$K_L \to \pi^0 \nu\nu$ Decay in the SM and Grossman-Nir limit

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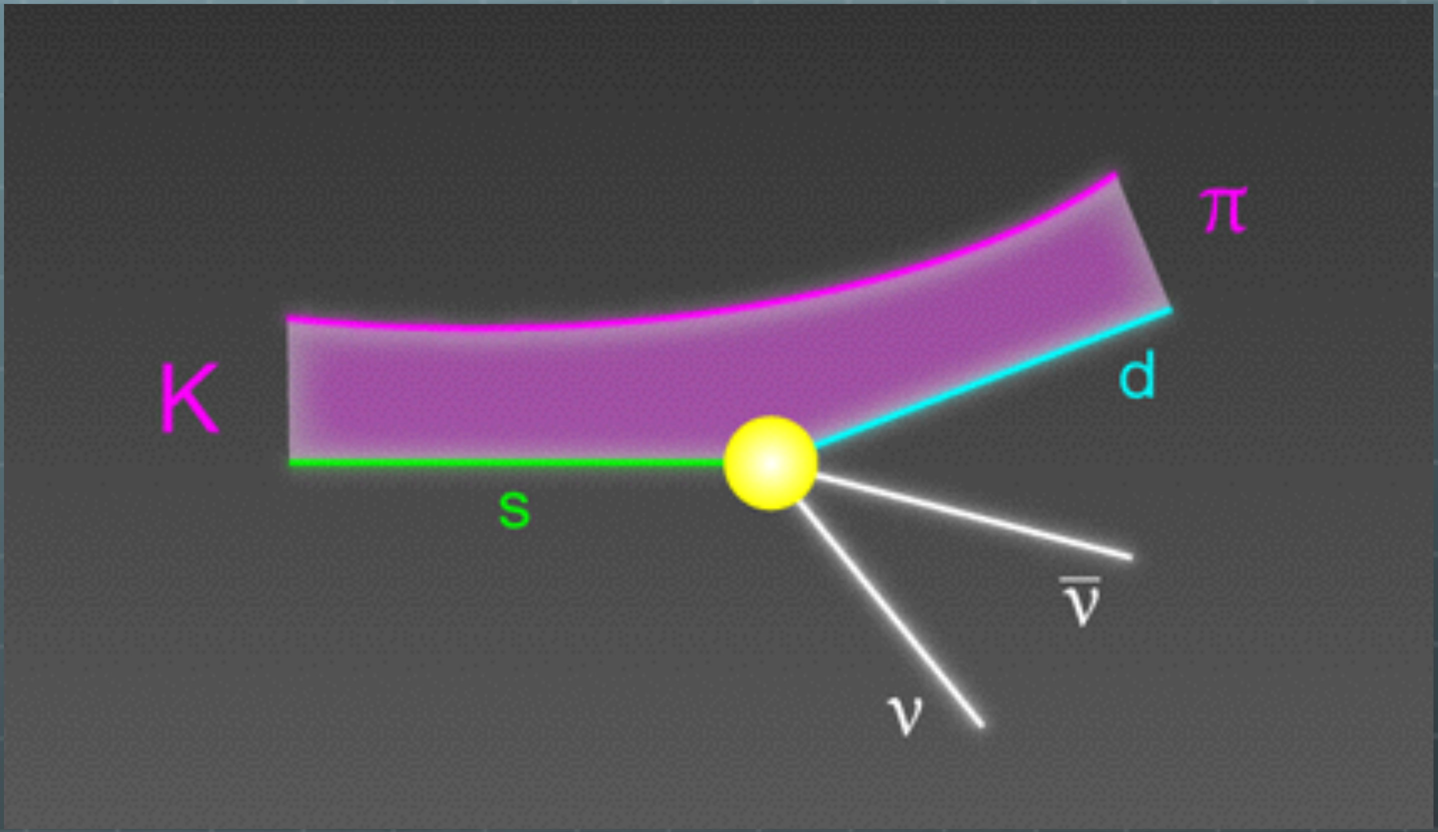
IPNS/KEK

KOTO-Korea Meeting at Korea Univ. 30th, Sep. 2016

$K_L \rightarrow \pi^0 \nu\bar{\nu}$ & $K^+ \rightarrow \pi^+ \nu\bar{\nu}$



Decay

- Transition from a (unstable) state to other (stable) state
- Conservation : Energy-momentum, charge, CPT, etc.
- Key observation : decay probability

Observable

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar} \cdot \int d\rho_f \cdot \mid M_{i\to f} \mid^2$$

Kinematics
How many momentum state
for given energy
$K_S \to \pi^0\pi^0$ **v.s.** $K_L \to \pi^0\pi^0\pi^0$

Dynamics
For given process to consider
Nature of (mixed) interaction
Coupling constants, mixing
etc.

Operator Production Expansion

$$A(M \to F) = \langle F|\mathcal{H}_{eff}|M\rangle = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle F|Q_i(\mu)|M\rangle$$

Operator Product Expansion, Renormalization Group and Weak Decays *

Andrzej J. Buras

arXiv:hep-ph/9901409v1 26 Jan 1999

PHYSICAL REVIEW VOLUME 179, NUMBER 5 25 MARCH 1969

Non-Lagrangian Models of Current Algebra*

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(Received 25 November 1968)

An alternative is proposed to specific Lagrangian models of current algebra. In this alternative there are no explicit canonical fields, and operator products at the same point [say, $j_\mu(x)j^\mu(x)$] have no meaning. Instead, it is assumed that scale invariance is a broken symmetry of strong interactions, as proposed by Kastrup and Mack. Also, a generalization of equal-time commutators is assumed: Operator products at short distances have expansions involving local fields multiplying singular functions. It is assumed that the dominant fields are the $SU(3)\times SU(3)$ currents and the $SU(3)\times SU(3)$ multiplet containing the pion field. It is assumed that the pion field scales like a field of dimension Δ, where Δ is unspecified within the range $1\leq\Delta<4$; the value of Δ is a consequence of renormalization. These hypotheses imply several qualitative predictions: The second Weinberg sum rule does not hold for the difference of the K^* and axial-K^* propagators, even for exact $SU(2)\times SU(2)$; electromagnetic corrections require one subtraction proportional to the $I=1$, $I_z=0$ σ field; $\eta\to 3\pi$ and $\pi_0\to 2\gamma$ are allowed by current algebra. Octet dominance of nonleptonic weak processes can be understood, and a new form of superconvergence relation is deduced as a consequence. A generalization of the Bjorken limit is proposed.

Amplitude for a decay of a given meson M into a final state F is given as

$$A(M \to F) = \langle F|\mathcal{H}_{eff}|M\rangle = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle F|Q_i(\mu)|M\rangle$$

G_F Fermi Constant : Universal gauge coupling of the Weak int.

V_{CKM} CKM Matrix Element : Quark mixing

$C_i(\mu)$ Wilson Coefficient : Short distance (perturbative) contribution

$< F|Q_i(\mu)|M >$ Hadronic matrix elements : Long distance (non-perturbative)

$$M : K_L \longrightarrow F : \pi^0 \nu \bar{\nu}$$

Branching ratio

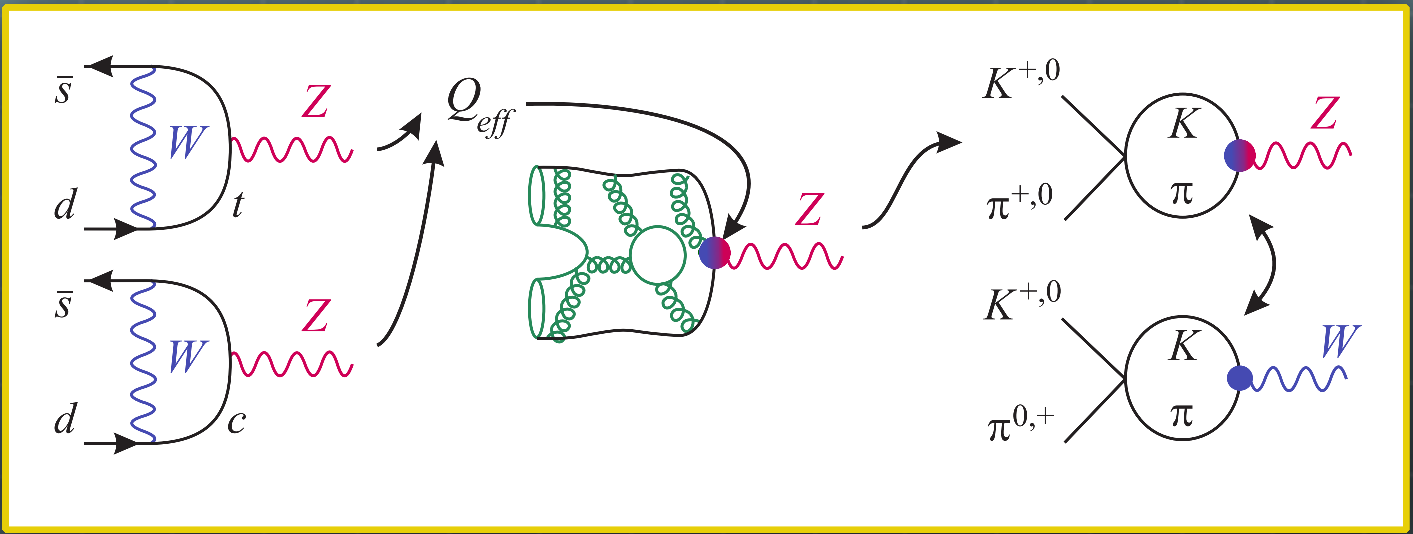
$$\mathbf{Br}(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L (\frac{\mathbf{Im}\lambda_t}{\lambda^5} X(x_t))^2$$

$\lambda_t = V_{ts}^* V_{td}$ ← **From measured values**

$\lambda = 0.2252(9)$ **Precise measured Cabibbo angle**

$$\kappa_L = 2.231 \pm 0.013 \cdot 10^{-10} [\frac{\lambda}{0.225}]$$

Hadronic matrix elements



K→πνν

K→πlν

C. Smith, arXiv:1409.6162

Branching ratio

$$\mathbf{Br}(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L (\frac{\mathbf{Im}\lambda_t}{\lambda^5} X(x_t))^2$$

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Long distance contribution

$X(x_t)$ **Lim-Inami function**
Short distance contribution

To get the matrix-elements of these operators, and to carry out the phase-space integration. parametrization wit Ke3 decay

Progress of Theoretical Physics, Vol. 65, No. 1, January 1981

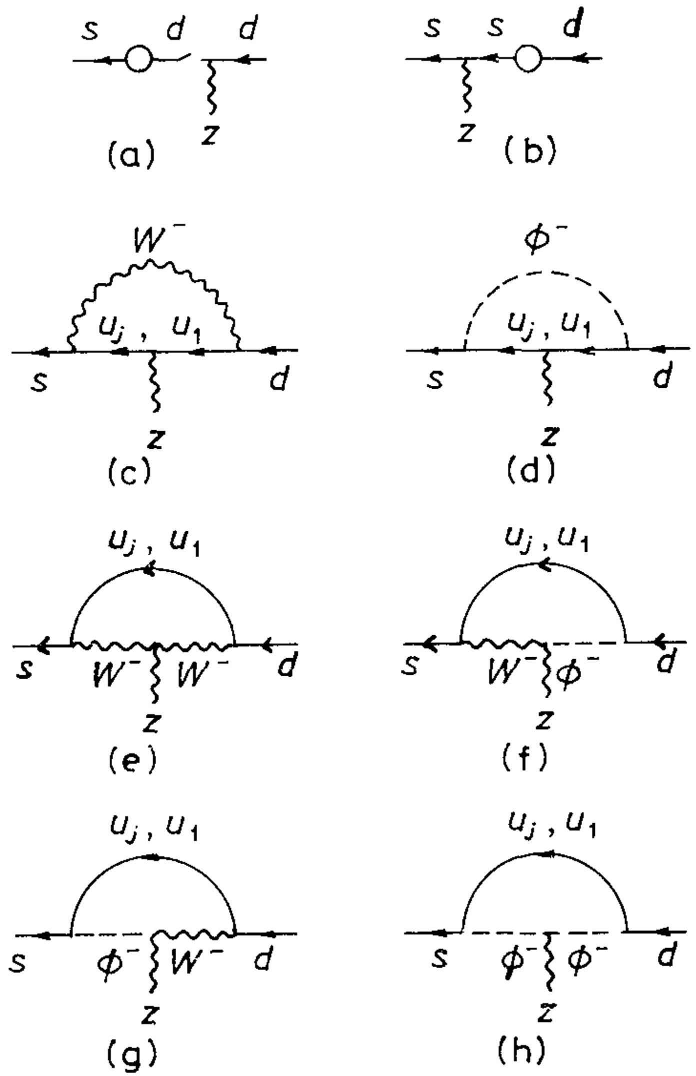
Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$

TAKEO INAMI and C. S. LIM

Institute of Physics, University of Tokyo, Komaba, Tokyo 153

(Received October 13, 1980)

We investigate potentially important effects due to the existence of superheavy quarks and leptons of the sequential type in higher-order weak processes at low energies. The second-order $\Delta S \neq 0$ neutral-current processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and K_L-K_S mass difference are analysed allowing for fermions of masses comparable to or larger than the weak-boson mass in the Kobayashi-Maskawa scheme and in the general sequential scheme with an arbitrary number of generations. Possible connection between heavy-quark masses and light-heavy quark mixing are also examined. The requirement that the rare decay processes such as $K_L \to \mu\bar{\mu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$ be absent up to order αG_F yields a rather stringent bound on the magnitude of light-heavy quark mixing: Such mixing has to be less than m_W / m_{Quark} times a factor much smaller than unity.



$$\Gamma_Z \equiv \sum_{i=1}^{h} \Gamma^{(i)} = \frac{1}{4} x_j - \frac{3}{8} \frac{1}{x_j - 1} + \frac{3}{8} \frac{2x_j^2 - x_j}{(x_j - 1)^2} \ln x_j + \gamma(x_j, \xi) - (x_j \to x_1).$$

$$x_j \equiv m_{uj}^2 / m_W{}^2$$

The $K_L \to \mu^+\mu^-$ decay and the $n\lambda Z$-vertex in the Weinberg–Salam model

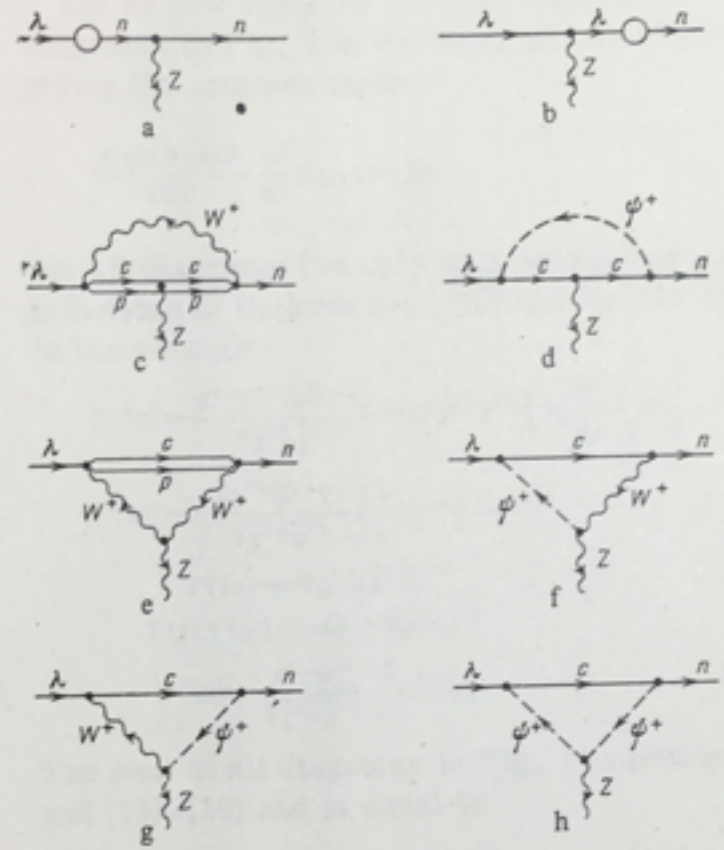
M. B. Voloshin

Moscow Physico-Technical Institute

(Submitted November 19, 1975)
Yad. Fiz. **24**, 810–819 (October 1976)

The $n\lambda Z$ vertex in the Weinberg–Salam model is calculated using two independent methods in the one-loop approximation for free quarks. The results of these calculations are used to estimate the $K_L \to \mu^+\mu^-$ decay amplitude and the mass of the c quark.

PACS numbers: 13.20.Eb, 12.30.Cx



Determination of the Br.
- Using tree-level observables -

$$\mathbf{Br}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t)\right)^2$$

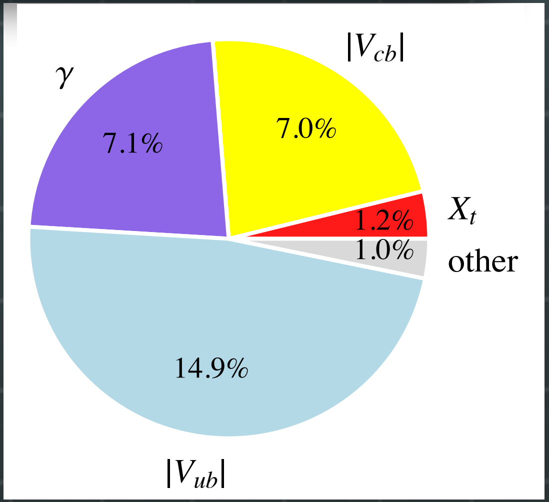
$$X(x_t) = 1.481 \pm 0.005_{th} \pm 0.008_{exp}$$

$$\mathrm{Im}\lambda_t = |V_{ub}||V_{cb}|\sin\gamma$$

$$|V_{ub}|_{\rm avg} = (3.88 \pm 0.29) \times 10^{-3}, \qquad |V_{cb}|_{\rm avg} = (40.7 \pm 1.4) \times 10^{-3}$$

$$\gamma = (73.2^{+6.3}_{-7.0})^\circ$$

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}.$$



A. Buras, arXiv:1503.02693

Determination of the Br.
- Using loop-level observables -

	$\{\lvert\varepsilon_K\rvert, \Delta M_d/\Delta M_s, S_{\psi K_S}\}_{\rm SM}$	$\{\Delta M_d, \Delta M_s, S_{\psi K_S}\}_{\rm SM}$	$\{\lvert\varepsilon_K\rvert, \Delta M_d, \Delta M_s, S_{\psi K_S}\}_{\rm SM}$
$\lvert V_{cb}\rvert$ $[10^{-3}]$	$42.59^{+1.41}_{-1.26}$	$41.30^{+2.65}_{-2.47}$	$42.35^{+1.25}_{-1.13}$
$\lvert V_{ub}\rvert$ $[10^{-3}]$	$3.62^{+0.15}_{-0.14}$	$3.51^{+0.27}_{-0.25}$	$3.61^{+0.15}_{-0.14}$
$\lvert V_{td}\rvert$ $[10^{-3}]$	$8.96^{+0.28}_{-0.28}$	$8.68^{+0.66}_{-0.62}$	$8.95^{+0.27}_{-0.28}$
$\lvert V_{ts}\rvert$ $[10^{-3}]$	$41.79^{+1.43}_{-1.27}$	$40.52^{+2.60}_{-2.42}$	$41.55^{+1.27}_{-1.14}$
$\mathcal{B}(K^+ \to \pi^+\nu\bar\nu)$ $[10^{-11}]$	$9.18^{+0.79}_{-0.71}$	$8.39^{+1.76}_{-1.41}$	$9.08^{+0.74}_{-0.68}$
$\mathcal{B}(K_L \to \pi^0\nu\bar\nu)$ $[10^{-11}]$	$3.01^{+0.33}_{-0.29}$	$2.66^{+0.84}_{-0.63}$	$2.98^{+0.32}_{-0.28}$
$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$	$3.69^{+0.30}_{-0.26}$	$3.46^{+0.49}_{-0.43}$	$3.64^{+0.27}_{-0.24}$
$\mathcal{B}(B_d \to \mu^+\mu^-)$ $[10^{-10}]$	$1.09^{+0.08}_{-0.08}$	$1.02^{+0.17}_{-0.15}$	$1.09^{+0.08}_{-0.08}$
${\rm Im}(\lambda_t)$ $[10^{-4}]$	$1.43^{+0.08}_{-0.07}$	$1.35^{+0.20}_{-0.17}$	$1.42^{+0.07}_{-0.07}$
${\rm Re}(\lambda_t)$ $[10^{-4}]$	$-3.46^{+0.18}_{-0.19}$	$-3.25^{+0.40}_{-0.45}$	$-3.43^{+0.17}_{-0.18}$

A. Buras, arXiv:1503.02693

Grossman-Nir Limit

$K_L \to \pi^0 \nu\bar{\nu}$ beyond the Standard Model ★

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Received 29 January 1997

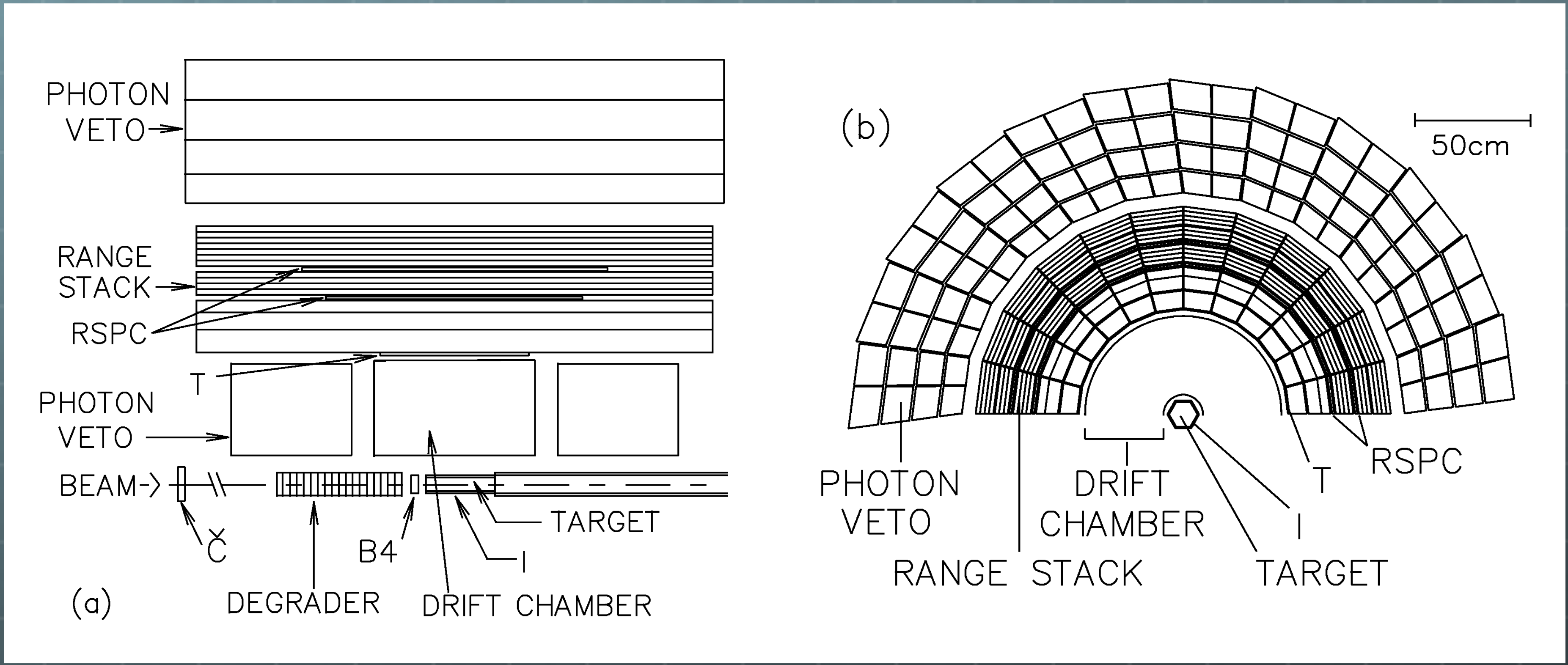
Editor: M. Dine

Abstract

We analyze the decay $K_L \to \pi^0 \nu\bar{\nu}$ in a model independent way. If lepton flavor is conserved the final state is (to a good approximation) purely CP even. In that case this decay mode goes mainly through CP violating interference between mixing and decay. Consequently, a theoretically clean relation between the measured rate and electroweak parameters holds in any given model. Specifically, $\Gamma(K_L \to \pi^0 \nu\bar{\nu})/\Gamma(K^+ \to \pi^+ \nu\bar{\nu}) = \sin^2\theta$ (up to known isospin corrections), where θ is the relative CP violating phase between the $K - \bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude. The experimental bound on $BR(K^+ \to \pi^+ \nu\bar{\nu})$ provides a model independent upper bound: $BR(K_L \to \pi^0 \nu\bar{\nu}) < 1.1 \times 10^{-8}$. In models with lepton flavor violation, the final state is not necessarily a CP eigenstate. Then CP conserving contributions can dominate the decay rate.

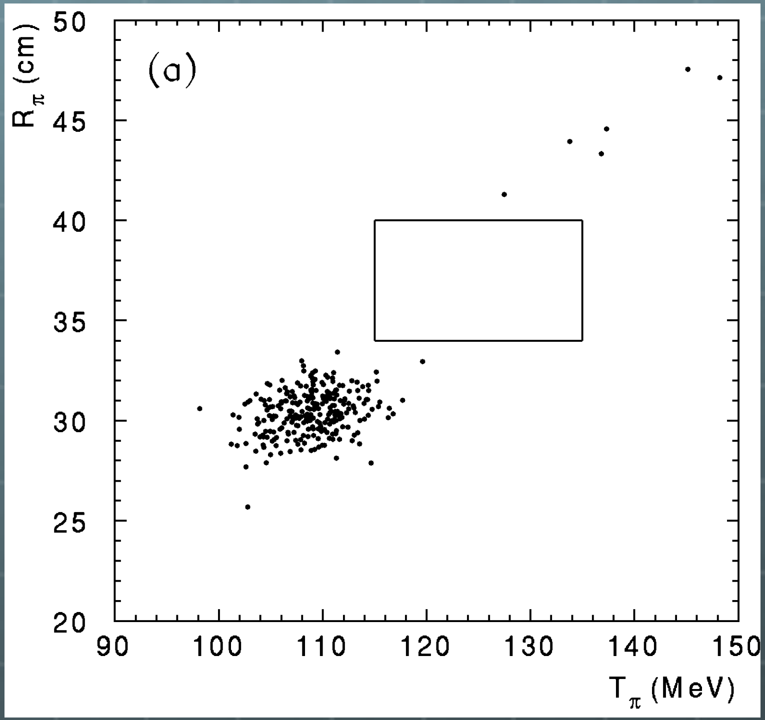
Physics Letters B 398 (1997) 163–168

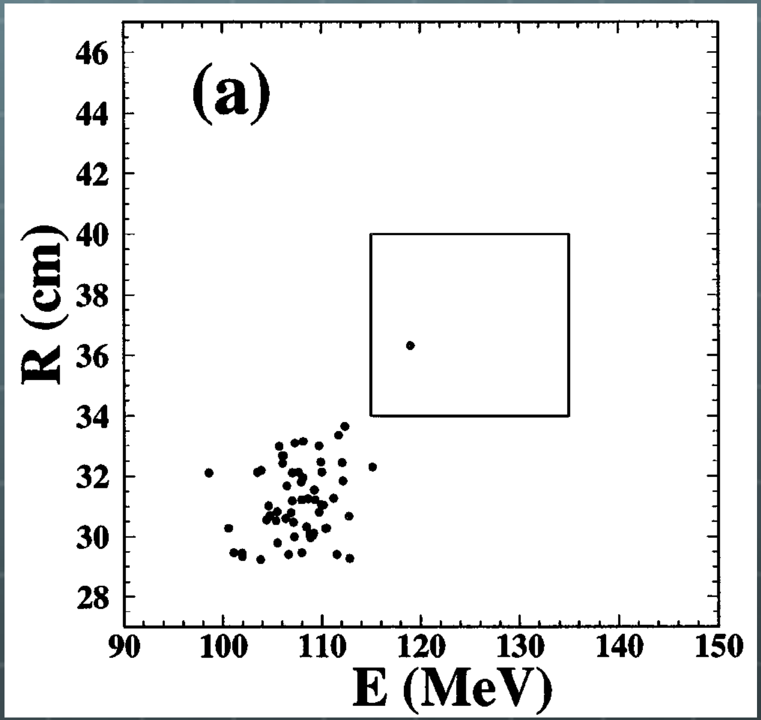
BNL E787 : Search for $K^+ \to \pi^+ \nu \bar{\nu}$



- Stopped Kaon (800 MeV/c) in target
- Main Backgrounds: $K^+ \to \pi^+ \pi^0$ $K^+ \to \mu^+ \nu$
- K⁺ incident, only one charged particle, π^+

BNL E787 Experiment





PRL 76, 1421 (1996)

$$\mathbf{Br}(K^+ \to \pi^+ \nu\bar{\nu}) < 2.4 \times 10^{-9}$$

PRL 79, 2204 (1997)

$$\mathbf{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = 4.2^{+9.7}_{-3.5} \times 10^{-10}$$

Decay amplitude

$$A = \langle \pi^0 \nu \bar{\nu} | H | K^0 \rangle, \quad \bar{A} = \langle \pi^0 \nu \bar{\nu} | H | \bar{K}^0 \rangle.$$

$|\bar{A}/A| = 1$ CP Limit (CP symmetry is exact)

$$|K1> = \frac{1}{\sqrt{2}}(|K^0> - |\bar{K^0}>) \quad |K2> = \frac{1}{\sqrt{2}}(|K^0> + |\bar{K^0}>)$$

CP EVEN

CP ODD

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle),$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle),$$

$$|q/p| \neq 1$$

Indirect CP Violation

$$|K_{L,S}\rangle = p|K^0\rangle \mp q|\bar{K}^0\rangle$$

$$\langle \pi^0 \nu \bar{\nu} | H | K_{L,S} \rangle = pA \mp q\bar{A}$$

$$\lambda \equiv \frac{q}{p} \frac{\bar{A}}{A}$$

$$\frac{\Gamma(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma(K_S \to \pi^0 \nu \bar{\nu})} = \frac{1 + |\lambda|^2 - 2\mathrm{Re}\,\lambda}{1 + |\lambda|^2 + 2\mathrm{Re}\,\lambda}$$

PHYSICAL REVIEW D VOLUME 39, NUMBER 11 1 JUNE 1989

CP-violating decay $K_L^0 \to \pi^0 \nu \bar{\nu}$

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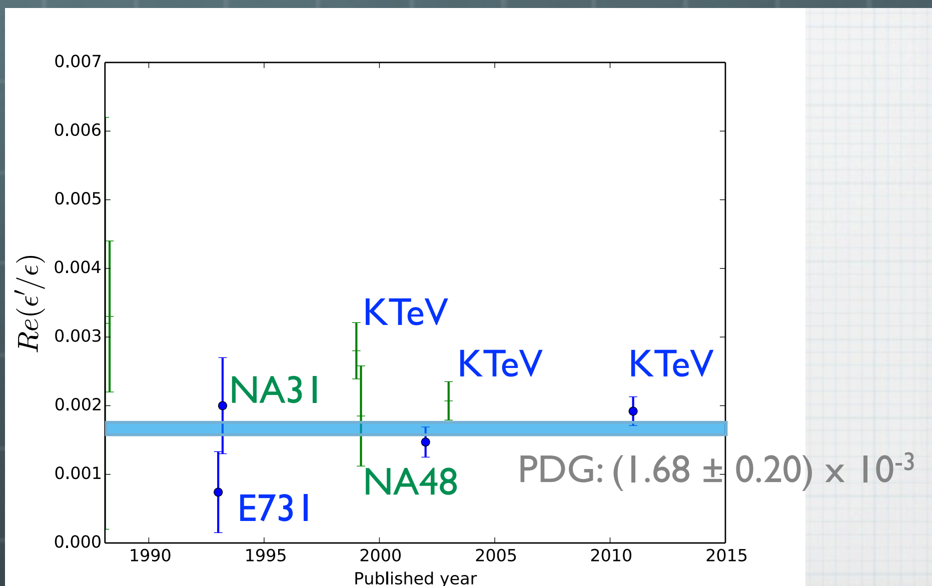
(Received 6 January 1989)

The process $K_L^0 \to \pi^0 \nu \bar{\nu}$ offers perhaps the clearest window yet proposed into the origin of *CP* violation. The largest expected contribution to this decay is a direct *CP*-violating term at $\approx$few$\times 10^{-12}$. The indirect *CP*-violating contribution is some *3 orders of magnitude* smaller, and *CP*-conserving contributions are also estimated to be extremely small. Although this decay has never been directly probed, a branching ratio upper limit of $\sim 1\%$ can be extracted from previous data on $K_L^0 \to 2\pi^0$. This leaves an enormous range in which to search for new physics. If the Kobayashi-Maskawa (KM) model prediction can be reached, a theoretically clean determination of the KM product $\sin\theta_2 \sin\theta_3 \sin\delta$ can be made.



Direct CP-violation

T. Yamanaka , 50 Years of CP Violation

$$|\lambda| = 1 \text{ to } \mathcal{O}(10^{-3}) \text{ accuracy.}$$

Leading CP Violation effect : Phase

Arbitrary phase between Direct and Indirect CP Violation

$$\lambda = e^{2i\theta}$$

$$\frac{\Gamma(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma(K_S \to \pi^0 \nu \bar{\nu})} = \frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2 \theta$$

$$a_{\mathrm{CP}} \equiv r_{\mathrm{is}} \frac{\Gamma(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma(K^+ \to \pi^+ \nu \bar{\nu})}$$

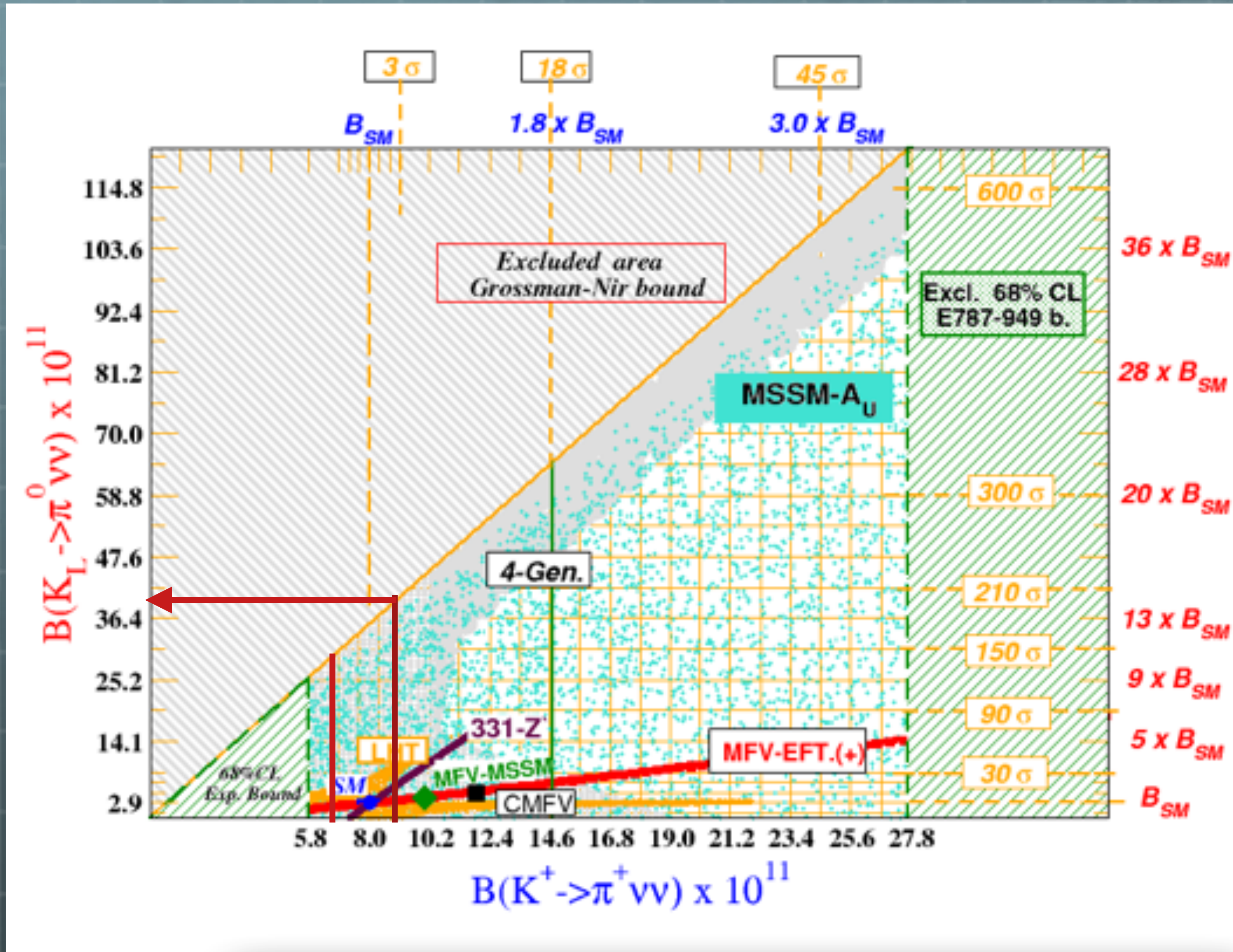
$$= \frac{1 - \cos 2\theta}{2} = \sin^2 \theta,$$

$$\frac{A(K^0 \to \pi^0 \nu \bar{\nu})}{A(K^+ \to \pi^+ \nu \bar{\nu})} = \frac{1}{\sqrt{2}}$$

$$\mathrm{BR}(K_L \to \pi^0 \nu \bar{\nu}) < 4.4 \times \mathrm{BR}(K^+ \to \pi^+ \nu \bar{\nu})$$

$$\tau_{K_L} / \tau_{K^+} = 4.17$$

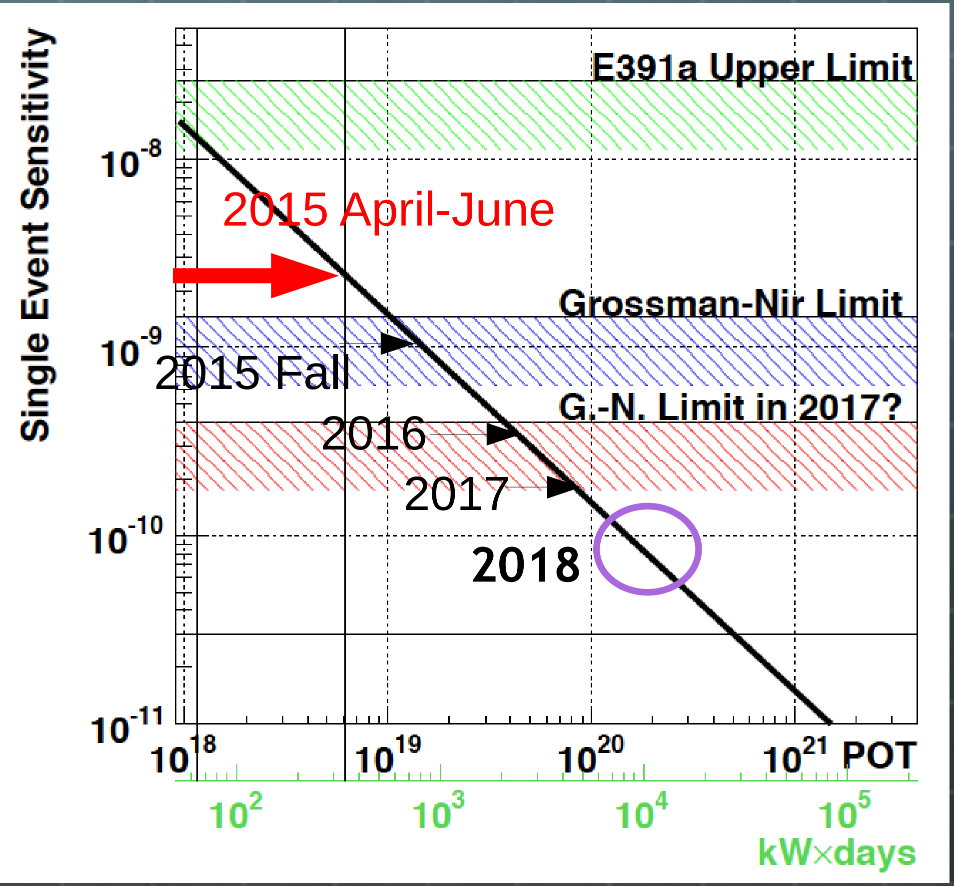
$$r_{is} = 0.954$$



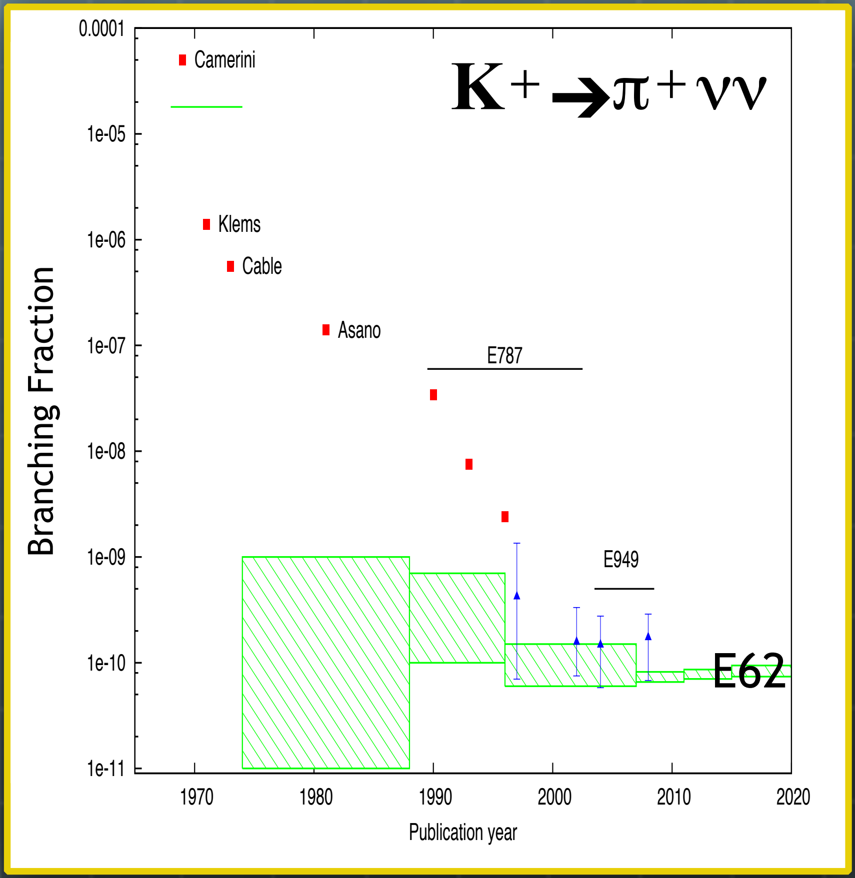
http://www.lnf.infn.it/wg/vus/content/Krare.html

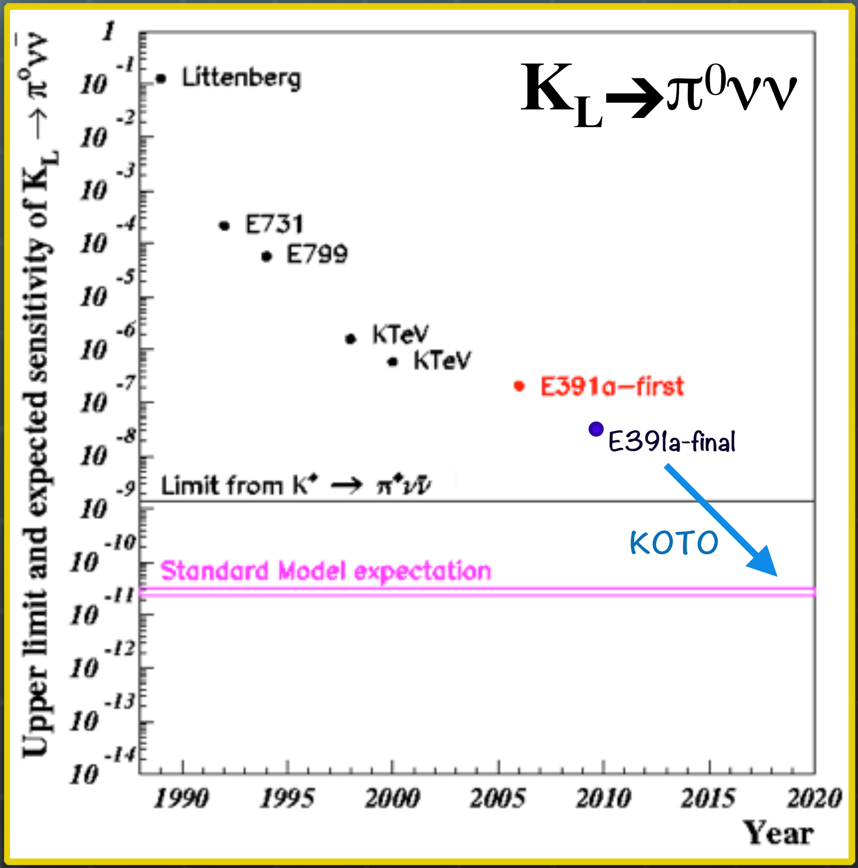
3 years from now

- Up-graded detector System
- Higher beam intensity of J-PARC



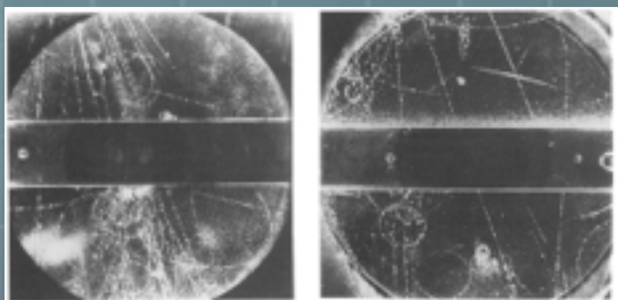
Experimental Status

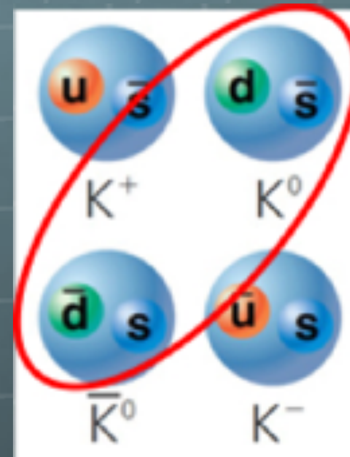


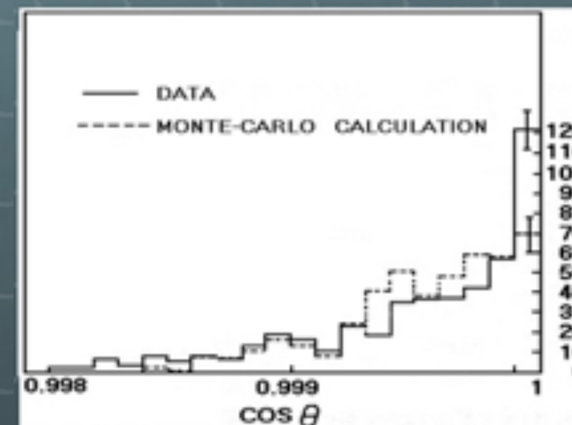


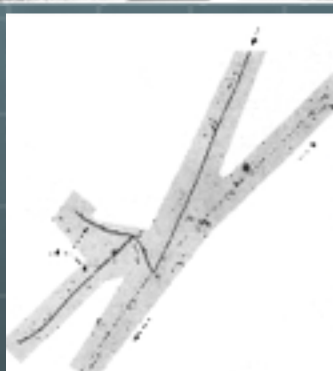
Still far from the goal ...

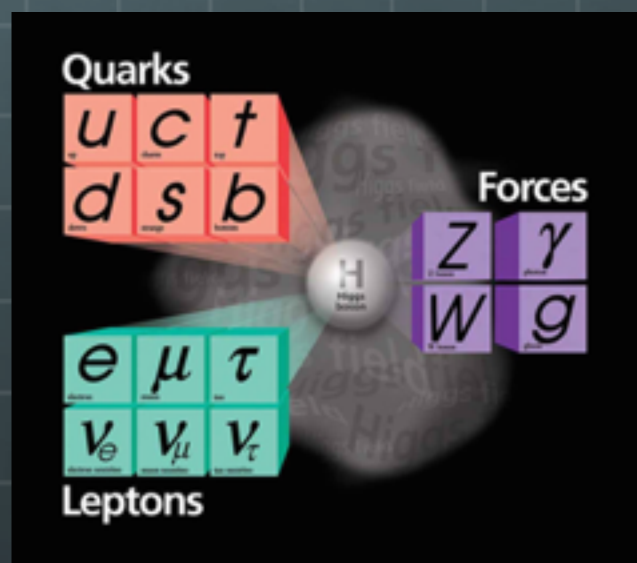
Studies on kaon decays
- Cornerstone of the SM -

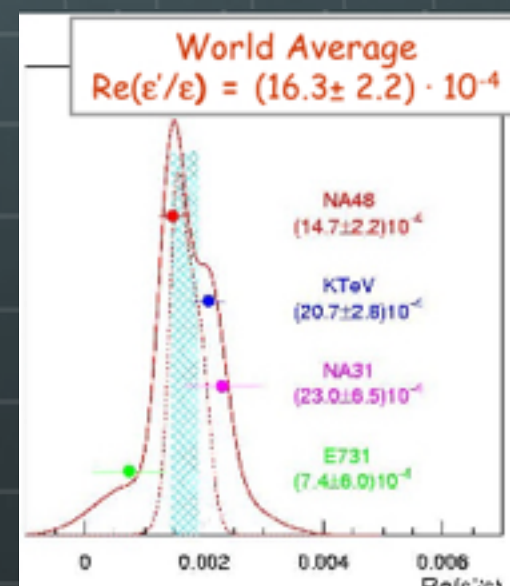


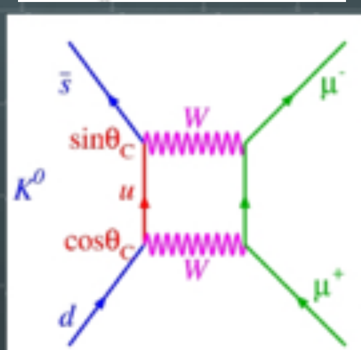






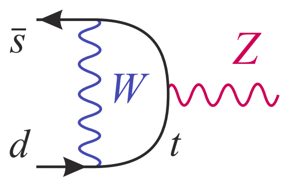
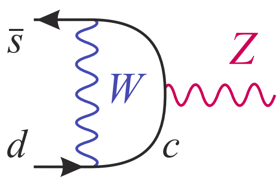
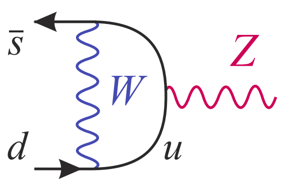






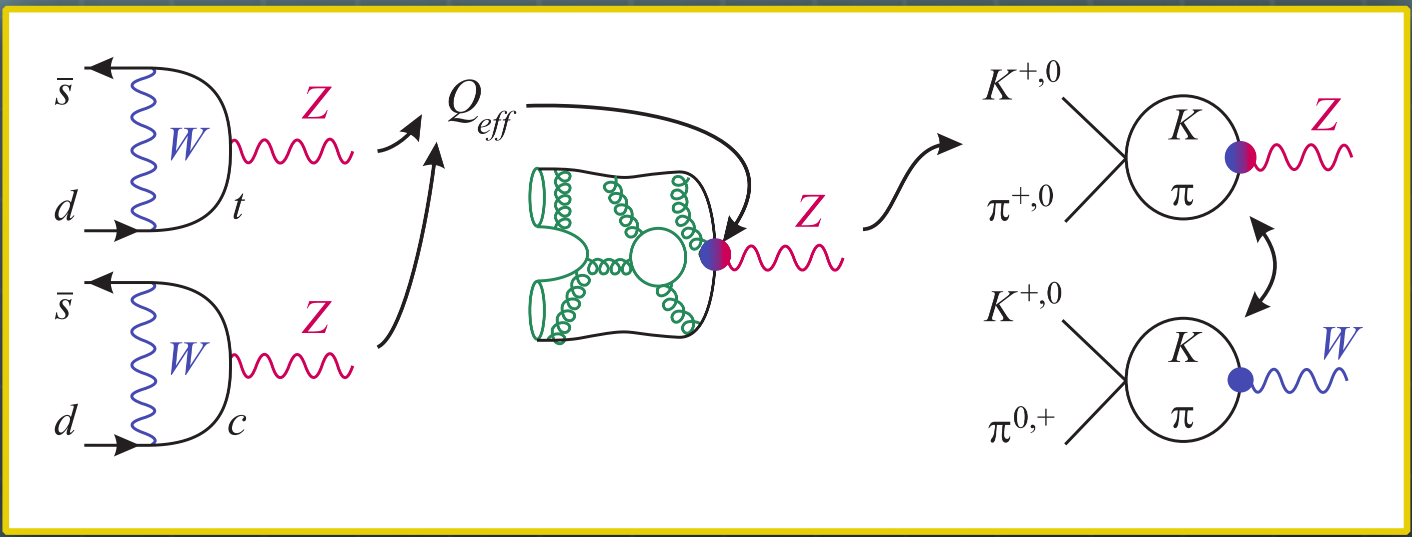
Highly suppressed process (in SM)

$$K^+ \to \pi^+ \nu\overline{\nu}$$

	$K_1 \to \pi^0 \nu\overline{\nu}$	$K_2 \to \pi^0 \nu\overline{\nu}$
	$\frac{m_t^2}{M_W^2}(\mathrm{Re}\, V_{ts}^\dagger V_{td} \sim \lambda^5)$	$\frac{m_t^2}{M_W^2}(\mathrm{Im}\, V_{ts}^\dagger V_{td} \sim \lambda^5)$
	$\frac{m_c^2}{M_W^2}(\mathrm{Re}\, V_{cs}^\dagger V_{cd} \sim \lambda)$	$\frac{m_c^2}{M_W^2}(\mathrm{Im}\, V_{cs}^\dagger V_{cd} \sim \lambda^5)$
	$\frac{m_u^2}{M_W^2}(\mathrm{Re}\, V_{us}^\dagger V_{ud} \sim \lambda)$	$\frac{m_u^2}{M_W^2}(\mathrm{Im}\, V_{us}^\dagger V_{ud} = 0)$

C. Smith, arXiv:1409.6162

Hadronic matrix elements



K→πνν

K→πlν

C. Smith, arXiv:1409.6162