

The role of Fock terms on nuclear symmetry energy in a relativistic framework

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in collaboration with

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8th International Symposium on Nuclear Symmetry Energy (NuSYM2018)
@ Hanwha Resort, Haeundae, Busan
September 10–13, 2018

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Nuclear symmetry energy

The **nuclear symmetry energy** is known to be an important physical quantity not only **in nuclear physics** but also **in astrophysics**.

- neutron skin and the different distributions of neutrons and protons in a nucleus
- isospin-asymmetric nuclear matter in the density region above the saturation density, experimentally performed by the heavy-ion collisions
- some astrophysical observations, for instance the mass-radius relations of neutron stars and the cooling processes of proto-neutron stars, strongly depend on the nuclear symmetry energy.

Theoretical point of view

Many theoretical calculations have been focused on the properties of asymmetric nuclear matter, including **the nuclear symmetry energy**.

- the phenomenological calculations based on the effective many-body interaction with the Skyrme or Gogny force
- the relativistic mean-field (RMF) models based on Quantum Hadrodynamics (QHD)
- microscopic studies, for examine, the so-called Dirac-Brueckner-Hartree-Fock (DBHF) approach and chiral perturbation theory based on realistic NN interactions

However, **different results for the high-dense behavior of the nuclear symmetry energy** have been reported so far and, in particular,

its density dependence above the saturation density is still unknown.

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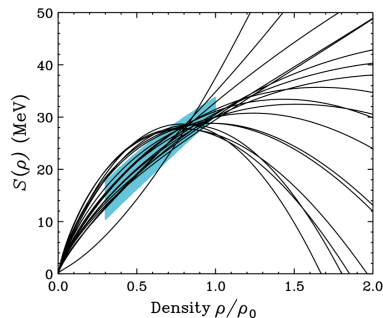


FIG. 1. (Color online) Density dependence of the symmetry energy from the Skyrme interactions used in Ref. [6]. The shaded region is obtained from HIC experiments as described in the text and corresponds to the shaded region in Fig. 2.

Motivation

We study **the Fock contribution** to the density dependence of **nuclear symmetry energy**, E_{sym} , and **its slope parameter**, L , in **relativistic theory**.

Relativistic mean-field (RMF) models:

- relativistic **Hartree (RH)** approximation
- relativistic **Hartree-Fock (RHF)** approximation

Since only **the direct diagram** is usually considered in **the RH models**, interacting mesons are treated within zero momentum transfer.

In contrast, the inclusion of the Fock terms automatically allows us to explore **the momentum dependence** at meson-nucleon vertices.

In addition, the pion contribution can be taken into account through **the exchange contribution**.

Model Lagrangian

Lagrangian density for uniform hadronic matter:

$$\mathcal{L} = \sum_{N=\rho,n} \bar{\psi}_N (i\gamma_\mu \partial^\mu - M_N) \psi_N + \mathcal{L}_M + \mathcal{L}_{\text{int}} - U_{\text{NL}}.$$

Interaction Lagrangian density: mesons (σ , ω , $\vec{\pi}$, and $\vec{\rho}$)

$$\mathcal{L}_{\text{int}} = \sum_{N=\rho,n} (\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\pi + \mathcal{L}_\rho),$$

$$\begin{aligned} \mathcal{L}_\sigma &= g_{\sigma N} \bar{\psi}_N \sigma \psi_N, & \mathcal{L}_\omega &= -g_{\omega N} \bar{\psi}_N \gamma_\mu \omega^\mu \psi_N, & \mathcal{L}_\pi &= -\frac{f_{\pi N}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma_\mu \partial^\mu \vec{\pi} \psi_N \cdot \vec{\tau}_N, \\ & \text{scalar} & \text{vector} & & \text{pseudovector} \\ \mathcal{L}_\rho &= -g_{\rho N} \bar{\psi}_N \gamma_\mu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N + \frac{f_{\rho N}}{2M} \bar{\psi}_N \sigma_{\mu\nu} \partial^\nu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N. \\ & \text{vector} & \text{tensor} & & \end{aligned}$$

The following nonlinear term is also introduced in order to reproduce the saturation properties of nuclear matter at the mean-field level:

$$U_{\text{NL}} = \frac{1}{3} g_2 \bar{\sigma}^3 + \frac{1}{4} g_3 \bar{\sigma}^4.$$

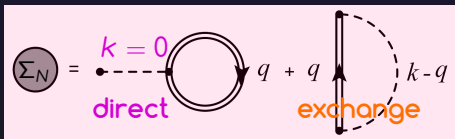
Nucleon self-energy

The nucleon self-energy is given by the Lorentz covariant form with scalar (s), time (0), and space (v) components.

$$\Sigma_N(k) = \Sigma_N^s(k) - \gamma_0 \Sigma_N^0(k) + (\vec{\gamma} \cdot \hat{k}) \Sigma_N^v, \quad N = n, \rho.$$

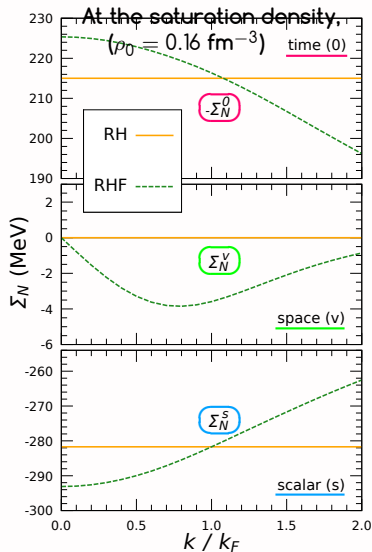
Within the relativistic Hartree-Fock approximation, the Σ_N is composed of the direct and exchange diagrams.

$$\Sigma_N^i(k) = \Sigma_N^{i,dir} + \Sigma_N^{i,ex}, \quad i = s, 0, v.$$



Inserting this form into the Dirac equation, we get the effective nucleon mass and momentum in nuclear matter.

$$M_N^*(k) = M_N + \Sigma_N^s(k), \quad k_N^*(k) = \left(k^0 + \Sigma_N^0(k), \vec{k} + \hat{k} \Sigma_N^v(k) \right).$$



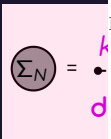
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Within the relativistic framework, the self-energy is composed of the contributions from the various meson exchanges:

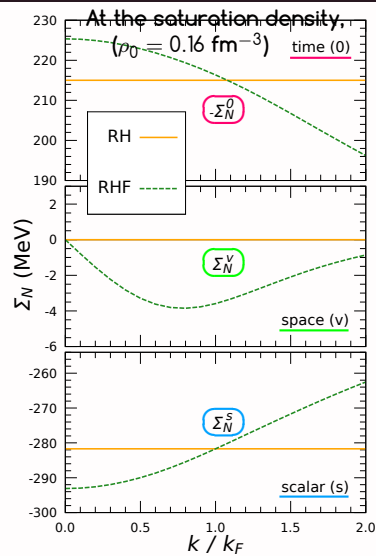
$$\Sigma_N^i(k)$$



		RH			RHF		
		Σ_N^s	$-\Sigma_N^0$	Σ_N^v	Σ_N^s	$-\Sigma_N^0$	Σ_N^v
Direct	σ	-282	0	0	-240	0	0
	ω	0	215	0	0	165	0
	ρ	0	0	0	0	0	0
Exchange	σ	-	-	-	26	27	-1
	ω	-	-	-	-57	30	-1
	ρ	-	-	-	-4	-4	-3
	ρ_{VV}	-	-	-	-1	1	0
	ρ_{TT}	-	-	-	-6	-2	0
	ρ_{VT}	-	-	-	0	0	2
	Total	-282	215	0	-282	216	-4

Inserting this form into the Dirac equation for the nucleon mass and momentum, we obtain the self-energy:

$$M_N^*(k) = M_N + \Sigma_N^s(k)$$



Experimental analyses on E_{sym}

The recent experimental analyses on the density dependence of E_{sym} have been performed by using the free Fermi Gas (FFG) model and it is separated into the **kinetic** and **potential** terms:

$$E_{\text{sym}}(\rho_B) = E_{\text{sym}}^{\text{kin}}(\rho_B) + E_{\text{sym}}^{\text{pot}}(\rho_B) \simeq E_{\text{sym}}^{\text{kin}}(\rho_0) \left(\frac{\rho_B}{\rho_0} \right)^{2/3} + E_{\text{sym}}^{\text{pot}}(\rho_0) \left(\frac{\rho_B}{\rho_0} \right)^\gamma,$$

where $\rho_{B(0)}$ is the total number (nuclear saturation) density.

In the theoretical calculations,

$$E_{\text{sym}}^{\text{kin}} = \frac{1}{6} \frac{k_F^*}{E_F^*} k_F, \quad E_{\text{sym}}^{\text{pot}} = \frac{1}{8} \rho_B \left(\frac{M_N^*}{E_F^*} \partial \Sigma_{\text{sym}}^{\text{s}} - \partial \Sigma_{\text{sym}}^{\text{0}} + \frac{k_F^*}{E_F^*} \partial \Sigma_{\text{sym}}^{\text{v}} \right) = E_{\text{sym}}^{\text{pot,s}} + E_{\text{sym}}^{\text{pot,0}} + E_{\text{sym}}^{\text{pot,v}},$$

with $k_F = k_{F\rho} = k_{F_n}$, $E_F^* = \sqrt{k_F^{*2} + M_N^{*2}}$, and

$$\partial \Sigma_{\text{sym}}^{\text{s(0)[v]}} \equiv \left(\frac{\partial}{\partial \rho_\rho} - \frac{\partial}{\partial \rho_n} \right) \left(\Sigma_\rho^{\text{s(0)[v]}} - \Sigma_n^{\text{s(0)[v]}} \right).$$

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Lorentz covariance of nucleon self-energy

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$$\begin{array}{ccc} \downarrow & & \downarrow \\ 0 & \frac{1}{2} \frac{g_\rho^2}{m_\rho^2} \rho_B & 0 \end{array}$$

(in Hartree limit.)

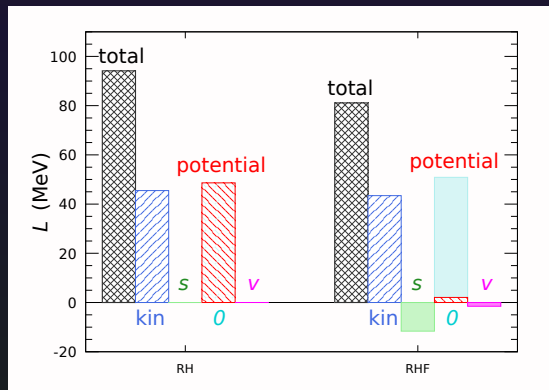
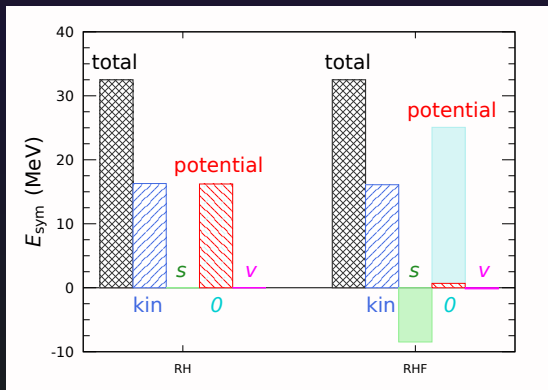
Symmetry energy and slope parameter at ρ_0

$$E_{\text{sym}} = E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot}}$$

$$= E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot},s} + E_{\text{sym}}^{\text{pot},0} + E_{\text{sym}}^{\text{pot},v}$$

$$L = L^{\text{kin}} + L^{\text{pot}}$$

$$= L^{\text{kin}} + L^{\text{pot},s} + L^{\text{pot},0} + L^{\text{pot},v}$$



Density dependence of nuclear symmetry energy

Nuclear symmetry energy, E_{sym}

Constraints:

- HIC: heavy ion collision

M. B. Tsang, et al., Phys. Rev. C **86**, 015803 (2012).

- EDP: electric dipole polarizability in ^{208}Pb experiment

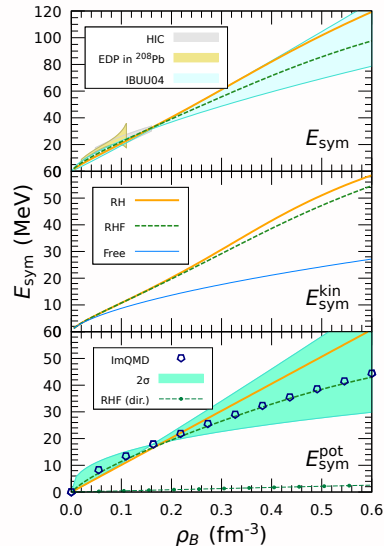
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- IBUU04: the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model

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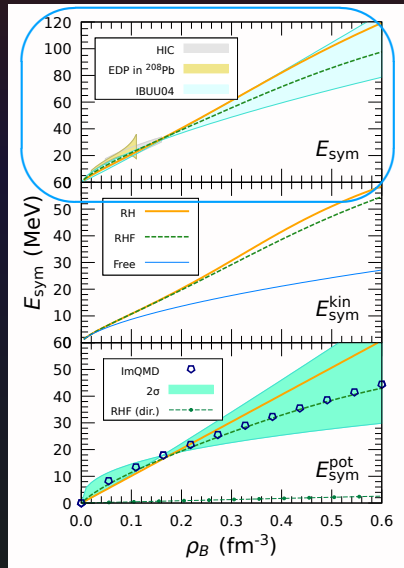
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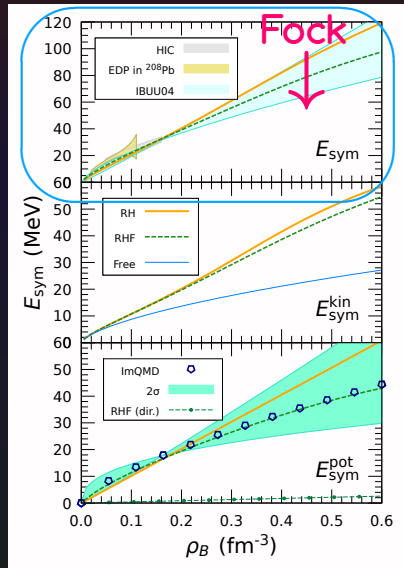
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Density dependence of nuclear symmetry energy

Kinetic term, $E_{\text{sym}}^{\text{kin}}$

- Hartree (RH) approximation

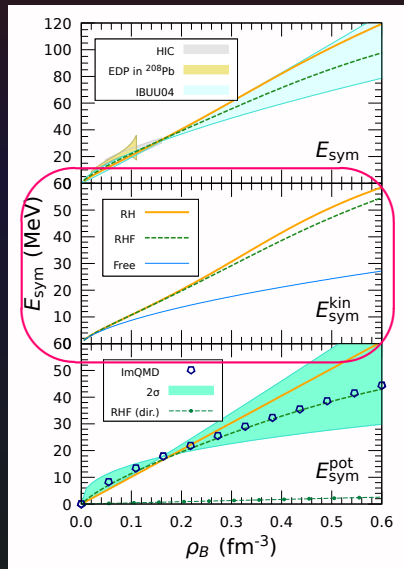
$$\frac{1}{6} \frac{k_F^2}{E_F^*} = \frac{1}{6} \frac{k_F^2}{\sqrt{k_F^2 + M_N^{*2}}}, \quad M_N^* = M_N + \Sigma_N^{s,\text{dir}} = M_N - g_{\sigma} N \bar{\sigma}$$

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- Free: interactions are ignored.

$$\frac{1}{6} \frac{k_F^2}{\sqrt{k_F^2 + M_N^2}}$$



Density dependence of nuclear symmetry energy

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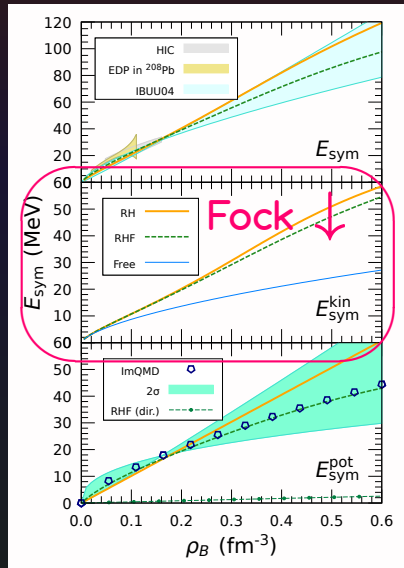
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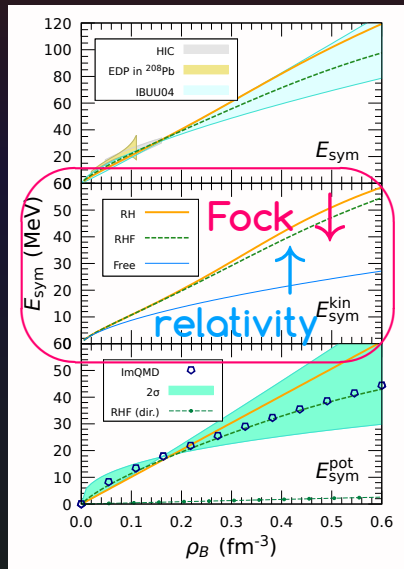
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Density dependence of nuclear symmetry energy

Potential term, $E_{\text{sym}}^{\text{pot}}$

$$\begin{aligned} E_{\text{sym}}^{\text{pot}} &= E_{\text{sym}}^{\text{pot,s}} + E_{\text{sym}}^{\text{pot,0}} + E_{\text{sym}}^{\text{pot,v}} \\ &= E_{\text{sym}}^{\text{pot,dir}} + E_{\text{sym}}^{\text{pot,ex}} \\ &= \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_B + E_{\text{sym}}^{\text{pot,ex}} \end{aligned}$$

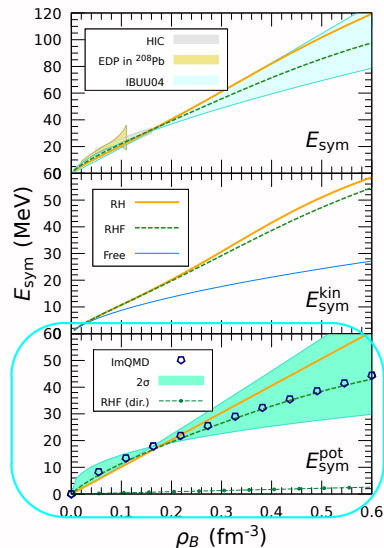
Constraints:

- **ImQMD: Improved quantum molecular dynamics transport model**

M. B. Tsang, et al., Phys. Rev. Lett. 102, 122701 (2009).

$$E_{\text{sym}}^{\text{pot}}(\rho_0) \left(\frac{\rho_B}{\rho_0} \right)^{\gamma} \text{ with } \gamma = 0.7^{+0.35}_{-0.3} \text{ (FFG model),}$$

- **Hartree (RH)** approximation: $\gamma = 1.00$
- **Hartree-Fock (RHF)** approximation: $\gamma = 0.74$



Density dependence of nuclear symmetry energy

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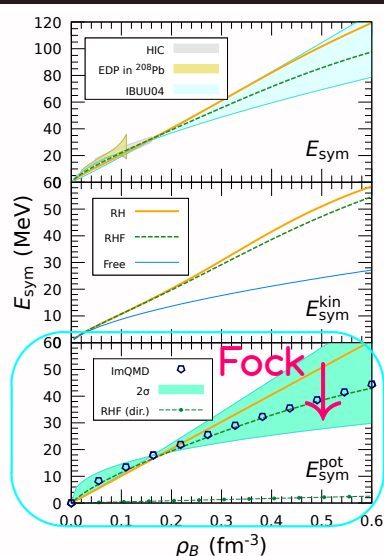
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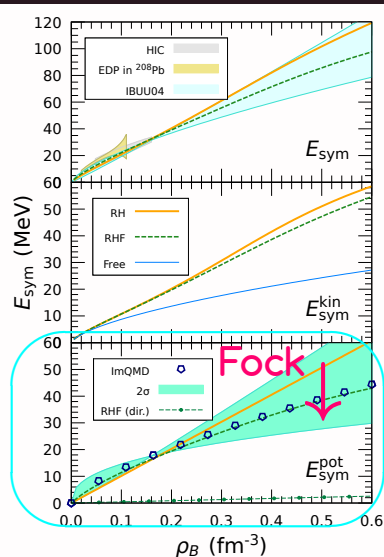
Cons: exchange contribution mainly affects $E_{\text{sym}}^{\text{pot}}$

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- Hartree-Fock (RHF) approximation: $\gamma = 0.74$



Lorentz decomposition and meson contributions

- Hartree (RH) approximation: only ρ meson

$$E_{\text{sym}}^{\text{pot}} = E_{\text{sym}}^{\text{pot,dir}} = \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_B$$

- Hartree-Fock (RHF) approximation:

$$E_{\text{sym}}^{\text{pot}} = E_{\text{sym}}^{\text{pot,dir}} + E_{\text{sym}}^{\text{pot,ex}}$$

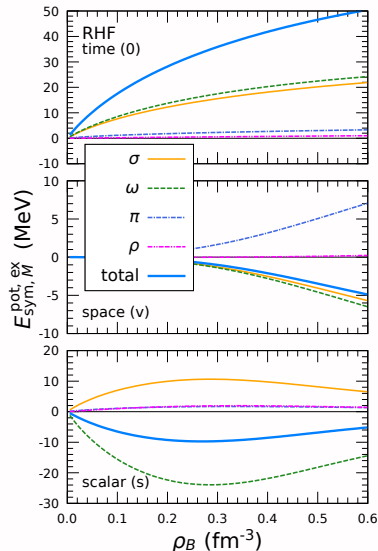
$$= \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_B + \sum_{i=s,0,v} \left(\sum_{M=\sigma,\omega,\rho,\pi} E_{\text{sym},M}^{\text{pot,ex}} \right)^i$$

Not only ρ meson but also σ , ω , and π mesons influence

$E_{\text{sym}}^{\text{pot,ex}}$.

The σ and ω mesons play a important role in $E_{\text{sym}}^{\text{pot,ex}}$.

The contributions due to ρ and π mesons are extremely small even at high densities.



Density dependence of slope parameter

$$L = L^{\text{kin}} + L^{\text{pot}}$$

with

$$L^{\text{kin}} = \frac{1}{6} k_F \left[\frac{k_F^*}{E_F^*} + \frac{k_F}{E_F^*} \left(\frac{M_N^*}{E_F^*} \right)^2 + \frac{2k_F^2}{\pi^2} \frac{k_F}{E_F^*} \frac{M_N^*}{E_F^*} \left(\frac{M_N^*}{E_F^*} \frac{\partial \Sigma_N^V}{\partial \rho_B} - \frac{k_F^*}{E_F^*} \frac{\partial \Sigma_N^S}{\partial \rho_B} \right) \right],$$

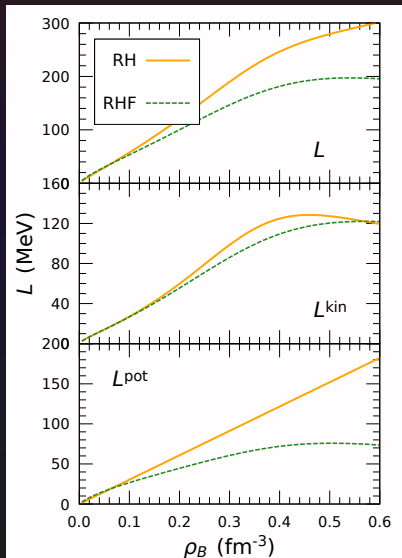
$$L^{\text{pot}} = 3E_{\text{sym}}^{\text{pot}} + \frac{3}{8} \rho_B \frac{\partial}{\partial \rho_B} \left(\frac{M_N^*}{E_F^*} \partial \Sigma_{\text{sym}}^S - \partial \Sigma_{\text{sym}}^0 + \frac{k_F^*}{E_F^*} \partial \Sigma_{\text{sym}}^V \right).$$

In RH approximation:

$$L^{\text{kin,dir}} = \frac{1}{3} \frac{k_F^2}{\sqrt{k_F^2 + M_N^{*2}}} \left[1 - \frac{k_F^2}{2(k_F^2 + M_N^{*2})} \left(1 + \frac{2M_N^* k_F}{\pi^2} \frac{\partial M_N^*}{\partial \rho_B} \right) \right],$$

$$L^{\text{pot,dir}} = \frac{3}{2} \frac{g_\rho^2}{m_\rho^2} \rho_B.$$

The **exchange contribution** prevents the slope parameter from increasing monotonically at high densities.



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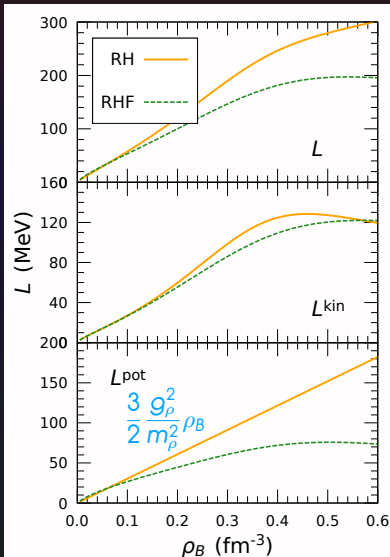
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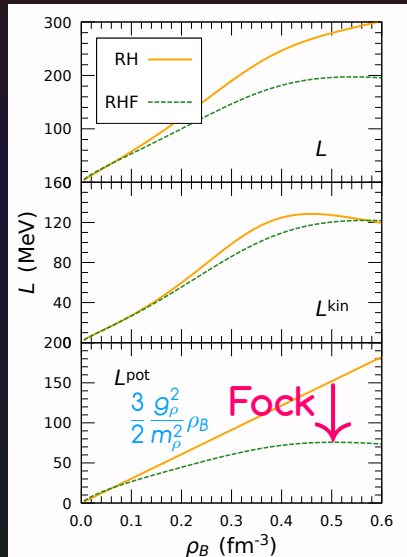
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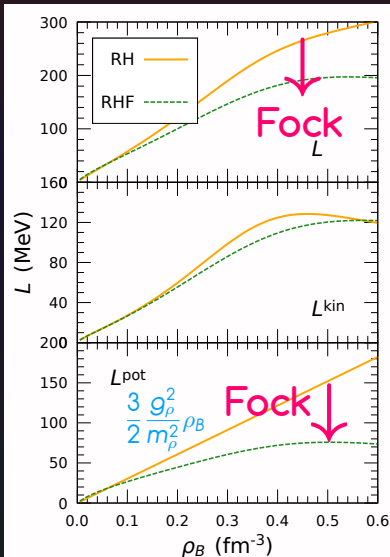
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$$L^{\text{pot,dir}} = \frac{3}{2} \frac{g_\rho^2}{m_\rho^2} \rho_B.$$

The exchange contribution prevents the slope parameter from increasing monotonically at high densities.



Summary

Motivation:

- Using **the relativistic Hartree-Fock (RHF)** approximation, we study the effect of **the Fock terms** on the nuclear properties not only around the saturation density but also at higher densities.
- In particular, by taking into account **the Lorentz-covariant decomposition of the nucleon self-energy**, we investigate how the momentum dependence due to **the exchange contribution** affects **the nuclear symmetry energy, E_{sym}** , and **its slope parameter, L** .

Summary

Results:

- We find that **the Fock contribution** suppresses the nuclear symmetry energy, E_{sym} , at the densities around and above the saturation density.
- Not only the isovector-vector (ρ) meson but also the isoscalar-scalar (σ) and isoscalar-vector (ω) mesons and pion make significant influence on the potential term of E_{sym} through **the exchange diagram**.
- **The exchange contribution** prevents **the slope parameter, L** , from increasing monotonically at high densities.

Thank You for Your Attention.