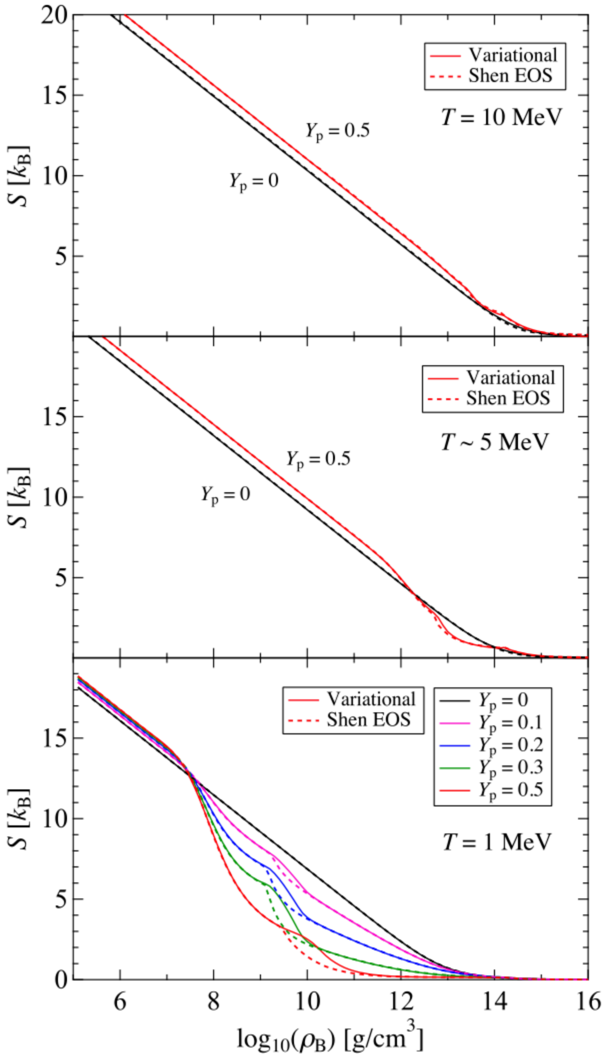
Thermodynamic Quantities



Free energy per nucleon



Pressure

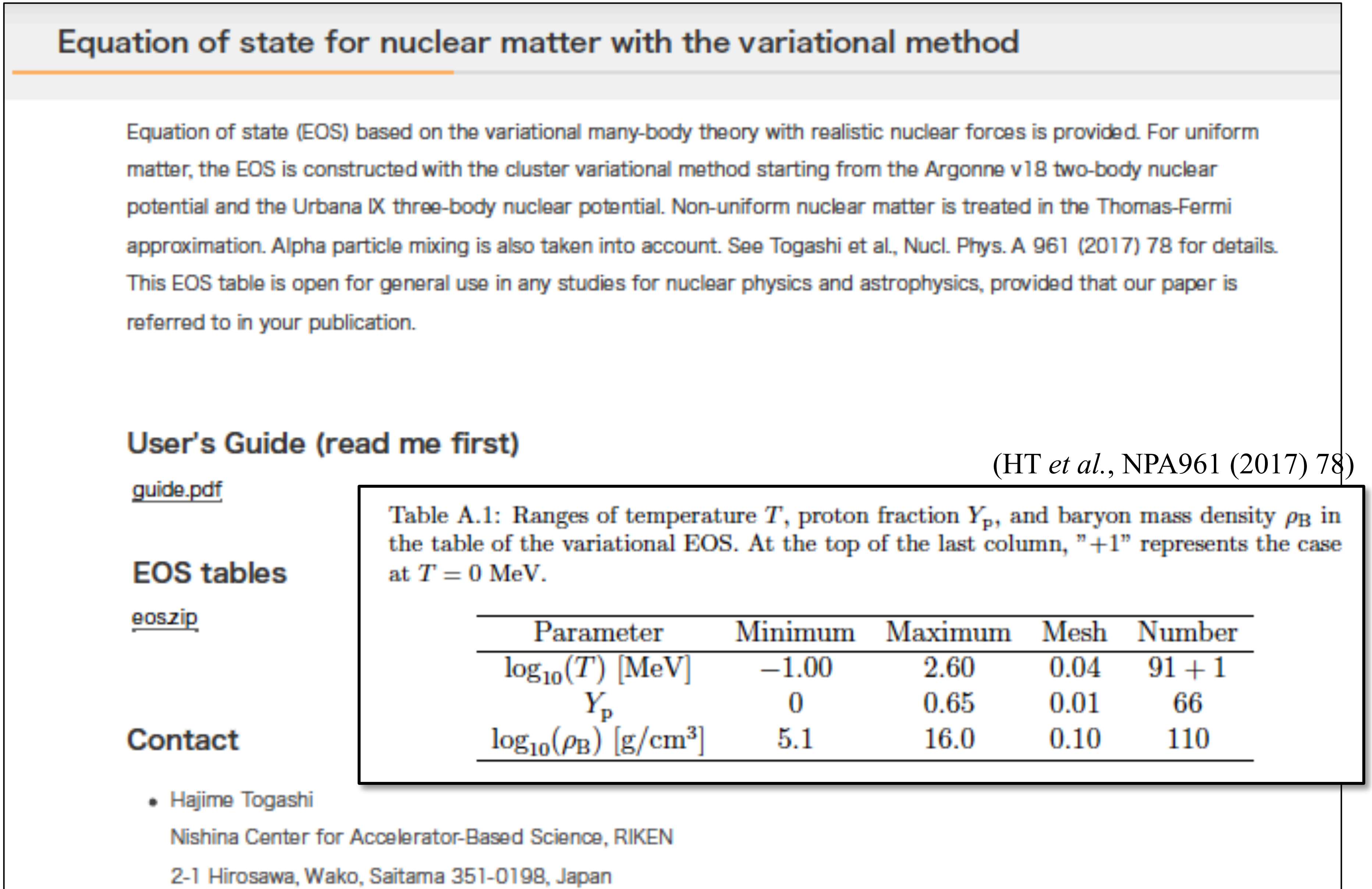


Entropy

HT *et al.*, NPA 961 (2017) 78

Home Page of Variational EOS Table

http://www.np.phys.waseda.ac.jp/EOS/



Equation of state for nuclear matter with the variational method

Equation of state (EOS) based on the variational many-body theory with realistic nuclear forces is provided. For uniform matter, the EOS is constructed with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. Non-uniform nuclear matter is treated in the Thomas-Fermi approximation. Alpha particle mixing is also taken into account. See Togashi et al., Nucl. Phys. A 961 (2017) 78 for details. This EOS table is open for general use in any studies for nuclear physics and astrophysics, provided that our paper is referred to in your publication.

User's Guide (read me first)

guide.pdf

EOS tables

eos.zip

Contact

- Hajime Togashi
  Nishina Center for Accelerator-Based Science, RIKEN
  2-1 Hirosawa, Wako, Saitama 351-0198, Japan

(HT *et al.*, NPA961 (2017) 78)

Table A.1: Ranges of temperature T, proton fraction Y_p, and baryon mass density ρ_B in the table of the variational EOS. At the top of the last column, "+1" represents the case at $T = 0$ MeV.

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91 + 1$
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_\mathrm{B})$ [g/cm^3]	5.1	16.0	0.10	110

Home Page of Variational EOS Table

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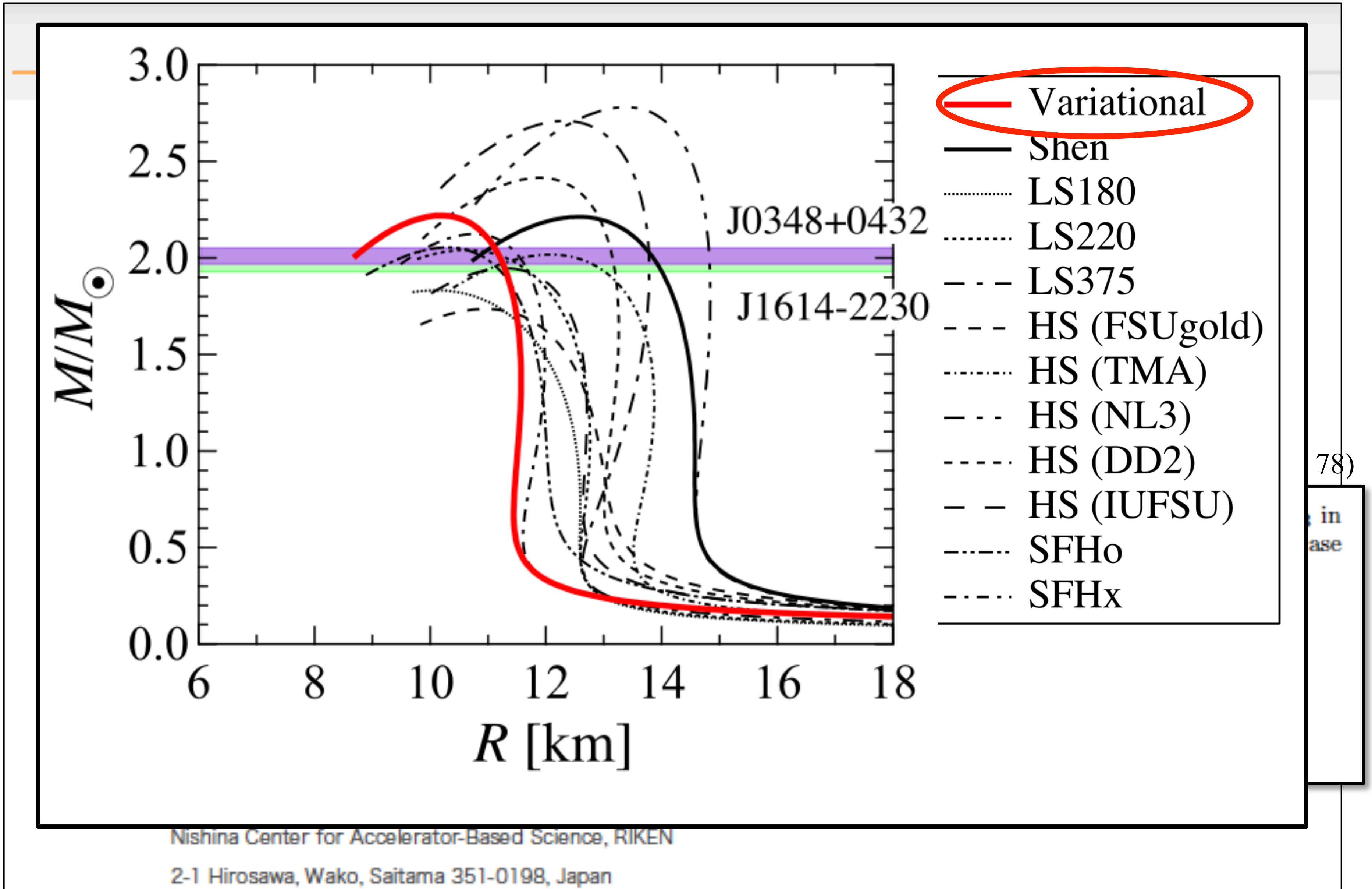

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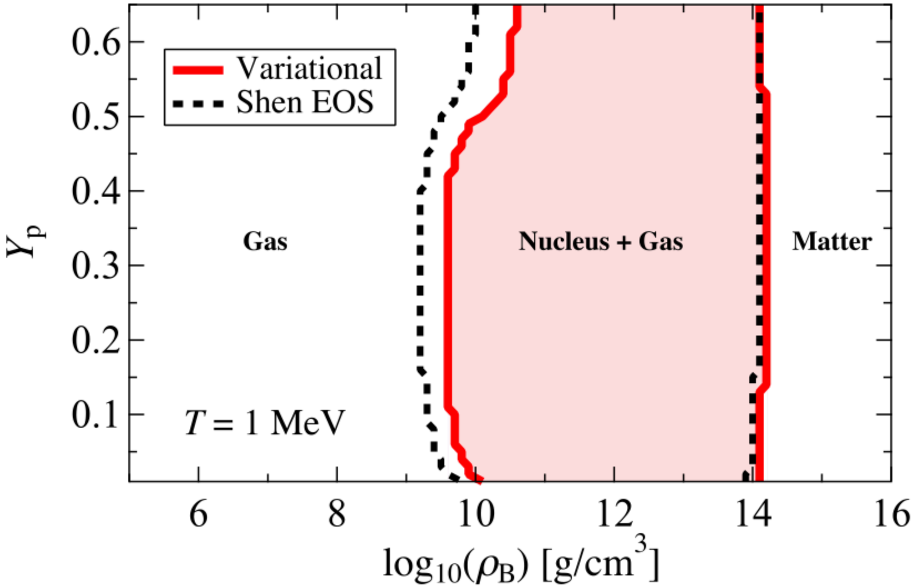
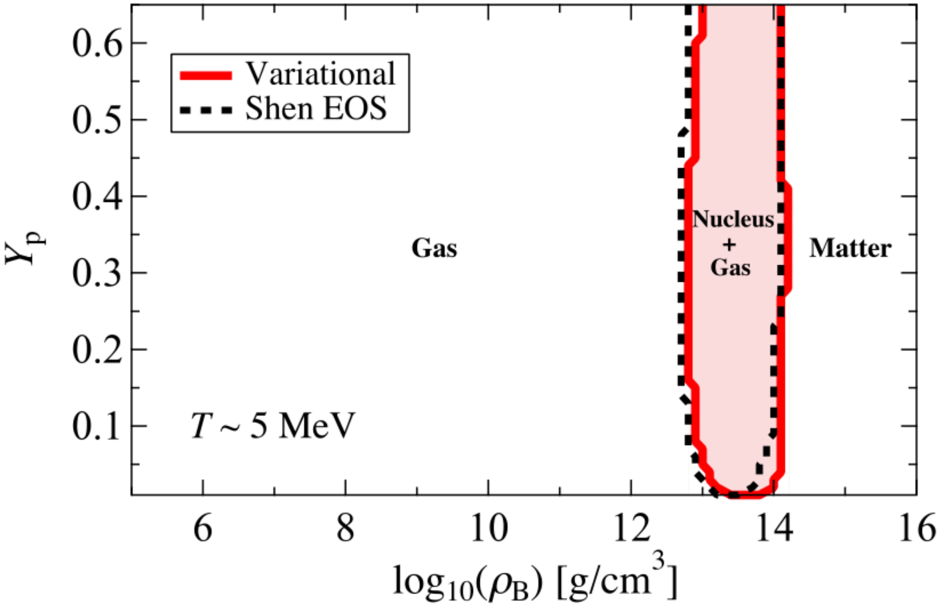
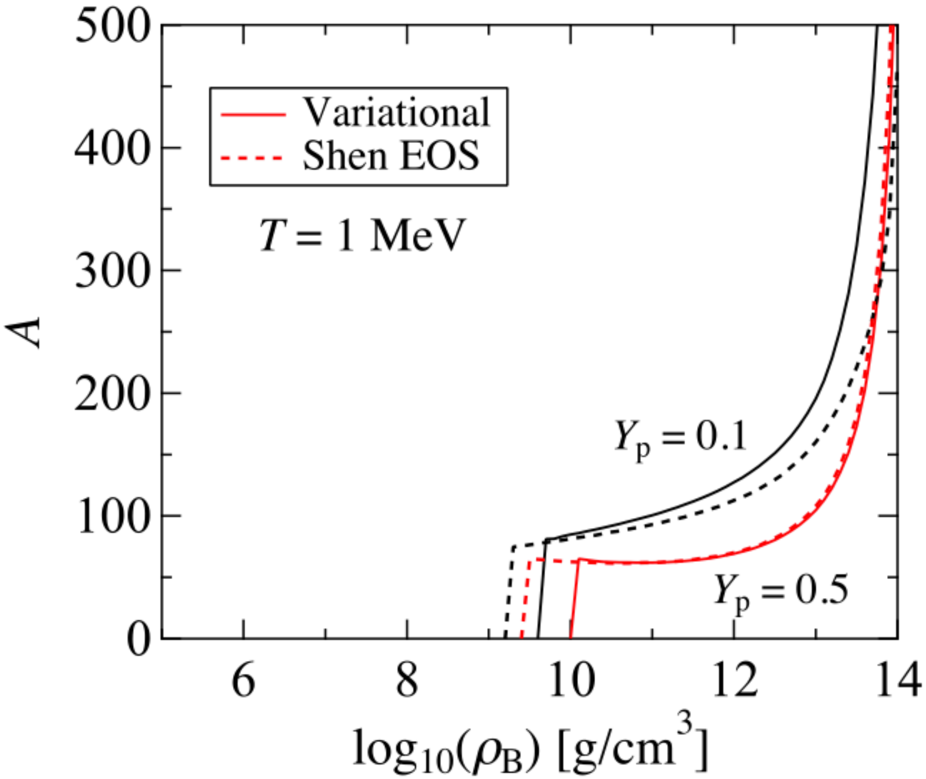
Phase Diagram and Mass Number





HT *et al.*, NPA 961 (2017) 78



→ Due to the smaller value of *L*

K. Oyamatsu & K. Iida PRC 75 (2007) 015801

Our EOS: $L = 35$ MeV

Shen EOS: $L = 111$ MeV

3. Systematic study for non-uniform matter

K. Oyamatsu & K. Iida PRC 75 (2007) 015801

Parameterized EOS for uniform matter at 0 MeV (energy per particle)

$$E(n_B, Y_p) = E_F + [1 - (1 - 2Y_p)^2] v_s(n_B) + (1 - 2Y_p)^2 v_n(n_B)$$

E_F: Kinetic energy per particle for Fermi-gas

Potential energy per particle for symmetric and neutron matter

$$v_s(n_B) = a_1 n_B + \frac{a_2 n_B^2}{1 + a_3 n_B}$$

$$v_n(n_B) = b_1 n_B + \frac{b_2 n_B^2}{1 + b_3 n_B}$$

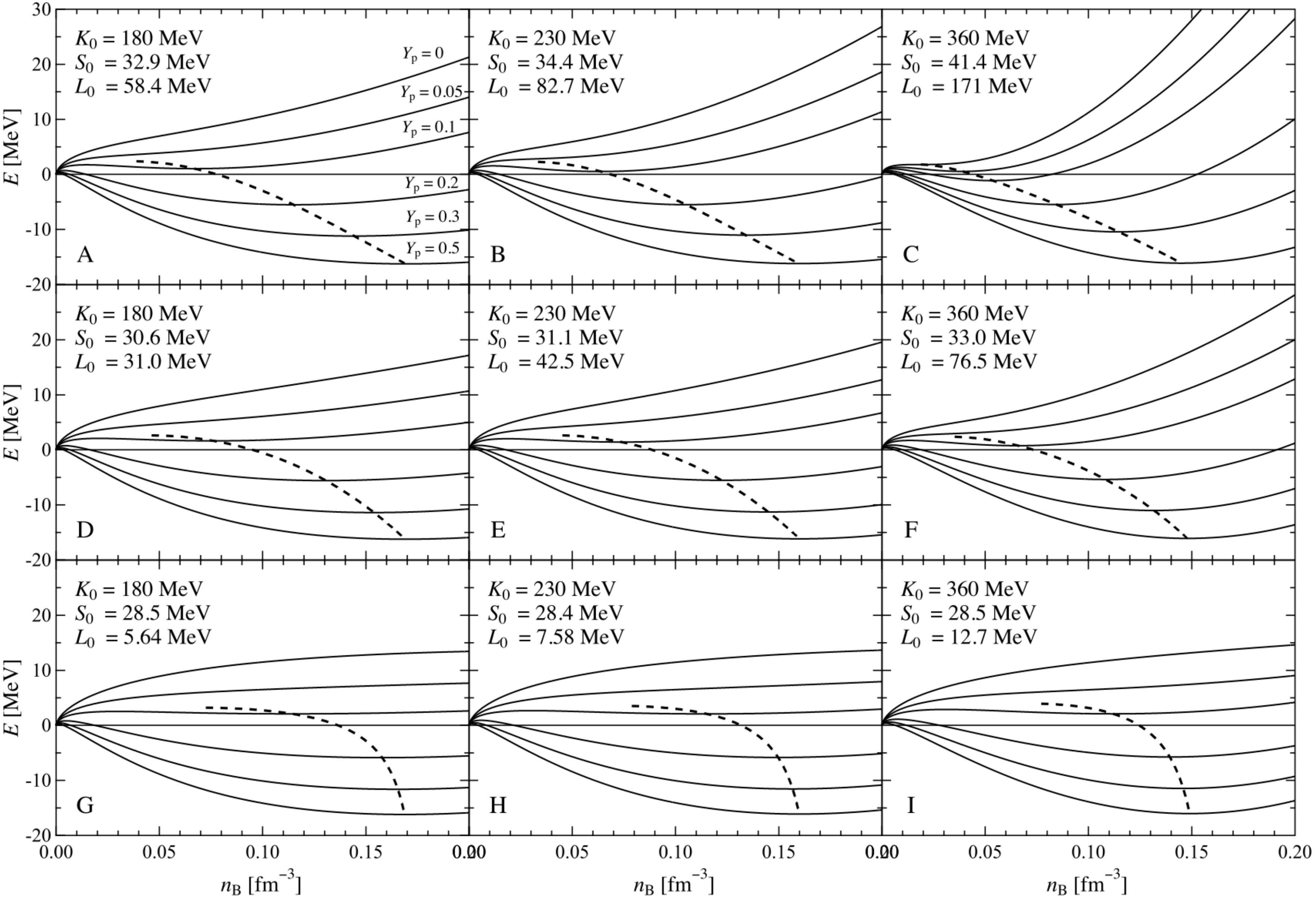
- Parameters (a_1, a_2, a_3, b_1, b_2, b_3) : Thomas-Fermi calculation for nuclei

Parameterized EOS for uniform matter (free energy per particle)

$$\boxed{F(n_B, Y_p, T) = F_F + [1 - (1 - 2Y_p)^2] v_s(n_B) + (1 - 2Y_p)^2 v_n(n_B)}$$

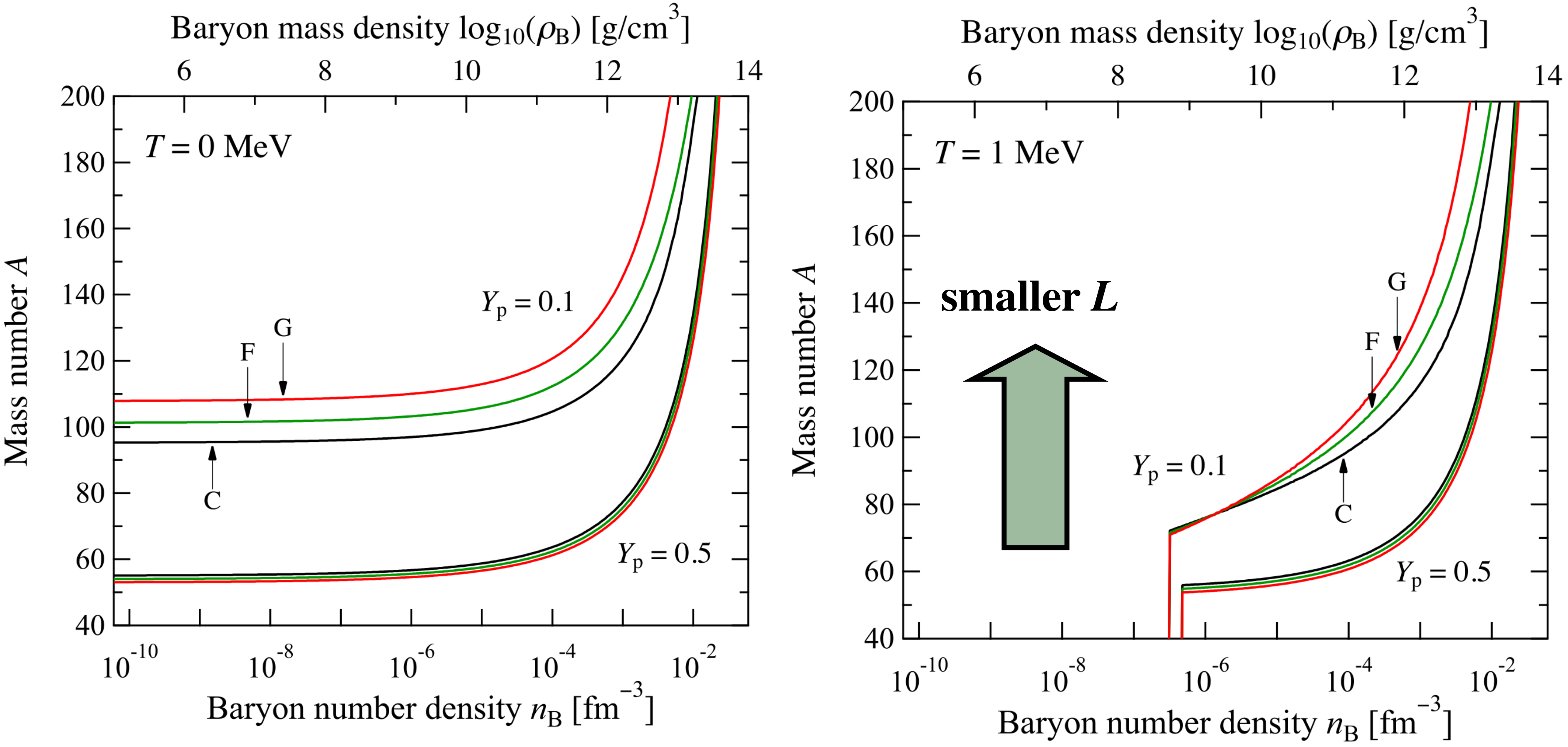
F_F: Free energy per particle for Fermi-gas

Parameterized EOS for uniform matter at 0 MeV



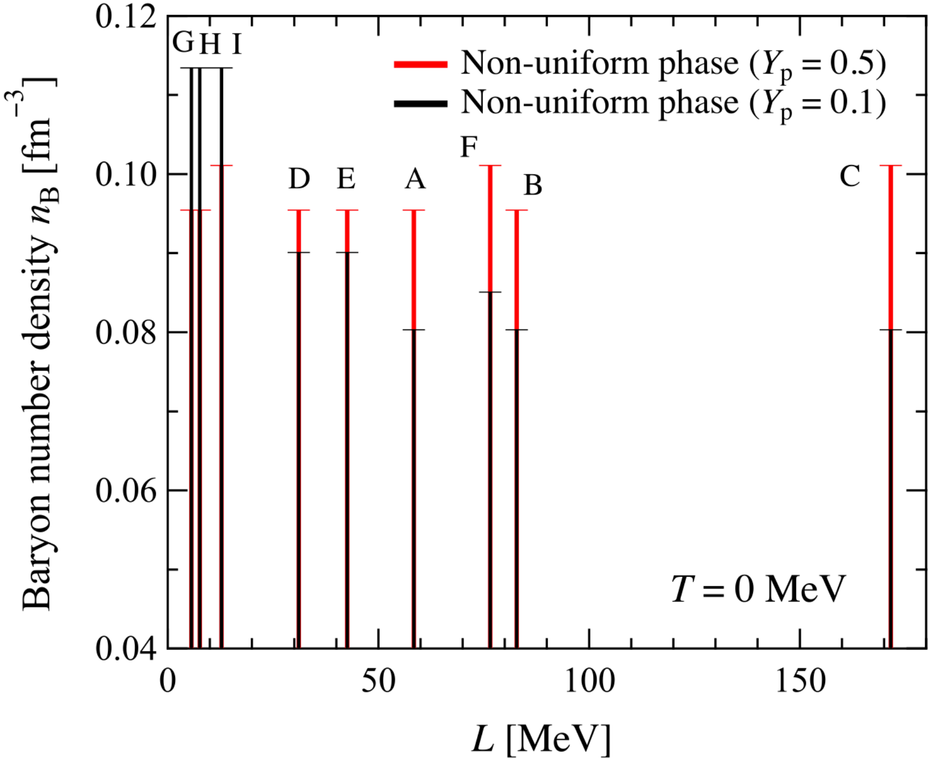
Thomas-Fermi calculations for non-uniform matter with these EOSs

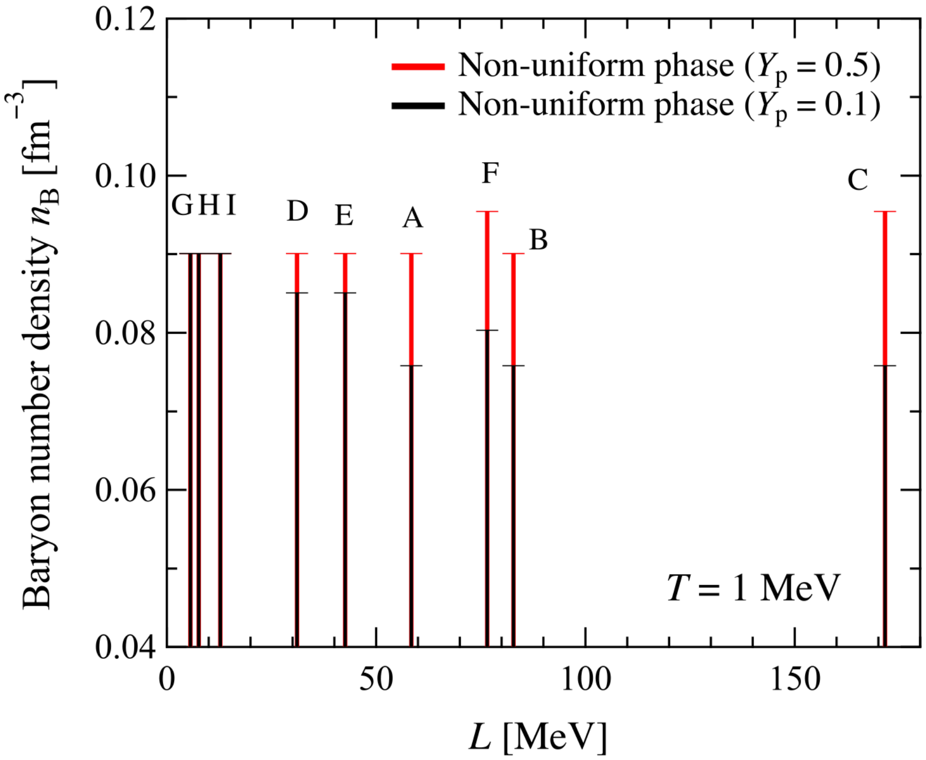
Mass Number of Nuclei in Supernova Matter



The smaller value of *L* → Larger mass number in neutron-rich matter

Critical density with respect to the phase transition





The smaller value of *L* → Higher critical density in neutron-rich matter

Summary

A new nuclear EOS for astrophysical simulations is constructed with realistic nuclear forces (AV18 + UIX).

- ***uniform nuclear matter : the cluster variational method***
- ***Non-uniform nuclear : the Thomas-Fermi calculation***

Due to the smaller value of *L*

- Masses of heavy nuclides are slightly larger in neutron-rich nuclear matter.
- The critical density from non-uniform matter to uniform matter is higher.

Our SN-EOS is available at

`http://www.np.phys.waseda.ac.jp/EOS/`

NSE model based on the variational EOS for uniform matter

`https://sites.google.com/site/furusawashun/eosdata`