Correlated Structure of Nuclear Symmetry Energy from Covariant Nucleon Self-Energy

Bao Yuan SUN 孙保元[1]

School of Nuclear Science and Technology, Lanzhou University,
Lanzhou 730000, Gansu Province, P. R. China

NuSYM2018
13 September 2018 @ Busan

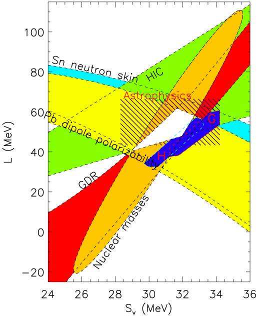
[1]E-mail address: sunby@lzu.edu.cn & sunbaoyuan@gmail.com

Symmetry Energy in Nuclear Matter

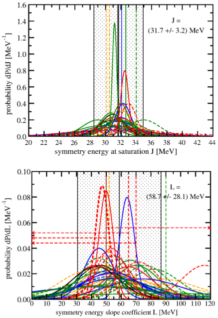
- Equation of state for isospin asymmetric nuclear matter

$$E_b(\rho_b,\delta) = E_0(\rho_b) + E_S(\rho_b)\delta^2 + S_4(\rho_b)\delta^4 + \mathcal{O}(4), \quad L = 3\rho_0 \left.\frac{\partial E_S(\rho_b)}{\partial \rho_b}\right|_{\rho_b=\rho_0},$$

- Important to understand
  - nuclear structure: fission properties, density distribution, collective excitation, ...
  - nuclear astrophysics: NS's radius, crust-core transition density, cooling rate, ...
  - heavy ion reactions: isospin diffusion, DR(n/p), ...



☼ *Lattimer:ApJ2013*



☼ *Oertel:RMP2017*

To improve the nuclear many-body models, information of symmetry energy is essential

New interest in studying ingredients of effective nuclear force: momentum-dependence, exchange term, tensor force, short range correlation, ...

Symmetry Energy Studied in CDF Theory

Covariant density functional (CDF) theory:

☼ *Walecka (1974), Serot (1986), Reihard (1989), Ring (1996), Bender (2003), Meng (2006), Liang (2015)*

- spin-orbit coupling
- pseudo-spin symmetry
- consistent treatment of time-odd fields
- connection to QCD

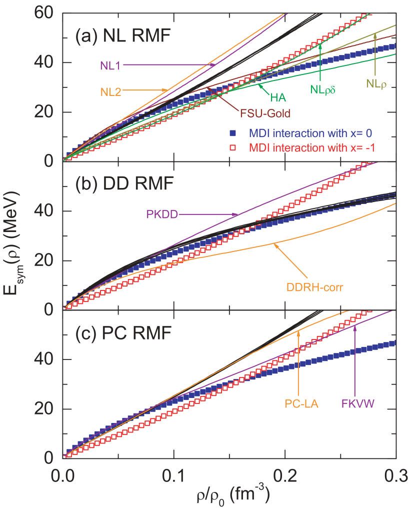
Relativistic Hartree-Fock (RHF) theory:

☼ *Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2006-2010)*

- nonlocal Fock terms
- π-PV and ρ-T couplings
- tensor force involved naturally

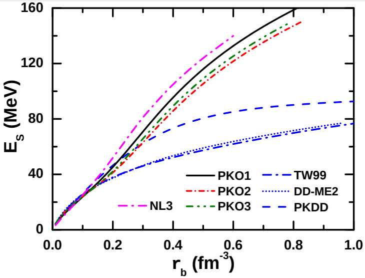
Improved isospin related structure descriptions

☼ *W.H.Long:PLB2006, H.Z.Liang:PRL2008, Q.Zhao:JPG2015*



☼ *L. W. Chen, C. M. Ko and B. A. Li, Phys. Rev. C* ***76****, 054316 (2007).*

Impact of Fock Terms on Nuclear Symmetry Energy



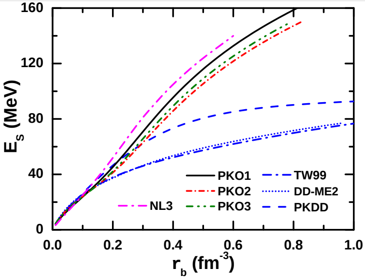
Not only the ρ meson but all the mesons take part in the isospin properties in the DDRHF theory

→ In charge of producing symmetry energy via Fock channel

→ Significant contributions from isoscalar σ and ω exchange diagram to the symmetry energy

☼ *BYS, W.H. Long, J. Meng, and U. Lombardo, PRC* **78**, *065805 (2008).*

Impact of Fock Terms on Nuclear Symmetry Energy



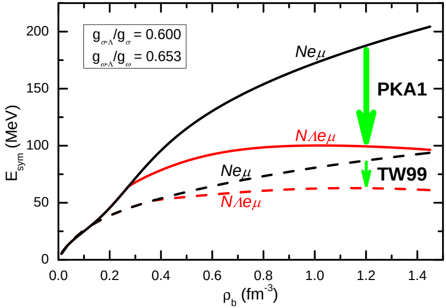
Not only the ρ meson but all the mesons take part in the isospin properties in the DDRHF theory

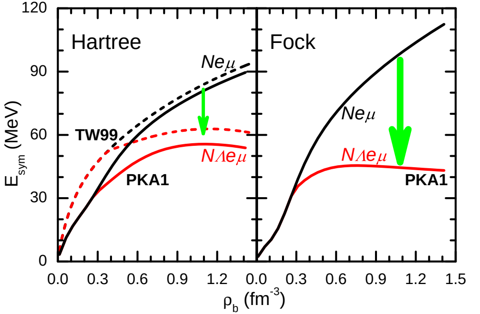
→ In charge of producing symmetry energy via Fock channel
→ Significant contributions from isoscalar σ and ω exchange diagram to the symmetry energy

☼ *BYS, W.H. Long, J. Meng, and U. Lombardo, PRC* **78**, *065805 (2008).*

- Hyperon effects: extra E_{sym} softening due to Fock terms

☼ *W. H. Long, BYS, K. Hagino et al, PRC* **85**, *025806 (2012); BYS, Science China PMA* **46**, *012018 (2016).*



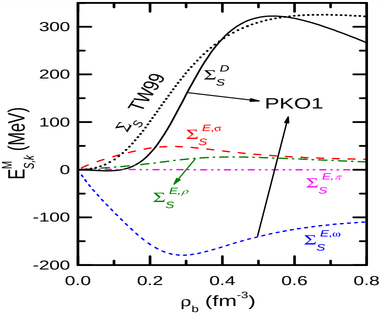


Impact of Fock Terms on Nuclear Symmetry Energy

- Kinetic and potential symmetry energy:
  - ☼ *Q. Zhao, BYS, W. H. Long, J. Phys. G* **42**, *095101 (2015).*
  - ☼ *BYS et al., EPJConf* **117**, *07011 (2016).*

		TW99	PKDD	PKO1	PKA1	BHF
J	$T=0$	51.0	50.8	38.8	42.4	44.2
	$T=1$	-26.2	-22.1	-8.1	-5.7	-9.0
	kin	8.0	8.1	3.7	0.5	-1.0

☼ *I. Vidaña et al., PRC* **84**, *062801(R) (2011).*



Impact of Fock Terms on Nuclear Symmetry Energy

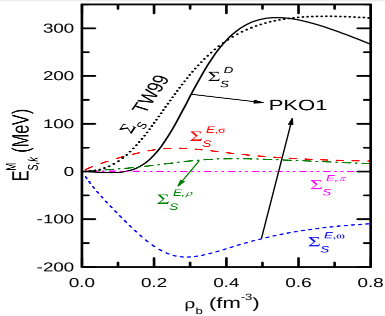
- Kinetic and potential symmetry energy:

  ☼ *Q. Zhao, BYS, W. H. Long, J. Phys. G* **42**, *095101 (2015).*

  ☼ *BYS et al., EPJConf* **117**, *07011 (2016).*

		TW99	PKDD	PKO1	PKA1	BHF
J	$T=0$	51.0	50.8	38.8	42.4	44.2
	$T=1$	-26.2	-22.1	-8.1	-5.7	-9.0
	kin	8.0	8.1	3.7	0.5	-1.0

☼ *I. Vidaña et al., PRC* **84**, *062801(R) (2011).*

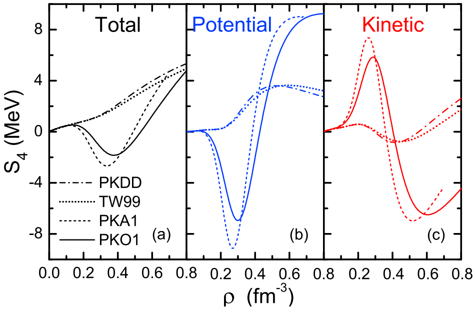


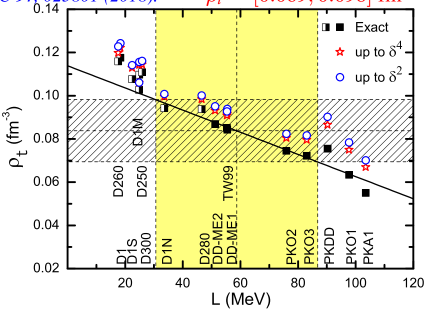
- Nuclear fourth-order symmetry energy:

  S_4 suppressed in RHF, but $S_{4,\mathrm{kin}}$ enhanced at ρ_0

  ☼ *Z.W.iu, Z.Qian, R.Y.Xing, J.R.Niu, and BYS, Phys. Rev. C* **97**, *025801 (2018).*

$\rho_t \sim [0.069, 0.098]$ fm^{-3}



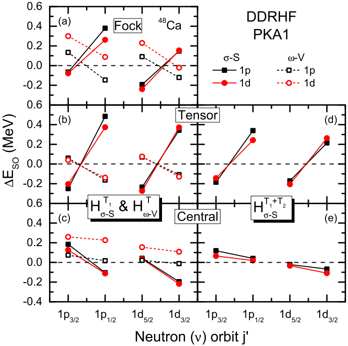


Nuclear Tensor Interaction: Relativistic Formalism

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, BYS, W. H. Long, et al., PRC* **91***, 034326 (2015).*

☼ *Y. Y. Zong and BYS, Chin. Phys. C* **42***, 024101 (2018).*



Second-Order Irreducible Tensor S_{12} for π-PV, σ-S:

$$S_{12} = 3\,(\gamma_0\boldsymbol{\Sigma}_1\cdot\boldsymbol{q})\,(\gamma_0\boldsymbol{\Sigma}_2\cdot\boldsymbol{q}) - (\gamma_0\boldsymbol{\Sigma}_1)\cdot(\gamma_0\boldsymbol{\Sigma}_2)\,\boldsymbol{q}^2$$

→ The tensors are involved naturally by the Fock diagrams and quantified by the relativistic formalism without introducing any additional free parameters.

Two evidence: Spin dependence, Tensor Sum Rule

Nuclear Tensor Interaction: Relativistic Formalism

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, BYS, W. H. Long, et al., PRC* **91***, 034326 (2015).*

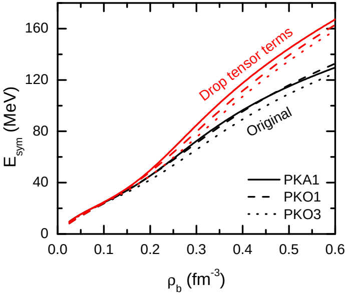
☼ *Y. Y. Zong and BYS, Chin. Phys. C* **42***, 024101 (2018).*

Second-Order Irreducible Tensor S_{12} for π-PV, σ-S:

$$S_{12} = 3\left(\gamma_0 \boldsymbol{\Sigma}_1 \cdot \boldsymbol{q}\right)\left(\gamma_0 \boldsymbol{\Sigma}_2 \cdot \boldsymbol{q}\right) - \left(\gamma_0 \boldsymbol{\Sigma}_1\right) \cdot \left(\gamma_0 \boldsymbol{\Sigma}_2\right) \boldsymbol{q}^2$$

→ The tensors are involved naturally by the Fock diagrams and quantified by the relativistic formalism without introducing any additional free parameters.

Two evidence: Spin dependence, Tensor Sum Rule



☼ *L. J. Jiang et al., PRC* **91***, 025802 (2015).*

Effects of Fock terms to E_S: Soften due to tensor part; Stiffen due to central part

Nuclear Tensor Interaction: Relativistic Formalism

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, BYS, W. H. Long, et al., PRC* **91***, 034326 (2015).*

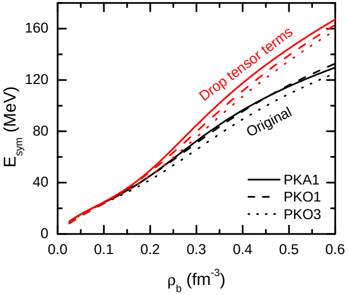
☼ *Y. Y. Zong and BYS, Chin. Phys. C* **42***, 024101 (2018).*

Second-Order Irreducible Tensor S_{12} for π-PV, σ-S:

$$S_{12} = 3\,(\gamma_0 \mathbf{\Sigma}_1 \cdot \boldsymbol{q})\,(\gamma_0 \mathbf{\Sigma}_2 \cdot \boldsymbol{q}) - (\gamma_0 \mathbf{\Sigma}_1) \cdot (\gamma_0 \mathbf{\Sigma}_2)\, \boldsymbol{q}^2$$

→ The tensors are involved naturally by the Fock diagrams and quantified by the relativistic formalism without introducing any additional free parameters.

Two evidence: Spin dependence, Tensor Sum Rule



☼ *L. J. Jiang et al., PRC* **91***, 025802 (2015).*

Effects of Fock terms to E_S: Soften due to tensor part; Stiffen due to central part

Tensor force effects on $E_{S,\mathrm{kin}}$:

Short Range Correlation?

		TW99	PKDD	PKO1	PKA1	BHF
J	kin	8.0	8.1	3.7	0.5	-1.0
	kin-T			-7.3	-9.7	

☼ *C. Xu and B. A. Li, PRC* **81***, 064612 (2010).*

☼ *I. Vidaña et al., PRC* **84***, 062801(R) (2011).*

☼ *Or Hen et al., PRC* **91***, 025803 (2015).* -10±7.5 MeV

☼ *B. J. Cai and B. A. Li, PRC* **93***, 014619 (2016).* -16.94±13.66 MeV

Properties of E_S at Saturation Density: J and L

Methods to analyse the structure of symmetry energy:

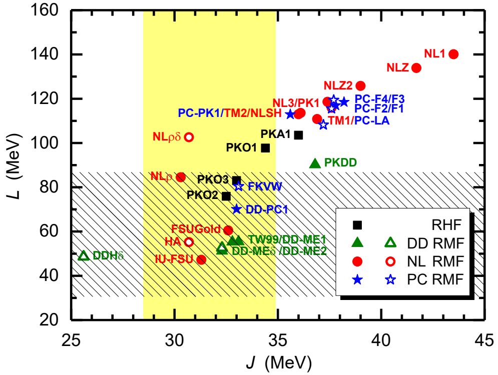
- In terms of energy density functional: kinetic and potential, spin-isospin ✓
- In terms of components of nuclear force: central and tensor part ✓
- In terms of single-particle energy: self-energy ×

Properties of E_S at Saturation Density: J and L

Methods to analyse the structure of symmetry energy:

- In terms of energy density functional: kinetic and potential, spin-isospin ✓
- In terms of components of nuclear force: central and tensor part ✓
- In terms of single-particle energy: self-energy ×

Properties of E_S at Saturation Density in CDF theory:



From a covariant nucleon self-energy, try to understand:

- Origin of model dependence of L
- Correlations between L and J

☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

Hugenholtz-Van Hove (HVH) Theorem

- Hugenholtz-Van Hove (HVH) theorem ☼ *N. M. Hugenholtz, L. Van Hove, Phys.* **24** *(1958) 363.*
  Relations between binding energy per nucleon and single-nucleon Fermi energy
- Symmetry Energy from HVH theorem ☼ *B. J. Cai, L. W. Chen, Phys. Lett. B* **711** *(2012) 104.*

$$E_b + \rho_b \frac{\partial E_b}{\partial \rho_b} = \varepsilon_F \xrightarrow[\text{to asymmetry}]{\text{expansion}} E_S(\rho_b) = \frac{1}{4}\frac{d}{d\delta}\left[\sum_\tau \tau \varepsilon_F^\tau(\rho_b, \delta, k_F^\tau)\right]\Bigg|_{\delta=0}$$

$$\frac{d\varepsilon_F^\tau}{d\delta}\Big|_{\delta=0} = \frac{\tau k_F}{3}\left(\frac{\partial \varepsilon}{\partial k}\right)\Bigg|_{k=k_F} + \frac{\partial \varepsilon_F^\tau}{\partial \delta}\Bigg|_{\delta=0} \quad \Longrightarrow E_S(\rho_b) = \boxed{\overset{\partial k}{E_S^{\rm kin}(\rho_b) + E_S^{\rm mon}(\rho_b)}} + \boxed{\overset{\partial \delta}{E_S^{\rm 1st}(\rho_b)}}$$

- Kinetic part, k-dependence of self-energy, δ-dependence of self-energy

$$E_S^{\rm kin}(\rho_b) = \frac{k_F k_F^*}{6E_F^*}, \qquad E_S^{\rm 1st}(\rho_b) = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_b + E_S^{1st,E}$$

$$E_S^{\rm mon}(\rho_b) = \frac{k_F k_F^*}{6E_F^*}\frac{\partial \Sigma_V}{\partial k}\Big|_{k=k_F} + \frac{k_F M_F^*}{6E_F^*}\frac{\partial \Sigma_S}{\partial k}\Big|_{k=k_F} + \frac{k_F}{6}\frac{\partial \Sigma_0}{\partial k}\Big|_{k=k_F},$$

- Symmetry energy at saturation density

$$J = J^{\rm kin} + J^{\rm mom} + J^{\rm 1st}$$

$$L = L^{\rm kin} + L^{\rm mom} + L^{\rm 1st} + L^{\rm cross} + L^{\rm 2nd}$$

J and L Expressed by Nucleon Self-Energy

$$J^{\rm kin} = \frac{k_F}{6}\frac{k_F^*}{\varepsilon_F^*}, \tag{1}$$

$$L^{\rm kin} = \frac{3}{2}J^{\rm kin} + \frac{k_F}{6}\left[\frac{M_F^{*2}}{\varepsilon_F^{*2}}\frac{k_F}{\varepsilon_F^*} - \frac{1}{2}\frac{k_F^*}{\varepsilon_F^*}\right], \tag{2}$$

$$J^{\rm mom} = \frac{k_F}{6}\left[\frac{M_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|} + \frac{\partial\Sigma_0}{\partial|\boldsymbol{k}|} + \frac{k_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right]_{|\boldsymbol{k}|=k_F}, \tag{3}$$

$$\begin{aligned}L^{\rm mom} =& \frac{k_F^2}{3\varepsilon_F^*}\frac{M_F^{*2}}{\varepsilon_F^{*2}}\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\bigg|_{|\boldsymbol{k}|=k_F} \\ &- \frac{k_F^2}{3\varepsilon_F^*}\left[\frac{M_F^*k_F^*}{\varepsilon_F^{*2}}\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|}\left(1+\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right)\right]_{|\boldsymbol{k}|=k_F} \\ &+ \frac{k_F}{6}\left[\frac{M_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|} + \frac{\partial\Sigma_0}{\partial|\boldsymbol{k}|} + \frac{k_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right]_{|\boldsymbol{k}|=k_F} \\ &+ \frac{k_F^2}{6\varepsilon_F^*}\left[\frac{k_F^{*2}}{\varepsilon_F^{*2}}\left(\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|}\right)^2 + \frac{M_F^{*2}}{\varepsilon_F^{*2}}\left(\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right)^2\right]_{|\boldsymbol{k}|=k_F} \\ &+ \frac{k_F^2}{6\varepsilon_F^*}\left[M_F^*\frac{\partial^2\Sigma_S}{\partial|\boldsymbol{k}|^2} + \varepsilon_F^*\frac{\partial^2\Sigma_0}{\partial|\boldsymbol{k}|^2} + k_F^*\frac{\partial^2\Sigma_V}{\partial|\boldsymbol{k}|^2}\right]_{|\boldsymbol{k}|=k_F},\end{aligned} \tag{4}$$

$$J^{\rm 1st} = \frac{1}{2}\left[\frac{M_F^*}{\varepsilon_F^*}\Sigma_S^{{\rm sym},1} + \Sigma_0^{{\rm sym},1} + \frac{k_F^*}{\varepsilon_F^*}\Sigma_V^{{\rm sym},1}\right]_{|\boldsymbol{k}|=k_F}, \tag{5}$$

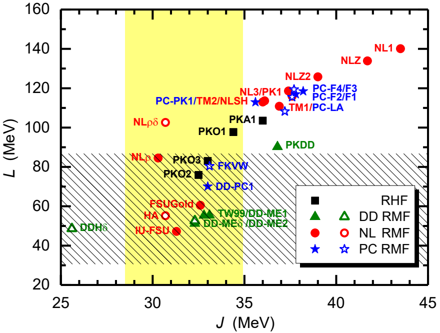
$$\begin{aligned}L^{\rm 1st} =& 3J^{\rm 1st} \\ &+ \frac{3}{2}\left[\frac{k_F^*}{\varepsilon_F^{*\frac{3}{2}}}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^{*\frac{3}{2}}}\Sigma_V^{{\rm sym},1}\right]^2_{|\boldsymbol{k}|=k_F} \\ &- \frac{k_F M_F^*}{\varepsilon_F^{*2}}\left[\frac{k_F^*}{\varepsilon_F^*}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^*}\Sigma_V^{{\rm sym},1}\right]_{|\boldsymbol{k}|=k_F},\end{aligned} \tag{6}$$

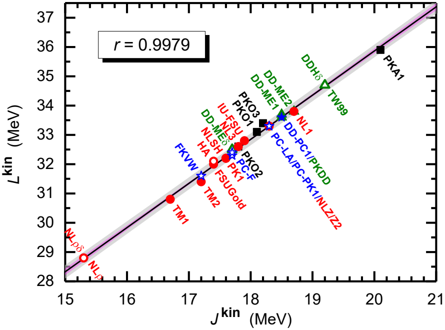
$$\begin{aligned}L^{\rm cross} =& k_F\left[\frac{M_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_S^{{\rm sym},1}}{\partial|\boldsymbol{k}|} + \frac{\partial\Sigma_0^{{\rm sym},1}}{\partial|\boldsymbol{k}|} + \frac{k_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_V^{{\rm sym},1}}{\partial|\boldsymbol{k}|}\right]_{|\boldsymbol{k}|=k_F} \\ &- \frac{k_F\Sigma_V^{{\rm sym},1}}{\varepsilon_F^*}\left[\frac{k_F^{*2}}{\varepsilon_F^{*2}}\left(\frac{M_F^*}{k_F^*}\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|} + \frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right) - \frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right]_{|\boldsymbol{k}|=k_F} \\ &- \frac{k_F\Sigma_S^{{\rm sym},1}}{\varepsilon_F^*}\left[\frac{M_F^{*2}}{\varepsilon_F^{*2}}\left(\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|} + \frac{k_F^*}{M_F^*}\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right) - \frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|}\right]_{|\boldsymbol{k}|=k_F},\end{aligned} \tag{7}$$

$$L^{\rm 2nd} = 3\left[\frac{M_F^*}{\varepsilon_F^*}\Sigma_S^{{\rm sym},2} + \Sigma_0^{{\rm sym},2} + \frac{k_F^*}{\varepsilon_F^*}\Sigma_V^{{\rm sym},2}\right]_{|\boldsymbol{k}|=k_F}. \tag{8}$$

Correlated Structure of Nuclear Symmetry Energy

$$J = \boxed{J^{\mathrm{kin}}} + J^{\mathrm{mom}} + J^{\mathrm{1st}}, \qquad L = \boxed{L^{\mathrm{kin}}} + L^{\mathrm{mom}} + L^{\mathrm{1st}} + L^{\mathrm{cross}} + L^{\mathrm{2nd}}$$

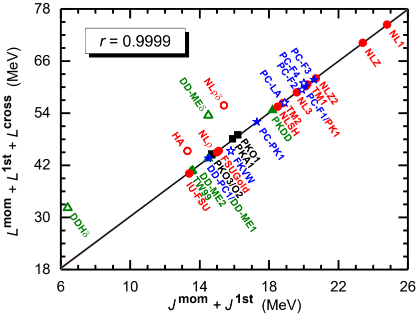




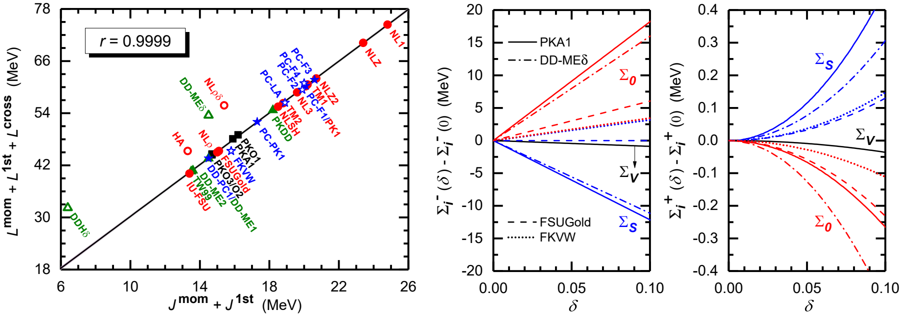
$$\begin{aligned} \boldsymbol{k}^{*,\tau} &= \boldsymbol{k} + \hat{\boldsymbol{k}}\Sigma_V^\tau(\rho,\delta,|\boldsymbol{k}|), \\ M_D^{*,\tau} &= M + \Sigma_S^\tau(\rho,\delta,|\boldsymbol{k}|), \\ \varepsilon^{*,\tau} &= \varepsilon^\tau - \Sigma_0^\tau(\rho,\delta,|\boldsymbol{k}|), \end{aligned}$$

$$L^{\mathrm{kin}} = \frac{3}{2}J^{\mathrm{kin}} + \frac{k_F}{6}\left[\frac{M_F^{*2}}{\varepsilon_F^{*2}}\frac{k_F}{\varepsilon_F^*} - \frac{1}{2}\frac{k_F^*}{\varepsilon_F^*}\right]$$

$$J = J^{\text{kin}} + \boxed{J^{\text{mom}} + J^{\text{1st}}}, \qquad L = L^{\text{kin}} + \boxed{L^{\text{mom}} + L^{\text{1st}} + L^{\text{cross}}} + L^{\text{2nd}}$$



$$J = J^{\rm kin} + \boxed{J^{\rm mom} + J^{\rm 1st}}, \qquad L = L^{\rm kin} + \boxed{L^{\rm mom} + L^{\rm 1st} + L^{\rm cross}} + L^{\rm 2nd}$$



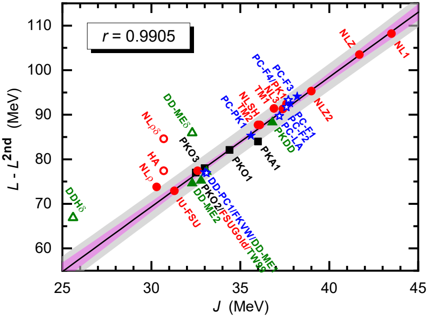
$$L^{\rm 1st} = 3J^{\rm 1st} + \frac{3}{2}\left[\frac{k_F^*}{\varepsilon_F^{*3/2}}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^{*3/2}}\Sigma_V^{{\rm sym},1}\right]^2_{|\boldsymbol{k}|=k_F} - \frac{k_F M_F^*}{\varepsilon_F^{*2}}\left[\frac{k_F^*}{\varepsilon_F^*}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^*}\Sigma_V^{{\rm sym},1}\right]_{|\boldsymbol{k}|=k_F}$$

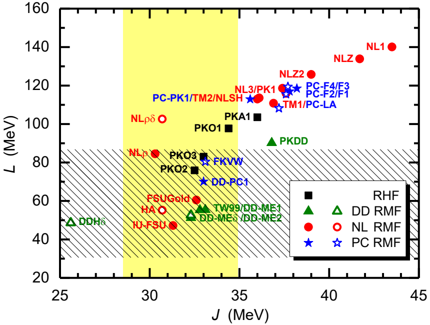
$$\Sigma_{\mathcal{O}}^{{\rm sym},1} = \frac{\partial}{\partial\delta}\frac{\Sigma_{\mathcal{O}}^-}{2}\bigg|_{\delta=0}, \qquad \Sigma_{\mathcal{O}}^- = \Sigma_{\mathcal{O}}^n - \Sigma_{\mathcal{O}}^p$$

☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

Two underlying linear correlations construct the fundamental correlation between L and J in CDF framework.

$$J = \boxed{J^{\text{kin}} + J^{\text{mom}} + J^{\text{1st}}}, \qquad L = \boxed{L^{\text{kin}} + L^{\text{mom}} + L^{\text{1st}} + L^{\text{cross}}} + L^{\text{2nd}}$$





$$\boxed{L - L^{\text{2nd}} = 2.91J - 17.95 \text{ MeV}}$$ For those without δ

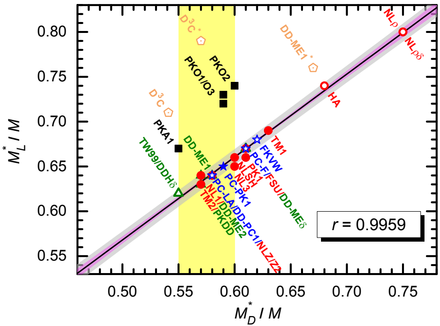
$$L^{\text{2nd}} = 3\left[\frac{M_F^*}{\varepsilon_F^*}\Sigma_S^{\text{sym},2} + \Sigma_0^{\text{sym},2} + \frac{k_F^*}{\varepsilon_F^*}\Sigma_V^{\text{sym},2}\right]_{|\boldsymbol{k}|=k_F}$$

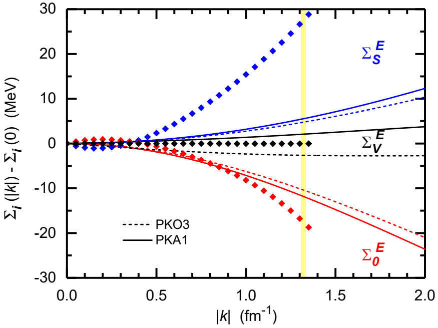
$$\Sigma_{\mathcal{O}}^{\text{sym},2} = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\frac{\Sigma_{\mathcal{O}}^+}{2}\Big|_{\delta=0}, \qquad \Sigma_{\mathcal{O}}^+ = \Sigma_{\mathcal{O}}^n + \Sigma_{\mathcal{O}}^p$$

Main factors to break the correlation:

- $\Sigma_{\mathcal{O}}^{\text{sym},1}$: isovector scalar coupling
- $\Sigma_{\mathcal{O}}^{\text{sym},2}$: strong model dependence

Landau Mass in CDF Theory





Landau Mass:

☼ *T. Katayama, K. Saito, PRC 88, 035805 (2013).*

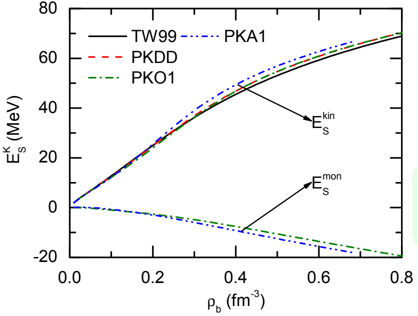
$$M_L^* = k_F \left[\frac{d|\boldsymbol{k}|}{d\epsilon^\tau}\right]_{|\boldsymbol{k}|=k_F} = k_F \left[\frac{k_F^*}{\varepsilon_F^*} + \frac{M_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_S}{\partial|\boldsymbol{k}|} + \frac{\partial\Sigma_0}{\partial|\boldsymbol{k}|} + \frac{k_F^*}{\varepsilon_F^*}\frac{\partial\Sigma_V}{\partial|\boldsymbol{k}|}\right]^{-1}_{|\boldsymbol{k}|=k_F}$$

➞ In RMF: Relation $M_L^* = \sqrt{k_F^2 + M_D^{*2}}$ approximated to a linear correlation.

➞ Systematically deviation due to *k*-dependence of the nucleon self-energies in RHF

☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

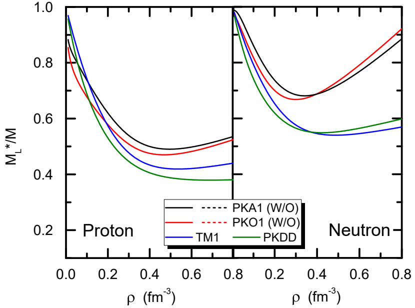
Impact of Landau Mass

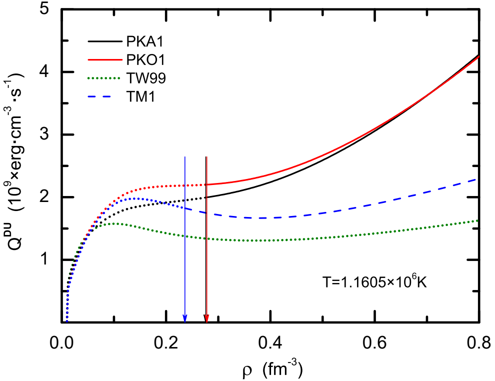


k-dependenct contribution and Landau mass:

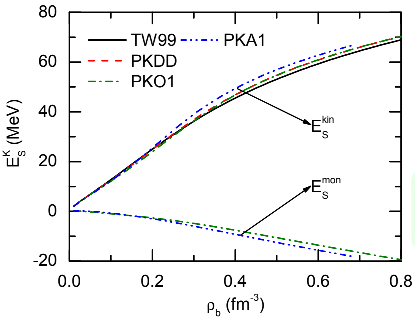
$$E_S^K = E_S^{\rm kin} + E_S^{\rm mon} = \frac{k_F^2}{6M_L^*}$$

The negative $E_S^{\rm mon}$ due to the *k*-dependence of self-energies in RHF, lead to larger M_L^*





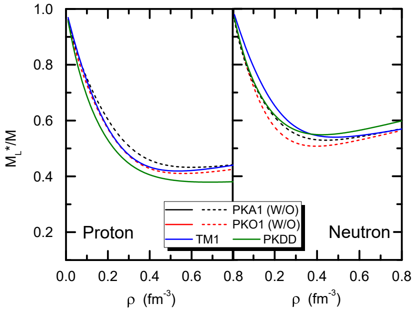
Impact of Landau Mass

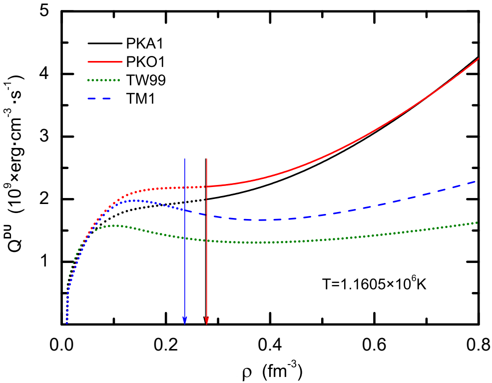


k-dependenct contribution and Landau mass:

$$E_S^K = E_S^{kin} + E_S^{mon} = \frac{k_F^2}{6M_L^*}$$

The negative E_S^{mon} due to the k-dependence of self-energies in RHF, lead to larger M_L^*





Summary and Outlook

- In terms of the covariant EDFs and the components of nuclear force, the symmetry energy is studied in CDF theory, illustrating the effects of exchange terms, also the tensor force effects in kinetic part
- By using HVH theorem, the correlated structure between L and J is revealed in terms of the nucleon self-energy
- Possible way to improve the CDFs in the market, from a viewpoint of isospin and momentum dependence of the nucleon self-energy, constrained by either microscopic predictions or experiments

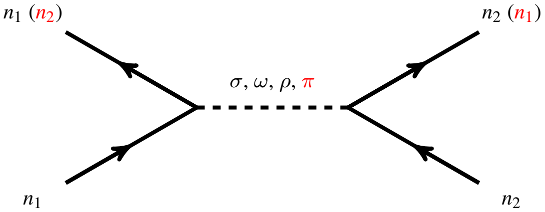
Summary and Outlook

- In terms of the covariant EDFs and the components of nuclear force, the symmetry energy is studied in CDF theory, illustrating the effects of exchange terms, also the tensor force effects in kinetic part
- By using HVH theorem, the correlated structure between L and J is revealed in terms of the nucleon self-energy
- Possible way to improve the CDFs in the market, from a viewpoint of isospin and momentum dependence of the nucleon self-energy, constrained by either microscopic predictions or experiments

Thank you for your attention!

RHF Lagrangian Density

- Relativistic Hartree & Fock (RHF): meson & photon exchanges



Hartree (Fock)

σ: Scalar I

ω: Vector γ^μ

ρ: Vector γ^μ, Tensor $\sigma^{\mu\nu}$

π: Pseudo-Vector $\gamma^\mu\gamma^5$

- RHF Lagrangian density: Nucleon (ψ), Hyperon Λ (ψ_Λ), Mesons (σ, ω, ρ, π)

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_M + \mathcal{L}_\Lambda + \mathcal{L}_I + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\pi \\
&= \bar{\psi}\left[i\gamma^\mu\partial_\mu - M\right]\psi + \bar{\psi}_\Lambda\left(i\gamma^\mu\partial_\mu - M_\Lambda - g_{\sigma\text{-}\Lambda}\sigma - g_{\omega\text{-}\Lambda}\gamma^\mu\omega_\mu\right)\psi_\Lambda \\
&\quad + \bar{\psi}\left[-g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\tau}\cdot\vec{\rho}_\mu + \frac{f_\rho}{2M}\sigma_{\mu\nu}\partial^\nu\vec{\rho}^\mu\cdot\vec{\tau} - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi \\
&\quad + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu \\
&\quad - \frac{1}{4}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu + \frac{1}{2}\partial^\mu\vec{\pi}\cdot\partial_\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi},
\end{aligned}$$

with $\Omega^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$, $\vec{R}^{\mu\nu} \equiv \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$.

Λ hyperon participates only in the interactions propagated by the isoscalar mesons.

RHF Energy Functional in Momentum Representation

- Energy functional in momentum representation: energy density in nuclear matter

$$\varepsilon = \frac{1}{\Omega}\langle\Phi_0|\,H\,|\Phi_0\rangle = \varepsilon_k + \sum_\phi \left(\varepsilon_\phi^D + \varepsilon_\phi^E\right),$$

with kinetic energy density ε_k, direct (ε_ϕ^D) and exchange (ε_ϕ^E) terms of the potential energy density,

$$\varepsilon_k = \sum_{p,s,\tau} \bar{u}(p,s,\tau)\left(\boldsymbol{\gamma}\cdot\boldsymbol{p} + M_\tau\right)u(p,s,\tau), \quad \text{with} \quad \tau_n = \frac{1}{2},\ \tau_p = -\frac{1}{2},\ \tau_\Lambda = 0,$$

$$\varepsilon_\phi^D = +\frac{1}{2}\sum_{p_1,s_1,\tau_1}\sum_{p_2,s_2,\tau_2} \bar{u}(p_1,s_1,\tau_1)\Gamma_\phi u(p_1,s_1,\tau_1)\frac{1}{m_\phi^2}\bar{u}(p_2,s_2,\tau_2)\Gamma^\phi u(p_2,s_2,\tau_2),$$

$$\varepsilon_\phi^E = -\frac{1}{2}\sum_{p_1,s_1,\tau_1}\sum_{p_2,s_2,\tau_2} \bar{u}(p_1,s_1,\tau_1)\Gamma_\phi u(p_2,s_2,\tau_2)\frac{1}{m_\phi^2+\boldsymbol{q}^2}\bar{u}(p_2,s_2,\tau_2)\Gamma^\phi u(p_1,s_1,\tau_1),$$

where ϕ represents σ-S, ω-V, ρ-V, ρ-T, ρ-VT, and π-PV couplings,

$$\Gamma_{\sigma\text{-S}} = ig_\sigma \text{ or } ig_{\sigma\text{-}\Lambda}, \qquad \Gamma_{\omega\text{-V}} = g_\omega\gamma_\mu \text{ or } g_{\omega\text{-}\Lambda}\gamma_\mu, \qquad \Gamma_{\rho\text{-V}} = g_\rho\gamma_\mu\vec{\tau},$$

$$\Gamma_{\rho\text{-T}} = \frac{f_\rho}{2M}q^\nu\sigma_{\mu\nu}\vec{\tau}, \qquad \Gamma_{\rho\text{-VT}} = \Gamma_{\rho\text{-V}} \text{ or } \Gamma_{\rho\text{-T}}, \qquad \Gamma_{\pi\text{-PV}} = \frac{f_\pi}{m_\pi}\boldsymbol{q}\cdot\boldsymbol{\gamma}\gamma_5\vec{\tau}.$$

- Self-energies in nuclear matter from variation: $\Sigma(p) = \Sigma_S(p) + \gamma_0\Sigma_0(p) + \boldsymbol{\gamma}\cdot\hat{\boldsymbol{p}}\Sigma_V(p)$

$$\Sigma(p)u(p,s,\tau) = \frac{\delta}{\delta\bar{u}(p,s,\tau)}\sum_{\sigma,\omega,\rho,\pi}\left[\varepsilon_\phi^D + \varepsilon_\phi^E\right].$$

Selected CDF Effective Lagrangians

Table: Bulk properties of symmetric nuclear matter at saturation point

	Fock	σ-NL	ω-NL	DD	π-PV	ρ-T	ρ_0 (fm^{-3})	E_B/A (MeV)	K (MeV)	J (MeV)	L (MeV)	Reference
PKA1	✓	×	×	✓	✓	✓	0.160	-15.83	230.0	36.0	104	*Long:2007*
PKO1	✓	×	×	✓	✓	×	0.152	-16.00	250.2	34.4	98	*Long:2006*
PKO2	✓	×	×	✓	×	×	0.151	-16.03	249.6	32.5	76	*Long:2008*
PKO3	✓	×	×	✓	✓	×	0.153	-16.04	262.5	33.0	83	*Long:2008*
NL1	×	✓	×	×	×	×	0.152	-16.43	211.2	43.5	140	*Reinhard:1986*
NL3	×	✓	×	×	×	×	0.148	-16.25	271.7	37.4	118	*Lalazissis:1997*
NL-SH	×	✓	×	×	×	×	0.146	-16.33	354.9	36.1	114	*Sharma:1993*
TM1	×	✓	✓	×	×	×	0.145	-16.26	281.2	36.9	111	*Sugahara:1994*
PK1	×	✓	✓	×	×	×	0.148	-16.27	282.7	37.6	116	*Long:2004*
TW99	×	×	×	✓	×	×	0.153	-16.25	240.3	32.8	55	*Typel:1999*
DD-ME1	×	×	×	✓	×	×	0.152	-16.20	244.7	33.1	56	*Nikšić:2002*
DD-ME2	×	×	×	✓	×	×	0.152	-16.11	250.3	32.3	51	*Lalazissis:2005*
PKDD	×	×	×	✓	×	×	0.150	-16.27	262.2	36.8	90	*Long:2004*

Relatively large values of K and J systematically in RMF with nonlinear self-coupling of mesons (NLRMF)

☼ *B. Y. Sun et al., PRC* **78***(2008)065805; W. H. Long et al., PRC* **85***(2012)025806; L. J. Jiang et al., PRC* **91***(2015)025802.*

Relativistic Formalism of Tensors

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C* **91**, *034326 (2015).*

$$\mathcal{H}^T_{\pi\text{-PV}} = -\frac{1}{2}\left[\frac{f_\pi}{m_\pi}\bar{\psi}\gamma_0\Sigma_\mu\vec{\tau}\psi\right]_1 \cdot \left[\frac{f_\pi}{m_\pi}\bar{\psi}\gamma_0\Sigma_\nu\vec{\tau}\psi\right]_2 D^{T,\,\mu\nu}_{\pi\text{-PV}}(1,2), \tag{10}$$

$$\mathcal{H}^T_{\sigma\text{-S}} = -\frac{1}{4}\left[\frac{g_\sigma}{m_\sigma}\bar{\psi}\gamma_0\Sigma_\mu\psi\right]_1 \left[\frac{g_\sigma}{m_\sigma}\bar{\psi}\gamma_0\Sigma_\nu\psi\right]_2 D^{T,\,\mu\nu}_{\sigma\text{-S}}(1,2), \tag{11}$$

$$\mathcal{H}^T_{\omega\text{-V}} = +\frac{1}{4}\left[\frac{g_\omega}{m_\omega}\bar{\psi}\gamma_\lambda\gamma_0\Sigma_\mu\psi\right]_1 \left[\frac{g_\omega}{m_\omega}\bar{\psi}\gamma_\delta\gamma_0\Sigma_\nu\psi\right]_2 D^{T,\,\mu\nu\lambda\delta}_{\omega\text{-V}}(1,2), \tag{12}$$

$$\mathcal{H}^T_{\rho\text{-T}} = +\frac{1}{2}\left[\frac{f_\rho}{2M}\bar{\psi}\sigma_{\lambda\mu}\vec{\tau}\psi\right]_1 \cdot \left[\frac{f_\rho}{2M}\bar{\psi}\sigma_{\delta\nu}\vec{\tau}\psi\right]_2 D^{T,\,\mu\nu\lambda\delta}_{\rho\text{-T}}(1,2), \tag{13}$$

where $\Sigma^\mu = \left(\gamma^5, \boldsymbol{\Sigma}\right)$, and D^T (ϕ for σ and π, ϕ' for ω and ρ) read as,

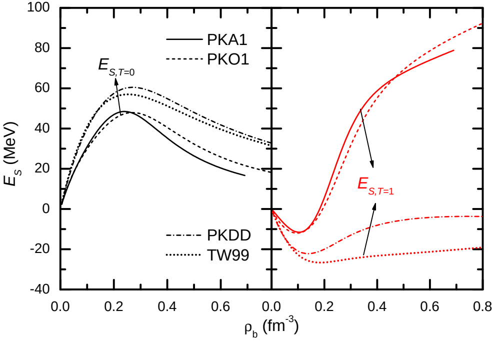
$$D^{T,\,\mu\nu}_{\phi}(1,2) = \left[\partial^\mu(1)\partial^\nu(2) - \frac{1}{3}g^{\mu\nu}m_\phi^2\right]D_\phi(1,2) + \frac{1}{3}g^{\mu\nu}\delta(x_1 - x_2),$$

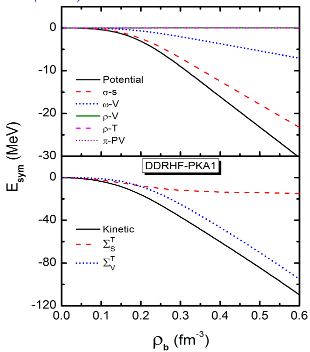
$$D^{T,\,\mu\nu\lambda\delta}_{\phi'}(1,2) = \left[\partial^\mu(1)\partial^\nu(2)g^{\lambda\delta} - \frac{1}{3}G^{\mu\nu\lambda\delta}m_{\phi'}^2\right]D_{\phi'}(1,2) + \frac{1}{3}G^{\mu\nu\lambda\delta}\delta(x_1 - x_2).$$

$$G^{\mu\nu\lambda\delta} \equiv \left(g^{\mu\nu}g^{\lambda\delta} - \frac{1}{3}g^{\mu\lambda}g^{\nu\delta}\right)$$

$$E_S(\rho_b) = E_{S,k} + E^D_{S,T=0} + E^E_{S,T=0} + E^D_{S,T=1} + E^E_{S,T=1}$$

☼ *Qian Zhao, Bao Yuan Sun, Wen Hui Long, J. Phys. G* **42**, *095101 (2015).*





		TW99	PKDD	PKO1	PKA1	BHF
	kin	5.9	5.0	-34.5	-69.6	14.9
L	$T=0$	62.2	78.2	67.5	71.3	69.1
	$T=1$	-12.8	7.0	64.8	103.2	-17.5

Large model dependence in kinetic and T=1 potential parts: Significant $E_S^{E,\sigma+\omega}$

Stiff E_S in RHF: Too strong T=1 ☹

☼ *I. Vidaña et al., PRC* **84**, *062801(R) (2011).*

Effective Mass in RHF Theory

- Definition of the Effective Mass:

$$\frac{M_\tau^*}{M} = 1 - \frac{dU_\tau(k,\epsilon(k))}{d\epsilon_\tau} = k\frac{dk}{d\epsilon_\tau} = \left[1 + \frac{M}{k}\frac{dU_\tau}{dk}\right]^{-1}$$

- Non-relativistic Mass: Shrödinger equivalent potential U_e^τ

$$E - mass : \frac{\bar{M}}{M} = \left[1 - \frac{\partial U^\tau}{\partial \epsilon^\tau}\right], \quad K - mass : \frac{\tilde{M}}{M} = \left[1 + \frac{M}{k}\frac{\partial U^\tau}{\partial k}\right]^{-1}$$

$$\frac{M_{NR}^*}{M} = \frac{\bar{M}}{M}\cdot\frac{\tilde{M}}{M} = \left[1 - \frac{\Sigma_0}{M}\right]\left[1 + \left(\frac{M^*}{k}\frac{\partial \Sigma_S}{\partial k} + \frac{E^*}{k}\frac{\partial \Sigma_0}{\partial k} + \frac{k^*}{k}\frac{\partial \Sigma_V}{\partial k} + \frac{\Sigma_V}{k}\right)\right]^{-1}$$

- Landau Mass: $\epsilon + M = E^* + \Sigma_0$

$$M_L^* = k\frac{dk}{d\epsilon^\tau} = k\left[\frac{k^*}{E^*} + \frac{M^*}{E^*}\frac{\partial \Sigma_{S,E}}{\partial k} + \frac{k^*}{E^*}\frac{\partial \Sigma_{V,E}}{\partial k} + \frac{\partial \Sigma_{0,E}}{\partial k}\right]^{-1}$$

- Relativistic Mass:

☼ *W. H. Long et al., Phys. Lett. B 640, 150 (2006).*

$$\frac{M_R^*}{M} = 1 - \frac{d}{d\epsilon}\left[U_e^\tau - \frac{\epsilon^2}{2M}\right], \quad M_R^* = M_{NR}^* + \epsilon = M_g^* \to \text{group mass}$$

S_4 at Saturation Density

	ρ_0	$S_2(\rho_0)$	$S_{2,\text{pot}}$	$S_{2,\text{kin}}$	$S_4(\rho_0)$	$S_{4,\text{pot}}$	$S_{4,\text{kin}}$
PKO1	0.152	34.37	30.66	3.71	0.52	-0.73	1.25
PKO2	0.151	32.49	28.09	4.40	0.58	-0.51	1.09
PKO3	0.153	32.99	29.72	3.27	0.47	-0.87	1.35
PKA1	0.160	36.02	35.55	0.46	0.35	-1.77	2.12
DD-ME1	0.152	33.07	24.69	8.37	0.65	0.17	0.48
DD-ME2	0.152	32.30	24.04	8.26	0.65	0.17	0.48
TW99	0.153	32.77	24.77	7.99	0.66	0.17	0.49
PKDD	0.150	36.79	28.66	8.13	0.65	0.17	0.48

- $S_4(\rho_0) \sim 0.5$ MeV comparable within RMF, but smaller than referred value of 20 ± 4.6 MeV.
- In comparison with RMF, the inclusion of the Fock terms decreases $S_{2,\text{kin}}$, enhances $S_{4,\text{kin}}$.

☼ *Z. W. Liu, Z. Qian, R. Y. Xing, J. R. Niu, and B. Y. Sun, Phys. Rev. C 97, 025801 (2018).*