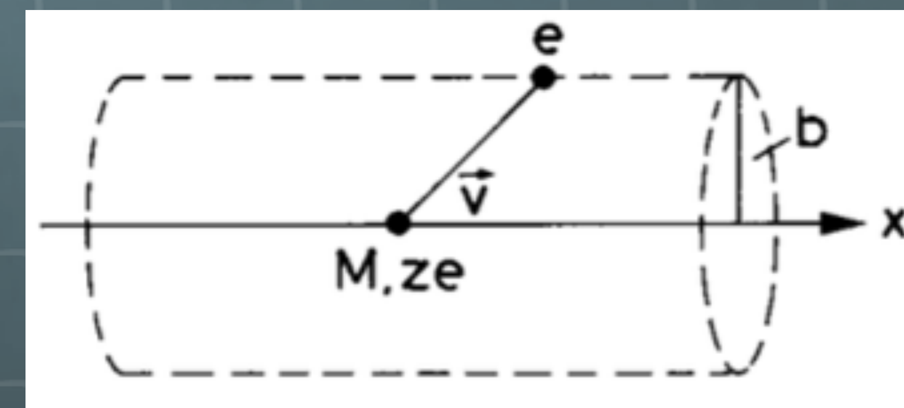
Energy Loss of (heavy) charged particles by atomic collision

Energy Loss

- **Charge ze, velocity v, mass M, Impact parameter b**
- **Momentum Transfer**



$$\Delta P_T = \int F dt = \int F_T dt = \int F_T \frac{dx}{v}$$

$$= \int \frac{ze^2}{(x^2+b^2)} \cdot \frac{b}{\sqrt{x^2+b^2}} \cdot \frac{1}{v} dx$$

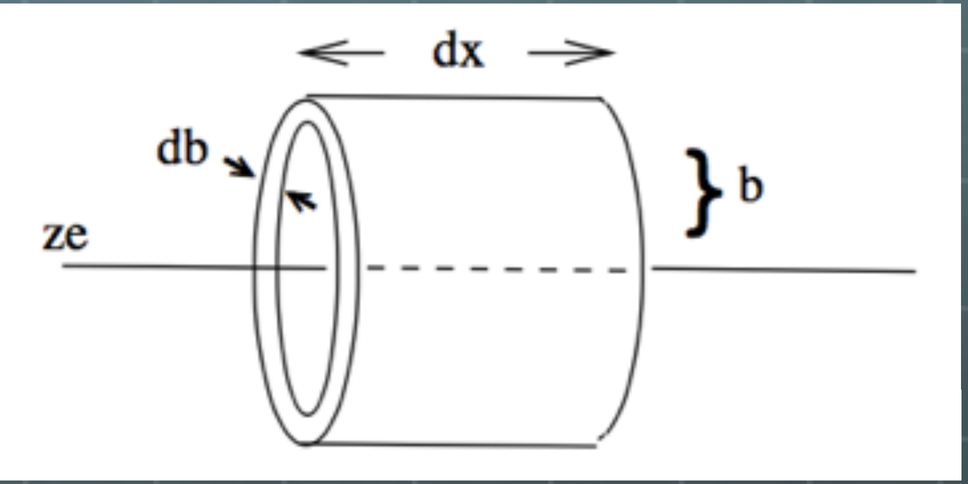
$$= \frac{2ze^2}{bv}$$

Consider a charged particle with a charge ze, velocity v, and mass M interacting with an atomic electron at distance at b.

Energy Loss (cont.)

- Consider a bulk of electrons
  - Number of electrons

$$N_e = \rho \cdot 2\pi b \cdot db\ dx$$



  - Energy loss becomes

$$dE(b) = N_e \cdot \Delta E(b) = \frac{4\pi z^2 e^4}{m_e v^2}\rho \frac{db}{b} dx$$

$$\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2}\rho \int \frac{db}{b} \quad \rightarrow \frac{4\pi z^2 e^4}{m_e v^2}\rho \cdot \mathbf{ln}(\frac{b_{max}}{b_{min}})$$

Bohr's Formula

b_{min} : head-on collision

$$2m_e v^2 = \frac{ze^2}{b_{min}} \longrightarrow b_{min} = \frac{ze^2}{2m_e v^2}$$

b_{max} : adiabatic invariance

$$F \cdot \tau = \Delta P$$

τ : Period of bound electron's orbital motion

$$\frac{ze^2\tau}{b_{max}^2} = \frac{2ze^2}{b_{max}v} \longrightarrow b_{max} = \frac{v\tau}{2} = \frac{v}{2\nu}$$

$$< -\frac{dE}{dx} >= \frac{4\pi z^2 e^4}{m_e v^2} N_A \frac{Z}{A} \mathbf{ln}(\frac{mv^3}{\nu z e^2})$$

PDG
particle data group
1957 2017

About PDG | PDG Authors | PDG Citation | Contact Us

The Review of Particle Physics (2017)

C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update.

pdgLive - Interactive Listings

Summary Tables

Reviews, Tables, Plots (2016)

Particle Listings

Search

Order: Book & Booklet

Download or Print: Book, Booklet, Website, Figures & more

Previous Editions (& Errata) 1957-2016	Physical Constants
Errata in current edition	Astrophysical Constants
Figures in reviews	Atomic & Nuclear Properties
Mirror Sites	Astrophysics & Cosmology

PDG Outreach

Particle Adventure & Apps | CPEP Charts | History book

Non-PDG Resources

HEP Papers | Databases & Info | Institutions & People

Funded by:

US DOE, CERN, MEXT (Japan), IHEP-CAS (China), INFN (Italy), MINECO (Spain), IHEP (Russia)

http://pdg.lbl.gov

33. PASSAGE OF PARTICLES THROUGH MATTER

Revised August 2015 by H. Bichsel (University of Washington), D.E. Groom (LBNL), and S.R. Klein (LBNL).

Bethe Equation

$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$$K = 4\pi N_A r_e^2 m_e c^2 = 0.307075 \text{ MeV } mol^{-1}\ cm^2$$

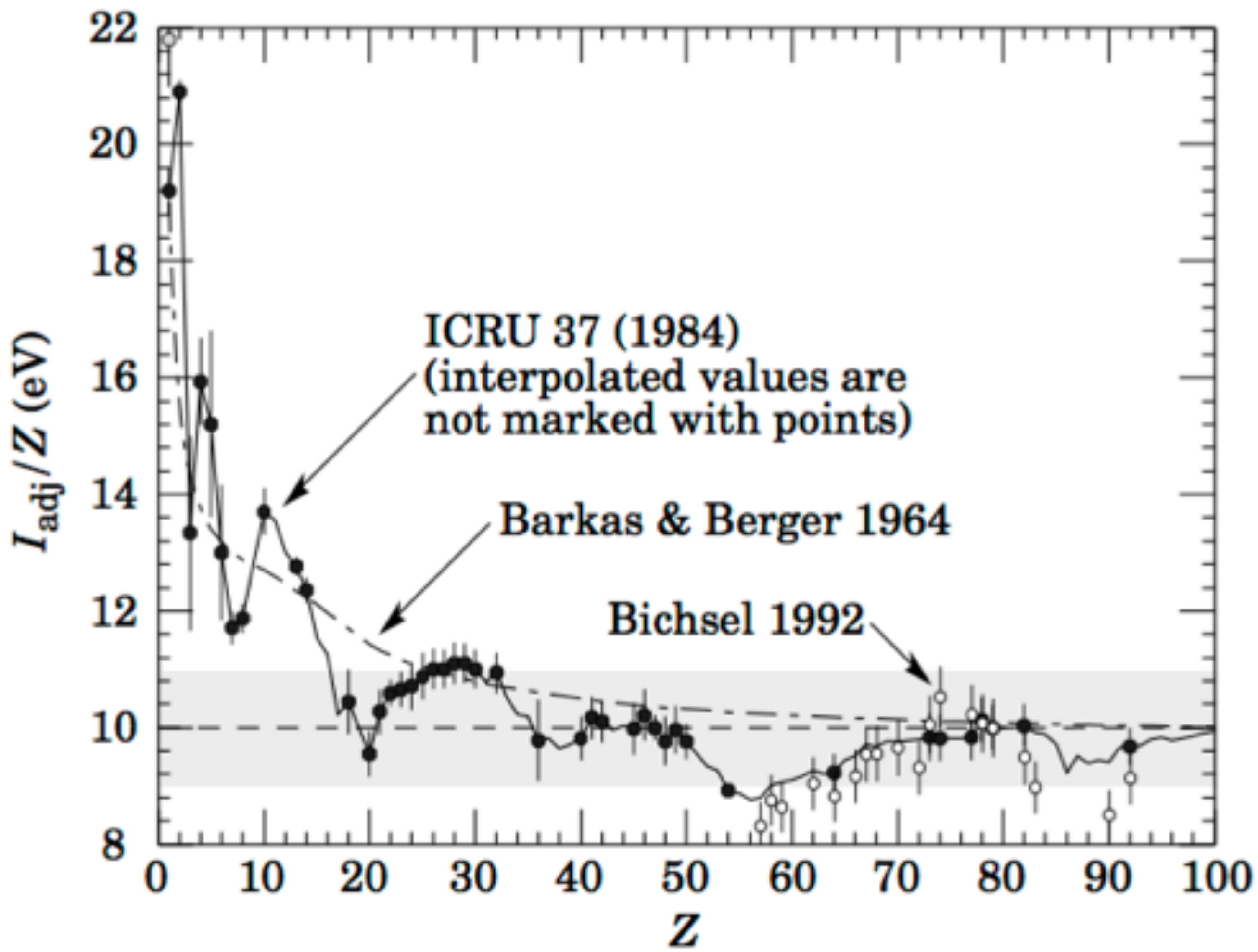
$$r_e = e^2/(4\pi\epsilon_0 m_e c^2) = 2.8179403267(27) fm$$

$$W_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

$$< -\frac{dE}{dx} > = \frac{4\pi z^2 e^4}{m_e v^2} N_A \frac{Z}{A} \ln\left(\frac{mv^3}{4\nu z e^2}\right)$$

하전 입자가 물질을 통과하면서 Inelastic collision에 의하여 에너지를 잃어 버리는 과정을 설명하는 방정식은 다음과 같은 Bethe Equation으로 표현된다. 이

Mean Excitation Energy



ICRU : International Commission of Radiation Units & Measurements
I : average excitation potential of atoms
Bloch's law : I = 10*Z

실제로 에너지를 전달한다는 것은 물질의 원자의 excitation energy에 관련하는 것이 되고, 초기의 실험에서는 이 Stopping power를 측정하는 실험이 많이 이루어졌으며, 이 결과로부터, 원자의 평균적인 Excitation potential 즉 mean excitation energy를 유도해 낸다.

먼저는 I/Z=10 Bloch's law 라고 하는 데, 내 눈으로 확인한 것이 아니라 불안함. 이 relation이 LOW-Z material에서 벗어나고, 이 correction을 Barkas correction. second-order correction이 Bloch임.

Atomic and nuclear properties of copper (Cu)

Quantity	Value	Units	Value	Units
Atomic number	29			
Atomic mass	63.546(3)	g mole^{-1}		
Specific gravity	8.960	g cm^{-3}		
Mean excitation energy	322.0	eV		
Minimum ionization	1.403	MeV g^{-1}cm^2	12.57	MeV cm^{-1}
Nuclear collision length	84.2	g cm^{-2}	9.393	cm
Nuclear interaction length	137.3	g cm^{-2}	15.32	cm
Pion collision length	109.3	g cm^{-2}	12.20	cm
Pion interaction length	165.9	g cm^{-2}	18.51	cm
Radiation length	12.86	g cm^{-2}	1.436	cm
Critical energy	19.42	MeV (for e^-)	18.79	MeV (for e^+)
Molière radius	14.05	g cm^{-2}	1.568	cm
Plasma energy $\hbar\omega_p$	58.27	eV		
Muon critical energy	317.	GeV		
Melting point	1358.	K	1085.	C
Boiling point @ 1 atm	2835.	K	2562.	C

For muons, $dE/dx = a(E) + b(E)\,E$. Tables of $b(E)$: PDF TEXT

Table of muon dE/dx and Range: PDF TEXT

Explanation of some entries

Table of isotopes Warning: may not be current

x ray mass attenuation coefficients

aterials

>) tables including radiative losses for muons, nuclear and pion
ius, plasma energy, and links to isotope and x-ray mass atteunation

naterials.

					$_2$He
$_5$B	$_6$C	$_7$N	$_8$O	$_9$F	$_{10}$Ne
$_{13}$Al	$_{14}$Si	$_{15}$P	$_{16}$S	$_{17}$Cl	$_{18}$Ar
$_{31}$Ga	$_{32}$Ge	$_{33}$As	$_{34}$Se	$_{35}$Br	$_{36}$Kr
$_{49}$In	$_{50}$Sn	$_{51}$Sb	$_{52}$Te	$_{53}$I	$_{54}$Xe
$_{81}$Tl	$_{82}$Pb	$_{83}$Bi	$_{84}$Po	$_{85}$At	$_{86}$Rn
$_{113}$Nh	$_{114}$Fl	$_{115}$Mc	$_{116}$Lv	$_{117}$Ts	$_{118}$Og
$_{68}$Er	$_{69}$Tm	$_{70}$Yb	$_{71}$Lu		
$_{100}$Fm	$_{101}$Md	$_{102}$No	$_{103}$Lr		

2016/AtomicNuclearProperties/

PDG
particle data group

Atomic and Nuclear Properties of Materials
for more than 300 materials

Click on element or other materal for properties of interest in high-energy physics: stopping power (<-dE/dx>) tables including radiative losses for muons, nuclear and pion collision and interaction lengths, electron, positron, and muon critical energies, radiation length, Moliere radius, plasma energy, and links to isotope and x-ray mass atteunation coefficient tables and plots.

This AtomicNuclearProperties page is upgraded as needed in response to suggestions and requests for new materials.
Suggestions and comments are welcome. Please report errors.

Chemical elements: For entries in red, a pull-down menu permits selection of the physical state.
Cryogenic liquid densties are at the boiling point at 1 atm.

0 n																	
1 Ps																	
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
		90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

Inorganic compounds (Al through Fe)	Aluminum oxide through ferrous oxide
Inorganic compounds (Freon through Pu)	Freon through plutonium oxide
Inorganic compounds (Potassium thru yttrium)	Potassium iodide through water
Inorganic scintillators (BaF2 through Y2SiO5)	Barium fluoride through Y2SiO5
Simple organic compounds	Acetone through Xylene
Polymers	Polymers
Mixtures	Aerogel t
Biological materials	A-mn-dimethyl_formamide through tissue-equivalent gas

http://pdg.lbl.gov/2016/AtomicNuclearProperties/

33. PASSAGE OF PARTICLES THROUGH MATTER

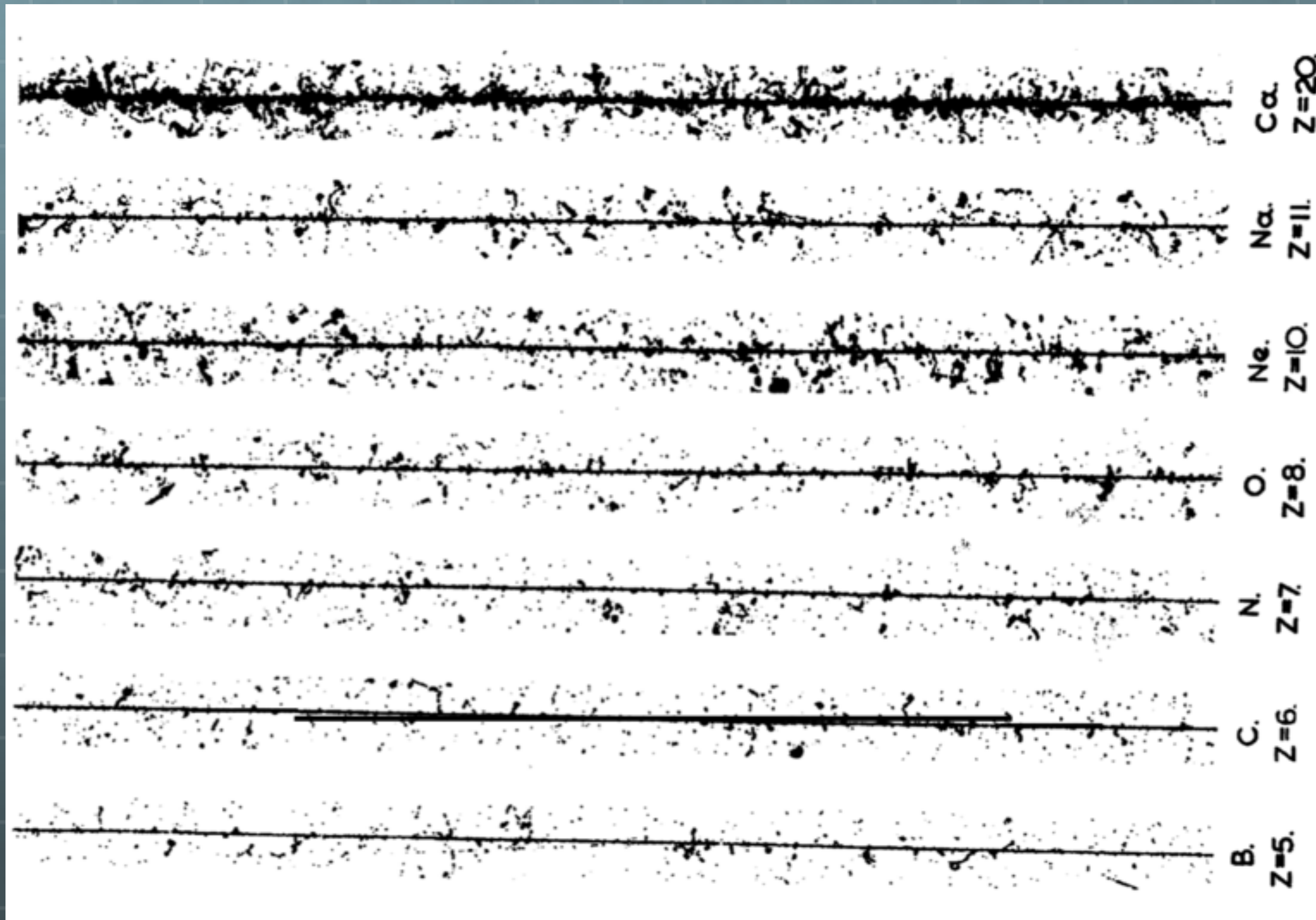
Revised August 2015 by H. Bichsel (University of Washington), D.E. Groom (LBNL), and S.R. Klein (LBNL).

Bethe Equation

$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

- charge square of incident particle (z^2)
- charge of matter (Z)
- Inverse of mass of matter (1/A)
  - Unit of MeV · cm^2/g

일반적인 방정식에서와 같이, 하나의 sacle을 나타내주는 Constant K로 표현하고 있고, 단위 길이당 평균적으로 잃어 버리는 에너지는 입사하는 입자의 전하의 제곱과 물질의 전하 Z 에 비례하는 값이 됨을 알 수 있습니다. 물질의 Mass Number에 반비례하는 표현으로 나타나고 있는 데, 이는 이 표현식이 MeV · cm ^ 2 /g의 단위를 갖게 함으로써, 좀 더 보편적인 표현식이 되도록 한 것입니다.



Cecil F. Powell Nobel Lecture 1950

33. PASSAGE OF PARTICLES THROUGH MATTER

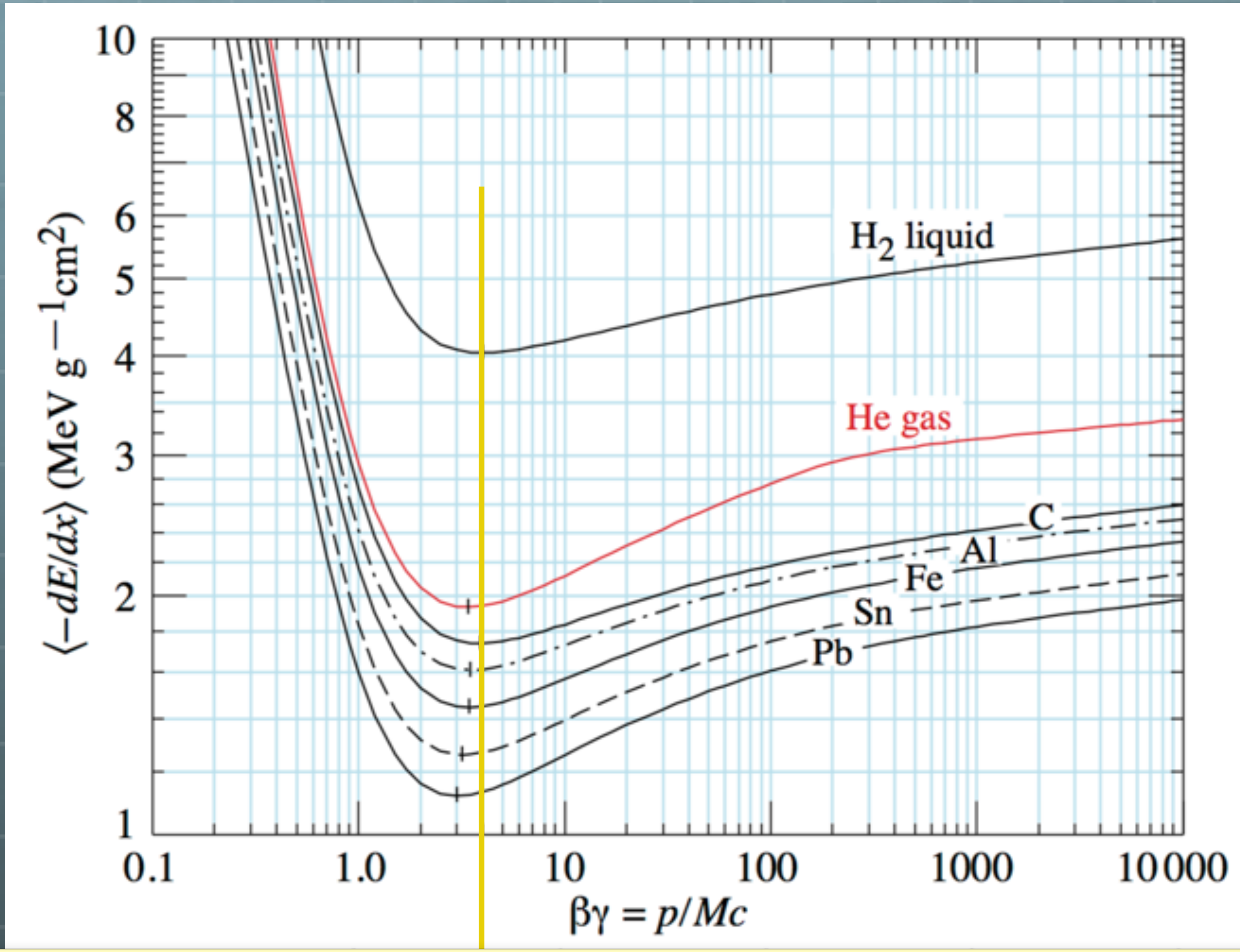
Revised August 2015 by H. Bichsel (University of Washington), D.E. Groom (LBNL), and S.R. Klein (LBNL).

Bethe Equation

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

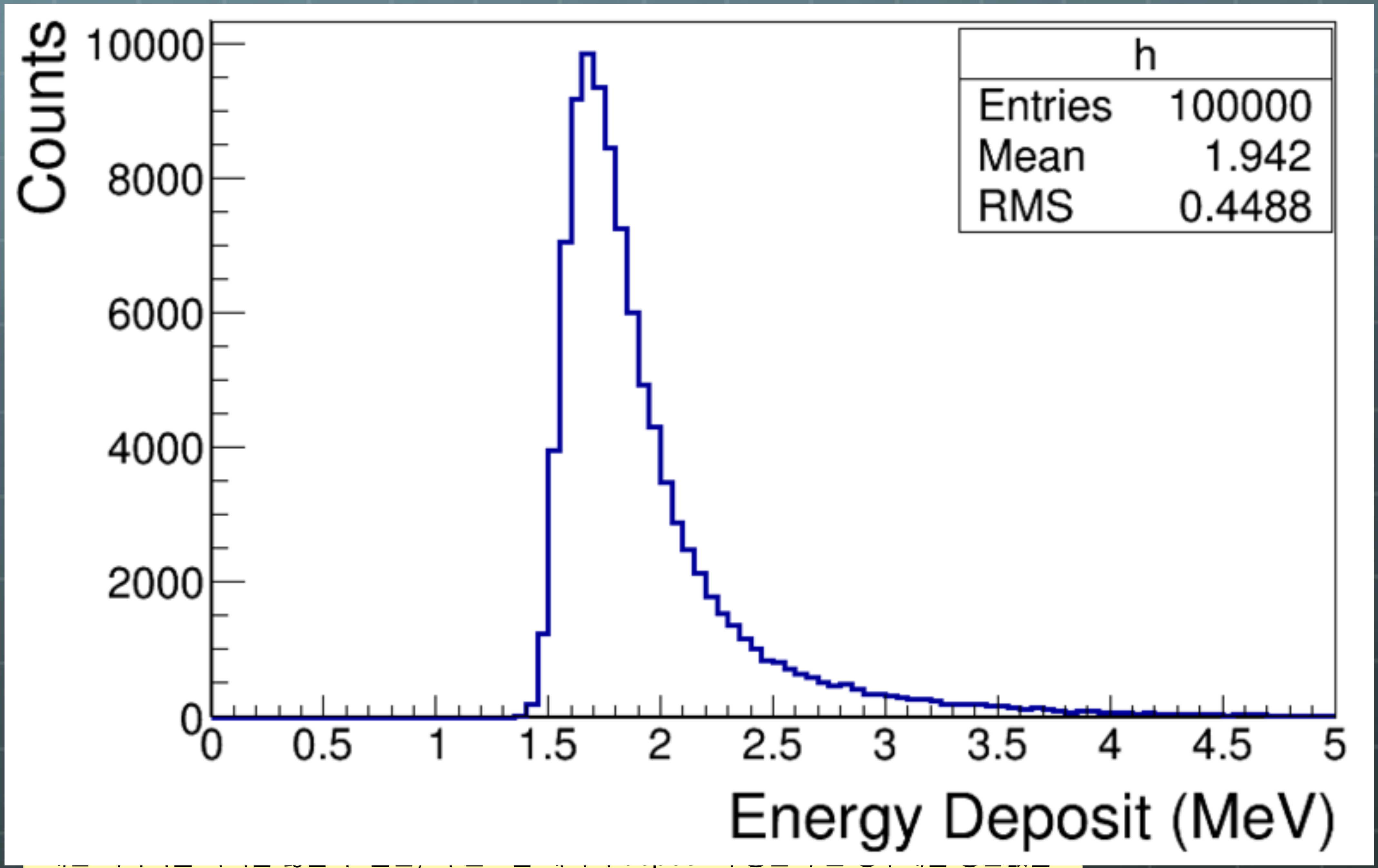
- charge square of incident particle (z^2)
- charge of matter (Z)
- Inverse of mass of matter (1/A)
- Mixture of different dependence on the velocity (β)
- Maximum energy transfer in a single collision

$$W_{\max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$



이 그림에서 보는 바와 같이, 넓은 영역에 있어서, Mass stopping power는 Mass number로 normalize한 energy loss식으로 나타낼 수 있게 되는 것입니다.이 그림에서보면 beta・gamma가 3에서 3.5사이의 영역에서 가장 작은 값을 갖게 되는 데, 이 운동량을 갖는 입자를 minimum Ionizing particle이라고 합니다. 이 값이 beat・gamma가 갖은 값을 갖는 것이 되니까,입사하는 입자의 운동량은 질량에 따라서 큰 차이를 나타내게 되는 데....가령 muon의 경우에는 약 400 MeV/c의 운동량을 갖는 경우가 MIP이 되지만, proton의 경우가 되면 약 3GeV/c의 운동량을 갖는 경우가 MIP가 된다.
실제 실험실에서 Cosmic muon (엄밀히 말하면 각 지점에 따라서 약간의 차이가 있고 (대기중에서 만들어 져서 실험실까지 들어오니까....)
(여러분의 실험실에 들어오는 Cosmic muon의 energy spectrum을 재보는 것도 재미있는 놀이가 될 지도 모르겠네요.....).
에너지 분포를 갖고 있으니까, 이 cosmic muon을 MIP이라고 하는 것은 약간은 다른 의미가 되겠지만, 300 MeV/c을 넘어서면 그다지 크게 변하지 않기때문에, 광범위한 의미에서 Cosmic muon을 MIP으로 취급하고, cosmic muon이 남기는 에너지를 하나의 값으로 나타내기도 합니다.

MIP energy deposit



중심으로하는 gaussian 으로 나타나겠지만, 지금 이 그림에서 보는 바와같이 비대칭적인 분포를 갖게된다.

6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1 Abridged from `pdg.lbl.gov/AtomicNuclearProperties` by D.E. Groom (2015). See web pages for more detail about entries in this table and for several hundred other substances. Parentheses in the dE/dx and density columns indicate gases at 20° C and 1 atm. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg$1 in brackets indicate $(n-1)\times 10^6$ for gases at 0° C and 1 atm.

Material	Z	A	$\langle Z/A\rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\|_{min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index @ Na D
H_2	1	1.008(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D_2	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.94(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.0121831(5)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N_2	7	14.007(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O_2	8	15.999(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F_2	9	18.998403163(6)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815385(7)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl_2	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.630(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(5)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	[289]
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glass			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard rock			0.50000	66.8	101.3	26.54	1.688	2.650			
Methane (CH_4)			0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C_2H_6)			0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Propane (C_3H_8)			0.58962	55.3	76.7	45.37	(2.262)	0.493(1.868)	85.52	231.0	
Butane (C_4H_{10})			0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C_8H_{18})			0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin ($CH_3(CH_2)_{n\approx 23}CH_3$)			0.57275	56.0	78.3	44.85	2.088	0.930			
Nylon (type 6, 6/6)			0.54790	57.5	81.6	41.92	1.973	1.18			
Polycarbonate (Lexan)			0.52697	58.3	83.6	41.50	1.886	1.20			
Polyethylene ($[CH_2CH_2]_n$)			0.57034	56.1	78.5	44.77	2.079	0.89			
Polyethylene terephthalate (Mylar)			0.52037	58.9	84.9	39.95	1.848	1.40			
Polyimide film (Kapton)			0.51264	59.2	85.5	40.58	1.820	1.42			
Polymethylmethacrylate (acrylic)			0.53937	58.1	82.8	40.55	1.929	1.19			1.49
Polypropylene			0.55998	56.1	78.5	44.77	2.041	0.90			
Polystyrene ($[C_6H_5CHCH_2]_n$)			0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetrafluoroethylene (Teflon)			0.47992	63.5	94.4	34.84	1.671	2.20			
Polyvinyltoluene			0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminum oxide (sapphire)			0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium flouride (BaF_2)			0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Bismuth germanate (BGO)			0.42065	96.2	159.1	7.97	1.251	7.130	1317.		2.15
Carbon dioxide gas (CO_2)			0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
Solid carbon dioxide (dry ice)			0.49989	60.7	88.9	36.20	1.787	1.563	Sublimes at 194.7 K		
Cesium iodide (CsI)			0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
Lithium fluoride (LiF)			0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
Lithium hydride (LiH)			0.50321	50.8	68.1	79.62	1.897	0.820	965.		
Lead tungstate ($PbWO_4$)			0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
Silicon dioxide (SiO_2, fused quartz)			0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
Sodium chloride (NaCl)			0.47910	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
Sodium iodide (NaI)			0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H_2O)			0.55509	58.5	83.3	36.08	1.992	1.000	273.1	373.1	1.33
Silica aerogel			0.50093	65.0	97.3	27.25	1.740	0.200	(0.03 H_2O, 0.97 SiO_2)		

- Polyvinyltoluene
  - $\langle Z/A\rangle = 0.54141$
  - $\lambda_T = 57.3\ g/cm^2$
  - $\lambda_I = 81.3\ g/cm^2$
  - $X_0 = 43.9\ g/cm^2$
  - $dE/dx = 1.965$

    $MeV \cdot cm^2/g$
  - $\rho = 1.03\ g/cm^3$

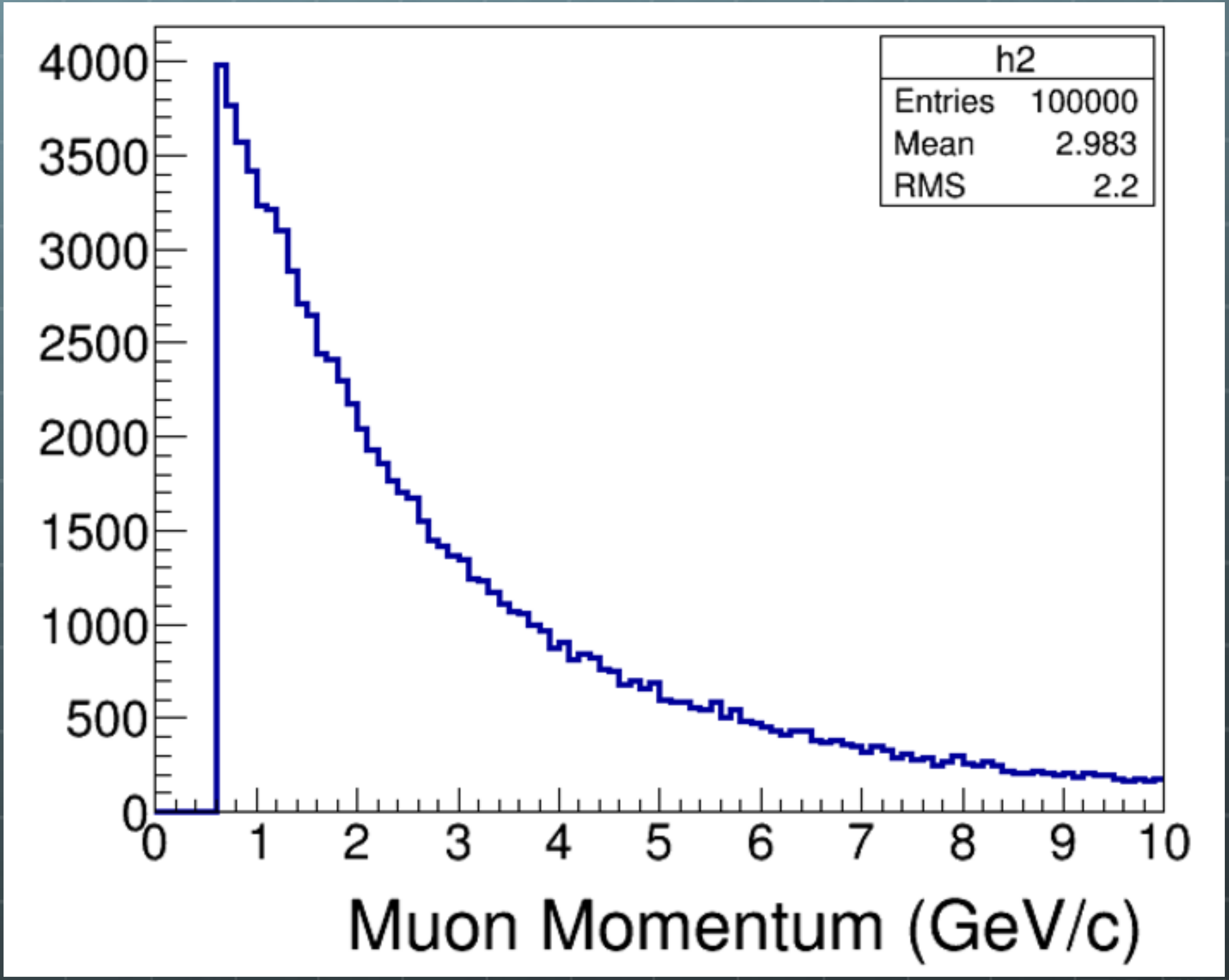
In case of 400 MeV/c muon entering the plastic scintillator

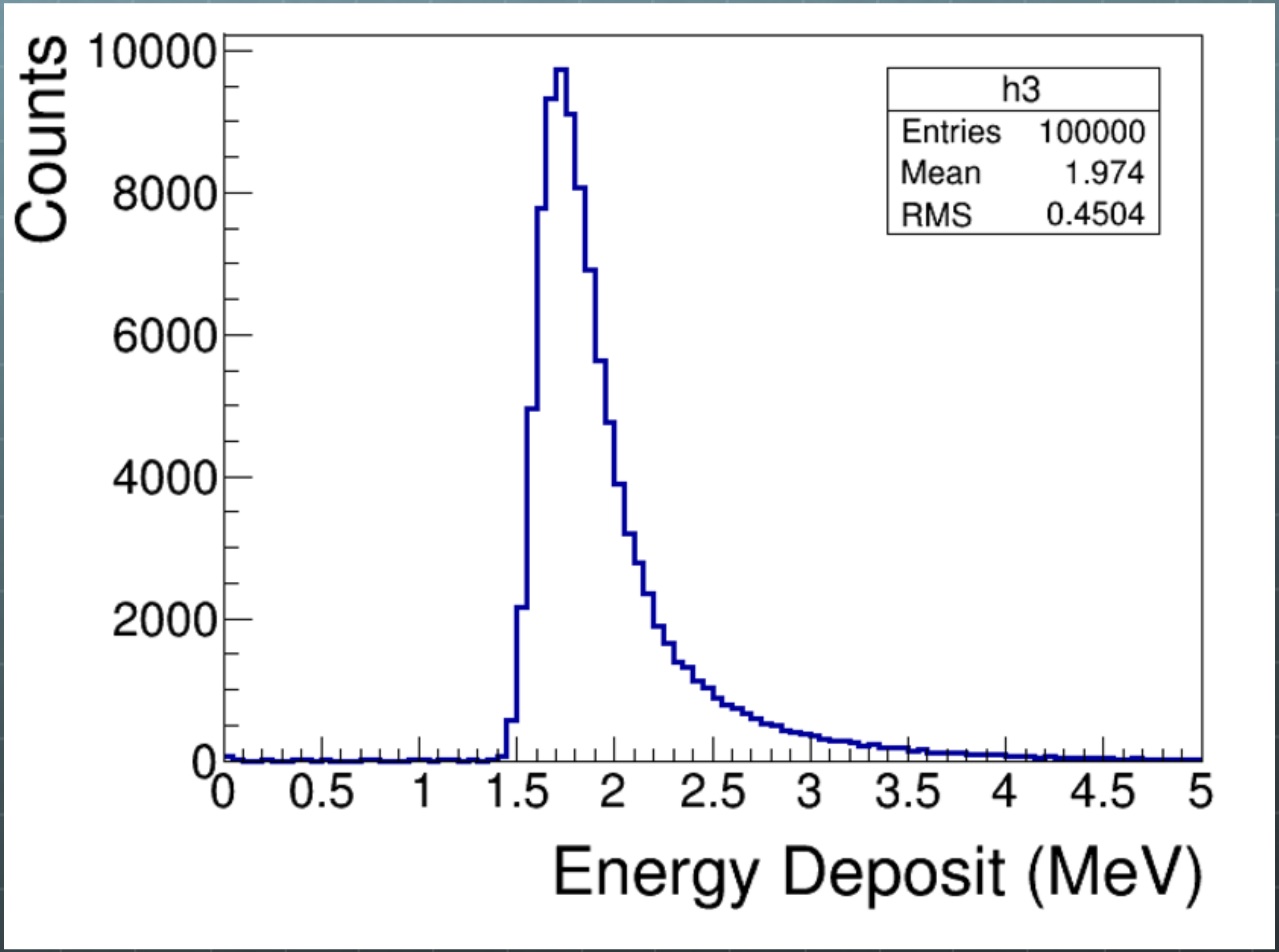
$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

- $K = 0.307075\ \mathrm{MeV} \cdot cm^2/mol$
- <Z/A> =0.54141
- $\beta = 0.9668$, $\beta\gamma = 3.7858$
- I = 64.7 eV
- $m_e c^2 = 0.511$ MeV
- $W_{max} = 14.109$ MeV

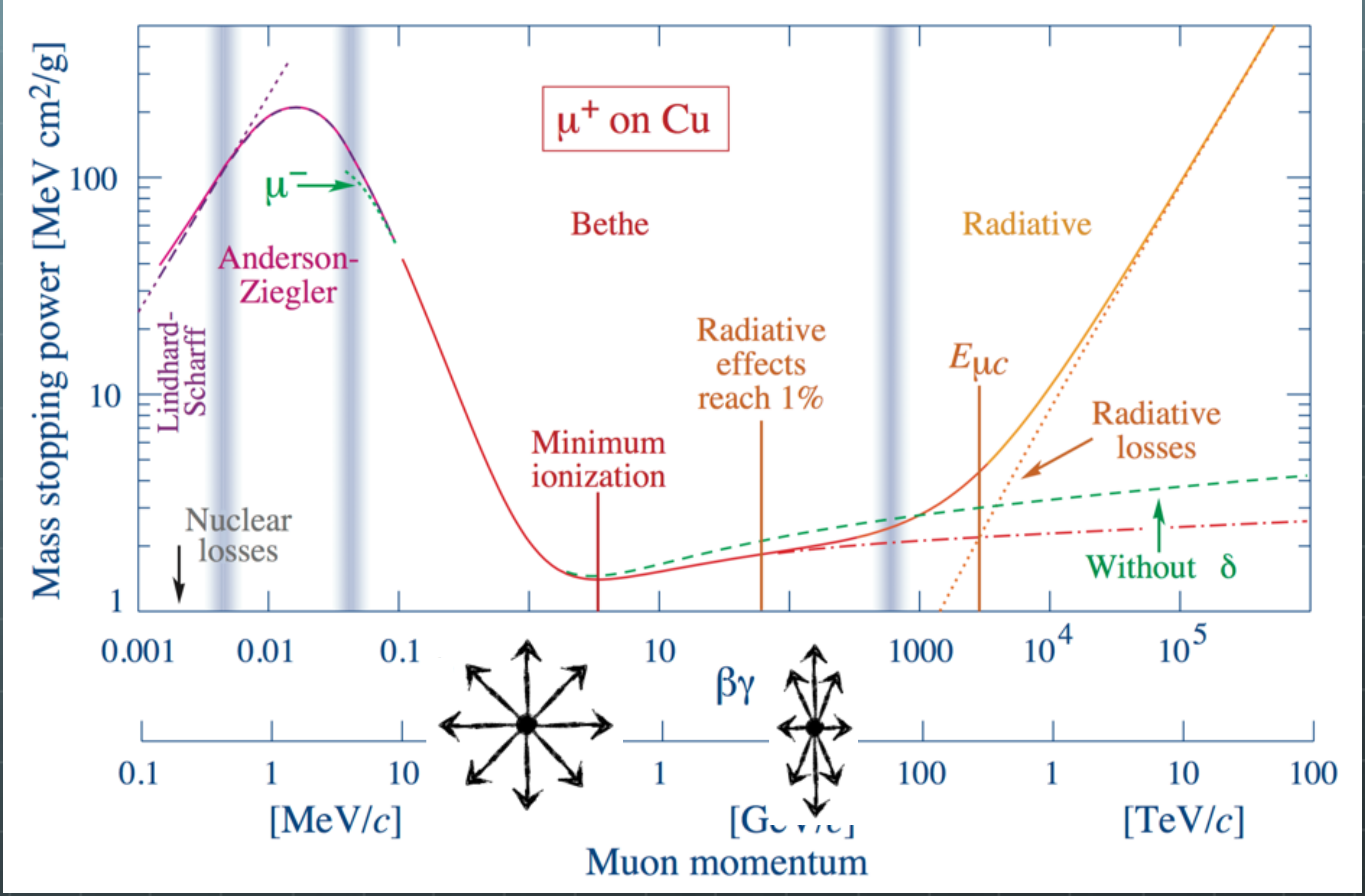
Quantity	Value	Units	Value	Units
<Z/A>	0.54141			
Specific gravity	1.032	$g\ cm^{-3}$		
Mean excitation energy	64.7	eV		
Minimum ionization	1.956	$MeV\ g^{-1}cm^2$	2.019	$MeV\ cm^{-1}$
Nuclear collision length	57.3	$g\ cm^{-2}$	55.56	cm
Nuclear interaction length	81.3	$g\ cm^{-2}$	78.80	cm
Pion collision length	84.8	$g\ cm^{-2}$	82.19	cm
Pion interaction length	113.3	$g\ cm^{-2}$	109.8	cm
Radiation length	43.90	$g\ cm^{-2}$	42.54	cm
Critical energy	94.11	MeV (for e^-)	91.62	MeV (for e^+)
Molière radius	9.89	$g\ cm^{-2}$	9.586	cm
Plasma energy $\hbar\omega_p$	21.54	eV		
Muon critical energy	1195.	GeV		
Index of refraction (Na D)	1.580			

Cosmic ray momentum spectrum @ KEK

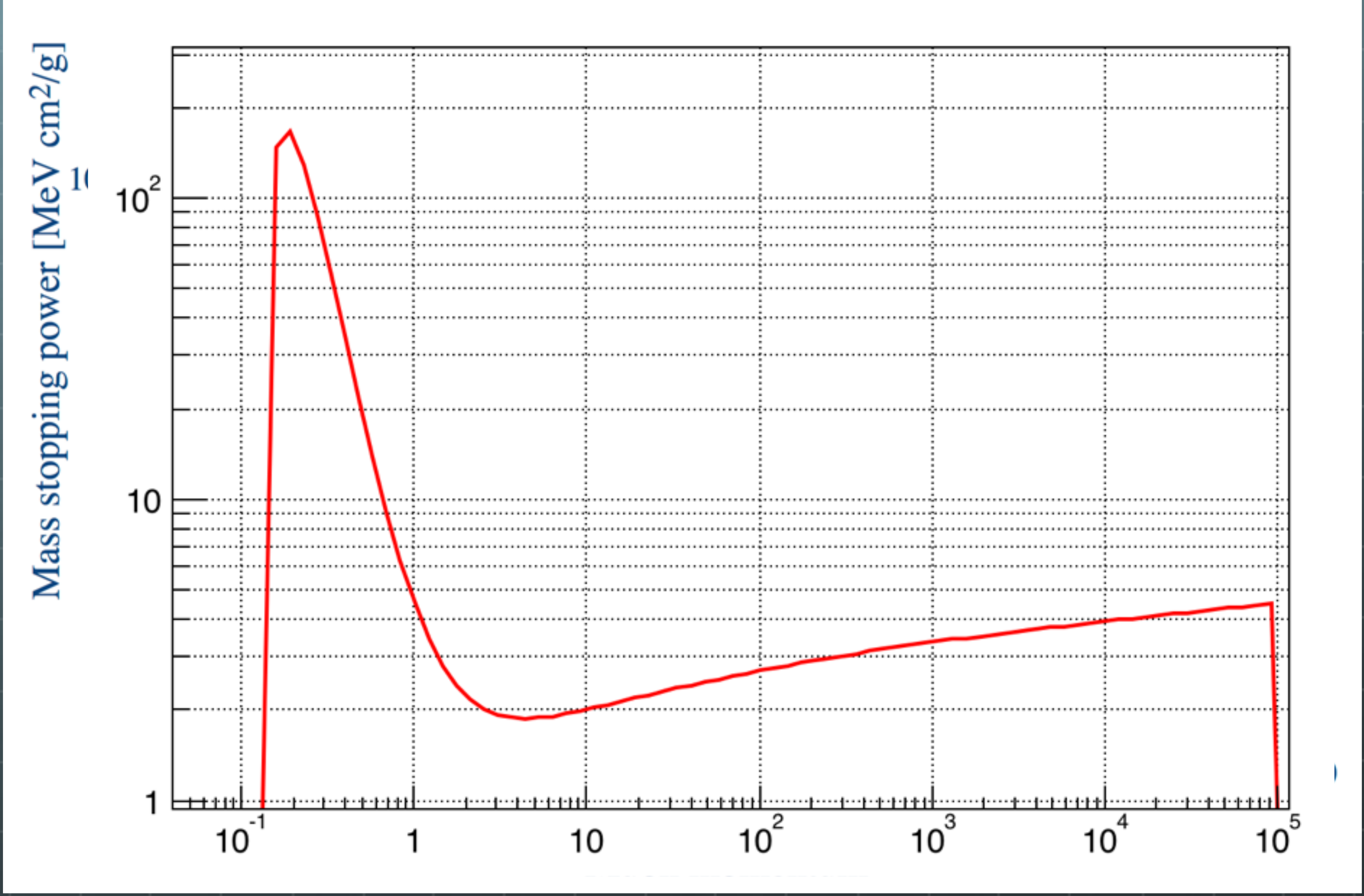



h3
Entries 100000
Mean 1.974
RMS 0.4504
Counts
10000
8000
6000
4000
2000
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Energy Deposit (MeV)

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$



$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$



Particle Energy Deposit

βγ > 3.5:

$$\left\langle \frac{dE}{dx} \right\rangle \approx \left. \frac{dE}{dx} \right|_{\mathrm{min}}$$

βγ < 3.5:

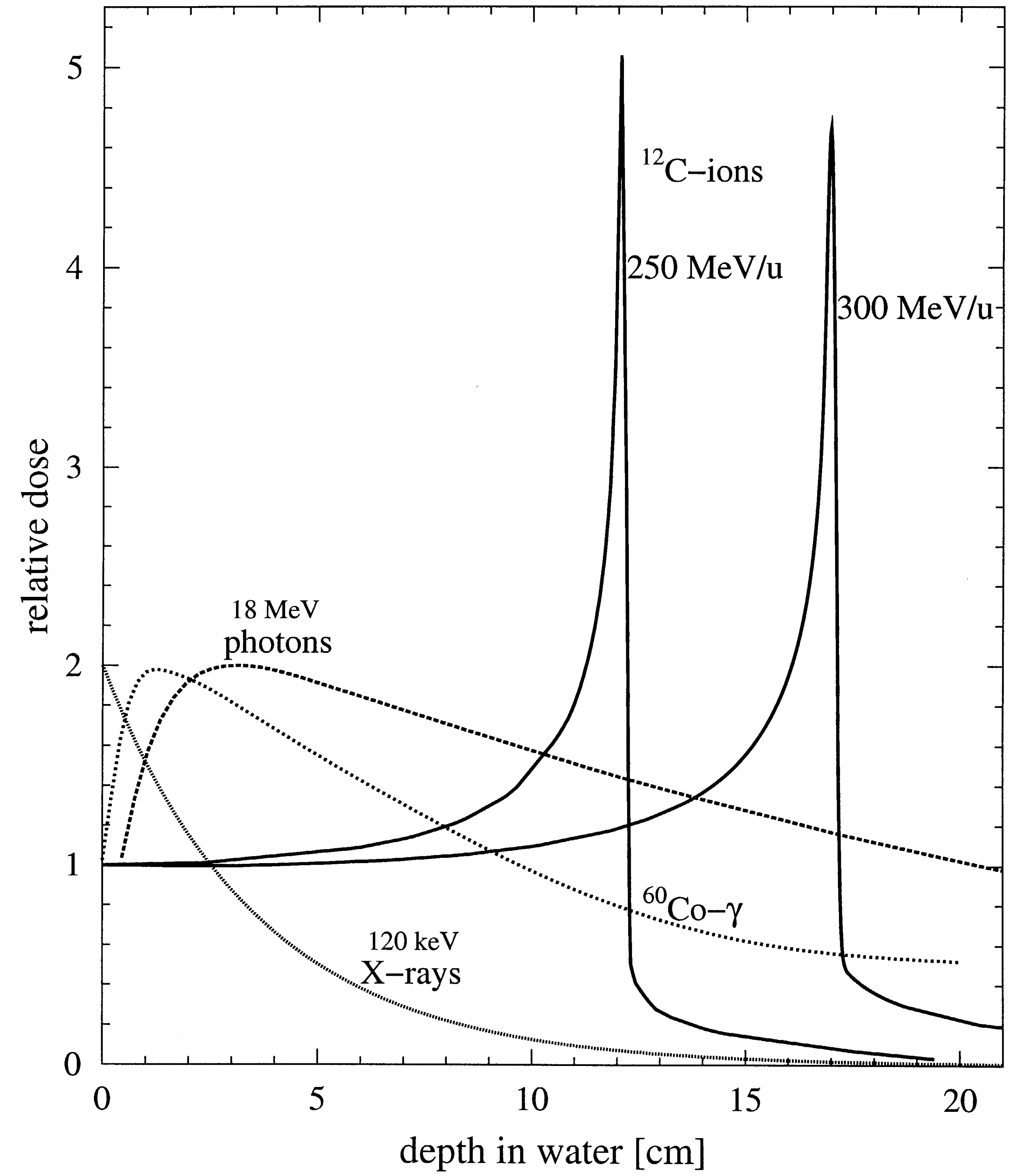
$$\left\langle \frac{dE}{dx} \right\rangle \gg \left. \frac{dE}{dx} \right|_{\mathrm{min}}$$

Applications:

Tumor therapy

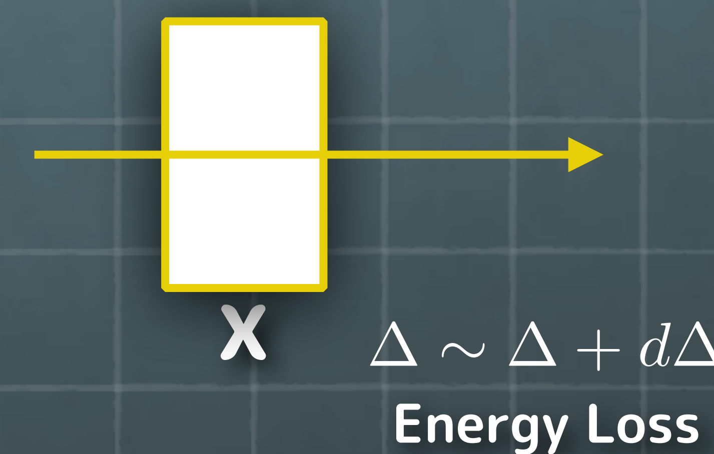
Possibility to precisely deposit dose at well defined depth by E_{beam} variation

[see Journal Club]



Straggling
-Landau Distribution-

- For a small energy loss, prob. of fluctuations
- Let unknown function f(x,Δ)



$$\frac{\partial f}{\partial x} = \int_0^\infty \omega(\varepsilon)[f(x, \Delta - \varepsilon) - f(x, \Delta)]d\varepsilon$$

$$f(x, \Delta) = \frac{1}{2\pi i}\int_{i\infty+\sigma}^{+i\infty+\sigma} e^{p\Delta}\phi(p, x)dp$$

두께 x의 매질을 통과하는 하전 입자가 delta~delta+ddelta사이리 에너지를 남길 확률을 f(x, D)라고 하자. 이 unknown 함수가 어떤 분포를 갖는 가를 계산한 사람이 Landau인데, Laplace transform을 포함하는 수학적 계산 과정은 그냥 믿기로 한다면,
함수 f를 다음과 같이 얻게되고, 확률함수 ㅈ를 제한된 과정에서 앞에서 얻은 에너지 loss를 주고, 이 값을 무한대까지 적분함으로써
얻을 수 있다는 것을 보여줍니다.

단위계를 거리에서 에너지 xi로 변환하면, probability function을 변수 lambda의 변수를 갖는 함수로 표현할 수 있고, 이 확률 함수가 최대가 되는 조건은 이와 같이 되고, 이 값을 most probable value라고 함. 이 함수는 이와 같이 주어지고, 이러한 분포를 Landau distribution이라고 함.

With a variable $\xi = x\dfrac{2\pi N e^2 \rho \sum Z}{mv^2 \sum A}$

$$f(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

$$\phi(\lambda) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{u \ln u + \lambda u} du$$

$$\lambda = \frac{\Delta - \xi(\ln\frac{\xi}{\varepsilon'} + 1 - C)}{\xi}$$

Most probable value of energy loss :

$$f(x,\Delta)d\Delta = \phi(\frac{\Delta - \Delta_0}{\xi})d(\frac{\Delta}{}$$

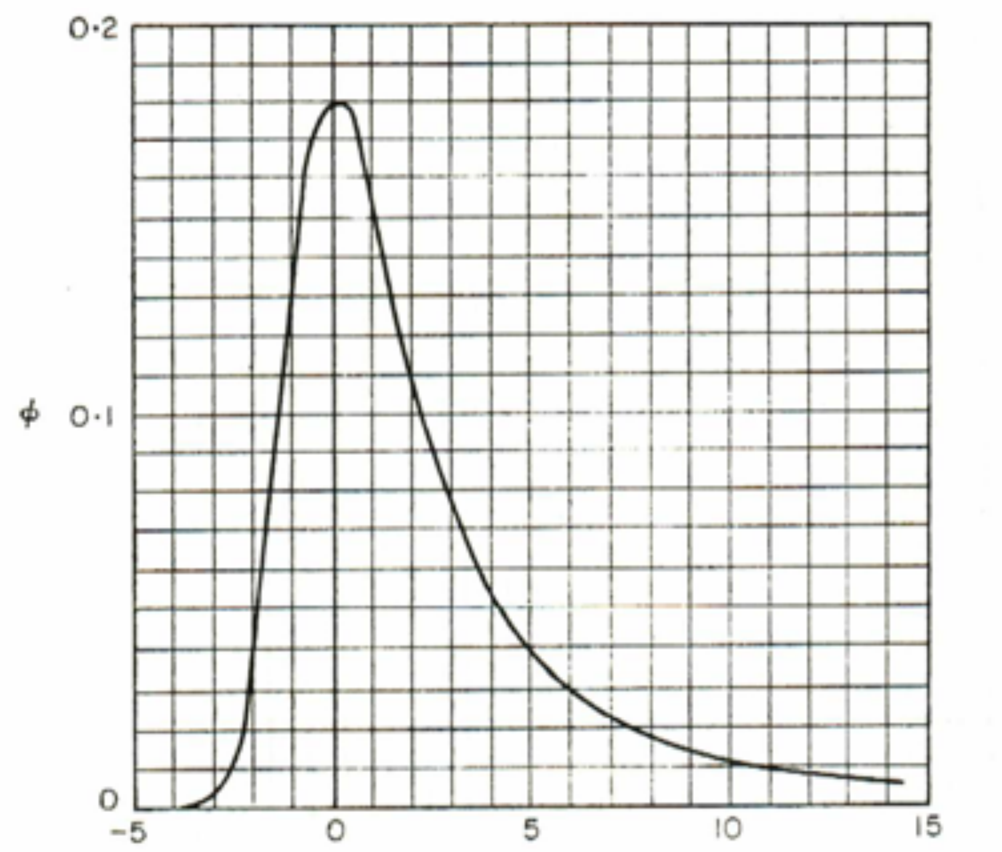


FIG. 1.

단위계를 거리에서 에너지 xi로 변환하면, probability function을 변수 lambda의 변수를 갖는 함수로 표현할 수 있고, 이 확률 함수가 최대가 되는 조건은 이와 같이 되고, 이 값을 most probable value라고 함. 이 함수는 이와 같이 주어지고, 이러한 분포를 Landau distribution이라고 함.

With a variable $\xi = x\frac{2\pi N e^2 \rho \sum Z}{mv^2 \sum A}$

$$f(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

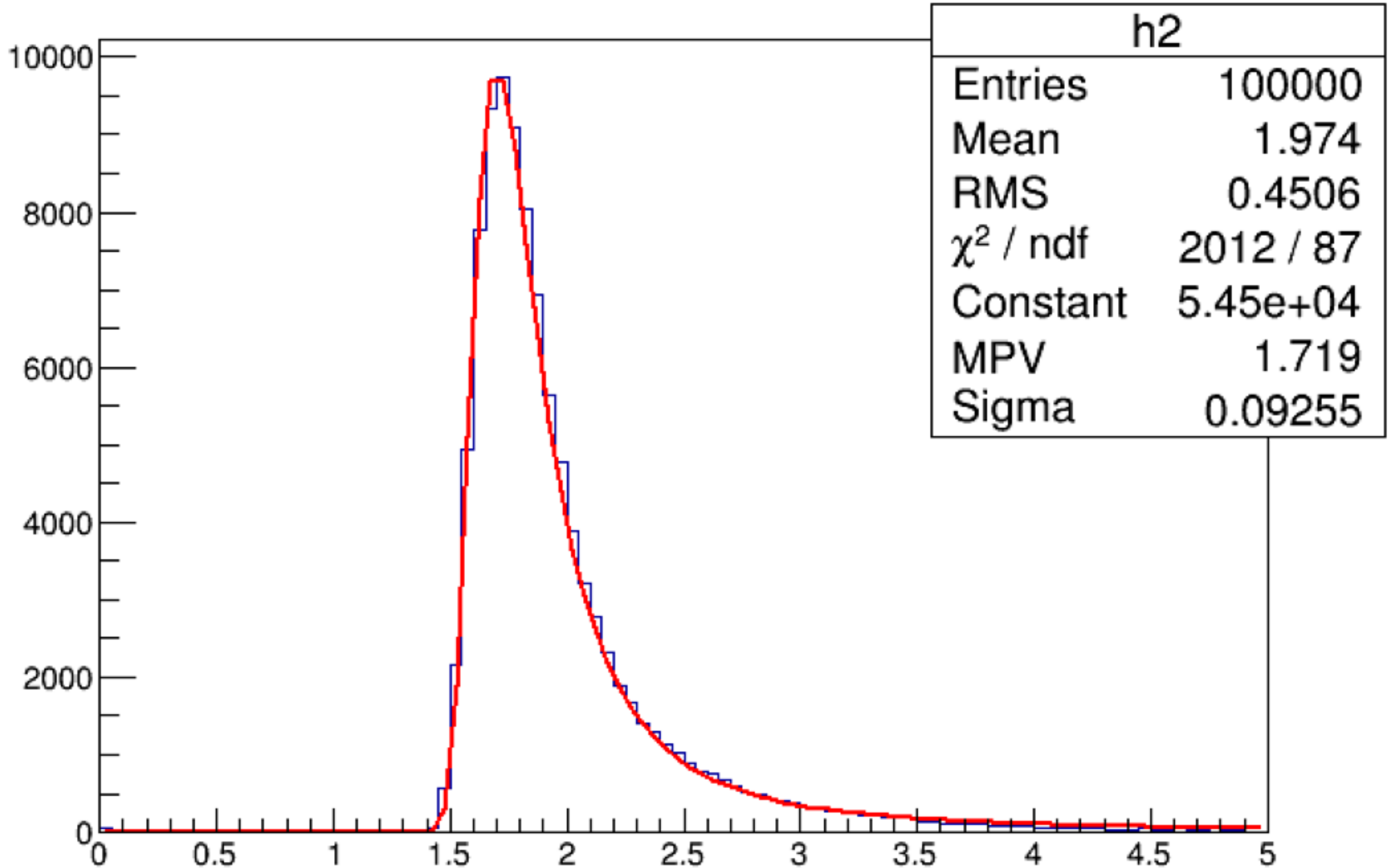
$$\phi(\lambda) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{u\ln u+\lambda u}du$$

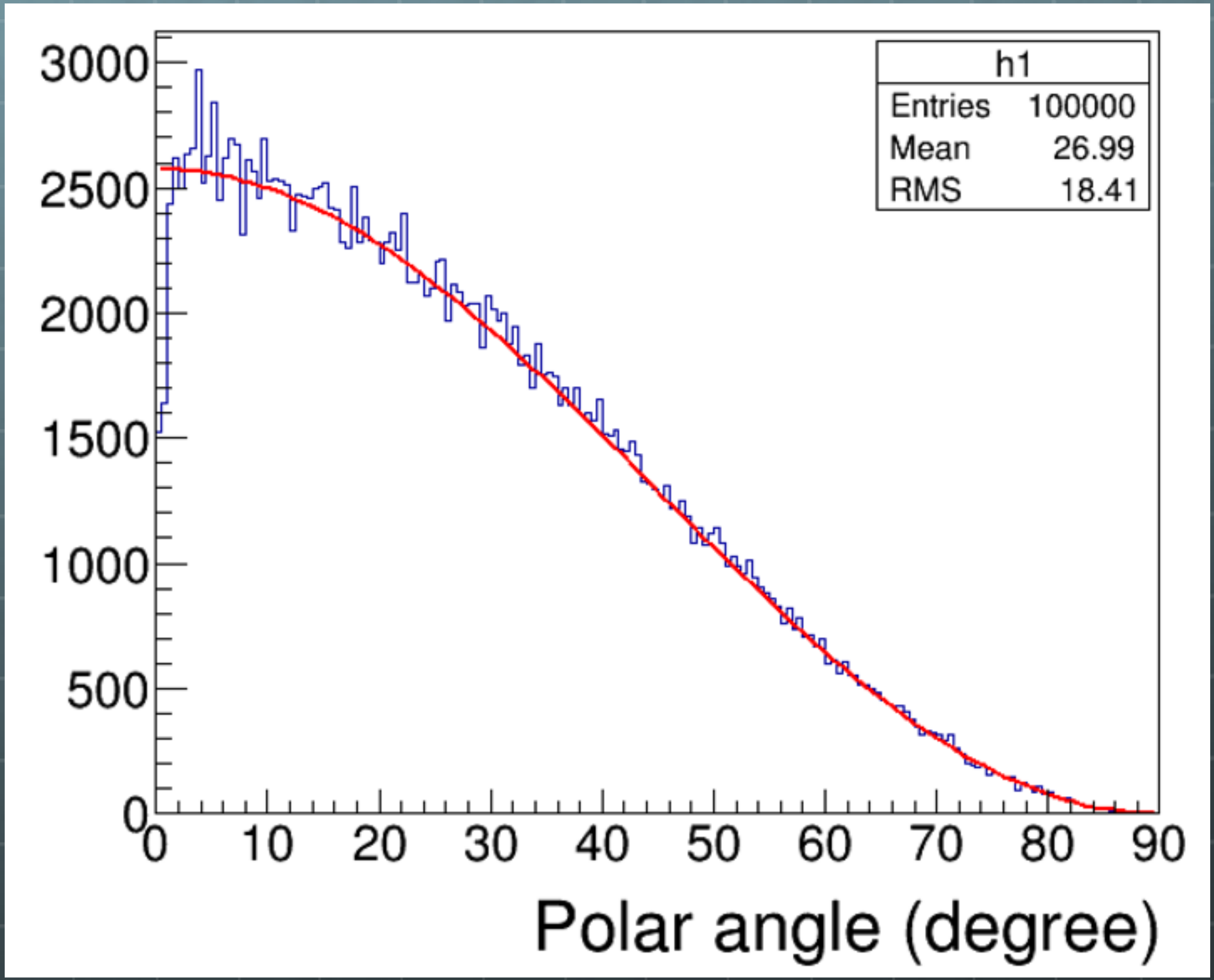
Maximum at λ=-0.05

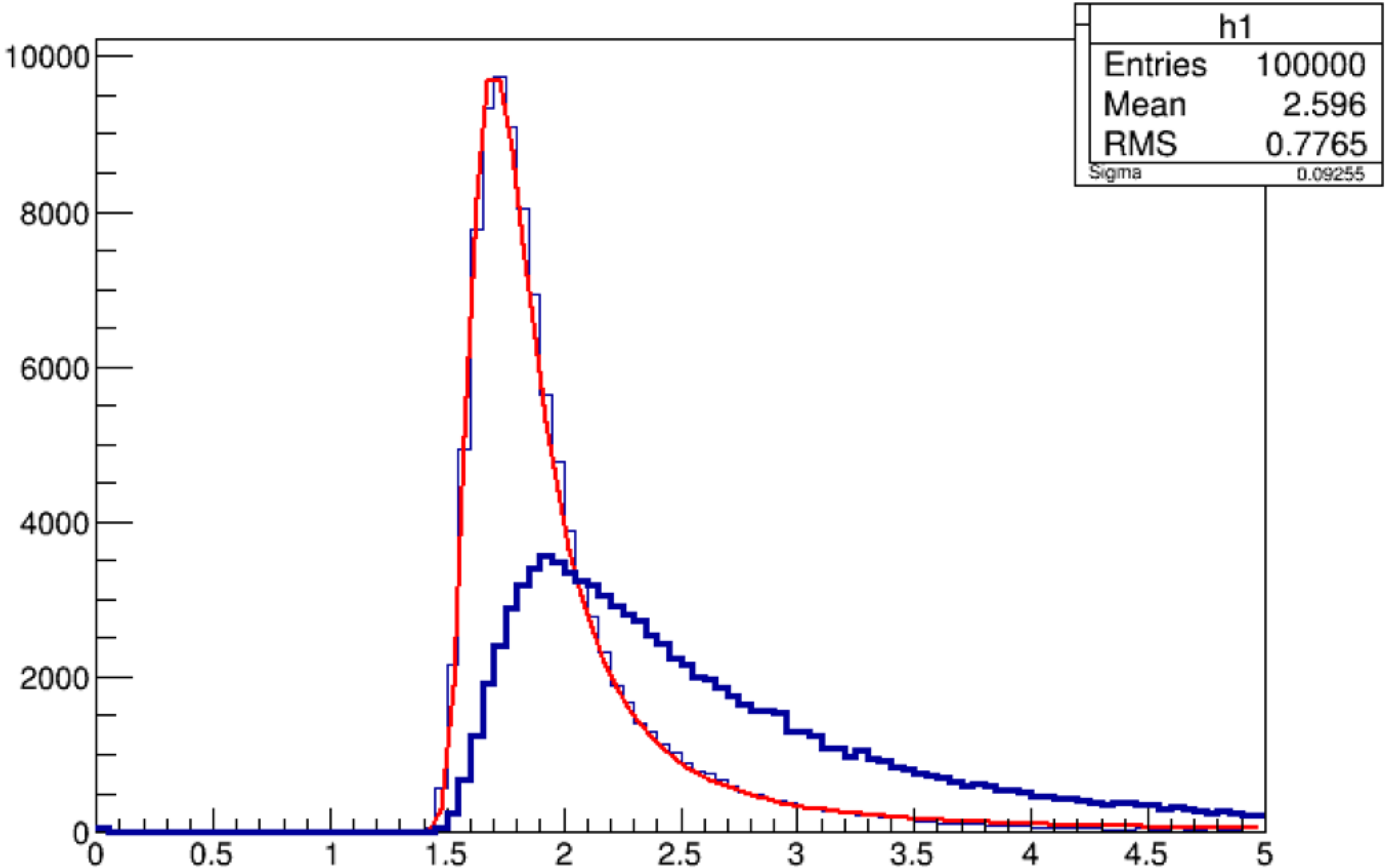
$$\lambda = \frac{\Delta - \xi(\ln\frac{\xi}{\varepsilon'} + 1 - C)}{\xi}$$

Most probable value of energy loss : $\Delta_0 = \xi(\ln\frac{\xi}{\varepsilon'} + 0.37)$

$$f(x,\Delta)d\Delta = \phi(\frac{\Delta - \Delta_0}{\xi})d(\frac{\Delta - \Delta_0}{\xi})$$


h2
Entries 100000
Mean 1.974
RMS 0.4506
χ² / ndf 2012 / 87
Constant 5.45e+04
MPV 1.719
Sigma 0.09255
10000
8000
6000
4000
2000
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5


h1
Entries 100000
Mean 26.99
RMS 18.41
3000
2500
2000
1500
1000
500
0
0 10 20 30 40 50 60 70 80 90
Polar angle (degree)

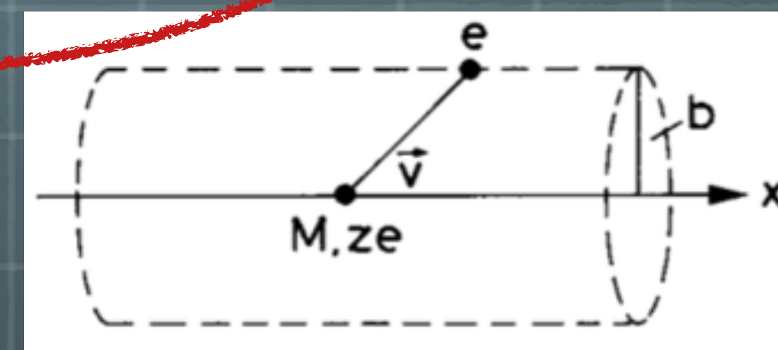

h1
Entries 100000
Mean 2.596
RMS 0.7765
Sigma 0.09255
10000
8000
6000
4000
2000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Energy Loss of $e^{\pm}$

Energy loss of $e^{\pm}$

$$\left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2W_{\max}}{I^2}-\beta^2-\frac{\delta(\beta\gamma)}{2}\right]$$

$$W_{\max} = \frac{2m_ec^2\,\beta^2\gamma^2}{1+2\gamma m_e/M+(m_e/M)^2}$$



$$\frac{1}{2}\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_ec^2)^2}+F(\tau)\right.$$

$$W_{max}=T/2$$

for e^-

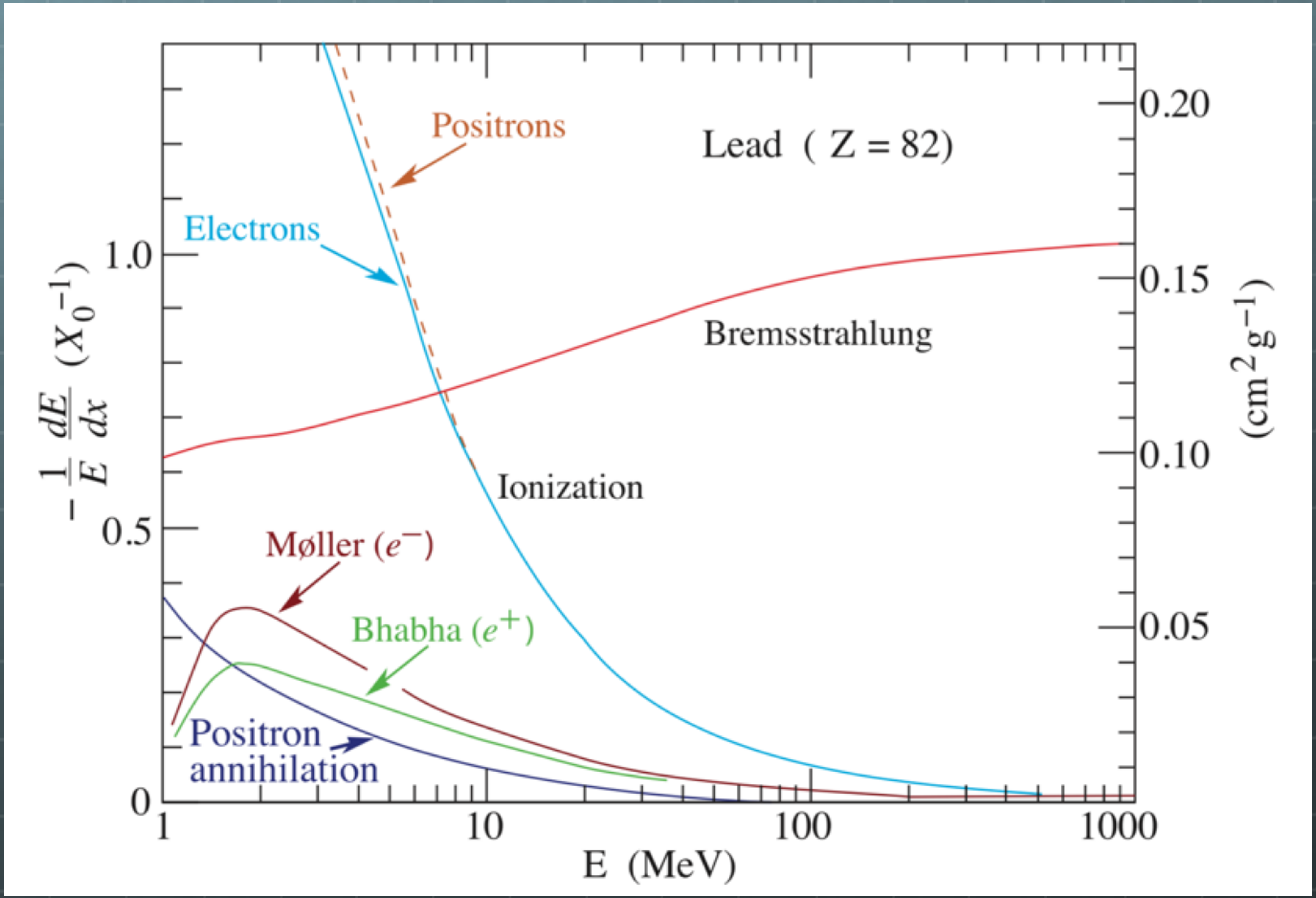
$$F(\tau)=1-\beta^2+\frac{(\tau^2/8)-(2r+1)\ln 2}{(\tau+1)^2}$$

for e^+

$$F(\tau)=2\ln 2-\frac{\beta^2}{12}\left(23+\frac{14}{\tau+2}+\frac{10}{(\tau+2)^2}+\frac{4}{(\tau+2)^3}\right)$$

$$W_{max}=T$$

Energy loss of e±

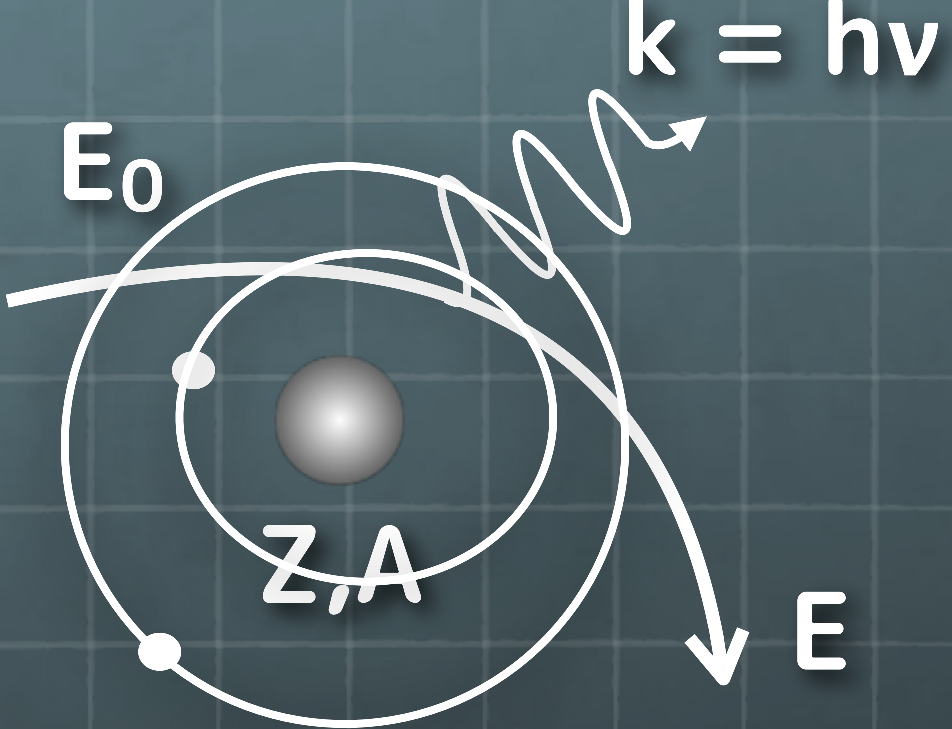


Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A z^2 \frac{Z^2}{A} (\frac{e^2}{mc^2})^2 E \ \mathbf{ln} \frac{183}{Z^{1/3}}$$

$$= K z^2 \frac{Z}{A} (\frac{Z}{m_e c^2}) \frac{\alpha}{\pi} E \cdot \mathbf{ln} \frac{183}{Z^{1/3}}$$

$$= X_0 E$$

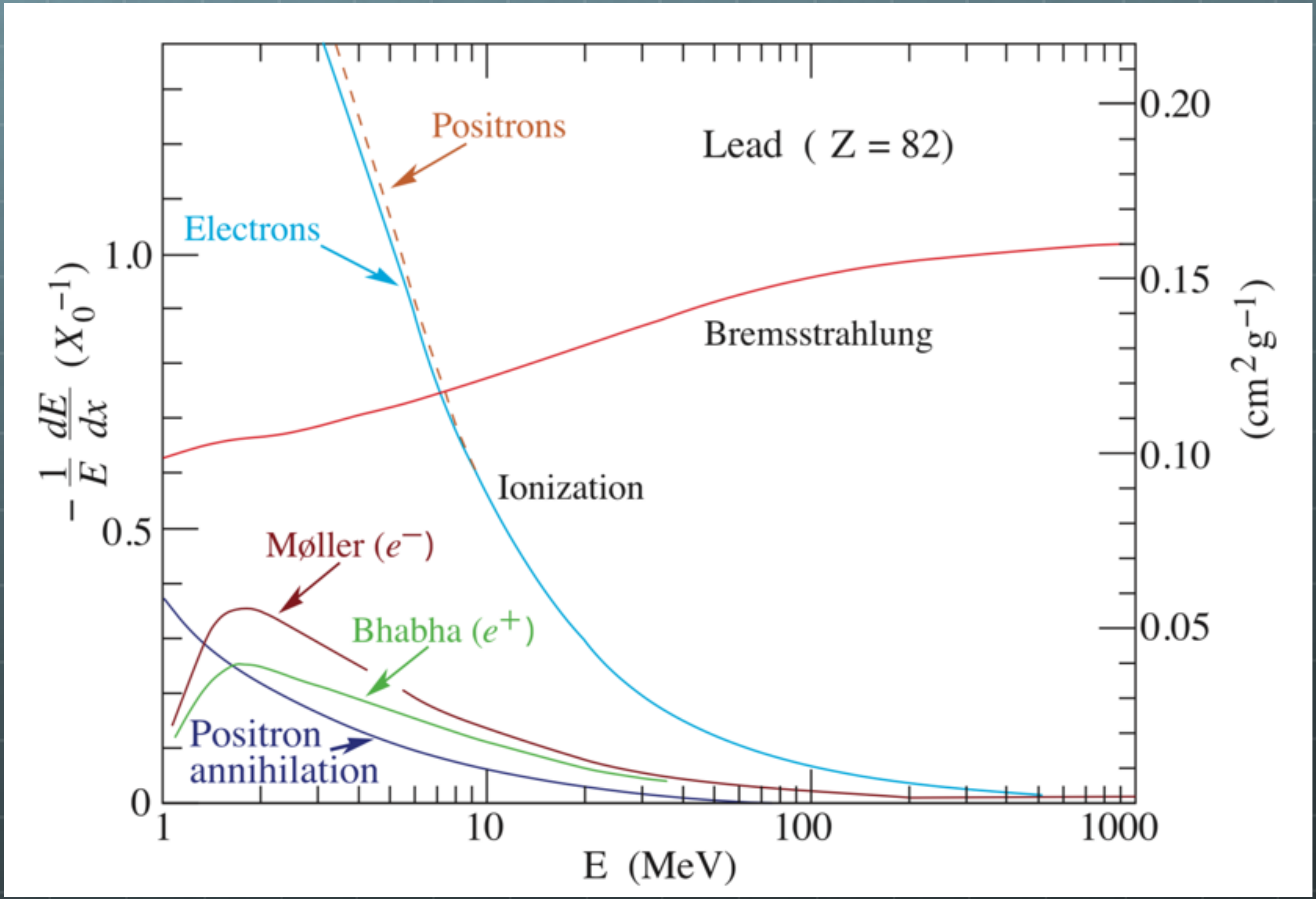


$$E(x) = E_0 e^{-x/X_0}$$

Radiation Length Xo

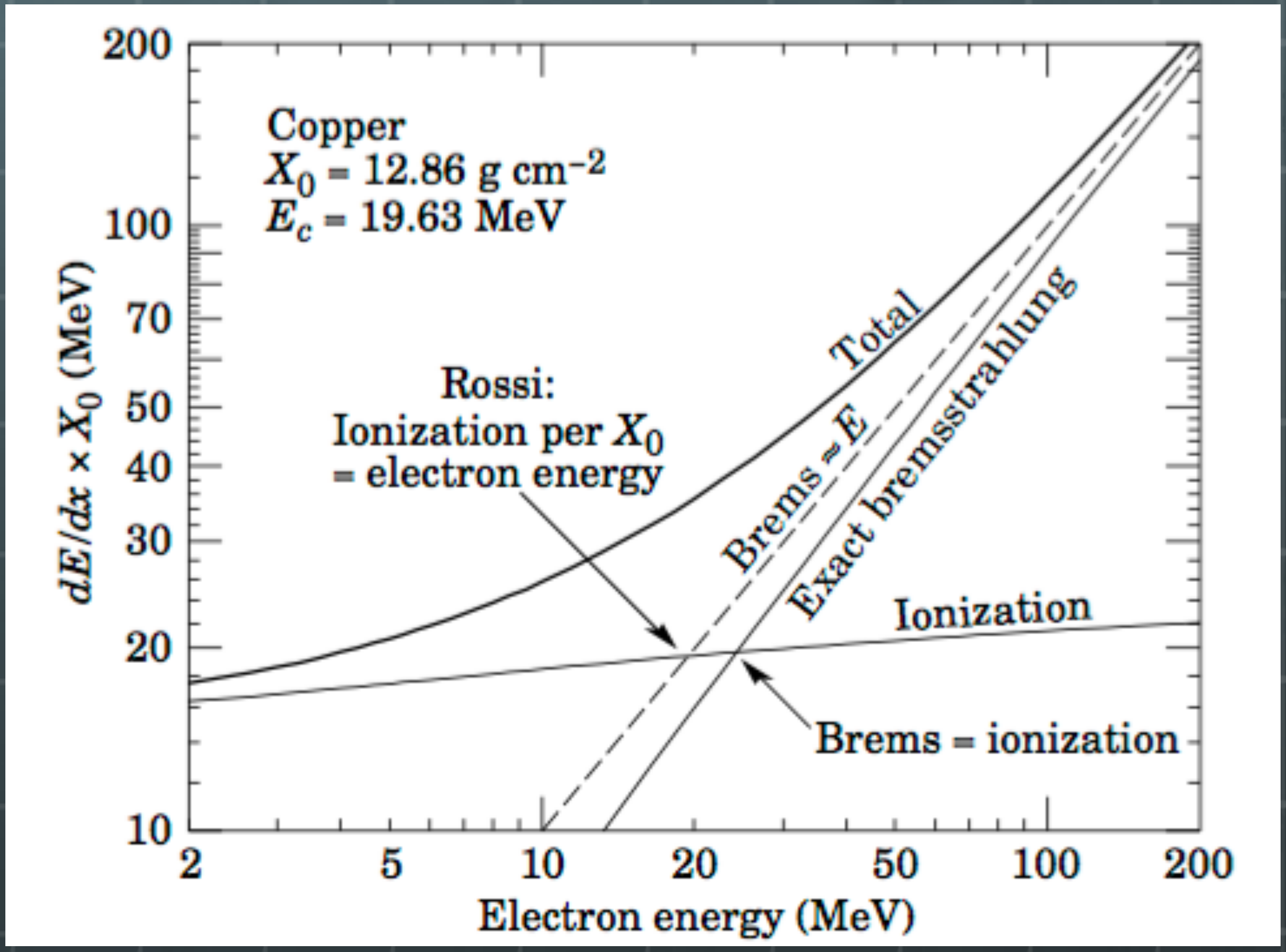
$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \mathbf{ln} \frac{183}{Z^{1/3}}}$$

Energy loss of $e^{\pm}$



Critical energy (Ec)

- Energy at which a electron losses its energy as same amount by bremsstrahlung and ionization.
- Energy at which the ionization loss per Xo is equal to the electron energy.



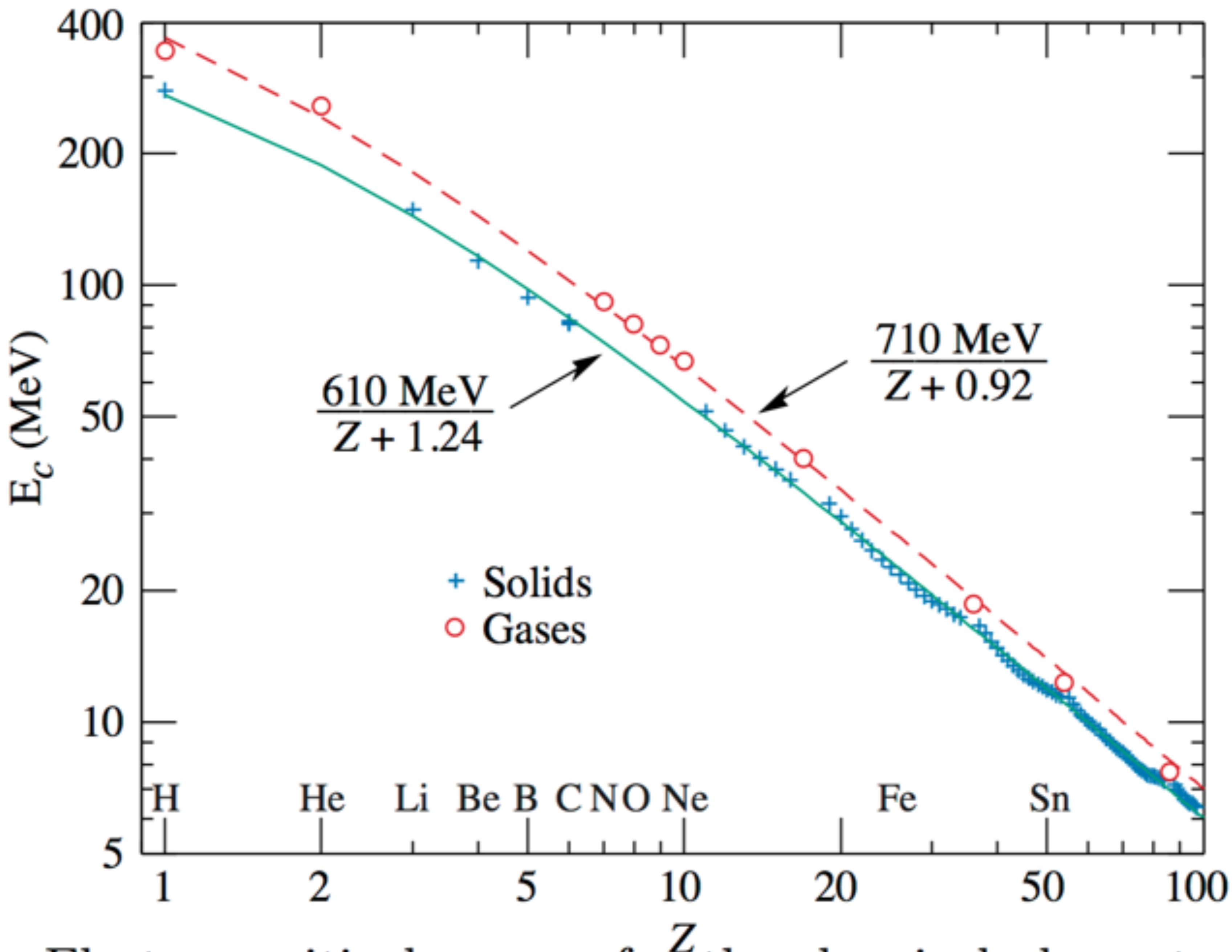
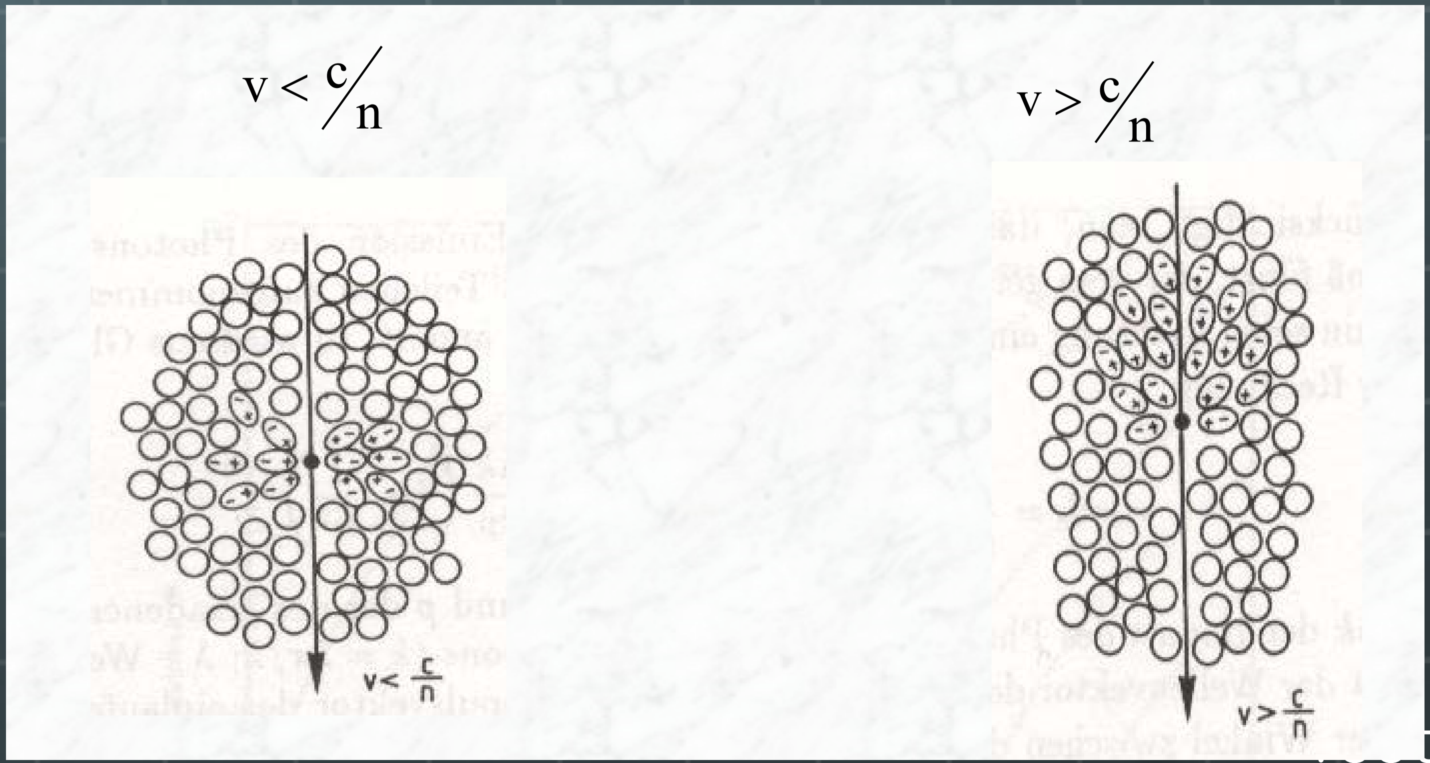
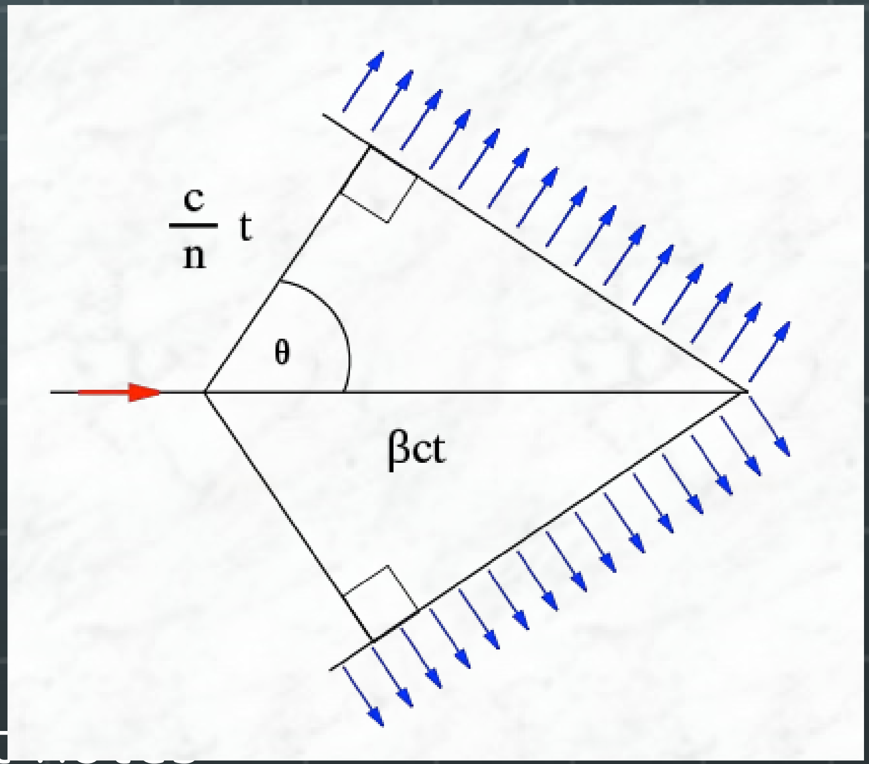


Figure 33.14: Electron critical energy for the chemical elements, using Rossi's definition [2]. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases. (Computed with code supplied by A. Fassó.)

Cherenkov radiation

- In 1934, P.A. Cherenkov observe new type of luminescence irradiating gamma rays into uranyl salt.
  - Originated by charged particle
  - Not to be radiative origin
  - Observed at a certain angle along particle direction





Refractive indices, Cherenkov threshold values

Material	$n-1$	β-Schwelle	γ-Schwelle
festes Natrium	3.22	0.24	1.029
Bleisulfit	2.91	0.26	1.034
Diamant	1.42	0.41	1.10
Zinksulfid ($ZnS(Ag)$)	1.37	0.42	1.10
Silberchlorid	1.07	0.48	1.14
Flintglas (SFS1)	0.92	0.52	1.17
Bleifluorid	0.80	0.55	1.20
Clerici-Lösung	0.69	0.59	1.24
Bleiglas	0.67	0.60	1.25
Thalliumformiat-Lösung	0.59	0.63	1.29
Szintillator	0.58	0.63	1.29
Plexiglas	0.48	0.66	1.33
Borsilikatglas	0.47	0.68	1.36
Wasser	0.33	0.75	1.52
Aerogel	0.025 - 0.075	0.93 - 0.976	4.5 - 2.7
Pentan (STP)	$1.7 \cdot 10^{-3}$	0.9983	17.2
CO_2 (STP)	$4.3 \cdot 10^{-4}$	0.9996	34.1
Luft (STP)	$2.93 \cdot 10^{-4}$	0.9997	41.2
H_2 (STP)	$1.4 \cdot 10^{-4}$	0.99986	59.8
He (STP)	$3.3 \cdot 10^{-5}$	0.99997	123

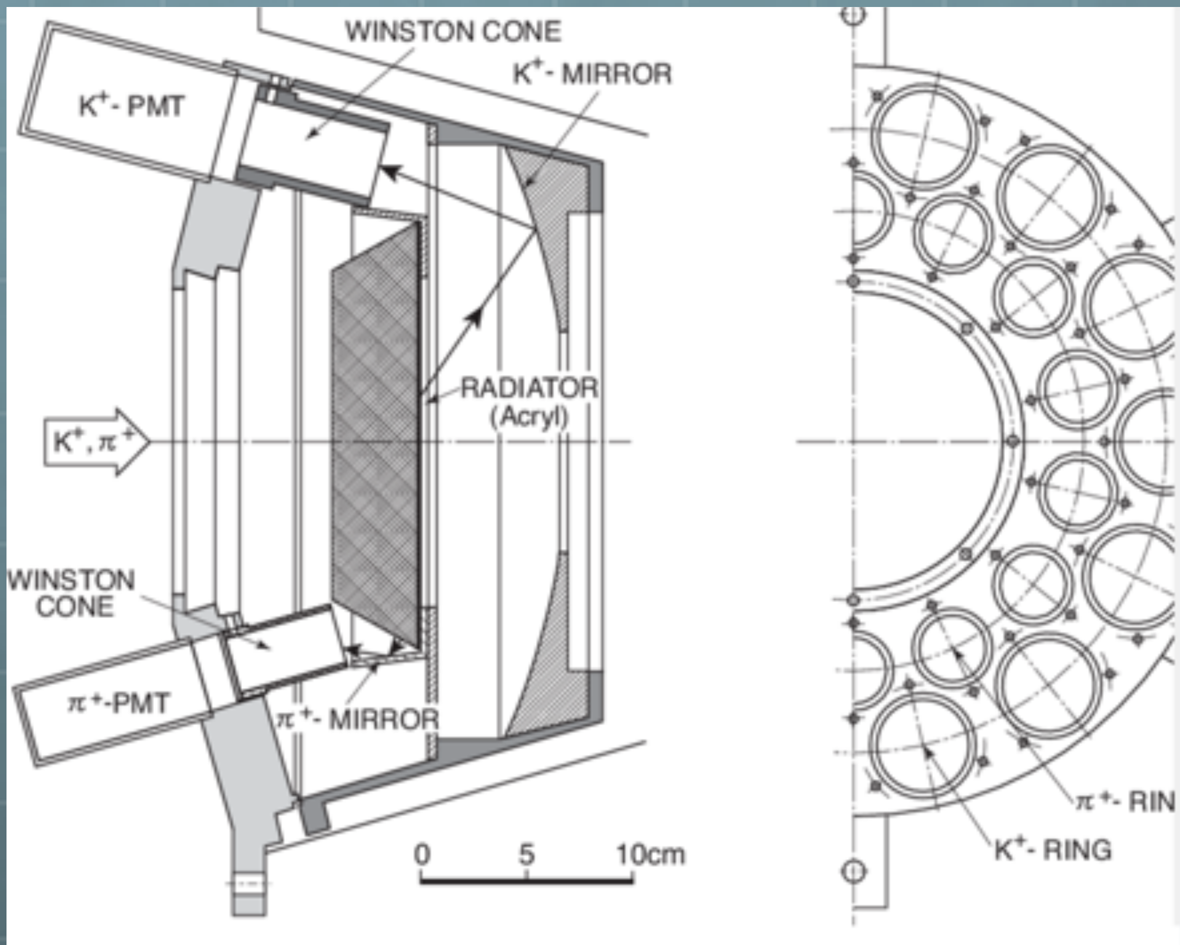
Tabelle 6.2: Cherenkov-Radiatoren [94, 32, 313]. Der Brechungsindex für Gase bezieht sich auf 0°C und 1 atm (STP). Festes Natrium ist für Wellenlängen unterhalb von 2000 Å transparent [373, 209].

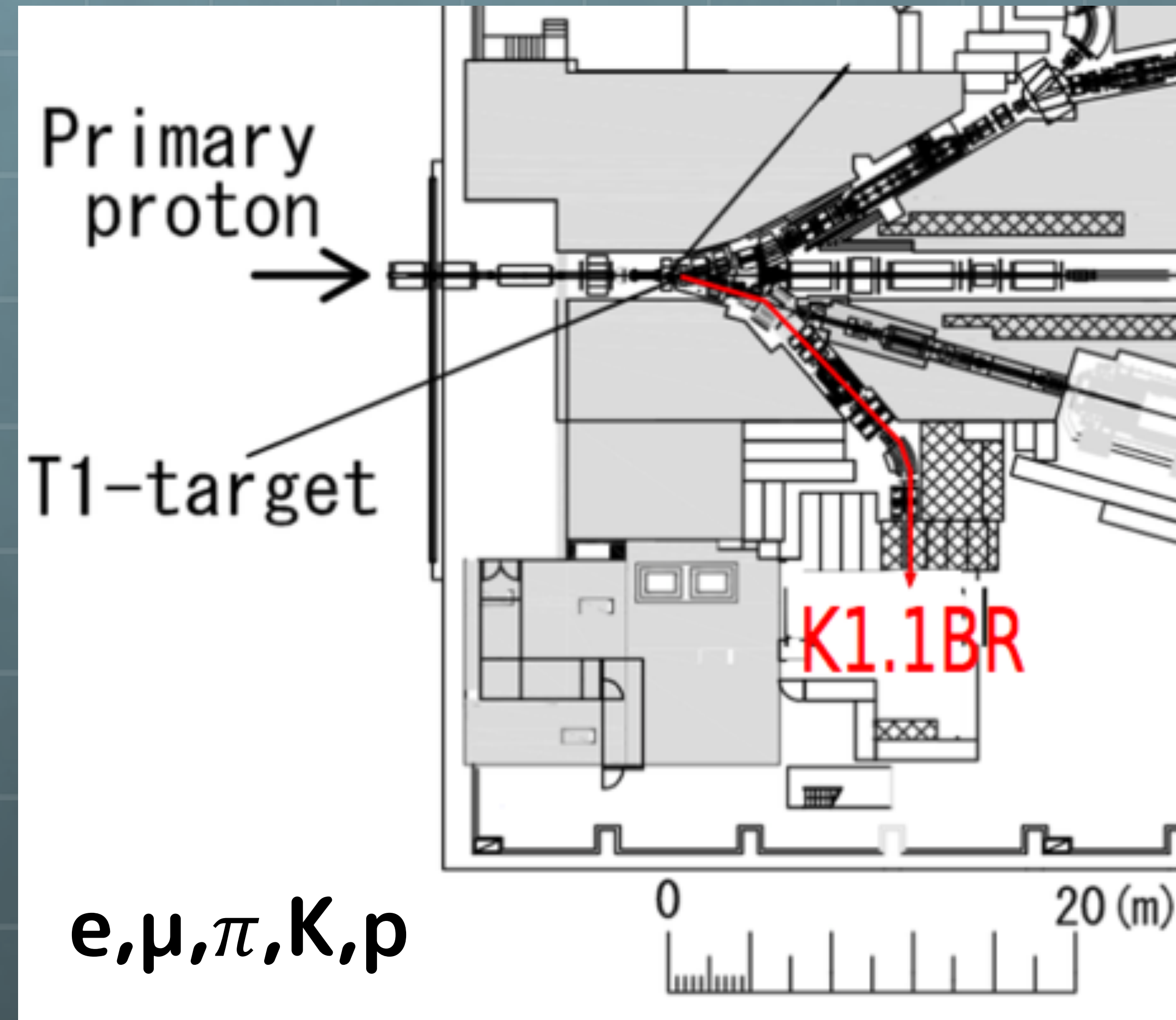
problematic: region between liquids and gases

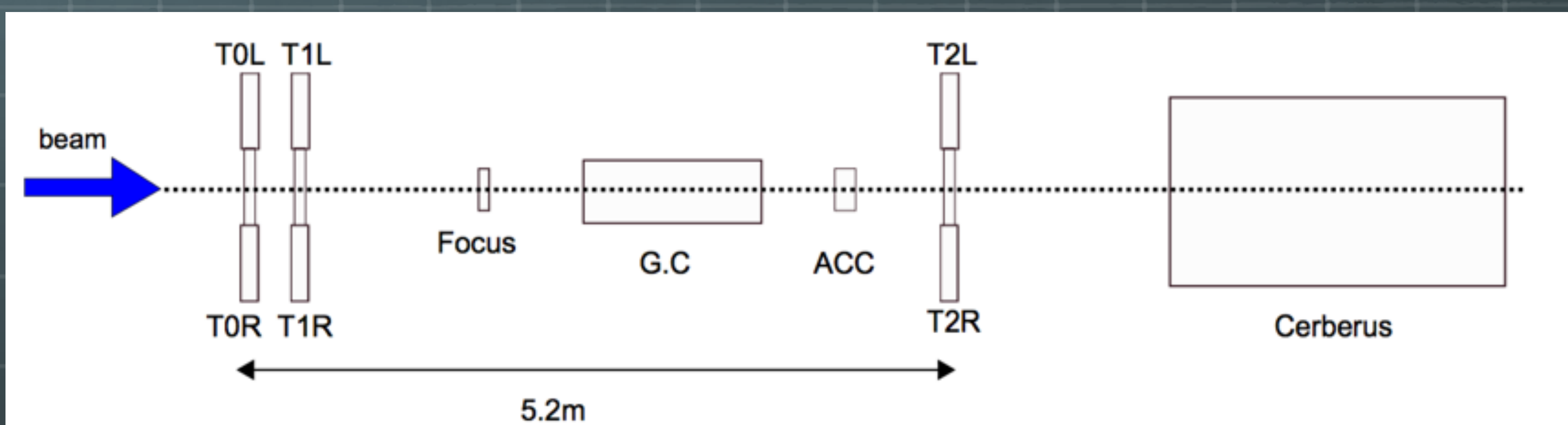
Aerogel: mixture of
m (SiO_2) + 2m (H_2O)

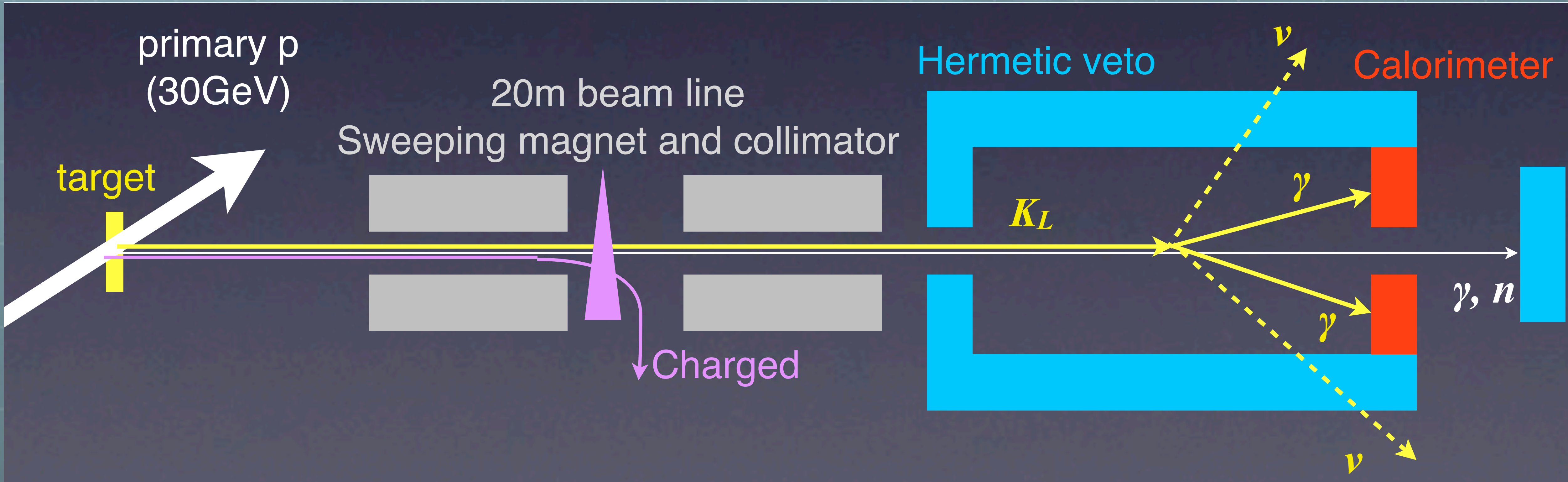
light structure with inclusions of air, bubbles with diameter < λ_{Licht}

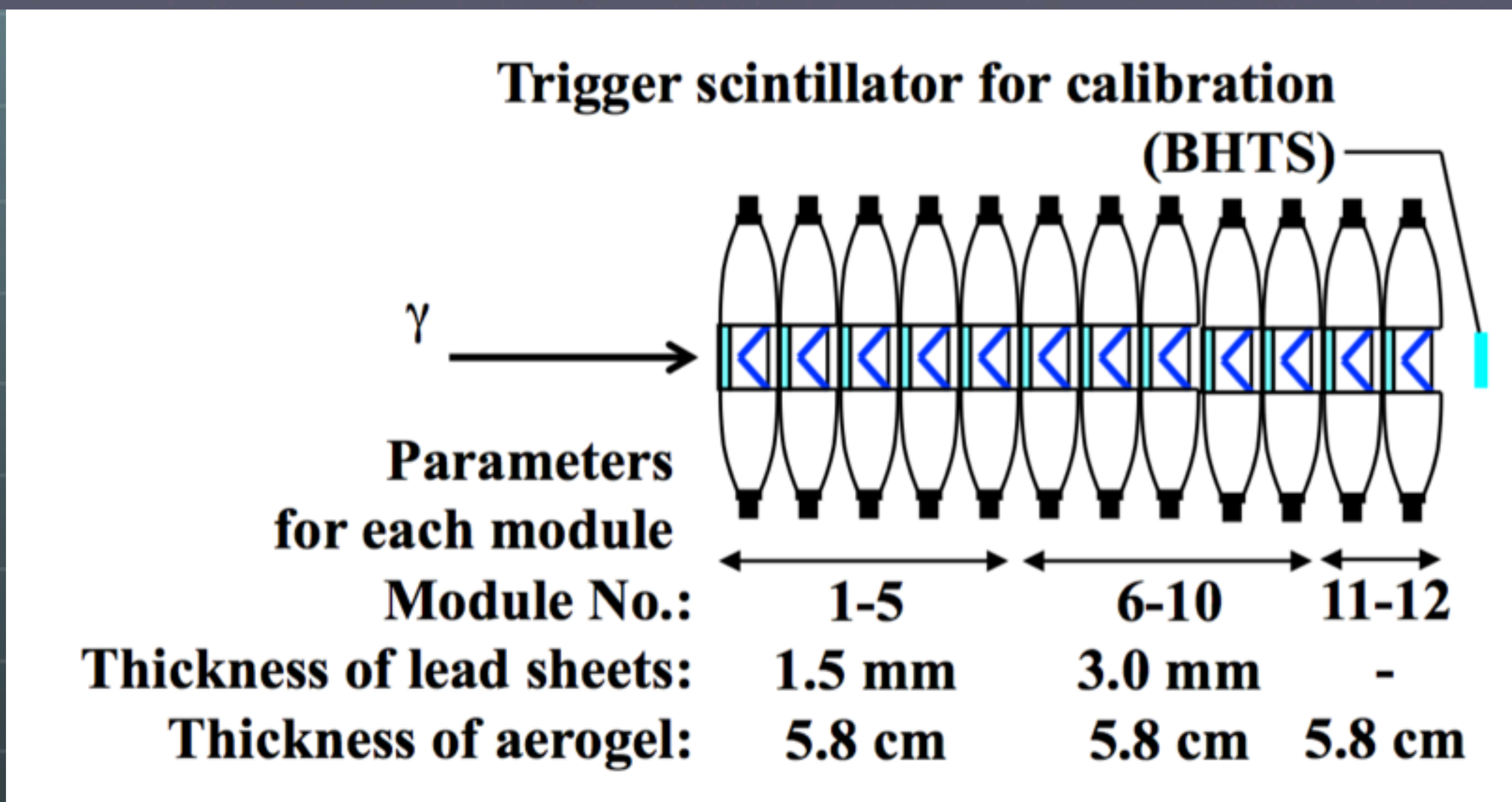
→ n: average from n_{air}, n_{SiO2}, n_{H2O}

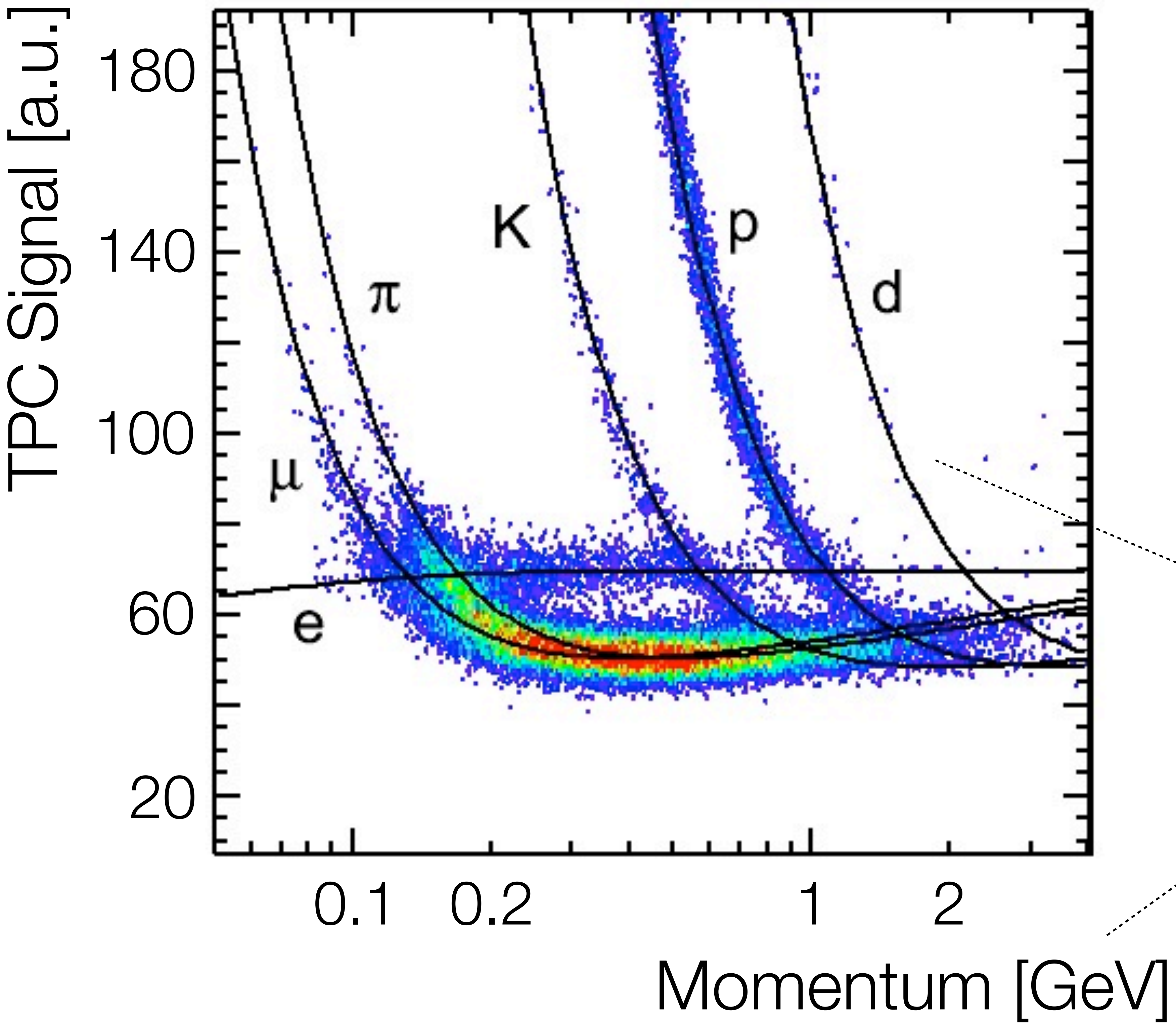

WINSTON CONE
K+- MIRROR
K+- PMT
RADIATOR (Acryl)
K+, π+
WINSTON CONE
π+-PMT
π+- MIRROR
0
5
10cm
π+- RIN
K+- RING


Primary proton
T1-target
K1.1BR
e,μ,π,K,p
0
20 (m)


T0L
T1L
T2L
beam
Focus
G.C
ACC
T0R
T1R
T2R
Cerberus
5.2m


primary p
(30GeV)
target
20m beam line
Sweeping magnet and collimator
Charged
Hermetic veto
Calorimeter
K_L
ν
γ
γ
ν
γ, n


Trigger scintillator for calibration
(BHTS)
γ
Parameters
for each module
Module No.: 1-5 6-10 11-12
Thickness of lead sheets: 1.5 mm 3.0 mm -
Thickness of aerogel: 5.8 cm 5.8 cm 5.8 cm



Measured
energy loss

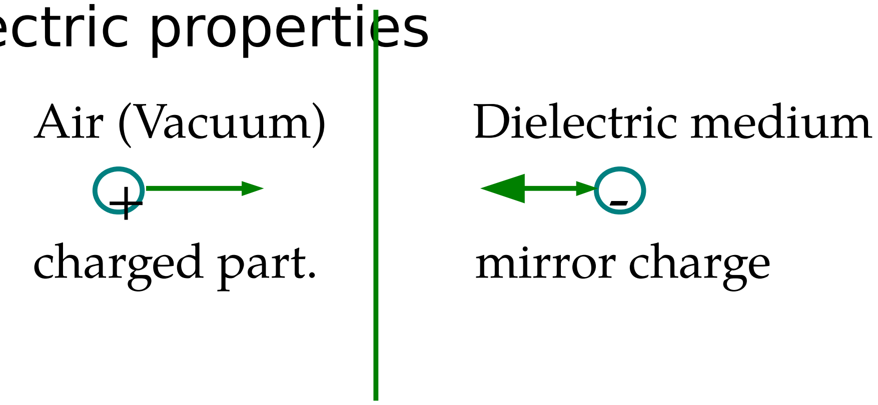
[ALICE TPC, 2009]

Bethe-Bloch

Remember:
dE/dx depends on β!

Transition radiation (TRD)

Charged particle passes through materials with different dielectric properties
→ particle forms dipole with the mirror charge
→ dipole changes with time
→ radiation



- radiated energy W proportional to the energy of particle!

$$W = \frac{1}{3} \alpha \hbar \omega_p \gamma$$

- with ω_p Plasma frequency

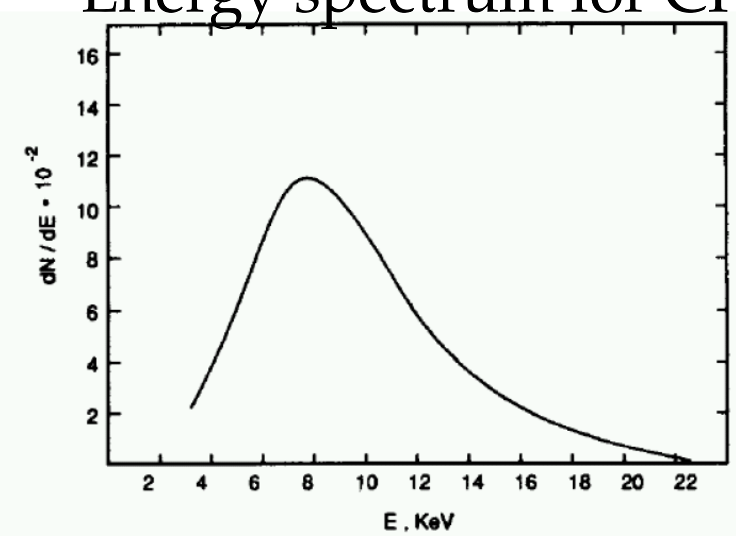
$$\omega_p = \sqrt{\frac{N_e e^2}{\epsilon_0 m_e}} \qquad \hbar \omega_p = 20\, eV$$

- only important for highly relativistic particles

- energy: keV (x-rays)

- $\theta \propto 1/\gamma$: emission in very forward direction

Energy spectrum for CH_2 foil



keV!

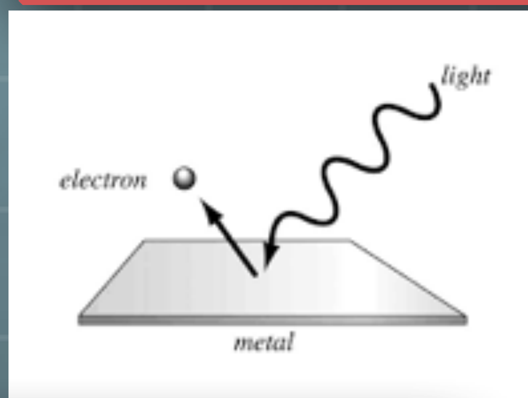
- probability for photon emission very small → many transitions (foils with gaps)
- # photons <N> ~ W / hν ~ O (α)=1/137 α fine structure constant
- energy loss due to TRD negligible for single transition
- important for particle ID at high energies, other effects used for PID ∝ β (β≃1)

  Review article: B. Dolgoshein; NIM A 326 (1993) 434

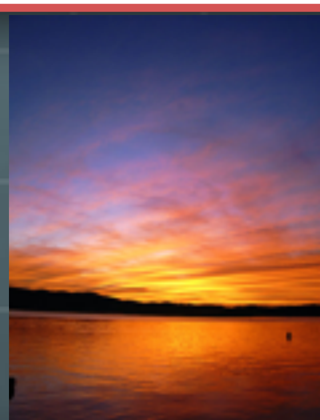
Interaction of photons

Interaction of photons

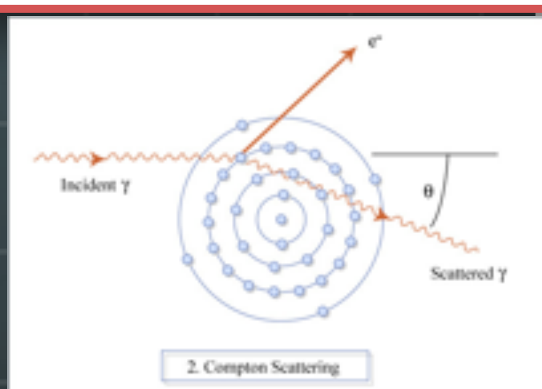
Photoelectric effect

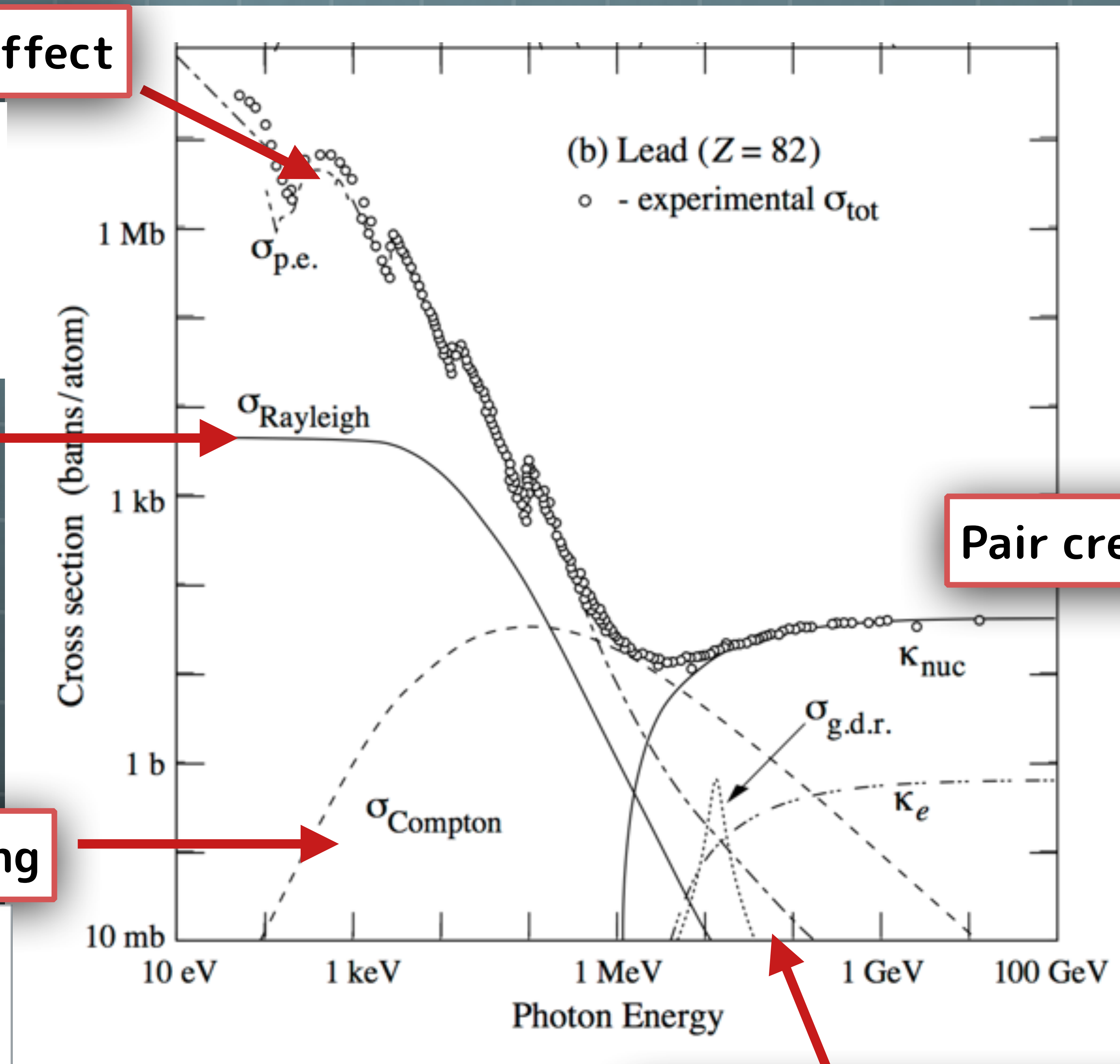


Rayleigh scattering



Compton scattering

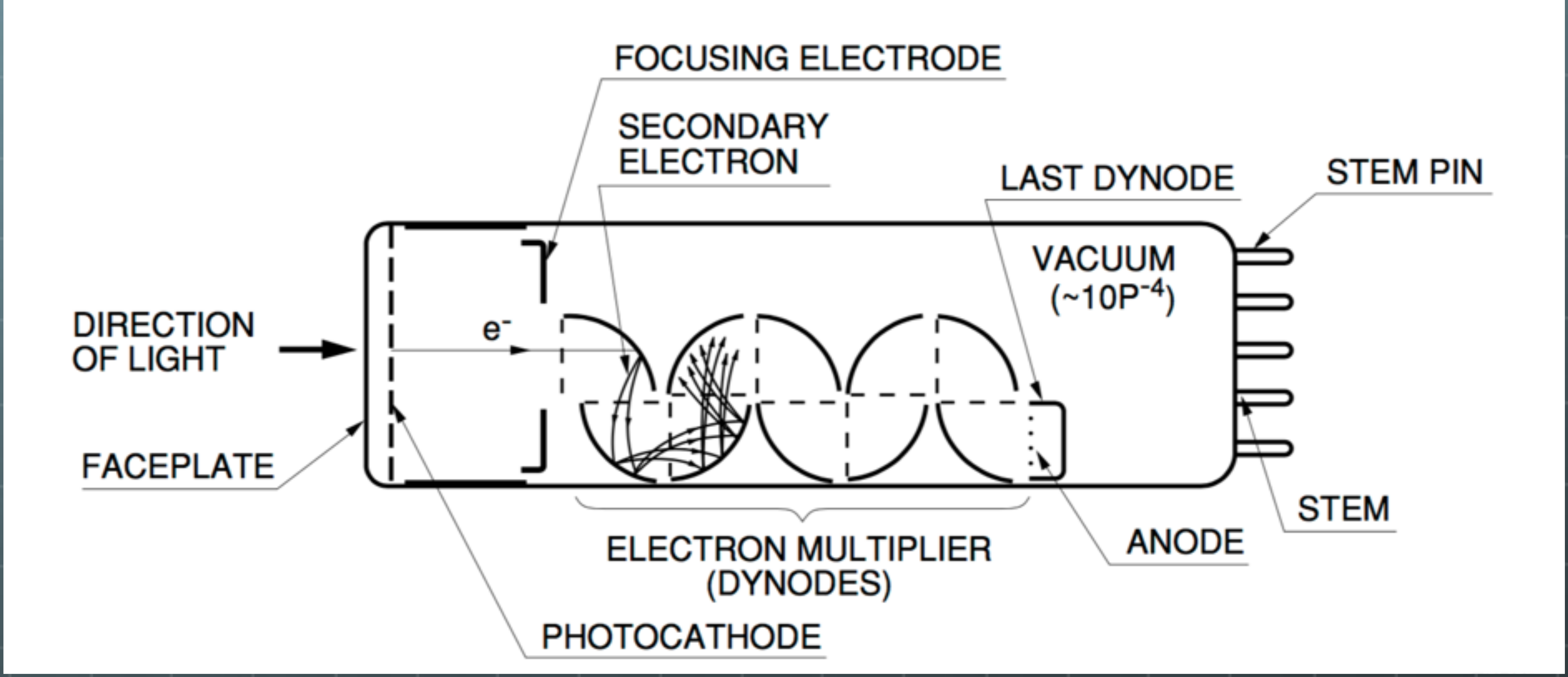




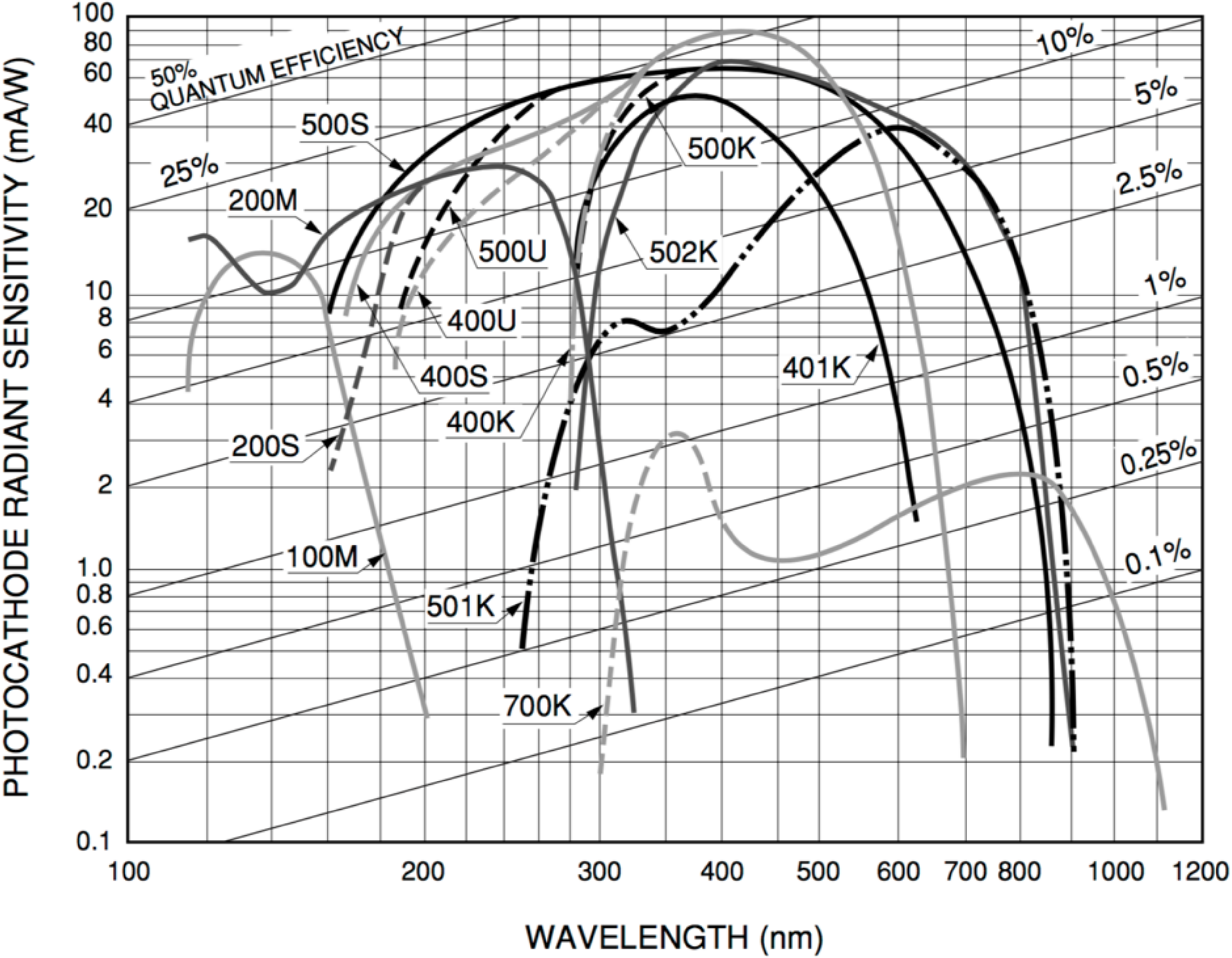
Pair creation

Photonuclear interaction

PMT


FOCUSING ELECTRODE
SECONDARY
ELECTRON
LAST DYNODE
STEM PIN
VACUUM
(~10P-4)
DIRECTION
OF LIGHT
e-
FACEPLATE
STEM
ANODE
ELECTRON MULTIPLIER
(DYNODES)
PHOTOCATHODE

Transmission Mode Photocathodes



Transmission mode photocathodes

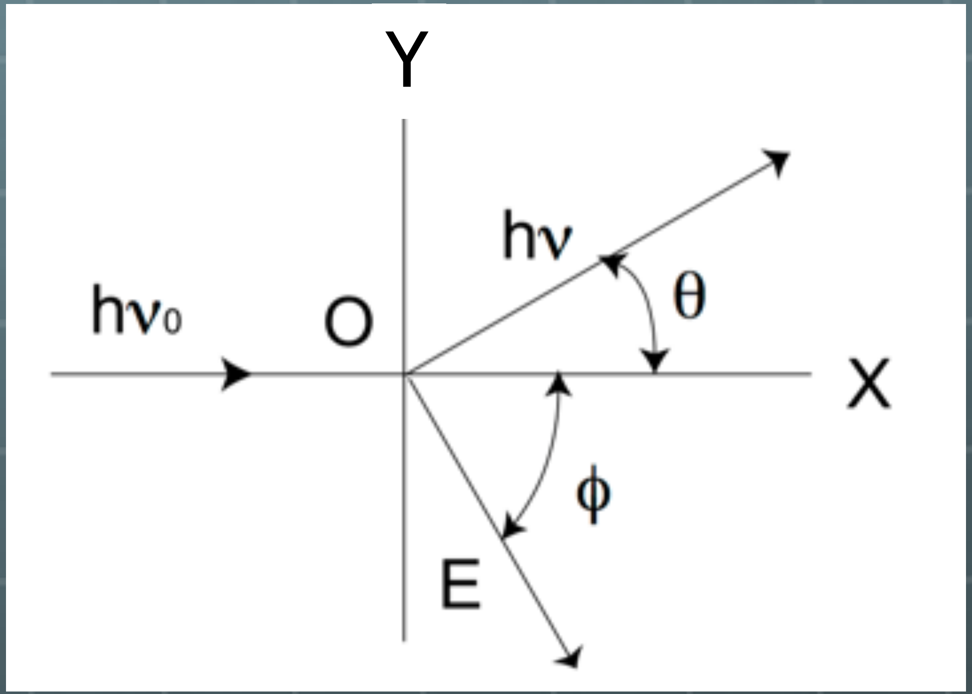
Curve Code (S number)	Photocathode Material	Window Material	Luminous Sensitivity (Typ.) (µA/lm)	Spectral Response					
				Spectral Range (nm)	Peak Wavelength				
					Radiant Sensitivity		Quantum Efficiency		
					(mA/W)	(nm)	(%)	(nm)	
100M	Cs-I	MgF_2	—	115 to 200	14	140	13	130
200S	Cs-Te	Quartz	—	160 to 320	29	240	14	210
200M	Cs-Te	MgF_2	—	115 to 320	29	240	14	200
400K	Bialkali	Borosilicate	95	300 to 650	88	420	27	390
400U	Bialkali	UV	95	185 to 650	88	420	27	390
400S	Bialkali	Quartz	95	160 to 650	88	420	27	390
401K	High temp. bialkali	Borosilicate	40	300 to 650	51	375	17	375
500K (S-20)	Multialkali	Borosilicate	150	300 to 850	64	420	20	375
500U	Multialkali	UV	150	185 to 850	64	420	25	280
500S	Multialkali	Quartz	150	160 to 850	64	420	25	280
501K (S-25)	Multialkali	Borosilicate	200	300 to 900	40	600	8	580
502K	Multialkali	Borosilicate (prism)	230	300 to 900	69	420	20	390
700K (S-1)	Ag-O-Cs	Borosilicate	20	400 to 1200	2.2	800	0.36	740
—	GaAsP(Cs)	—	—	300 to 720	180	580	40	540
—	GaAs(Cs)	—	—	380 to 890	85	800	14	760
—	InP/InGaAsP(Cs)	—	—	950 to 1400	21	1300	2.0	1000 to 1300
—	InP/InGaAs(Cs)	—	—	950 to 1700	24	1500	2.0	1000 to 1550

Compton Scattering

$$h\nu = \frac{h\nu_0}{1 + \left(\frac{h\nu_0}{m_e c^2}\right)(1 - \cos\theta)},$$

$$E = h\nu_0 - h\nu = m_e c^2 \frac{2(h\nu_0)^2 \cos^2\phi}{(h\nu_0 + m_e c^2)^2 - (h\nu_0)^2 \cos^2\phi},$$

$$\tan\phi = \frac{1}{1 + \left(\frac{h\nu_0}{m_e c^2}\right)} \cot\frac{\theta}{2},$$



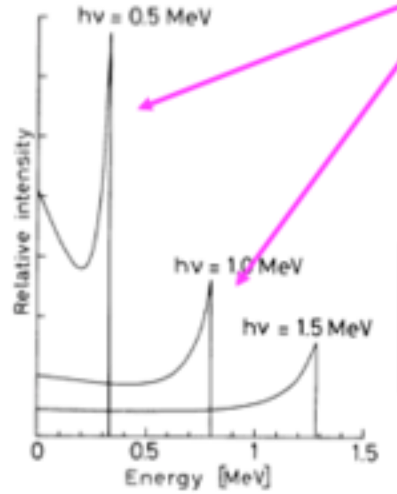
We can also calculate the recoil kinetic energy (T) spectrum of the electron:

$$\frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_e c^2 \gamma^2}\left(2 + \frac{s^2}{\gamma^2(1-s)^2} + \frac{s}{(1-s)}\left(s - \frac{2}{\gamma}\right)\right) \quad \text{with } s = T / E_{\gamma,in}$$

This cross section is strongly peaked around T_{max}:

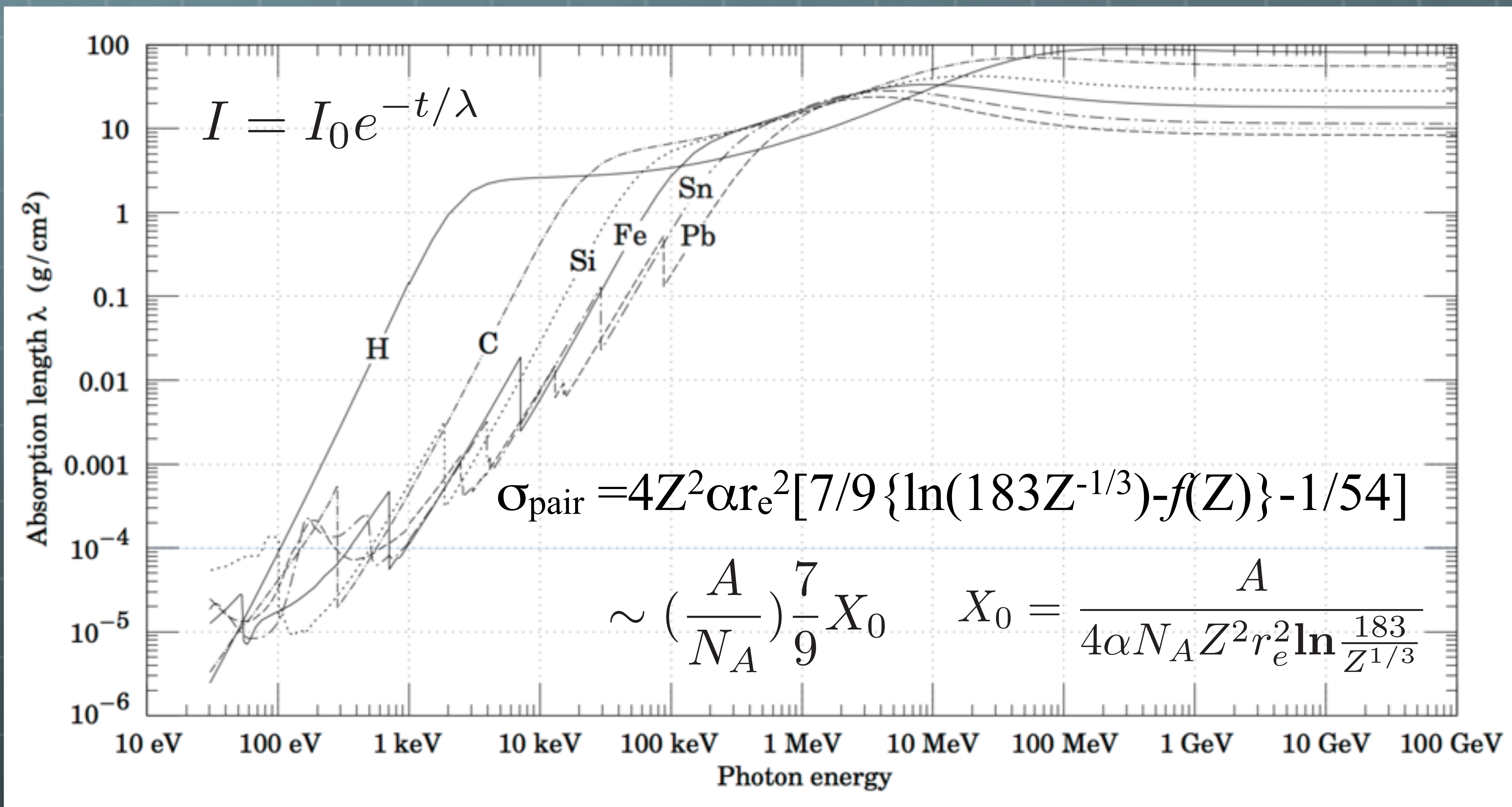
$$T_{max} = E_{\gamma,in} \frac{2\gamma}{1+2\gamma}$$

T_{max} is known as the Compton Edge



Kinetic energy distribution of Compton recoil electrons

Pair Production



$$I = I_0 e^{-t/\lambda}$$

$$\sigma_{pair} = 4Z^2\alpha r_e^2[7/9\{\ln(183Z^{-1/3})-f(Z)\}-1/54]$$

$$\sim (\frac{A}{N_A})\frac{7}{9}X_0 \qquad X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \mathbf{ln}\frac{183}{Z^{1/3}}}$$

RESOLUTION OF THE Σ^--MASS ANOMALY

Walter H. Barkas, John N. Dyer,* and Harry H. Heckman
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 29 May 1963)

- **Mass of hyperons are measured by range in the emulsion**

Table I. Observed ranges.

Process	Measured	Mean range	Number of emulsion stacks used
(1) $\Sigma^+ \to p + \pi^0$	144 p	1677 ±2 μ	5
(2) $\Sigma^+ \to n + \pi^+$	48 π^+	92.74 ±0.34 mm	2
(3) $K + p \to \Sigma^+ + \pi^-$	40 Σ^+	818.8 ±1.7 μ	6
	24 π^-	88.58 ±0.51 mm	[illegible]
(4) $K + p \to \Sigma^- + \pi^+$	94 Σ^-	708.9 ±1.5 μ	
	63 π^+	78.45 ±0.25 mm	

expected as 684 (5)

It is well known that stopping theory based on the first Born approximation fails when the particle velocity becomes comparable to the velocities of many of the electrons in the stopping material. It is perhaps not surprising that there

foot notes