Isospin asymmetry in perspective of QCD symmetry breaking

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Outline

I. QCD symmetry breaking

II. Hadron phase – QCD sum rules

- Symmetry energy
- Nucleon and hyperons
- Delta resonances
- III. Another pairing in cold dense limit
	- 2-color superconductivity

Parton in hadron

In Bjorken limit (large-momentum transferred region), there are no resonances \rightarrow the scattering can be approximated by point-like free particles (parton)

OPE (at $-q^2 \rightarrow \infty$, $-q^2/(2M(k))$ λ_0 = ζ) reproduces Bjorken scaling \rightarrow quantum number of hadron can be interpolated with explicit quark current in QCD

QCD symmetry breaking

• Classical symmetry

$$
\mathcal{L}_{\rm E}^{\rm QCD} = \frac{1}{2} \text{Tr} [F^{\mu\nu} F_{\mu\nu}] + \sum_{i} \bar{q}_i D q_i + \mathcal{L}_{\rm ghost} + \mathcal{L}_{\rm g.f.} + \cdots \qquad \text{U(1)} \times \frac{\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(3)_R \times \text{SU}(3)_R \times \text{SU}(4)_R}{\text{flavor}}
$$

After quantum correction

$$
\mathcal{L}_{\rm E}^{\rm QCD} = \frac{1}{2} \text{Tr} [F^{\mu\nu} F_{\mu\nu}] + \sum_{i} \bar{q}_{i} p q_{i} + \mathcal{L}_{\rm ghost} + \mathcal{L}_{\rm g.f.} + \frac{ig^{2}}{8\pi^{2}} \text{Tr} [F^{\mu\nu} \tilde{F}_{\mu\nu}] \theta + \cdots
$$
\n
$$
\partial^{\mu} j_{5\mu}^{a} = \partial^{\mu} (\bar{q} \gamma_{5} \gamma_{\mu} \tau^{a} q) = \frac{g^{2}}{8\pi^{2}} \text{Tr} [F^{\mu\nu} \tilde{F}_{\mu\nu}] \equiv \boxed{\nu = n_{L} - n_{R}} \left(\tilde{F}_{\mu\nu} \equiv \frac{\epsilon_{\mu\nu\alpha\beta}}{2!} F^{\alpha\beta} \right)
$$
\n
$$
\text{Axial charge is not conserved}
$$
\n
$$
\text{V=1}
$$

Axial charge is not conserved

For ν=1 configuration, classical solution (instanton) can be found as

 $A_\mu^{\text{inst.}a}(x) = \frac{2}{q} \frac{\eta_{a\mu\nu}(x-x_0)^\nu}{(x-x_0)^2 + \lambda^2}$, $O(4) \simeq SU(2) \times SU(2) \rightarrow$ trapped color already breaks symmetry which resides in closed Wilson loop $\exp \oint_{c} dx^{\mu} A_{\mu}(x)$, $F^{\mu\nu} = 0$ (zero curvature on the contour) (Belavin, Polyakov, Schwartz, Tyupkin, PLB59(1975)85)

QCD symmetry breaking

• Fermionic zero-modes in $v>0$ configuration

 $iD_{\text{inst.}}q = 0$ $(iD_{\text{inst.}})^2q = 0$ $(iD_{\text{inst.}})^2 q = 0$ leads $\left(-D_{\text{inst.}}^2 + \frac{1}{2} \sigma_{\mu\nu} F_{\text{inst.}}^{\mu\nu} \right) q_L = 0$ and $-D_{\text{inst.}}^2 q_R = 0$ $\left(-D_{\text{inst.}}^2 > 0 \right)$ different topological configuration measures different axial charge

• Θ vacuum

$$
\mathcal{Z}_{\theta} = \sum_{\nu} \exp\left[-\int d^4 x_E \mathcal{L}_{E,\nu}^{\text{QCD}}\right] \to \text{ all possible topological configuration contributes}
$$

helicity bases can be correlated via Instantons

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QCD Sum Rules - overview

• Correlator for baryon current

$$
\Pi(q) \equiv i \int d^4x \ e^{iqx} \langle \Psi_0 | \mathcal{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle
$$

= $\Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u)q + \Pi_u(q^2, q \cdot u)q$

Correlation of the quantum number contained in **n** q stands for external momentum u stands for medium velocity \rightarrow (1,0) in rest frame

• Energy dispersion relation and OPE (in **QCD degree of freedom**)

$$
\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials},
$$

• Phenomenological Ansatz (in hadronic degree of freedom)

 $\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^{\mu} - \tilde{\Sigma}_{v}^{\mu})\gamma_{\mu} - M_{N}^{*}}$

Mean-field type pole structure is adopted

• Weighting - Borel transformation

$$
\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n \to \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|)
$$

QCD SR - operator product expansion

• Operator product expansion (Example: 2-quark condensate diagram)

- Separation scale is set to be hadronic scale (\leq 1 GeV)
	- **Wilson coefficient** contains perturbative contribution above separation scale short-ranged partonic propagation in hadron
	- **Condensate** contains non-perturbative contribution below separation scale long ranged correlation in low energy part of hadron
	- Quark confinement inside hadron is low energy QCD phenomenon
	- Genuine properties of hadron are reflected in the condensates

Interpolating current for baryons

To obtain physical information

- a. Quasi-particle state will be extracted from the overlap
- b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
- c. Our object: N, P, $Λ$, Σ, and $Δ$ resonances

Constructing **proton** current

Required quantum number: $I = 1/2$, $J^P = (1/2)⁺$ Simplest structure: $[I = 0, I = 0$ di-quark structure] X [single quark with $I = 1/2, J = 1/2$] Di-quark structure: $\epsilon_{abc}(u_a^T C \gamma_5 d_b)$, $\epsilon_{abc}(u_a^T C d_b)$ u and d flavor in antisymmetric combination Positive parity matching: $\eta_1 = \epsilon_{abc} (u_a^T C d_b) \gamma_5 u_c$, $\eta_2 = \epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c$

$$
\eta_{p(t)} = 2\epsilon_{abc} \left([u_a^T C d_b] \gamma_5 u_c + t [u_a^T C \gamma_5 d_b] u_c \right)
$$

= $\left(\frac{1-t}{2} \right) \epsilon_{abc} [u_a^T C \gamma_\mu u_b] \gamma_5 \gamma^\mu d_c + \left(\frac{1+t}{4} \right) \epsilon_{abc} [u_a^T C \sigma_{\mu\nu} u_b] \gamma_5 \sigma^{\mu\nu} d_c$

Ioffe's choice (t=-1) \rightarrow chiral order parameter $\langle \bar{q}q \rangle \neq 0$ appears in the leading order of OPE

Interpolating current for baryons

• Current for **Δ resonances**

Requirement: spin-3/2, isospin-3/2 configuration

 $\left[\left(\frac{1}{2},0\right)\oplus \left(0,\frac{1}{2}\right)\right]^3=\left[\left(\frac{3}{2},0\right)\oplus \left(0,\frac{3}{2}\right)\right]+3\left[\left(\frac{1}{2},1\right)\oplus \left(1,\frac{1}{2}\right)\right]+\left[\left(I=\frac{1}{2}\right)\text{ representations}\right]$

For $\left(\frac{3}{2},0\right)\oplus\left(0,\frac{3}{2}\right)$ $\frac{3}{2}$) representation

 \rightarrow chiral condensate appears as $(\alpha_s)^n$ correction in OPE

For $\left(\frac{1}{2},1\right)\oplus\left(1,\frac{1}{2}\right)$ $\frac{1}{2}$) representation \rightarrow chiral condensate appears as the leading order in OPE

• Δ in Rarita-Schwinger field description $(q^{\mu}\psi_{\mu}^{(s)}(q) = 0, \gamma^{\mu}\psi_{\mu}^{(s)}(q) = 0)$ $S_{\mu\nu}^{3/2}(q) = \frac{1}{q-m_{\Delta}}\left(g_{\mu\nu} - \frac{1}{3}\gamma_{\mu}\gamma_{\nu} - \frac{2q_{\mu}q_{\nu}}{3a^2} - \frac{1}{3a^2}(q\gamma_{\mu}q_{\nu} + q_{\mu}\gamma_{\nu}q)\right) \Rightarrow \frac{\lambda^{*2}}{q-\sum_{\nu}q_{\nu}-m_{\Delta}-\sum_{\nu}}g_{\mu\nu} + \cdots$ quasiparticle pole extracted from $g_{\mu\nu}$ dependent term

 $s=3/2$, $l=3/2$ configuration can couple to π -N system the continuum contribution is subtracted by using $J_{\pi N}^{\mu}(x) = \epsilon^{\mu\alpha\nu\beta}\gamma_5\gamma_\alpha\partial_\nu\Psi_p(x)\partial_\beta\pi(x)$

Interpolating current for baryons

 \cdot \sum interpolating current

 $\eta_{\Sigma^{0}} = \epsilon_{abc} \left([u_a^T C s_b] \gamma_5 d_c + [d_a^T C s_b] \gamma_5 u_c + t \left([u_a^T C \gamma_5 s_b] d_c + [d_a^T C \gamma_5 s_b] u_c \right) \right)$ $= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C \gamma_\mu d_b]\gamma_5\gamma^\mu s_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C \sigma_{\mu\nu}d_b]\gamma_5\sigma^{\mu\nu}s_c.$

Two possible diquark $(I = 1)$ structure

$$
\epsilon_{abc}[u_a^T C \gamma_\mu d_b] \gamma_5 \gamma^\mu s_c = 2 \epsilon_{abc} \left([u_{R,a}^T C s_{R,b}] d_{L,c} + [d_{R,a}^T C s_{R,b}] u_{L,c} - (L \leftrightarrow R) \right)
$$

$$
\epsilon_{abc}[u_a^T C \sigma_{\mu\nu} d_b] \gamma_5 \sigma^{\mu\nu} s_c = 4 \epsilon_{abc} \left([u_{R,a}^T C s_{R,b}] d_{R,c} + [d_{R,a}^T C s_{R,b}] u_{R,c} - (L \leftrightarrow R) \right)
$$

• Special case Λ

Possible $I = 0$ combination in terms of spin-0 diquark structure

 $\{\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c, \ \epsilon_{abc}[u_a^T C \gamma_5 d_b] s_c, \ \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right), \ \epsilon_{abc}\left([u_a^T C \gamma_5 s_b] d_c - [d_a^T C \gamma_5 s_b] u_c\right)\}$

4th basis can be expressed in terms of the first two and 3rd basis \rightarrow basis set can be reduced to 3 independent bases set

General form of Λ interpolating field

$$
\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})} \epsilon_{abc} \left([u_a^T C d_b] \gamma_5 s_c + \tilde{a} [u_a^T C \gamma_5 d_b] s_c + \tilde{b} [u_a^T C \gamma_5 \gamma_\mu d_b] \gamma^\mu s_c \right)
$$

 $\tilde{a} \sim -0.2$ and $\tilde{b} \sim -0.2$ is determined by requiring stable Borel behavior

In-medium condensates

• Simplest guess: linear Fermi gas approximation

 $\langle \hat{O} \rangle_{\rho,I} = \langle \hat{O} \rangle_{\text{vac}} + \langle n | \hat{O} | n \rangle \rho_n + \langle p | \hat{O} | p \rangle \rho_p$ $= \langle \hat{O} \rangle_{\text{vac}} + \frac{1}{2} (\langle n | \hat{O} | n \rangle + \langle p | \hat{O} | p \rangle) \rho$ $+\frac{1}{2}(\langle n|\hat{O}|n\rangle - \langle p|\hat{O}|p\rangle)I\rho.$

[Vacuum condensate] + [nucleon expectation value] x [density] Iso-spin symmetric and asymmetric part

Example: chiral condensates

 $\langle \bar{q}q \rangle_{\rho} = \langle \bar{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_e} \rho$ Iso-spin symmetric part Nucleon-pion sigma term $\sigma_N = \frac{1}{3} \sum_{i=1}^{3} (\langle \tilde{N} | [Q_A^a, [Q_A^a, H_{QCD}]] | \tilde{N} \rangle - \langle 0 | [Q_A^a, [Q_A^a, H_{QCD}]] | 0 \rangle)$ $= 2 m_q \int d^3x \left(\langle \tilde{N} | \bar{q} q | \tilde{N} \rangle - \langle 0 | \bar{q} q | 0 \rangle \right) \equiv 2 m_q \langle N | \bar{q} q | N \rangle$ where, $H_{QCD} = \int d^3x (2m_q\bar{q}q + m_s\bar{s}s + \cdots)$ With Hellman-Feynman theorem

$$
2m_q\langle\psi|\bar{q}q|\psi\rangle = m_q \frac{d}{dm_q} \langle\psi|H_{QCD}|\psi\rangle
$$

and linear density approximation $\mathcal{E} \sim M_N \rho$

In-medium condensates

• Asymmetric part

From trace anomaly and heavy quark expansion

$$
T^{\mu}_{\ \mu} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum_{h=c,t,b} m_h \bar{h}h + \cdots
$$

$$
= \left(-\frac{9\alpha_s}{8\pi}\right)G^2 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + O(\mu^2/4m_h^2)
$$

Low-lying baryon octet mass relation

$$
m_p = A + m_u B_u + m_d B_d + m_s B_s
$$

\n
$$
m_n = A + m_u B_d + m_d B_u + m_s B_s
$$

\n
$$
m_{\Sigma^+} = A + m_u B_u + m_d B_s + m_s B_d
$$

\n
$$
m_{\Sigma^-} = A + m_u B_s + m_d B_u + m_s B_d
$$

\n
$$
m_{\Sigma^0} = A + m_u B_d + m_d B_s + m_s B_u
$$

\n
$$
m_{\Sigma^-} = A + m_u B_s + m_d B_d + m_s B_u
$$

\nwhere $A \equiv \langle (\bar{\beta}/4\alpha_s)G^2 \rangle_p$, $B_u \equiv \langle \bar{u}u \rangle_p$,

Strange contents

$$
\bar{s}s\rangle_{\rho} = \langle \bar{s}s\rangle_{\text{vac}} + \langle \bar{s}s\rangle_{N}\rho
$$

$$
= (0.8)\langle \bar{q}q\rangle_{\text{vac}} + y\frac{\sigma_{N}}{2m_{q}}\rho
$$

$$
y = \langle \bar{s}s\rangle_{N}/\langle \bar{q}q\rangle_{N}
$$

Ratio 0.8 is determined from vacuum sum rule for hyperon y can be determined from direct lattice QCD \rightarrow recent lattice results says γ should be small ^y~0.05 (PRD87 074503, PRD91 051502, PRD94 054503)

We confined $y \rightarrow 0.1$

Sum rule result I – Nucleons

• Neutron sum rules and symmetry energy

- 1. The quasi-**neutron** energy is slightly reduced \rightarrow represents bounding at $\rho = \rho_0$
- 2. Large cancelation mechanism in both of the quasi-neutron and the symmetry energy
- 3. Twist-4 matrix elements enhance the strength of cancelation mechanism

Sum rule result II – Λ hyperon

• A sum rules with new interpolating field

- 1. The quasi- Λ energy is slightly reduced \rightarrow represents bounding at normal nuclear density
- 2. Weak attraction and weak repulsion \rightarrow scalar: VsA / VsN ~ 0.31 vector: VvA / VvN ~ 0.26
	- \rightarrow naïve quark counting for determination of N-H force strength may not be good
- 3. Constant negative anti-**Λ** pole case (2nd graph) and density dependent case (3rd graph)

$$
\bar{E}_q = \Sigma_v(\bar{E}_q) - \sqrt{\bar{q}^2 + M^*(\bar{E}_q)^2}
$$
 (anti- Λ pole)

Sum rule result III – density behavior

• Comparison of density behavior (neutron matter)

- 1. Constant negative pole case: the quasi energy of **A** and **neutron** crosses at $\rho/\rho_0 = 1.8$
- 2. Density dependent case: never crosses
- 3. In Σ + sum rules, there is only small difference between constant- and density dependent-case

4. Within new interpolating field for Λ, the early onset of the hyperon in the dense nuclear matter is unlikely

Sum rule result IV – Δ resonance

• Δ^{++} in neutron matter (considering π -N continuum)

- 1. Negative mass shift in 100 MeV order (120 MeV in sym. matter, 150 MeV in neutron matter)
- 2. Current in $\left(\frac{1}{2},1\right)\bigoplus\left(1,\frac{1}{2}\right)$ $\frac{1}{2}$) representation does not strongly couple with π -N continuum
- 3. Weak isospin dependence \rightarrow comparing with quasi-neutron case, $x_{\rho} \equiv g_{\rho A}/g_{\rho N}$ ~0.13 \rightarrow it is very likely for early appearance of Δ resonances (PRC92.105802 (B. J. Cai et al.))

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- I. QCD symmetry breaking
- II. QCD approaches QCD sum rules
	- Symmetry energy
	- Nucleon and hyperons

III. Another pairing in cold dense limit

• 2-color superconductivity

At extremely low temperature

• At T~0 limit, quark is mainly confined near Fermi sea

If one scales longitudinal momentum to near Fermi surface $\int d^4p \rightarrow \mu_f^2 \int d\Omega \int dl^2 s^2$ where $l = (l_0, (\vec{l} \cdot \vec{v}_f) \vec{v}_f)$

Free fermion part should be invariant under scaling $\int d^2 l s^2 \psi_{\vec{v}_f}^{\dagger} s(l_0 - l_{\parallel}) \psi_{\vec{v}_f} \longrightarrow \psi \sim s^{-\frac{3}{2}}$

Four-quark interaction

General scattering

 $\int \Pi_{i}^{4} (dk_{\perp}^{2} dl^{2})_{i} [\psi^{\dagger}(k_{3})\psi(k_{1})V(k)\psi^{\dagger}(k_{4})\psi(k_{2})] \delta(k_{1} + k_{2} - (k_{3} + k_{4}))$

scales as s^2 : irrelevant in $s \to 0$ scaling

Interaction between opposite velocity (BCS type)

 $\int \Pi_{i}^{4} (dk_{\perp}^{2} dl^{2})_{i} [\psi^{\dagger}(k_{3})\psi(k_{1})V(k)\psi^{\dagger}(-k_{3})\psi(-k_{1})] \delta(l_{1} + l_{2} - (l_{3} + l_{4}))$

scales as s^0 : **marginal** in $s \to 0$ scaling

In QCD, there is no relevant interaction which scales as s^{-n} \rightarrow BCS type interaction becomes most important at scaling

Quasi-quark states in 2SC phase

• 2SC description in linear combination of Gellman matrices

Gapped (A=0,1,2,3) and un-gapped ($A=4,5$) quasi-state

 $\psi_{+,\alpha i} = \sum_{i=0}^{5} \frac{(\tilde{\lambda}_A)_{\alpha i}}{\sqrt{2}} \psi^A_+$ $\chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}$ + and – represents direction of Fermi velocity $\tilde{\lambda}_0 = \frac{1}{\sqrt{3}} \lambda_8 + \frac{2}{3} I; \ \tilde{\lambda}_A = \lambda_A (A = 1, 2, 3); \ \tilde{\lambda}_4 = \frac{1}{\sqrt{2}} (\lambda_4 - i \lambda_5); \ \tilde{\lambda}_5 = \frac{1}{\sqrt{2}} (\lambda_6 - i \lambda_7),$

These Hermitian representations $(\tilde{\lambda}_A)_{\alpha i}$ are color(α)-flavor(i) matrix

Color interaction can mediate transition of quasi-quark state

Paring pattern is quite similar with the chiral breaking via instantons

Some phenomenological anticipation

• Iso-spin distillation and π^{-}/π^{+} ratio (in agreement with PRD81 (2010) 094024)

Large symmetry energy leads iso-spin evaporation $E_{\text{sym}}^{\text{nuclear}}(\mu) \gg E_{\text{sym}}^{\text{quark}}(\mu)$ (*NL_Dδ* model and this calculation) \rightarrow Iso-spin distillation can occur at mixed phase At 2SC phase the distillation will be reduced Eventually, π^{-}/π^{+} ratio will be reduced

Further understanding and experimental observation is needed