Isospin asymmetry in perspective of QCD symmetry breaking

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Outline

I. QCD symmetry breaking

II. Hadron phase – QCD sum rules

- Symmetry energy
- Nucleon and hyperons
- Delta resonances
- III. Another pairing in cold dense limit
 - 2-color superconductivity

Parton in hadron



In Bjorken limit (large-momentum transferred region), there are no resonances \rightarrow the scattering can be approximated by point-like free particles (parton)

OPE (at $-q^2 \rightarrow \infty$, $-q^2/(2M(k'_0 - k_0)) = \xi$) reproduces Bjorken scaling \rightarrow quantum number of hadron can be interpolated with explicit quark current in QCD

QCD symmetry breaking

Classical symmetry

$$\mathcal{L}_{\rm E}^{\rm QCD} = \frac{1}{2} \operatorname{Tr}[F^{\mu\nu}F_{\mu\nu}] + \sum_{i} \bar{q}_{i} D q_{i} + \mathcal{L}_{\rm ghost} + \mathcal{L}_{\rm g.f.} + \cdots \qquad \mathrm{U}(1) \times \mathrm{SU}(2)_{L} \times \mathrm{SU}(2)_{R} \times \mathrm{SU}(3)_{R}$$
flavor

After quantum correction

For v=1 configuration, classical solution (instanton) can be found as

 $A^{\text{inst.}a}_{\mu}(x) = \frac{2}{q} \frac{\eta_{a\mu\nu}(x-x_0)^{\nu}}{(x-x_0)^2 + \lambda^2}$, $O(4) \simeq SU(2) \times SU(2) \rightarrow \text{trapped color already breaks symmetry}$ which resides in closed Wilson loop $\exp \oint dx^{\mu}A_{\mu}(x)$, $F^{\mu\nu} = 0$ (zero curvature on the contour) (Belavin, Polyakov, Schwartz, Tyupkin, PLB59(1975)85)

QCD symmetry breaking

Fermionic zero-modes in $\nu > 0$ configuration ۲

 $i \mathcal{D}_{\text{inst.}} q = 0$, $(i \mathcal{D}_{\text{inst.}})^2 q = 0$ $(i D_{\text{inst.}})^2 q = 0$ leads $\left(-D_{\text{inst.}}^2 + \frac{1}{2} \sigma_{\mu\nu} F_{\text{inst.}}^{\mu\nu} \right) q_L = 0$ and $-D_{\text{inst.}}^2 q_R = 0$ $(-D_{\text{inst.}}^2 > 0)$ different topological configuration measures different axial charge

 Θ vacuum •

$$\mathcal{Z}_{\theta} = \sum_{\nu} \exp\left[-\int d^4 x_E \mathcal{L}_{\mathrm{E},\nu}^{\mathrm{QCD}}\right] \rightarrow \text{ all possible topological configuration contributes}$$

helicity bases can be correlated via Instantons



color

For a given gauge configuration $\langle \bar{q}q \rangle \neq 0$ breaks chiral symmetry

 $U_V(1) \times \frac{U_A(1)}{V_A(1)} \times SU(2)_V \times \frac{SU(2)_A}{V_A} \times SU(3)$ flavor

hadron property can be described via the symmetry breaking pattern

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QCD Sum Rules - overview

• Correlator for baryon current

$$\Pi(q) \equiv i \int d^4x \ e^{iqx} \langle \Psi_0 | \mathbf{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$

= $\Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u) \not q + \Pi_u(q^2, q \cdot u) \not q$

Correlation of the quantum number contained in η stands for external momentum *u* stands for medium velocity \rightarrow (1,**0**) in rest frame

• Energy dispersion relation and OPE (in QCD degree of freedom)

 $\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials},$

• Phenomenological Ansatz (in hadronic degree of freedom)

 $\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$

Mean-field type pole structure is adopted

• Weighting - Borel transformation

$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n \to \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|)$$



QCD SR - operator product expansion

• Operator product expansion (Example: 2-quark condensate diagram)



- Separation scale is set to be hadronic scale (\leq 1 GeV)
 - Wilson coefficient contains perturbative contribution above separation scale short-ranged partonic propagation in hadron
 - Condensate contains non-perturbative contribution below separation scale long ranged correlation in low energy part of hadron
 - Quark confinement inside hadron is low energy QCD phenomenon
 - Genuine properties of hadron are reflected in the condensates

Interpolating current for baryons

• To obtain physical information



- a. Quasi-particle state will be extracted from the overlap
- b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
- c. Our object: N, P, Λ , Σ , and Δ resonances

Constructing proton current

Required quantum number: I = 1/2, $J^P = (1/2)^+$ Simplest structure: [I = 0, J = 0 di-quark structure] X [single quark with I = 1/2, J = 1/2] Di-quark structure: $\epsilon_{abc}(u_a^T C \gamma_5 d_b)$, $\epsilon_{abc}(u_a^T C d_b)$ u and d flavor in antisymmetric combination Positive parity matching: $\eta_1 = \epsilon_{abc}(u_a^T C d_b)\gamma_5 u_c$, $\eta_2 = \epsilon_{abc}(u_a^T C \gamma_5 d_b)u_c$

$$\eta_{p(t)} = 2\epsilon_{abc} \left([u_a^T C d_b] \gamma_5 u_c + t [u_a^T C \gamma_5 d_b] u_c \right) \\ = \left(\frac{1-t}{2} \right) \epsilon_{abc} [u_a^T C \gamma_\mu u_b] \gamma_5 \gamma^\mu d_c + \left(\frac{1+t}{4} \right) \epsilon_{abc} [u_a^T C \sigma_{\mu\nu} u_b] \gamma_5 \sigma^{\mu\nu} d_c$$

loffe's choice (t=-1) \rightarrow chiral order parameter $\langle \bar{q}q \rangle \neq 0$ appears in the leading order of OPE

Interpolating current for baryons

- Current for $\boldsymbol{\Delta}$ resonances

Requirement: spin-3/2, isospin-3/2 configuration

 $\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]^{3} = \left[\left(\frac{3}{2},0\right)\oplus\left(0,\frac{3}{2}\right)\right] + 3\left[\left(\frac{1}{2},1\right)\oplus\left(1,\frac{1}{2}\right)\right] + \left[\left(I = \frac{1}{2}\right) \text{ representations}\right]$

For $\left(\frac{3}{2},0\right) \oplus \left(0,\frac{3}{2}\right)$ representation $\eta_{\mu\nu}^{\Delta} \equiv (u^T C \sigma^{\alpha\beta} u) \sigma_{\alpha\beta} \sigma_{\mu\nu} u$

 \rightarrow chiral condensate appears as $(\alpha_s)^n$ correction in OPE

For $\left(\frac{1}{2},1\right) \bigoplus \left(1,\frac{1}{2}\right)$ representation $\eta_{\mu}^{(1)} \equiv (u^T C \sigma^{\alpha\beta} u) \sigma_{\alpha\beta} \gamma_{\mu} u = 4 \eta_{\mu}^{(2)} = 4 (u^T C \gamma_{\mu} u) u$ \rightarrow chiral condensate appears as the leading order in OPE

• Δ in Rarita-Schwinger field description $\left(q^{\mu}\psi_{\mu}^{(s)}(q)=0, \gamma^{\mu}\psi_{\mu}^{(s)}(q)=0\right)$ $S_{\mu\nu}^{3/2}(q) = \frac{1}{q-m_{\Delta}}\left(g_{\mu\nu}-\frac{1}{3}\gamma_{\mu}\gamma_{\nu}-\frac{2q_{\mu}q_{\nu}}{3q^{2}}-\frac{1}{3q^{2}}(q\gamma_{\mu}q_{\nu}+q_{\mu}\gamma_{\nu}q)\right) \Rightarrow \frac{\lambda^{*2}}{q-\Sigma_{\nu}q(-m_{\Delta}-\Sigma_{s})}g_{\mu\nu}+\cdots$

quasiparticle pole extracted from $g_{\mu\nu}$ dependent term

s=3/2, I=3/2 configuration can couple to π -*N* system the continuum contribution is subtracted by using $J^{\mu}_{\pi N}(x) = \epsilon^{\mu \alpha \nu \beta} \gamma_5 \gamma_{\alpha} \partial_{\nu} \Psi_p(x) \partial_{\beta} \pi(x)$

Interpolating current for baryons

• Σ interpolating current

 $\eta_{\Sigma^0} = \epsilon_{abc} \left([u_a^T C s_b] \gamma_5 d_c + [d_a^T C s_b] \gamma_5 u_c + t \left([u_a^T C \gamma_5 s_b] d_c + [d_a^T C \gamma_5 s_b] u_c \right) \right) \\ = \left(\frac{1-t}{2} \right) \epsilon_{abc} [u_a^T C \gamma_\mu d_b] \gamma_5 \gamma^\mu s_c + \left(\frac{1+t}{4} \right) \epsilon_{abc} [u_a^T C \sigma_{\mu\nu} d_b] \gamma_5 \sigma^{\mu\nu} s_c.$

Two possible diquark (I = 1) structure

$$\epsilon_{abc} [u_a^T C \gamma_\mu d_b] \gamma_5 \gamma^\mu s_c = 2\epsilon_{abc} \left([u_{R,a}^T C s_{R,b}] d_{L,c} + [d_{R,a}^T C s_{R,b}] u_{L,c} - (L \leftrightarrow R) \right)$$

$$\epsilon_{abc} [u_a^T C \sigma_{\mu\nu} d_b] \gamma_5 \sigma^{\mu\nu} s_c = 4\epsilon_{abc} \left([u_{R,a}^T C s_{R,b}] d_{R,c} + [d_{R,a}^T C s_{R,b}] u_{R,c} - (L \leftrightarrow R) \right)$$

Special case ∧

Possible I = 0 combination in terms of spin-0 diquark structure

 $\{\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c, \ \epsilon_{abc}[u_a^T C \gamma_5 d_b] s_c, \ \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right), \ \epsilon_{abc}\left([u_a^T C \gamma_5 s_b] d_c - [d_a^T C \gamma_5 s_b] u_c\right)\}$

 4^{th} basis can be expressed in terms of the first two and 3^{rd} basis \rightarrow basis set can be reduced to 3 independent bases set

General form of Λ interpolating field

$$\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})} \epsilon_{abc} \left([u_a^T C d_b] \gamma_5 s_c + \tilde{a} [u_a^T C \gamma_5 d_b] s_c + \tilde{b} [u_a^T C \gamma_5 \gamma_\mu d_b] \gamma^\mu s_c \right)$$

 \tilde{a} ~ - 0.2 and \tilde{b} ~ - 0.2 is determined by requiring stable Borel behavior

In-medium condensates

• Simplest guess: linear Fermi gas approximation

 $\langle \bar{q}q \rangle_{\rho} = \langle \bar{q}q \rangle_{\rm vac} + \frac{\sigma_N}{2m_a}\rho$

$$\begin{split} \langle \hat{O} \rangle_{\rho,I} &= \langle \hat{O} \rangle_{\text{vac}} + \langle n | \hat{O} | n \rangle \rho_n + \langle p | \hat{O} | p \rangle \rho_p \\ &= \frac{\langle \hat{O} \rangle_{\text{vac}} + \frac{1}{2} (\langle n | \hat{O} | n \rangle + \langle p | \hat{O} | p \rangle) \rho}{+ \frac{1}{2} (\langle n | \hat{O} | n \rangle - \langle p | \hat{O} | p \rangle) I \rho. \end{split}$$

[Vacuum condensate] + [nucleon expectation value] **x** [density] Iso-spin symmetric and asymmetric part

• Example: chiral condensates

Iso-spin symmetric part Nucleon-pion sigma term

 $\sigma_N = \frac{1}{3} \sum_{a=1}^3 \left(\langle \tilde{N} | [\mathcal{Q}_A^a, [\mathcal{Q}_A^a, H_{QCD}]] | \tilde{N} \rangle - \langle 0 | [\mathcal{Q}_A^a, [\mathcal{Q}_A^a, H_{QCD}]] | 0 \rangle \right)$

$$= 2m_q \int d^3x \left(\langle \tilde{N} | \bar{q}q | \tilde{N} \rangle - \langle 0 | \bar{q}q | 0 \rangle \right) \equiv 2m_q \langle N | \bar{q}q | N \rangle$$

where,
$$H_{QCD} = \int d^3x (2m_q \bar{q}q + m_s \bar{s}s + \cdots)$$

With Hellman-Feynman theorem

$$2m_q \langle \psi | \bar{q}q | \psi \rangle = m_q \frac{d}{dm_q} \langle \psi | H_{QCD} | \psi \rangle$$

and linear density approximation $\mathcal{E} \sim M_N \rho$



In-medium condensates

• Asymmetric part

From trace anomaly and heavy quark expansion

$$T^{\mu}_{\ \mu} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum_{h=c,t,b} m_h \bar{h}h + \cdots$$
$$= \left(-\frac{9\alpha_s}{8\pi}\right)G^2 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + O(\mu^2/4m_h^2)$$

Low-lying baryon octet mass relation

$$\begin{split} m_p &= A + m_u B_u + m_d B_d + m_s B_s \\ m_n &= A + m_u B_d + m_d B_u + m_s B_s \\ m_{\Sigma^+} &= A + m_u B_u + m_d B_s + m_s B_d \\ m_{\Sigma^-} &= A + m_u B_s + m_d B_u + m_s B_d \\ m_{\Xi^0} &= A + m_u B_d + m_d B_s + m_s B_u \\ m_{\Xi^-} &= A + m_u B_s + m_d B_d + m_s B_u \\ \end{split}$$
where $A \equiv \langle (\bar{\beta}/4\alpha_s)G^2 \rangle_p, \ B_u \equiv \langle \bar{u}u \rangle_p,$

$$xpansion$$

$$m_{h}\bar{h}h + \cdots$$

$$m_{s}\bar{s}s + O(\mu^{2}/4m_{h}^{2})$$

$$\Rightarrow \frac{1}{2} (\langle p|\bar{u}u|p \rangle - \langle p|\bar{d}d|p \rangle) = \frac{1}{2} \left(\frac{(m_{\Xi^{0}} + m_{\Xi^{-}}) - (m_{\Sigma^{+}} + m_{\Sigma^{-}})}{2m_{s} - (m_{u} + m_{d})} \right)$$

• Strange contents

$$\begin{split} \bar{s}s\rangle_{\rho} &= \langle \bar{s}s \rangle_{\text{vac}} + \langle \bar{s}s \rangle_{N}\rho \\ &= (0.8)\langle \bar{q}q \rangle_{\text{vac}} + y \frac{\sigma_{N}}{2m_{q}}\rho \\ y &= \langle \bar{s}s \rangle_{N} / \langle \bar{q}q \rangle_{N} \end{split}$$

Ratio **0.8** is determined from vacuum sum rule for hyperon y can be determined from direct lattice QCD \rightarrow recent lattice results says y should be small

y~0.05 (PRD87 074503, PRD91 051502, PRD94 054503)

We confined $y \rightarrow 0.1$

Sum rule result I – Nucleons

• **Neutron** sum rules and symmetry energy



- 1. The quasi-**neutron** energy is slightly reduced \rightarrow represents bounding at $\rho = \rho_0$
- 2. Large cancelation mechanism in both of the quasi-neutron and the symmetry energy
- 3. Twist-4 matrix elements enhance the strength of cancelation mechanism

Sum rule result II − ∧ hyperon

• Λ sum rules with new interpolating field



- 1. The quasi- Λ energy is slightly reduced \rightarrow represents bounding at normal nuclear density
- 2. Weak attraction and weak repulsion \rightarrow scalar: Vsn / Vsn ~ 0.31 vector: Vvn / Vvn ~ 0.26
 - \rightarrow naïve quark counting for determination of N-H force strength may not be good
- 3. Constant negative anti- Λ pole case (2nd graph) and density dependent case (3rd graph)

$$\bar{E}_q = \Sigma_v(\bar{E}_q) - \sqrt{\bar{q}^2 + M^*(\bar{E}_q)^2}$$
 (anti- Λ pole)

Sum rule result III – density behavior

• Comparison of density behavior (neutron matter)



- 1. Constant negative pole case: the quasi energy of Λ and **neutron** crosses at $\rho/\rho_0 = 1.8$
- 2. Density dependent case: never crosses
- 3. In Σ + sum rules, there is only small difference between constant- and density dependent-case
- 4. Within new interpolating field for Λ , the early onset of the hyperon in the dense nuclear matter is unlikely

Sum rule result IV – Δ resonance

• Δ^{++} in neutron matter (considering π -N continuum)



- 1. Negative mass shift in 100 MeV order (120 MeV in sym. matter, 150 MeV in neutron matter)
- 2. Current in $\left(\frac{1}{2}, 1\right) \oplus \left(1, \frac{1}{2}\right)$ representation does not strongly couple with π -N continuum
- 3. Weak isospin dependence \rightarrow comparing with quasi-neutron case, $x_{\rho} \equiv g_{\rho\Delta}/g_{\rho N} \sim 0.13$ \rightarrow it is very likely for early appearance of Δ resonances (PRC92.105802 (B. J. Cai et al.))

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- II. QCD approaches QCD sum rules
 - Symmetry energy
 - Nucleon and hyperons

III. Another pairing in cold dense limit

• 2-color superconductivity

At extremely low temperature

• At T~0 limit, quark is mainly confined near Fermi sea







If one scales longitudinal momentum to near Fermi surface $\int d^4p \to \mu_f^2 \int d\Omega \int dl^2 s^2 \quad \text{where} \quad l = (l_0, (\vec{l} \cdot \vec{v}_f) \vec{v}_f)$ Free fermion part should be invariant under scaling

 $\int d^2 l s^2 \psi_{\vec{v}_f}^{\dagger} s(l_0 - l_{\parallel}) \psi_{\vec{v}_f} \quad \rightarrow \quad \psi \sim s^{-\frac{3}{2}}$

Four-quark interaction

General scattering

 $\int \Pi_i^4 \left(dk_\perp^2 dl^2 \right)_i \left[\psi^{\dagger}(k_3) \psi(k_1) V(k) \psi^{\dagger}(k_4) \psi(k_2) \right] \delta(k_1 + k_2 - (k_3 + k_4))$

scales as s^2 : irrelevant in $s \rightarrow 0$ scaling

Interaction between opposite velocity (BCS type)

 $\int \Pi_i^4 \left(dk_{\perp}^2 dl^2 \right)_i \left[\psi^{\dagger}(k_3) \psi(k_1) V(k) \psi^{\dagger}(-k_3) \psi(-k_1) \right] \delta(l_1 + l_2 - (l_3 + l_4))$

scales as s^0 : marginal in $s \to 0$ scaling

In QCD, there is no relevant interaction which scales as $s^{-n} \rightarrow BCS$ type interaction becomes most important at scaling

Quasi-quark states in **2SC phase**

• **2SC** description in linear combination of Gellman matrices

Gapped (A=0,1,2,3) and un-gapped (A=4,5) quasi-state

 $\psi_{+,\alpha i} = \sum_{A=0}^{5} \frac{(\tilde{\lambda}_{A})_{\alpha i}}{\sqrt{2}} \psi_{+}^{A} \qquad \chi = \begin{pmatrix} \psi_{+} \\ C\psi_{-}^{*} \end{pmatrix} + \text{ and } - \text{ represents direction of Fermi velocity}$ $\tilde{\lambda}_{0} = \frac{1}{\sqrt{3}} \lambda_{8} + \frac{2}{3} I; \quad \tilde{\lambda}_{A} = \lambda_{A} \ (A = 1, 2, 3); \quad \tilde{\lambda}_{4} = \frac{1}{\sqrt{2}} (\lambda_{4} - i\lambda_{5}); \quad \tilde{\lambda}_{5} = \frac{1}{\sqrt{2}} (\lambda_{6} - i\lambda_{7}),$

These Hermitian representations $(\tilde{\lambda}_A)_{\alpha i}$ are color(α)-flavor(i) matrix

Color interaction can mediate transition of quasi-quark state



Paring pattern is quite similar with the chiral breaking via instantons

Some phenomenological anticipation

• Iso-spin distillation and π^-/π^+ ratio (in agreement with PRD81 (2010) 094024)





Large symmetry energy leads iso-spin evaporation $E_{\rm sym}^{\rm nuclear}(\mu) \gg E_{\rm sym}^{\rm quark}(\mu) \ (NL\rho\delta \text{ model and this calculation})$ \rightarrow Iso-spin distillation can occur at mixed phase At **2SC phase** the distillation will be **reduced** Eventually, π^-/π^+ ratio will be **reduced**

Further understanding and experimental observation is needed