Pauli-blocking effects on pion production in heavy-ion collisions

Natsumi Ikeno (Tottori University)

A. Ono (Tohoku Univ.),
Y. Nara (Akita International Univ.),
A. Ohnishi (YITP)

NuSYM2018 (8th International Symposium on Nuclear Symmetry Energy)
10-13 September 2018, Hanwha Resort, Haeundae, Busan

Pion and Symmetry energy

Symmetry energy
soft / stiff

↔

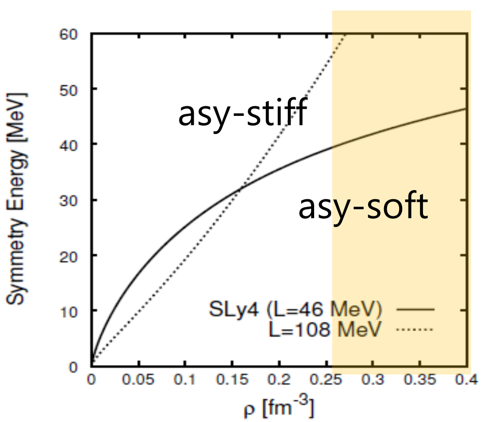
Nucleon
N/Z

↔

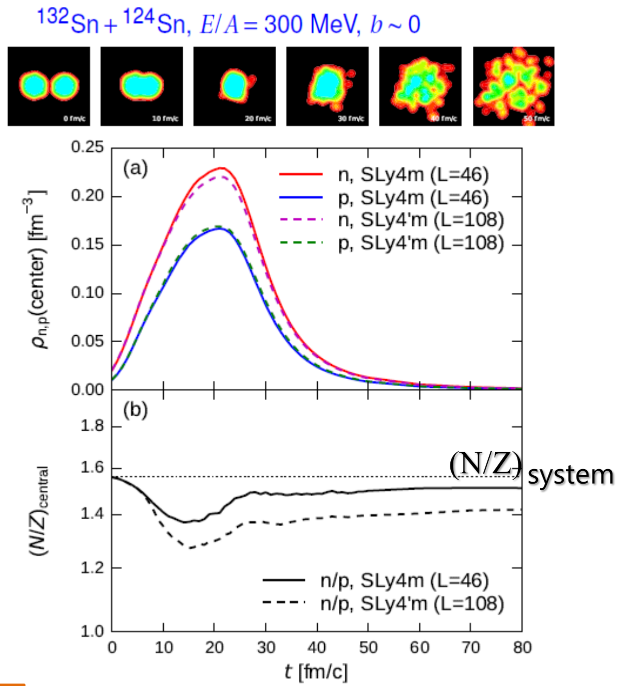
Δ resonance, Pion
Δ^-/Δ^{++}, π^-/π^+

Nucleon dynamics

NN ↔ NΔ
Δ ↔ Nπ

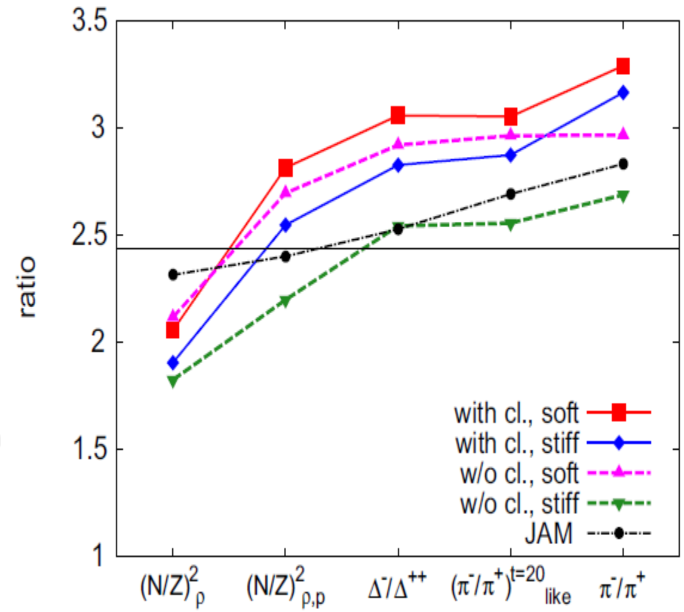


Interest:
High density $\rho \sim 2\rho_0$



Clear difference of N/Z in high density due to different **S(ρ)**

AMD + JAM



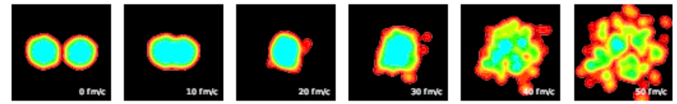
N. Ikeno, A. Ono, Y. Nara, A. Ohnishi,
PRC93, 044612 (2016);
PRC97, 069902(E) (2018)

- ✓ Delta threshold energy
- ✓ Pion potential
- ✓ Clustering
- ✓ Pauli blocking ←

etc.

Pion production and Pauli-blocking effect

＊Production of **Pions, Δ resonances:**



Formation in NN collisions at early times in the compressed part of the system

π^- production (main reaction)

$$nn \rightarrow p\Delta^-$$

$$\Delta^- \rightarrow n\pi^-$$

π^+ production (main)

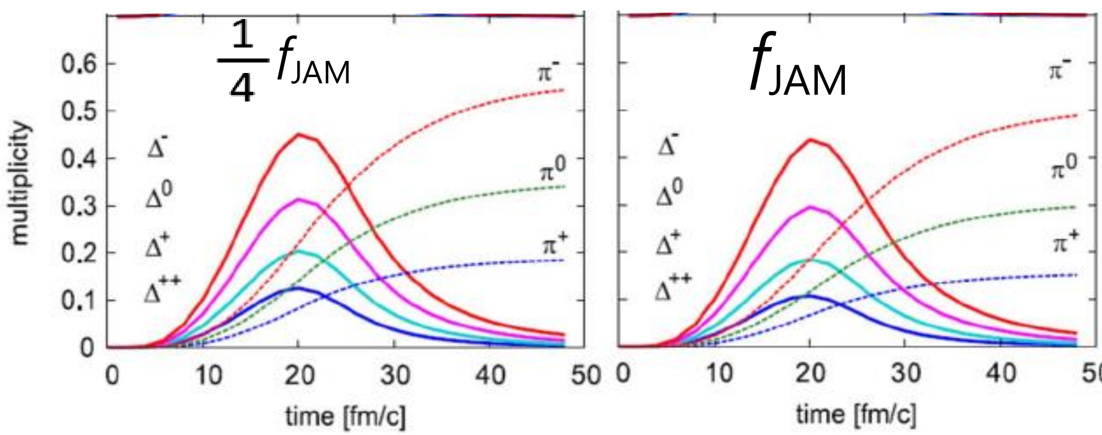
$$pp \rightarrow n\Delta^{++}$$

$$\Delta^{++} \rightarrow p\pi^+$$

Pauli blocking factor (1-*f*) for the final nucleon

ex)

Effect of Pauli-blocking is stronger
-> Δ and π numbers are smaller



N. Ikeno, A. Ono, Y. Nara, A. Ohnishi, PRC93, 044612 (2016)
PRC97, 069902(E) (2018)

Pauli blocking may play some important role on the pion observables.
⇒ We need to estimate the **Pauli blocking factor (1-f)** precisely

Motivation of our study

- Improved Pauli-blocking procedure by using Wigner function calculated in AMD

  for $NN \rightarrow N\Delta$,

  $N\Delta \rightarrow NN$, $\Delta \rightarrow N\pi$

- We like to see how the pion number and ratio change by Pauli-blocking effect

^{132}Sn + ^{124}Sn Collision @E/A=300, 270 MeV

- Experiment at RIKEN/RIBF SπRIT project
- Neutron rich system (N/Z) = 1.56

-> Final neutron is blocked more strongly than proton

-> π^-/π^+ may change

π^- production (main reaction)

$nn \rightarrow p\Delta^-$

$\Delta^- \rightarrow n\pi^-$

π^+ production (main)

$pp \rightarrow n\Delta^{++}$

$\Delta^{++} \rightarrow p\pi^+$

Transport model (AMD + JAM)

➢ Our model: JAM coupled with AMD

- **Nucleon f_N : Zeroth order equation**

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{p}} = I_N^{\mathrm{el}}[f_N^{(0)}, 0]$$

Solved by AMD

- **Δ particle f_Δ and pion f_π : First order equation**

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]$$

Solved by JAM for given $f_N^{(0)}$

➢ Coupled equations for $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ ($\alpha = N, \Delta, \pi$)

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N}{\partial \boldsymbol{p}} = I_N[f_N, f_{\Delta,\pi}]$$

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N, f_{\Delta,\pi}]$$

$I_N[f_N, f_{\Delta,\pi}]$:collision term

(
N N → N N
N N → N Δ
N Δ → N N
Δ → N π
N π → Δ ... etc.
)

Perturbative treatment of pion and Δ particle production

$$I_N = I_N^{\mathrm{el}}[f_N, 0] + \lambda I_N'[f_N, f_{\Delta,\pi}]$$

$f_{\Delta,\pi} = O(\lambda)$: Δ and pion productions are rare

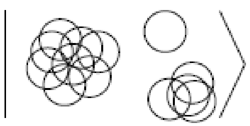
$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \ldots$

Transport model (AMD)

➢ AMD (Antisymmetrized Molecular Dynamics)

A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185

- AMD wave function at a time *t* for an event



$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\boldsymbol{r}_j - \frac{\boldsymbol{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

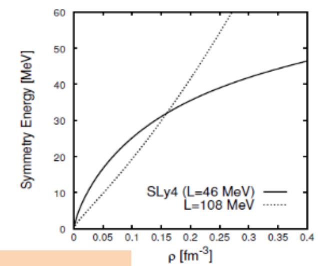
$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Solve the time evolution of the wave packet centroids Z

✓ Effective interaction

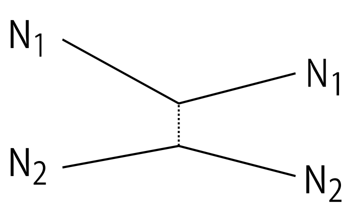


Skyrme force

- Turn on/off Cluster correlation

  - Without Cluster

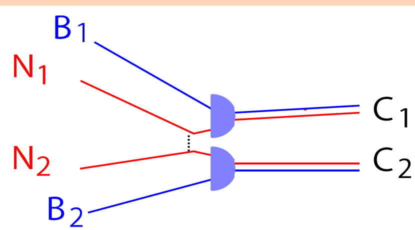
N1 + N2 -> N1 + N2



N1, N2 : Colliding nucleons

  - With Cluster

N1 + B1 + N2 + B2 -> C1 + C2



N1, N2: Colliding nucleons

B1, B2: Spectator nucleons/clusters

C1, C2: N, (2N), (3N), (4N) (up to α cluster)

Transport model (AMD + JAM)

- Nucleon test Particles

$$f_{\rm AMD}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

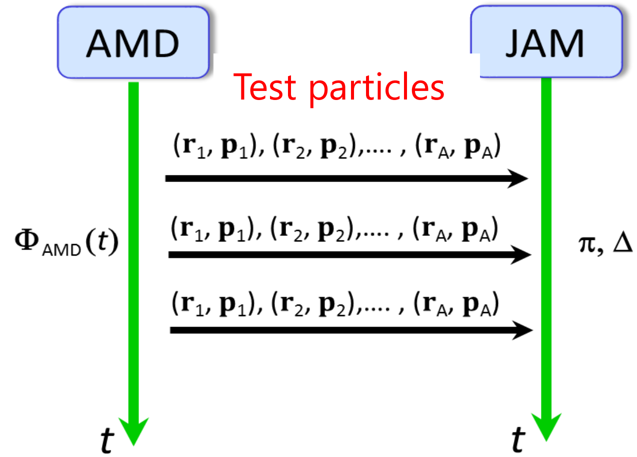
$$\boldsymbol{R}_{jk} = (\boldsymbol{Z}^*_j + \boldsymbol{Z}_k)/\sqrt{\nu}$$

$$\boldsymbol{P}_{jk} = 2i\hbar\sqrt{\nu}(\boldsymbol{Z}^*_j - \boldsymbol{Z}_k)$$

$$B_{jk} = \langle\varphi_j|\varphi_k\rangle$$

Test particles $(\mathbf{r}_1, \mathbf{p}_1), (\mathbf{r}_2, \mathbf{p}_2), \ldots, (\mathbf{r}_A, \mathbf{p}_A)$

- generated following the Wigner function $f_{\rm AMD}(\mathbf{r}, \mathbf{p})$
- sent from AMD to JAM at every 2 fm/c with corrections for the conservation of baryon number and charge



➢ JAM (Jet AA Microscopic transport model)

Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901

- Applied to high-energy collisions (1 ～ 158 A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default)
- *s*-wave pion production ($NN \to NN\pi$) is turned off. … etc.

Methods for Pauli-blocking factor *f*

- **Do Pauli blocking within JAM** (Natural prescription in AMD+JAM)

$$f_{\mathrm{JAM}}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2}\times 2^3 \sum_{j\in\tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L-2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

L=2.0 fm^2

N N → N Δ
Δ → N π etc.

Pauli blocking factor 1 - $f_{\mathrm{JAM}}(\mathbf{r}_i, \mathbf{p}'_i)$ calculated for Test particles {($\mathbf{r}_j$, $\mathbf{p}_j$); , j=1,2, … ,A}

A problem is that fluctuation of *f* seems to be large.
(Y. Zhang et al., PRC97, 034625 (2018) : Box Homework 1)

→ Blocking is less effect

- **Use *f* of AMD for Pauli blocking** (reasonable in principle)

Wigner function calculated for the AMD wave function,
for τ = neutron or proton, is

$$f_{\mathrm{AMD}}(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{2}\times 2^3 \sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

$$\boldsymbol{R}_{jk} = (\boldsymbol{Z}_j^* + \boldsymbol{Z}_k)/\sqrt{\nu}$$
$$\boldsymbol{P}_{jk} = 2i\hbar\sqrt{\nu}(\boldsymbol{Z}_j^* - \boldsymbol{Z}_k)$$
$$B_{jk} = \langle\varphi_j|\varphi_k\rangle$$

Pauli-blocking factor 1 - $f_{\mathrm{AMD}}(\mathbf{r}_i, \mathbf{p}'_i)$ for the final phase-space point ($\mathbf{r}_i$, $\mathbf{p}'_i$).

Calculated system and parameters/options

- $^{132}Sn + ^{124}Sn$ @E/A=300, 270 MeV $0<b<1$

- 4 options: Pauli blocking procedures

(1) $\frac{1}{4} f_{JAM}$: Pauli blocking factor f_{JAM} is artificially reduced by factor 4

(2) f_{JAM} : Do Pauli blocking within JAM

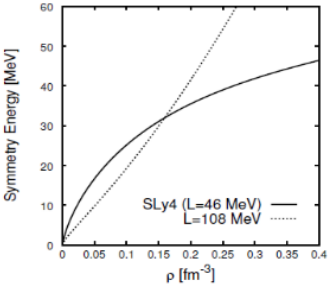
(3) $f_{AMD}^{NNN\Delta}$: Use Wigner function of AMD for Pauli blocking
only for $NN \leftrightarrow N\Delta$, $\Delta \rightarrow N\pi$ is JAM

(4) f_{AMD} : Use Wigner function of AMD for Pauli blocking
both for $NN \leftrightarrow N\Delta$ and $\Delta \rightarrow N\pi$

- Calculation model:

AMD (4 different nucleon dynamics)

1. with cluster (asy-soft)
2. with cluster (asy-stiff)
3. without cluster (asy-soft)
4. without cluster (asy-stiff)

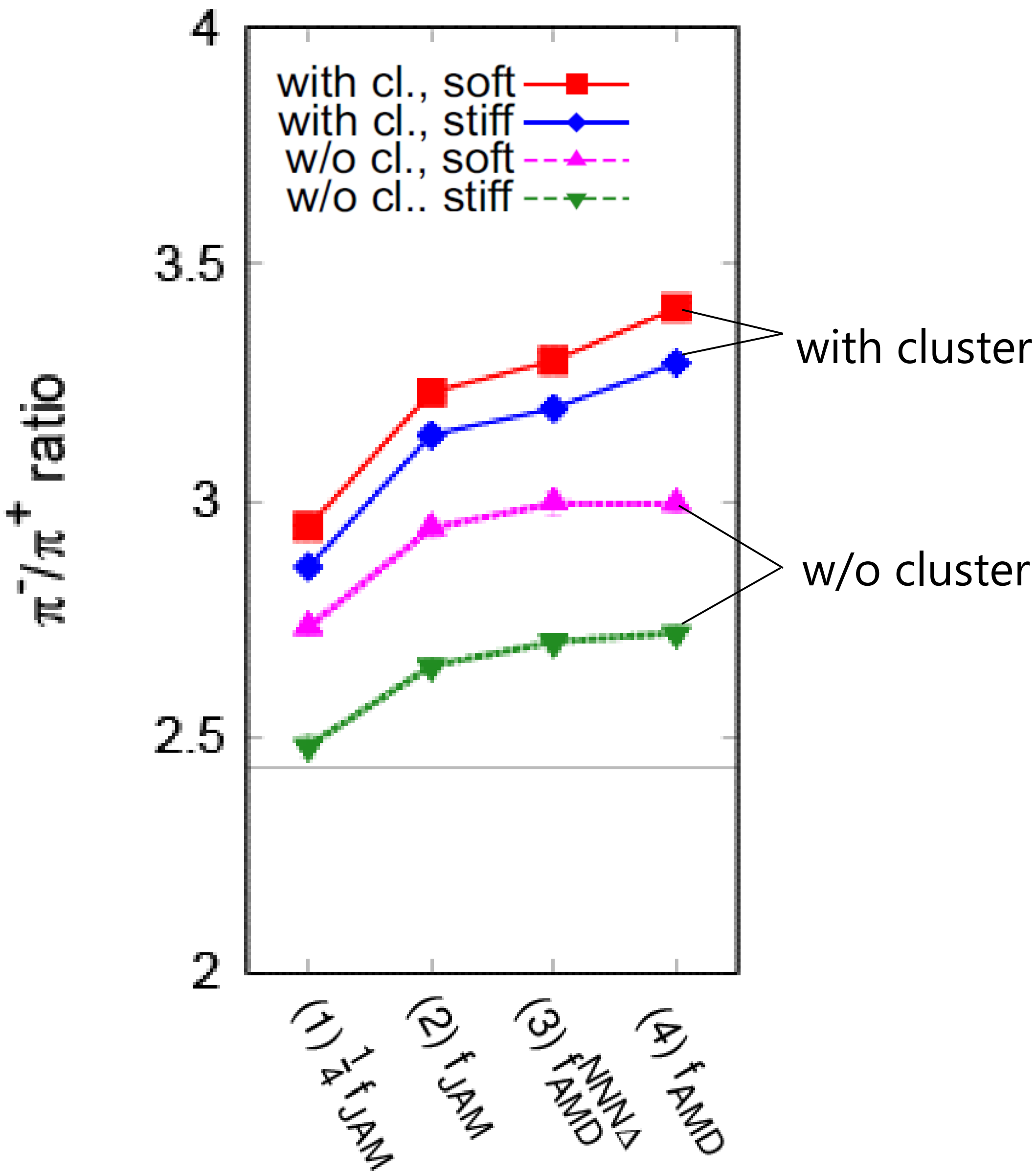


Effective interaction:

Skyrme force

asy-soft : L=46 (SLy4)
asy-stiff : L=108

Final π^-/π^+ ratio @ E/A=300 MeV

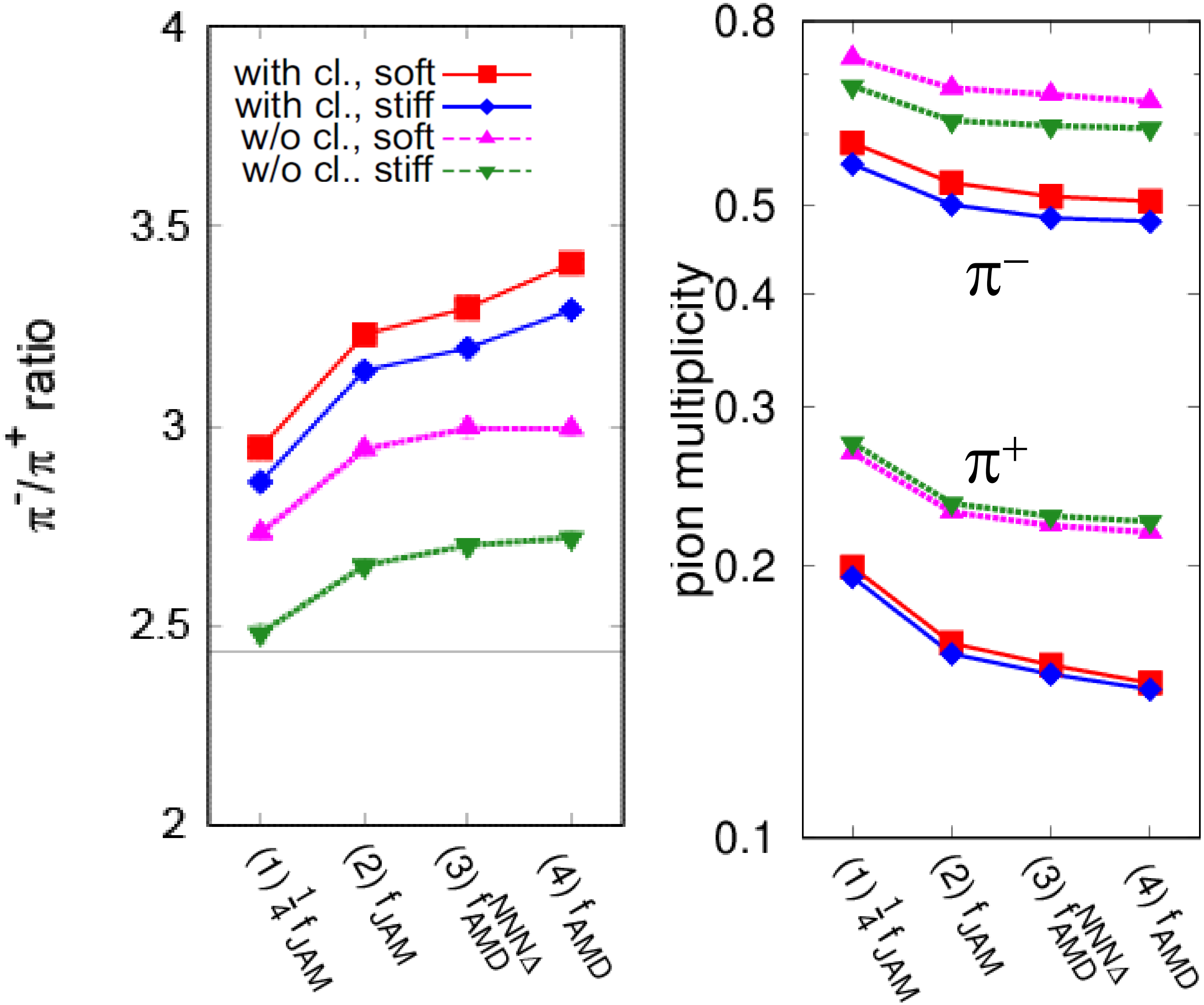


Clear dependence on Pauli blocking

Pion ratios become larger in precise treatment.

In particular when cluster correlation is switched on.

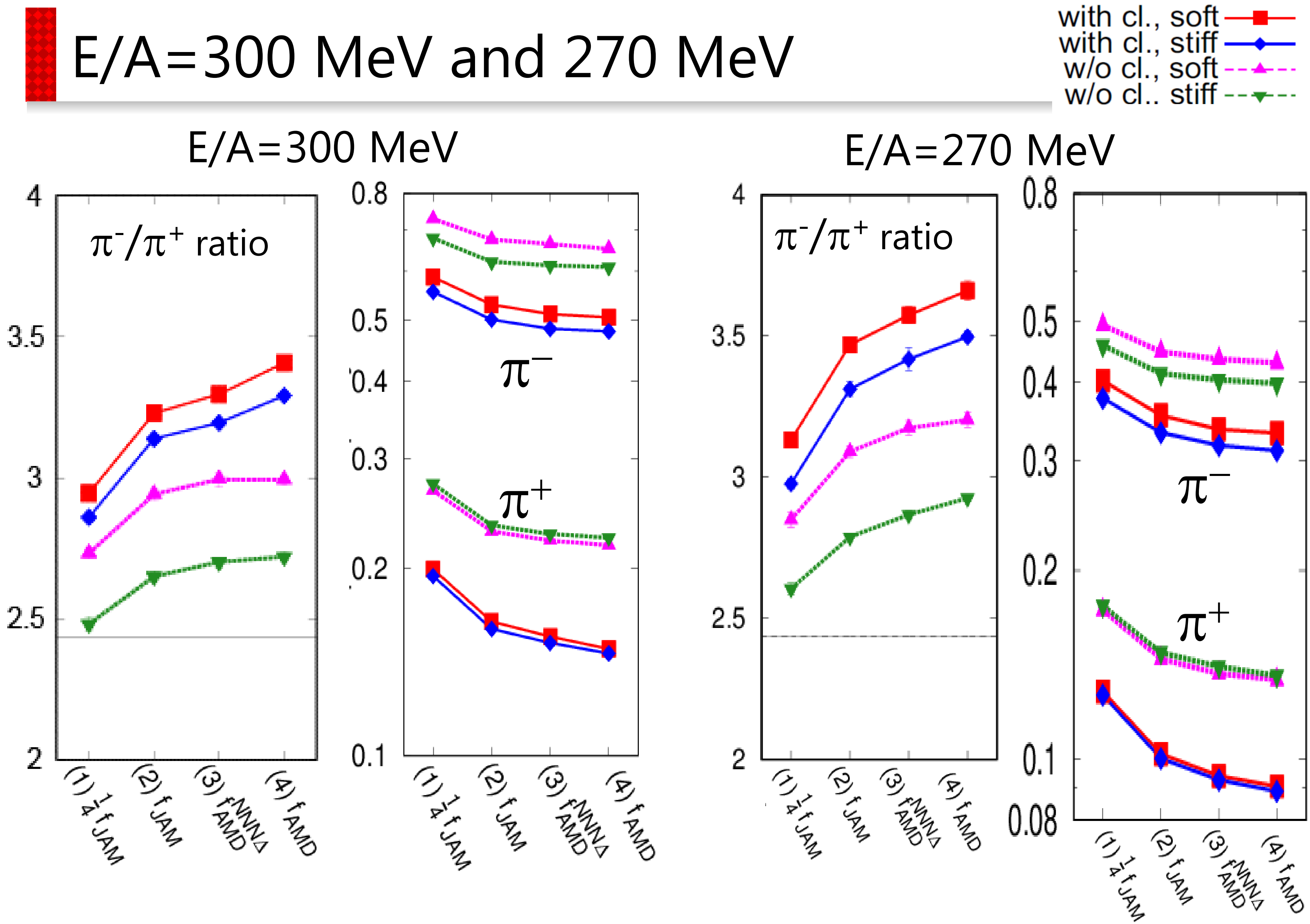
Final pion @ E/A=300 MeV



Pauli-blocking effect is stronger for π^+ than π^-.

-> Pion ratio goes up by Pauli-blocking effect

E/A=300 MeV and 270 MeV



Dynamics of pion production

✓ Pauli-blocking effect is stronger for π^+ than π^-.

Why?

- $NN \to N\Delta$ is easy to understand.

- $\Delta \to N\pi$ is more complicated.

π^- production

$nn \leftrightarrow p\Delta^-$ $\qquad \Delta^- \leftrightarrow n\pi^-$

$pn \leftrightarrow p\Delta^0$

$nn \leftrightarrow n\Delta^0$ $\qquad \Delta^0 \leftrightarrow p\pi^-$

π^+ production

$pp \leftrightarrow n\Delta^{++}$ $\qquad \Delta^{++} \leftrightarrow p\pi^+$

$pp \leftrightarrow p\Delta^+$

$pn \leftrightarrow n\Delta^+$ $\qquad \Delta^+ \leftrightarrow n\pi^+$

We compare these cases in reaction process.

(2) f_{JAM} vs. (3) $f_{\mathrm{AMD}}^{\mathrm{NNN}\Delta}$

$N\,N \to N\,\Delta$

(3) $f_{\mathrm{AMD}}^{\mathrm{NNN}\Delta}$ vs. (4) f_{AMD}

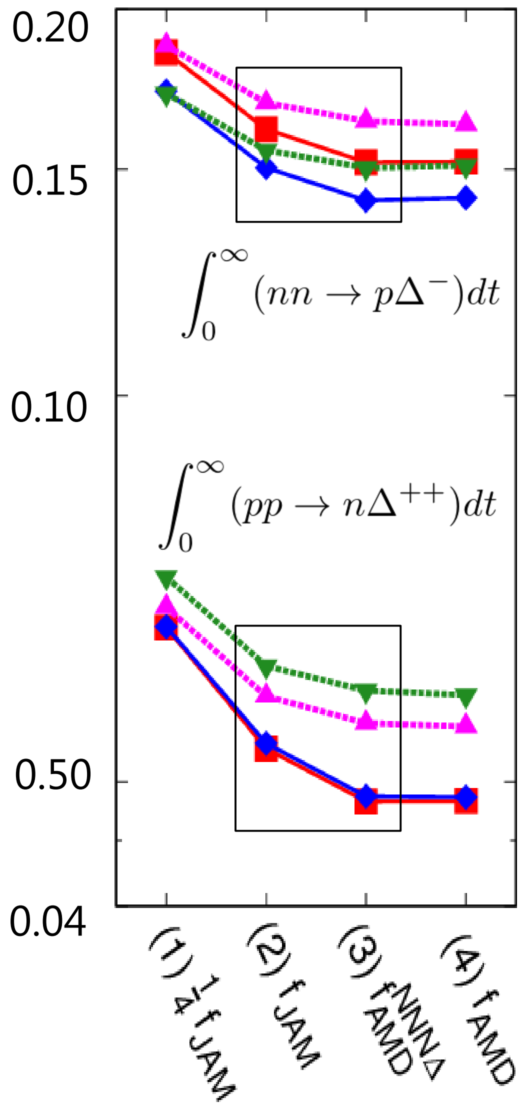
$\Delta \to N\pi$

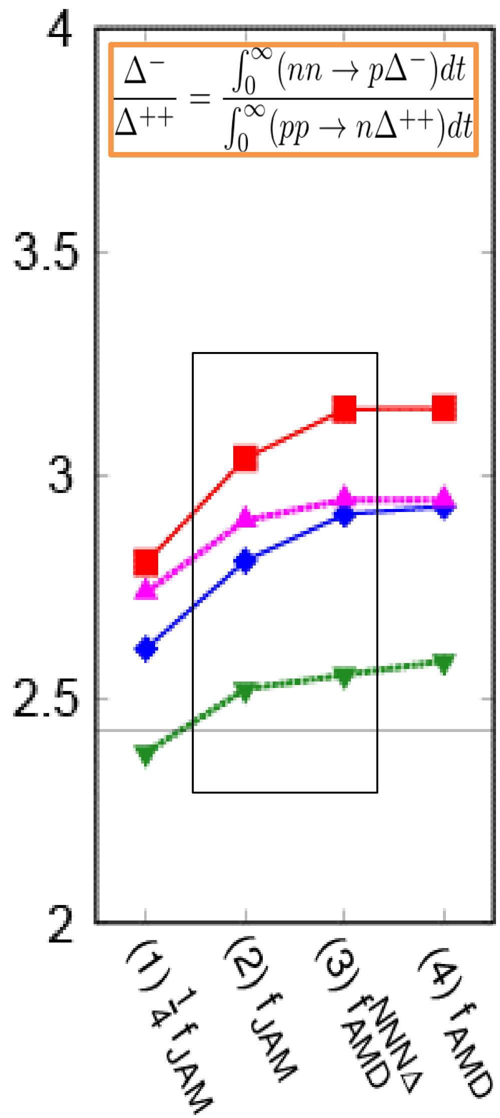
Pauli-blocking effect for NN→NΔ

with cl., soft
with cl., stiff
w/o cl., soft
w/o cl., stiff

Comparison of (2) f_{JAM} and (3) $f_{AMD}^{NNN\Delta}$

E/A=300MeV

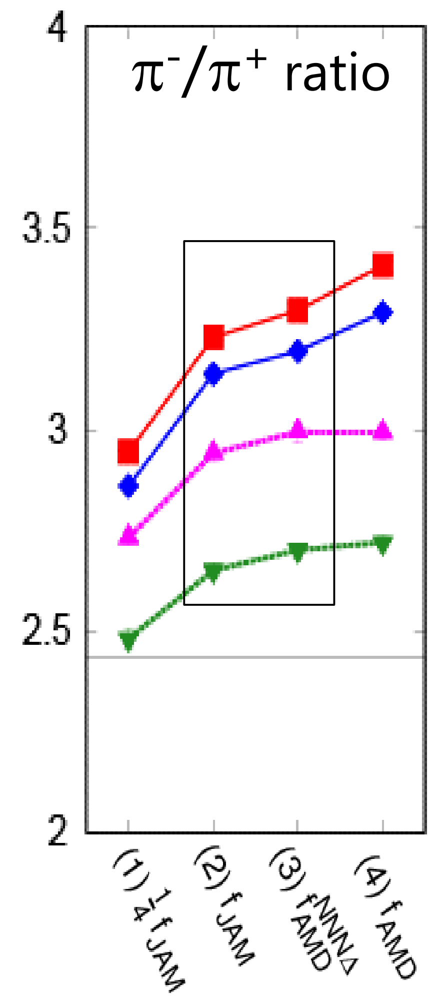




Pauli-blocking effect is stronger for the production of Δ^{++} than Δ^-

n-rich system -> final neutron is blocked more strongly

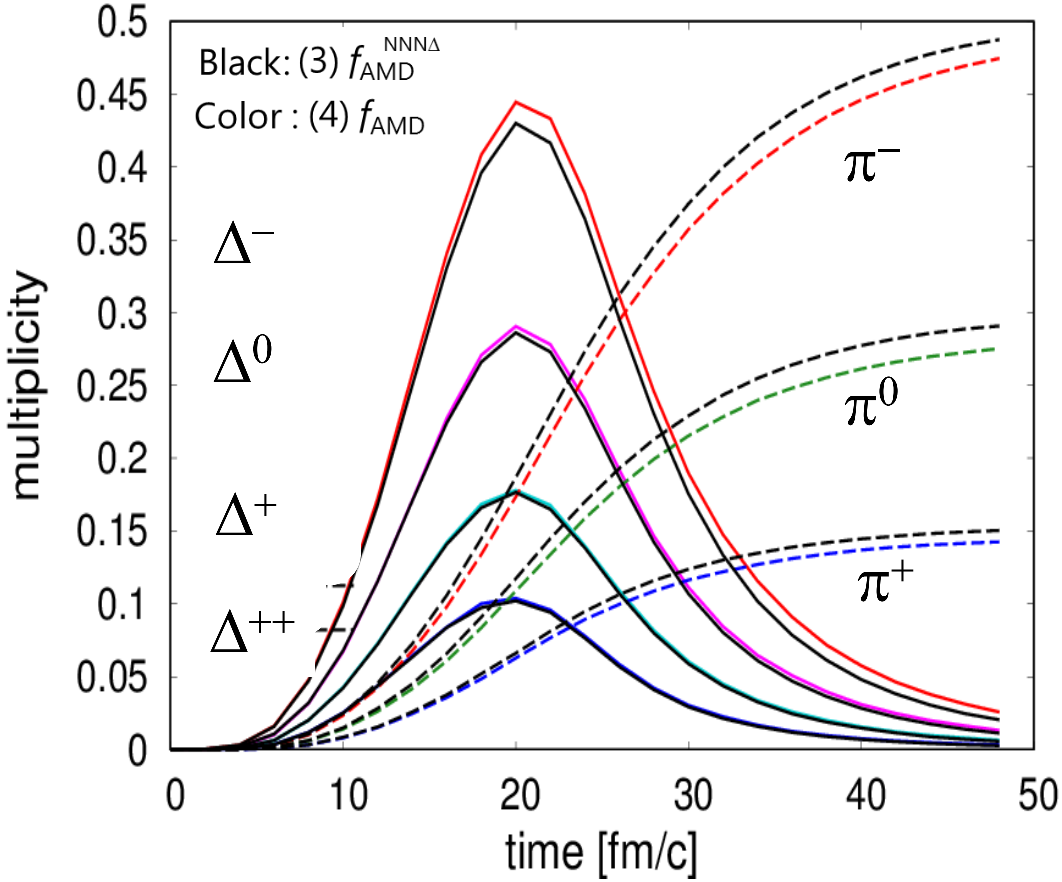
$pp \to n\Delta^{++}$



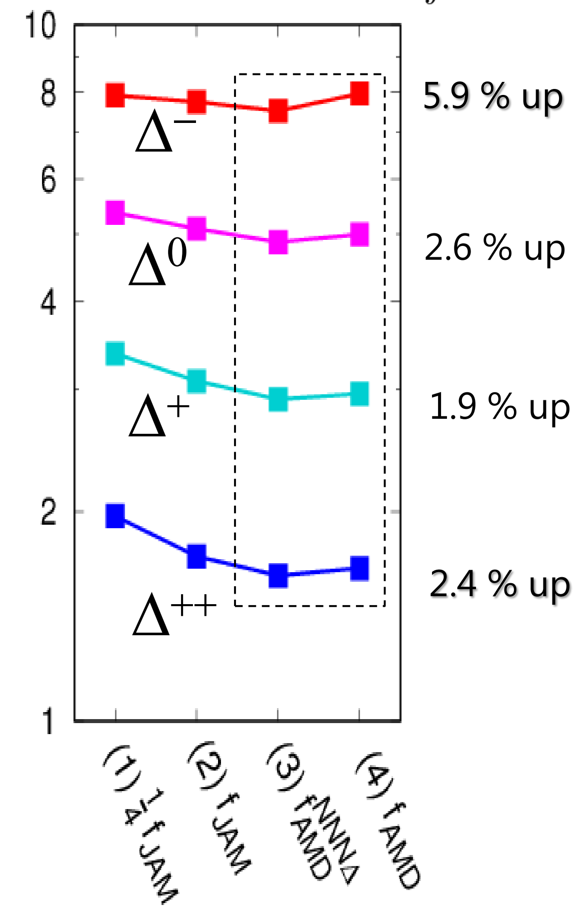
Pauli-blocking effect for Δ→Nπ

E/A=300MeV

Comparison of (3) $f_{AMD}^{NNN\Delta}$ and (4) f_{AMD}



Total numbers of Δ. $\int \Delta(t)dt$



Improved Pauli blocking for Δ -> Nπ

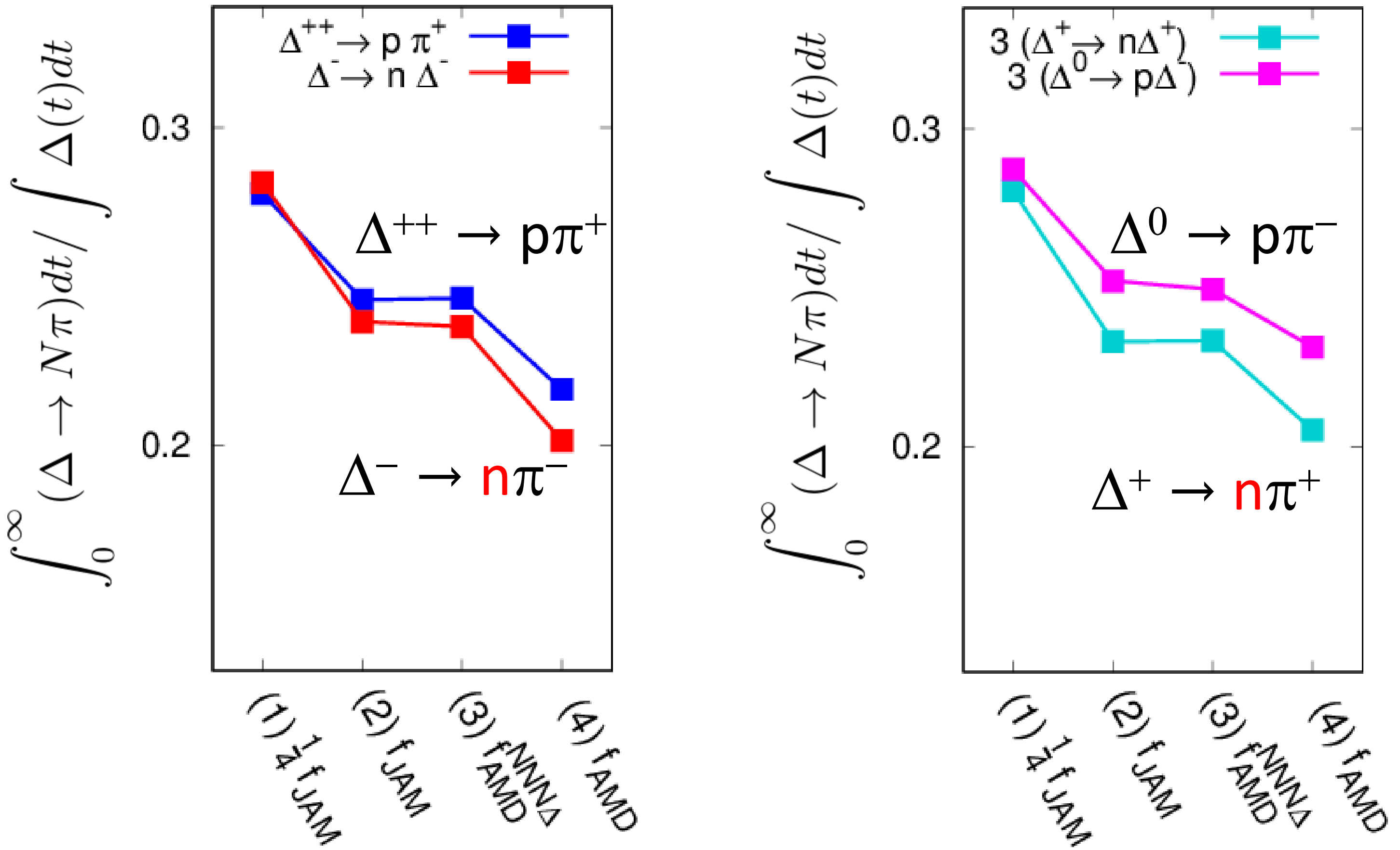
- Δ increases
- π decreases

Increase of numbers is different for the different Δ .

Especially, Δ^- increases largely.

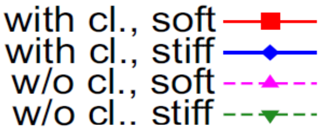
Pauli-blocking effect for $\Delta \to N\pi$

Comparison of (3) $f_{AMD}^{NNN\Delta}$ and (4) f_{AMD}

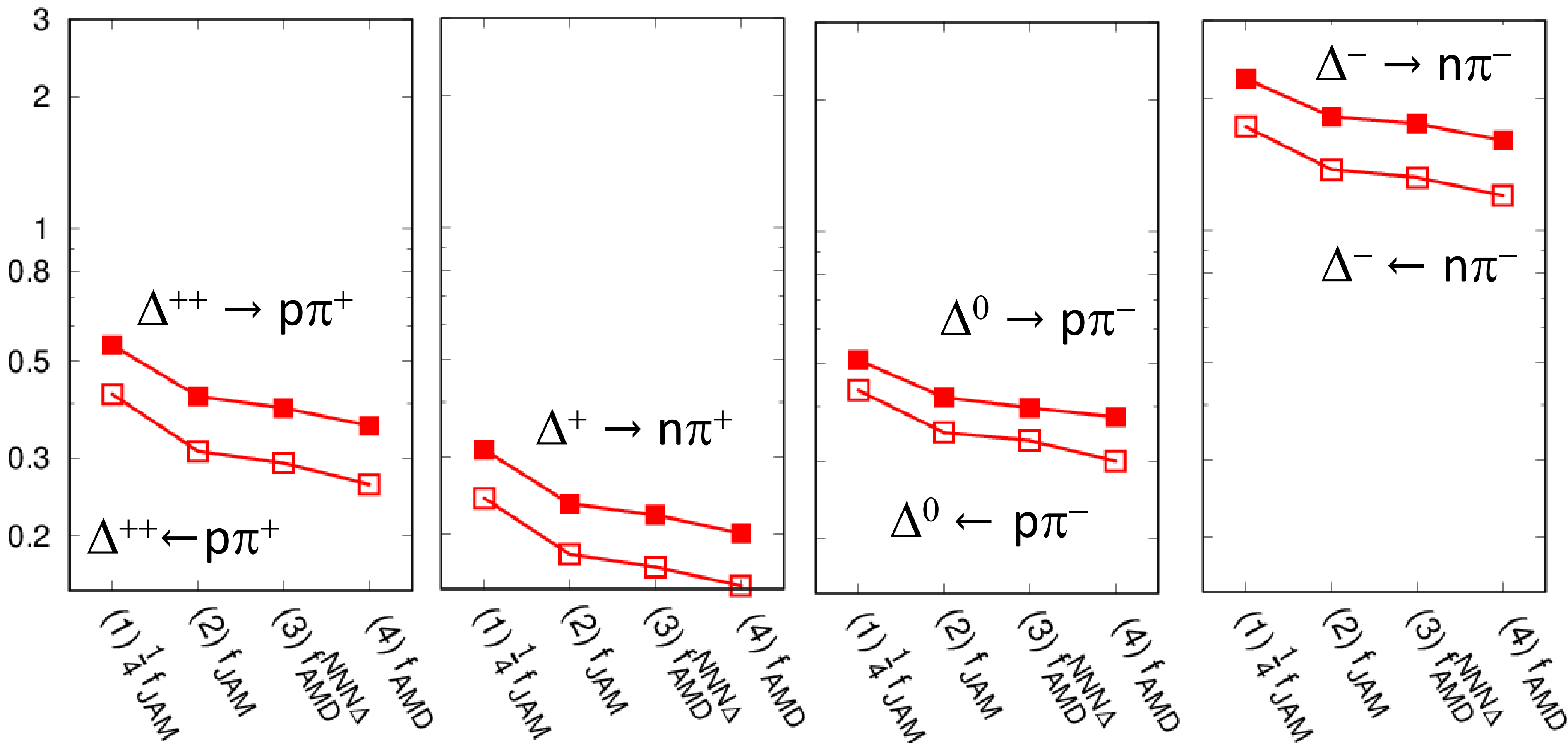


- Final proton is not blocked so strongly as a neutron
- The changes from (3) to (4) for $\Delta^0 \to p\pi^-$ and $\Delta^+ \to n\pi^+$ are smaller than those for $\Delta^{++} \to p\pi^+$ and $\Delta^- \to n\pi^-$.

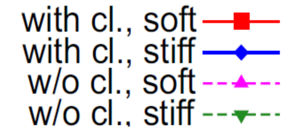
$\int_0^\infty (\Delta \to N\pi)dt$ and $\int_0^\infty (N\pi \to \Delta)dt$

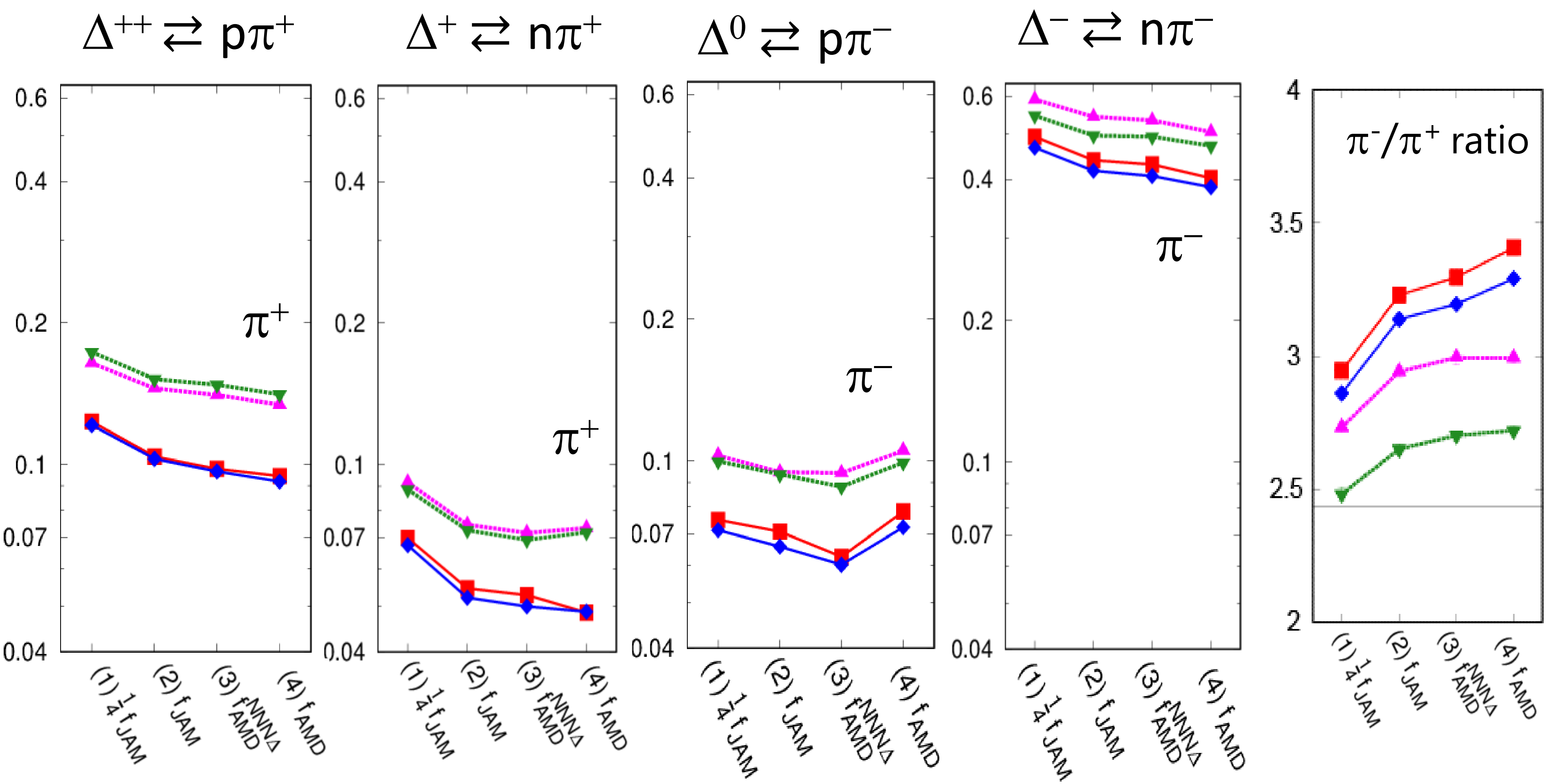


- Final pions are considered to be the subtraction of $\Delta \to N\pi$ and $\Delta \leftarrow N\pi$



π production by $\int_0^\infty (\Delta \to N\pi)dt$ - $\int_0^\infty (N\pi \to \Delta)dt$





In the balance of $\Delta^0 \rightleftarrows p\pi^-$ reaction, π^- increases.

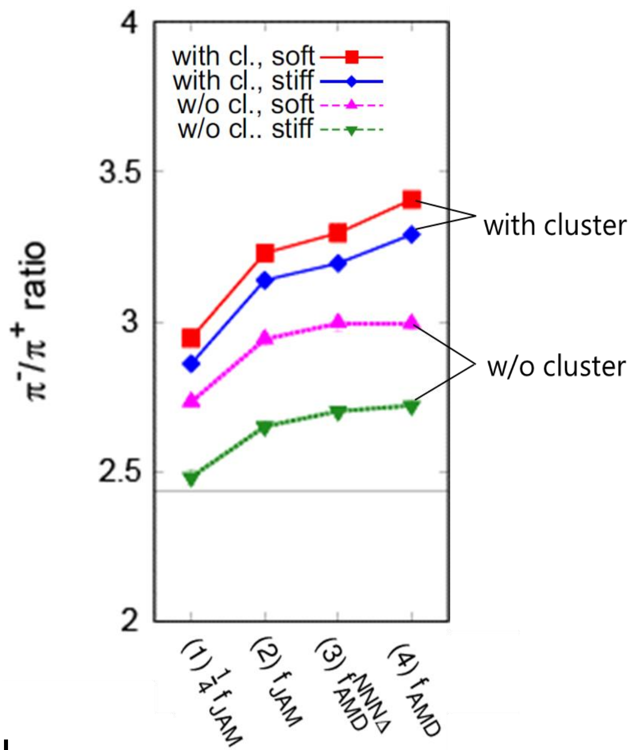
A small effect in $\Delta \to N\pi$ can result in a large change of the balance of $\Delta \rightleftarrows N\pi$.

Summary

- We improved Pauli blocking procedure for NN<->NΔ, Δ –> Nπ
  - AMD Winger, - AMD Wigner (NN<->NΔ), - JAM, - 1/4 JAM
- We have seen the Pauli-blocking effect for pion production

We found that

- Pion multiplicities and ratios depend on Pauli-blocking effect
- Pauli-blocking effect is stronger for π^+ (Δ^{++}) than π^- (Δ^-) in n-rich system
- The effect of blocking for decay (Δ –> Nπ) must be understood well.



Future work:

We need to study other treatments for pion observables

- Δ resonance production threshold ...

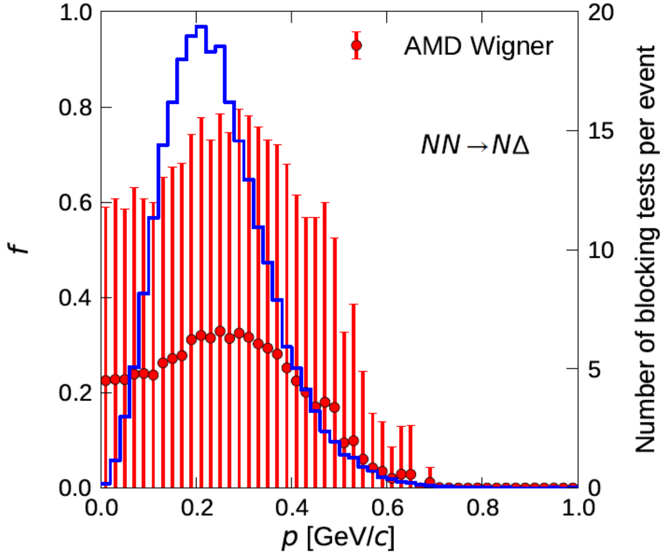
Phase space distribution f_{AMD}

Wigner function

^{132}Sn + ^{124}Sn@E/A=300 MeV

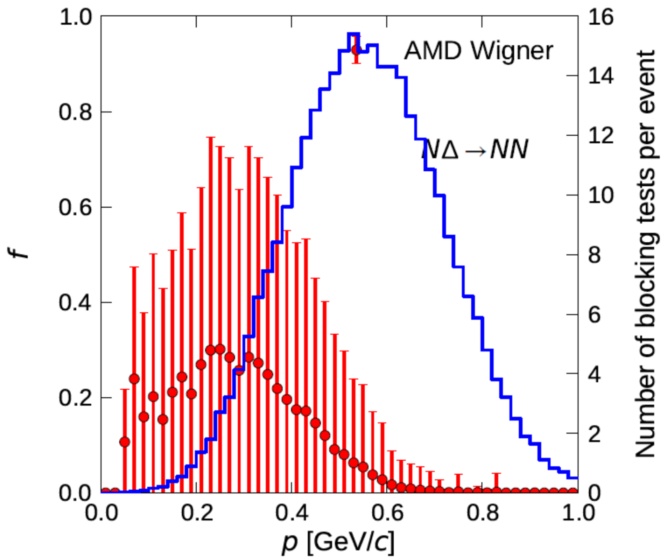
— momentum distribution of final nucleons

● blocking probability *f*



NN->NΔ

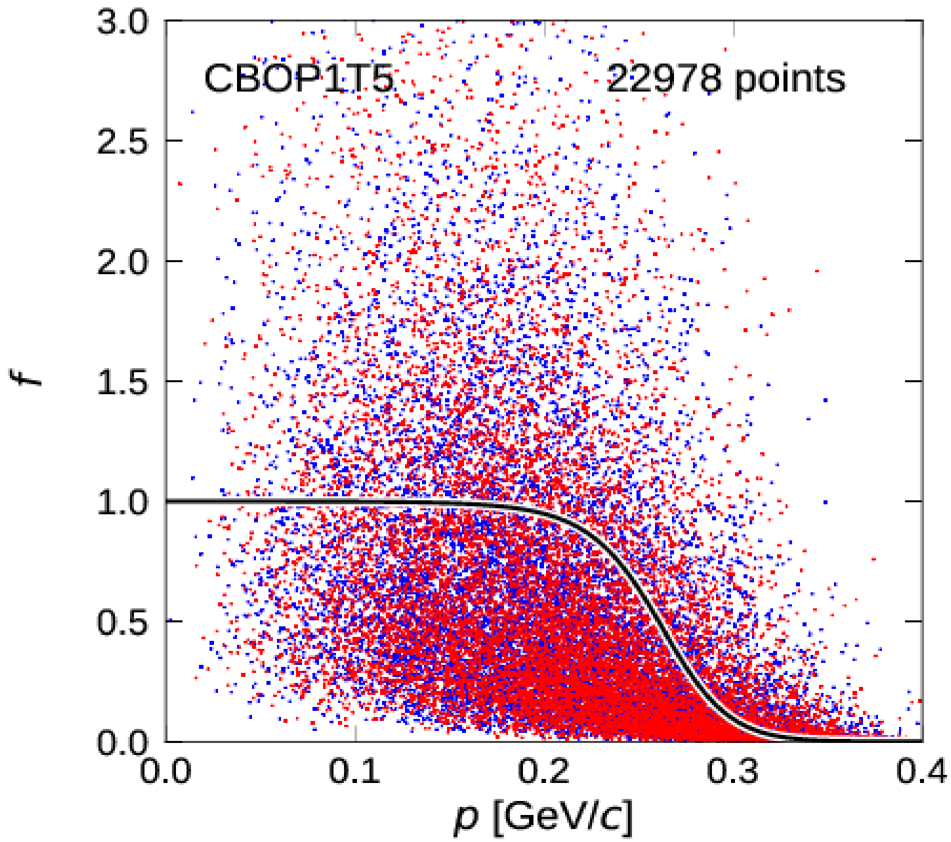
Pauli blocking is important for NN->NΔ because the final momentum is relatively low



NΔ->NN

Box HW1 test for JAM

JAM



Fermi distribution

$$f = \frac{1}{1 + e^{(E-\mu)/T}}$$

Test particles are generated from f

Then

$$f_{\mathrm{JAM}}(\boldsymbol{r}, \boldsymbol{p}) = \frac{1}{2} \times 2^3 \sum_{j \in \tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L - 2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

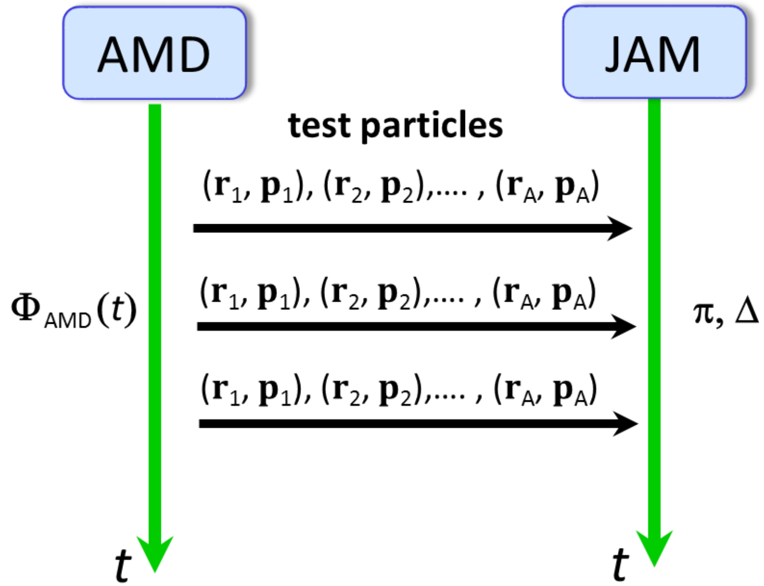
It is impossible to reconstruct the original *f* from test particles (1 test particle per nucleon)

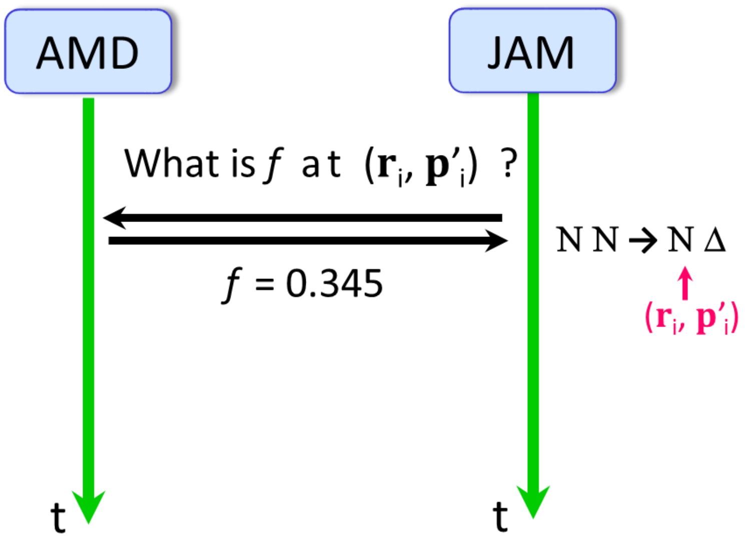
- ✓ Fluctuation of *f* is large
- ✓ *f* does not reproduce Fermi distribution

✓ *f* is larger than 1 => Pauli blocking is underestimated

f_{AMD} is free from this problem of fluctuation

Communication between AMD and JAM





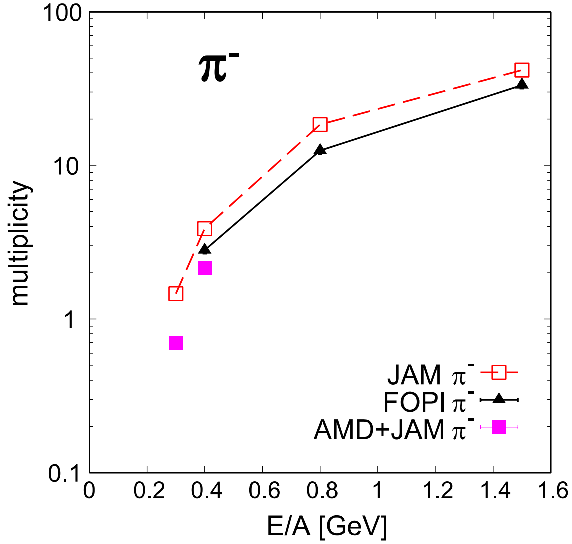
- send nucleon information from AMD to JAM in one direction
- AMD accepts a question from JAM, calculates f, and answers it to JAM

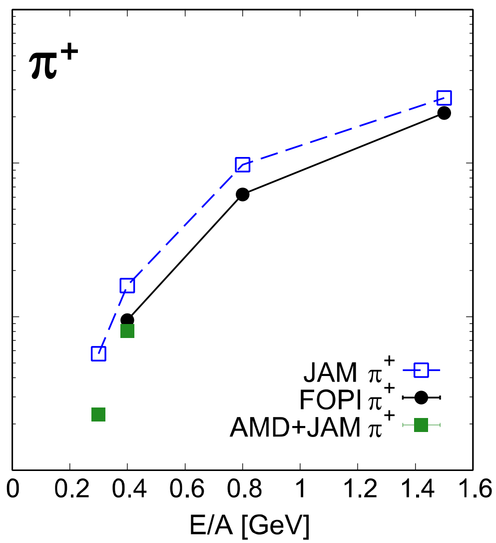
Pion Calcutions in central Au+Au collisions

by transport model (AMD + JAM)

with cluster (asy-soft)

- Pion multiplicity

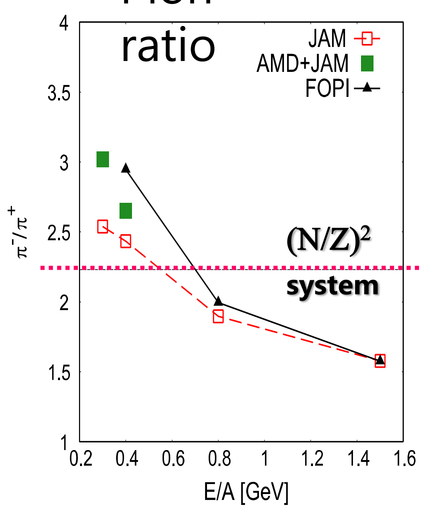


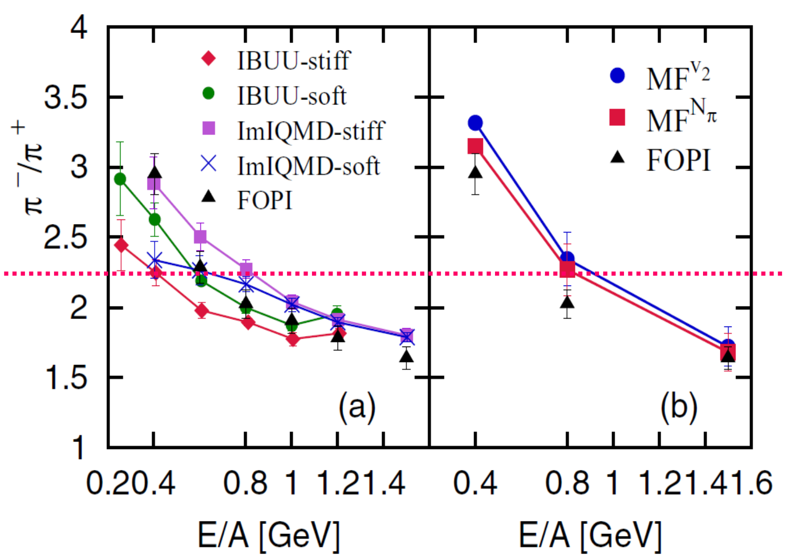


✓ Our calculation almost reproduces the experimental data reasonably well

✓ Pion ratios are also larger than $(N/Z)^2_{system}$

- Pion ratio



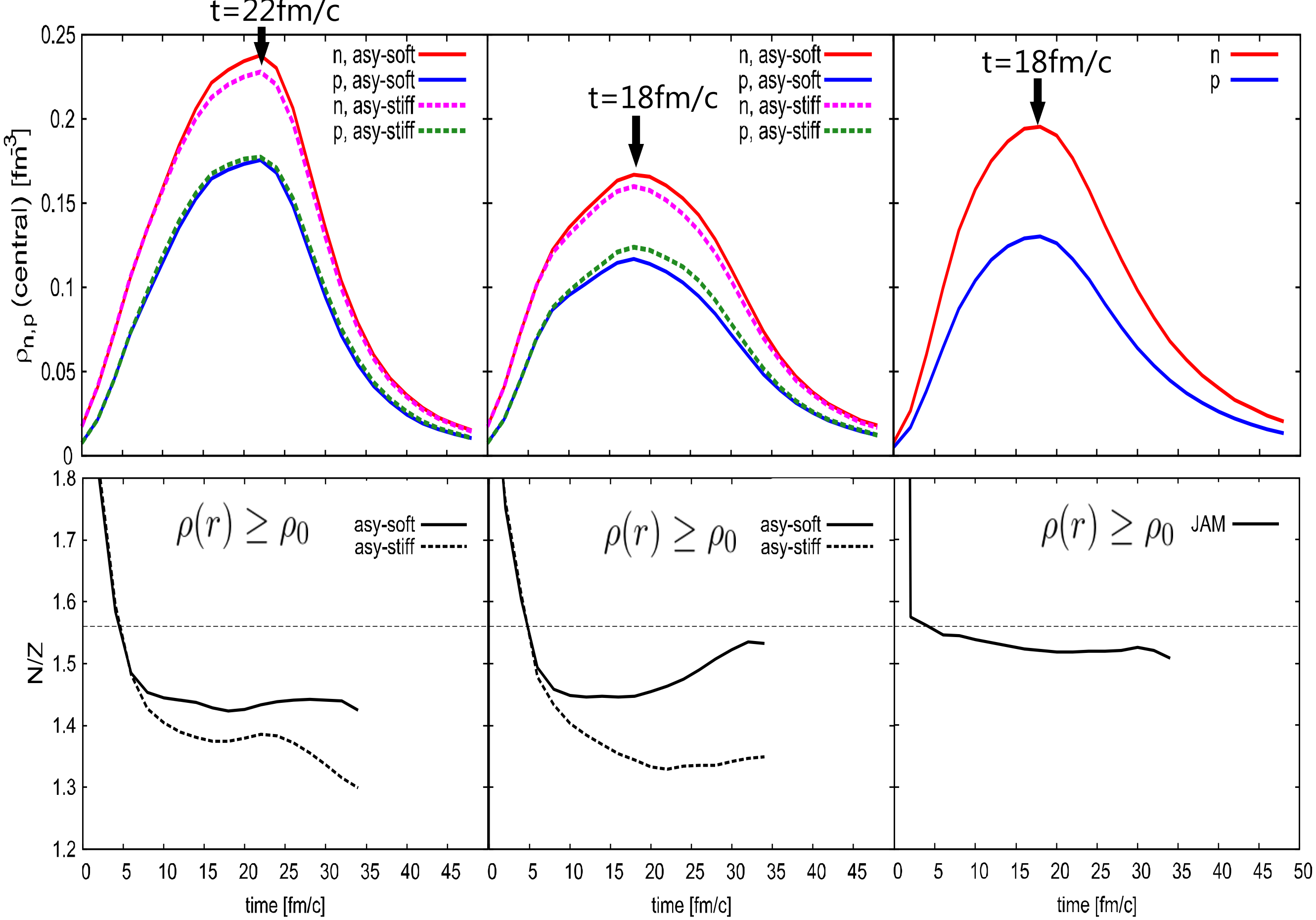


Exp. Data: Reisdorf *et al.*, NPA 848 (2010) 366

J. Hong and P. Danielewicz , PRC90 (2014) 024605

^{132}Sn + ^{124}Sn Collision @E/A=300 MeV

- Dynamics of neutrons and protons
  - with cluster
  - without cluster
  - JAM



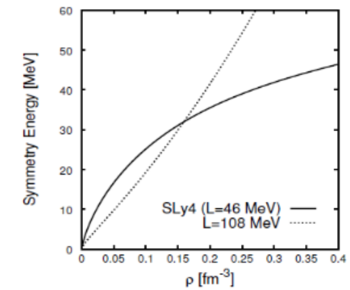
Calculation set:

AMD + JAM

1. with cluster (asy-soft)
2. with cluster (asy-stiff)
3. without cluster (asy-soft)
4. without cluster (asy-stiff)

5. JAM (no mean field)

asy-soft : L=46 (SLy4)
asy-stiff : L=108



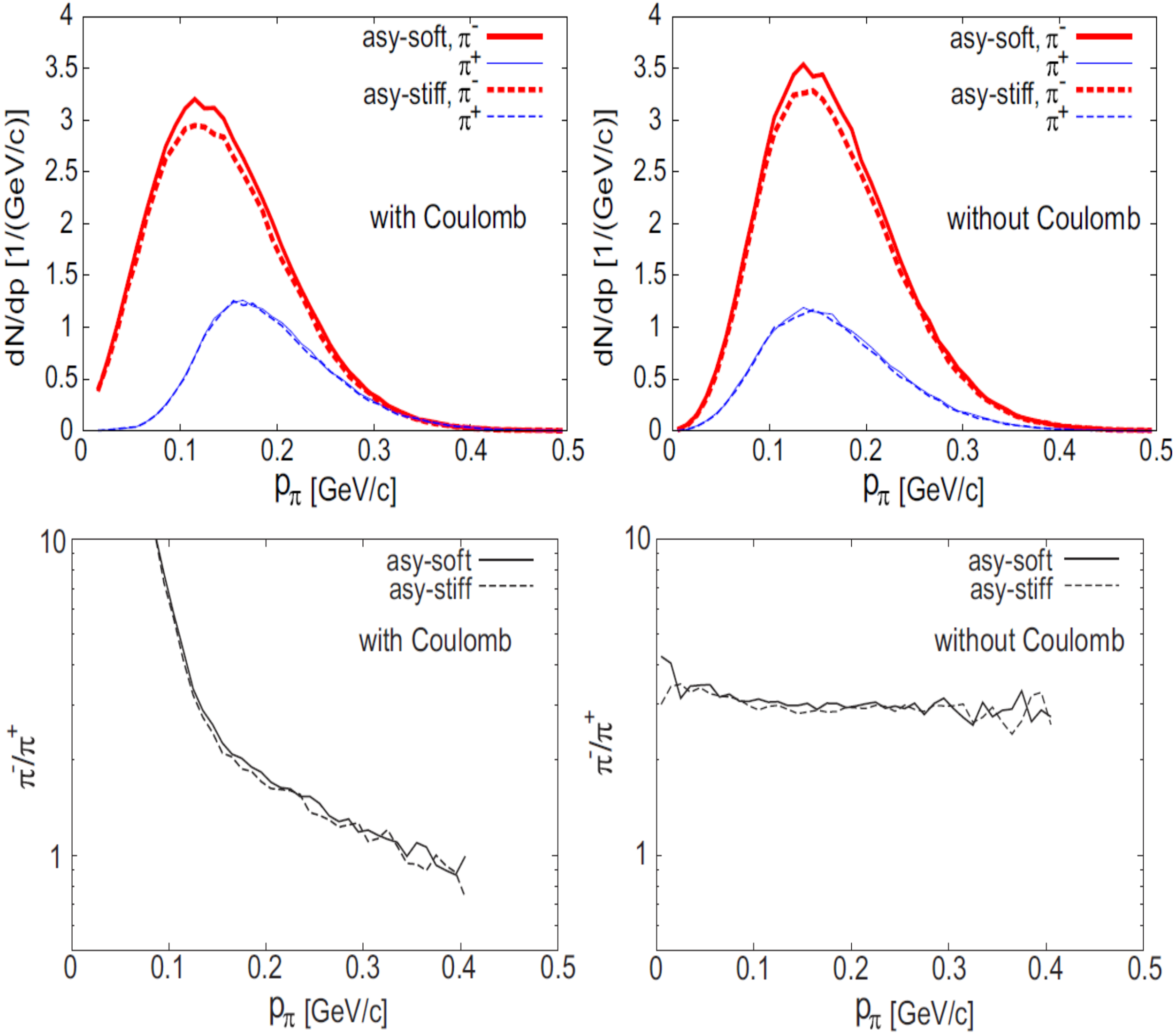
Effective interaction:
Skyrme force

- ✓ Density maximum is different for cases with or without cluster
- ✓ Clear difference of N/Z ratio due to different symmetry energy
- ✓ Especially symmetry energy effect is weaker if there is cluster correlation

Pion spectra

AMD + JAM with cluster (asy-soft)

- With Coulomb
- Without Coulomb



- Coulomb effect: Acceleration of π^+ Deceleration of π^-

→ Changes of pion spectra

	π^-	π^+	π^-/π^+
with Coulomb	0.577	0.192	**3.01(1)**
w/o Coulomb	0.582	0.193	**3.02(1)**

→ Coulomb effect has almost no effect on the pion multiplicities and the pion ratio.

Potential for Δ and pion

In JAM, reaction thresholds are the same as in free space.

(The production and absorption reactions for Δ and pions occur in the JAM calculation as in the free space)

Nucleons feel potential in the AMD calculation.

Therefore AMD+JAM assumes

$$\mathrm{NN} \leftrightarrow \mathrm{N}\Delta \qquad \Delta \leftrightarrow \mathrm{N}\pi$$

$$U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} = U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \qquad U^{(\Delta)}_{\tau_1} = U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is equivalent to the choice in the pBUU calculation

c.f. Hong and Danielewicz, PRC 90 (2014) 024605

$$\begin{aligned}
v_{asy}(\Delta^-) &= 2v_{asy}(n) - v_{asy}(p) = 3v_{asy}(n),\\
v_{asy}(\Delta^0) &= v_{asy}(n),\\
v_{asy}(\Delta^+) &= v_{asy}(p) = -v_{asy}(n),\\
v_{asy}(\Delta^{++}) &= 2v_{asy}(p) - v_{asy}(n) = -3v_{asy}(n).
\end{aligned}$$

* Different choice,
  cf. Bao-An Li

$$\begin{aligned}
v_{asy}(\Delta^-) &= v_{asy}(n),\\
v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n),\\
v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n),\\
v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n).
\end{aligned}$$