모르는 것들

1.12하고 1.13의 유도방법?

Direct CP-violation

Another source of CP-violation is possible, where the CP-odd component K_2 *directly* decays into CP-even two pion systems due to CP-violation in the decay process:

$$\langle\pi\pi|\,\mathscr{H}\,|K_L\rangle \sim \langle\pi\pi|\,\mathscr{H}\,|K_2\rangle + \epsilon\,\langle\pi\pi|\,\mathscr{H}\,|K_1\rangle\,, \tag{1.11}$$

where $\mathscr{H}$ denotes Hamiltonian of weak interaction. This "direct CP-violation," as given by the first term in the right hand side of Eq. (1.11), was formalized by T. T. Wu and C. N. Yang [17] soon after the discovery of the *indirect* CP-violation. Contribution of the direct CP-violation in decay into $\pi^+\pi^-$ final states is usually denoted as ϵ' and this gives

$$\frac{\langle\pi^+\pi^-|\,\mathscr{H}\,|K_L\rangle}{\langle\pi^+\pi^-|\,\mathscr{H}\,|K_S\rangle} = \eta_{+-} \sim \epsilon + \epsilon'. \tag{1.12}$$

Due to isospin difference in the $\pi^0\pi^0$ modes, different contribution of the direct CP-violation is expected as

$$\frac{\langle\pi^0\pi^0|\,\mathscr{H}\,|K_L\rangle}{\langle\pi^0\pi^0|\,\mathscr{H}\,|K_S\rangle} = \eta_{00} \sim \epsilon - 2\epsilon'. \tag{1.13}$$

Although it is difficult to measure the small effect of the direct CP-violation only by observing the $K_L \to \pi^+\pi^-$ decay, Equations (1.12) and (1.13) allow to extract its effects by simultaneous measurements of the four decays, $K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$, $K_S \to \pi^+\pi^-$ and $K_L \to \pi^0\pi^0$ as

$$\frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)} \sim \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 \sim 1 + 6\mathrm{Re}(\epsilon'/\epsilon). \tag{1.14}$$

관련자료?

CP violation in $K^0 \to \pi\pi$

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \frac{\varepsilon'}{1+\omega/\sqrt{2}} \approx \varepsilon + \varepsilon'$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - \frac{2\varepsilon'}{1-\omega\sqrt{2}} \approx \varepsilon - 2\varepsilon'$$

$$\varepsilon = \bar{\varepsilon} + i\frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)}$$

$$\varepsilon' = \frac{i}{\sqrt{2}} \frac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)} \left[\frac{\mathrm{Im}(a_2)}{\mathrm{Re}(a_2)} - \frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)} \right] e^{i(\delta_2 - \delta_0)}$$

$$\omega = \frac{A[K_S \to \pi\pi(I=2)]}{A[K_S \to \pi\pi(I=0)]} \approx 1/22$$

$\Delta I=1/2$ "rule": hadronic amplitudes with $\Delta I > 1/2$ are smaller: $\Gamma(K^+ \to \pi^+\pi^0) \ll \Gamma(K_S \to \pi^+\pi^-)$

Inami-Lim function, K_L

Another important key of this mode is small theoretical uncertainty in branching ratio prediction. The branching ratio of the $K_L \to \pi^0 \nu\bar{\nu}$ decay is expressed as follows [24]:

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2, \tag{1.35}$$

where $\lambda_t = V_{ts}^* V_{td}$ in the CKM matrix, x_t is the square of the mass ratio of the top quark to the W boson and $x_t = m_t^2/M_W^2$, $X(x_t)$ is the Inami-Lim loop function [25] with QCD higher order corrections, and the factor κ_L includes other effects which is given as

$$\kappa_L = (2.231 \pm 0.013) \times 10^{-10} \left(\frac{\lambda}{0.225} \right)^8. \tag{1.36}$$

The loop effect, as given in the function $X(x_t)$, is reliably calculated. This is because the internal states of this decay process involves only heavy particles such as top quark, W and Z bosons as shown in Fig. 1.3 hence effects from long-distance interactions from light quarks are negligible. Although there could be an uncertainty in calculation of hadron matrix element, which is included in κ_L, it is canceled by using the well-measured branching ratio of the $K^+ \to \pi^0 e^+ \nu_e$ decay, whose matrix element is identical to that of the $K_L \to \pi^0 \nu\bar{\nu}$ decay due to isospin symmetry [26].

Taking the above discussion into account, the SM prediction of the $K_L \to \pi^0 \nu\bar{\nu}$ branching fraction with two-loop electroweak corrections is given as [27]

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = (3.00 \pm 0.30) \times 10^{-11}, \tag{1.37}$$

R_is

of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay and the $K_L \to \pi^0 \nu\bar{\nu}$ decay,

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \lesssim \frac{1}{r_{\mathrm{is}}} \frac{\tau_{K_L}}{\tau_{K^+}} \times \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) \tag{1.38}$$

$$\sim 4.3 \times \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}), \tag{1.39}$$

where τ_{K_L} and τ_{K^+} indicate the lifetimes of K_L and K^+, respectively, and $r_{\mathrm{is}} = 0.954$ is the isospin breaking factor [31]. This equation assumes only isospin symmetry and hence independent of new physics models. This limit, called "Grossman-Nir bound," is obtained from measurements of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay and new physics effects are allowed to give as large branching fraction as this bound at maximum. The branching fraction of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay was obtained by the E787/E949 experiment in the Brookhaven National Laboratory (BNL) [32] as

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-11}. \tag{1.40}$$