The Symmetry Energy in Neutron Stars: Constraints from GW170817 and Direct Urca Cooling
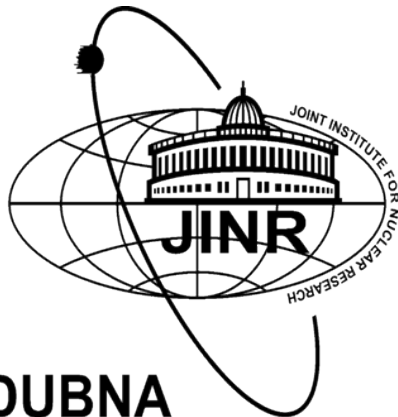
David E. Álvarez Castillo
Joint Institute for Nuclear Research
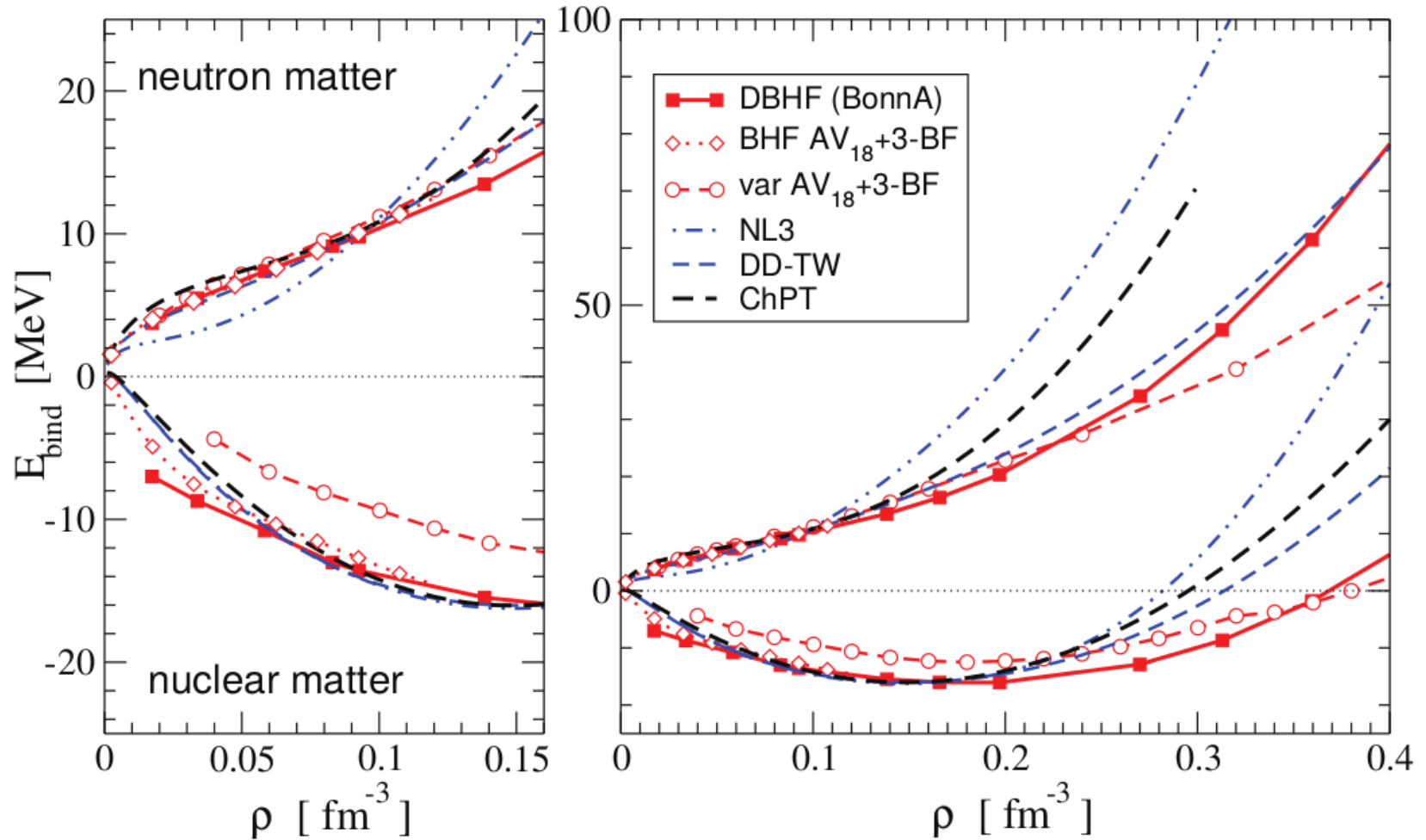
**NuSYM2018
Busan, South Korea
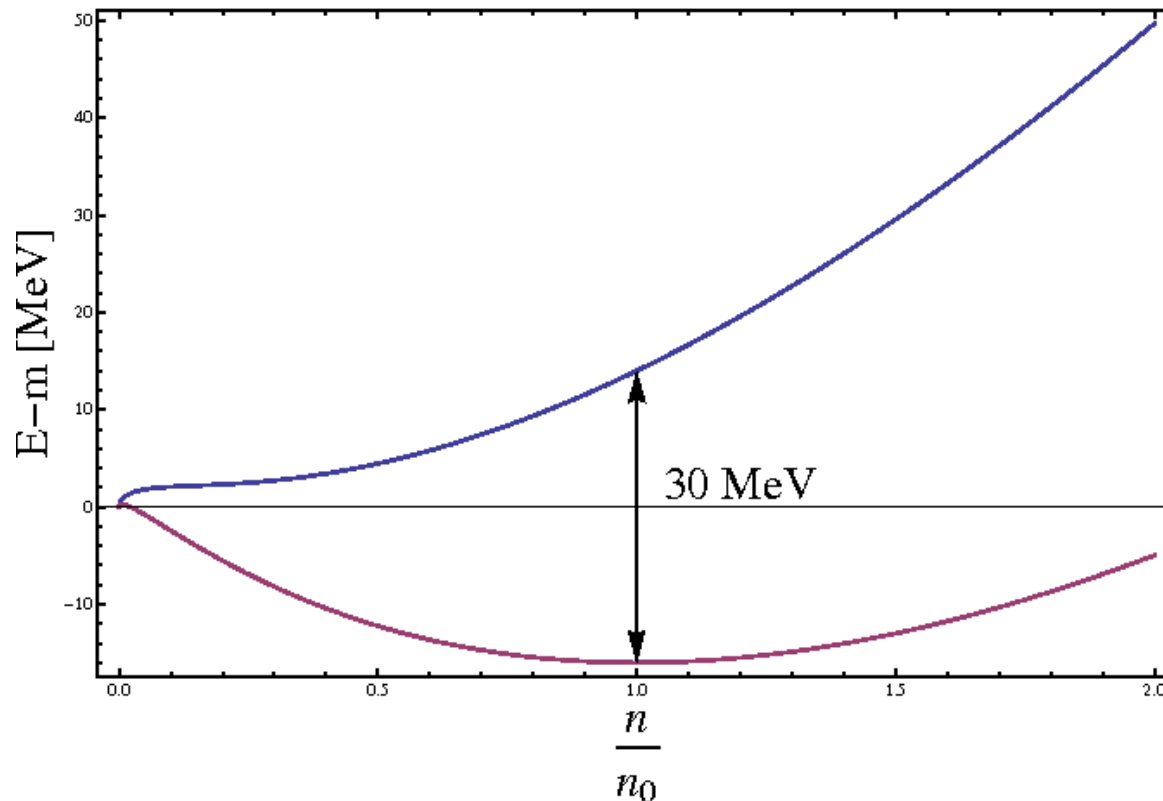September 10, 2018**

Outline

- A few words on the nuclear symmetry.

- A brief introduction to the neutron star equation of state.

- Astrophysics measurements of compact stars: multi-messenger astronomy.

- Astrophysical implications and perspectives.

Nuclear Matter

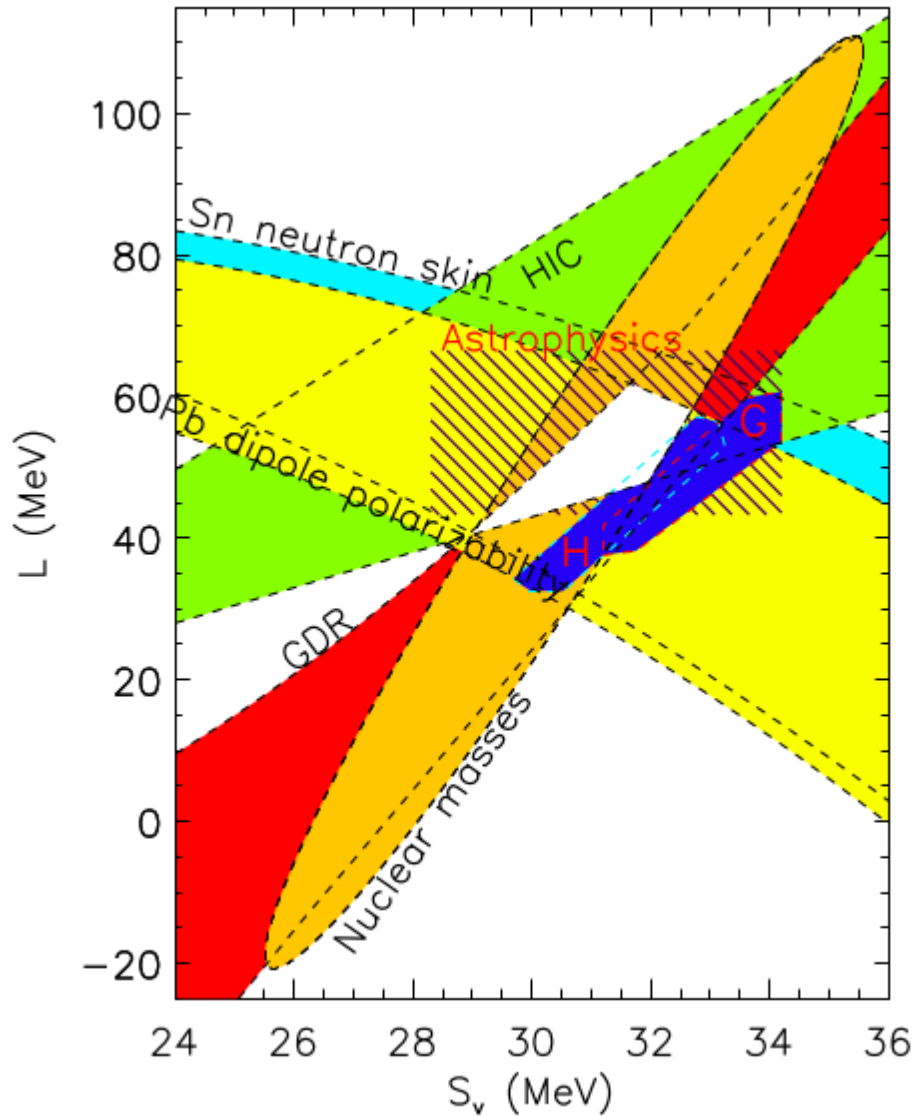C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Nuclear Symmetry Energy

is the difference between symmetric nuclear matter and pure neutron matter:

$$E(n,x) = E(n, x = 1/2) + E_s(n) * \alpha^2(x) + E_q(n) * \alpha^4(x) + O(\alpha^6(x))$$

where α=1-2x

Measuring the symmetry energy

Lattimer and Lim (2013) ApJ 771 51

Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$
\begin{aligned}
E_{\text{tot}}(n, \{x_i\}) &= E_b(n, x_p) + E_{\text{lep}}(n, x_e, x_\mu) , \\
E_b(n, x_p) &= E_0(n) + S(n, x_p) \\
E_{\text{lep}}(n, x_e, x_\mu) &= E_e(n, x_e) + E_\mu(n, x_\mu) ,
\end{aligned}
$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$
\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,
$$

resulting in $S(n, x_p) = (1 - 2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$
E_l(n, x_l) = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} \left[\sqrt{1 + z_l^2} \left(1 + \frac{z_l^2}{2}\right) - \frac{z_l^4}{2} \operatorname{Arsinh}\left(\frac{1}{z_l}\right) \right] ,
$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$
E_l(n, x_l)\big|_{m_l=0} = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4} \left(3\pi^2 n\right)^{1/3} x_l^{4/3} .
$$

Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i,j = n,p,e,\mu \ ,$$

where $\varepsilon_i = n \, E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x)E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3}(x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\text{tot}}}{\partial n} \right) \ .$

Neutron Star EoS

- Nuclear interaction:

$$E(n, x) = E(n, x = 1/2) + E_s(n) * \alpha^2(x),$$

- Beta equilibrium: $\quad \mu_n - \mu_p = \mu_e = \mu_\mu$

- 2 phase construction under Gibbs
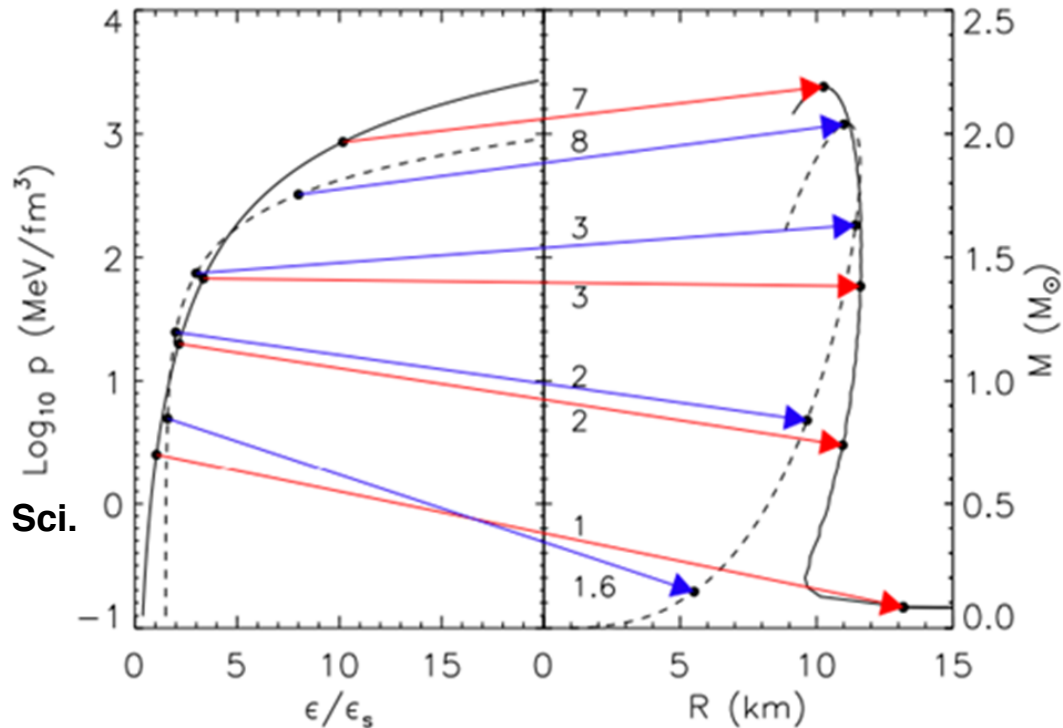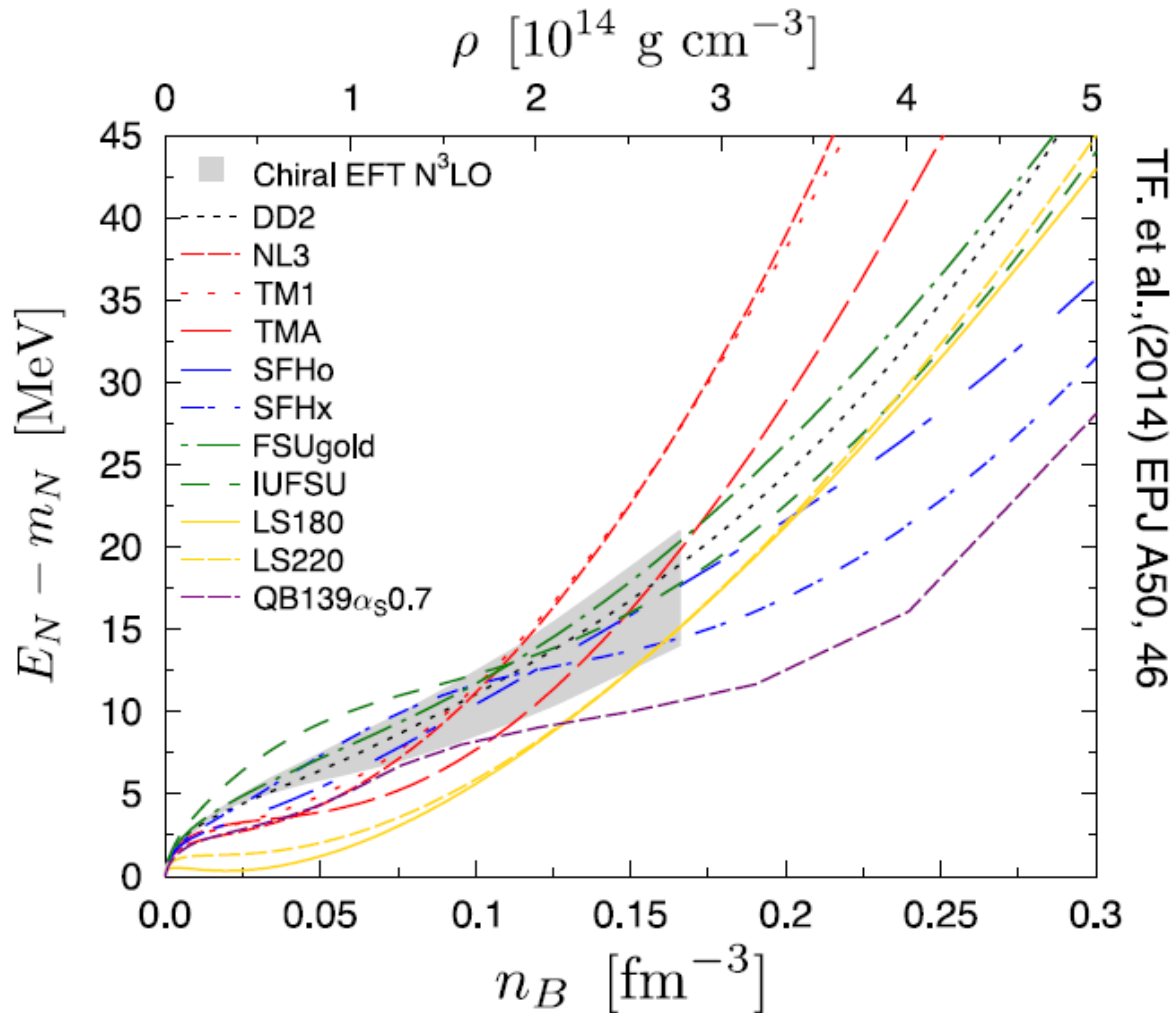 conditions: $\quad p^I = p^{II} \qquad \mu_n^I = \mu_n^{II} \qquad \mu_e^I = \mu_e^{II}$

- Charge neutrality: $\quad x_p = x_e$

- TOV equations + Equation of State

$$\frac{dp}{dr} = -\frac{(\rho + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)} \qquad p(\rho)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

Compact Star Sequences
(M-R ↔ EoS)

- TOV Equations

- Equation of State (EoS)

$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$
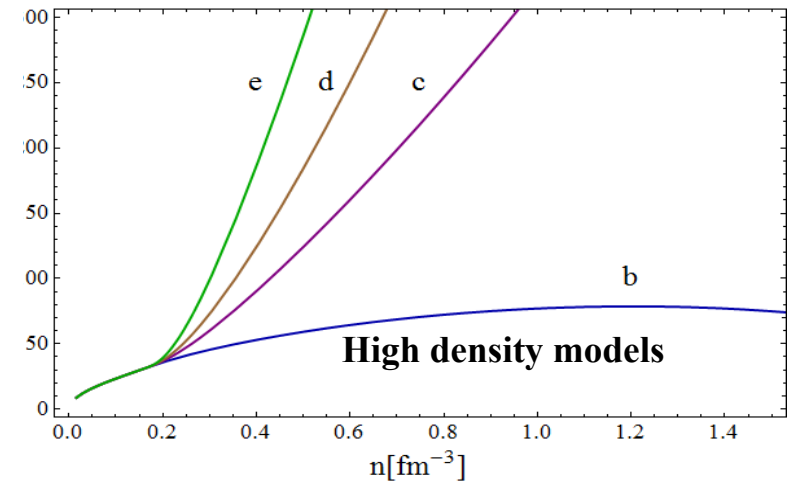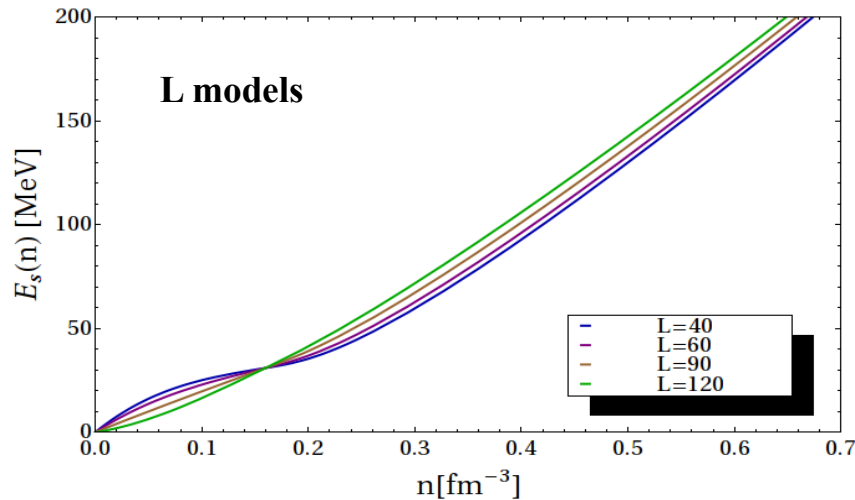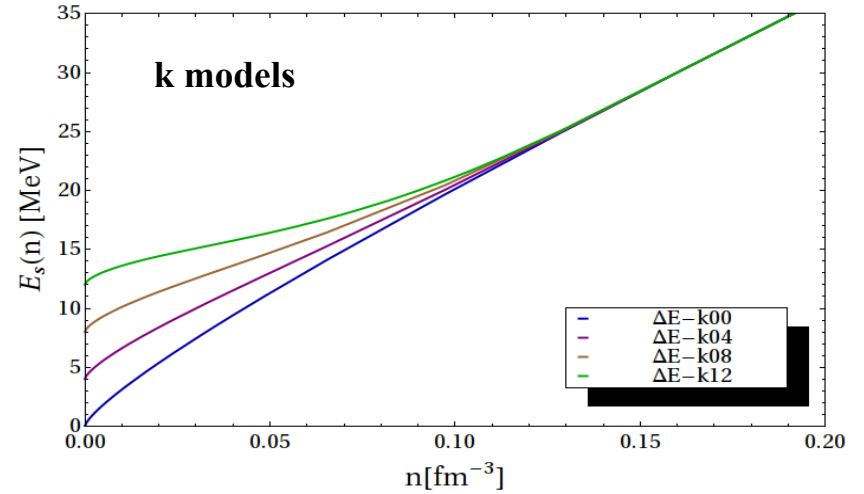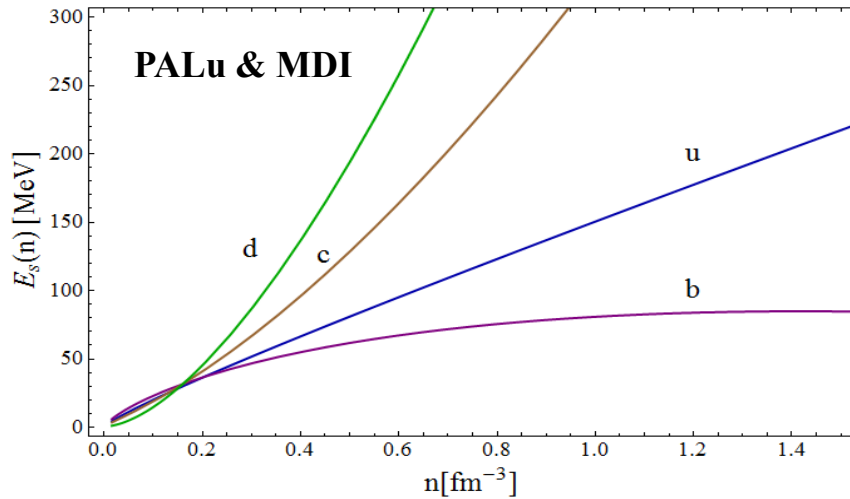
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

$$p(\varepsilon)$$
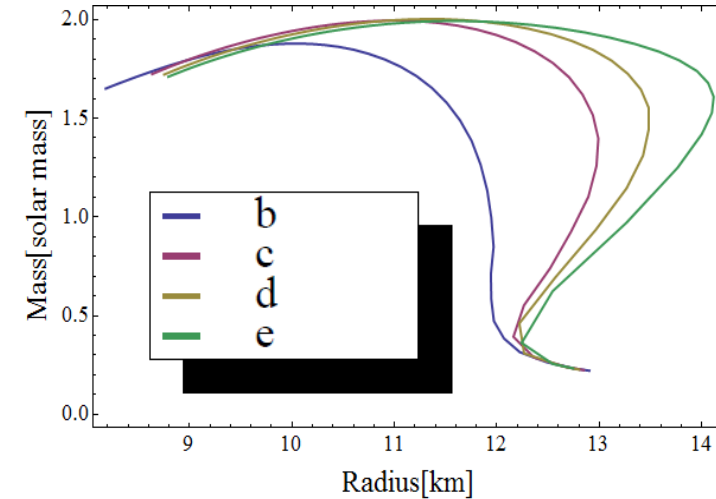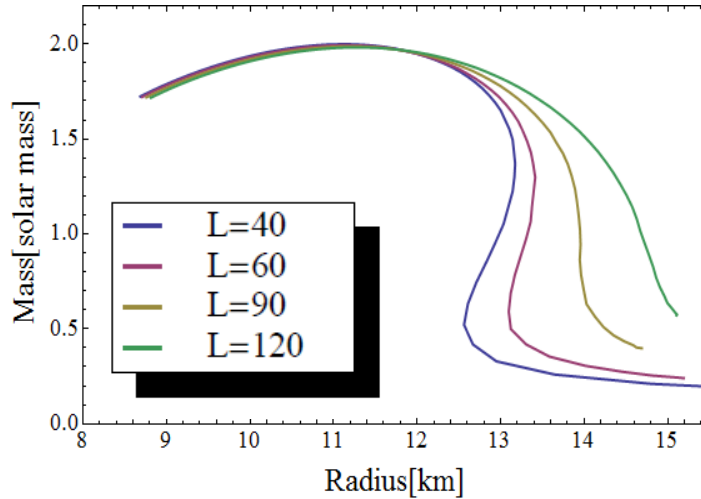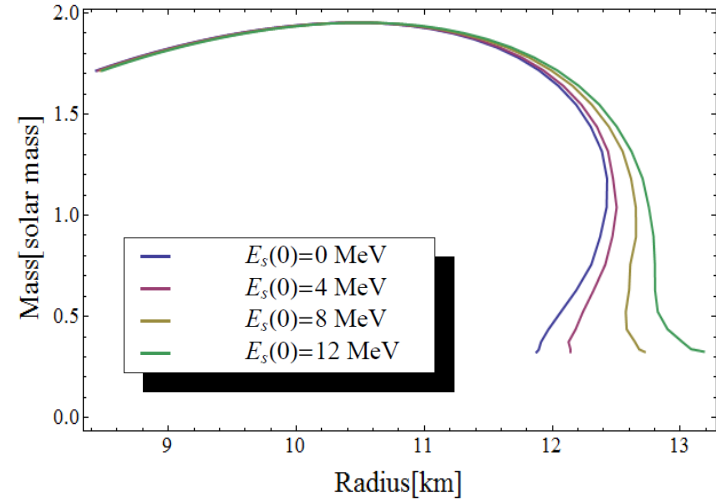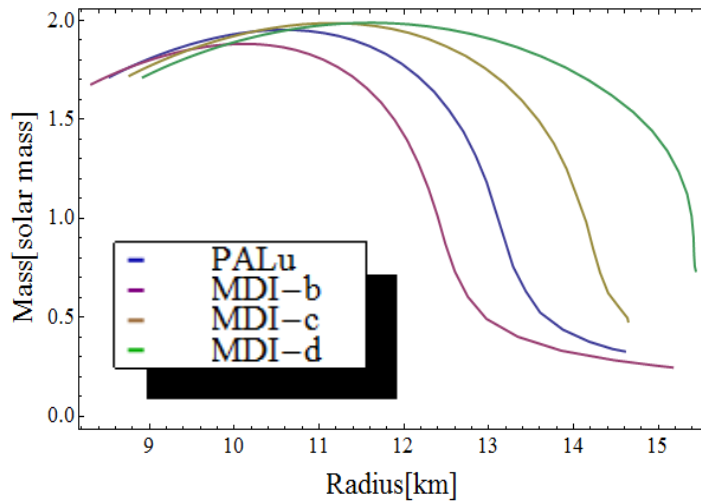
Nuclear Equation of State

Compilation of Neutron matter Equations of State; T. Fischer et al., EPJA 50, 46 (2014)

DD2 equation of state (dotted line) [S. Typel et al., Phys. Rev. C 81 (2010)] compares very well with chiral EFT N3LO (grey band)
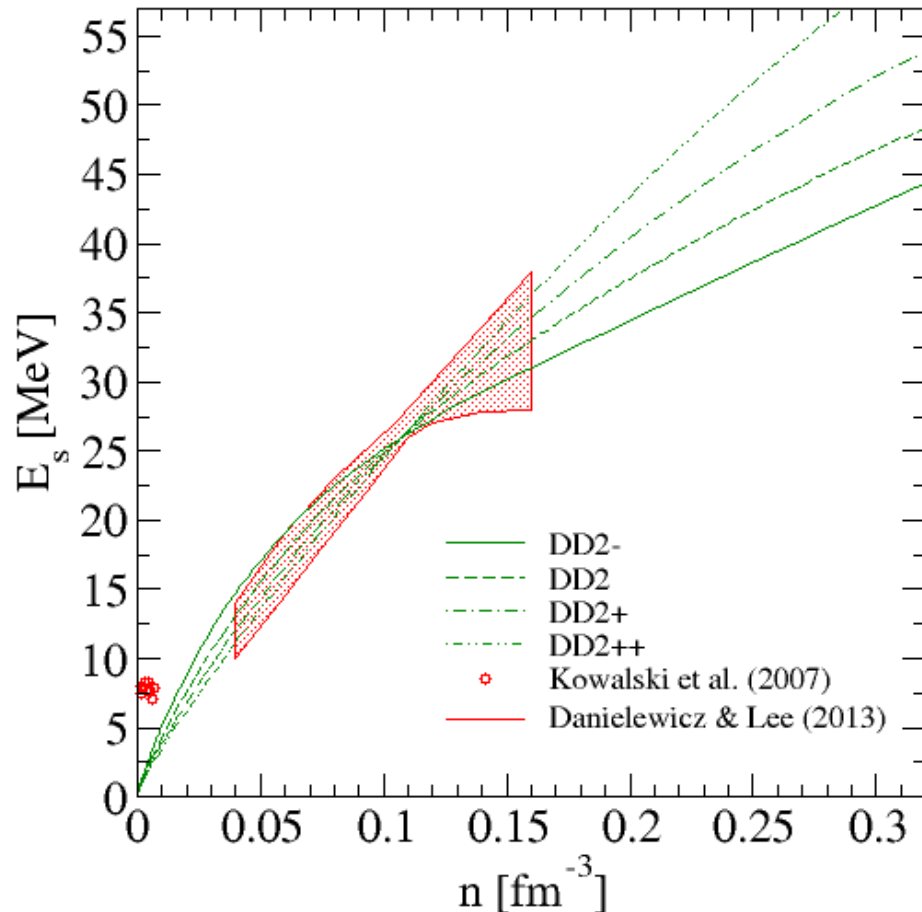
Symmetry energy effects

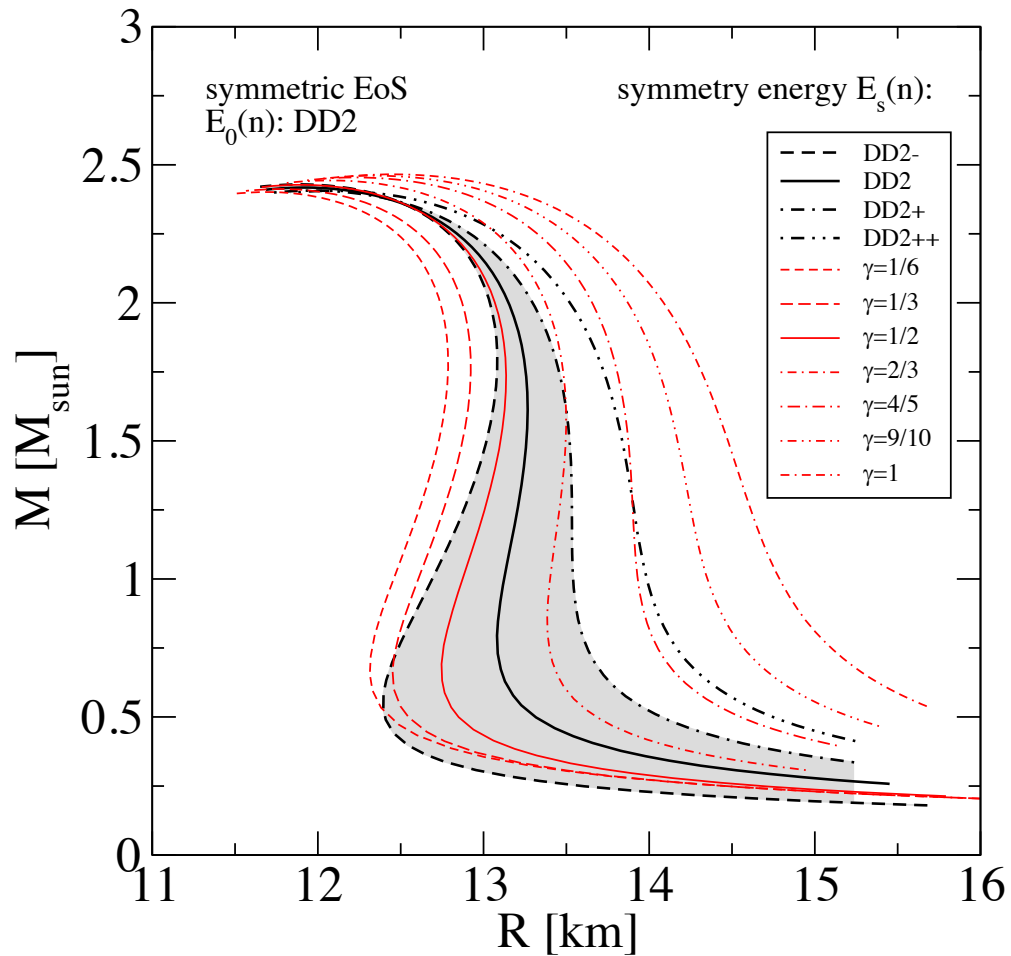**S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368**

Symmetry energy effects

S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368

Nuclear Symmetry Energy

$E_s(n)$	Parametrization		$\Gamma_\rho(n_{\text{sat}})$	a_ρ
Stiff	DD2+	DD2F+	3.806504	0.342181
Medium	DD2	DD2F	3.626940	0.518903
Soft	DD2−	DD2F−	3.398486	0.742082

S. Typel, Phys. Rev. C 89, 064321 (2014)

Symmetry energy effects

Neutron Star Cooling Processes

Process Name	Process	Emissivity Q_ν (erg cm^{-3} s^{-1})	Reference
Bremsstrahlung	$n + n \rightarrow n + n + \nu_e + \bar{\nu}_e$ $n + p \rightarrow n + p + \nu_e + \bar{\nu}_e$ $p + p \rightarrow p + p + \nu_e + \bar{\nu}_e$	$\simeq 10^{19} T_9^8$	Page, Geppert and Weber [92]
Modified Urca	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$ $n + p + e^- \rightarrow n + n + \nu_e$	$\simeq 10^{20} T_9^8$	Friman and Maxwell [93]
Direct Urca	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\simeq 10^{27} T_9^6$	Lattimer et al. [94]
Quark Urca	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$	$\simeq 10^{26} \alpha_c T_9^6$	Iwamoto [95]
Kaon Condensate	$n + K^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + K^- + \nu_e$	$\simeq 10^{24} T_9^6$	Brown et al. [96]
Pion Condensate	$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + \pi^- + \nu_e$	$\simeq 10^{26} T_9^6$	Maxwell et al. [97]

Direct Urca is the fastest cooling process.
Threshold for onset: $p_{F,n} < p_{F,p+}\ p_{F,e}$. For electrons only then $x_{DU} = 1/9$.

DUrca Process Constraint

E_s plays an important role in determination of the activation of the direct Urca (DU) cooling process

$$n \rightarrow p + e + \bar{\nu}_e.$$

If the central density in a neutron star exceeds the critical value which allows the DU process to operate then this process triggers a dramatic drop of the core temperature due to rapid energy loss by neutrino emission. This process can therefore not be operative in typical neutron stars as we do observe cooling neutron stars much older than the typical transport timescale (~ 1000 years) with surface temperatures that are not compatible with DU cooling.

The DU threshold condition is derived from the triangle inequality for the Fermi momenta of neutron, proton and electron (neutrinos are neglected):

$$n_n^{1/3} < n_p^{1/3} + n_e^{1/3} \, ,$$

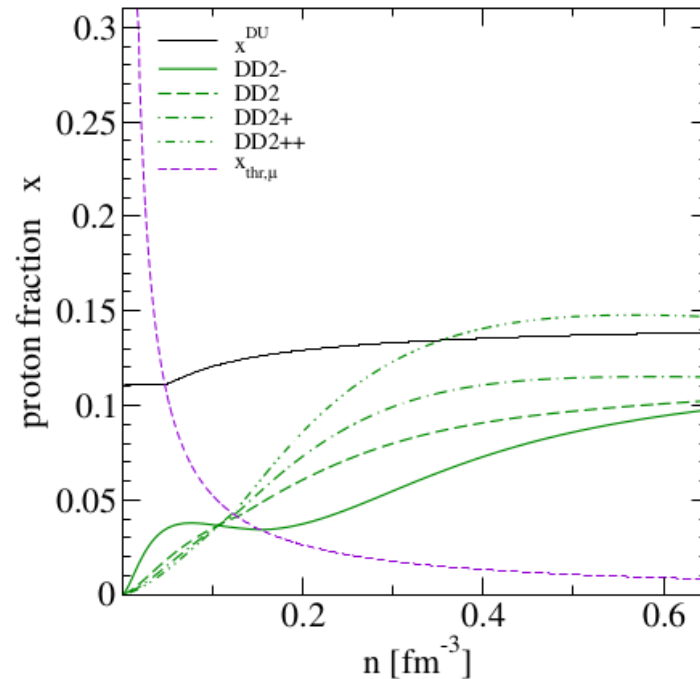which can be formulated in terms of proton and muon fractions as

$$(1-x)^{1/3} < x^{1/3} + (x - x_\mu)^{1/3} \, .$$

Below the muon threshold $x^{\mathrm{DU}} = 1/9 = 11.1$.

DUrca Process Constraint

E_s	$n_{DU}[\text{fm}^{-3}]$	$n_c[\text{fm}^{-3}]$				
		1.25	1.40	1.60	1.80	2.00
DD2-	-	0.331	0.352	0.385	0.423	0.472
DD2	-	0.331	0.354	0.387	0.426	0.478
DD2+	-	0.325	0.349	0.384	0.425	0.479
DD2++	0.354	0.314	0.339	**0.375**	**0.416**	**0.469**

D. E. Alvarez-Castillo, D. Blaschke and T. Klahn. (2016)
arXiv: 1604.08575

Symmetry energy Conjecture
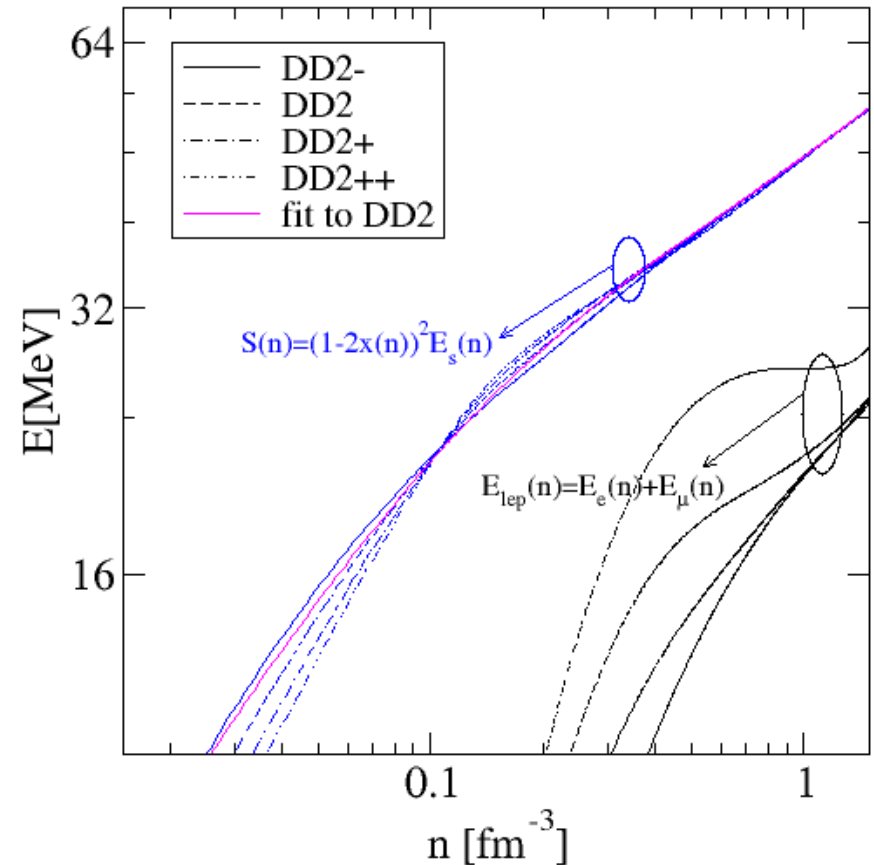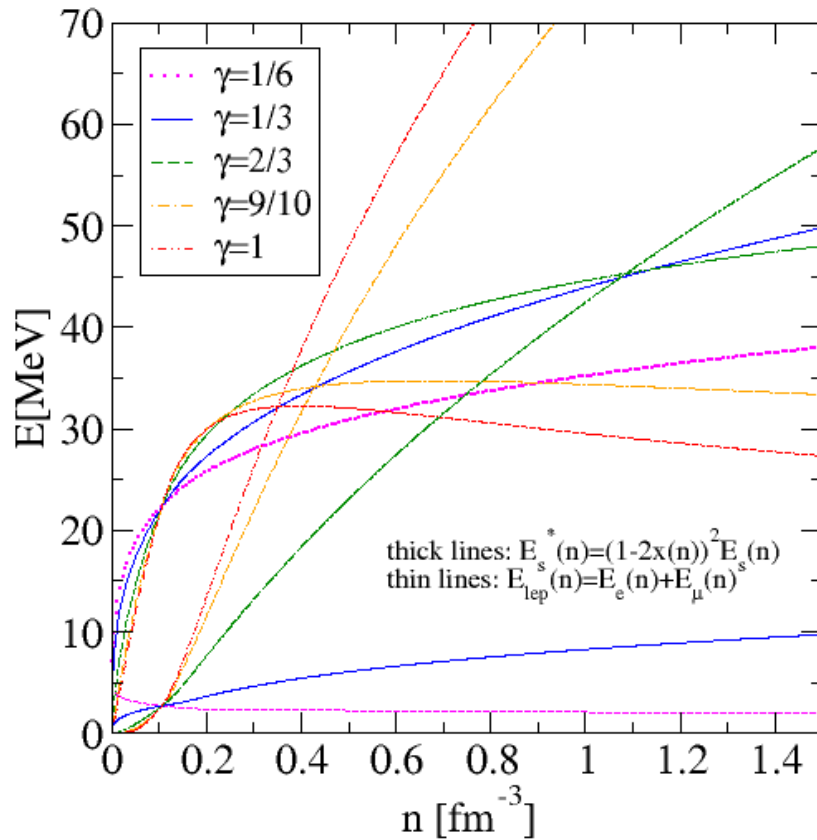
Klaehn et al. PhysRev C74 (2006)

FIG. 7. (Color online) Density dependence of the asymmetry contribution to the energy per particle (left panel) and of the proton fraction (right panel) in NSM. Encircled curves correspond to EoSs that violate the DU-constraint.
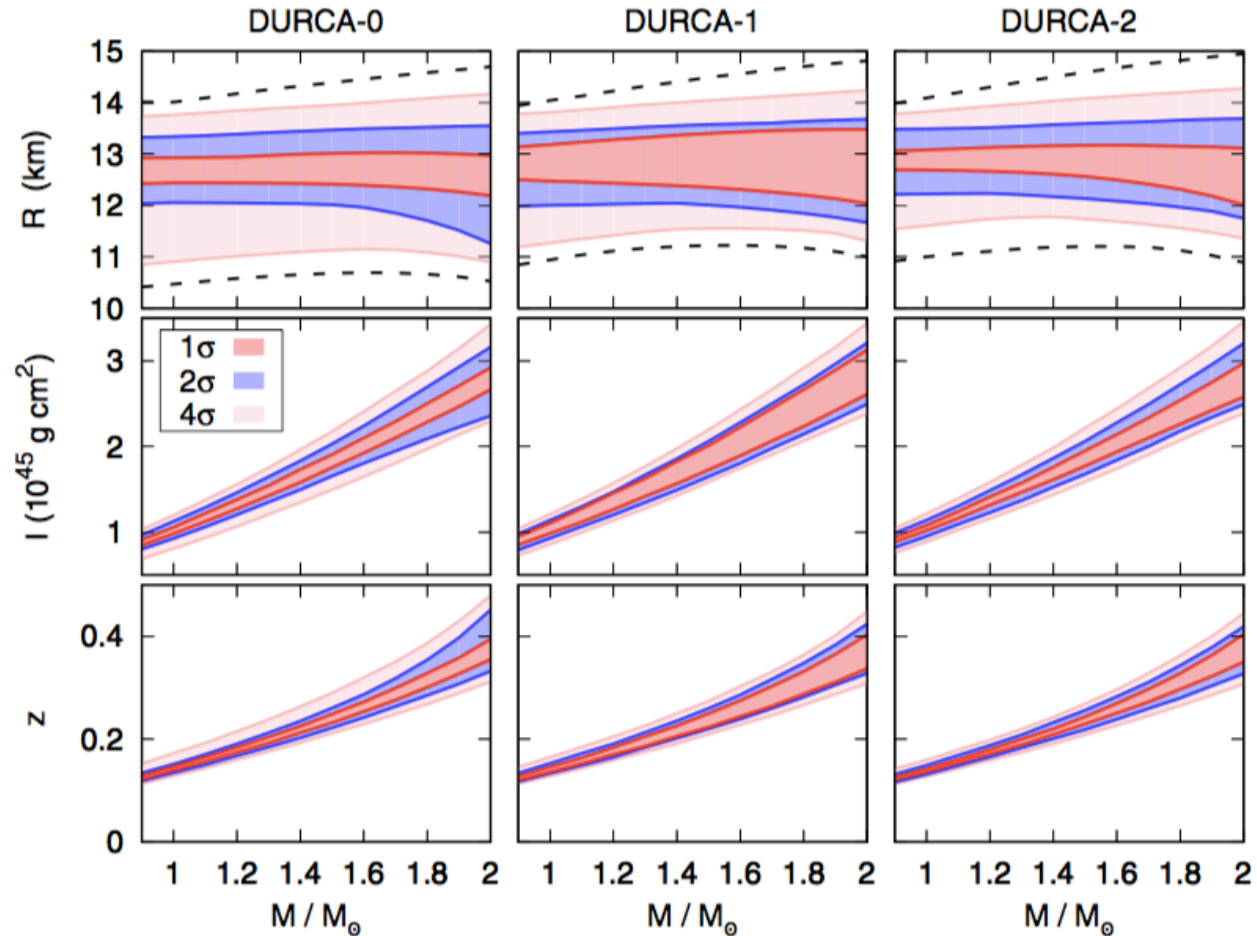
Universal symmetry energy contribution

D. E. Alvarez-Castillo, D. Blaschke and T. Klahn. (2016)
arXiv: 1604.08575

The symmetry energy contribution to the neutron star EoS behaves universal!
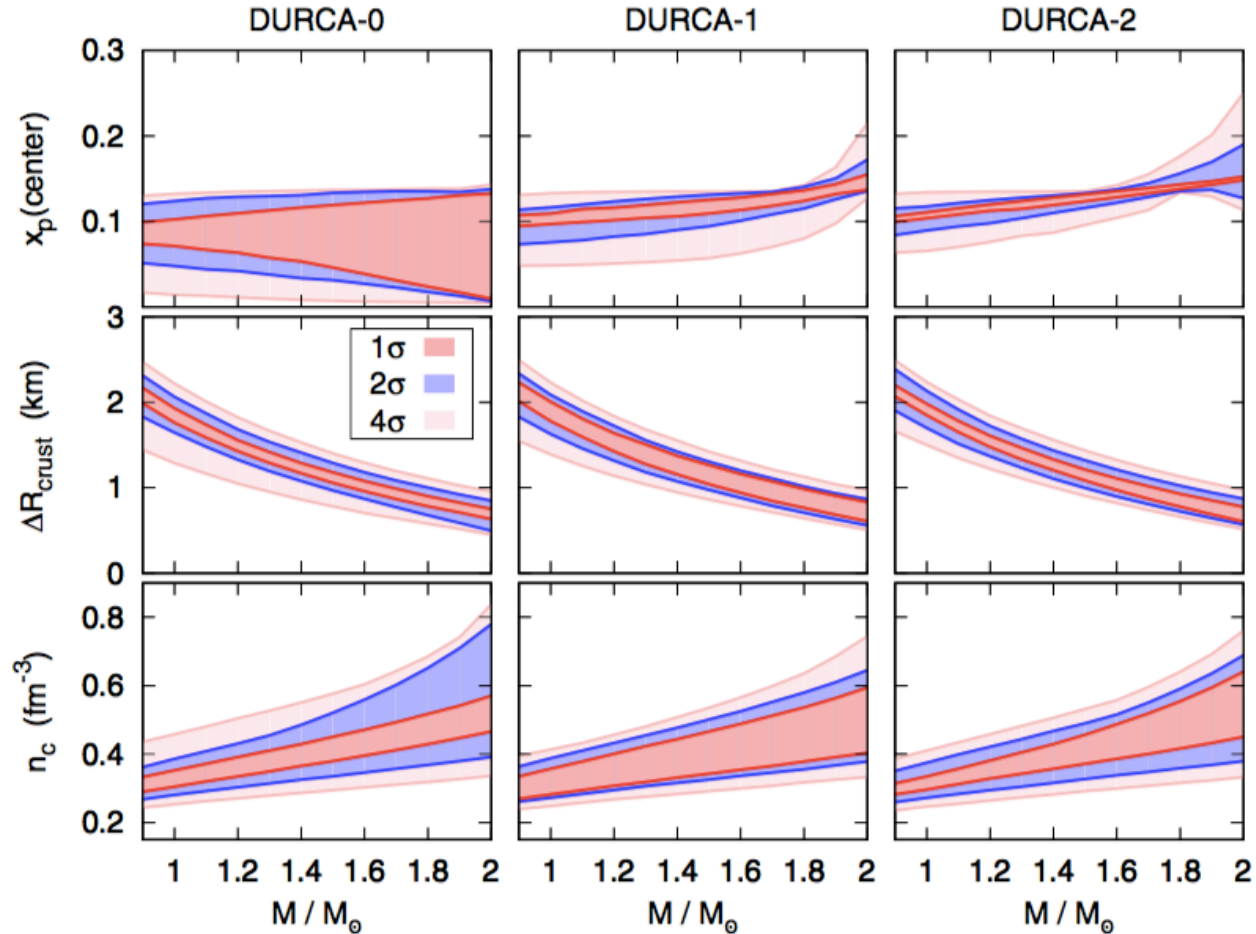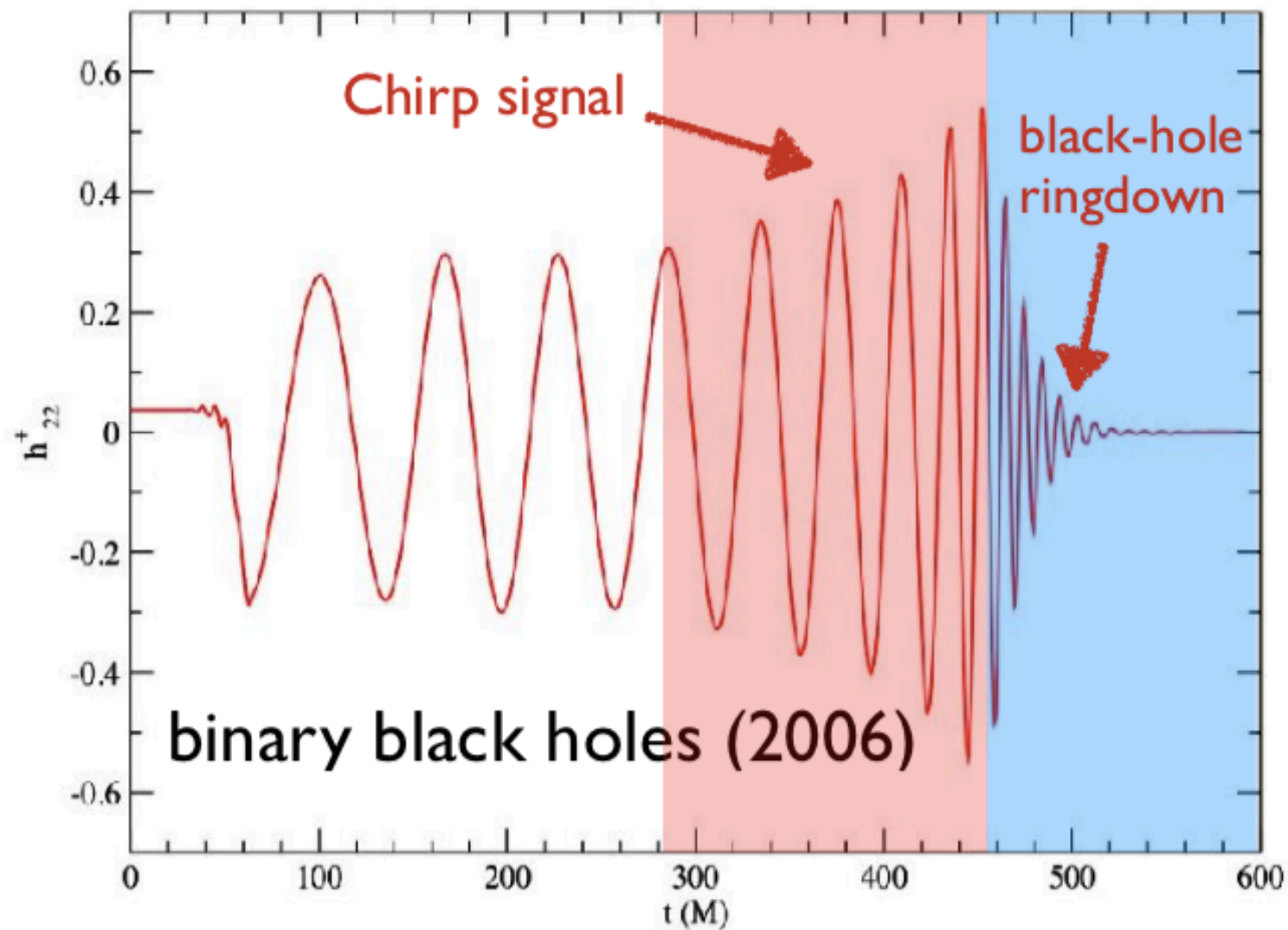
Predictions for neutron stars properties

If composed exclusively of nucleons and leptons, our prediction is that neutron stars have a radius of 12.7 ± 0.4 km for masses between 1 and $2M_\odot$

J. Margueron, R. Hoffmann Casali, F. Gulminelli - Phys. Rev. C 97, 025806 (2018)

Predictions for neutron stars properties

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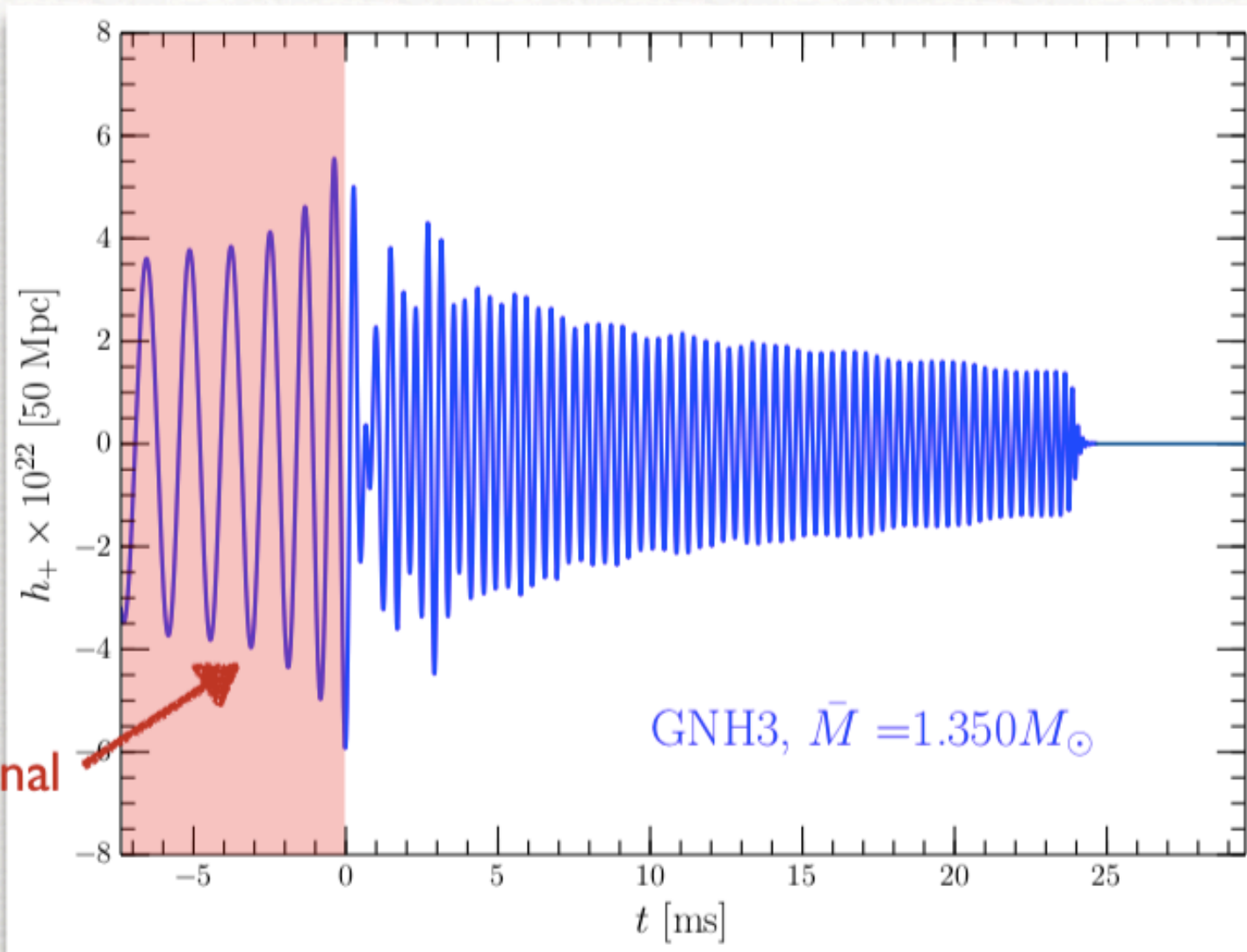
J. Margueron, R. Hoffmann Casali, F. Gulminelli - Phys. Rev. C 97, 025806 (2018)

GW170817
and
Tidal deformability

Anatomy of the GW signal

Anatomy of the GW signal

GNH3, $\bar{M} = 1.350 M_\odot$

Chirp signal

What can we learn from the inspiral II

- Waveforms incl. finite-size effects are described by tidal deformability (how a star reacts on an external tidal field)

- Offer possibility to constrain EoS because tidal deformability depends on EoS

$$\Lambda \equiv \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$$

- Corresponding to ~10 % error in radius R for nearby events (<100Mpc) (e.g. Read et al. 2013)

- Note: faithful templates to be constructed

R/M compactness (EoS dependent)

k_2 tidal love number (EoS dependent)

Computing the love number/tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

$$ds^2 = -e^{2\Phi(r)}\left[1 + H(r)Y_{20}(\theta, \varphi)\right]dt^2$$
$$+e^{2\Lambda(r)}\left[1 - H(r)Y_{20}(\theta, \varphi)\right]dr^2$$
$$+r^2\left[1 - K(r)Y_{20}(\theta, \varphi)\right]\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Following Hinderer et al. 2010

Integrate standard TOV system:

$$e^{2\Lambda} = \left(1 - \frac{2m_r}{r}\right)^{-1},$$

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon + p}\frac{dp}{dr},$$

$$\frac{dp}{dr} = -(\epsilon + p)\frac{m_r + 4\pi r^3 p}{r(r - 2m_r)},$$

$$\frac{dm_r}{dr} = 4\pi r^2\epsilon.$$

EoS to be provided ε(p)

And additional eqs. for perturbations:

$$\frac{dH}{dr} = \beta \tag{11}$$

$$\frac{d\beta}{dr} = 2\left(1 - 2\frac{m_r}{r}\right)^{-1}H\left\{-2\pi\left[5\epsilon + 9p + f(\epsilon + p)\right]\right.$$

$$\left. +\frac{3}{r^2} + 2\left(1 - 2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2} + 4\pi rp\right)^2\right\}$$

$$+\frac{2\beta}{r}\left(1 - 2\frac{m_r}{r}\right)^{-1}\left\{-1 + \frac{m_r}{r} + 2\pi r^2(\epsilon - p)\right\}.$$

(K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20

Love number

$$y = \frac{R\,\beta(R)}{H(R)}.$$

$$
\begin{aligned}
k_2 =\ & \frac{8C^5}{5}(1 - 2C)^2[2 + 2C(y - 1) - y] \\
& \times \Big\{ 2C[6 - 3y + 3C(5y - 8)] \\
& \quad + 4C^3[13 - 11y + C(3y - 2) + 2C^2(1 + y)] \\
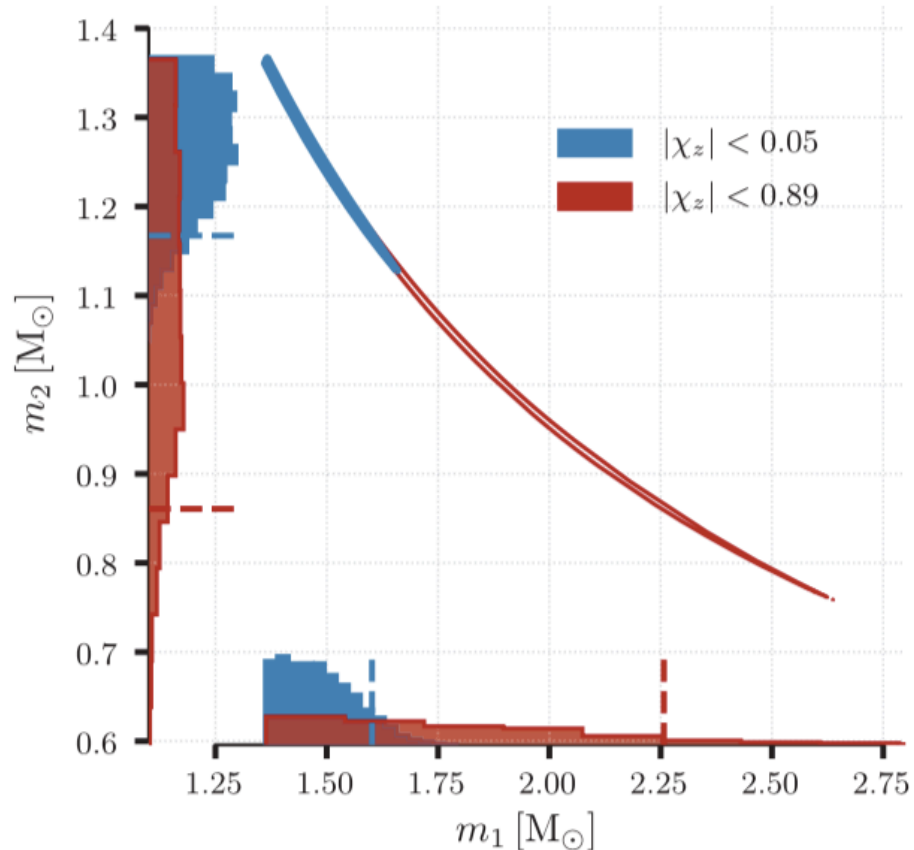& \quad + 3(1 - 2C)^2[2 - y + 2C(y - 1)]\ln(1 - 2C) \Big\}^{-1}.
\end{aligned}
$$

where $C = M/R$ is the compactness of the star.

Love number

For fixed compactness k_2 depends on EoS => tidal deformability is not a unique function of compactness for different EoSs

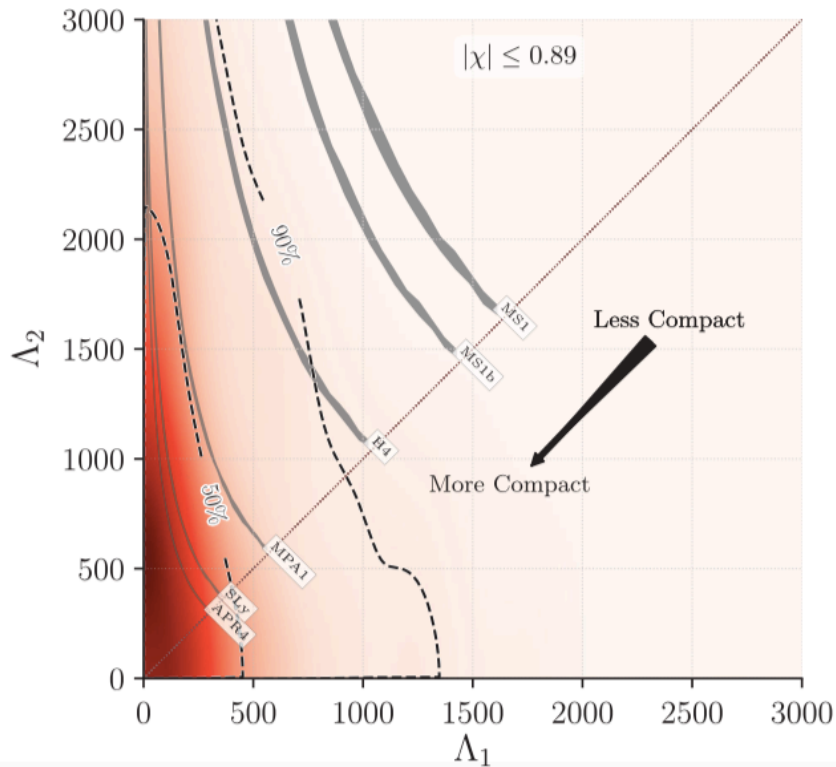Hinderer et al. 2010

Implications from GW170817

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral
B. P. Abbott et al. arXiv:1712.00451
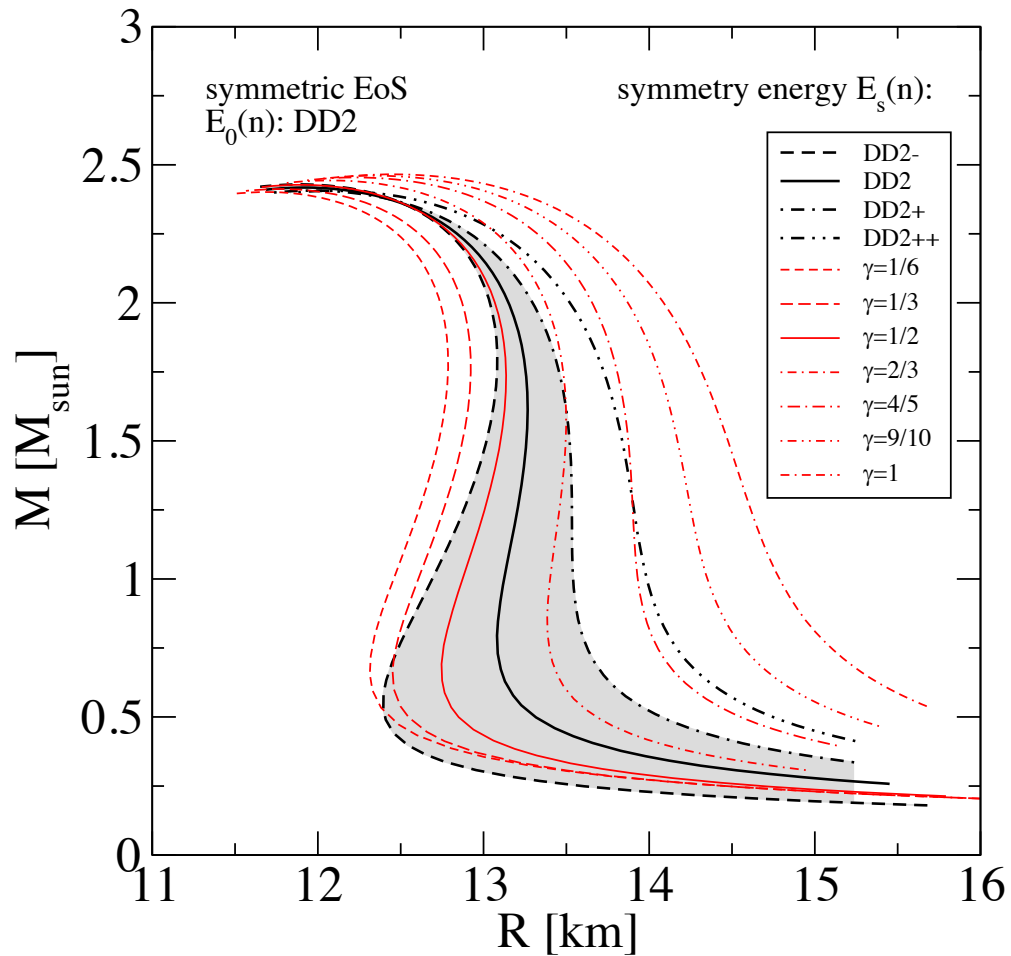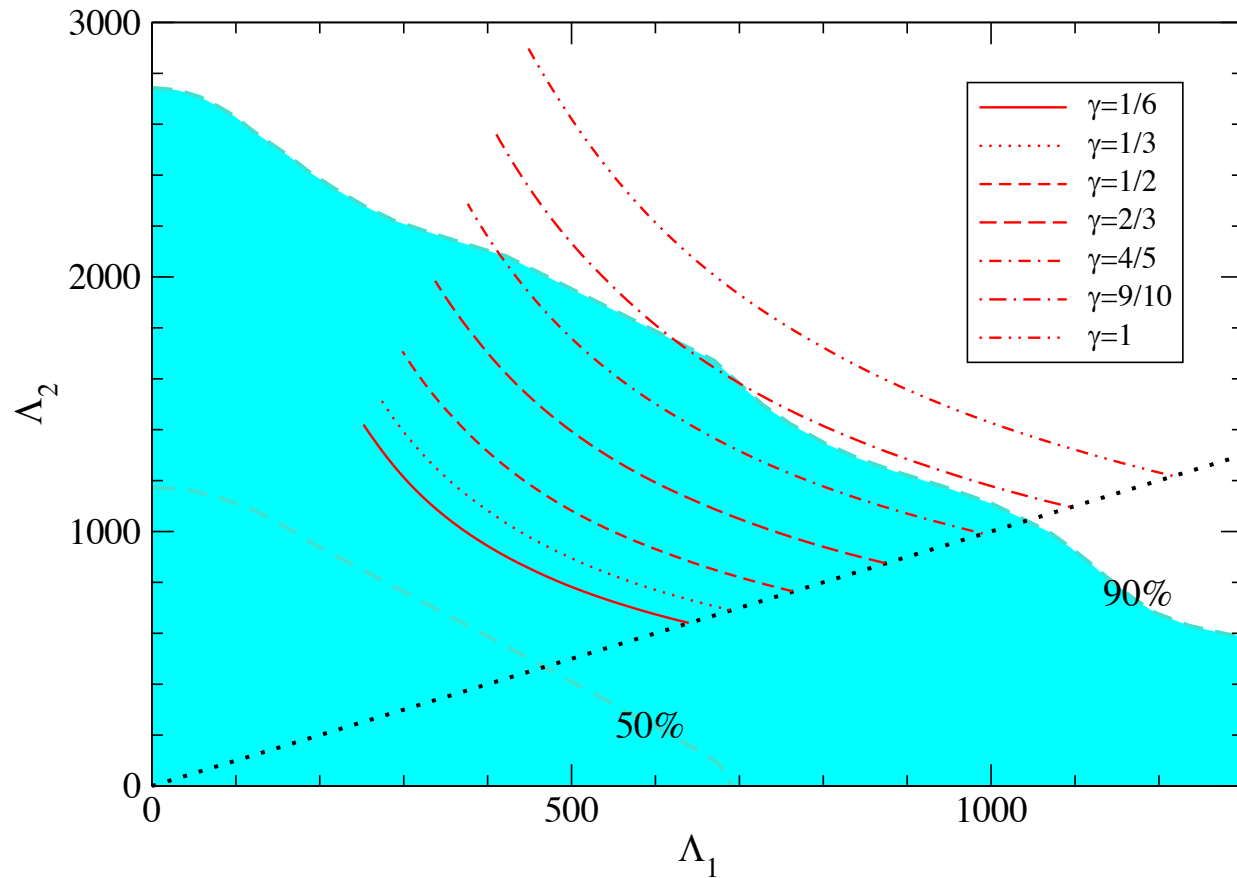
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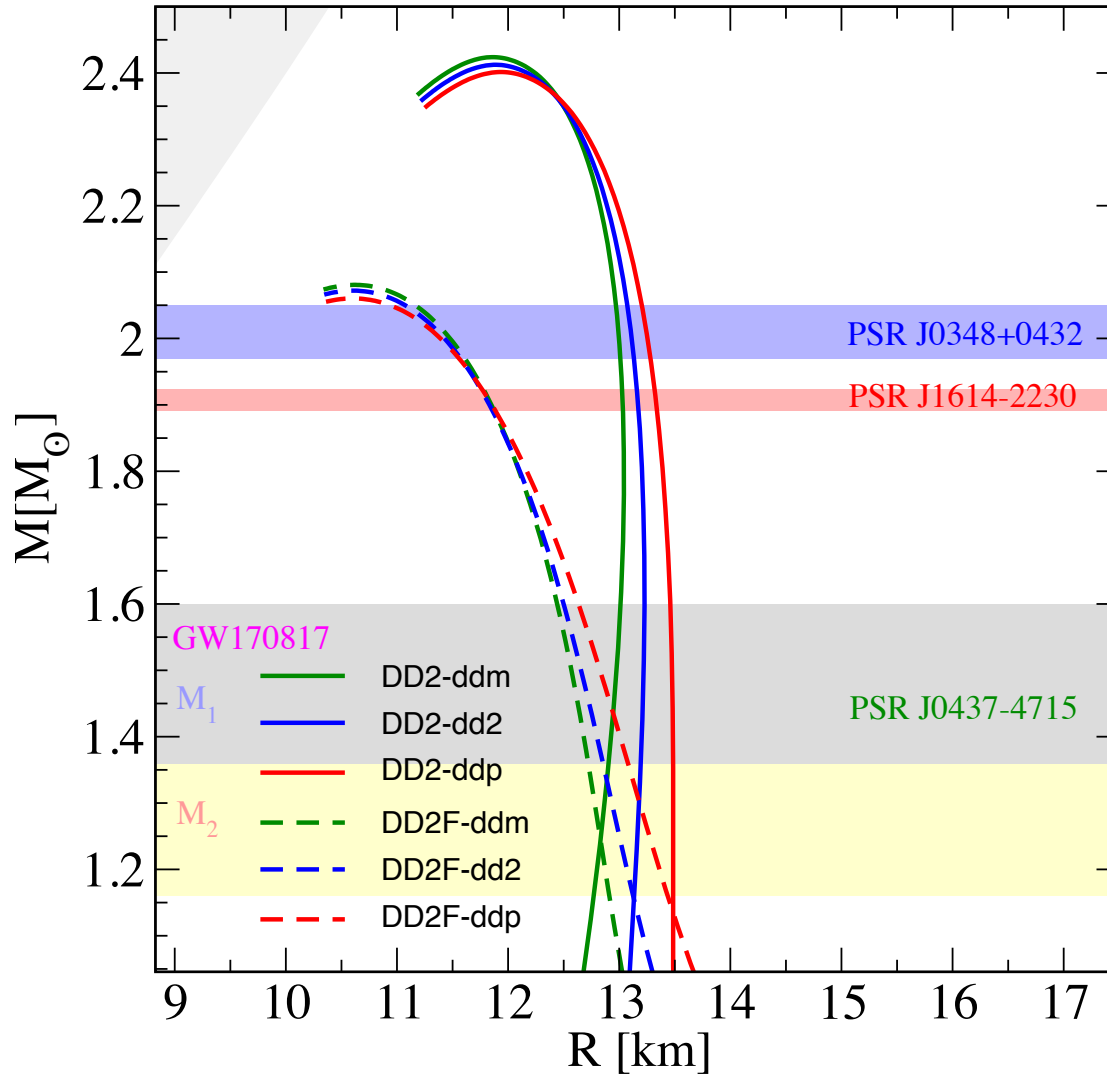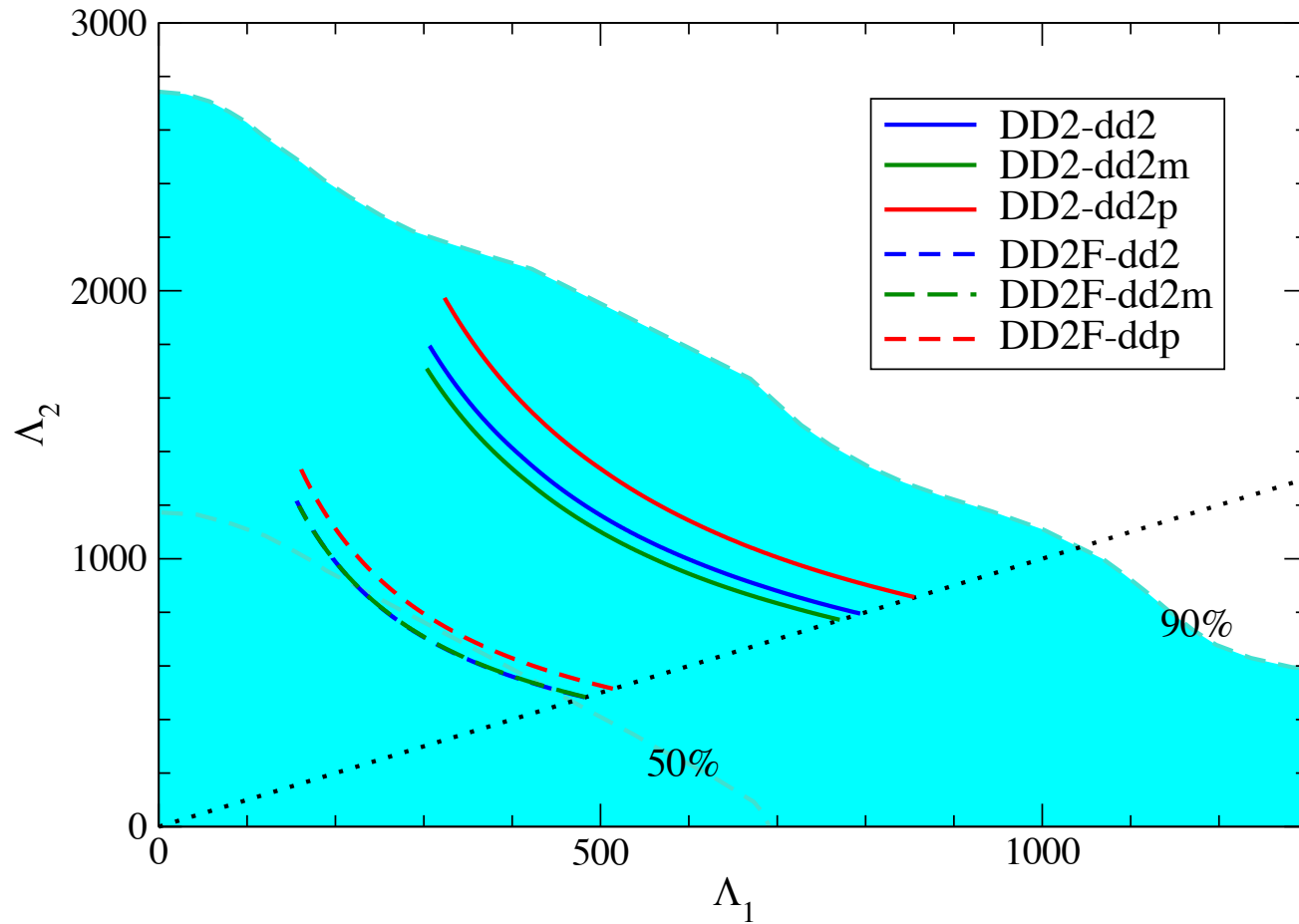# Symmetry energy effects

Implications from GW170817

Implications from GW170817

Implications from GW170817

Perspectives

NEUTRON-STAR RADIUS CONSTRAINTS FROM GW170817 AND FUTURE DETECTIONS

Andreas Bauswein,[1] Oliver Just,[2] Hans-Thomas Janka,[3] and Nikolaos Stergioulas[4]

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[2] *Astrophysical Big Bang Laboratory, RIKEN, Saitama 351-0198, Japan*
[3] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85748 Garching, Germany*
[4] *Department of Physics, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece*
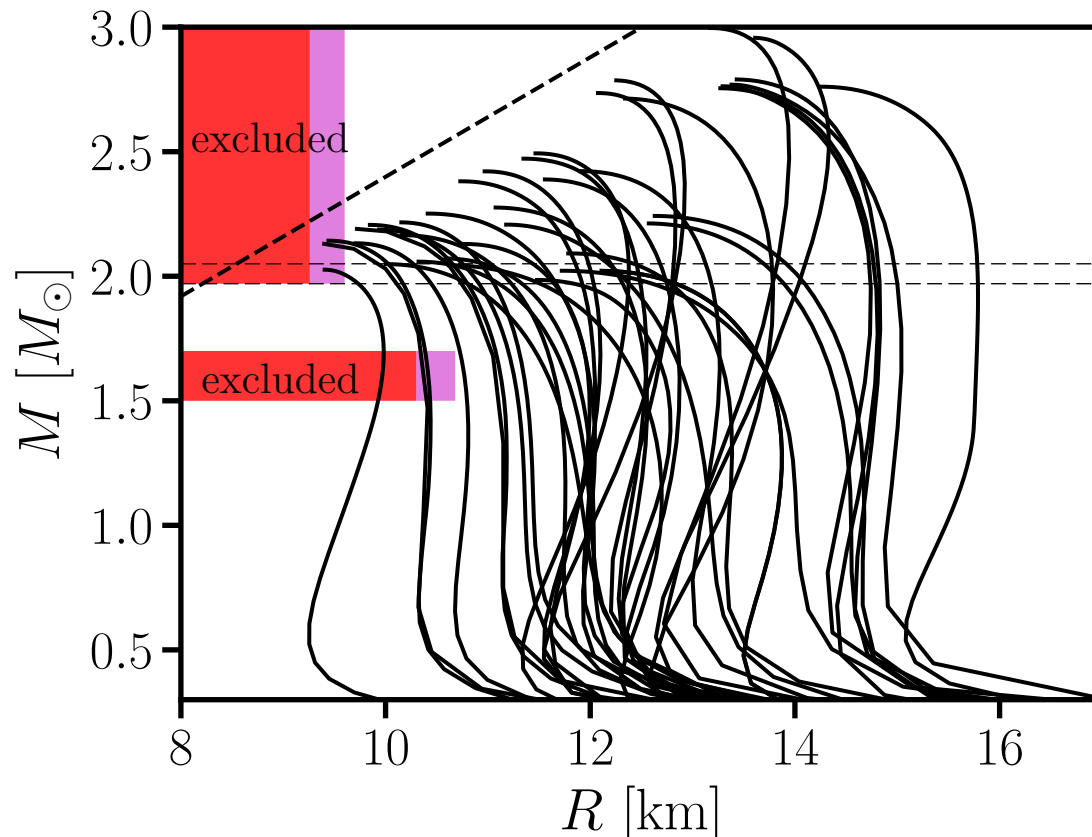
ABSTRACT

We introduce a new, powerful method to constrain properties of neutron stars (NSs). We show that the total mass of GW170817 provides a reliable constraint on the stellar radius if the merger did not result in a prompt collapse as suggested by the interpretation of associated electromagnetic emission. The radius $R_{1.6}$ of nonrotating NSs with a mass of 1.6 $M_\odot$ can be constrained to be larger than $10.68^{+0.15}_{-0.04}$ km, and the radius $R_{\rm max}$ of the nonrotating maximum mass configuration must be larger than $9.60^{+0.14}_{-0.03}$ km. We point out that detections of future events will further improve these constraints. Moreover, we show that a future event with a signature of a prompt collapse of the merger remnant will establish even stronger constraints on the NS radius from above and the maximum mass $M_{\rm max}$ of NSs from above. These constraints are particularly robust because they only require a measurement of the chirp mass and a distinction between prompt and delayed collapse of the merger remnant, which may be inferred from the electromagnetic signal or even from the presence/absence of a ringdown gravitational-wave (GW) signal. This prospect strengthens the case of our novel method of constraining NS properties, which is directly applicable to future GW events with accompanying electromagnetic counterpart observations. We emphasize that this procedure is a new way of constraining NS radii from GW detections independent of existing efforts to infer radius information from the late inspiral phase or postmerger oscillations, and it does not require particularly loud GW events.

$$M_{\rm thres} > M_{\rm tot}^{\rm GW170817} = 2.74^{+0.04}_{-0.01} \ M_\odot,$$

$$M_{\rm thres} = \left(-3.606 \frac{G M_{\rm max}}{c^2 R_{1.6}} + 2.38 \right) \cdot M_{\rm max}$$
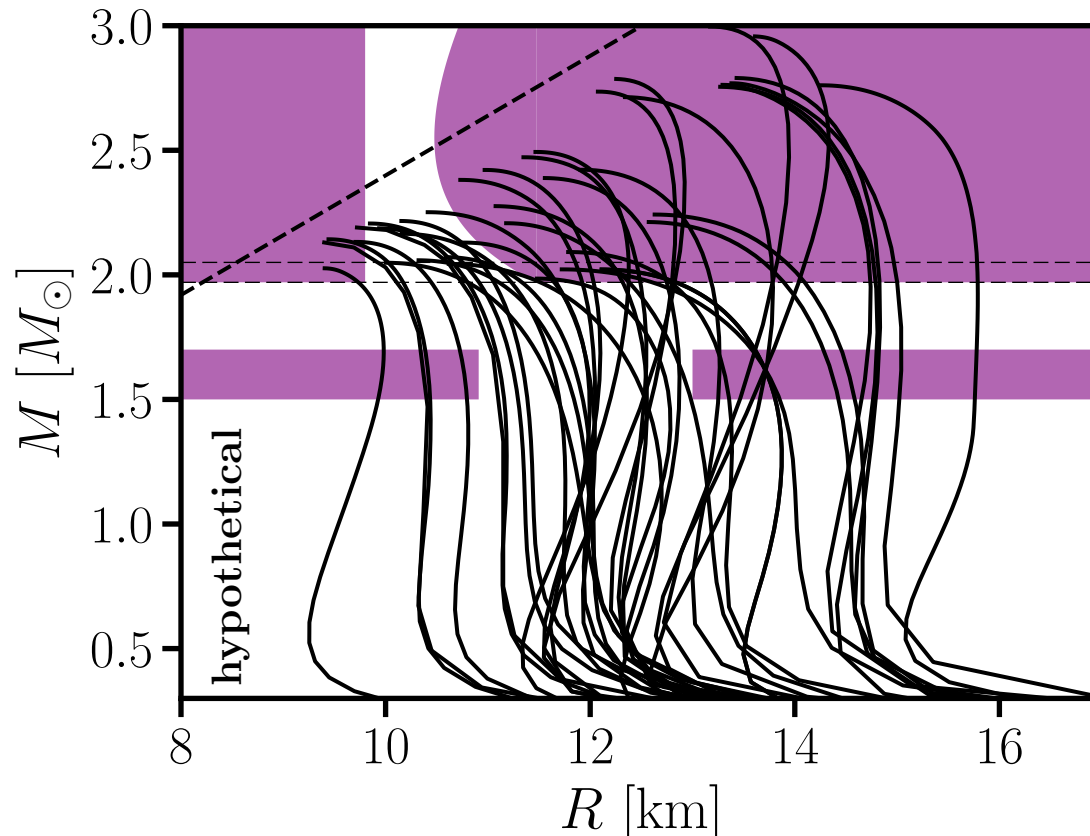
$$M_{\rm thres} = \left(-3.38 \frac{G M_{\rm max}}{c^2 R_{\rm max}} + 2.43 \right) \cdot M_{\rm max}$$

GW170817 Radius Constraints

Andreas Bauswein, Oliver Just, Hans-Thomas Janka and Nikolaos Stergioulas
arXiv: 1710.06843

Fictitious GW constraints

Andreas Bauswein, Oliver Just, Hans-Thomas Janka and Nikolaos Stergioulas
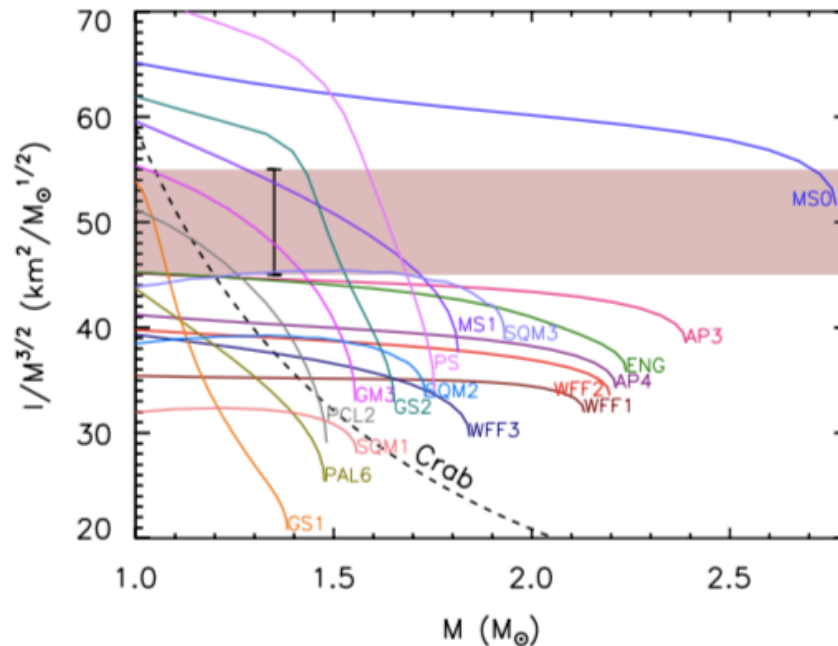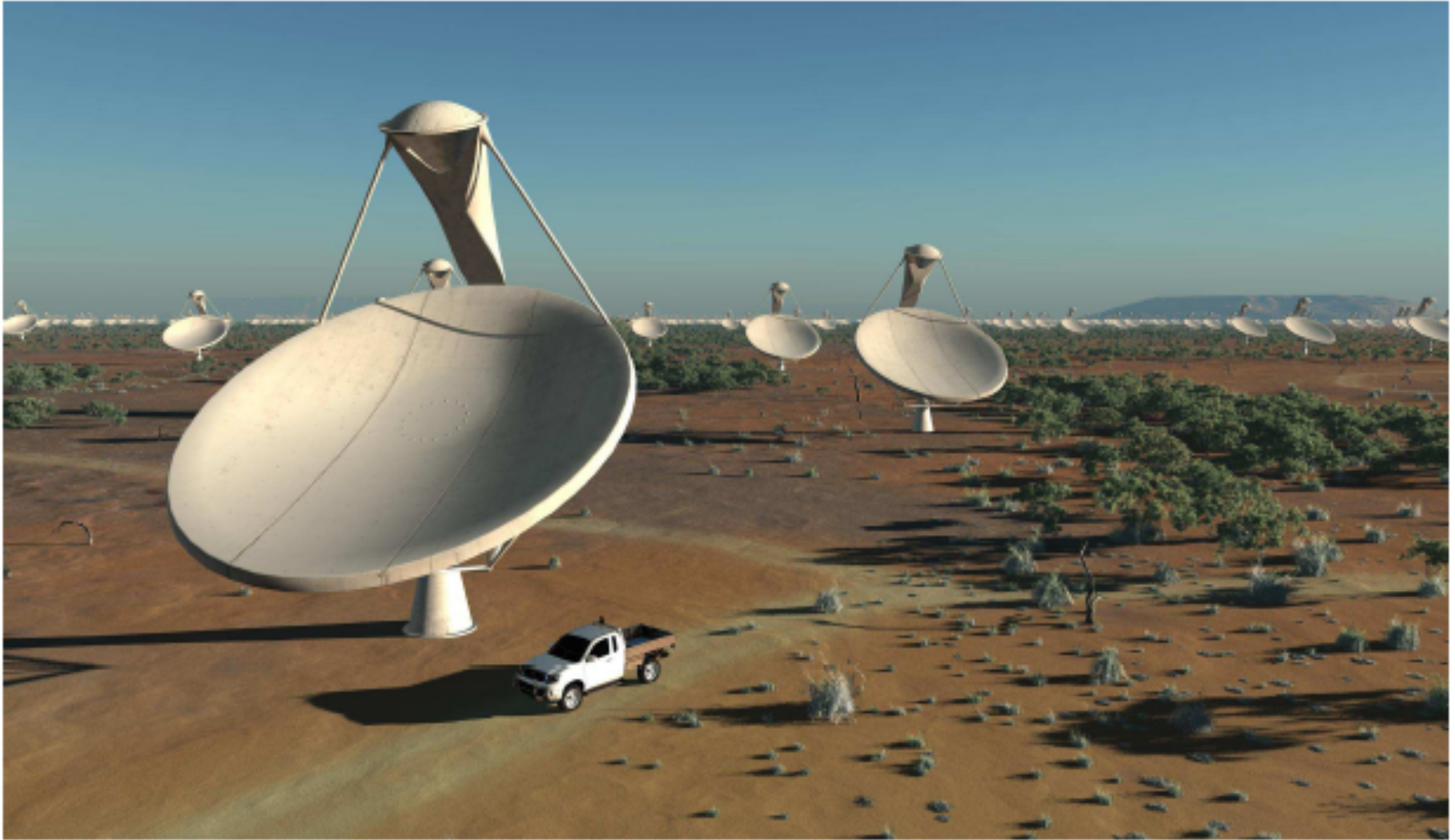
Moments of Inertia

Fig. 9. The moment of inertia scaled by $M^{3/2}$ as a function of stellar mass M for EOSs described in [6]. The shaded band illustrates a $\pm 10\%$ error on a hypothetical $I/M^{3/2}$ measurement with centroid $50\,\mathrm{km}^2\,\mathrm{M_\odot^{-1/2}}$; the error bar shows the specific case in which the mass is $1.34\,\mathrm{M_\odot}$ with essentially no error. The dashed curve labelled "Crab" is the lower limit derived by [123] for the Crab pulsar.

$$I \simeq \frac{J}{1 + 2GJ/R^3c^2}, \quad J = \frac{8\pi}{3} \int_0^R r^4 \left(\rho + \frac{p}{c^2}\right) \Lambda dr, \quad \Lambda = \frac{1}{1 - 2Gm/rc^2}$$
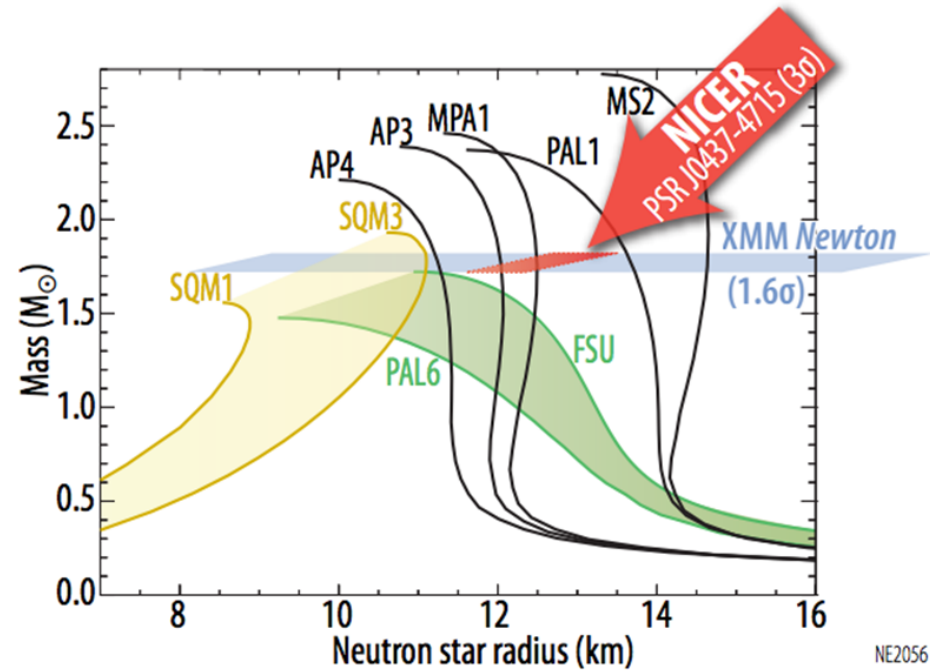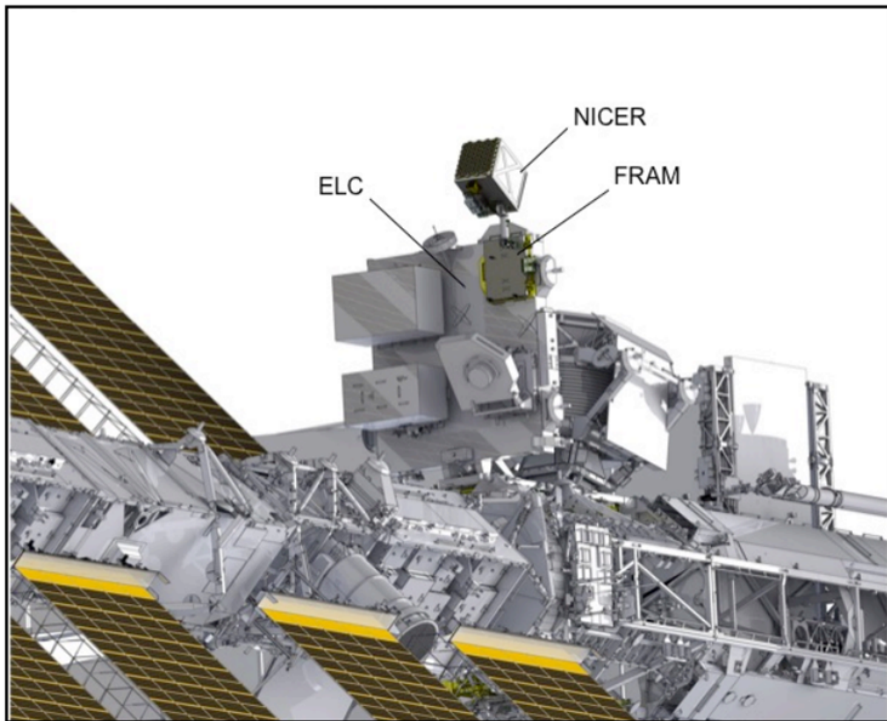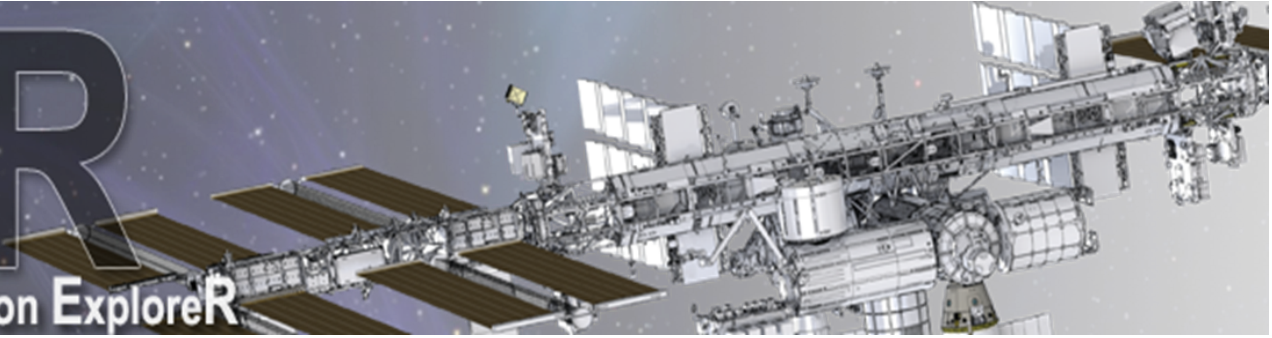
Perspectives for new Instruments?

THE FUTURE: SKA - SQUARE KILOMETER ARRAY

NICER 2017

Gendreau, K. C., Arzoumanian, Z., & Okajima, T. 2012, Proc. SPIE, 8443, 844313

Hot Spots

Conclusions

- The symmetry energy strongly determines the NS radius.

- USEC conjecture has been corroborated and E_s related quantities found to be correlated with the NS radius.

- GW170817 favours softer EoS and together with the Durca constraint DD2F-like EoS are favoured.

- Future GW observations, NICER and SKA will soon result into stronger NS EoS constraints.

Gracias