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Symmetry energy in the chiral soliton model

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Strategy and Motivation

How to construct a theoretical framework (model of ``nuclear physics”)?

Our guiding principles are

- **simplicity (easy to analyse, transparent, etc…) <=> e.g. small number of terms in the Lagrangian;**
- **relation to phenomenology in a attractive way — as much as possible the peculiarities of strong interactions should be taken into account using as less as possible the number of parameters;**
- **universality <=> applicability to**
  - **hadron structure and spectrum studies (from light to heavy sector);**
  - **analysis of NN interactions;**
  - **nuclear many body problems <=> nucleonic systems (finite nuclei) and nuclear matter properties (EOS);**
  - **relation to mesonic atoms;**
  - **hadron structure changes in nuclear environment;**
  - **extreme density phenomena (e.g. neutron stars);**
  - **etc.**

Two possible ways (in the sense of choice):

- **to construct completely new approach;**
- **a bit fresh look to old ideas (e.g. putting a bit more phenomenological information).**

Content

- Topological soliton models
- Medium modifications
- Nucleon in nuclear matter
- Nuclear matter
    - symmetric matter
    - asymmetric matter
- Neutron stars
- Summary and Outlook

Topological soliton models

Why topological models?

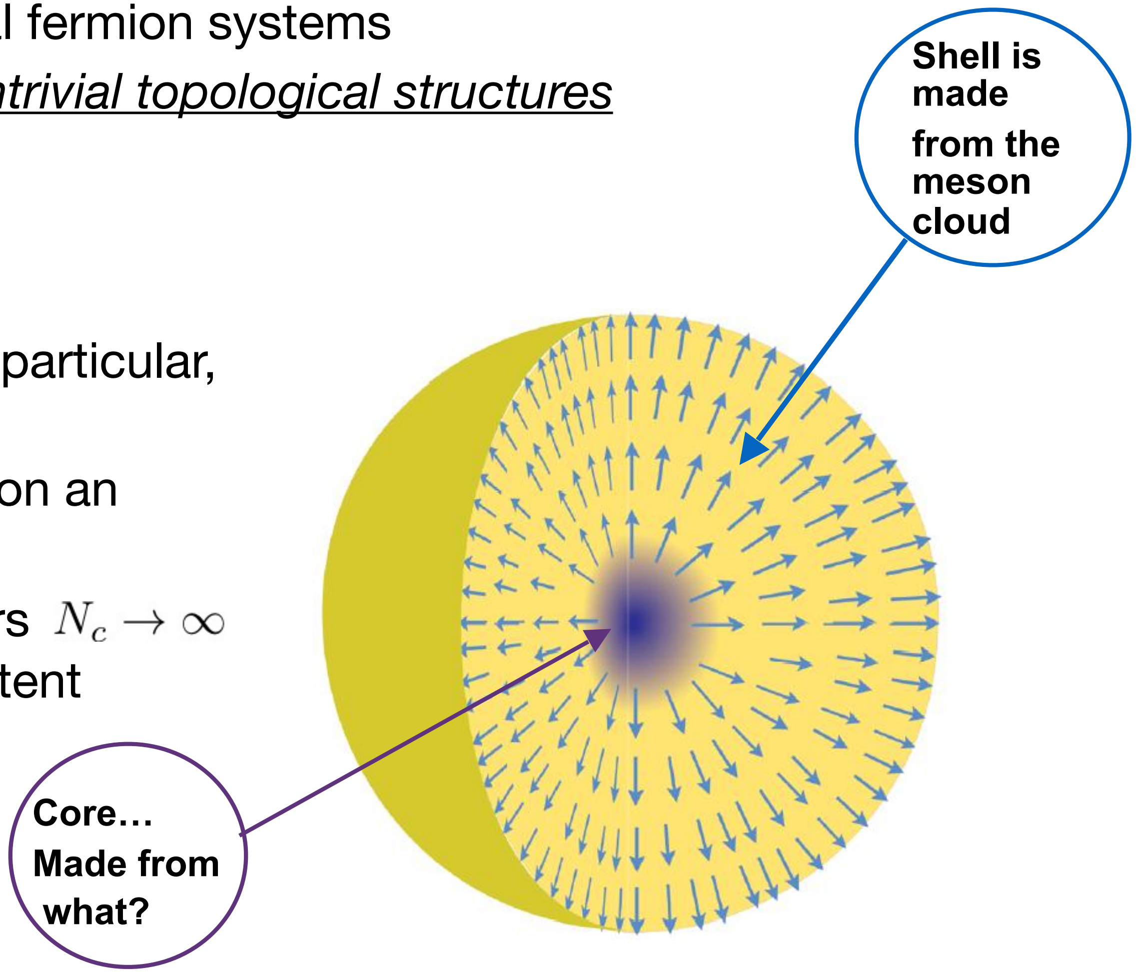
At fundamental level we may have

- fermions -> then bosons are trivial fermion systems
- bosons -> then fermions are *nontrivial topological structures*

Structure

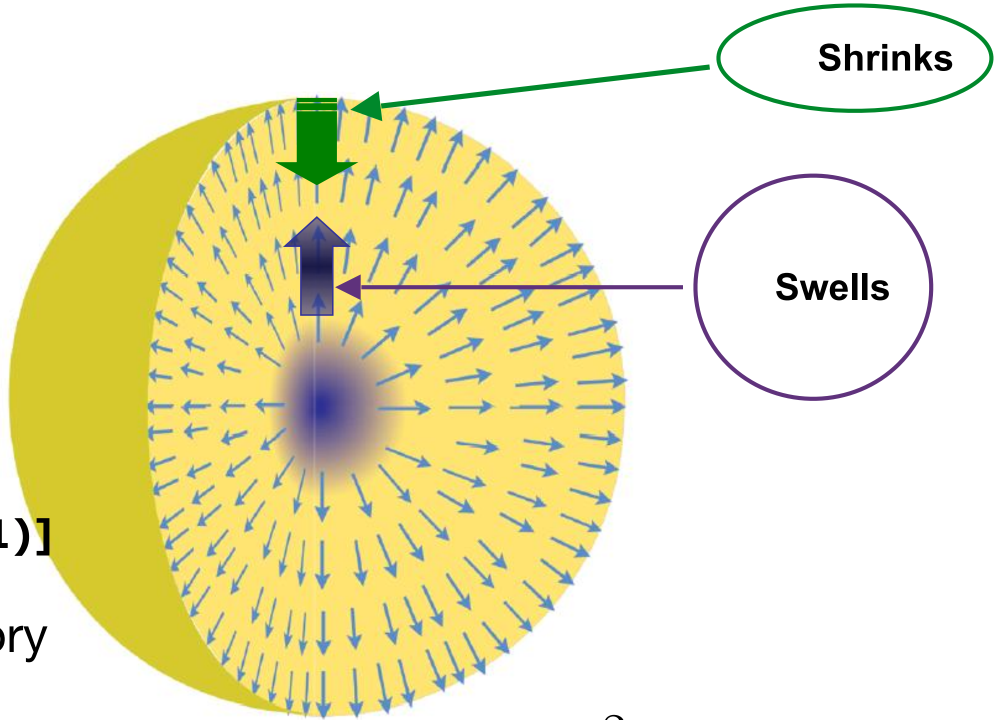
From what is made a nucleon and, in particular, its core in a boson picture approach?

- The structure treatment depends on an energy scale
- At the limit of large number colours $N_c \to \infty$ the core still has the mesonic content



Stabilisation mechanism

- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian



Prototype: **Skyrme model**

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2$$

Shrinking term ← → Swelling term

- *Hedgehog* solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

Topological soliton models

The free space Lagrangian (which was widely in use)
[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}\left(U + U^\dagger - 2\right)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) ***A***

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

- Nucleon is quantized state of the classical soliton-skyrmion which rotates in the ordinary and internal spaces

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|\,S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

What happens in the nuclear medium?

The possible medium effects

- Deformations (swelling or shrinking, multipole deformations) of nucleons
- Characteristic changes in: effective mass, charge distributions, all possible form factors
- NN interactions may change
- etc.

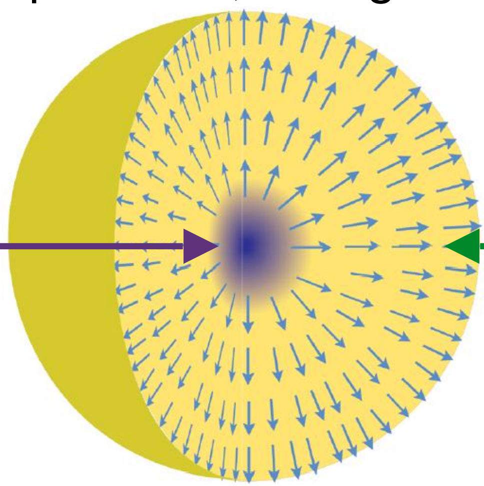
One should be able to describe all those phenomena

Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)

Inner core modifications in the nuclear medium may be related to:

- **vector meson properties in the nuclear medium**
- **nuclear matter properties at saturation density**



Meson cloud modifications in the nuclear medium:

Pion physics in the nuclear medium

Medium modifications

“Outer shell” modifications

- In free space three types of pions can be treated separately: isospin breaking

$$(\partial^\mu \partial_\mu + m_\pi^2)\vec{\pi}^{(\pm,0)} = 0$$

- In nuclear matter: there are three types of polarization operators

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Optic potential approach: parameters from the pion-nucleon scattering (including the isospin dependents)

	π-atom	$T_\pi = 50$ MeV
b_0 $[m_\pi^{-1}]$	- 0.03	- 0.04
b_1 $[m_\pi^{-1}]$	- 0.09	- 0.09
c_0 $[m_\pi^{-3}]$	0.23	0.25
c_1 $[m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

"Outer shell" modifications in the Lagrangian
[U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16}\alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \mathrm{Tr}\left(2 - U - U^\dagger\right)$$

$$\hat{\Pi}^0 = 2\omega U_{\mathrm{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla}\cdot\chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Due to the non-locality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters, the following parts of the kinetic term are modified in different forms:
  - Temporal part
  - Space part

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
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Medium modifications

“Inner core” modifications
[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau}\mathrm{Tr}\,\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\,\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

may be related to

- Vector meson properties in nuclear matter;
- Nuclear matter properties.

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \text{parameters})$$

Medium modifications

Final Lagrangian
[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}$$

- Isoscalar part

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_m$$

- Isovector part

$$\mathcal{L}^*_{\rm asym} = \mathcal{L}^*_{\delta m} + \mathcal{L}^*_{\delta\rho}$$

- **Nuclear matter stabilization** (→ $\mathcal{L}^*_2$, $\mathcal{L}^*_4$)
- **Asymmetric matter properties** (→ $\mathcal{L}^*_{\delta\rho}$)

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}\mathrm{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

$$\mathcal{L}^*_m = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \mathrm{Tr}\left(2 - U - U^+\right)$$

$$\mathcal{L}^*_{\delta m} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m_{\pi^\pm}^2 - m_{\pi^0}^2)\mathrm{Tr}\left(\tau_a U\right)\mathrm{Tr}\left(\tau_a U^\dagger\right)$$

$$\mathcal{L}^*_{\delta\rho} = -\frac{F_\pi^2}{16} m_\pi \alpha_e\ \varepsilon_{ab3}\mathrm{Tr}\left(\tau_a U\right)\mathrm{Tr}\left(\tau_b \partial_0 U^\dagger\right)$$

Medium modifications

Reparametrization
[UY, PRC88 (2013)]

- Five density dependent parameters
- Rearrangment (technical simplification)

Outer shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau},\quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau,\quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

$$1 + C_1\lambda = f_1(\lambda) \equiv \sqrt{\frac{\alpha_p^0}{\zeta_s}},$$

$$1 + C_2\lambda = f_2(\lambda) \equiv \frac{\alpha_s^{00}}{\left(\alpha_p^0\right)^2 \zeta_s},$$

$$1 + C_3\lambda = f_3(\lambda) \equiv \frac{\left(\alpha_p^0 \zeta_s\right)^{3/2}}{\alpha_s^{02}},$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Nucleon in nuclear matter

Structure studies1: EMT FF of in-medium nucleons
[H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

Structure studies 2: Transverse EM charge densities of in-medium nucleons
[UY, H.C.Kim, Phys.Lett. B726 (2013)]

Nucleon in nuclear matter

Static properties (e.g. mass) [UY, PRC88 (2013)]

- Isoscalar effective mass

$$m^*_{N,s} = M^*_S + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2}\left(a^{*2} + \frac{\Lambda^{*2}_{\mathrm{env}}}{\Lambda^{*2}}\right)$$

- Isovector effective mass (relevant to: universe evolution in Early stage; stability of drip line nuclei; mirror nuclei; transport in heavy-ion collisions; asymmetric nuclear matter)

$$\Delta m^*_{np} = a^* + \frac{\Lambda^*_{\mathrm{env}}}{\Lambda^*}$$

- Effective masses of the nucleons

$$m^*_{n,p} = m^*_{N,s} - \Delta m^*_{np} T_3$$

Nuclear matter

Bethe-Weizsacker formula for the binding energy per nucleon

$$\varepsilon\,(A, Z) = -a_V + a_S \frac{(N - Z)^2}{A^2} + \text{[illegible]}$$

Its terms can be obtained in the framework of present model

We are ready to reproduce (Volume term, Asymmetry term)

- Volume term
  - considering symmetric infinite nuclear matter
- Asymmetry term
  - considering isospin asymmetric environment
- Surface and Coulomb terms
  - considering the nucleons in a finite volume
- Finite nuclei properties
  - using the local density approximation

Nuclear matter

The volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda,\delta) = \varepsilon_V(\lambda) + \varepsilon_S\delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda,\delta)$$

  - λ is normalised nuclear matter density
  - δ is asymmetry parameter
  - ε_S is symmetry energy

- Our model calculations
  - Symmetric matter
  - Asymmetric matter

$$\varepsilon_V(\lambda) = m^*_{N,s}(\lambda,0) - m_N^{\text{free}}$$

$$\varepsilon_A(\lambda,\delta) = \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda)$$

$$= m^*_{N,s}(\lambda,\delta) - m^*_{N,s}(\lambda,0) + m^*_{N,V}(\lambda,\delta)\delta$$

Nuclear matter

Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left. \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda} \right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left. \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2} \right|_{\rho=\rho_0} \quad Q = 27\lambda^3 \left. \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

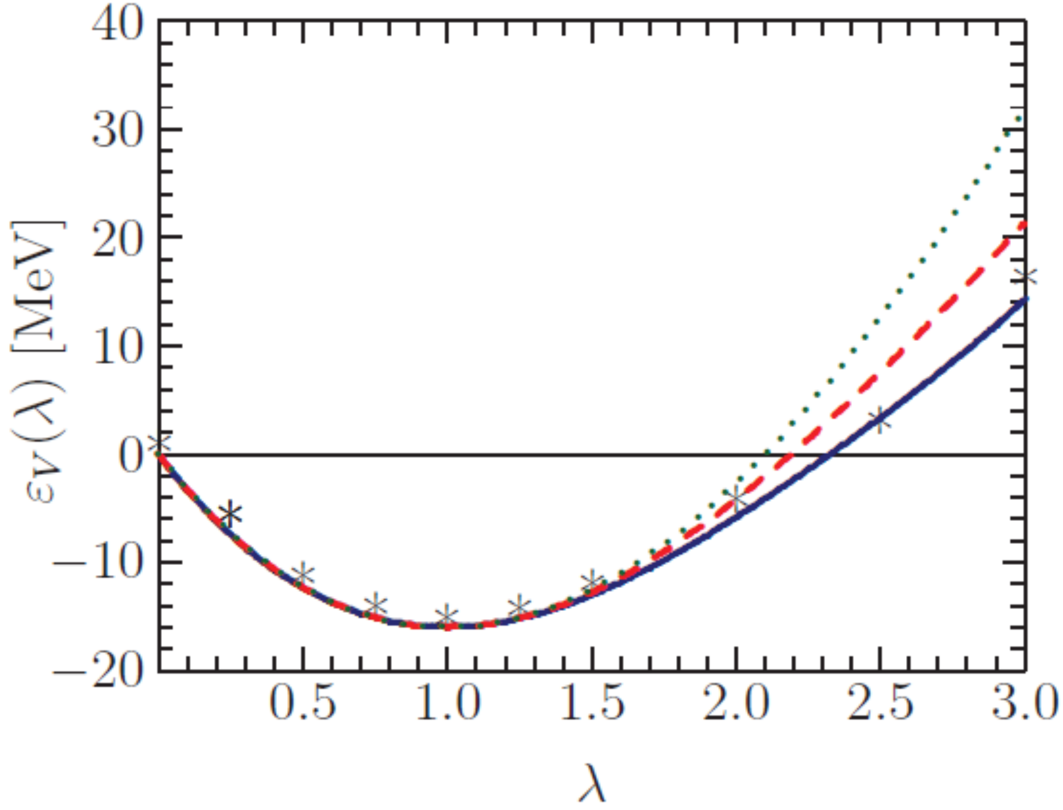
$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 +$$

Volume energy
[UY, PRC88 (2013)]

- Set I – solid
- Set II – dashed
- Set III – dotted

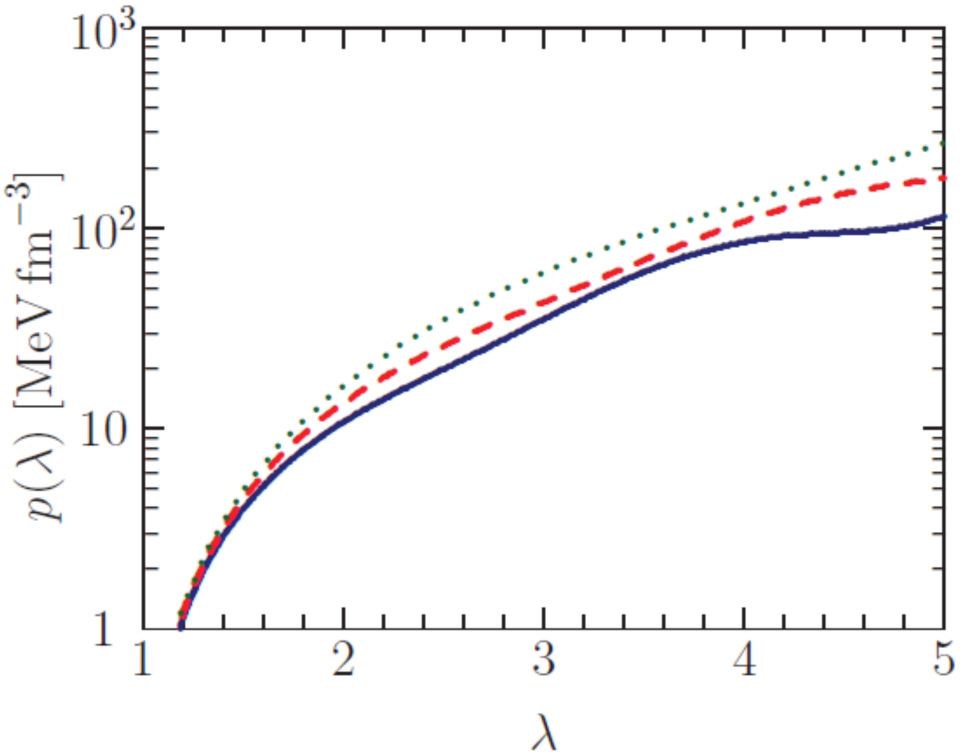
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.

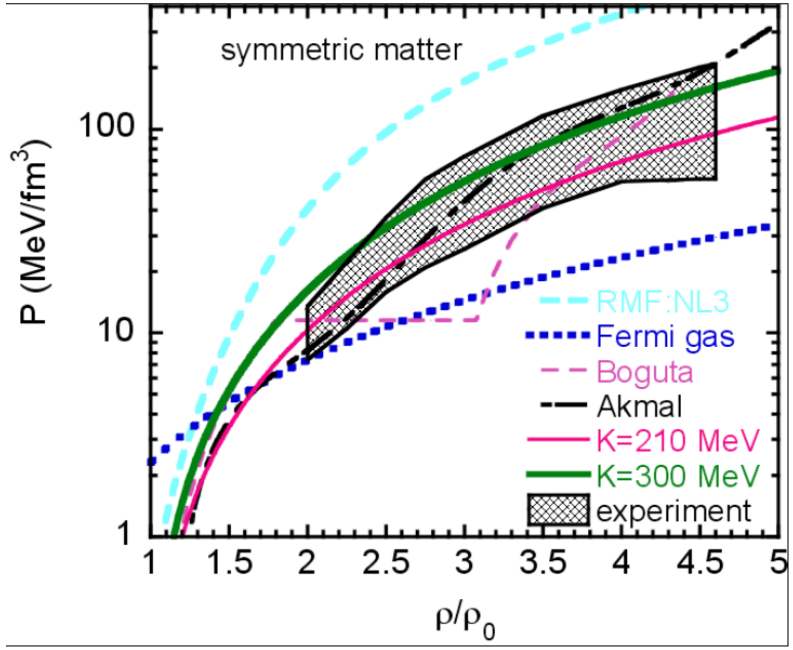
(From Arigonna 2 body interactions + 3 body interactions)



Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

Pressure
[UY, PRC88 (2013)]





For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

Asymmetric matter

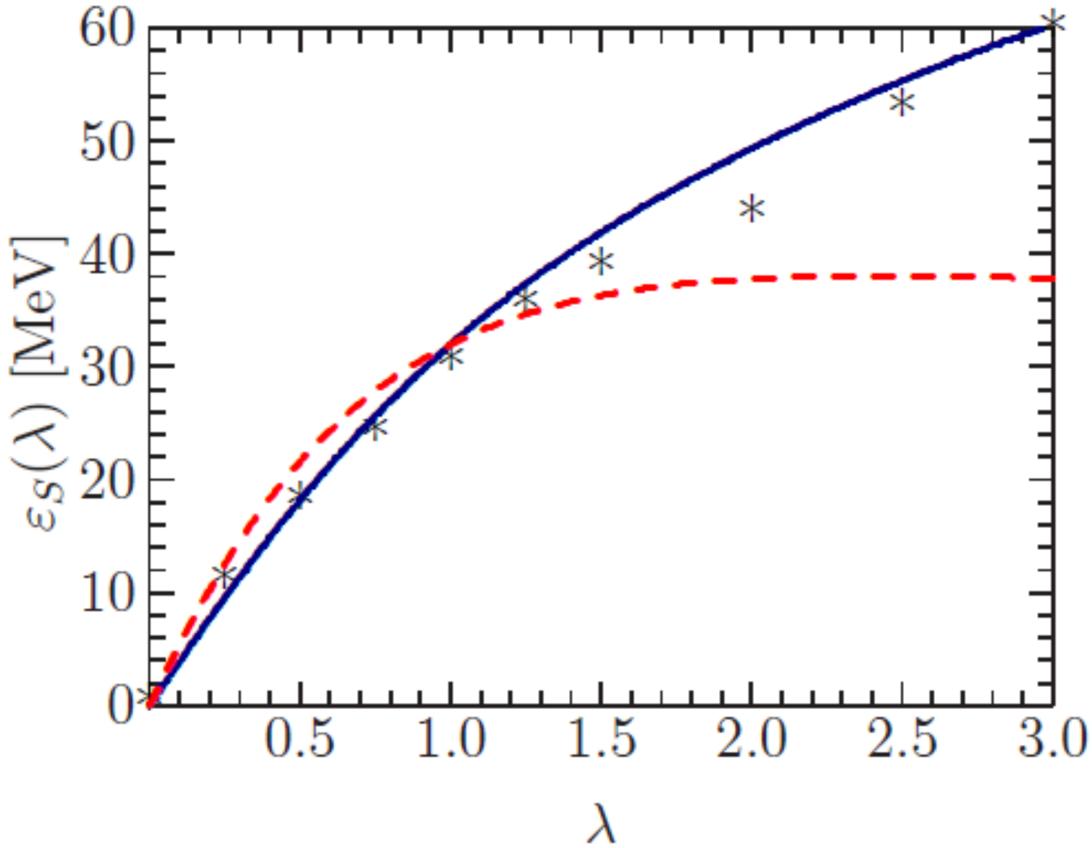
Symmetry energy

- Solid $L_s = 70\,\mathrm{MeV}$
- Dashed $L_s = 40\,\mathrm{MeV}$

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions

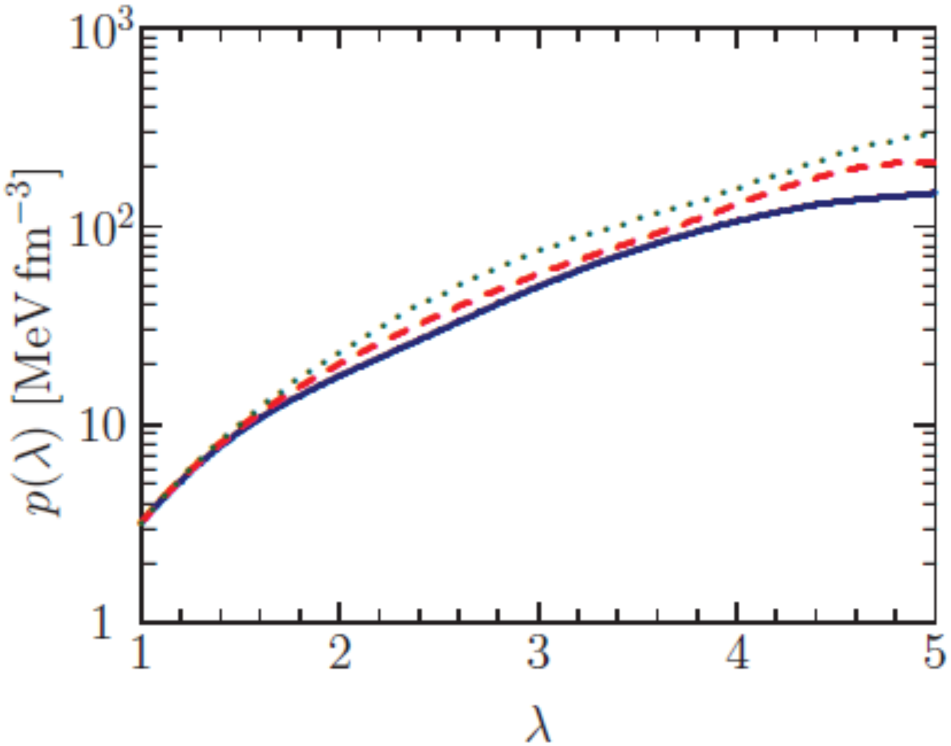
[PRC 58, 1804 (1998)] are given by stars.

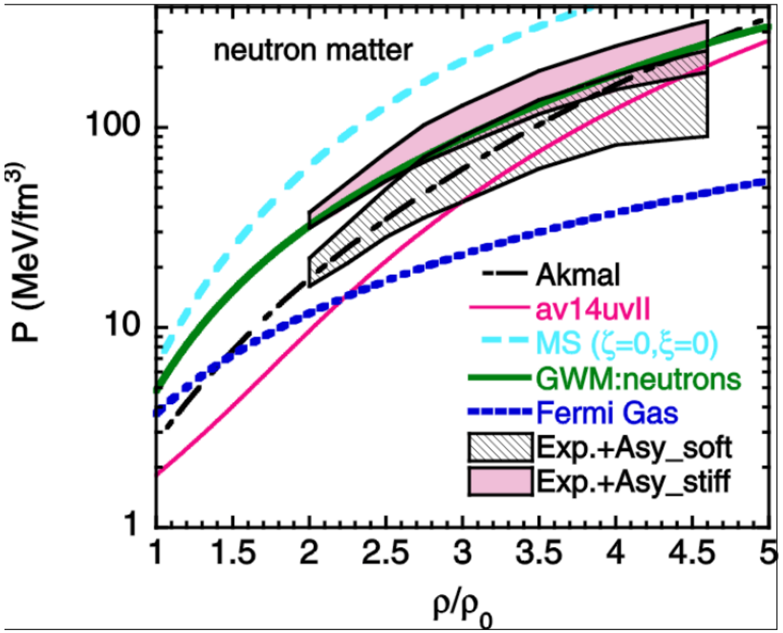
(From arigonna 2 body interactions + 3 body interactions)



Asymmetric matter

Pressure in neutron matter
[UY, PRC88 (2013)]





For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

Asymmetric matter

Low density behaviour of symmetry energy

For comparison:
Trippa-Colo-Vigezzi
[PRC 77, 061304 (2008)];
From analysis of GDR
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\mathrm{fm}^{-3}) < 24.9\ \mathrm{MeV}$$

Consequently one can predict in this model:

$$K_\tau = K_s - 6L_s$$

$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_S$$

$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\mathrm{fm}^{-3})$ [MeV]
32	40	−181	−301	−257	25.15
32	50	−160	−310	−254	24.15
32	60	−126	−306	−239	23.22
32	70	−80	−290	−211	22.37
32	80	−21	−261	−172	21.57
32	90	50	−220	−119	20.82
32	100	134	−166	−55	20.13

Neutron star properties

- **TOV equations**

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1-\frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1+\frac{P(r)}{\mathcal{E}(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

- **Energy-pressure relation**

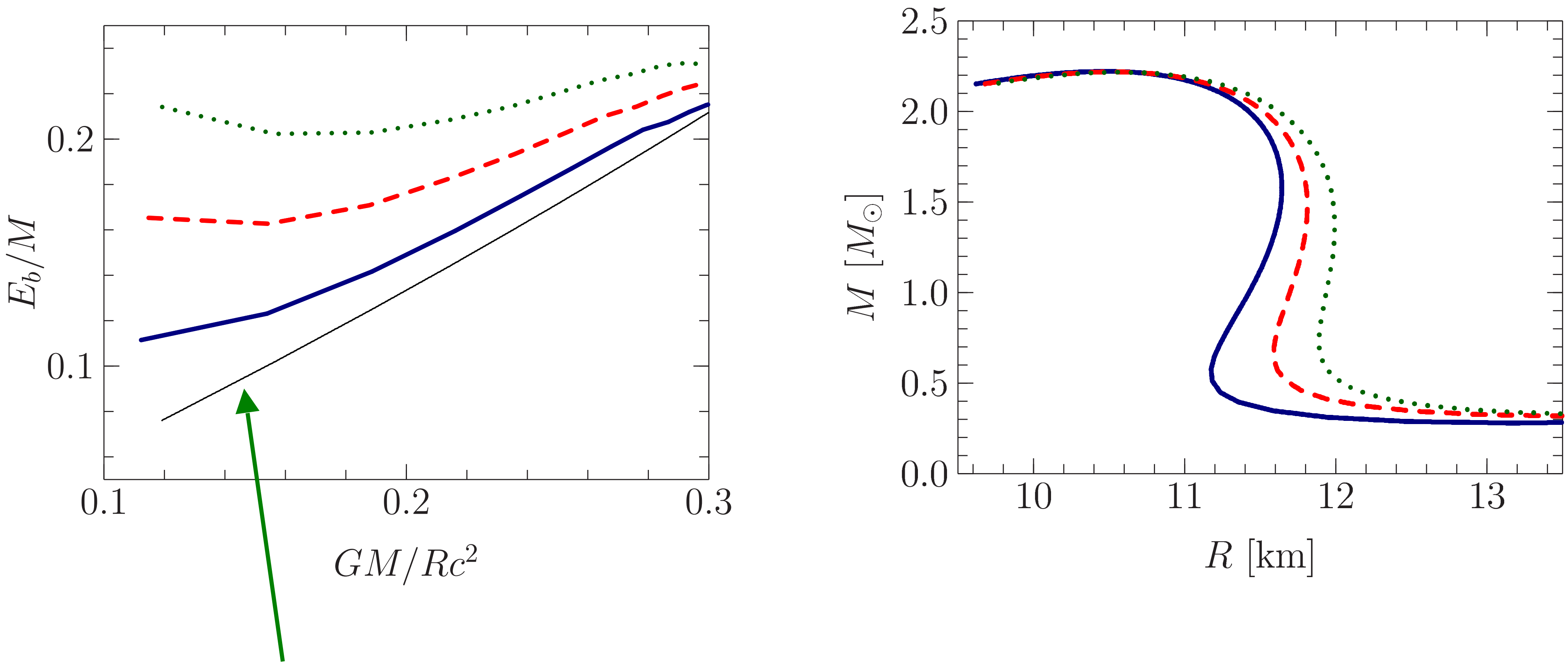
$$P = P(\mathcal{E})$$

$$\begin{aligned} P(\lambda) &= \rho_0\lambda^2\frac{\partial\varepsilon(\lambda,1)}{\partial\lambda}\,, \\ \mathcal{E}(\lambda) &= [\varepsilon(\lambda,1)+m_N]\lambda\rho_0. \end{aligned}$$

- **Neutron star's mass**

$$\mathcal{M}(r) = 4\pi\int_0^r \mathrm{d}r\, r^2\mathcal{E}(r)\,.$$

Neutron star properties [UY, PLB749 (2015)]



From Ref. [J.M. Lattimer & M. Prakash, Astrophys. J. 550 (2001)].

Neutron star properties

[UY, PLB749 (2015)]

TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters): n_c is central number density, ρ_c is central energy-mass density, R is radius of the neutron star, $M_{\max}$ is possible maximal mass, A is number of baryons in the star, E_b is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass $M_{\max}$ and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	$M_{\max}$ [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	M [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

Summary and Outlook

The present model describes at same footing (the corresponding phenomenology always qualitatively and in several cases quantitatively too)

the single nucleon properties

- **in free space considering it as a structure-full system**
- **in nuclear matter (EM and EMT form factors)**

as well as the properties of the whole nucleonic systems

- **infinite nuclear matter properties (volume and symmetry energy properties)**
- **matter under extreme conditions (e.g. neutron stars)**
- **few/many nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)**
- **nucleon knock-out reactions (lepton-nucleus scattering experiments)**
- **possible changes in in-medium NN interactions**
- **etc**

Thank you very much for your attention!