Interplay between cluster correlations and collision dynamics

Akira Ono

Tohoku University

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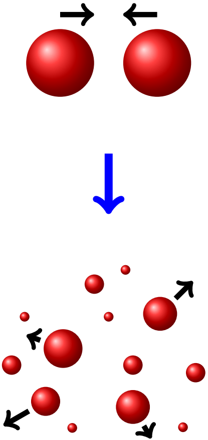
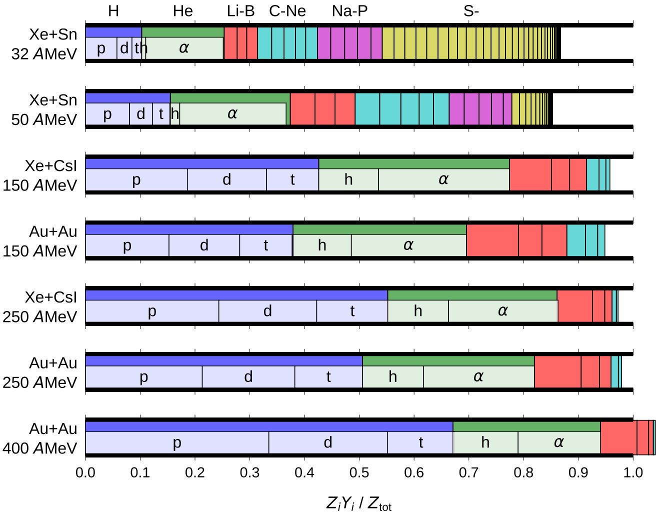
Fraction of protons in clusters and fragments in heavy-ion collisions

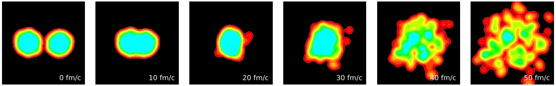




INDRA: Hudan et al., PRC67 (2003) 064613.

FOPI: Reisdorf et al., NPA 848 (2010) 366.

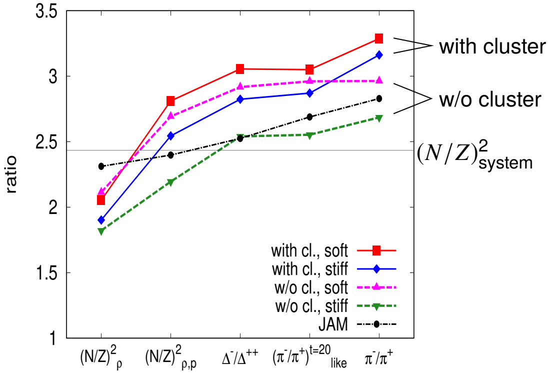
π^-/π^+ ratio in central ^{132}Sn + ^{124}Sn collisions at 300 MeV/nucleon



Ikeno, Ono, Nara, Ohnishi,

PRC 93 (2016) 044612,

Erratum PRC 97 (2018) 069902.



Final π^-/π^+

$\Updownarrow$

$(\pi^-/\pi^+)_{\text{like}}$ at $t = 20$ fm/c

$\Updownarrow$

$(N/Z)^2$ at high density &

high momentum

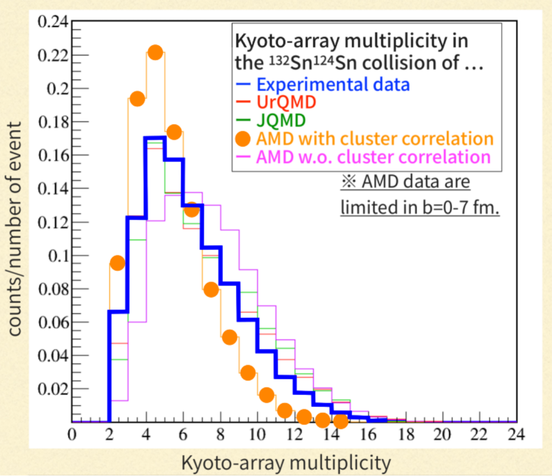
$\Updownarrow$

- Cluster correlations
- Symmetry energy

Ideal Fermi gas of nucleons v.s. Mixed gas of nucleons and clusters

Kyoto-Array Multiplicity for the SπRIT system

Masanori Kaneko, NuSYM17



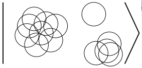
AMD with clusters predicts less multiplicity on Kyoto Array than experimental data.

⇓

- Too strong cluster correlations?
- Wrong angular distribution?

Needs more careful study of cluster correlations in AMD.

Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} \quad + \quad \text{(NN collisions)}$$

$\{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}}$: Motion in the mean field

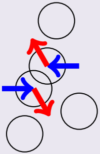
$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

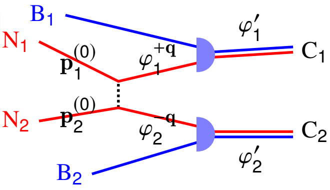


Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

NN collisions with cluster correlations

$\mathrm{N}_1 + \mathrm{B}_1 \ + \ \mathrm{N}_2 + \mathrm{B}_2 \ \rightarrow \mathrm{C}_1 + \mathrm{C}_2$

- $\mathrm{N}_1, \mathrm{N}_2$: Colliding nucleons
- $\mathrm{B}_1, \mathrm{B}_2$: Spectator nucleons/clusters
- $\mathrm{C}_1, \mathrm{C}_2$: N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



Transition probability

$$W(\mathrm{NBNB} \rightarrow \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

$$v\,d\sigma \propto |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2 |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 |M|^2 \delta(E_f - E_i) p_{\mathrm{rel}}^2 dp_{\mathrm{rel}} d\Omega$$

$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}}\hat{\mathbf{\Omega}}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_2})\varphi_2^{(0)}$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

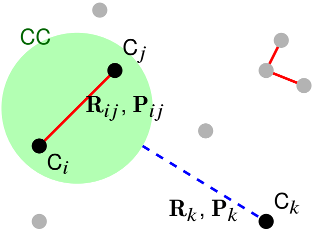
$\Leftarrow (d\sigma/d\Omega)_{\mathrm{NN}}$ in free space (or in medium)

Correlations to bind several clusters

Several clusters may form a loosely bound state.

e.g., $^7\mathrm{Li} = \alpha + t - 2.5$ MeV

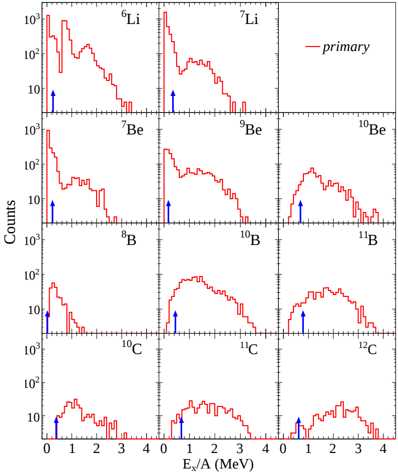
Need more probability of $|\alpha + t\rangle \longrightarrow |^7\mathrm{Li}\rangle$ etc.



Production of light nuclei

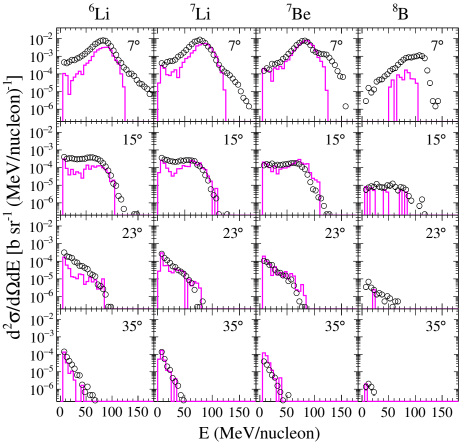
$^{12}C + ^{12}C$ at 95 MeV/nucleon

Tian et al., PRC 97 (2018) 034610.



Excitation energy per nucleon

Some light nuclei are emitted at large angles ($\theta_{\mathrm{lab}} > 20^\circ$) almost in their ground states, at $t = 300$ fm/c.



More about cluster production in NN collisions

$$\text{N}_1 + \text{B}_1 \; + \; \text{N}_2 + \text{B}_2 \; \to \text{C}_1 + \text{C}_2$$

$$v d\sigma \propto |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2 |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 dp_{\text{rel}} d\Omega \qquad E_i,\ E_f = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle}$$

$$\Rightarrow P(\text{C}_1, \text{C}_2, p_{\text{rel}}, \Omega) \times \left| M(p_{\text{rel}}^{(0)}, p_{\text{rel}}, \Omega) \right|^2 \times \frac{p_{\text{rel}}{}^2 d\Omega}{\partial E_f / \partial p_{\text{rel}}}$$

- Gaussian width $\nu_{\text{cl}} = 0.24\ \text{fm}^{-2}$ for the overlap factors.
- There are a huge number of final cluster configurations (C_1, C_2).

$$\sum_{\text{C}_1\text{C}_2} P(\text{C}_1, \text{C}_2, p_{\text{rel}}, \Omega) = 1 \quad \text{for any fixed } (p_{\text{rel}}, \Omega)$$

- The energy-conserving final momentum depends on the cluster configuration

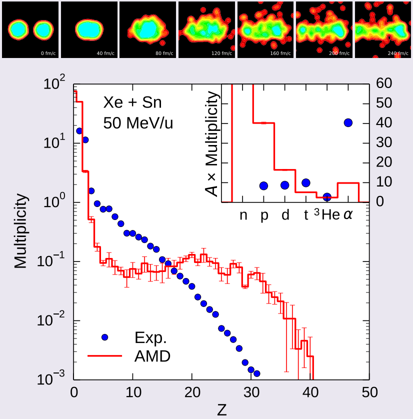
$$p_{\text{rel}} = p_{\text{rel}}(\text{C}_1, \text{C}_2, \Omega)$$

When clusters are formed, p_{rel} tends to be large, and the effect of collisions will increase.

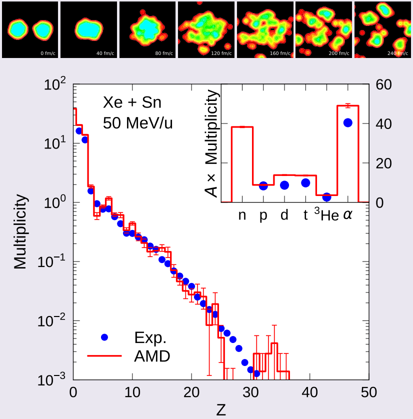
  - the phase space factor ↑
  - Pauli blocking ↓ (collision probability ↑)
  - momentum transfer ↑

Effect of cluster correlations: central Xe + Sn at 50 MeV/u

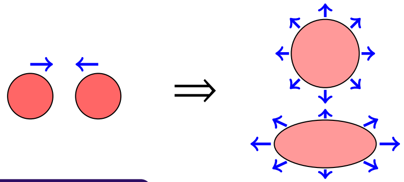
Without clusters



With clusters



Stopping



A quantity to represent stopping

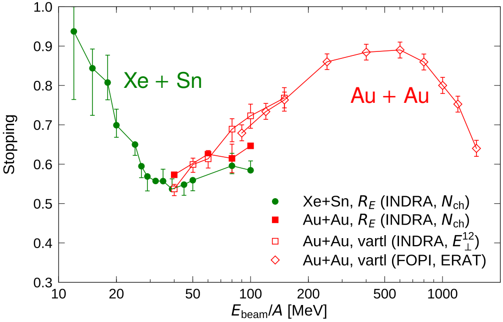
$$R_E = \frac{\sum(E_x + E_y)}{2\sum E_z}$$

$\sum$: for all charged products ($Z \geq 1$)

Stopping should depend on

- Inmedium NN cross sections
- Treatment of Pauli blocking
- Effective interaction (EOS)
- How to select central events

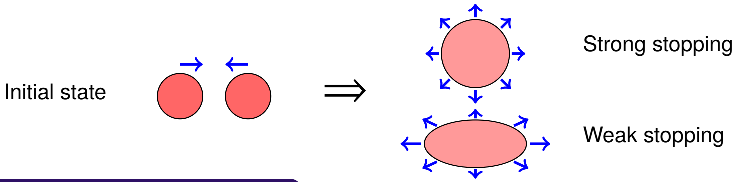
It is also a many-body quantity.



INDRA: Lehaut et al., PRL104 (2010) 232701.

FOPI: Reisdorf et al., NPA 848 (2010) 366.

Stopping



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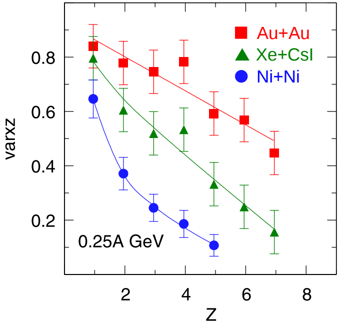
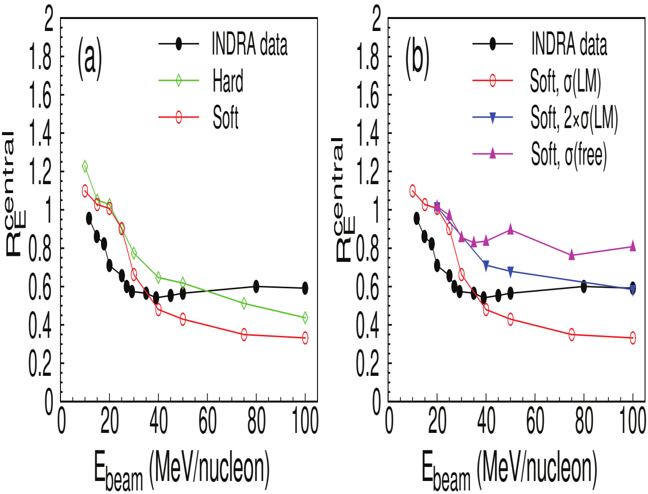


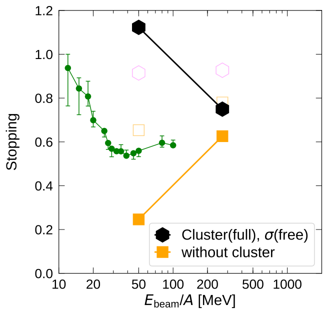
FOPI: Reisdorf et al., NPA 848 (2010) 366.

Zhao et al., PRC 89 (2014) 037001.

Dependence on EOS | Dependence on σ_{NN}



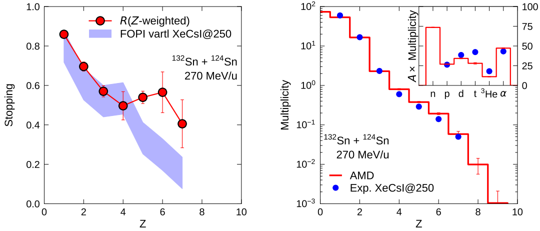
Results with full clusters



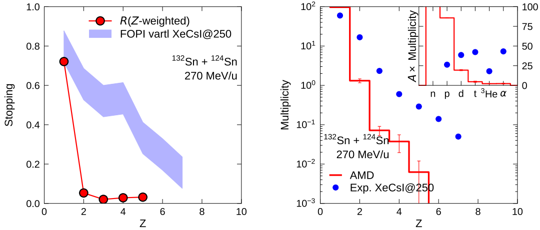
Central collisions (b < 1-2 fm)

- Xe + Sn for $E \leq 50A$ MeV
- ^{132}Sn + ^{124}Sn at 270A MeV

Cluster(full) and σ_{NN}(free)

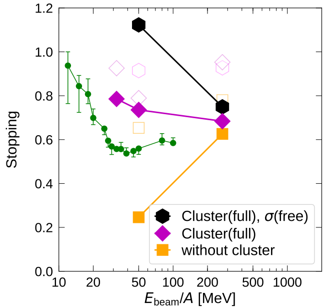


No cluster, and σ_{NN}(in-medium)



FOPI data: Xe + CsI at 250A MeV

Results with full clusters



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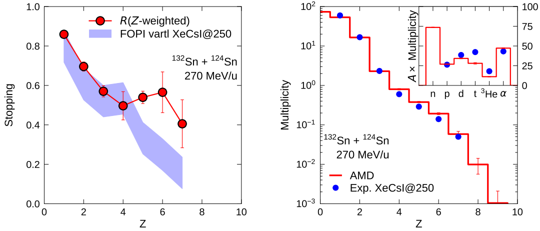
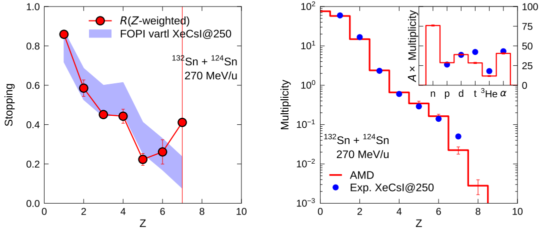
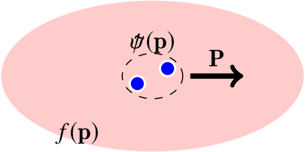


Cluster(full) and σ_{NN}(in-medium)



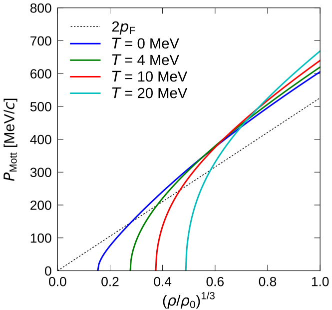
FOPI data: Xe + CsI at 250A MeV

A cluster in medium



Equation for a deuteron in uncorrelated medium

$$\Big[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\psi(\mathbf{p})$$
$$+\Big[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\psi(\mathbf{p}')$$
$$=E\psi(\mathbf{p})$$



Formula from Röpke, NPA867 (2011) 66.

- A bound deuteron cannot exist inside the Fermi sphere, except at very low densities.
- A deuteron can exist if its momentum is high enough.
- In AMD, Pauli blocking has already been considered for NN collisions. More suppression of clusters may be introduced. c.f. $\langle f\rangle_d < 0.2$ by Danielewicz et al., NPA533 (1991) 712.

Density-dependent suppression of clusters

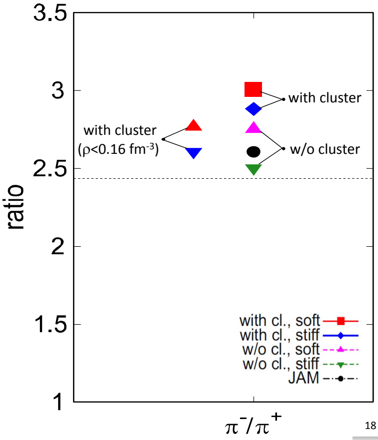
Ikeno, NuSYM16

Preliminary result with cluster ($\rho < 0.16$ fm^{-3})

- Final π^-/π^+ ratio

- With cluster ($\rho < 0.16$fm^{-3})

  Somewhere between
  the calculation with cluster and
  that without cluster



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Condition to switch on/off clusters

With or without clusters

$$N_1 + N_2 + B_1 + B_2 \longrightarrow C_1 + C_2 \qquad \text{or} \qquad N_1 + N_2 \longrightarrow N_1 + N_2$$

Density with a momentum cut for the nucleon N_i ($i = 1, 2$)

$$\rho_i'^{(\text{ini})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i - \mathbf{P}_k|\big)\ e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

$$\rho_i'^{(\text{fin})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i^{(\text{fin})} - \mathbf{P}_k|\big)\ e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

$$\rho' = \big(\rho_1'^{(\text{ini})}\rho_1'^{(\text{fin})}\rho_2'^{(\text{ini})}\rho_2'^{(\text{fin})}\big)^{\frac{1}{4}}$$

An energy-dependent momentum cut was chosen, $p_{\text{cut}} = (375\ \text{MeV}/c)\ e^{-\epsilon/(225\ \text{MeV})}$, where ϵ is the collision energy (i.e. the sum of the kinetic energies of N_1 and N_2 in their c.m. frame).

The condition to switch on clusters

$$\rho' < \rho_{\text{c}}, \qquad \rho_{\text{c}} = 0.125\ \text{fm}^{-3} \text{ or } 0.060\ \text{fm}^{-3} \text{ etc.}$$

Parametrization of in-medium NN cross section

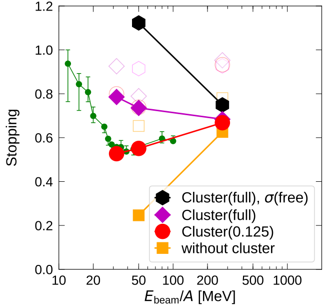
The matrix element $|M|^2$ is obtained from the NN cross section.

- Free cross section $\sigma_{\text{free}}(\epsilon)$, taken from the JAM code.
- In-medium cross section which depends on ρ' (with momentum cut).

$$\sigma(\rho', \epsilon) = \sigma_0 \tanh\left(\sigma_{\text{free}}(\epsilon)/\sigma_0\right), \qquad \sigma_0 = 0.5 \times (\rho')^{-2/3}$$

parametrization by Danielewicz

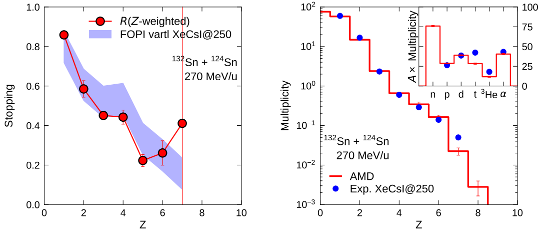
Effects of in-medium cluster suppression, with σ_{NN}(in-medium)



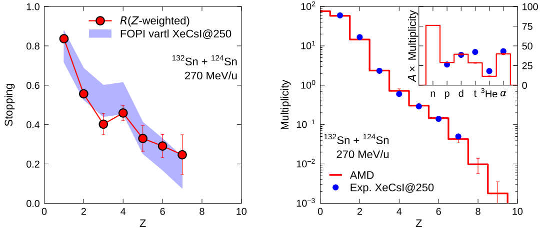
Central collisions (b < 1-2 fm)

- Xe + Sn for $E \leq 50A$ MeV
- ^{132}Sn + ^{124}Sn at 270A MeV

Cluster(full) and σ_{NN}(in-medium)

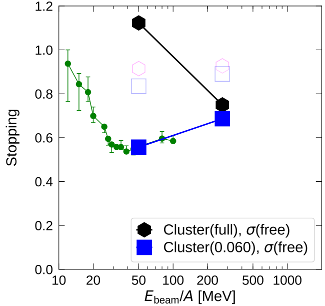


Cluster(0.125), and σ_{NN}(in-medium)



FOPI data: Xe + CsI at 250A MeV

Effects of in-medium cluster suppression, with σ_{NN}(free)



Central collisions ($b <$ 1-2 fm)

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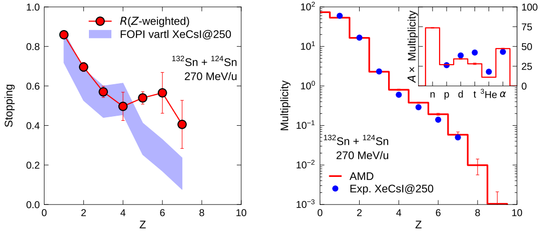
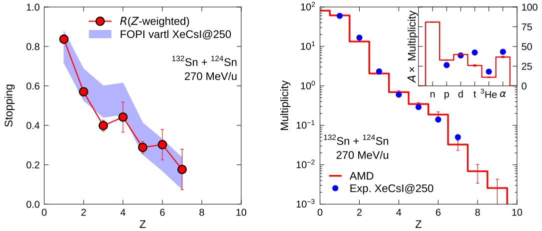
Cluster(full) and σ_{NN}(free)



Cluster(0.060) and σ_{NN}(free)



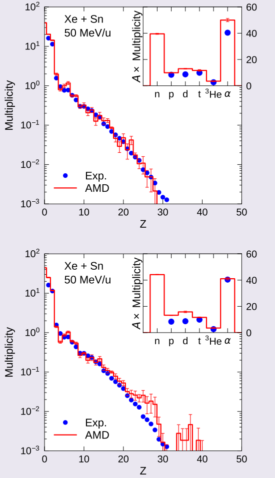
FOPI data: Xe + CsI at 250A MeV

Combinations of σ_{NN} and the in-medium cluster suppression

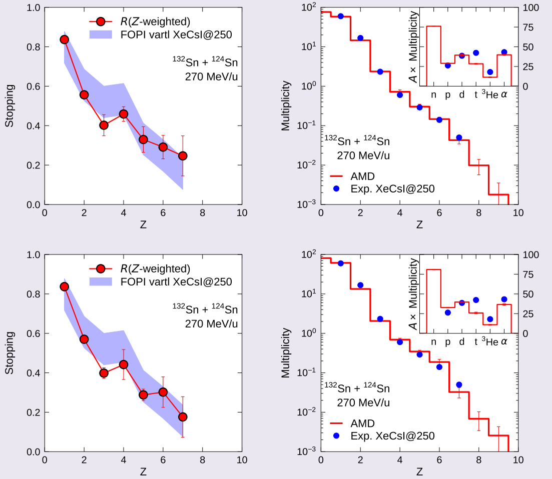
σ_{NN}(in-medium) Cluster(0.125)

σ_{NN}(free) Cluster(0.060)

Xe + Sn at 50A MeV



^{124}Sn + ^{132}Sn at 270A MeV



FOPI data: Xe + CsI at 250A MeV

Summary

- Cluster correlations can have strong impacts, not only on the cluster emission, but also on the collision dynamics (e.g. stopping) in central heavy-ion collisions.
- Suppression of clusters at high (phase-space) densities was considered, in the cluster production process in AMD.
- Information on stopping can give a constraint on a combination of the in-medium suppression of clusters and the in-medium NN cross section (and ...).
  - Too strong suppression (e.g. without clusters ■) cannot be compatible with the experimental data of the cluster yield and the cluster-size dependence of stopping.
  - In some range of the degree of suppression (∼ ● ∼ ■ ∼), the fragment yield can be roughly consistent with data.

- What can fix the degree of suppression of clusters (and in-medium NN cross section)?
- How is the isospin dynamics, i.e. the difference of neutrons and protons?

Time evolution of the central density