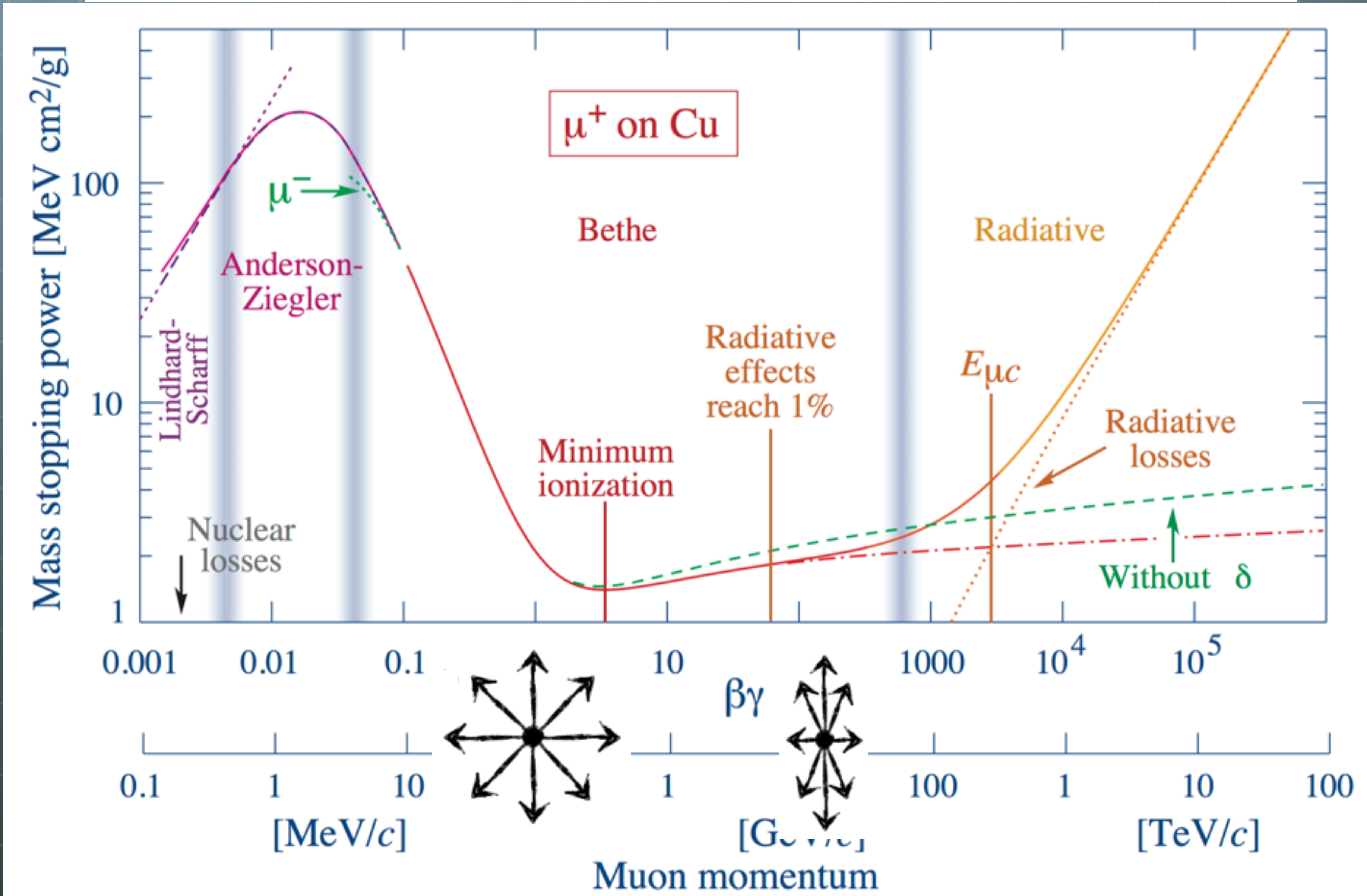
$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$



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The Ionization Loss of Energy in Gases and in Condensed Materials*

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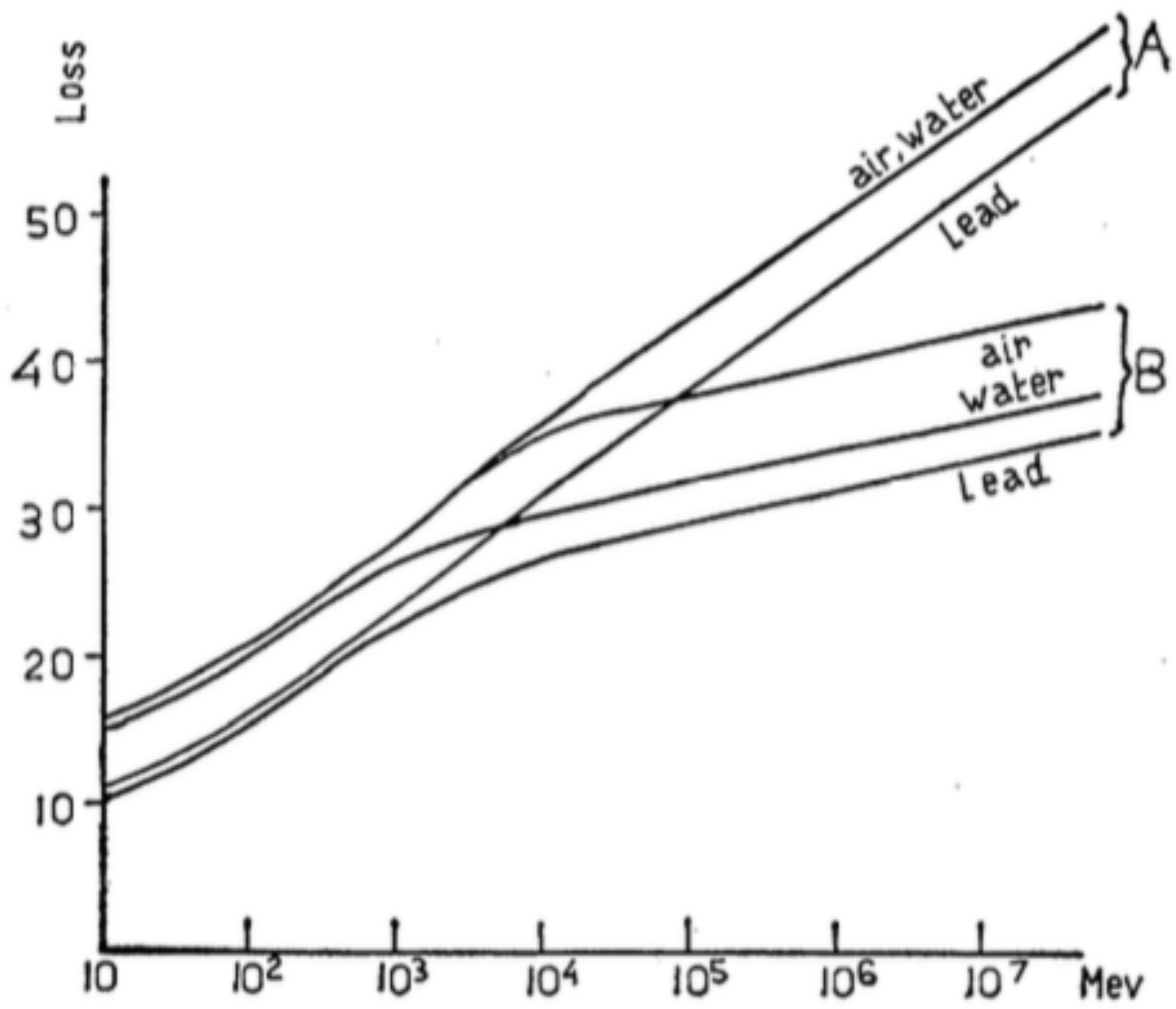


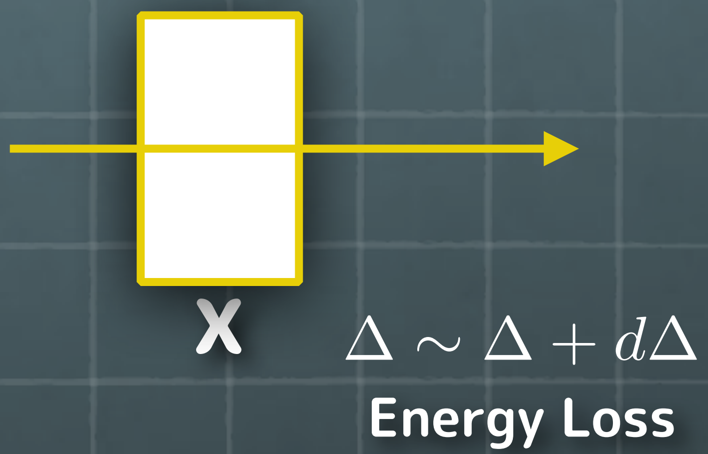
FIG. 2. Energy loss in units of $2\pi ne^4/mv^2$ for mesotrons of various energies in air, water, and lead. Curves *A* are calculated with Bloch's formula and curves *B* are corrected for the polarization effects according to the present theory.

$$\frac{2\pi ne^4}{mc^2}\log\frac{mc^2W}{(1-v^2/c^2)\hbar^2\nu_0^2}.$$

$$\frac{2\pi ne^4}{mc^2}\left[\log\frac{\pi m^2c^2W}{ne^2h^2}-1\right].$$

Straggling
-Landau Distribution-

- For a small energy loss, prob. of fluctuations
- Let unknown function f(x,Δ)



$$\frac{\partial f}{\partial x} = \int_0^\infty \omega(\varepsilon)[f(x,\Delta-\varepsilon) - f(x,\Delta)]d\varepsilon$$

$$f(x,\Delta) = \frac{1}{2\pi i}\int_{i\infty+\sigma}^{+i\infty+\sigma} e^{p\Delta}\phi(p,x)dp$$

$$f(x,\Delta) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{p\Delta - x\int_0^\infty \omega(\varepsilon)(1-e^{-p\varepsilon})d\varepsilon} dp$$

$$\omega(\varepsilon) = \frac{2\pi N e^2 \rho \sum Z}{m v^2 \sum A}\frac{1}{\varepsilon^2}$$ **for** $\varepsilon_0 \ll \varepsilon \ll \varepsilon_{max}$

Straggling
-Landau Distribution-

With a variable $\xi = x\frac{2\pi N e^2 \rho \sum Z}{m v^2 \sum A}$

$$f(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

$$\phi(\lambda) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{u\ln u+\lambda u} du$$

Maximum at λ=-0.05

$$\lambda = \frac{\Delta - \xi(\ln\frac{\xi}{\varepsilon'} + 1 - C)}{\xi}$$

Most probable value of energy loss : $\Delta_0 = \xi(\ln\frac{\xi}{\varepsilon'} + 0.37)$

$$f(x,\Delta)d\Delta = \phi(\frac{\Delta - \Delta_0}{\xi})d(\frac{\Delta - \Delta_0}{\xi})$$

Straggling
-Landau Distribution-

With a variable $\xi = x\frac{2\pi N e^2 \rho \sum Z}{m v^2 \sum A}$

$$f(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

$$\phi(\lambda) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{u \ln u + \lambda u} du$$

$$\lambda = \frac{\Delta - \xi(\ln\frac{\xi}{\varepsilon'} + 1 - C)}{\xi}$$

Most probable value of energy loss :

$$f(x,\Delta)d\Delta = \phi(\frac{\Delta - \Delta_0}{\xi})d(\frac{\Delta$$

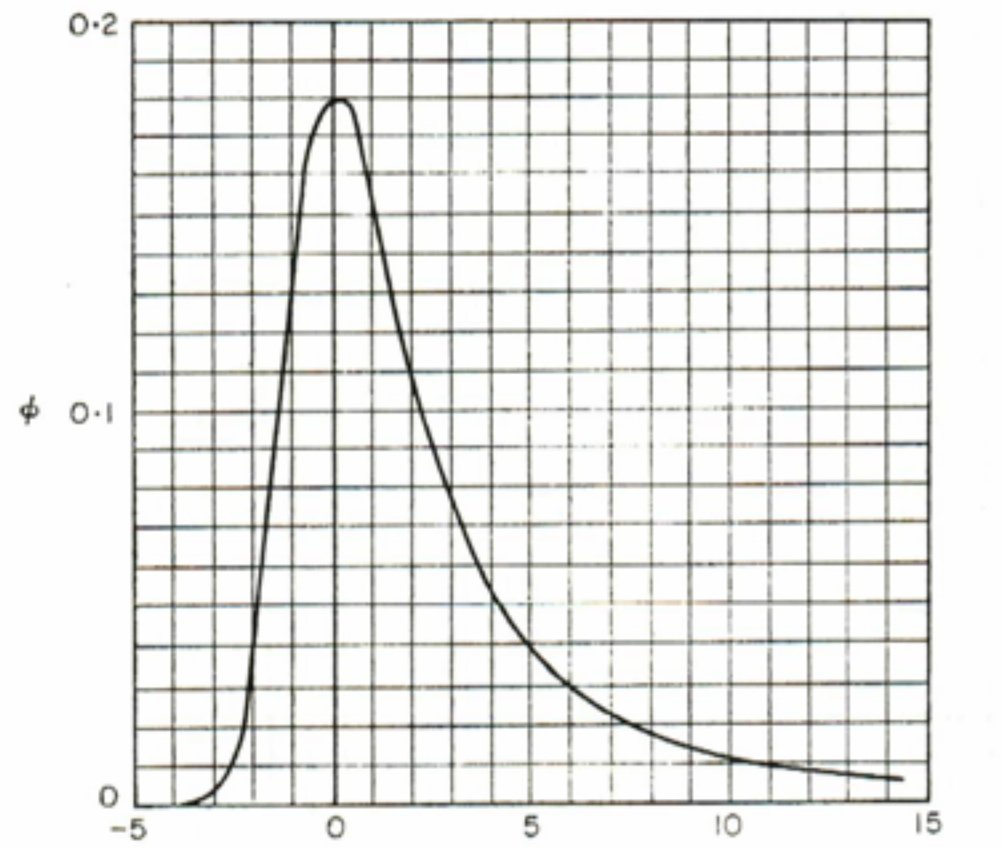
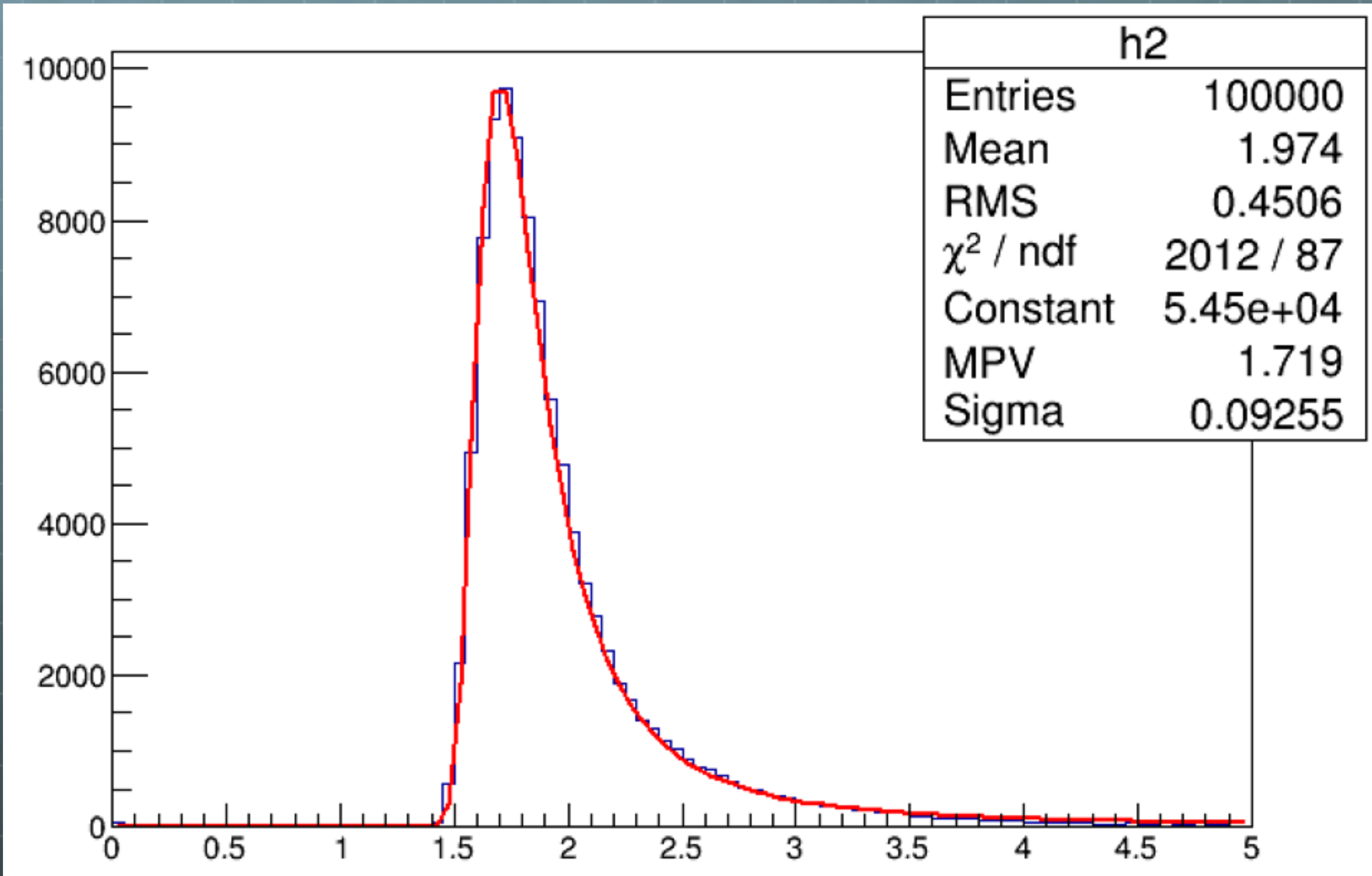


FIG. 1.

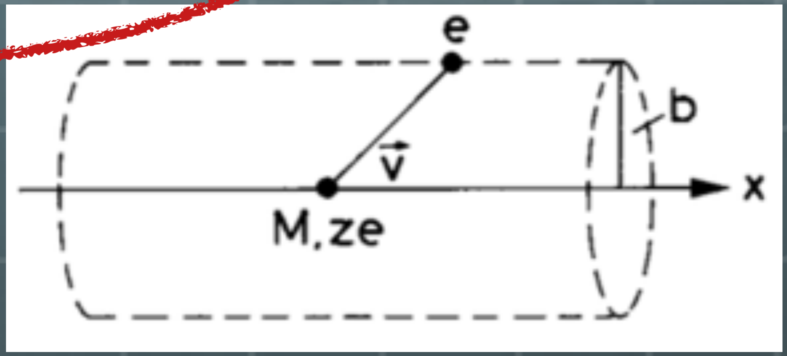

h2
Entries 100000
Mean 1.974
RMS 0.4506
χ² / ndf 2012 / 87
Constant 5.45e+04
MPV 1.719
Sigma 0.09255
10000
8000
6000
4000
2000
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Energy Loss of $e^{\pm}$

Energy loss of $e^{\pm}$

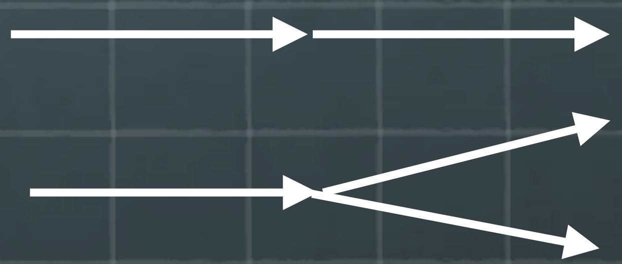
$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$$W_{\max} = \frac{2m_e c^2\, \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$



$$\frac{1}{2}\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau)$$

$$W_{max} = T/2$$



for e^-

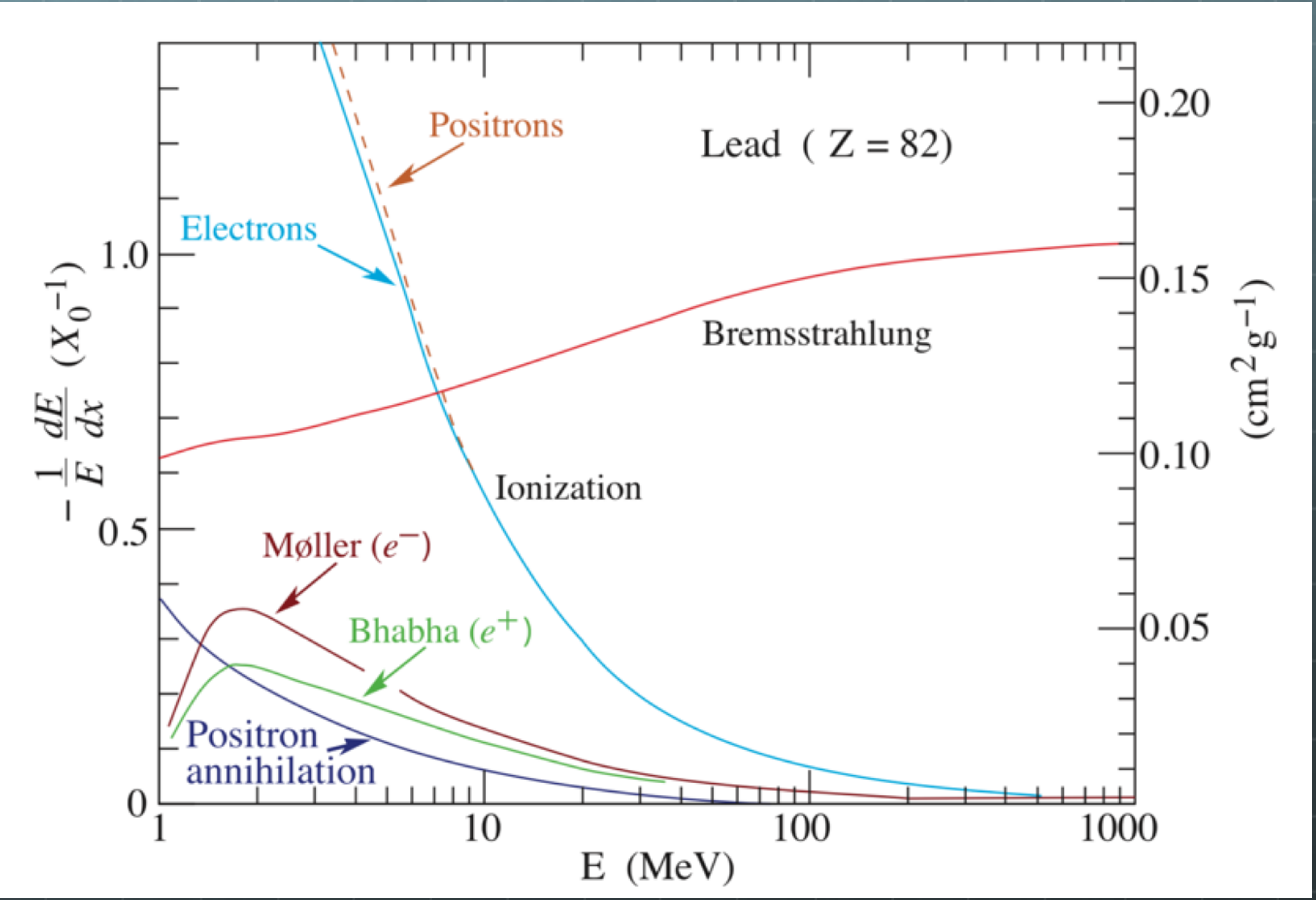
$$F(\tau) = 1 - \beta^2 + \frac{(\tau^2/8) - (2r+1)\ln 2}{(\tau+1)^2}$$

for e^+

$$F(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right)$$

$$W_{max} = T$$

Energy loss of $e^{\pm}$

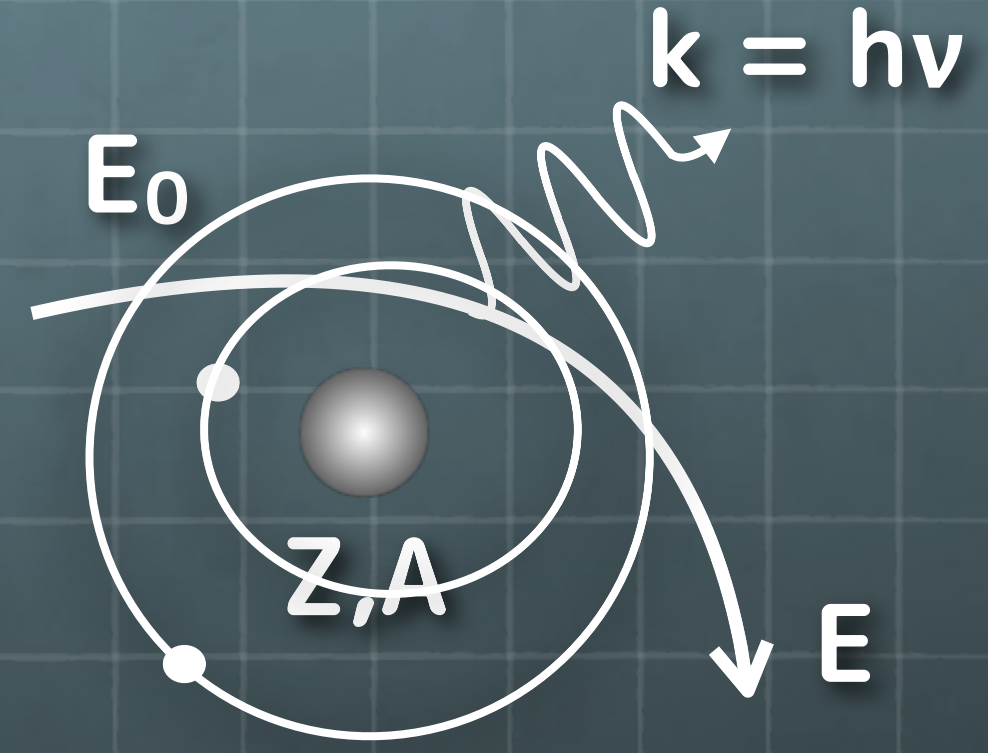


Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A z^2 \frac{Z^2}{A}\left(\frac{e^2}{mc^2}\right)^2 E \,\mathbf{ln}\frac{183}{Z^{1/3}}$$

$$= K z^2 \frac{Z}{A}\left(\frac{Z}{m_e c^2}\right)\frac{\alpha}{\pi} E \cdot \mathbf{ln}\frac{183}{Z^{1/3}}$$

$$= X_0 E$$

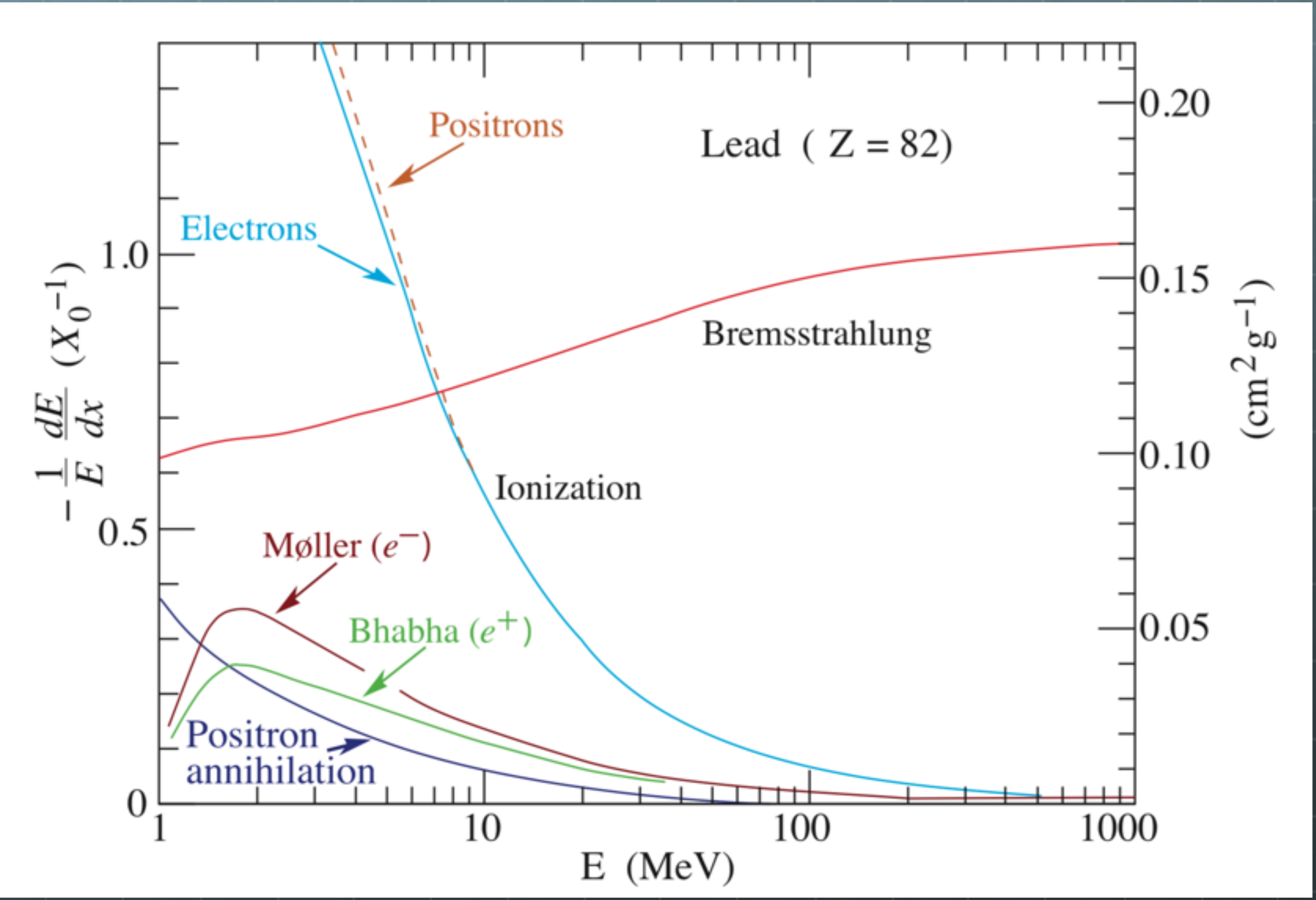


$$E(x) = E_0 e^{-x/X_0}$$

Radiation Length Xo

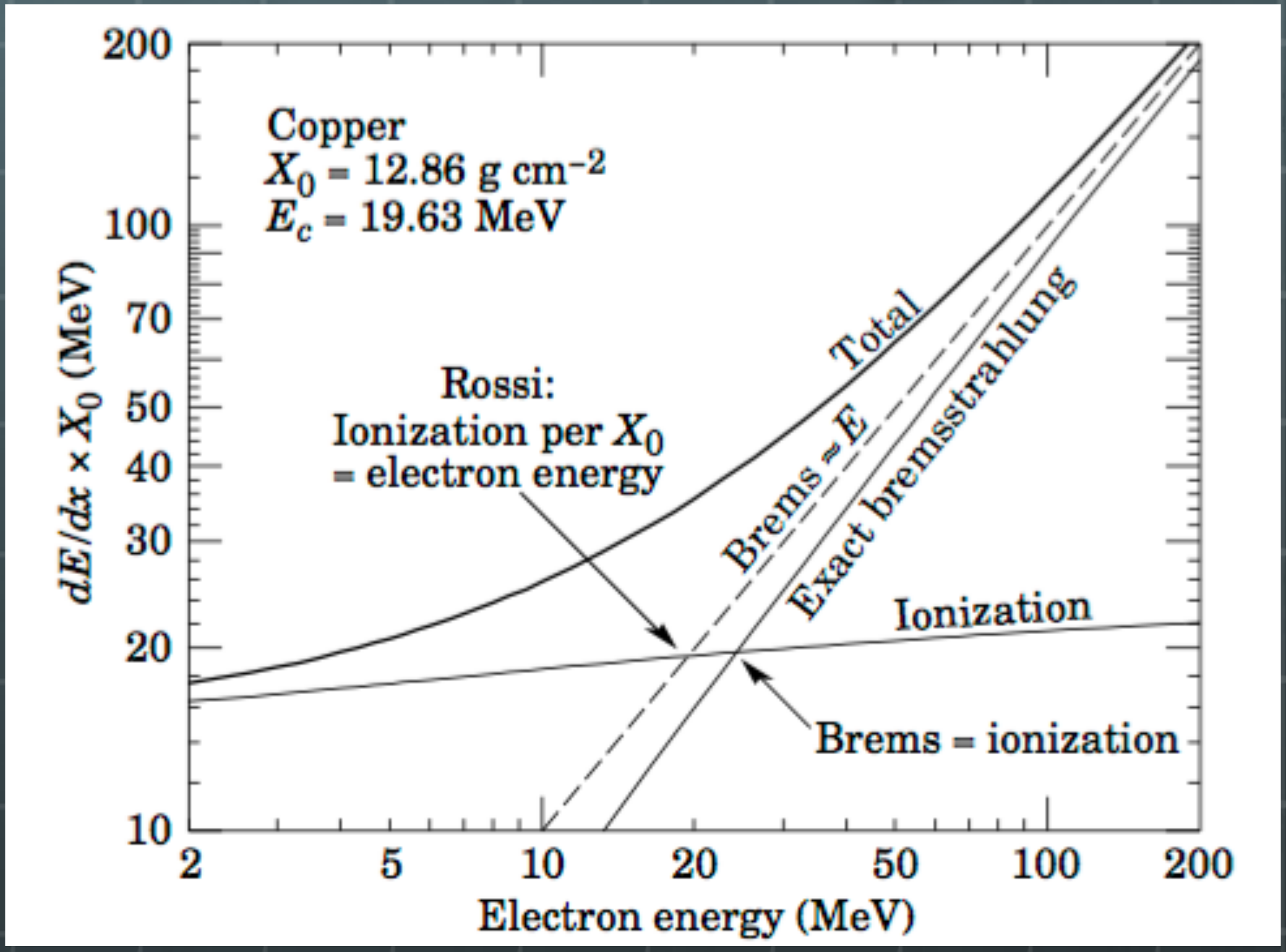
$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \mathbf{ln}\frac{183}{Z^{1/3}}}$$

Energy loss of $e^{\pm}$



Critical energy (Ec)

- Energy at which a electron losses its energy as same amount by bremsstrahlung and ionization.
- Energy at which the ionization loss per Xo is equal to the electron energy.



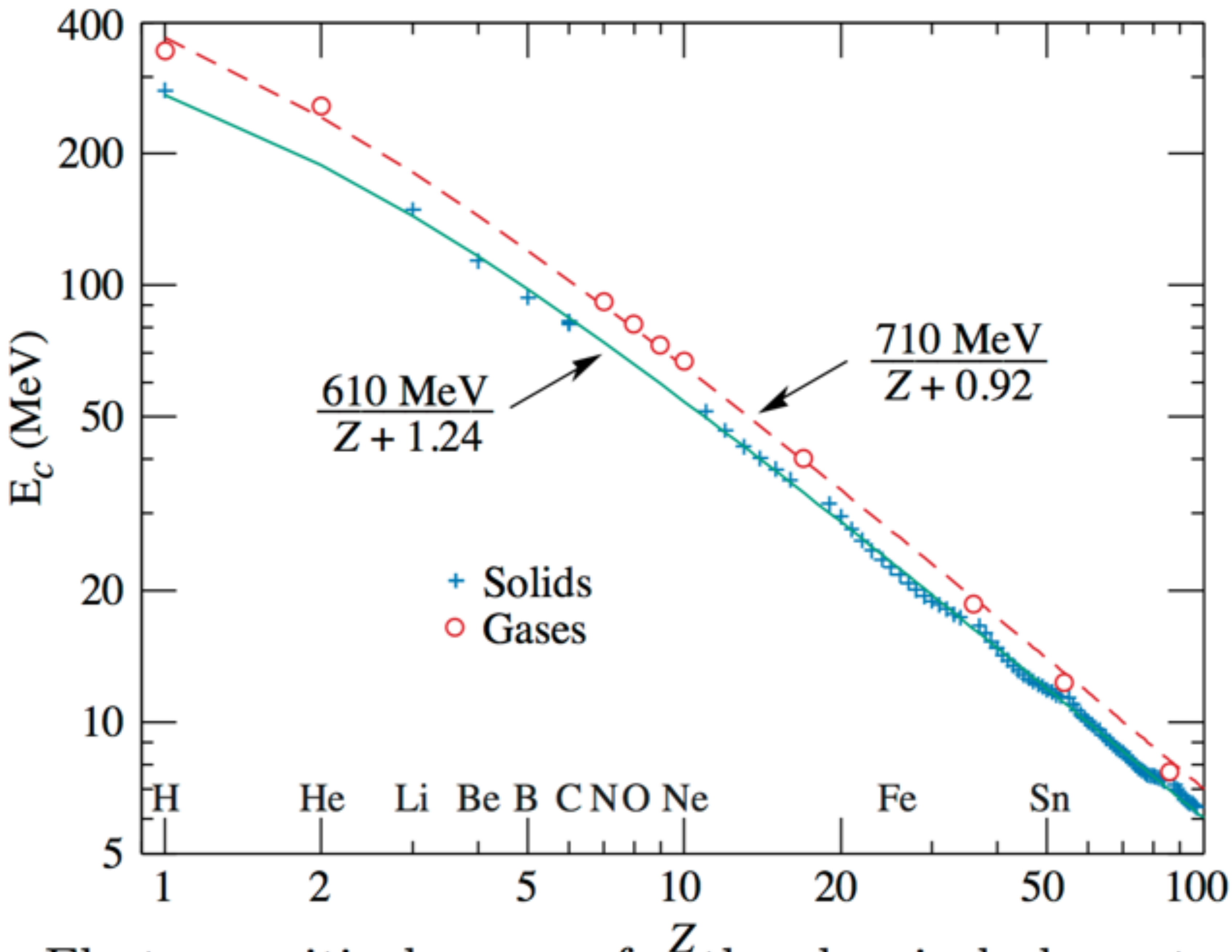


Figure 33.14: Electron critical energy for the chemical elements, using Rossi's definition [2]. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases. (Computed with code supplied by A. Fassó.)