Pion production in heavy ion collisions

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- Pion and nuclear EOS and symmetry energy
- Status of transport models for pion production
- Symmetry potential effect in collision terms
- Pion in-medium effect
- Short-range correlations vs thermal excitations
- Pion production in χBUU

Supported by US Department of Energy and The Welch Foundation

Density in neutron stars

Mass of Sun: $M_S = 1.99\times 10^{30}$ kg

Number of protons in Sun: $N_p = \frac{M_S}{M_p} = \frac{1.99\times 10^{30}}{1.7\times 10^{-27}} = 1.17\times 10^{57}$

Number of neutrons in neutron star of 1.4 solar mass

$$N_n = 1.64\times 10^{57}$$

Nuclear density in neutron star of 1.4 solar mass and 10 km radius

$$\rho = \frac{N_n}{4\pi R^3/3} = \frac{1.64\times 10^{57}}{4.2\times (10^{19})^3} = 0.39\ \mathrm{fm}^{-3} = 2.44\ \rho_0$$

which is a nuclear matter but will become a quark matter if the mass is twice the solar mass when the density is 3.49 ρ_0.

Microscopic Theory of Pion Production and Sidewards Flow in Heavy-Ion Collisions

H. Kruse,[a] B. V. Jacak, and H. Stöcker
Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory,
Michigan State University, East Lansing, Michigan 48824
(Received 9 July 1984)

Nuclear collisions from 0.3 to 2 GeV/nucleon are studied in a microscopic theory based on Vlasov's self-consistent mean field and Uehling-Uhlenbeck's two-body collision term which respects the Pauli principle. The theory explains simultaneously the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

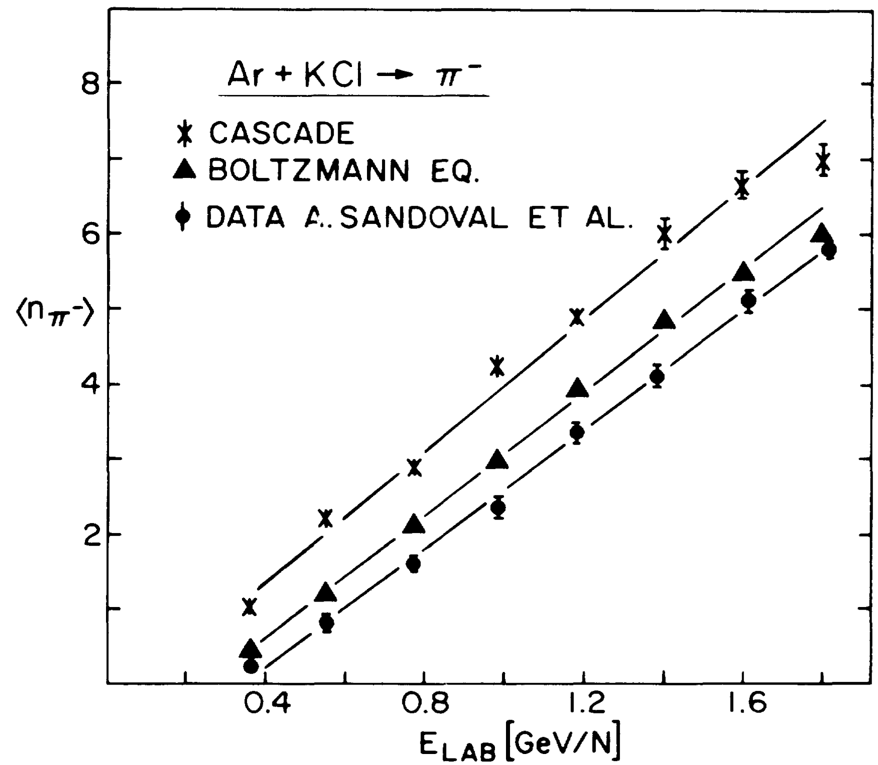
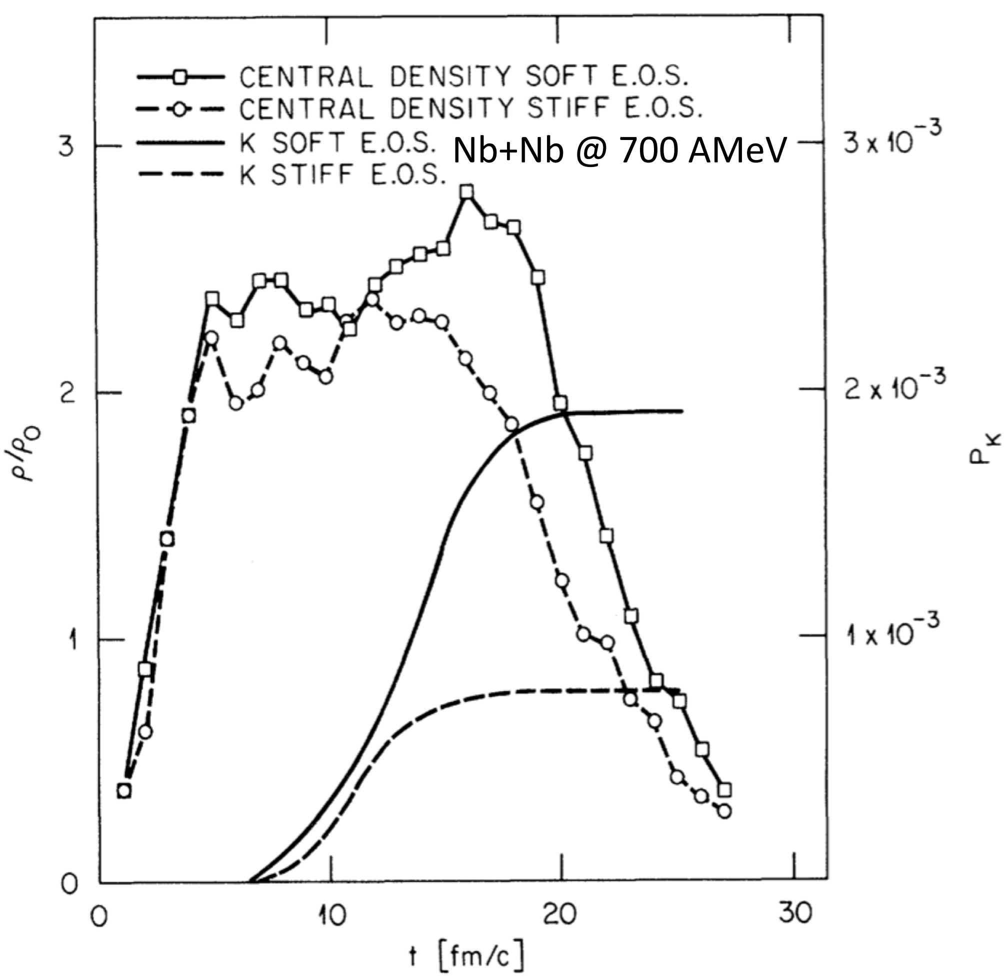


FIG. 2. Pion multiplicity for central collisions ($b < 2.4$ fm) of Ar+KCl. The data (Ref. 6, circles) are compared to the present theory in the "cascade mode" (crosses) and to the same theory with compression energy and phase-space Pauli blocking included (triangles).

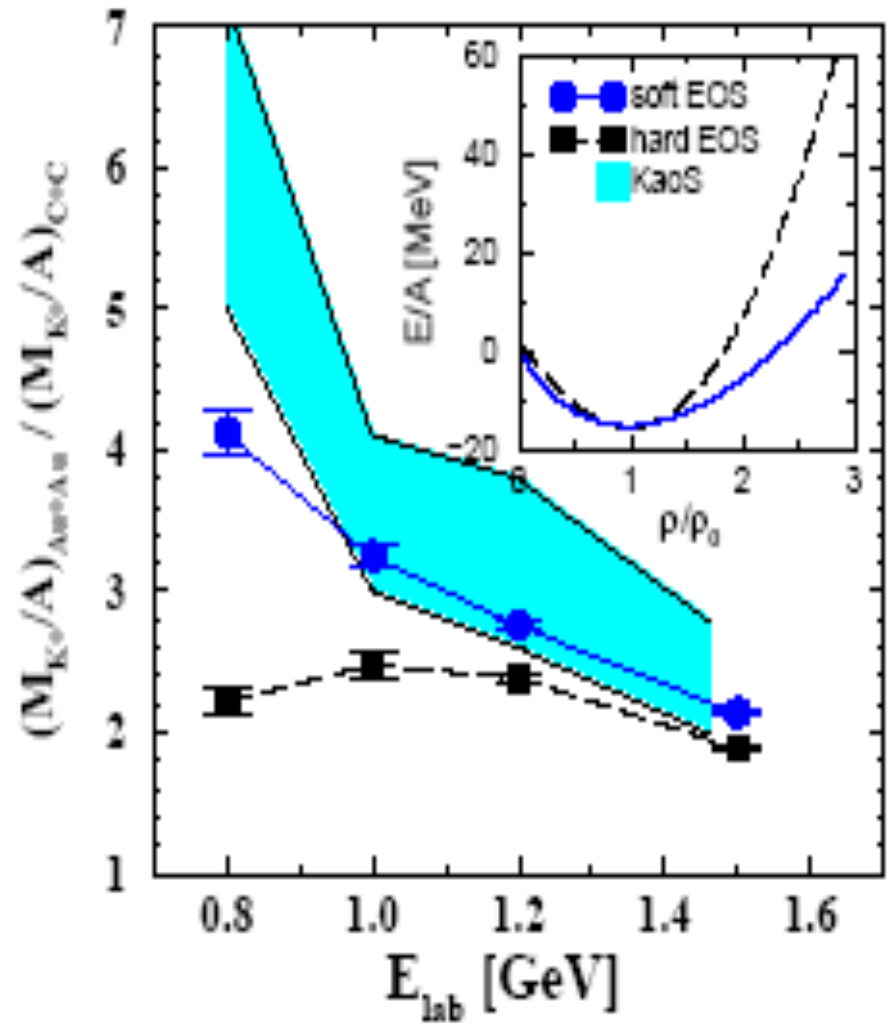
- Although difference between cascade and and BUU with mean fields can be 40%, difference between stiff and soft EOS is much smaller.
- No definitive conclusion on stiffness of nuclear EOS from pion yield!

Subthreshold kaon production in high-energy HIC

Aichelin & Ko, PRL 55, 2661 (1985)



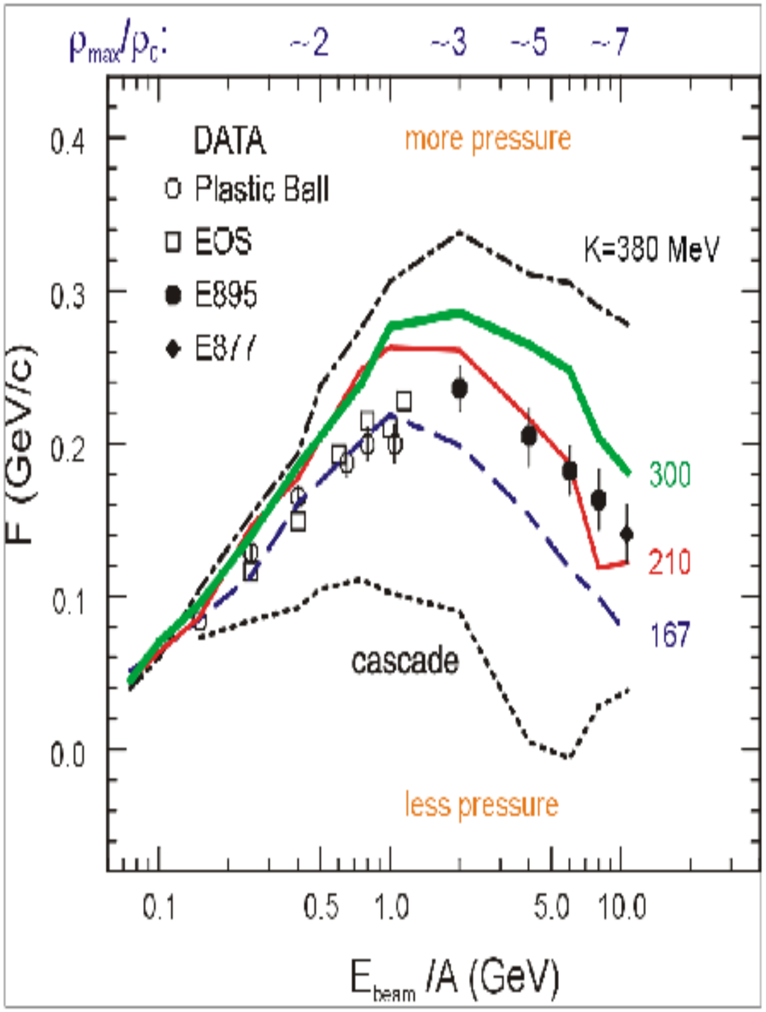
Fuchs, PRL 86, 1974 (2001)

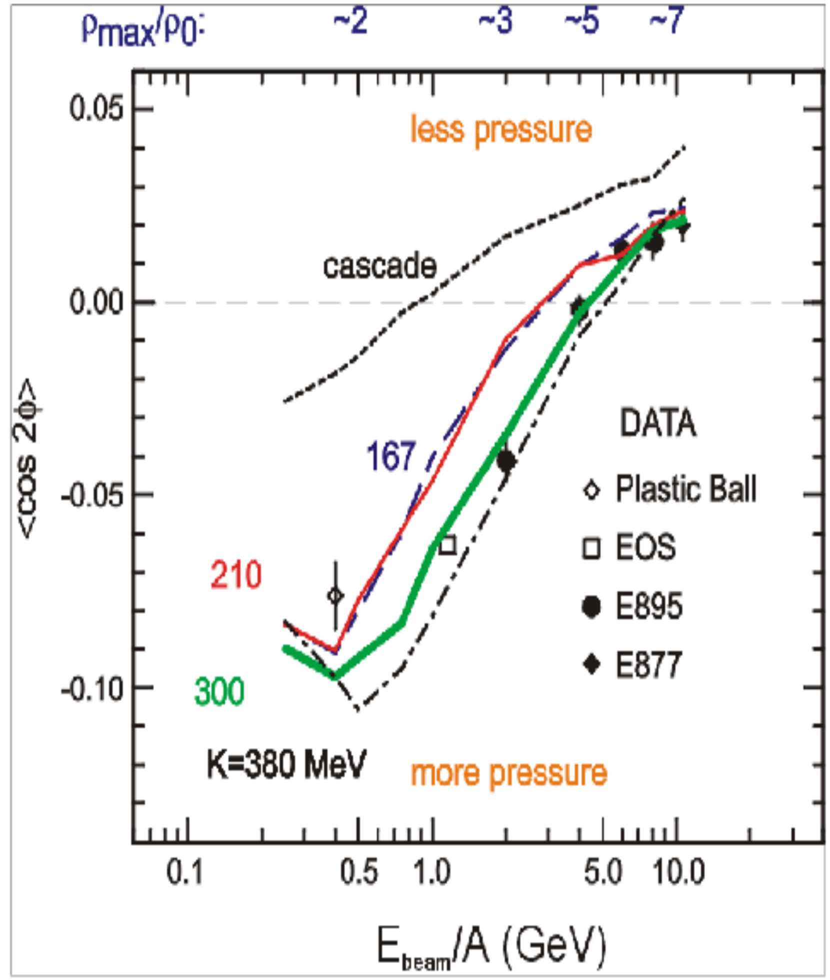


- Kaon production at subthreshold energy in HI collisions is sensitive to nuclear EOS and data are consistent with a soft one (K = 200 MeV)

Direct and elliptic flows

Danielewicz, Lacey & Lynch, Science 298, 1592 (2002)





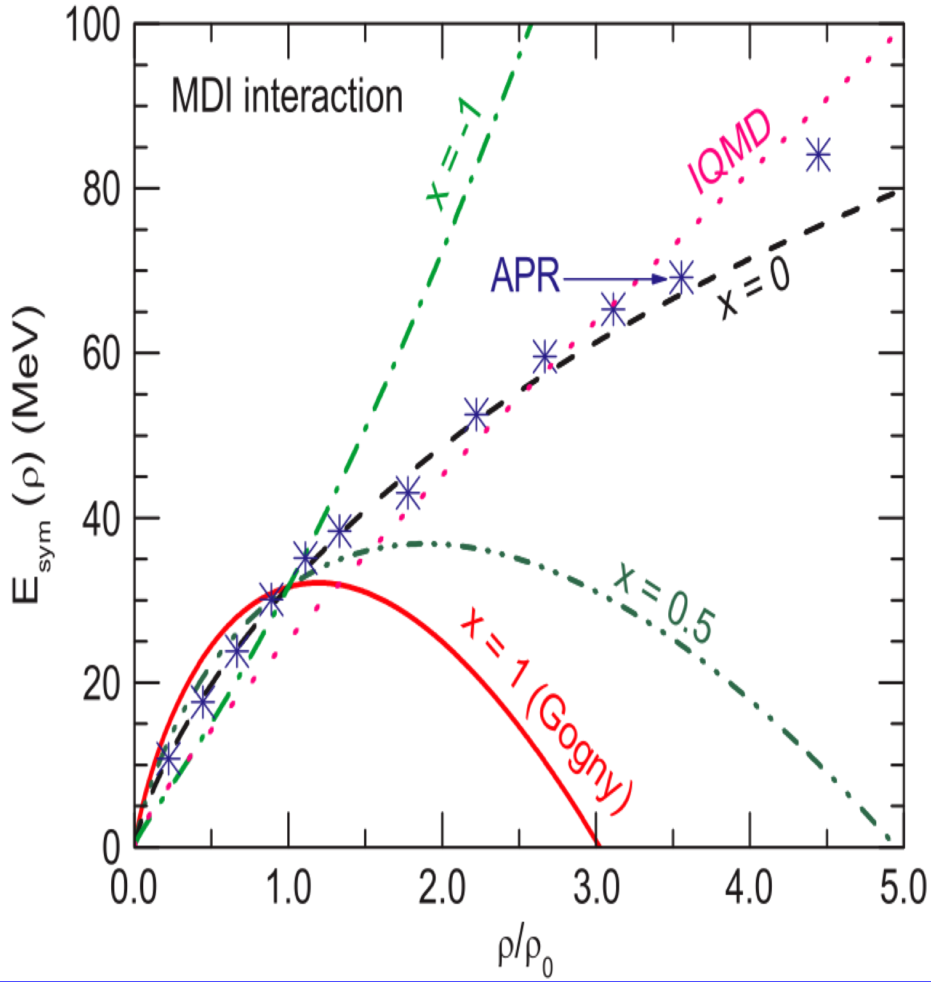
Direct flow $F = \left(\frac{dp_x}{dy}\right)_{y_{cm}}$

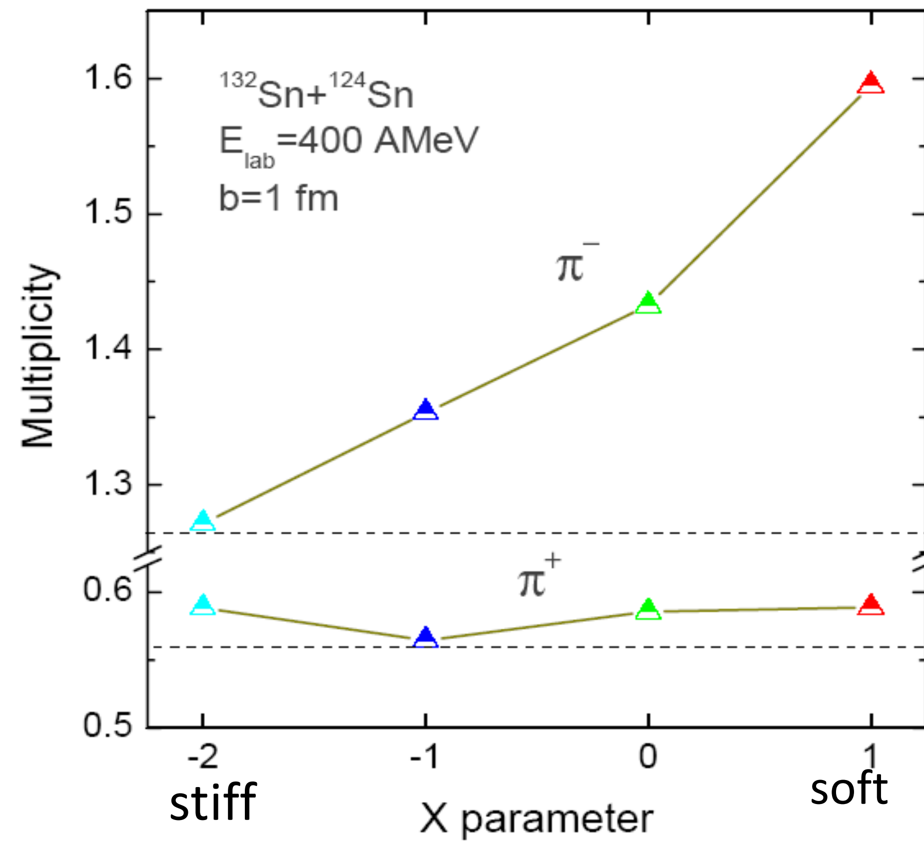
Elliptic flow $v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$

- Data consistent with a nuclear EOS with K ≈ 200 MeV.

Near-threshold pion production with high energy radioactive beams (IBUU)

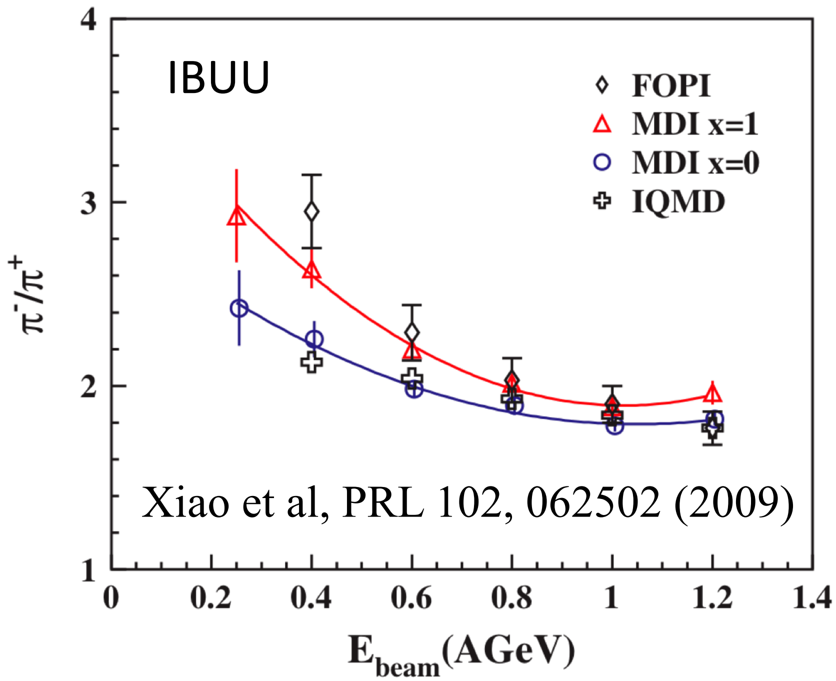
B. A. Li, PRL 88, 192701 (2002)

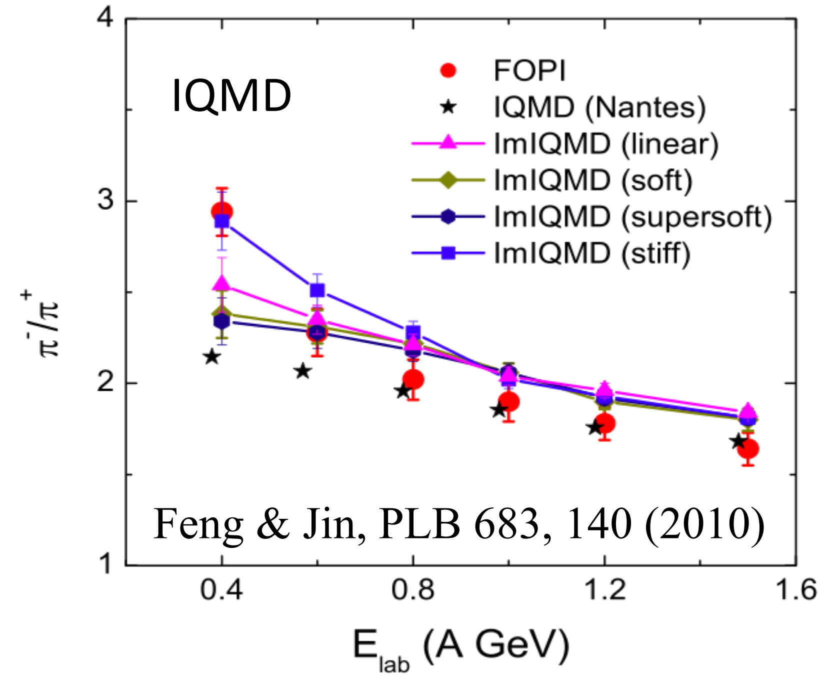


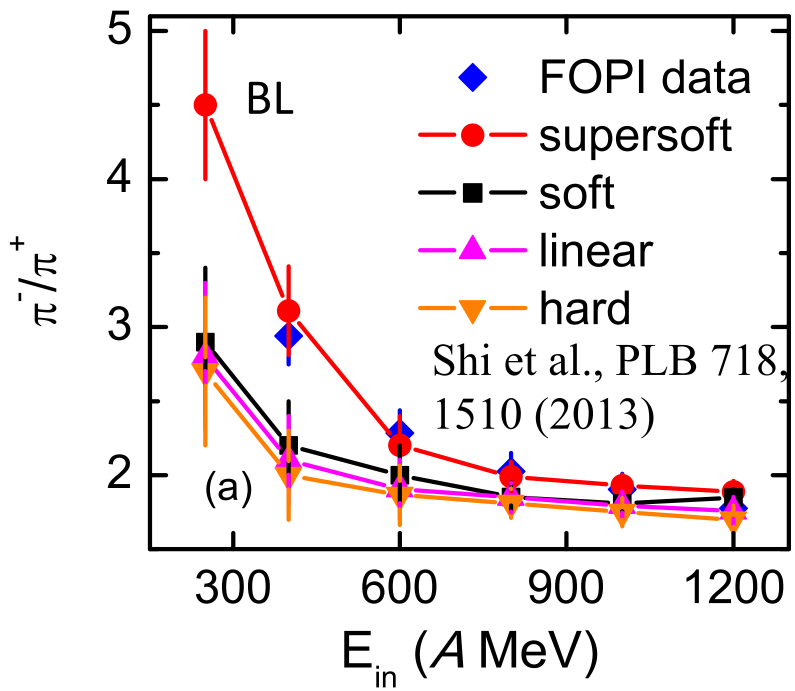


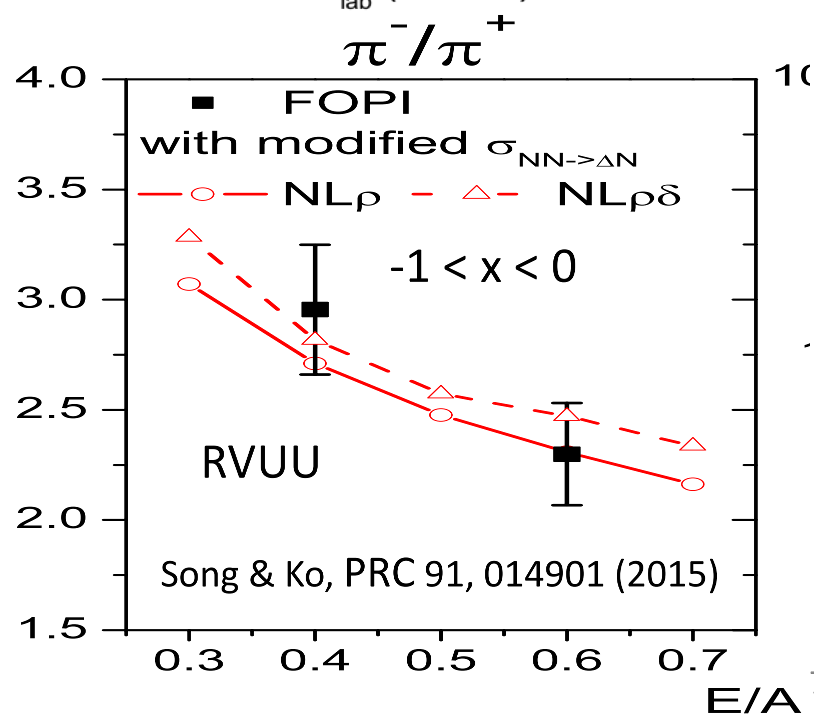
π^- yield is sensitive to the symmetry energy $E_{sym}(\rho)$ since they are mostly produced in the neutron-rich region, with softer one giving more π^- than stiffer one. Difference between super soft (x = 1) and super stiff (x = -1) is only about 30%, which makes it very challenging to determine from data using transport models.

Conflicting results on symmetry energy from charged pion ratio









Pion production in transport models

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **31** (2005) S741–S757 doi:10.1088/0954-3899/31/6/015

Transport theories for heavy-ion collisions in the 1 *A* GeV regime

E E Kolomeitsev[1,2], C Hartnack[3], H W Barz[4], M Bleicher[5], E Bratkovskaya[5], W Cassing[6], L W Chen[7,8], P Danielewicz[9], C Fuchs[10], T Gaitanos[11], C M Ko[7], A Larionov[6,13], M Reiter[5], Gy Wolf[12] and J Aichelin[3,14]

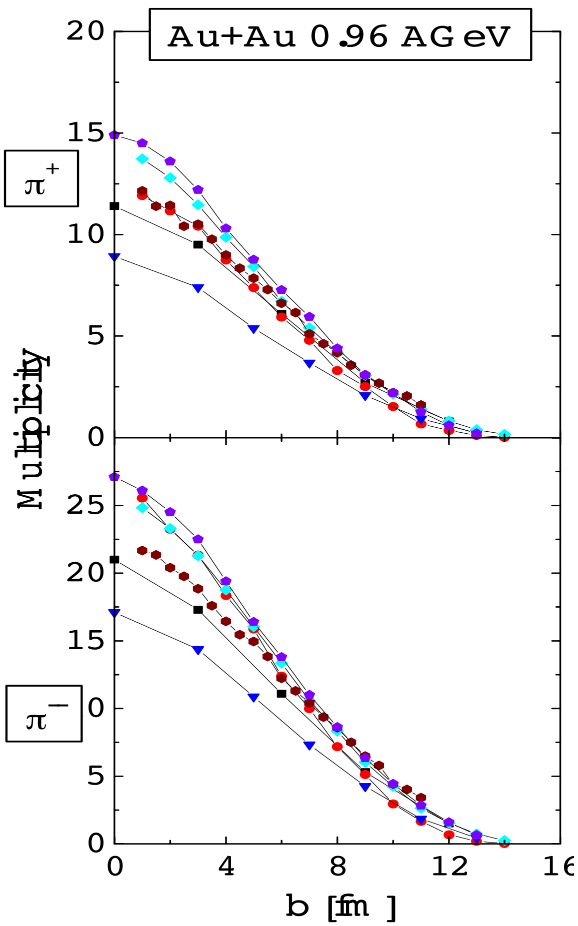
Abstract

We compare multiplicities as well as rapidity and transverse momentum distributions of protons, pions and kaons calculated within presently available transport approaches for heavy-ion collisions around 1 *A* GeV. For this purpose, three reactions have been selected: Au+Au at 1 and 1.48 *A* GeV and Ni+Ni at 1.93 *A* GeV.

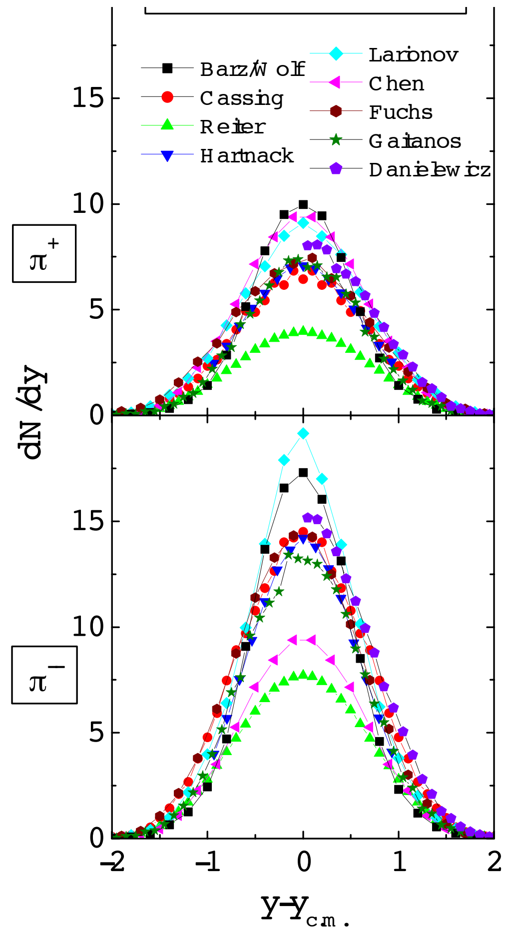
Transport model predictions for pion production in HIC

Kolomeitsev et al., J. Phys. G 31, S741 (2005)

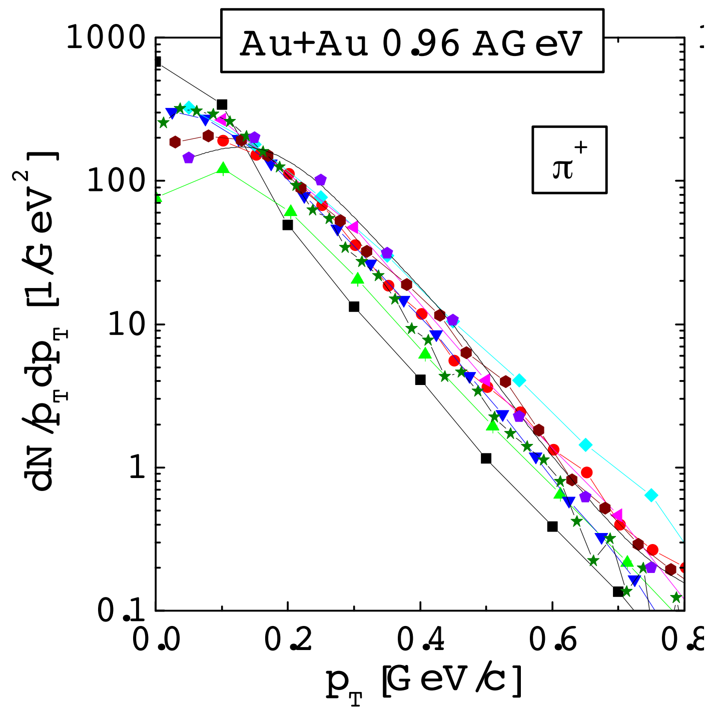
Centrality dependence



Rapidity distributions



Momentum spectra



Impact parameter b = 1fm

Delta width Γ_Δ = 120 MeV

- Results from different transport models can differ by ~ 2.

Transport Code Evaluation Project (TCEP)

- Started at first transport workshop in Shanghai (2014), followed in Lanzhou (2014), Xingxiang (2016), Beijing (2016), East Lansing (2017), Ganil (2017) and Zhuhai (2018); led to two publications. (31 scientists and 24 institutions)
  1) Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions [Xu et al., PRC 93, 044609, (2016)]
  2) Comparison of heavy-ion transport simulations: Collision integral in a box [Zhang et al., PRC 97, 034625 (2018)]
- **Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box** [Ono et al., in preparation]
  1) Symmetric or asymmetric (isospin asymmetry δ=0.2) nuclear matter at normal density and temperature of 60 MeV: Reaction rates for $NN \rightarrow N\Delta$ and time evolution of Δ number; $\sqrt{s}$ dependence of $NN \rightarrow N\Delta$ and $N\Delta \rightarrow NN$ reactions; most of 10 codes (4 BUU + 6 QMD) give results comparable to those from kinetic equations.
  2) Uncertainties among codes become large for time evolution of Δ and π numbers after including $\Delta \leftrightarrow N\pi$.
- **Pion production in $\mathbf{Sn^{132} + Sn^{124}}$ and $\mathbf{Sn^{112} + Sn^{108}}$ at 270A MeV and b = 4 fm** (in progress)
  1) Momentum-independent mean fields; stiff symmetry energy; self-consistent initial distributions; constant *BB* elastic scattering cross section ($\sigma = 40$ mb); $NN \leftrightarrow N\Delta$ and $\Delta \leftrightarrow N\pi$.
  2) Assess uncertainties on pion yield among codes and establish likelihood for extracting stiffness of nuclear symmetry energy from charged pion ratio.

(Isospin-dependent) Boltzmann-Uehling-Uhlenbeck model

Bertsch & Das Gupta, Phys. Rep. 160, 189 (1988)

$$\frac{\partial f_{n/p}(\boldsymbol{x},\boldsymbol{k},t)}{\partial t} = \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{n/p} - \nabla_{\boldsymbol{x}} U_{n/p}\cdot\nabla_{\boldsymbol{k}} f_{n/p} = \left(\frac{\partial f_{n/p}}{\partial t}\right)_{\text{coll}}$$

- $f_{n/p}(\boldsymbol{x},\boldsymbol{k},t)$: neutron/proton phase-space distribution functions
- $U_{n/p}$: neutron/proton mean-field potentials, e.g., for momentum independent Skyrme potentials

$$U_{n/p} = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^{\gamma} \pm S_{\text{pot}}(\rho/\rho_0)^{\delta}$$

- $\left(\frac{\partial f_{n/p}}{\partial t}\right)_{\text{coll}}$: collision integral

$$\begin{aligned}\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = & -\frac{1}{(2\pi)^3}\int\int d\mathbf{k}_2 d\mathbf{k}_3 \int d\Omega |v_{12}| \frac{d\sigma}{d\Omega}\delta^{(3)}(\mathbf{k}+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4) \\ & \times \{f(\mathbf{x},\mathbf{k},t)f(\mathbf{x},\mathbf{k}_2,t)[1-f(\mathbf{x},\mathbf{k}_3,t)][1-f(\mathbf{x},\mathbf{k}_4,t)] \\ & - f(\mathbf{x},\mathbf{k}_3,t)f(\mathbf{x},\mathbf{k}_4,t)[1-f(\mathbf{x},\mathbf{k},t)][1-f(\mathbf{x},\mathbf{k}_2,t)]\}\end{aligned}$$

Equilibrium requires $f_1 f_2(1-f_3)(1-f_4) = f_1 f_2(1-f_3)(1-f_4)$ or $f_1 f_2 = f_3 f_4$ if fermi statistics is neglected. Then

$$\ln f_1 + \ln f_2 = \ln f_3 + \ln f_4$$

Energy conservation

$$E_1 + U_1 + E_2 + U_2 = E_3 + U_3 + E_4 + U_4$$

with $E_i = \sqrt{m_i^2 + k_i^2}$ and U_i being the potential, implies that

$$\ln f_i \propto E_i + U_i \rightarrow f_i = e^{-\frac{\sqrt{m_i^2+k_i^2}+U_i}{T}}$$

- For elastic scatterings $nn \rightarrow nn, np \rightarrow np, pp \rightarrow pp,\ U_1 + U_2 = U_3 + U_4$. So $\mathbf{k}_3$ and $\mathbf{k}_4$ are determined from $E_1 + E_2 = E_3 + E_4$, and potentials have no effect on collisions.

- For inelastic collisions such as $pn \rightarrow p\Delta^0$ when $U_{\Delta^0} = \frac{1}{3}U_p + \frac{2}{3}U_n$. $\mathbf{k}_3$ and $\mathbf{k}_4$ are determined from

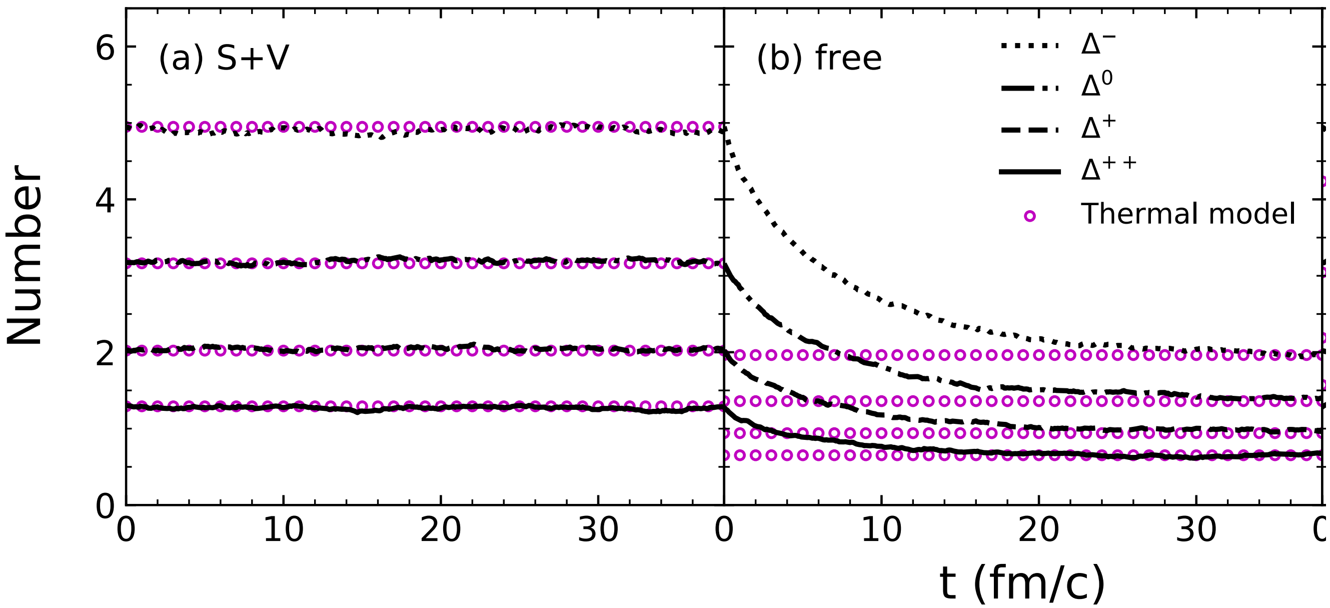
$$E_3 + E_4 = E_1 + E_2 + \frac{1}{3}U_n - \frac{1}{3}U_p$$

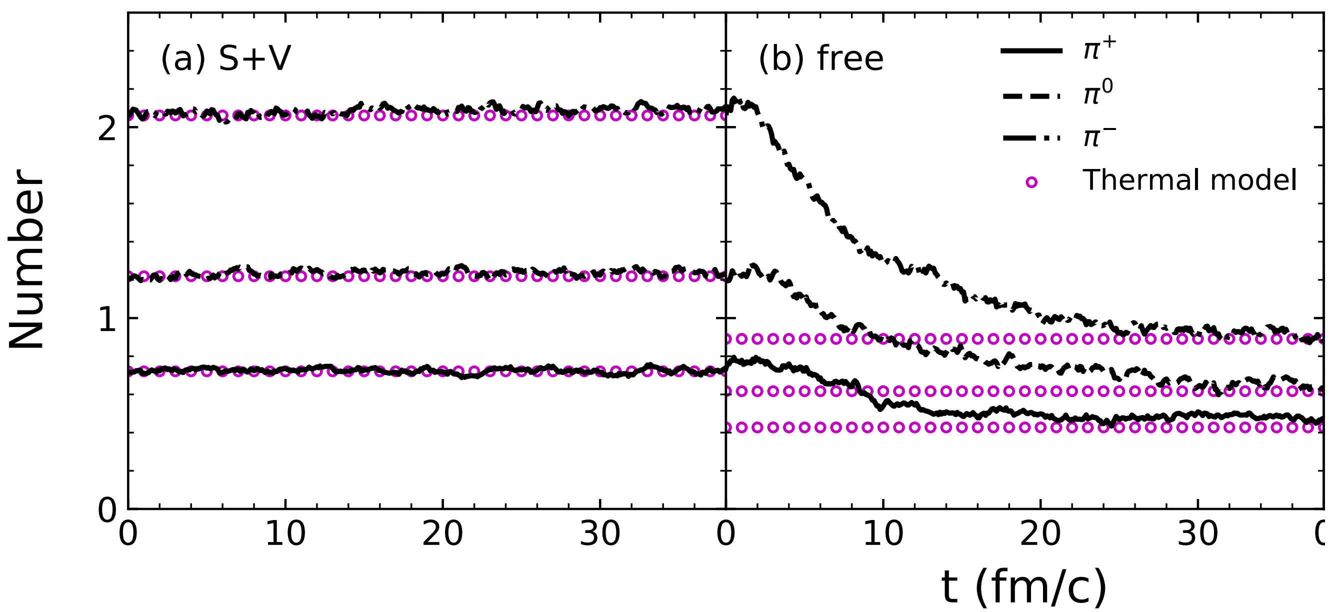
Also, potentials affect the threshold for inelastic collisions

$$\begin{aligned} E_{\rm cm} - E_{\rm cm}^{\rm th} &\approx m_1 + m_2 - m_3 - m_4 \\ &+ \frac{k_1^2}{m_1} + \frac{k_2^2}{m_2} + U_1 + U_2 - U_3 - U_4 \end{aligned}$$

So potentials affect both propagation and collisions. The latter is not included in essentially all transport models.

Effects of energy conservation on chemical equilibrium in hot dense symmetric nuclear matter (RVUU with $NL\rho$)





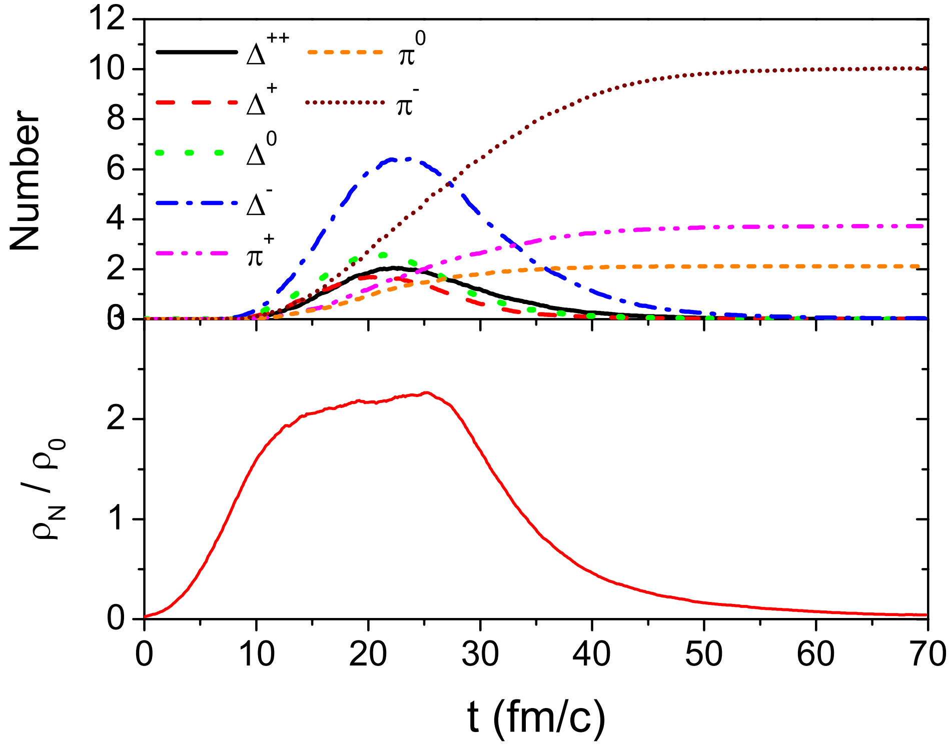
Zhang & Ko, PRC 97, 014910 (2018)

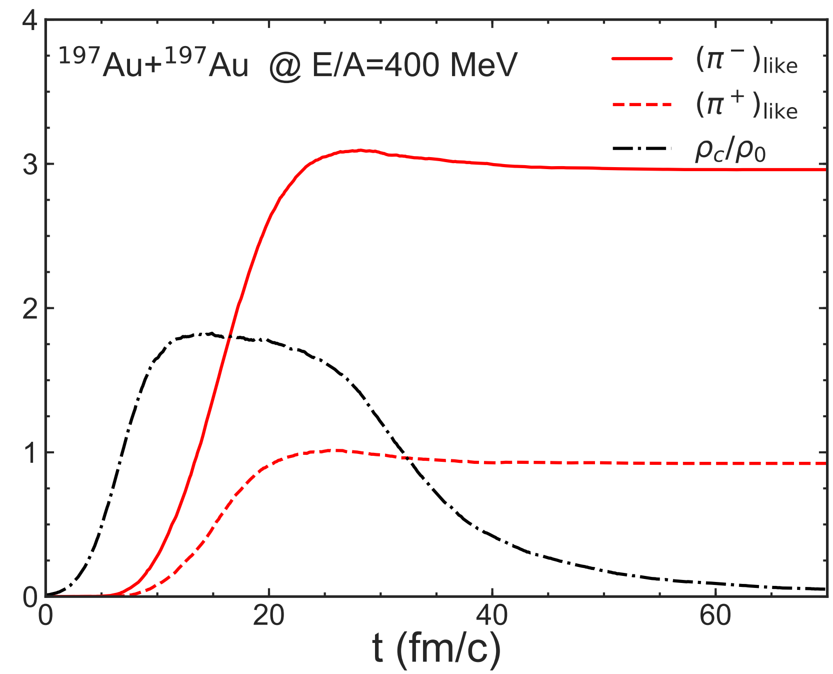
Nucleons, Deltas and pions in a box at T= 60 MeV, $\rho = 0.24\ \mathrm{fm}^{-3}$, $\rho_I = 0.096\ \mathrm{fm}^{-3}$

- Including potentials in the energy conservation during collisions keeps correct equilibrium distributions.
- Treating collisions as in free space, as done in essentially all transport models, leads to equilibrium distributions without potential effects.

Pion production in Au+Au collisions at E = 400 AMeV and b= 1fm (RVUU)

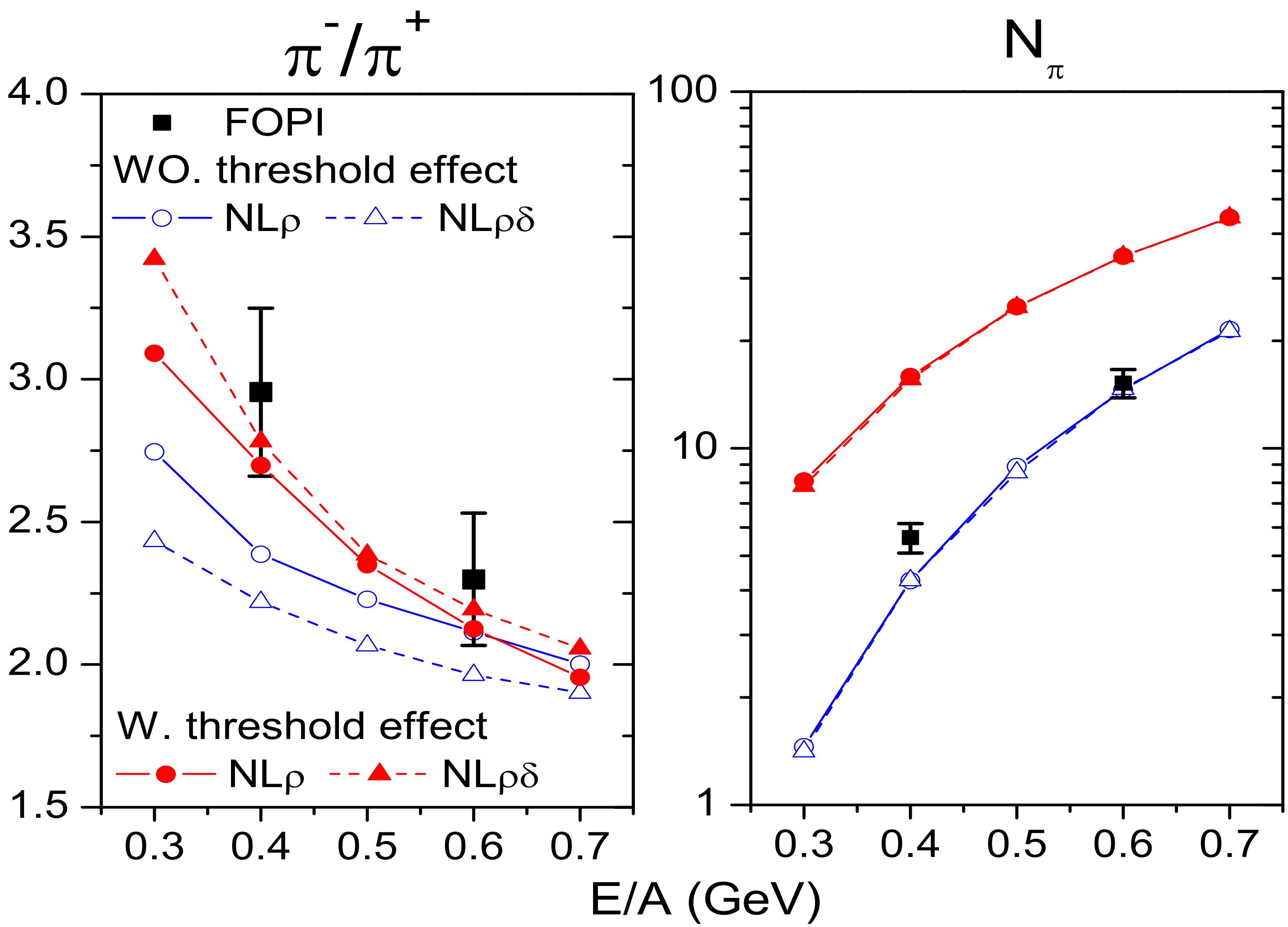
Song & Ko, PRC 91, 014901 (2015)





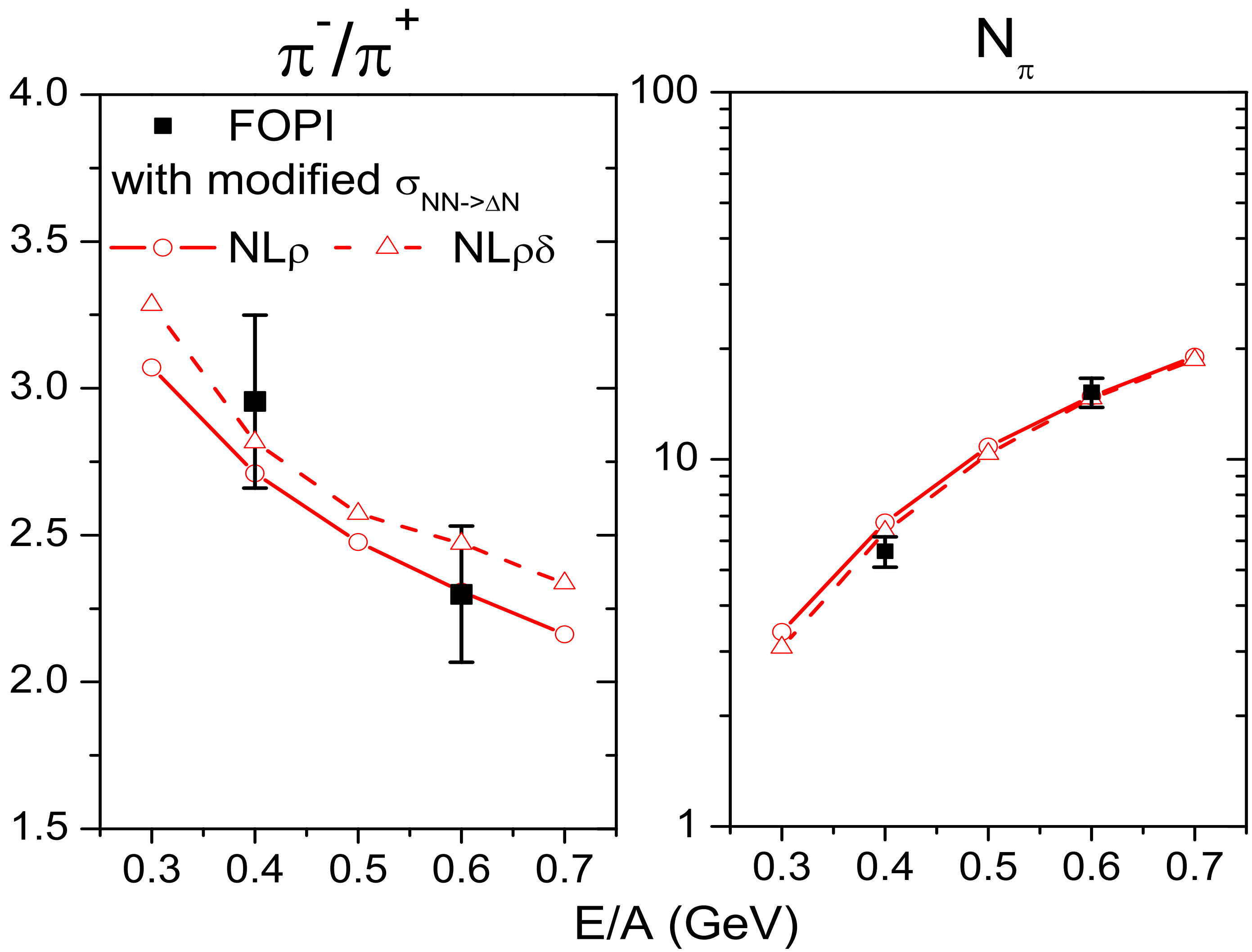
- Deltas are produced during high density stage and decay to pions as the matter expands.
- Both effective π^- and π^+ numbers (including those in Delta resonances remain unchanged after maximum compression (chemical freeze out), due to constancy of entropy per particle (Xu & Ko, PLB 772, 290 (2017)).

In-medium threshold effects on π^-/π^+ ratio



- In-medium threshold effects increase the total pion yield, the π^-/π^+ ratio, and reverse the effect of symmetry energy.

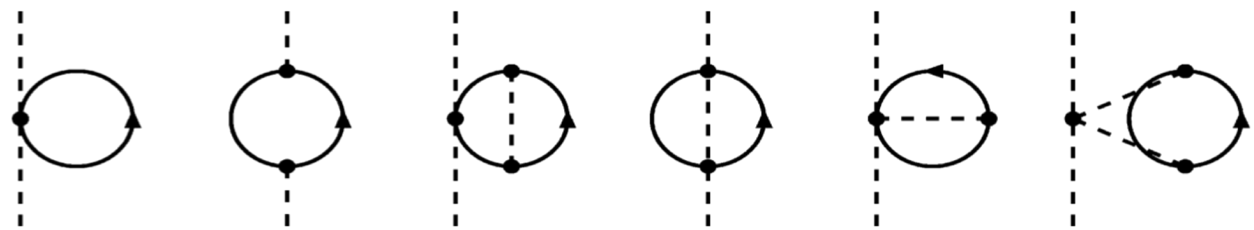
Effects of in-medium Delta production cross sections



- Reproducing the total pion yield requires density-dependent Delta production cross section $\sigma_{NN\to N\Delta}(\rho) = \sigma_{NN\to N\Delta}(0)\exp(-1.65\rho/\rho_0)$, similar to those by Larionov and Mosel, NPA 728, 135 (2003) and Prassa et al., NPA 789, 311 (2007).

Pion in nuclear matter (I)

- Pion s-wave selfenergies: Kaiser & Weise, PLB 512, 283 (2001)



$$\Pi^-(\rho_n,\rho_p)=\rho_n[T^-_{\pi N}-T^+_{\pi N}]-\rho_p[T^-_{\pi N}+T^+_{\pi N}]+\Pi^-_{rel}(\rho_n,\rho_p)+\Pi^-_{cor}(\rho_n,\rho_p)$$

$$\Pi^+(\rho_p,\rho_n)=\Pi^-(\rho_n,\rho_p)$$

$$\Pi^0(\rho_n,\rho_p)=-(\rho_p+\rho_n)T^+_{\pi N}+\Pi^0_{cor}(\rho_n,\rho_p)$$

Isospin even and odd πN-scattering matrices extracted from energy shift and width of 1s level in pionic hydrogen atom

$$T^+_{\pi N}\approx 1.847\,\mathrm{fm}\quad\text{and}\quad T^-_{\pi N}\approx -0.045\,\mathrm{fm}$$

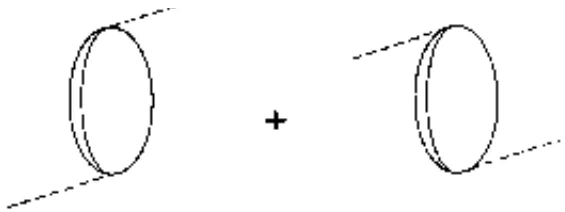
At normal nuclear density $\rho=0.165\ \mathrm{fm}^{-3}$ and isospin asymmetry $\delta=0.2$ such as in Pb,

$$U_\pi=\Pi/(2m_\pi)\qquad U_{\pi^-}=14\ \mathrm{MeV},\ U_{\pi^+}=-1\ \mathrm{MeV},\ U_{\pi^0}=6\ \mathrm{MeV}$$

Pion in nuclear matter (II)

Brown & Weise, PR 22, 279 (1975)

- Pion p-wave selfenergy

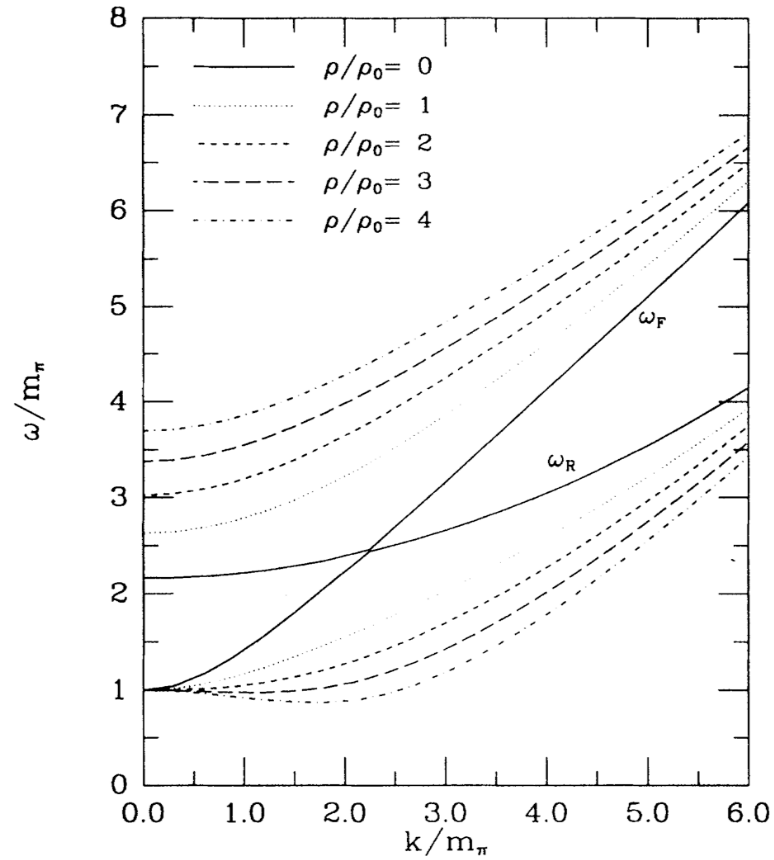


$$\Pi_0(\omega,k) \approx \frac{4}{3}\left(\frac{f_\Delta}{m_\pi}\right)^2 k^2 F^2(k)\rho \frac{\omega_0}{\omega^2-\omega_0^2}$$

$$\omega_0 \approx \frac{k^2}{2m_\Delta} + m_\Delta - m_N$$

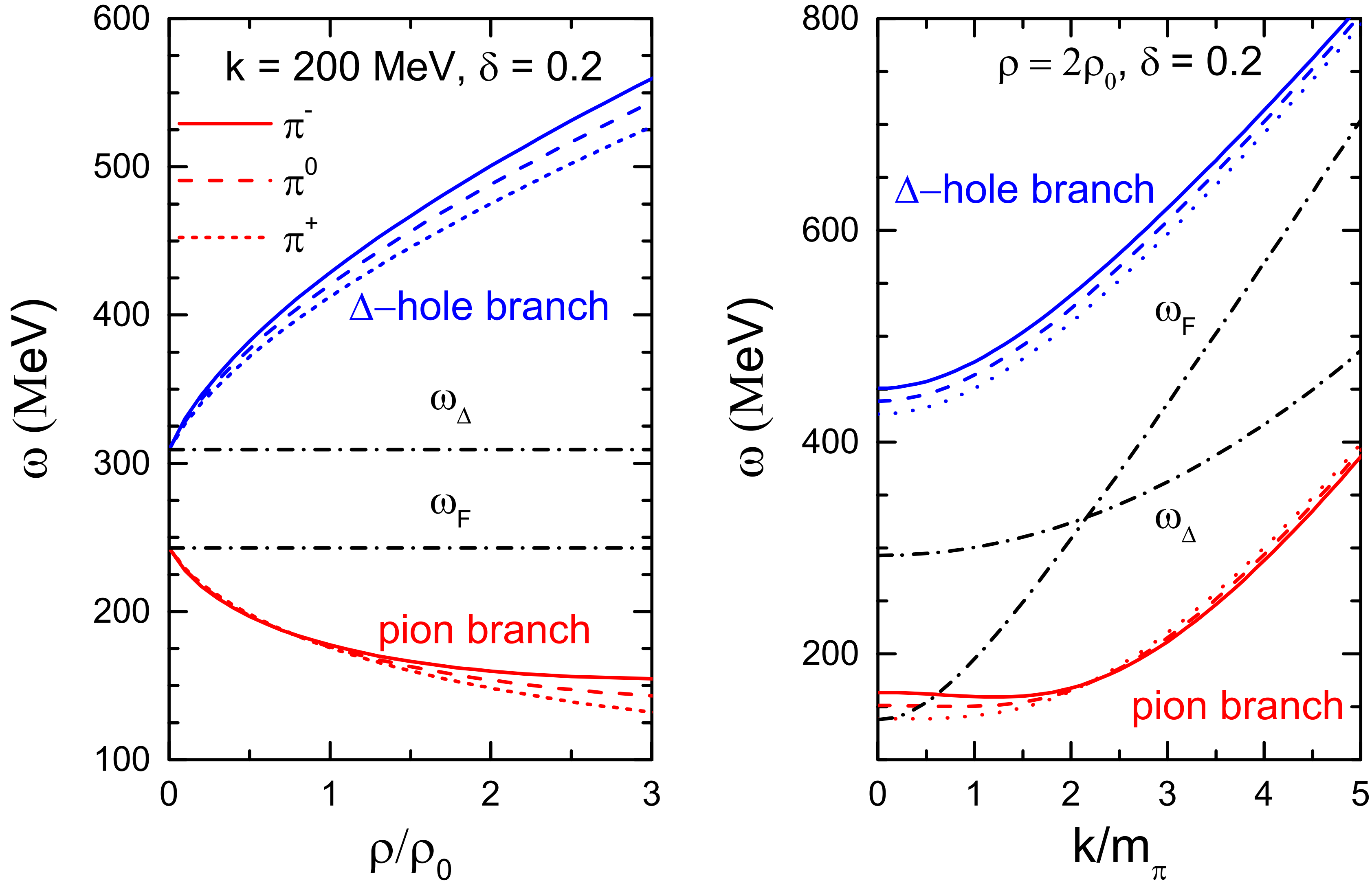
Including short-range repulsion through the Migdal parameter g' ~ 0.3

$$\Pi^{m_t}(\omega,k) = \frac{\Pi_0^{m_t}}{1-g'\Pi_0^{m_t}/k^2}$$



- Leads to a softening of the pion dispersion relation

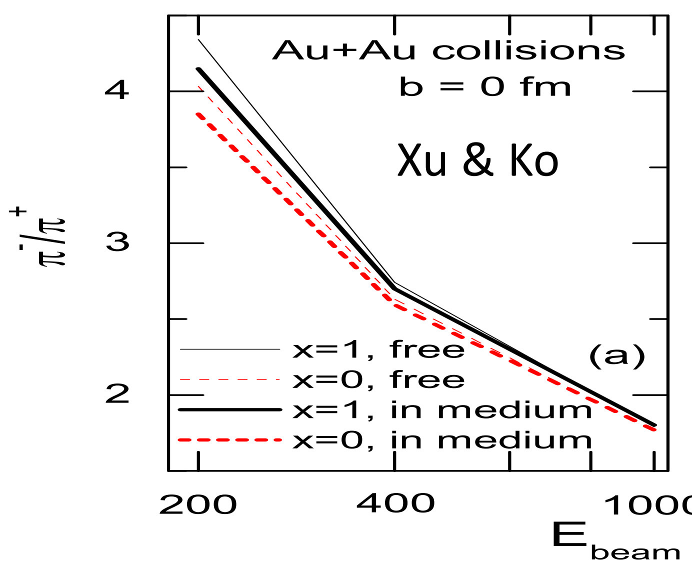
Pion energy in asymmetric nuclear matter

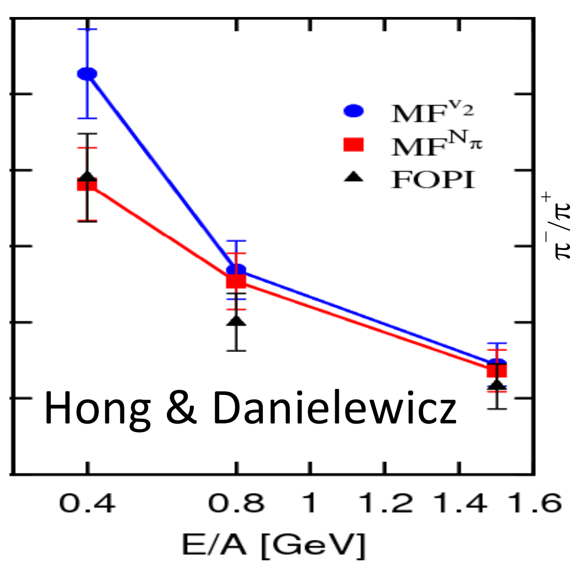


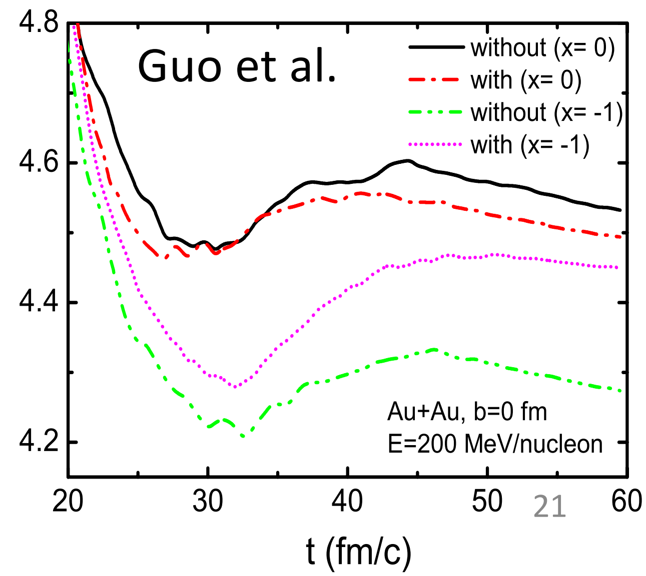
- Pion branch is lower in energy and thus more important.
- π^+ is lower than π^- and thus reduced π^-/π^+ ratio, opposite to that due to stiffness of symmetry energy.

Pion potential effects on charged pion ratio

- Xu & Ko, PRC 81, 024910 (2010); Xu, Chen, Ko, Li & Ma, PRC 87, 067601 (2013): Thermal model → Including both pion s- and p-wave interactions, which have opposite effects, decreases the π^-/π^+ ratio.
- Hong and Danielewicz, PRC 90, 024605 (2014): pBUU → π^-/π^+ ratio is insensitive to stiffness of symmetry energy after including pion s-wave potential.
- Guo, Yong, Liu & Zuo, PRC 91, 054616 (2015): IBUU → pion s- and p-wave potentials and symmetry potential have opposite effects. (p-wave potential essentially vanishes in this study because of average over the pion and Delta-hole branches.)
- Feng, EJPA 53, 30 (2017): LQMD → similar to Guo et al.

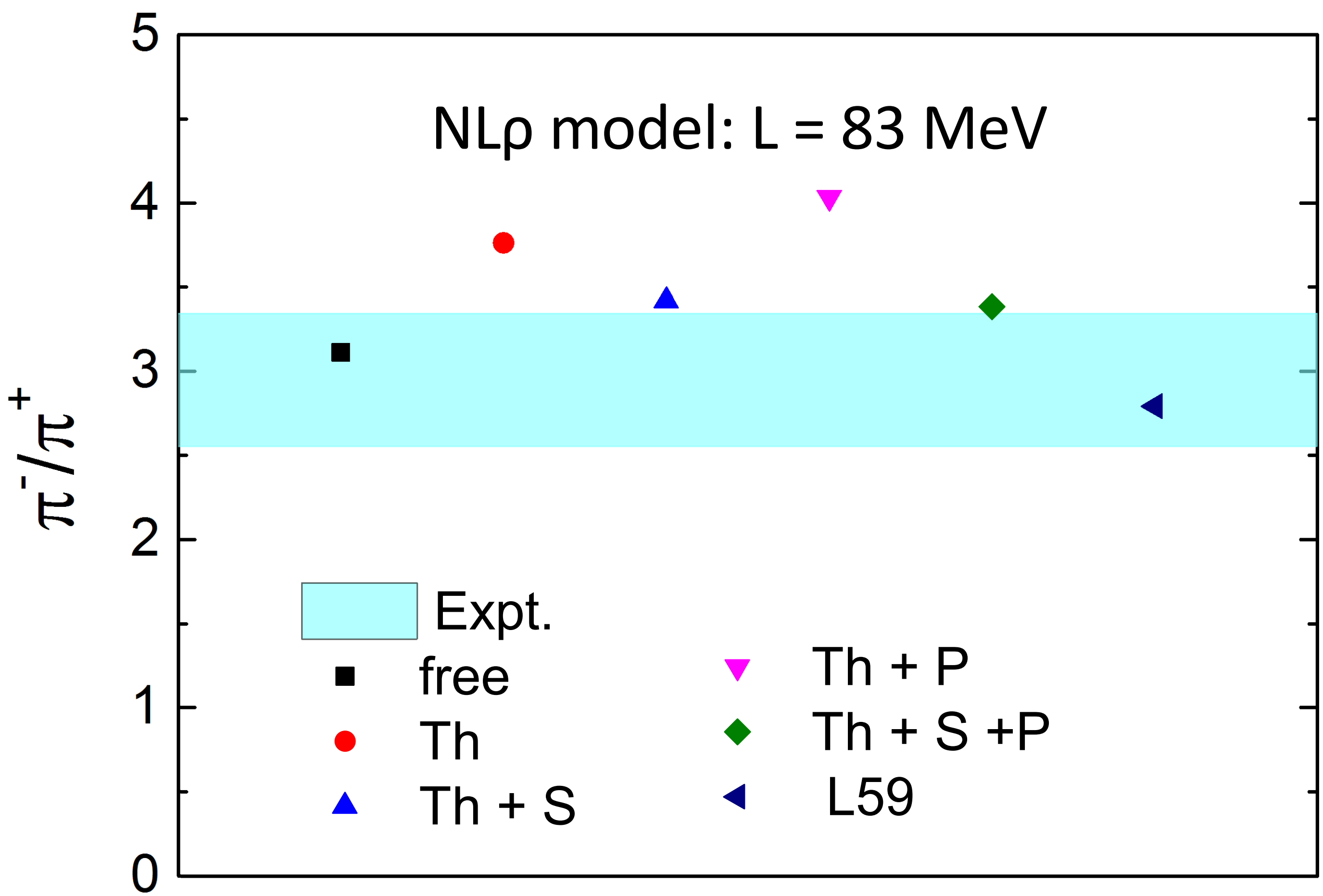






Charged pion ratio in Au+Au @ 400A MeV

Zhang& Ko, PRC 95, 064904 (2017)

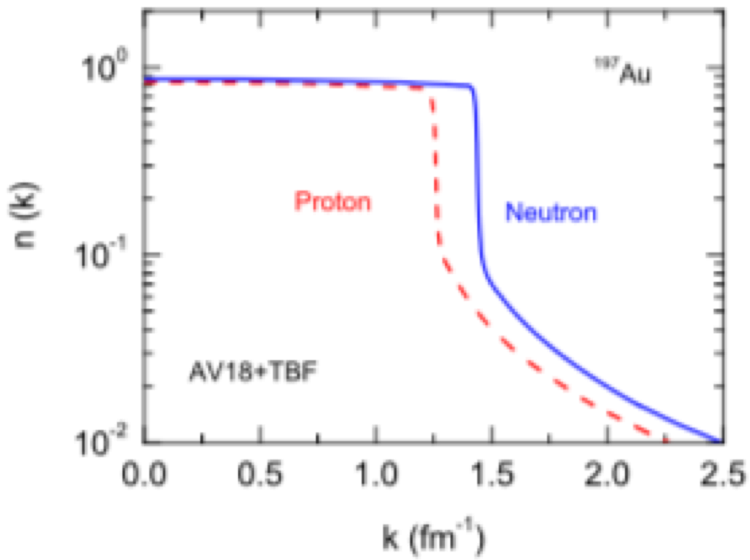


- Charged pion ratio is increased by threshold effect, reduced by s-wave potential, increased by p-wave potential, leading to a somewhat lager ratio compared to that without any medium effects.
- Reproducing FOPI data requires a small symmetry energy slope parameter L comparable with the constraints from nuclear structure and reactions as well as neutron star properties.

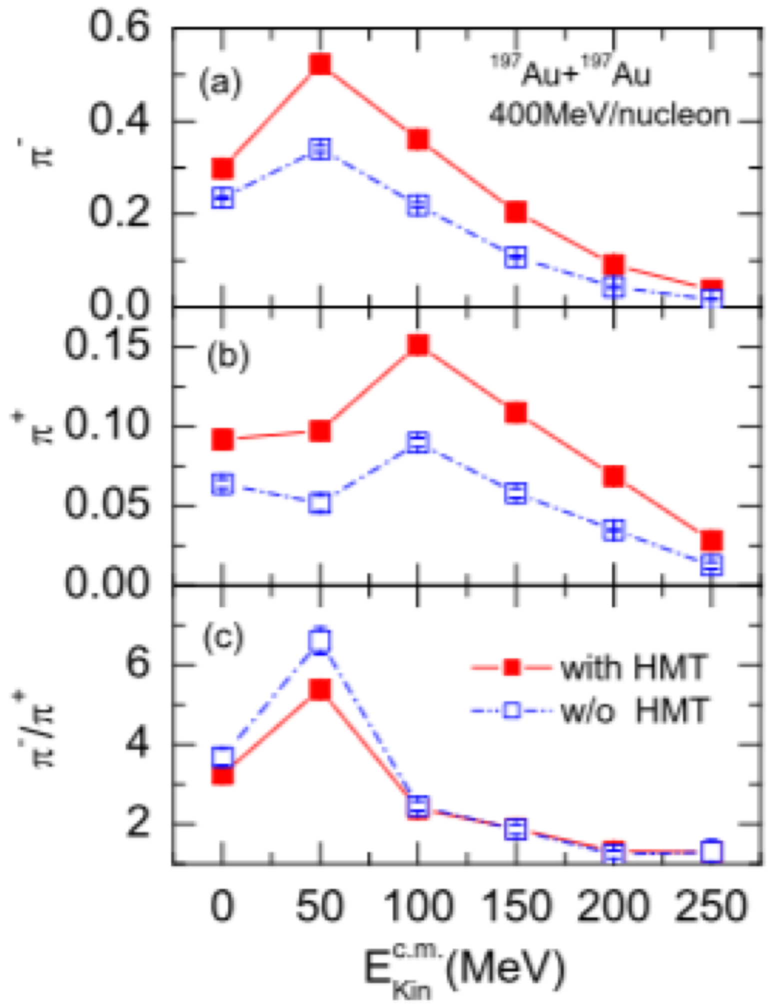
Effects of short-range correlations

Zhang & Yong & Zuo, EJPA 52, 350 (2016)

Nucleon momentum distributions

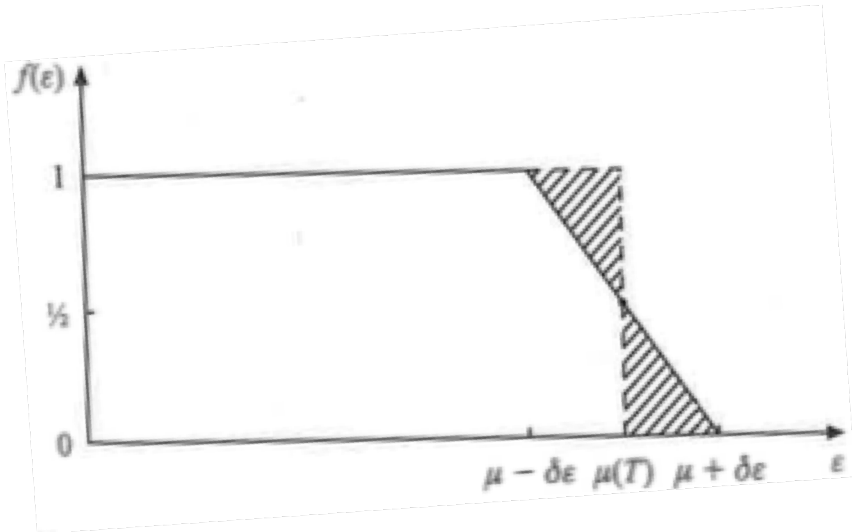


Spectra of charged pions and their ratio



- Fraction of high momentum nucleons ≈ 25 %
- Enhanced pion production is likely due to the larger incident energy as a result of the high momentum tail.

High momentum tail due to thermal excitation



Fraction of shifted nucleons ≈ density of states at Fermi energy times average single particle excitation

$$\frac{\Delta N}{N} = \frac{3\pi}{4\sqrt{6}}\frac{T}{\varepsilon_F} \approx \frac{T}{\varepsilon_F}$$

- At normal density, $\varepsilon_F \approx 35$ MeV, so 25% nucleons are above Fermi energy when $T \approx 9$ MeV, much smaller than the typical temperature of ≈ 60 MeV reached in HIC at intermediate energies of 300 – 400A MeV.
- Effect of short-range correlations is not expected to be important for pion production except at extremely subthreshold energy, where its production due to multinucleon collisions become important.

Chiral effective theory inspired transport model χBUU

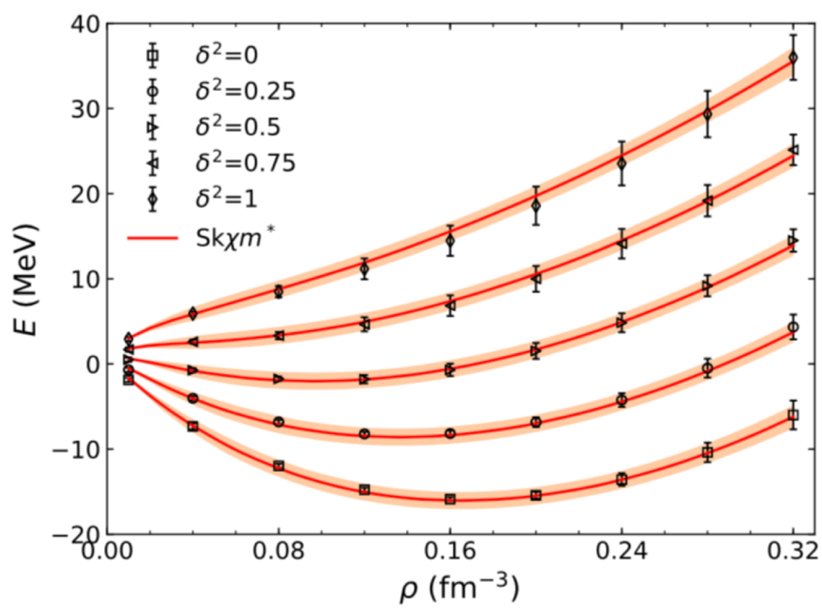
- Skχm* interaction (K_0=230 MeV, E_{sym}=31 MeV, L=45.6 MeV)

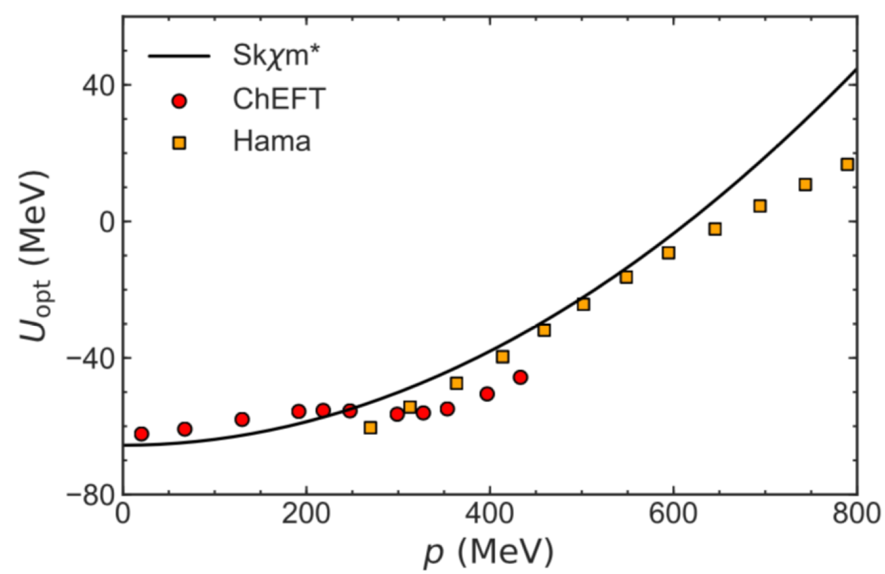
Zhang, Liam, Holt & Ko, PLB 777, 73 (2018), Zhang & Ko, submitted to PRC

$$
\begin{aligned}
v(\mathbf{r_1},\mathbf{r_2}) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r_1}-\mathbf{r_2}) + \frac{1}{2}\mathbf{t_1}(\mathbf{1}+\mathbf{x_1}\mathbf{P}_\sigma)[\mathbf{k'^2}\delta(\mathbf{r_1}-\mathbf{r_2}) + \text{c.c.}] \\
&+ t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r_1}-\mathbf{r_2})\mathbf{k} + \frac{1}{6}\mathbf{t_3}(\mathbf{1}+\mathbf{x_3}\mathbf{P}_\sigma)\rho^\alpha\left(\frac{\mathbf{r_1}+\mathbf{r_2}}{2}\right)\delta(\mathbf{r_1}-\mathbf{r_2}) \\
&+ iW_0(\sigma_1+\sigma_2)\cdot[\mathbf{k}'\times\delta(\mathbf{r_1}-\mathbf{r_2})\mathbf{k}]
\end{aligned}
$$

- Parameters fitted to EOS from chiral effective theory and binding energies of 7 doubly magic nuclei

- Good description of momentum dependence of optical potential below 600 MeV, dipole polarizability, and neutron skin thickness





Pion production from ^{197}Au+^{197}Aucollisions in χBUU

TABLE II: π^- and π^+ yields in ^{197}Au+^{197}Au collisions at the impact parameter of 1 fm and the incident energy of E/A=400 MeV. Experimental data from the FOPI Collaboration [50] are also listed for comparison.

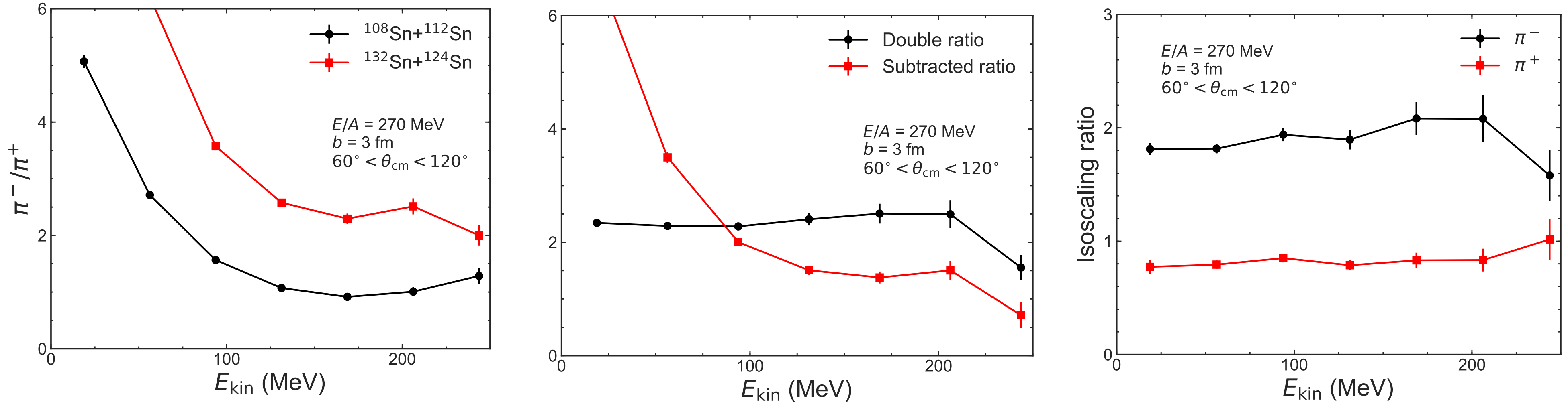
(Without pion potentials)

	FOPI [50]	free	Skχm*	
		$A=0$	$A=0$	$A=1.9$
π^-	2.80(14)	2.70(3)	7.84(4)	2.96(2)
π^+	0.95(8)	0.90(2)	2.57(2)	0.92(1)
$\pi^- + \pi^+$	3.75(22)	3.60(5)	10.41(6)	3.88(3)
π^-/π^+	2.95(29)	3.02(6)	3.05(3)	3.2(5)

- Without mean-field effects in collision terms can describe data.
- With mean-field effects in collision terms overestimates total pion yield, although give reasonable charged pion ratio.
- Introducing also reduced cross section for $NN \to N\Delta$ in medium can also reproduce data.

Pion production from Sn+Sn collisions in χBUU

Pion kinetic energy dependence of charged pion ratio, double and Subtracted ratios, and isoscaling ratios



- Double ratio: $(\pi^-/\pi^+)_{132+124}/(\pi^-/\pi^+)_{108+112}$
- Subtracted ratio: $(\pi^-/\pi^+)_{132+124} - (\pi^-/\pi^+)_{108+112}$
- Isoscaling ratio: $(\pi^-)_{132+124}/(\pi^-)_{108+112}$,
  $(\pi^+)_{132+124}/(\pi^+)_{108+112}$

Confirmation by data from SpiRIT would validate the EOS from nuclear chiral effective theory.

Summary

- Nuclear symmetry energy affects the π^-/π^+ ratio in HIC, but effect is only 30% between supersoft and superstiff ones, which is larger than current uncertainty among transport models.
- Transport Code Evaluation Project aims to reduce theory uncertainties.
- Effect of potentials needs to be included in scattering to ensure correct equilibrium pion abundance.
- In-medium threshold effect increases the total pion yield and the π^-/π^+ ratio, and reverse the effect of symmetry energy.
- Charged pion ratio is reduced by pion s-wave potential and increased by pion p-wave potential. The net effect is a reduction of the ratio if keeping the total pion number unchanged.
- Effect of short-range correlation is not important compared to thermal effects except at extremely subthreshold energies.
- Effects of clusters and offshell transport need to be studied.