Universal correlations in the nuclear symmetry energy, slope parameter, and curvature

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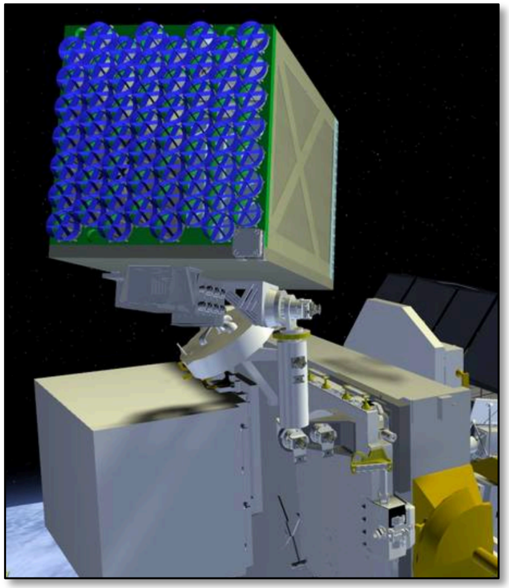
Supported by:

NuSYM2018, September 10, 2018

Next-generation observational campaigns of neutron stars

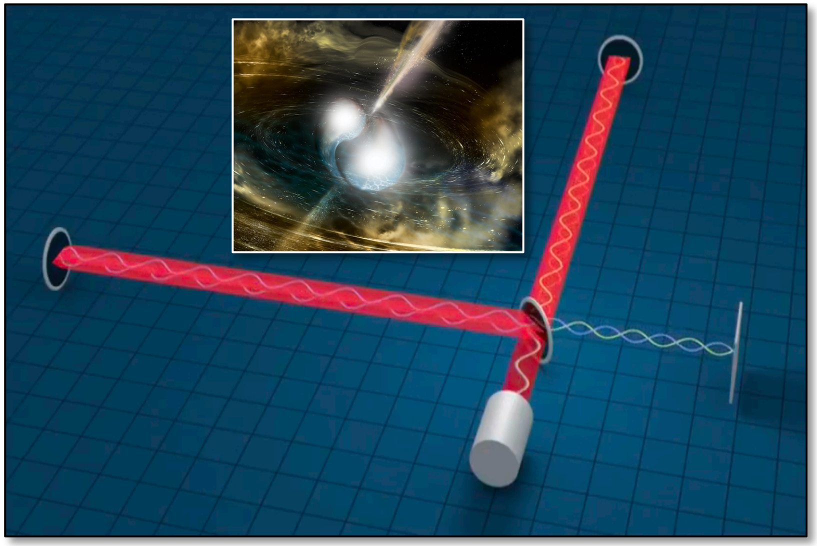


Neutron Star Interior Composition Explorer (NICER)

- Combined timing and spectral resolution in the soft X-ray band
- First dedicated targets: { PSR_J0437-4715, PSR_J0030+0451 }
- Neutron star radii: $\pm 5\%$
- Neutron star masses: $\pm 10\%$

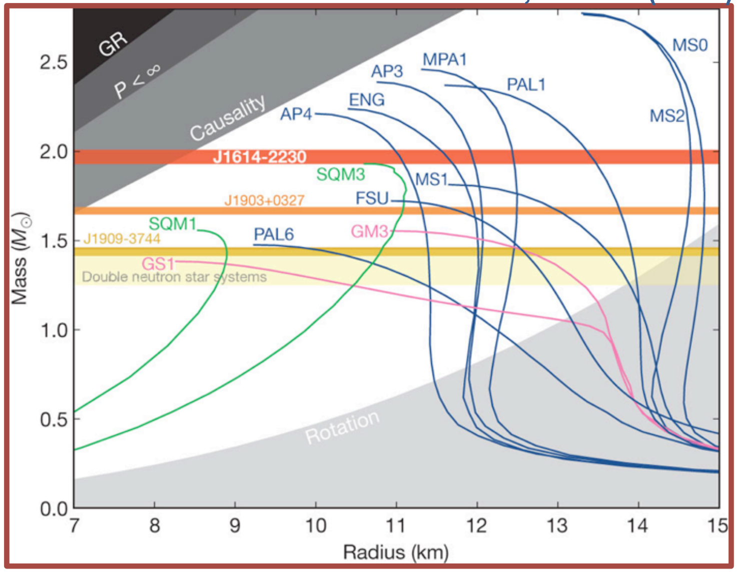
LIGO/VIRGO

- Late-inspiral gravitational waveform related to neutron star tidal deformability
- Poster-merger peak frequency sensitive to neutron star radius



Equation of state constraints from simultaneous neutron star mass and radius measurements

Demorest et al., Nature (2010)



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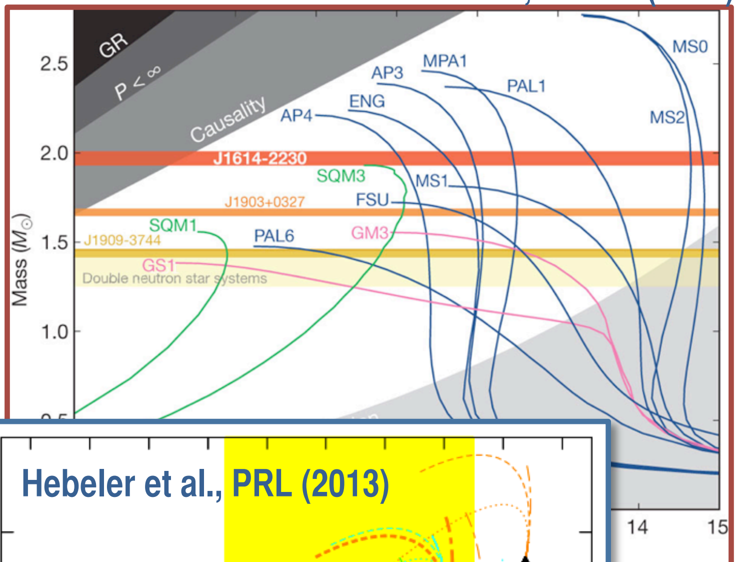

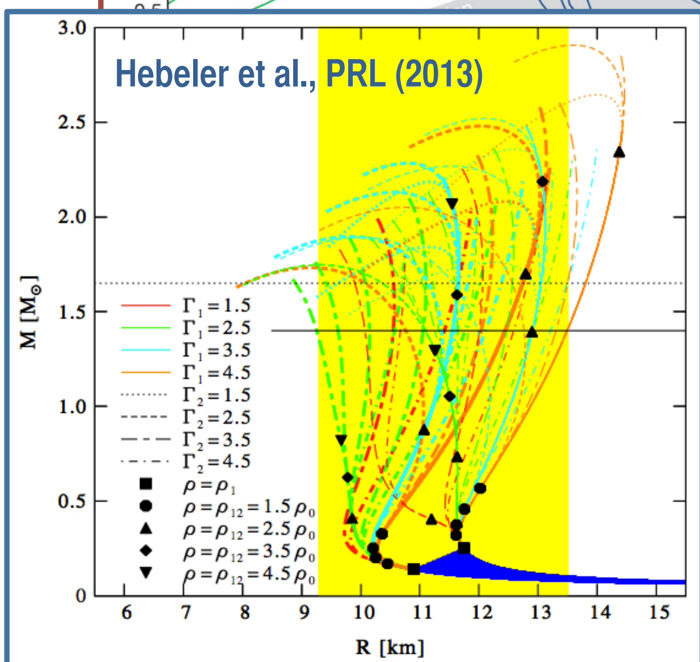

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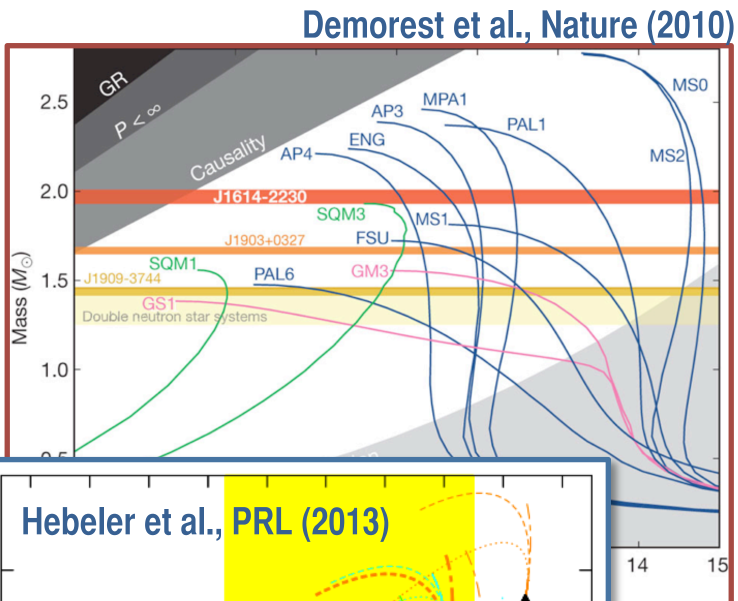

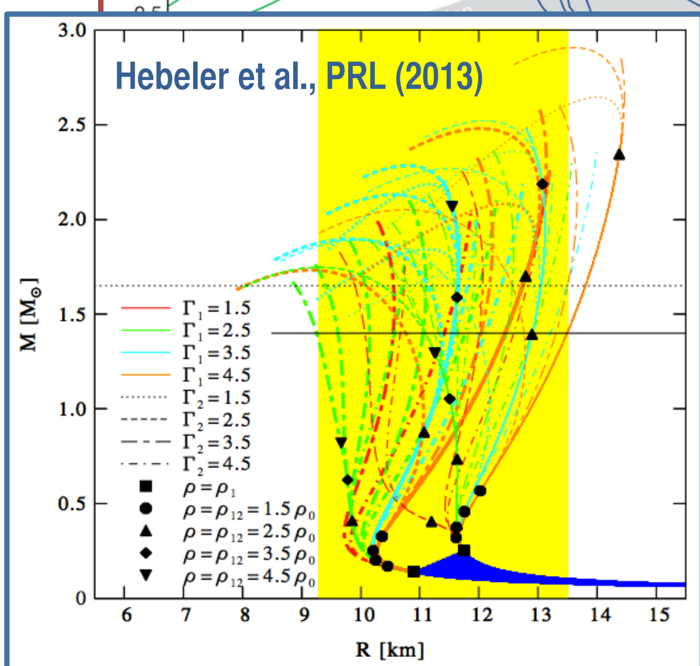

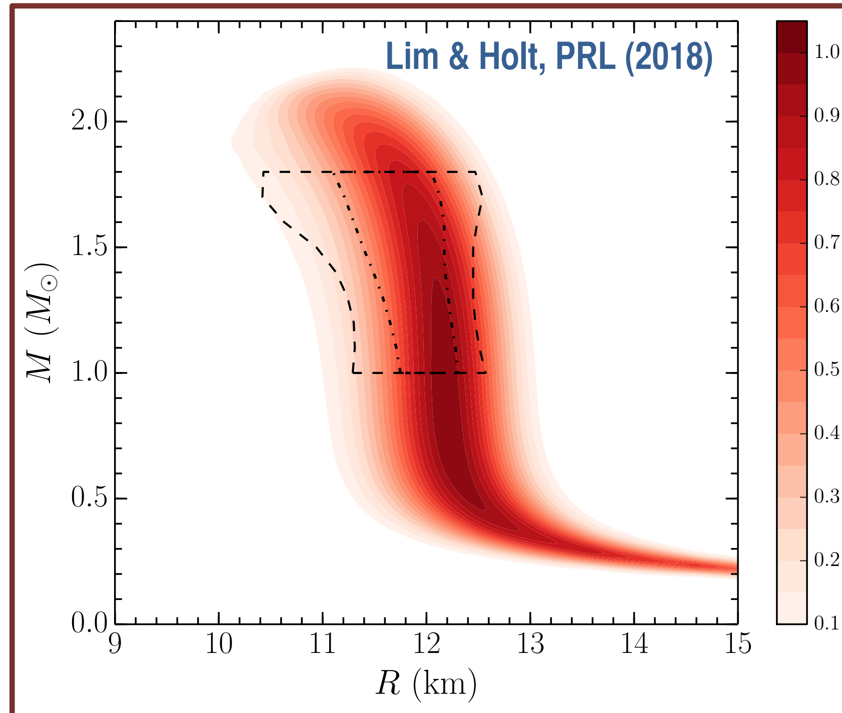

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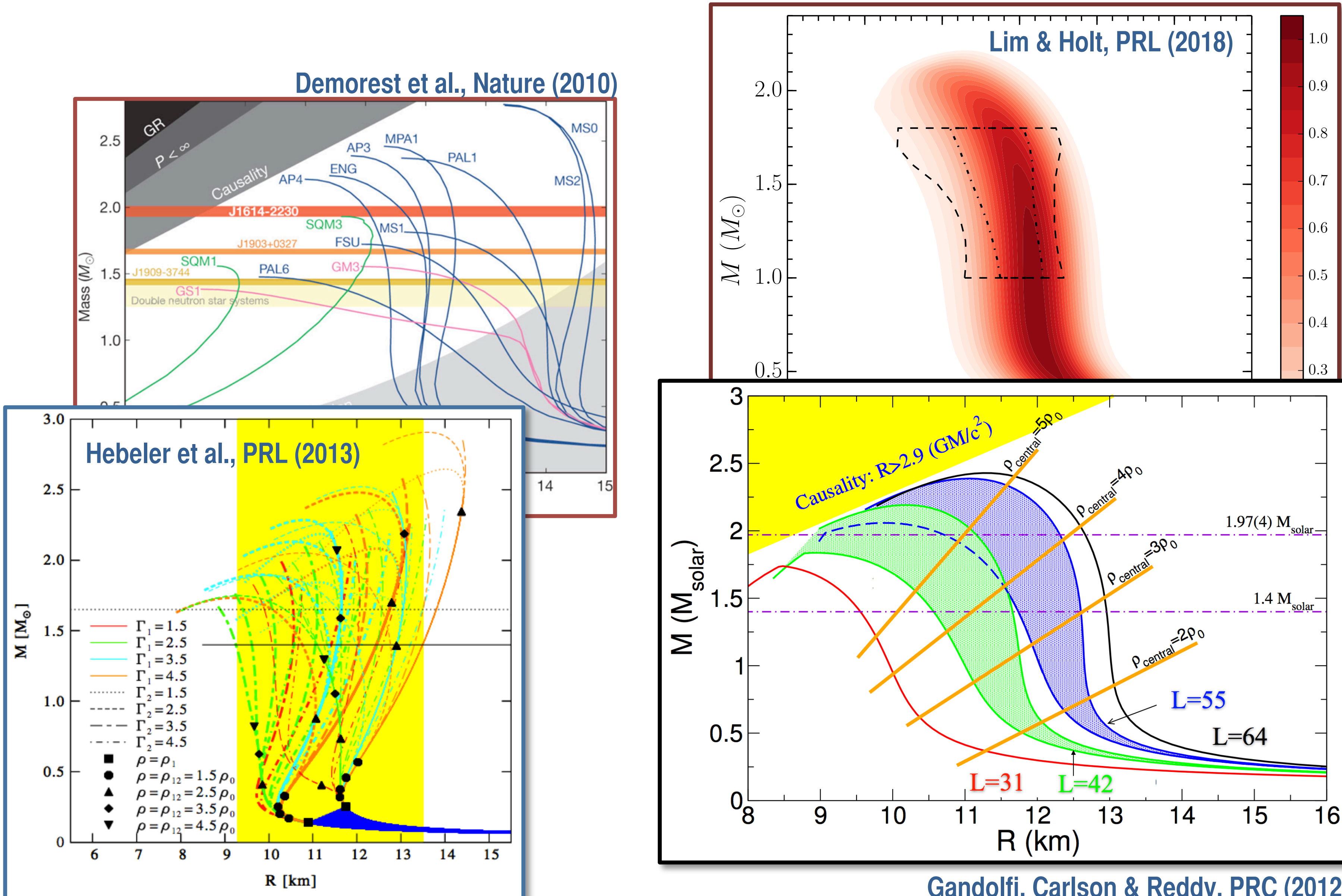

Parametrizing the zero-temperature equation of state

$$\frac{E}{A}(\rho, \delta_{np}) = A_0(\rho) + S_2(\rho)\delta_{np}^2 + \underbrace{\sum_{n=2}^{\infty}(S_{2n} + L_{2n}\ln|\delta_{np}|)\,\delta_{np}^{2n}}_{\text{small}}$$

- Traditionally expand symmetry energy about saturation density: $S_2(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{\rm sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \cdots$

- In Fermi liquid theory, symmetry energy related to 2 Landau parameters: $S_2(k_F) = \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2}\left[3f_0'(k_F) - f_1(k_F)\right]$

- The expression leads to two correlation equations **[Holt & Lim, PLB (2018)]** :

$$L = 3\rho_0 \left.\frac{dS_2}{d\rho}\right|_{\rho_0} = 3J - S_0 + \frac{\rho_0}{6}\left.\left(3k_F\frac{df_0'}{dk_F} - k_F\frac{df_1}{dk_F}\right)\right|_{k_F^0}$$

$$K_{\rm sym} = 9\rho_0^2 \left.\frac{d^2S_2}{d\rho^2}\right|_{\rho_0} = 4L - 12J + 2S_0 + \frac{\rho_0}{6}\left.\left(3k_F^2\frac{d^2f_0'}{dk_F^2} - k_F^2\frac{d^2f_1}{dk_F^2}\right)\right|_{k_F^0}$$

Parametrizing the zero-temperature equation of state

- Expand about a small reference Fermi momentum k_r:

$$S_2(k_F) = \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2}\left[3f_0'(k_F) - f_1(k_F)\right]$$

$$= \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2}\left[c_0 + c_1\beta + \frac{1}{2}c_2\beta^2\right]$$

$$\beta = \frac{k_F - k_r}{k_r}$$

- At low densities, Fermi liquid parameters should be well constrained by **chiral effective field theory**

- Logarithmic terms $\sim \log(1 + 4k_F^2/m_\pi^2)$ in the symmetry energy require $k_r > 0.9\,\mathrm{fm}^{-1}$ for the Taylor series to be convergent at saturation density

- Perturbation theory expansion breaks down below similar scale

Therefore choose $\beta_0 = \frac{k_F^0 - k_r}{k_r} \simeq 0.5$

Absorb density dependence of FLP's into correlations

- Symmetry energy slope parameter [Holt & Lim, PLB (2018)] :

$$L = 3\rho_0 \left. \frac{dS_2}{d\rho} \right|_{\rho_0} = 3J - S_0 + \frac{\rho_0}{6} \left. \left(3k_F \frac{df_0'}{dk_F} - k_F \frac{df_1}{dk_F} \right) \right|_{k_F^0}$$

$$= (3+\gamma)J - (1+\gamma)S_0 - \gamma \frac{\rho_0}{6} \left(c_0 - \eta_1 c_1 + \eta_1 c_2 \right)$$

- Likewise for the symmetry energy incompressibility [Holt & Lim, PLB (2018)] :

$$K_{\rm sym} = 9\rho_0^2 \left. \frac{d^2 S_2}{d\rho^2} \right|_{\rho_0} = 4L - 12J + 2S_0 + \frac{\rho_0}{6} \left. \left(3k_F^2 \frac{d^2 f_0'}{dk_F^2} - k_F^2 \frac{d^2 f_1}{dk_F^2} \right) \right|_{k_F^0}$$

$$= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma \frac{\rho_0}{6} \left(c_0 - \eta_2 c_1 + \eta_2 c_2 \right)$$

- Expect a_0 to be well constrained from chiral EFT

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$$\gamma = 3.7 \qquad \eta_1 = -0.08$$

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$$= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma\frac{\rho_0}{6}(c_0 - \eta_2 c_1 + \eta_2 c_2)$$

$$\eta_2 = -0.16$$

- Expect a_0 to be well constrained from chiral EFT

Absorb density dependence of FLP's into correlations

- Symmetry energy slope parameter [Holt & Lim, PLB (2018)] :

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$$= (3+\gamma)J - (1+\gamma)S_0 - \gamma\frac{\rho_0}{6}(c_0 - \eta_1 c_1 + \eta_1 c_2)$$

Universal slope parameter

Model-dependent scale shift

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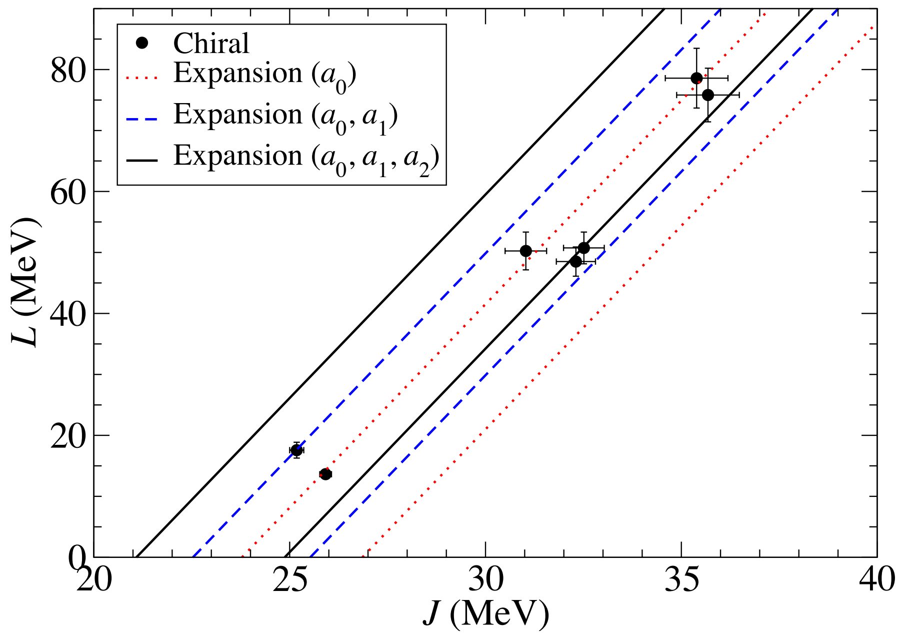
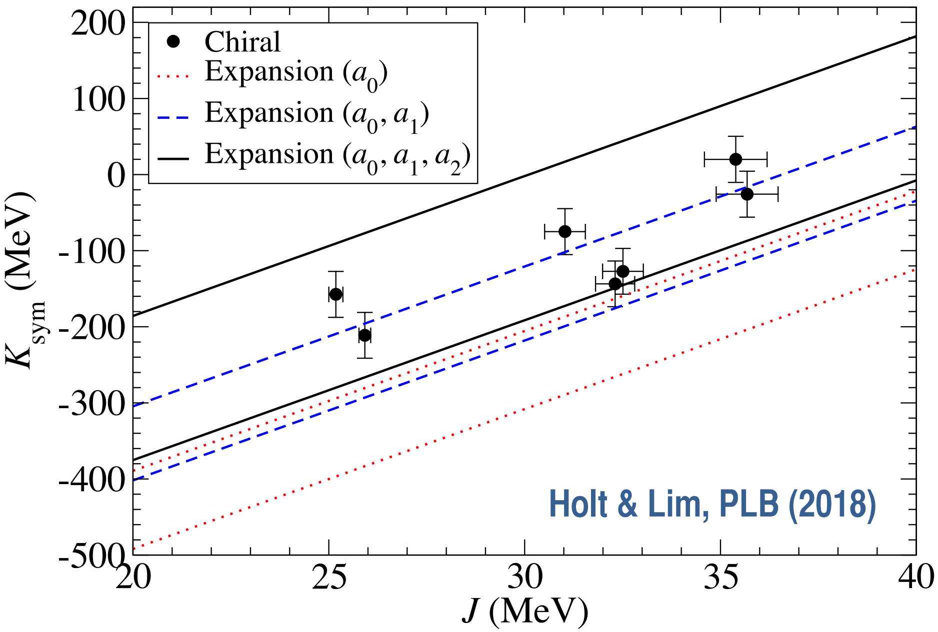
Universal slope parameter

Model-dependent scale shift

- Expect a_0 to be well constrained from chiral EFT

Comparison to chiral EFT results





- NLO, N2LO, and N3LO potentials (plus N2LO three-body force)
- Predicted correlation slopes agree well with explicit chiral EFT results
- Better theory constraints on low-density Fermi liquid parameters may reduce correlation uncertainties

Application: Infer properties of the nuclear equation of state from neutron star observations… a Bayesian approach

- Construct a model with parameters $\vec{a}$
- Bayes' Theorem:

$$P(\vec{a}|data) \sim \underline{P(data|\vec{a})}\,\underline{P(\vec{a})}$$

"Posterior"

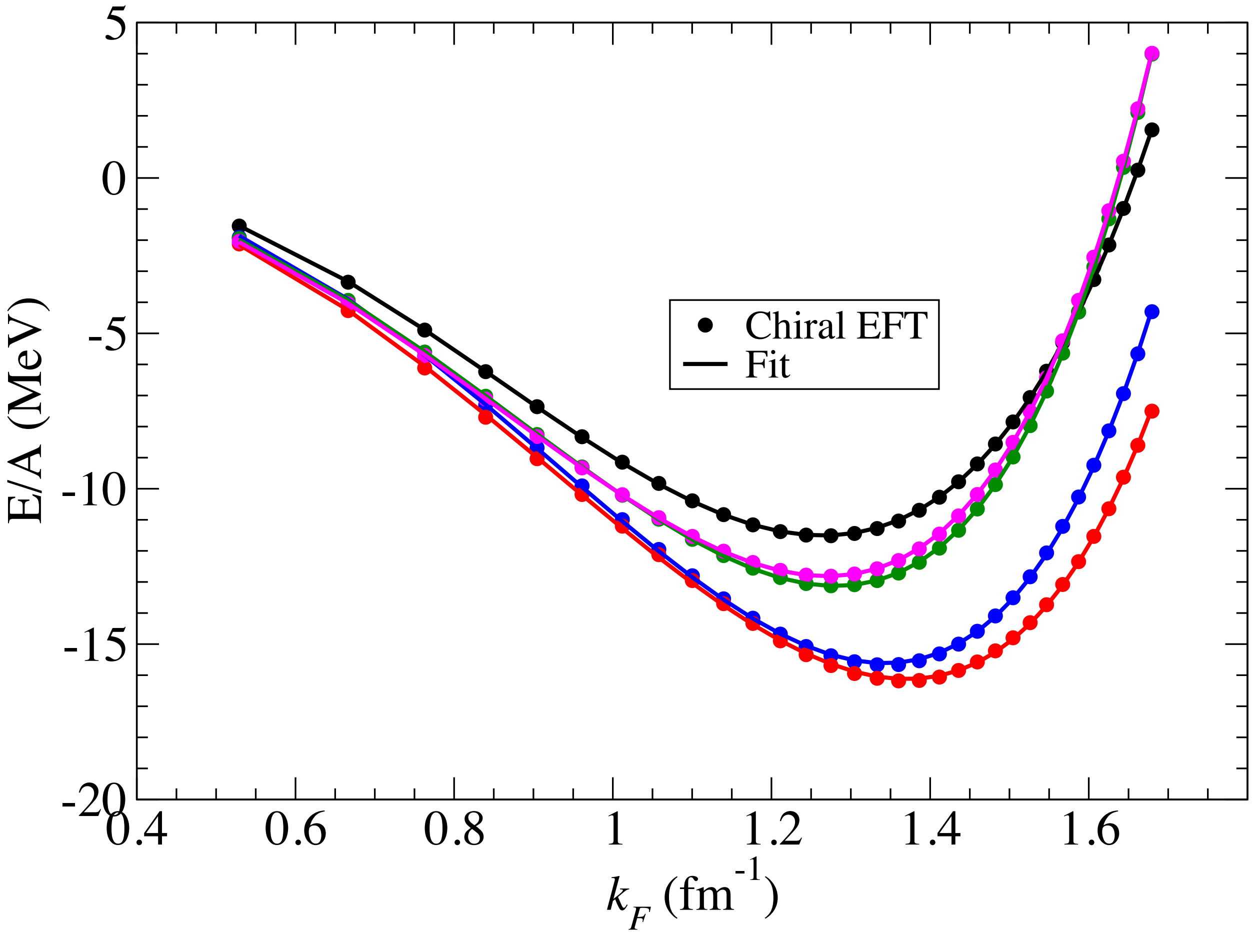
Likelihood of data given a probability distribution for $\vec{a}$

Beliefs about parameters $\vec{a}$ before measurements ("Prior")

- Strategy:
  - Find useful parametrizations for the equation of state
  - Obtain priors from chiral EFT predictions
  - Use laboratory measurements of finite nuclei to obtain likelihood functions and posteriors

Prior distributions from chiral EFT

$$\frac{E}{A}(\rho,\delta=0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2}\left(a_0 + a_1\,\beta + \frac{1}{2}a_2\,\beta^2 + \frac{1}{6}a_3\,\beta^3\right)$$



$$a_0 = -3.41 \pm 0.20\,\mathrm{fm}^2$$

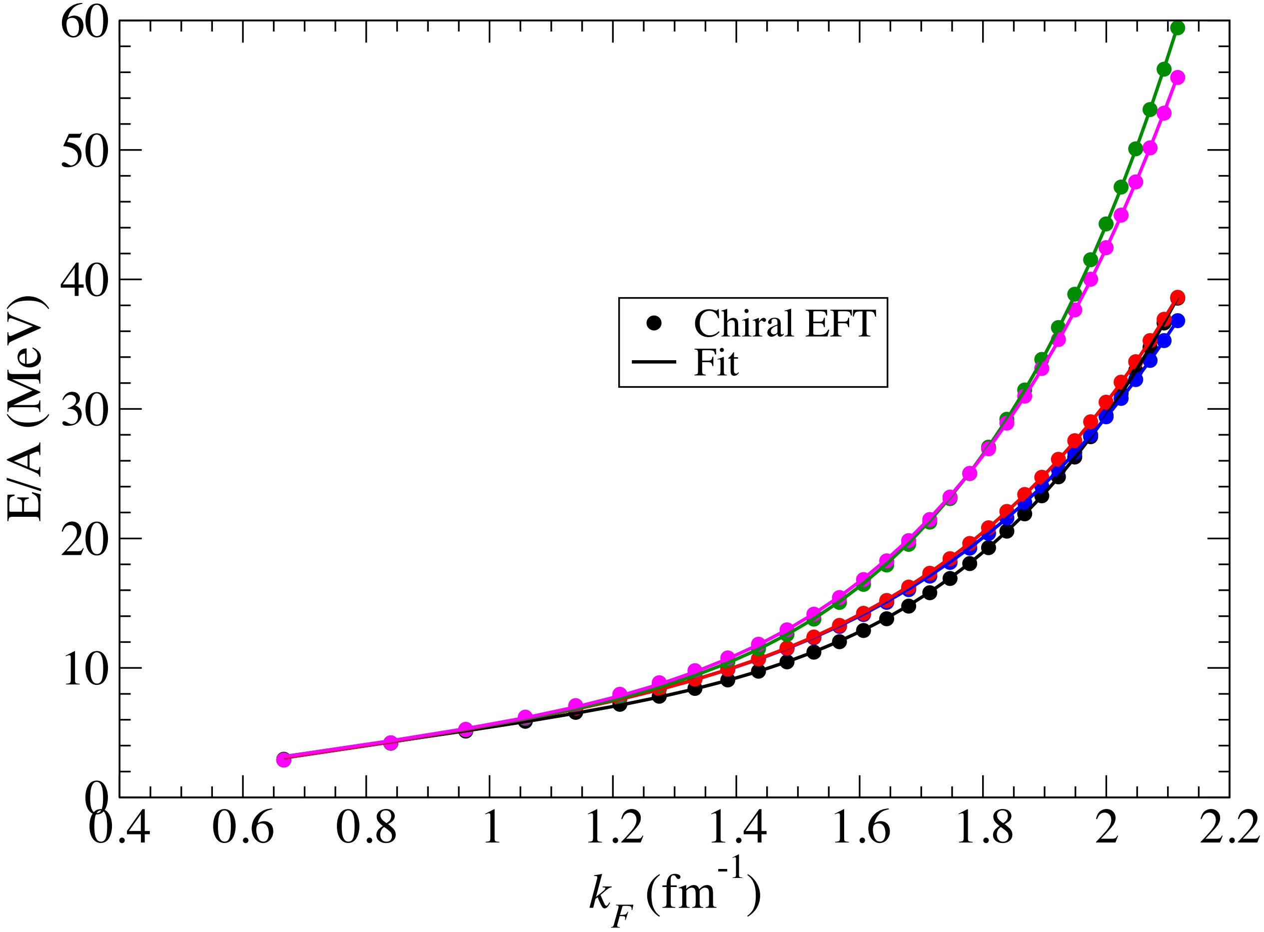
$$a_1 = 6.44 \pm 0.25\,\mathrm{fm}^2$$

$$a_2 = -1.02 \pm 0.96\,\mathrm{fm}^2$$

$$a_3 = 21.92 \pm 8.98\,\mathrm{fm}^2$$

Prior distributions from chiral EFT

$$\frac{E}{A}(\rho, \delta = 1) = 2^{2/3}\frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2}\left(b_0 + b_1\,\beta + \frac{1}{2}b_2\,\beta^2 + \frac{1}{6}b_3\,\beta^3\right)$$



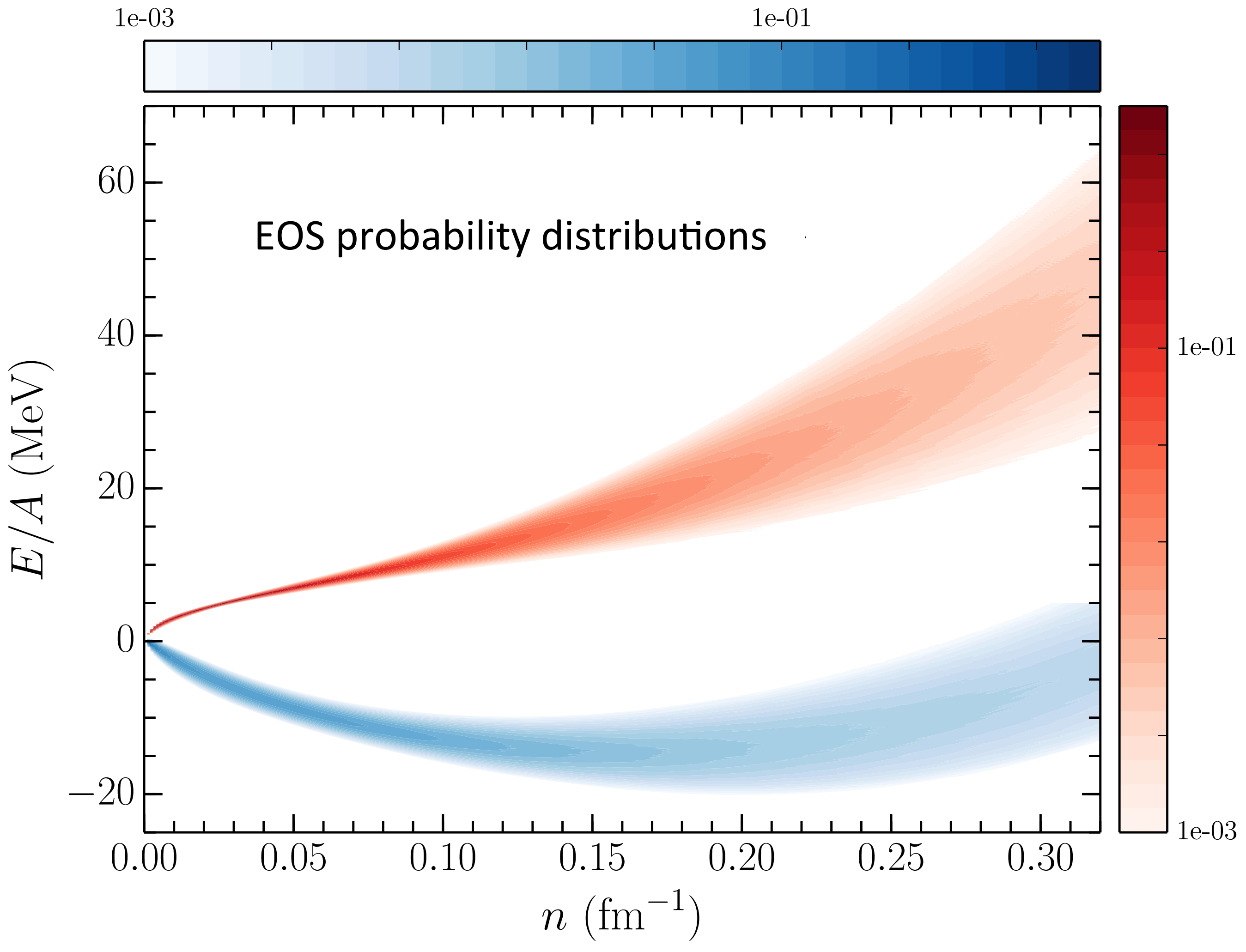
$b_0 = -1.68 \pm 0.22\,\mathrm{fm}^2$

$b_1 = 4.14 \pm 0.90\,\mathrm{fm}^2$

$b_2 = 3.81 \pm 2.56\,\mathrm{fm}^2$

$b_3 = 5.11 \pm 2.84\,\mathrm{fm}^2$

Equations of state from chiral EFT priors

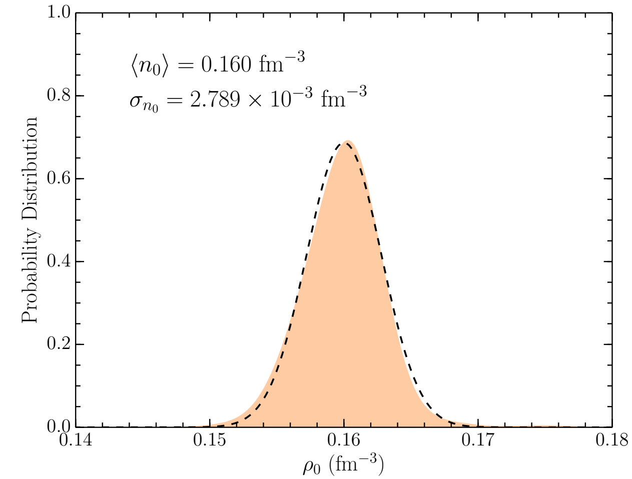


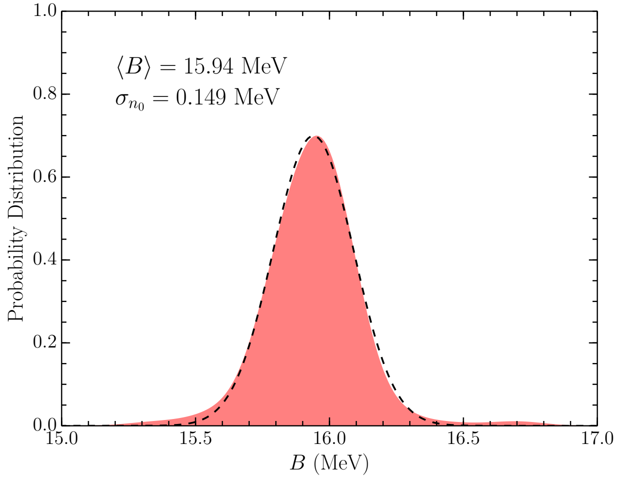
Likelihood functions for symmetric nuclear matter

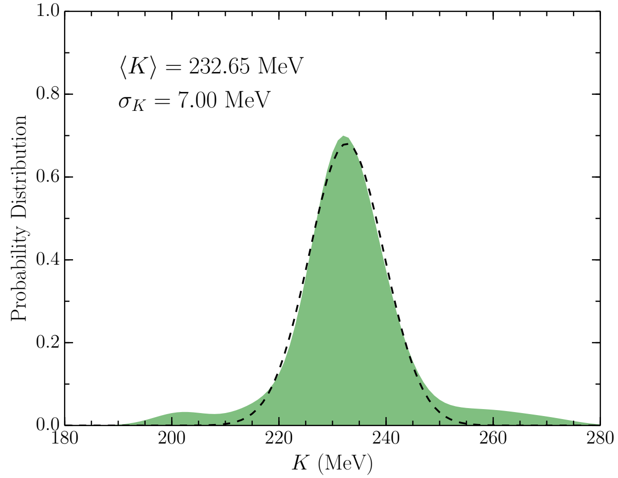
- Parametrization: $$\frac{E}{A}(\rho, \delta = 0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2}\left(a_0 + a_1\,\beta + \frac{1}{2}a_2\,\beta^2 + \frac{1}{6}a_3\,\beta^3\right)$$

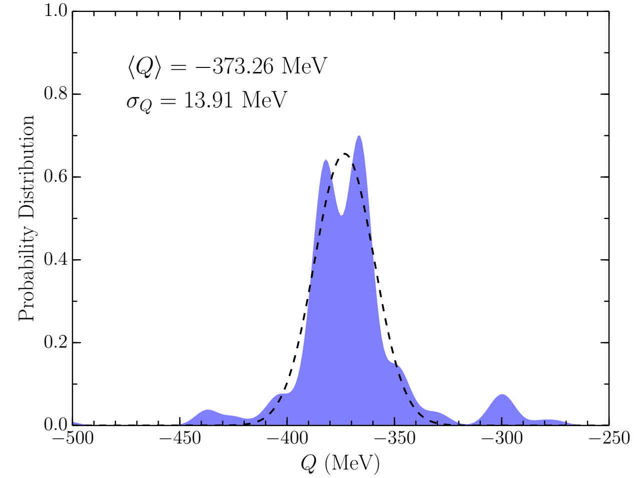
- Average values of $\vec{a}$ and full covariance matrix from analysis of 200 Skyrme mean field models fitted to nuclear properties

[M. Dutra et al., PRC (2012)]









Likelihood functions for pure neutron matter

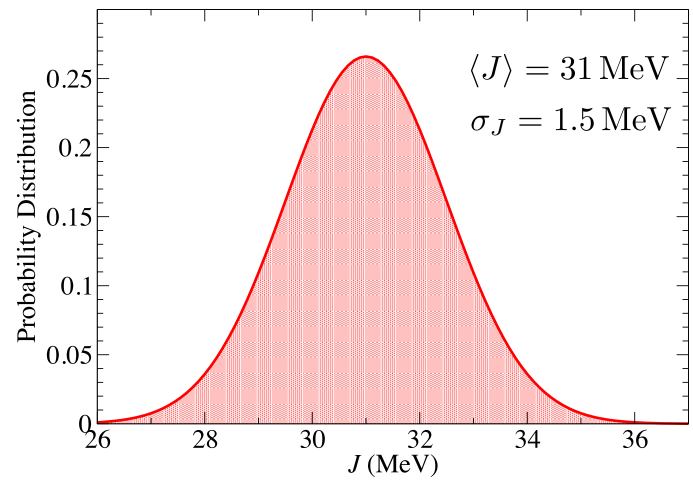
- Parametrization: $\frac{E}{A}(\rho, \delta = 1) = 2^{2/3}\frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2}\left(b_0 + b_1\,\beta + \frac{1}{2}b_2\,\beta^2 + \frac{1}{6}b_3\,\beta^3\right)$

$$S_2(\rho) = \frac{k_F^2}{6m} + \underbrace{\frac{k_F^3}{9\pi^2}\left(c_0 + c_1\,\beta + \frac{1}{2}c_2\,\beta^2 + \frac{1}{6}c_3\,\beta^3\right)}$$

Correlations among $J, L, K_{\rm sym}$

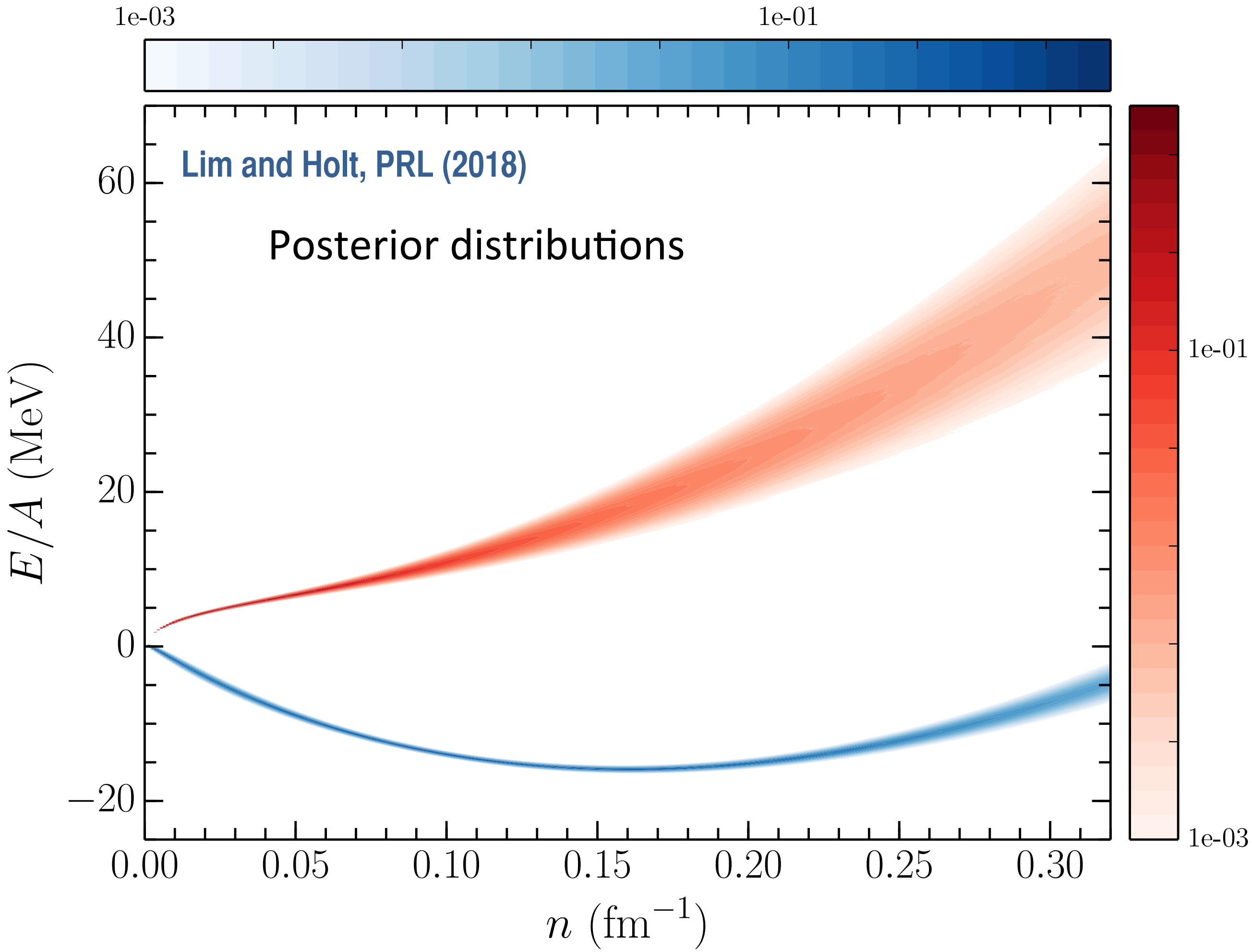
$$L = (3+\gamma)J - (1+\gamma)S_0 - \gamma\frac{\rho_0}{6}\left(c_0 - \eta_1 c_1 + \eta_1 c_2\right)$$

$$K_{\rm sym} = 5\gamma J - (5\gamma+2)S_0 - 5\gamma\frac{\rho_0}{6}\left(c_0 - \eta_2 c_1 + \eta_2 c_2\right)$$



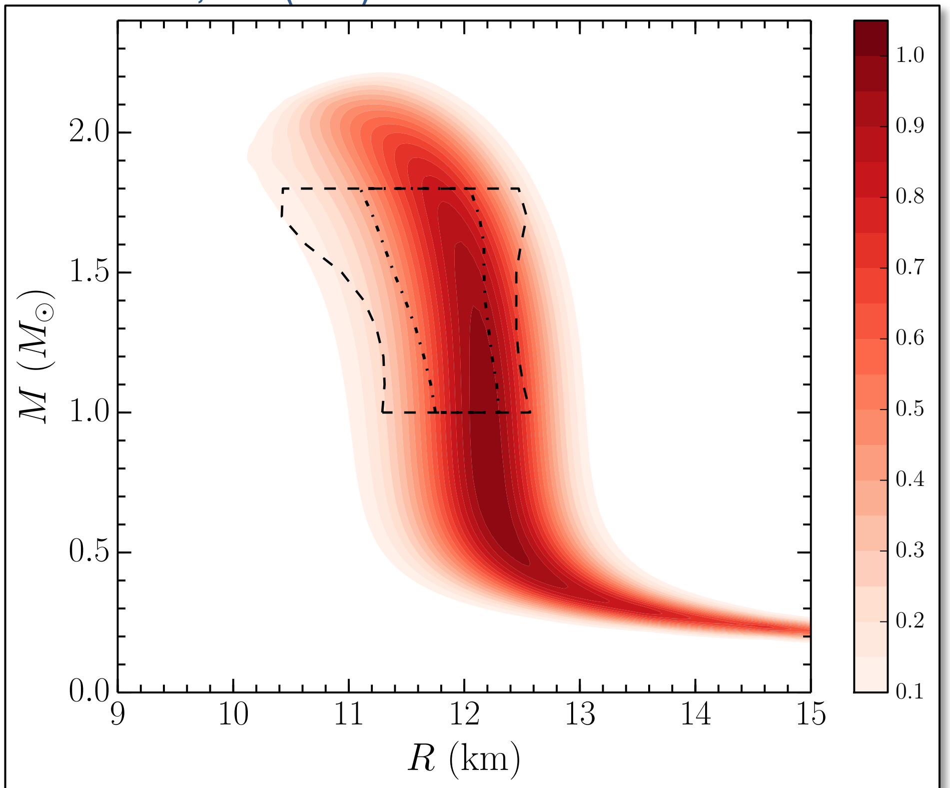
- Derive likelihood functions involving $\{b_0, b_1, b_2, b_3\}$ for subsequent Bayesian posterior probability distribution

Equations of state from posterior probability distributions



Derived probability distributions

Lim and Holt, PRL (2018)



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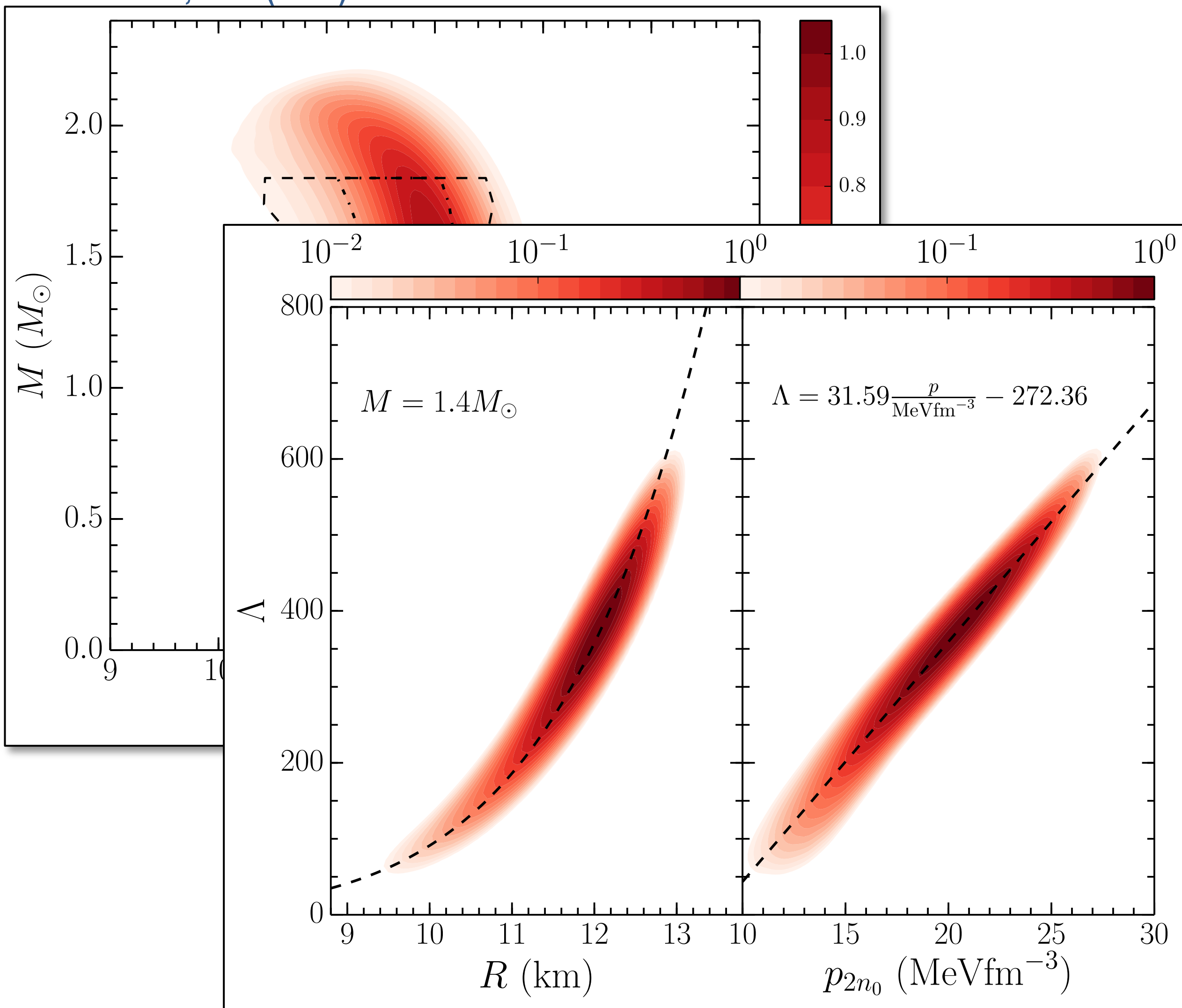

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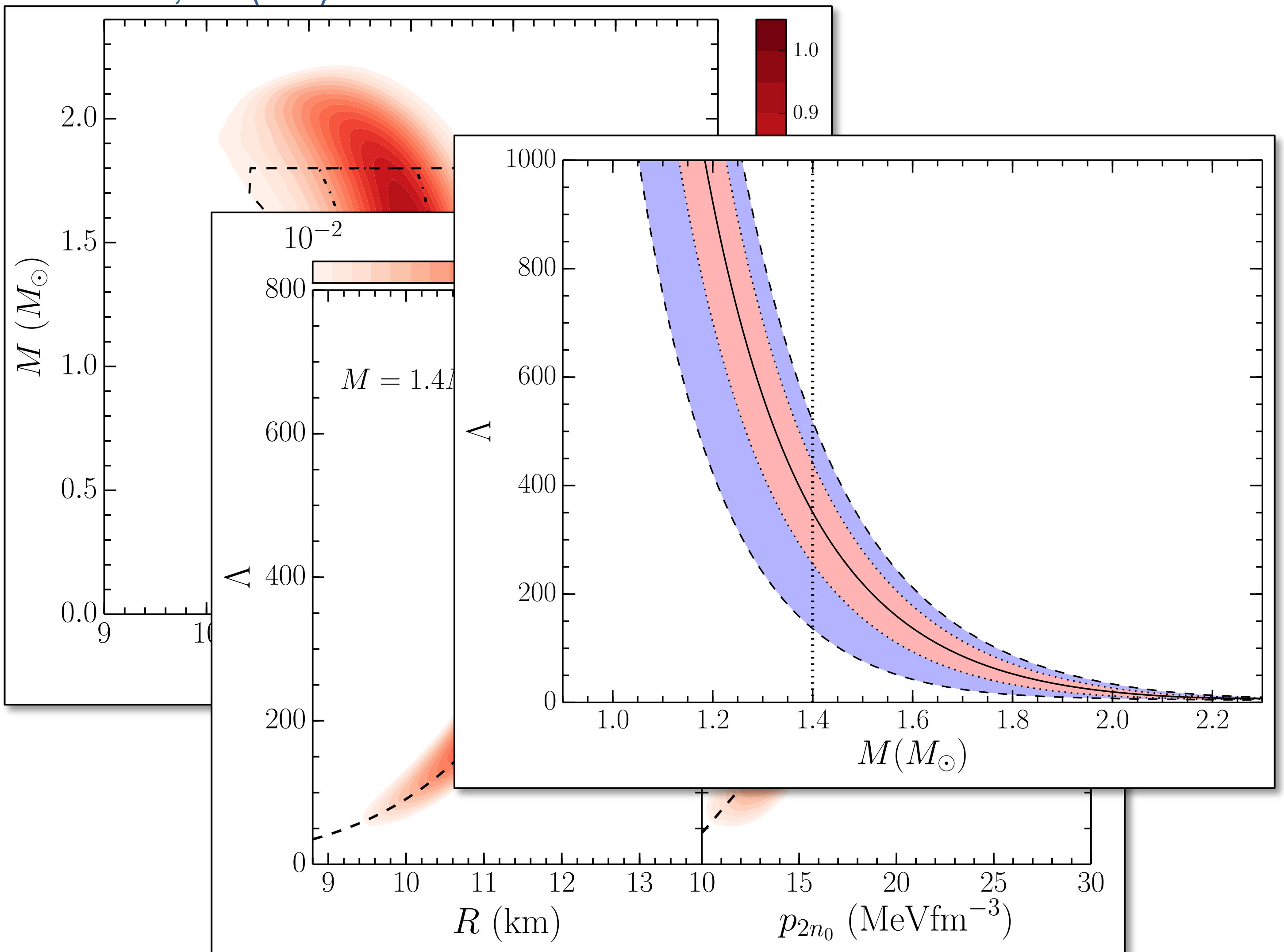

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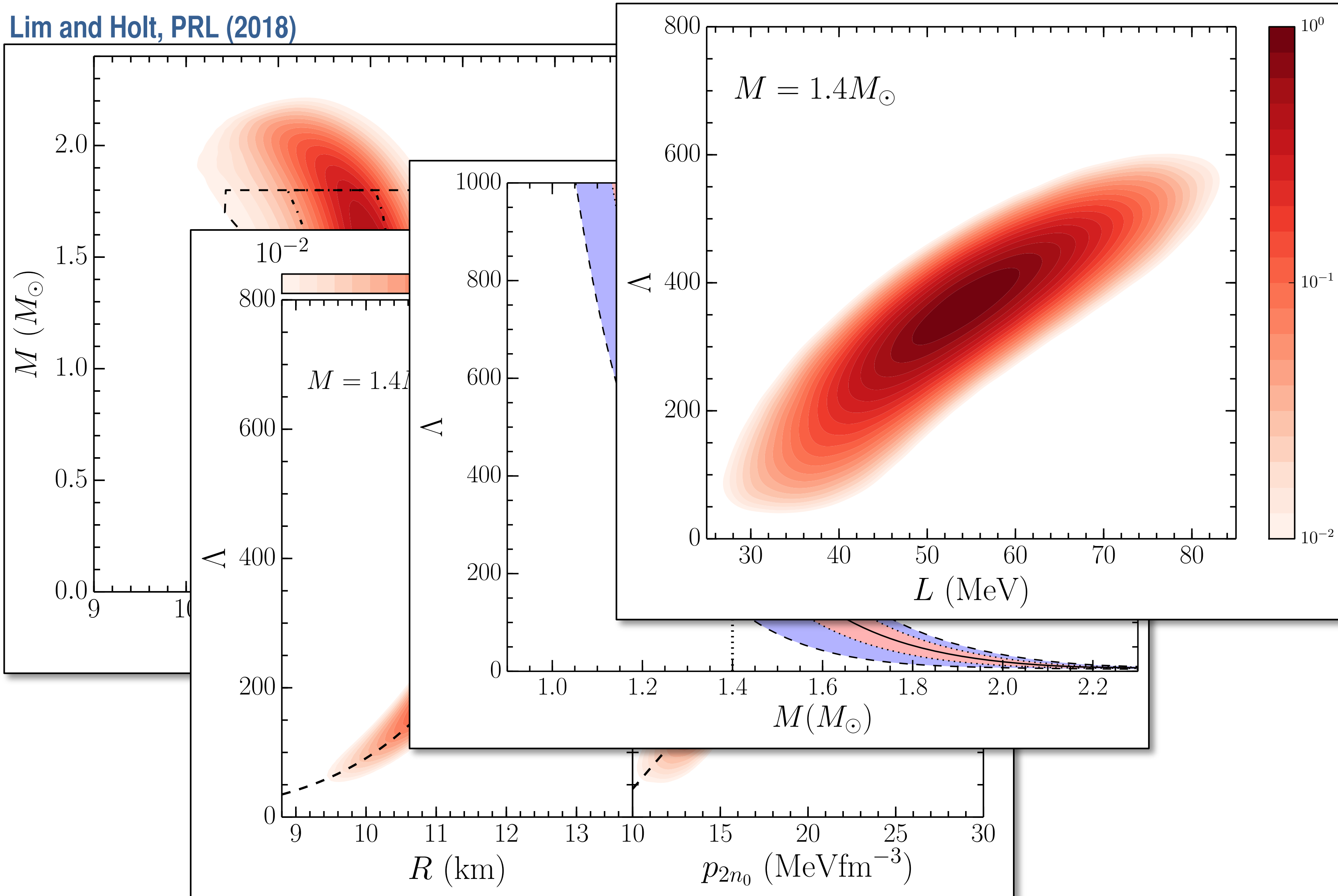


Summary and outlook

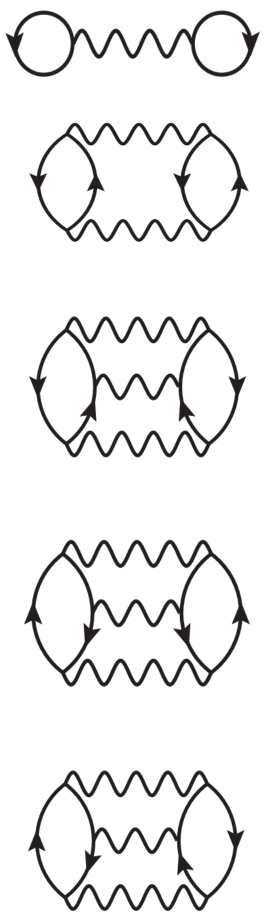
- New era of major observational campaigns to study the properties of neutron stars

- Complementary theoretical models with accurate nuclear physics inputs needed to guide and interpret observations

- Combine properties of finite nuclei with “model independent” predictions from chiral EFT to obtain posterior distribution function for model parameters

- Ultra high-density matter a challenging frontier for ***any*** theoretical, experimental, or observation investigation

Priors from chiral EFT EOS calculations



$$\rho E^{(1)} = \frac{1}{2} \sum_{12} n_1 n_2 \langle 12 | \left(\overline{V}_{NN} + \overline{V}_{NN}^{\text{med}}/3 \right) | 12 \rangle,$$

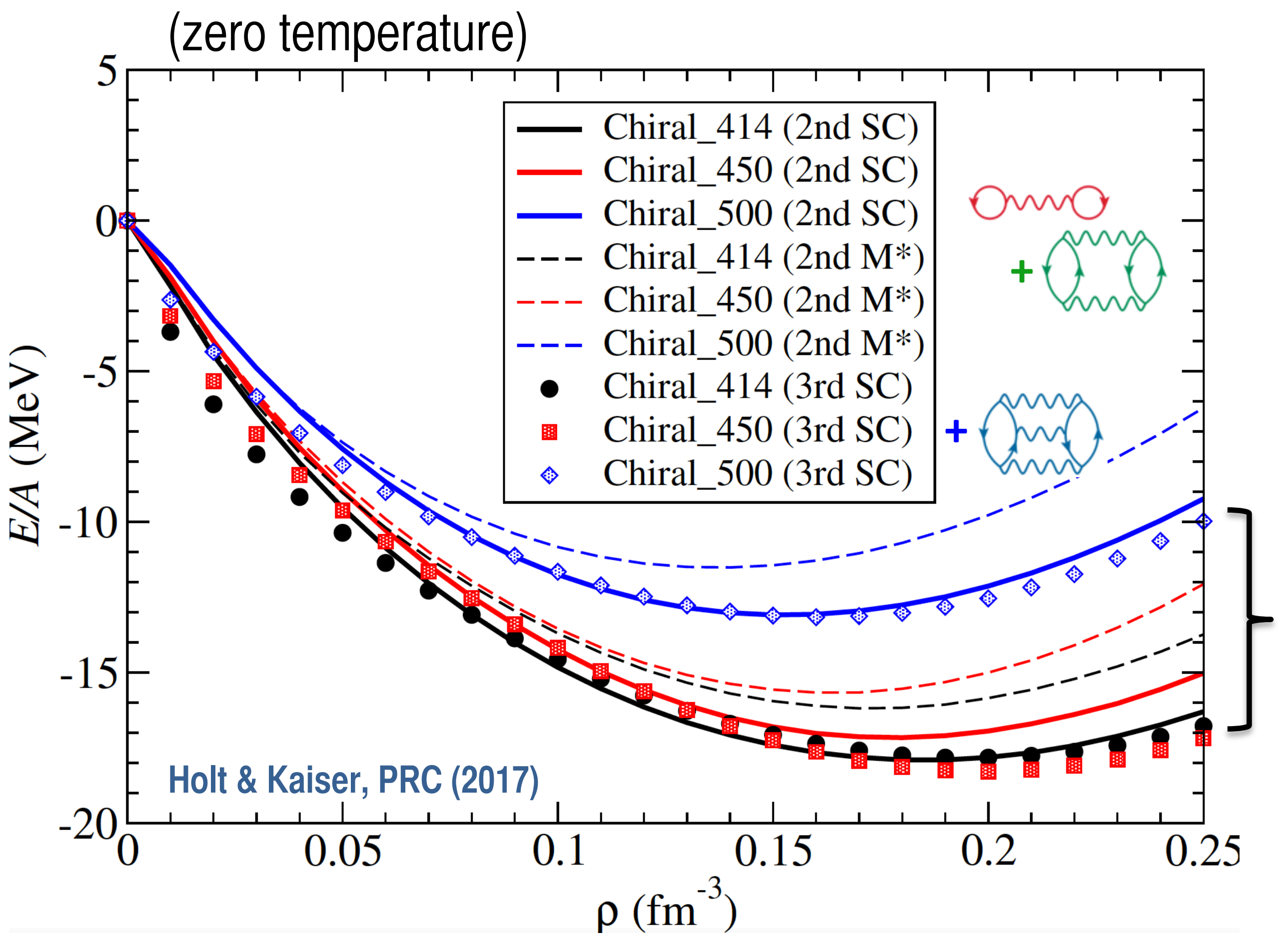
$$\rho E^{(2)} = -\frac{1}{4} \sum_{1234} |\langle 12 | \overline{V}_{\text{eff}} | 34 \rangle|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2},$$

$$\rho E^{(3)}_{\text{pp}} = \frac{1}{8} \sum_{123\,456} \langle 12 | \overline{V}_{\text{eff}} | 34 \rangle \langle 34 | \overline{V}_{\text{eff}} | 56 \rangle \langle 56 | \overline{V}_{\text{eff}} | 12 \rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 \bar{n}_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)},$$

$$\rho E^{(3)}_{\text{hh}} = \frac{1}{8} \sum_{123\,456} \langle 12 | \overline{V}_{\text{eff}} | 34 \rangle \langle 34 | \overline{V}_{\text{eff}} | 56 \rangle \langle 56 | \overline{V}_{\text{eff}} | 12 \rangle \times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)},$$

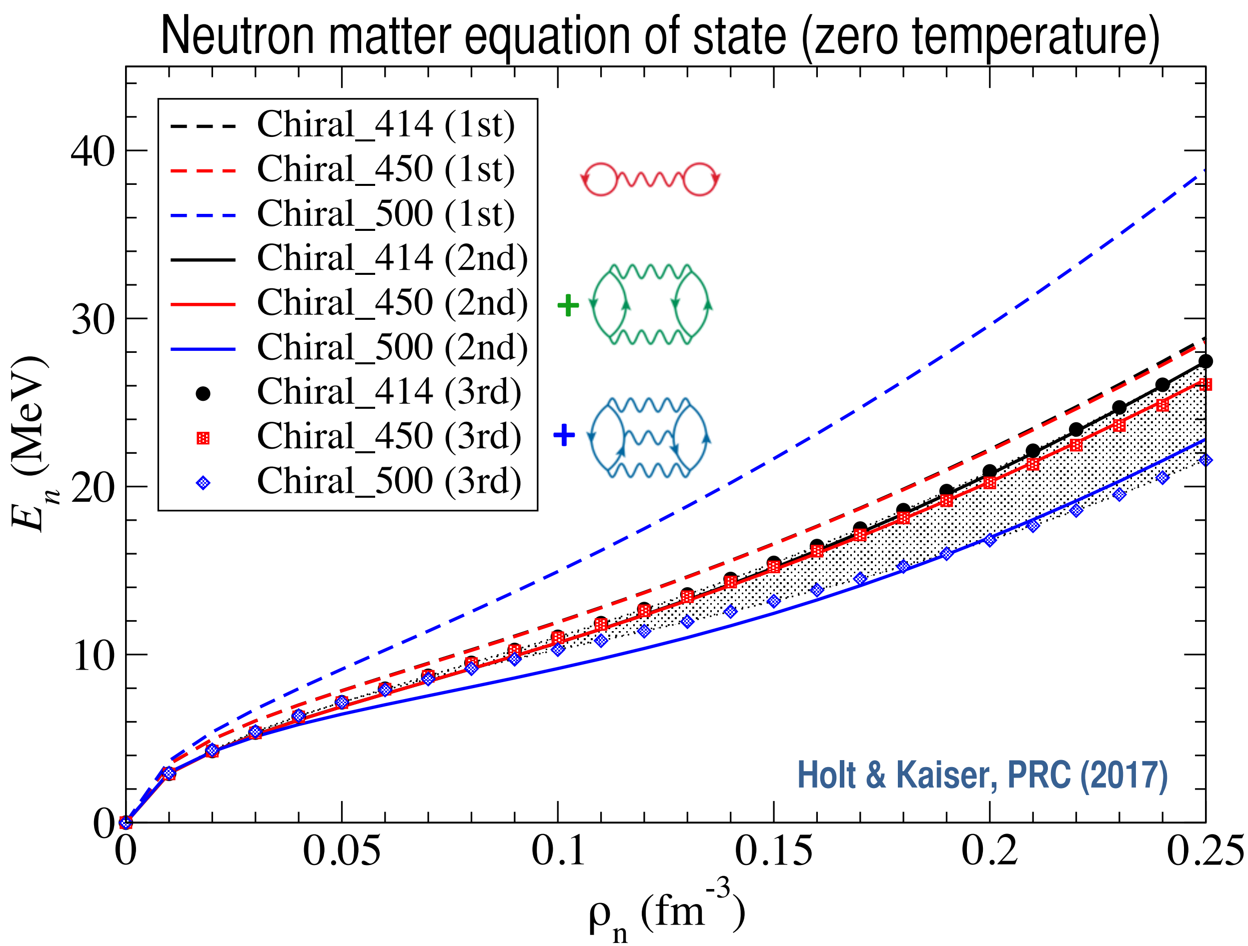
$$\rho E^{(3)}_{\text{ph}} = - \sum_{123\,456} \langle 12 | \overline{V}_{\text{eff}} | 34 \rangle \langle 54 | \overline{V}_{\text{eff}} | 16 \rangle \langle 36 | \overline{V}_{\text{eff}} | 52 \rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 n_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)},$$

Symmetric nuclear matter equation of state



Reduced uncertainty but grows with density

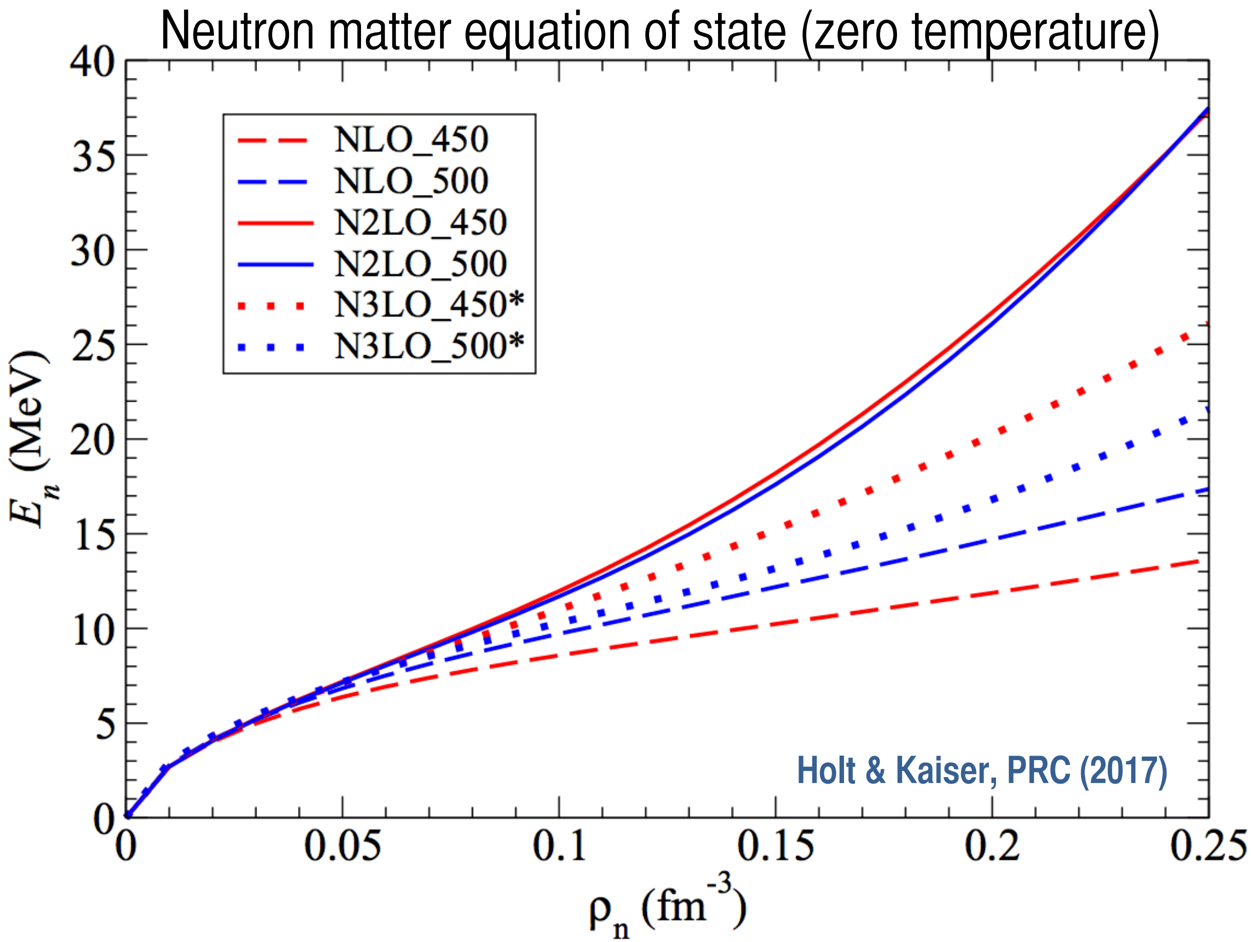
Pure neutron matter uncertainty estimates



Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory

Pure neutron matter convergence in the chiral expansion



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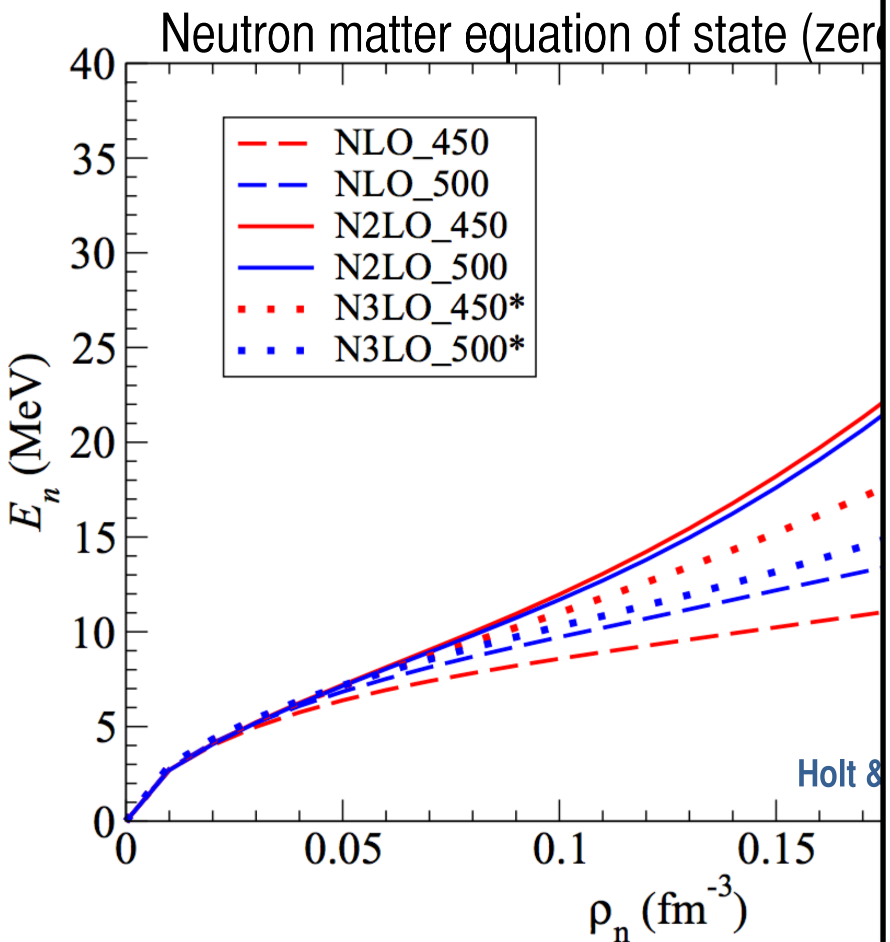
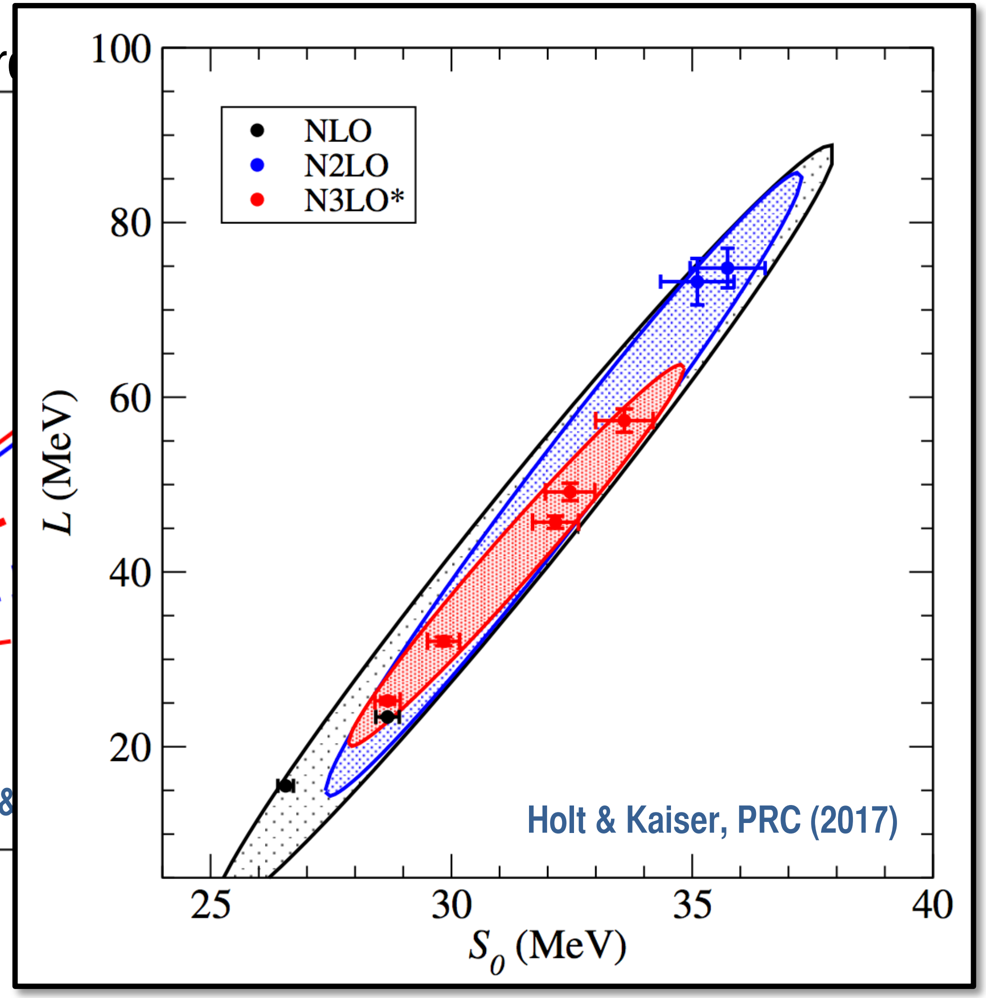
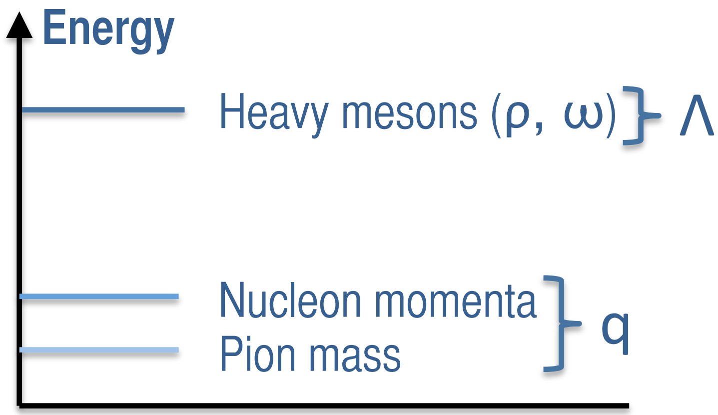




Modern theory of nuclear forces

NATURAL SEPARATION OF SCALES



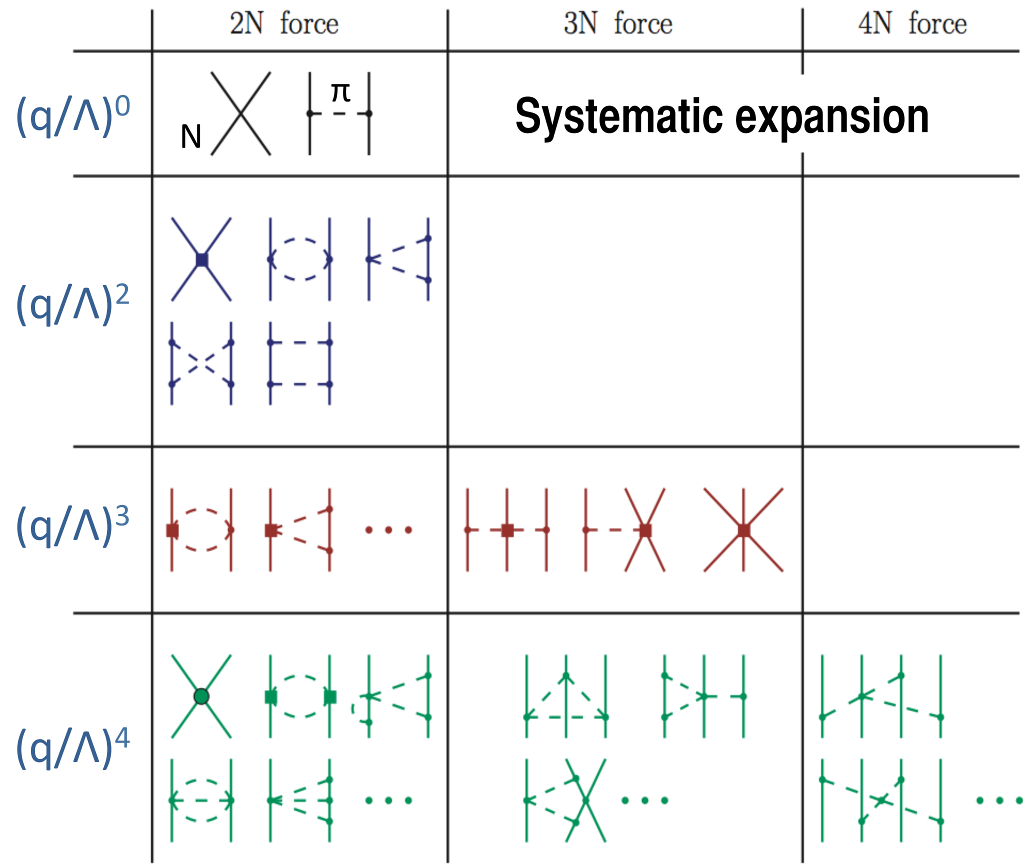
Pions weakly-coupled at low momenta

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2f_\pi^2}(\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi}\gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi}\right) N$$

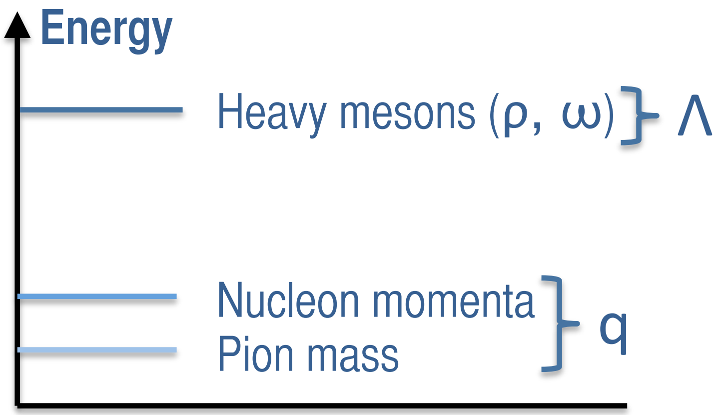
CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



Modern theory of nuclear forces

NATURAL SEPARATION OF SCALES



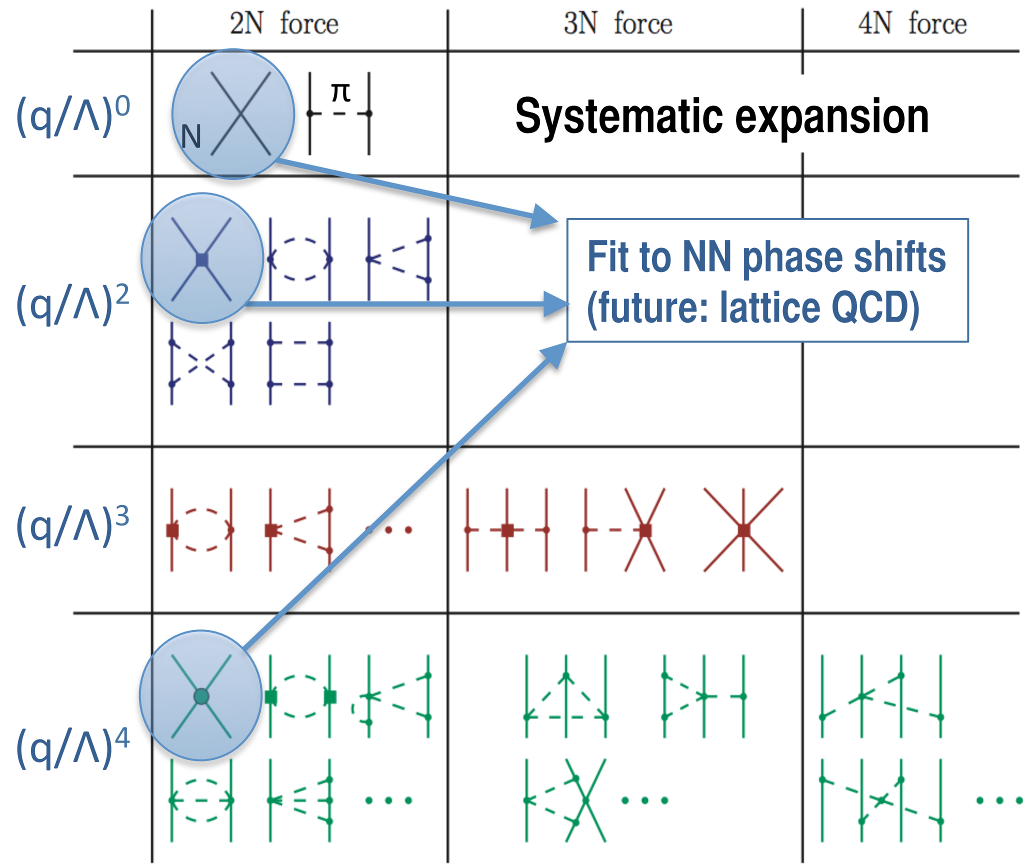
Pions weakly-coupled at low momenta

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{1}{2f_\pi^2}(\partial_\mu\vec{\pi}\cdot\vec{\pi})^2$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\right)N$$

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



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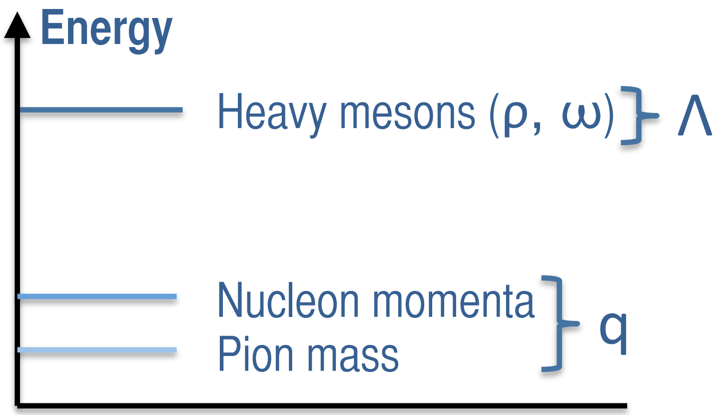

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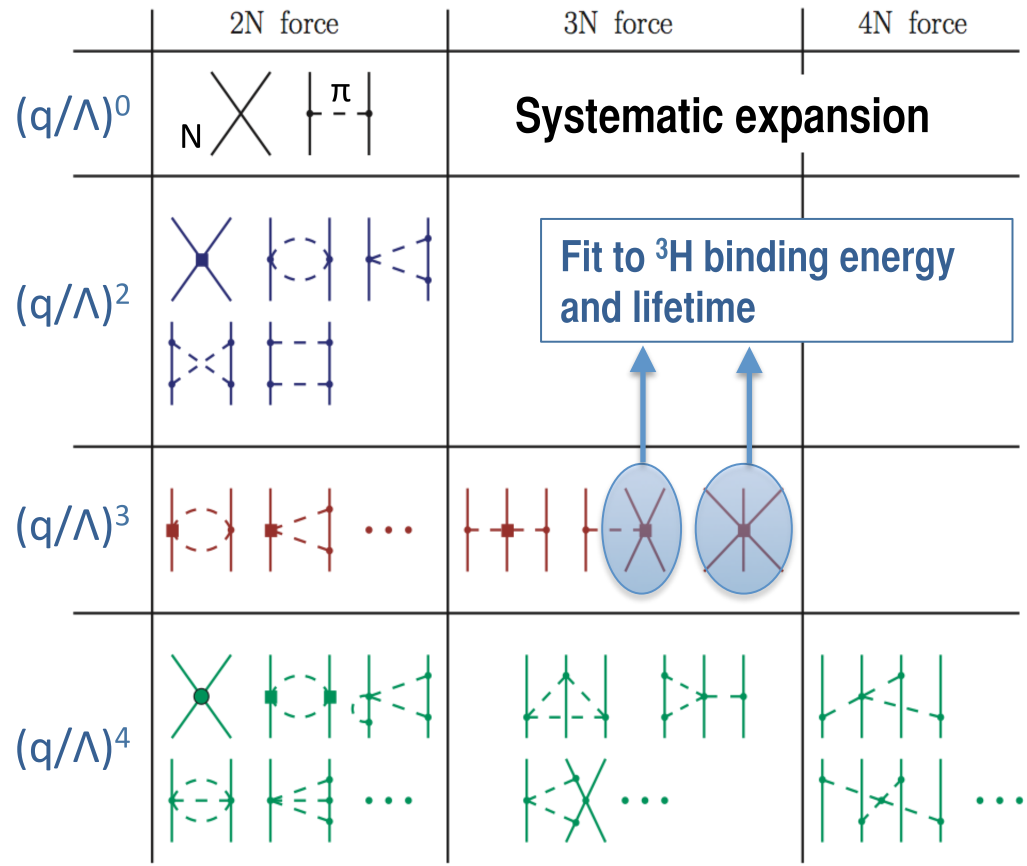

Symmetric nuclear matter at Hartree-Fock level