

Hadron tomography studies by generalized parton distributions and distribution amplitudes

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GPD (Generalized Parton Distribution)
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GDA (Generalized Distribution Amplitude)
- **Summary**

Motivations

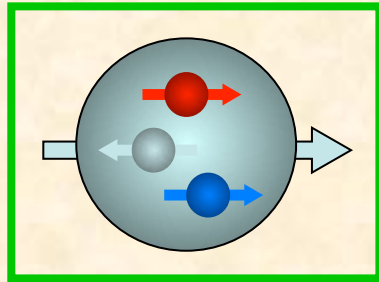
- **Nucleon spin structure**
- **Exotic hadrons**

...

I discuss future possibilities at J-PARC and KEKB; however, they can be investigated at high-energy lepton and hadron facilities (BNL, JLab, Fermilab, CERN, GSI, IHEP@China & Russia, EIC, LHeC, ...).

Recent progress on origin of nucleon spin

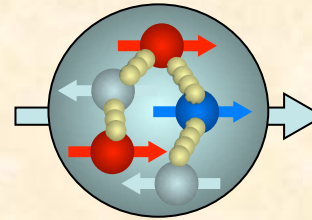
“old” standard model



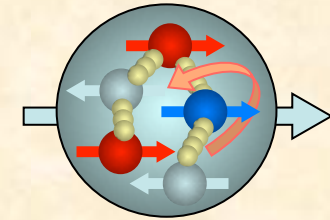
$$p_{\uparrow} = \frac{1}{3\sqrt{2}} \left(uud \left[2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

$$\Delta\Sigma = \sum_i \int dx [\Delta q_i(x) + \Delta \bar{q}_i(x)] \rightarrow 1 \text{ (100\%)}$$

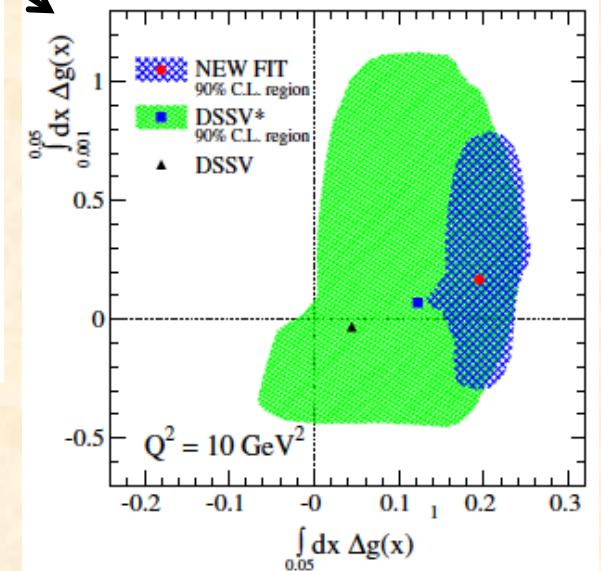
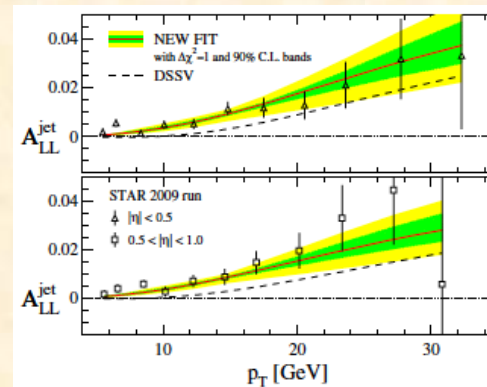


gluon spin

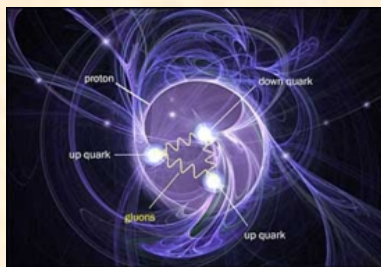


angular momentum

next pages



CNN (2014)

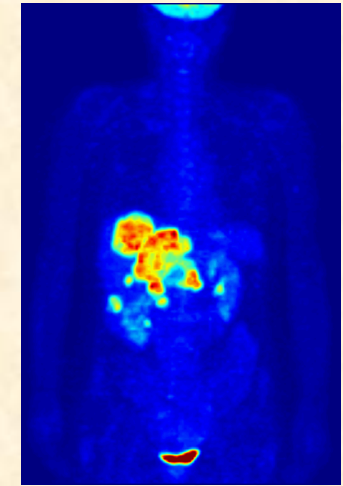


Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_{q,g}$$

Nucleon (hadron) tomography

PET (Positron Emission Tomography)



Classical density distribution

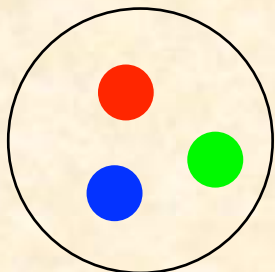
3D picture of nucleon
(Density distribution of quantum system:
Quantum tomography)

1D(Bjorken-x) picture@HERA

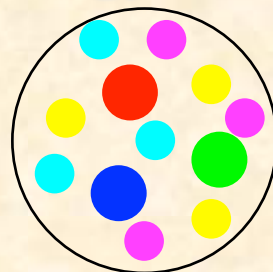


© DESY

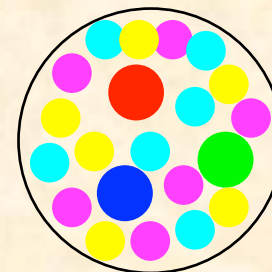
Low energy



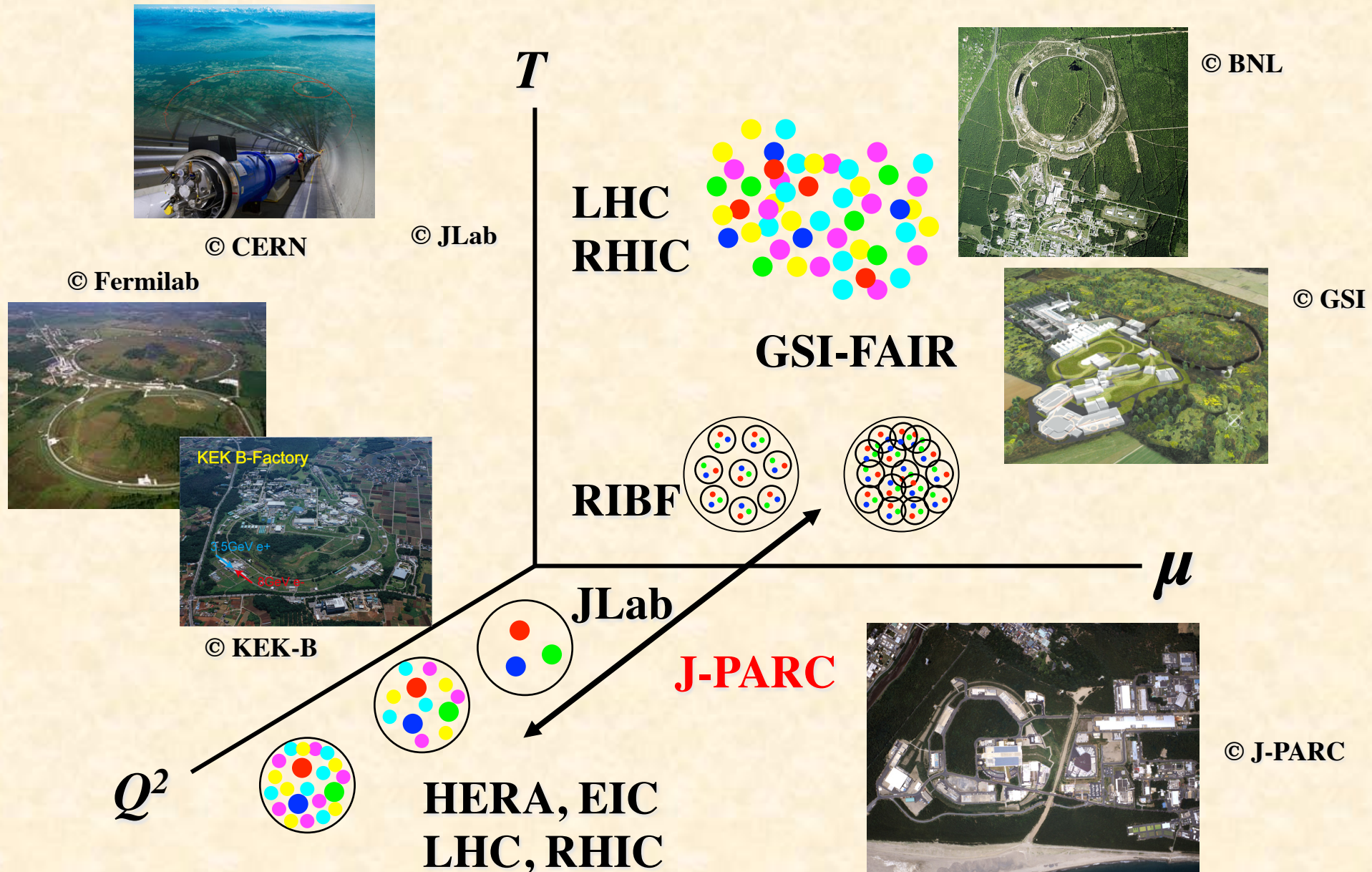
Intermediate energy



High energy



Test apparatus corresponds to ‘PET’



Wigner distribution

One-dimensional quantum mechanics with wave function $\psi(x)$.

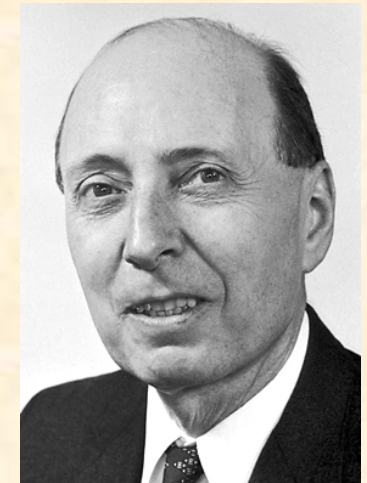
The Wigner distribution is defined by

$$W(x, p) \equiv \int d\xi e^{ip\xi/\hbar} \psi^*(x - \xi/2) \psi(x + \xi/2) = \text{phase-space distribution}$$

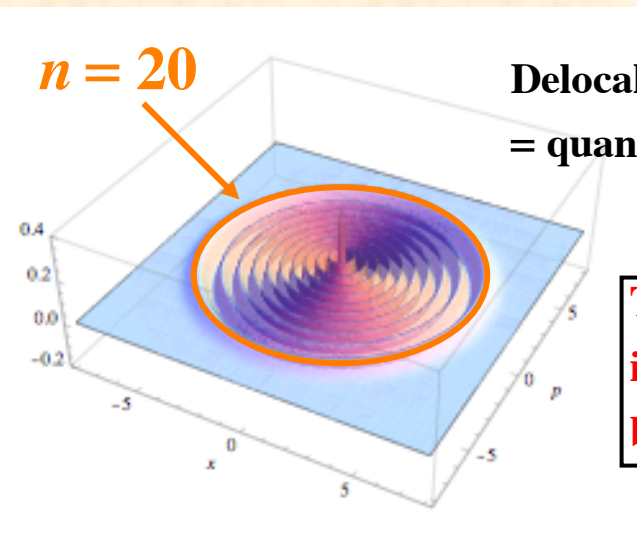
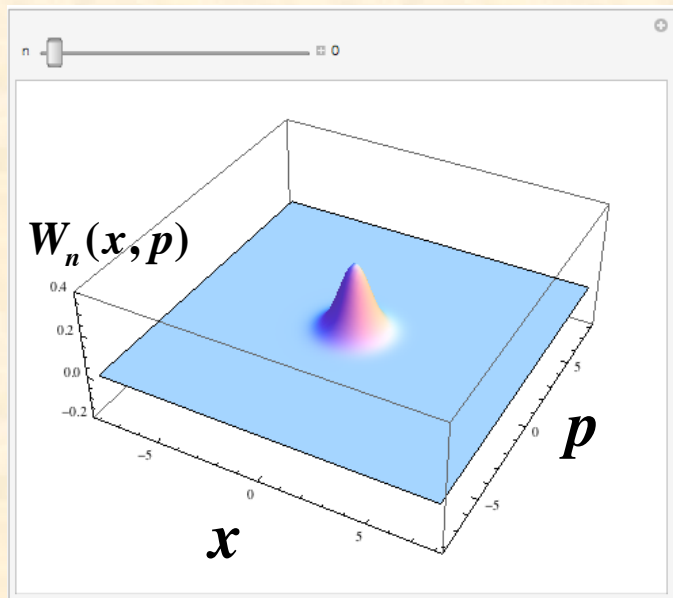
Example: One-dimensional harmonic oscillator: $H(x, p) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$

$$W_n(x, p) = \frac{(-1)^n}{\pi \hbar} e^{-2H/(\hbar\omega)} L_n\left(\frac{4H}{\hbar\omega}\right), \quad E_n = \hbar\omega\left(n + \frac{1}{2}\right), \quad L_n = \text{Laguerre polynomials}$$

$\rightarrow \delta(H(p, x) - E_n)$ as $\hbar \rightarrow 0, n \rightarrow \infty$ Classical trajectory with E_n .



© Nobel Foundation



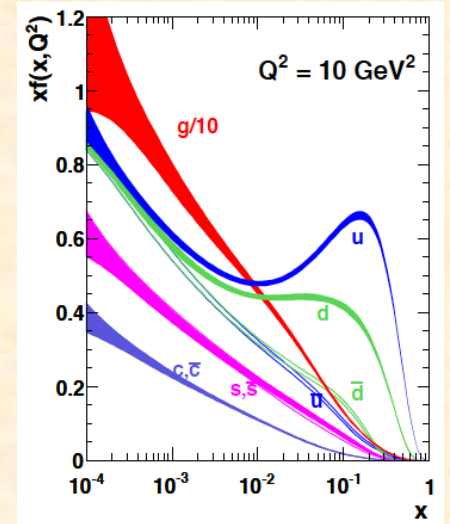
Delocalization of the Wigner distribution
= quantum effect (uncertainty principle)

The Wigner distribution provides information on quantum states by using phase-space concept.

Wigner distribution and various structure functions

Wigner operator: $\hat{w}(k_+, \vec{k}_\perp, \vec{r}) \equiv \int d\xi_- d^2\xi_\perp e^{i(\xi_- k_+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \bar{\psi}(\vec{r} - \vec{\xi} / 2) \psi(\vec{r} + \vec{\xi} / 2)$

Wigner distribution: $W(x, \vec{k}_\perp, \vec{r}) \equiv \int \frac{d^3q}{(2\pi)^3} \langle \vec{q} / 2 | \hat{w}(\vec{r}, k_+, \vec{k}_\perp) | -\vec{q} / 2 \rangle, \quad x = k_+ / p_+$



Form factor

PDF (Parton Distribution Function)

$$\int dx d^2k_\perp$$

$$\int d^2k_\perp d^3r$$

Wigner distribution $W(x, \vec{k}_\perp, \vec{r})$

3D world



$$\int d^3r$$

TMD (Transverse Momentum Dependent) parton distribution

$$\int d^2k_\perp dz$$

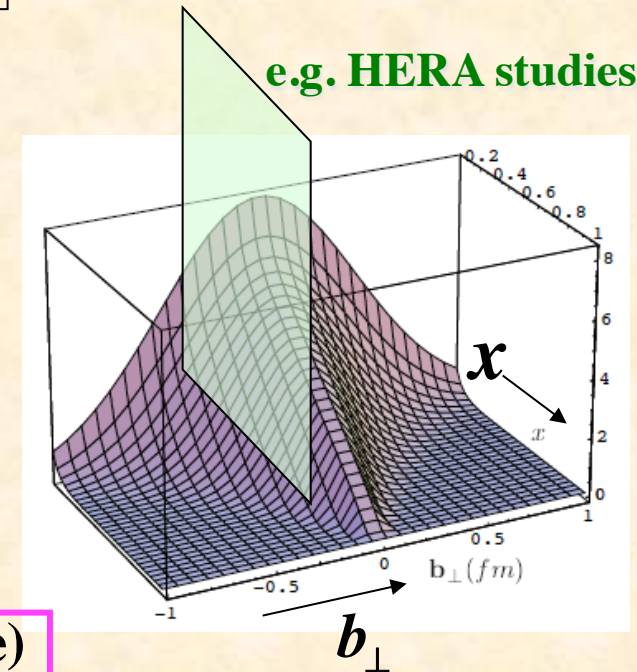
GPD (Generalized Parton Distribution)

s - t crossing $\rightarrow$

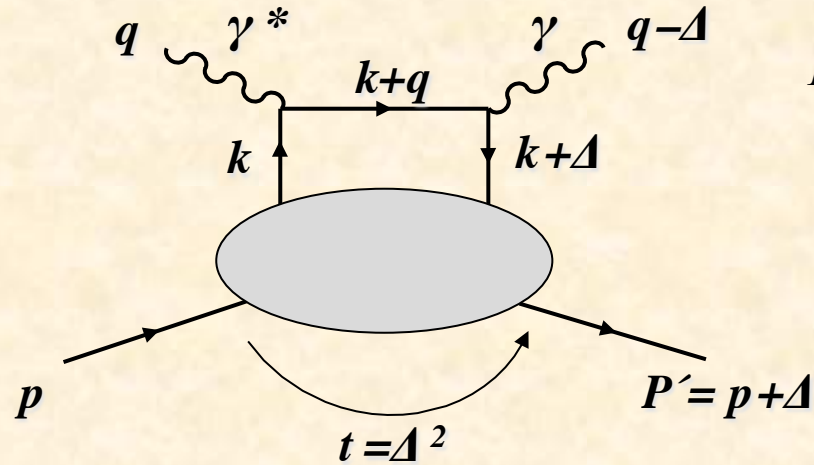
$\gamma\gamma \rightarrow h\bar{h}$

GDA (Generalized Distribution Amplitude)

e.g. HERA studies



Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

$$\text{Bjorken variable} \quad x = \frac{Q^2}{2p \cdot q}$$

$$\text{Momentum transfer squared} \quad t = \Delta^2$$

$$\text{Skewness parameter} \quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

Progress in exotic hadrons

$q\bar{q}$ Meson
 q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
 $q^4\bar{q}$ Pentaquark
 q^6 Dibaryon

...
 $q^{10}\bar{q}$ e.g. Strange tribaryon

...
 gg Glueball

...

- $\Theta^+(1540)???:$ LEPS

Pentaquark?

- **Kaonic nuclei?**: KEK-PS, ...
 Strange tribaryons, ...

$uudd\bar{s}$?

$K^- pnn, K^- ppn$?
 $K^- pp$?

- **X (3872), Y(3940)**: Belle
 Tetraquark, $D\bar{D}$ molecule

$c\bar{c}$
 $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
 $D^+(c\bar{d})D^-(\bar{c}d)$?

- **$D_{sJ}(2317), D_{sJ}(2460)$** : BaBar, CLEO, Belle
 Tetraquark, DK molecule

$c\bar{s}$
 $D^0(c\bar{u})K^+(u\bar{s})$
 $D^+(c\bar{d})K^0(d\bar{s})$?

- **Z (4430)**: Belle

Tetraquark, ...

$c\bar{c}u\bar{d}$, D molecule?

- **$P_c(4380), P_c(4450)$** : LHCb

- ...

$u\bar{c}udc, \bar{D}(u\bar{c})\Sigma_c^*(udc), \bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?

Scalar mesons $J^P=0^+$ at $M \sim 1$ GeV

Naïve quark-model

$a_1(1230)$

1.0 GeV

$a_0(980)$

$f_0(980)$

$\rho(770)$

0.5 GeV

$f_0(600) = \sigma$

$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \rightarrow \text{denote } f_0 \text{ in this talk}$$

$$a_0(980) = u\bar{d}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

⇕ contradiction

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

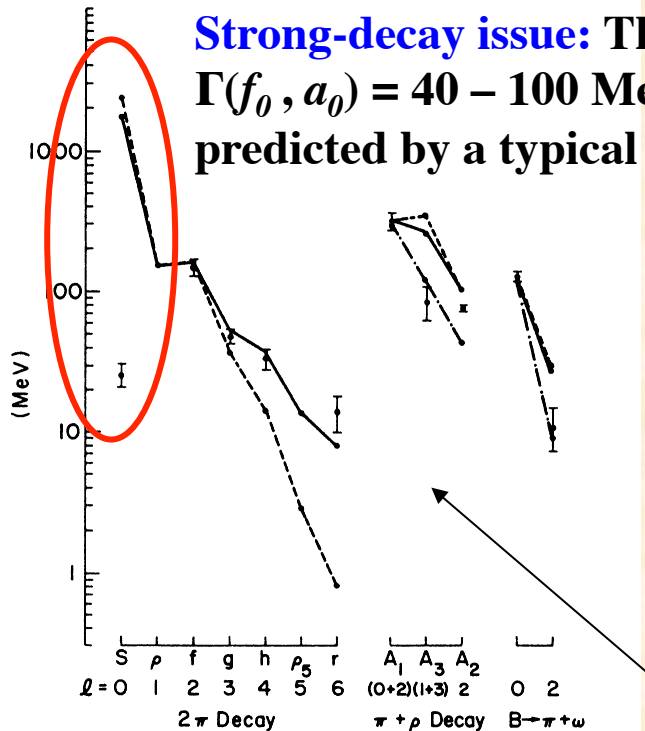
Strong-decay issue: The experimental widths $\Gamma(f_0, a_0) = 40 - 100$ MeV are too small to be predicted by a typical quark model.

These issues could be resolved

if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule, namely an "exotic" hadron.

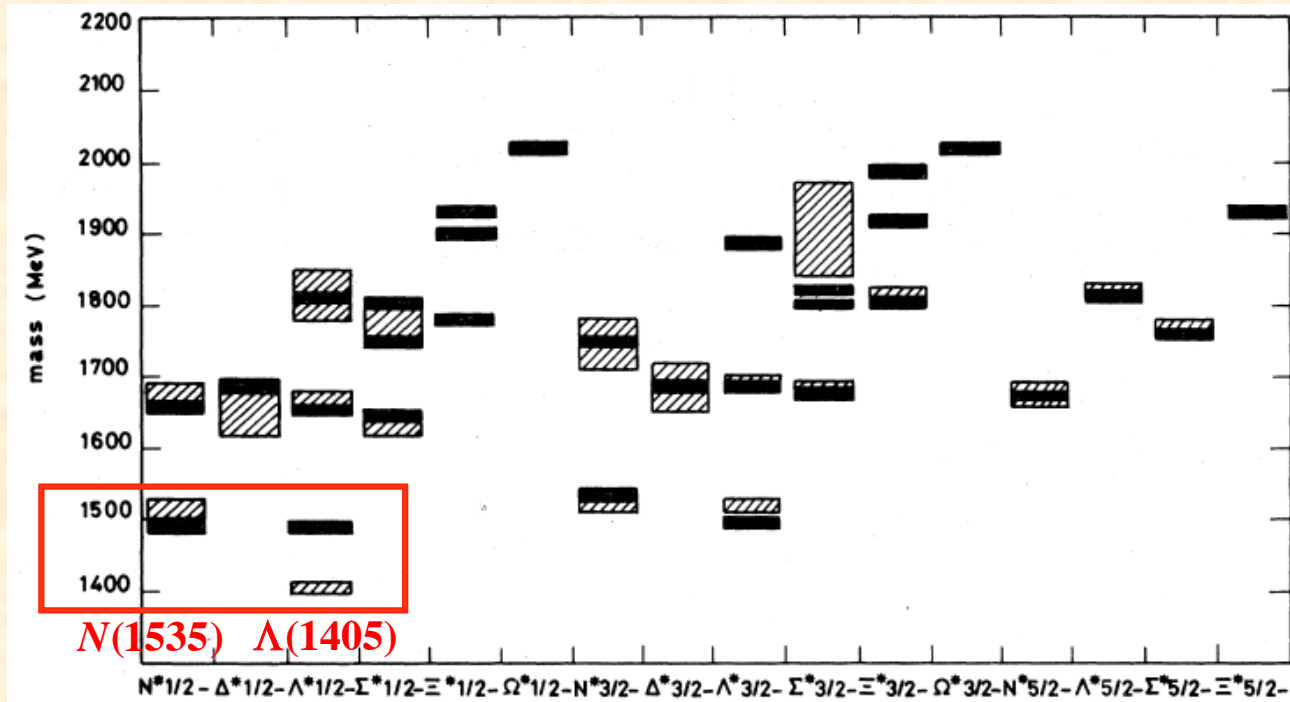
Radiative decay: F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.



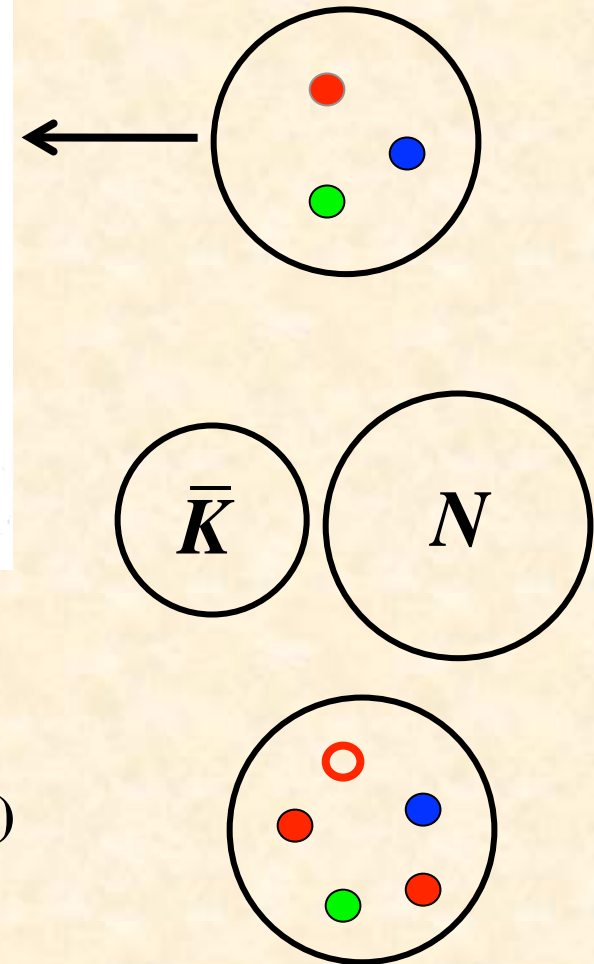
$\Lambda(1405)$: exotic hadron?

Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.

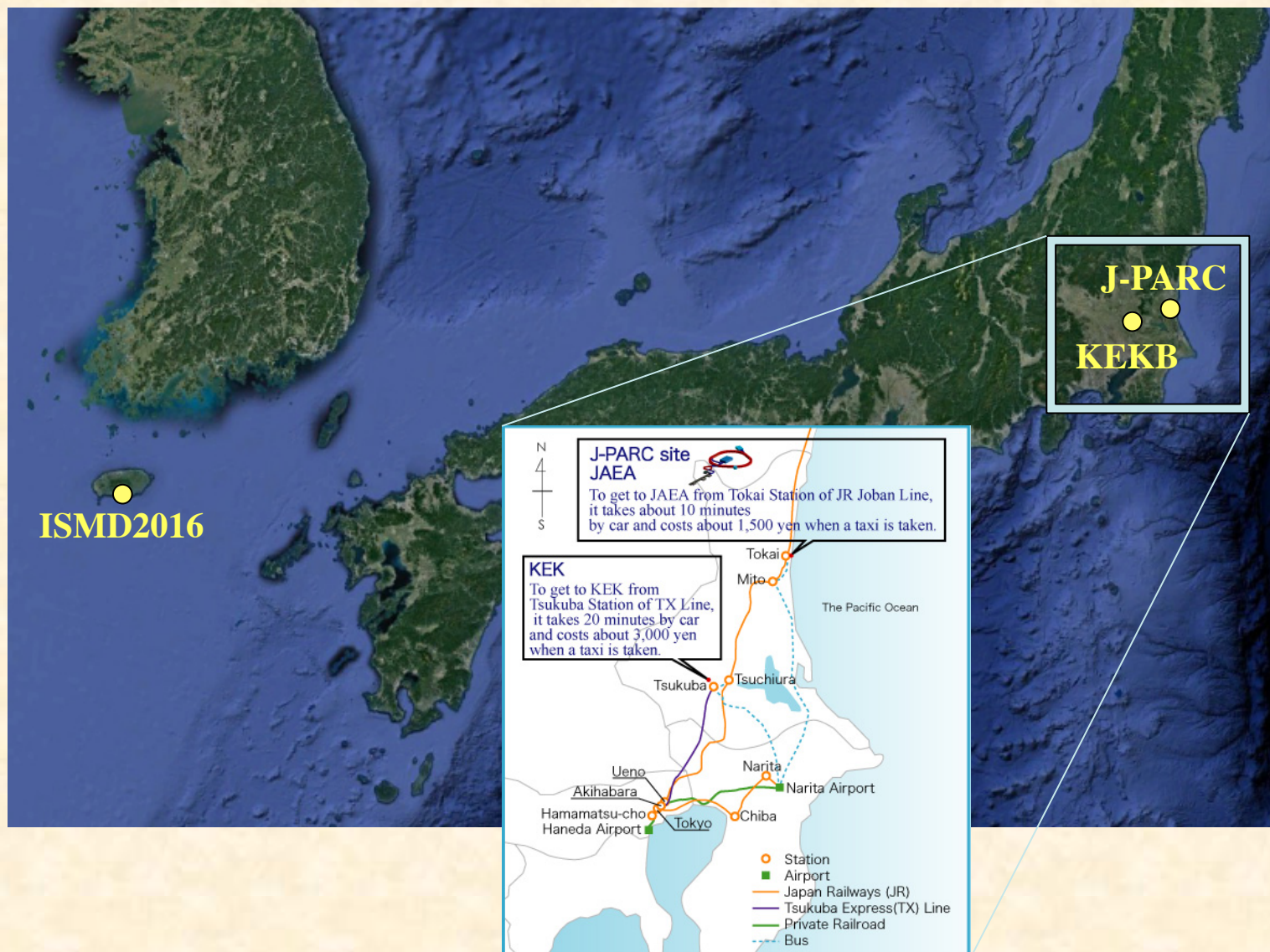


Most spectra agree with the ones by a $3q$ -picture

- Only $\Lambda(1405)$ deviates from the measurement.
- Difficult to understand the small mass of $\Lambda(1405)$ in comparison with $N(1535)$.
→ $\bar{K}N$ molecule or penta-quark ($qqqq\bar{q}$)?



Possible hadron-tomography studies at J-PARC & KEKB



Constituent-counting rule for exotic hadrons

**H. Kawamura, S. Kumano, T. Sekihara,
Phys. Rev. 88 (2013) 034010.**

**W.-C. Chang, S. Kumano, and T. Sekihara,
Phys. Rev. D 93 (2016) 034006.**

Research purposes

It is not easy to find undoubted evidence for exotic hadrons by global observables (mass, spin, parity, decay width) at low energies.

- (1) Determination of internal structure of exotic hadrons by high energy processes, where quark-gluon degrees of freedom appear.

Constituent-counting rule could be used because it counts internal constituents.

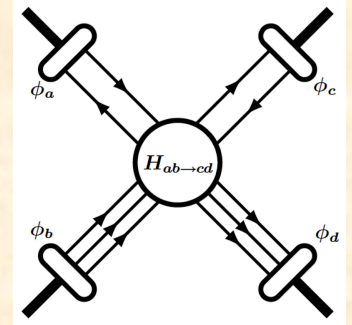
- (2) Investigation on transition from hadron degrees of freedom to quark-gluon degrees of freedom for exotic hadrons.

$$\frac{d\sigma_{a+b \rightarrow c+d}}{dt} \simeq \frac{1}{16\pi s^2} \sum_{pol} |M_{a+b \rightarrow c+d}|^2 \Rightarrow \frac{d\sigma_{a+b \rightarrow c+d}}{dt} = \frac{1}{s^{n-2}} f_{a+b \rightarrow c+d}(t/s) \quad \text{constituent-counting rule}$$

$n = n_a + n_b + n_c + n_d$

Constituent-counting rule in perturbative QCD:

Hard exclusive processes $a + b \rightarrow c + d$



Consider the hard exclusive hadron reaction $a + b \rightarrow c + d$

$$M_{ab \rightarrow cd} = \int d[x_a] d[x_b] d[x_c] d[x_d] \phi_c([x_c]) \phi_d([x_d]) H_M([x_a], [x_b], [x_c], [x_d], Q^2) \phi_a([x_a]) \phi_b([x_b])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)

Rule for estimating $M_{ab \rightarrow cd}$

- (1) Feynman diagram: Draw leading and connected Feynman diagram by connecting $n / 2$ quark lines by gluons.
- (2) Gluon propagators: The factor $1/P^2$ is assigned for each gluon propagator.
- (3) Quark propagators: The factor $1/P$ is assigned for each quark propagator.

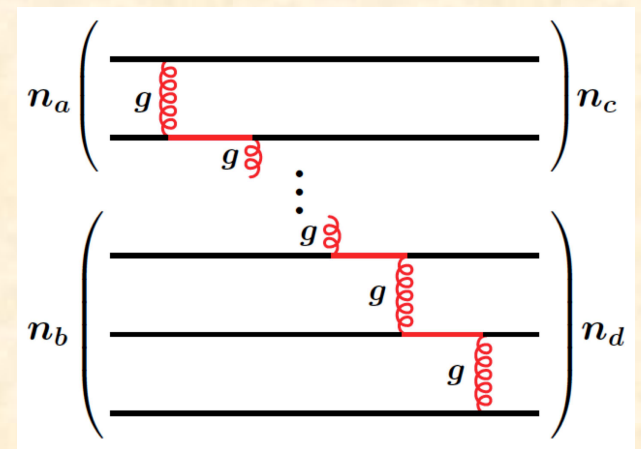
There are $n / 2 - 1$ gluon propagators $\sim 1/(P^2)^{n/2-1}$.

There are $n / 2 - 2$ gluon propagators $\sim 1/(P)^{n/2-2}$.

- (4) External quarks: The factor $\sqrt{P}$ is assigned for each external quark.

There are n gluon propagators $\sim (\sqrt{P})^n$.

$$M_{ab \rightarrow cd} \sim \frac{1}{(P^2)^{n/2-1}} \frac{1}{(P)^{n/2-2}} (\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2} (P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$



Cross section: $\frac{d\sigma_{ab \rightarrow cd}}{dt} \simeq \frac{1}{16\pi^2} \sum_{\text{pol}} |M_{ab \rightarrow cd}|^2 \sim \frac{1}{s^{n-2}}$

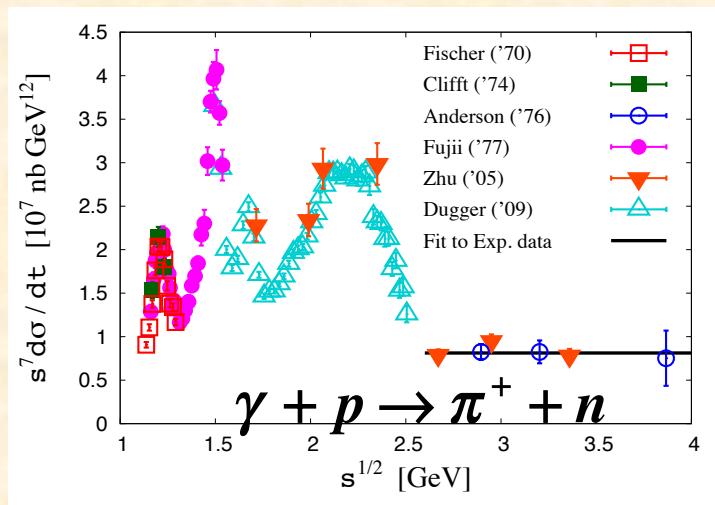
Constituent-counting rule, Transition from hadron degrees of freedom to quark-gluon ones

Typical current situation

- Transition from hadron d.o.f to quark d.o.f.
- (Looks like) Constituent-counting scaling

BNL experiment

C. White et al., PRD 49 (1994) 58.

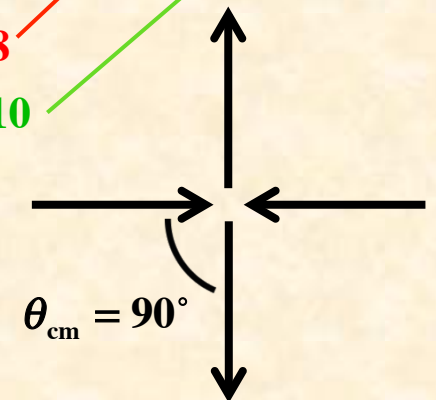


No.	Interaction	Cross section		$n-2$ ($\frac{d\sigma}{dt} \sim 1/s^{n-2}$)
		E838	E755	
1	$\pi^+ p \rightarrow p \pi^+$	132 ± 10	4.6 ± 0.3	6.7 ± 0.2
2	$\pi^- p \rightarrow p \pi^-$	73 ± 5	1.7 ± 0.2	7.5 ± 0.3
3	$K^+ p \rightarrow p K^+$	219 ± 30	3.4 ± 1.4	$8.3^{+0.6}_{-1.0}$
4	$K^- p \rightarrow p K^-$	18 ± 6	0.9 ± 0.9	≥ 3.9
5	$\pi^+ p \rightarrow p \rho^+$	214 ± 30	3.4 ± 0.7	8.3 ± 0.5
6	$\pi^- p \rightarrow p \rho^-$	99 ± 13	1.3 ± 0.6	8.7 ± 1.0
13	$\pi^+ p \rightarrow \pi^+ \Delta^+$	45 ± 10	2.0 ± 0.6	6.2 ± 0.8
15	$\pi^- p \rightarrow \pi^+ \Delta^-$	24 ± 5	≤ 0.12	≥ 10.1
17	$pp \rightarrow pp$	3300 ± 40	48 ± 5	9.1 ± 0.2
18	$\bar{p}p \rightarrow \bar{p}p$	75 ± 8	≤ 2.1	≥ 7.5

JLab: L.Y. Zhu *et al.*, PRL 91, 022003 (2003);
PRC 71, 044603 (2005);
W. Chen *et al.*, PRL 103, 012301 (2009).

$$n-2: (2+3+2+3)-2=8$$

$$(3+3+3+3)-2=10$$



see R. A. Schumacher and M. M. Sargsian,
PRC 83 (2011) 025207 for hyperon production

Exotic-hadron production $\pi^- + p \rightarrow K^0 + \Lambda(1405)$

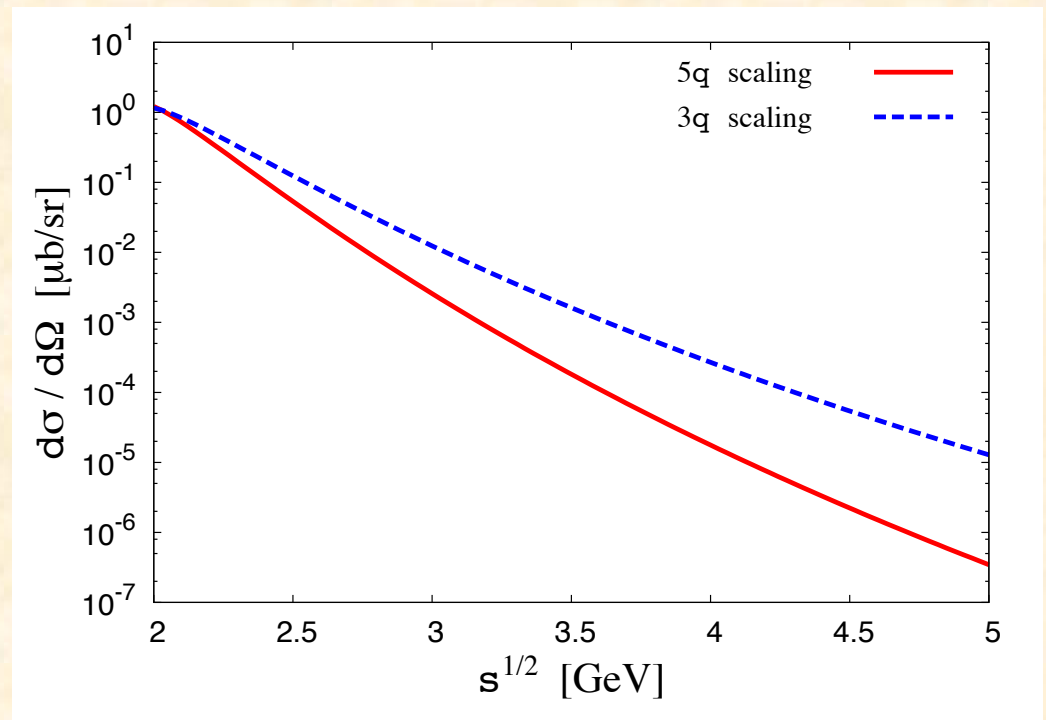
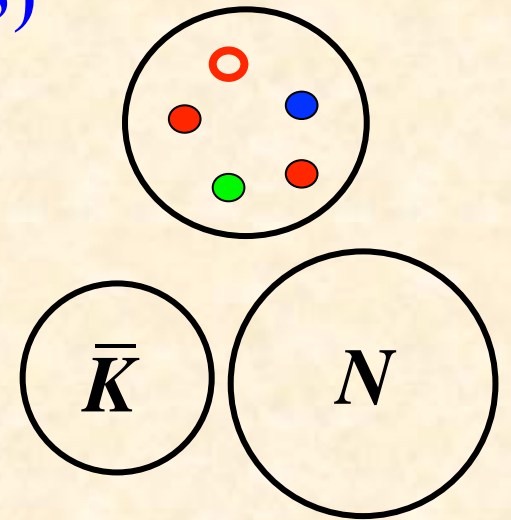
Theoretical and experimental situation
is no as good as the one for the ground Λ .

$$\begin{aligned} n &= 2 + 3 + 2 + 3 = 10 \quad \text{if } \Lambda(1405) = \text{three-quark state} \\ &= 2 + 3 + 2 + 5 = 12 \quad \text{if } \Lambda(1405) = \text{five-quark state} \\ &\quad \text{(including } \bar{K}N \text{ molecule)} \end{aligned}$$

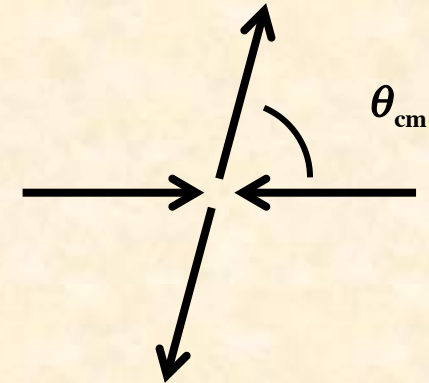
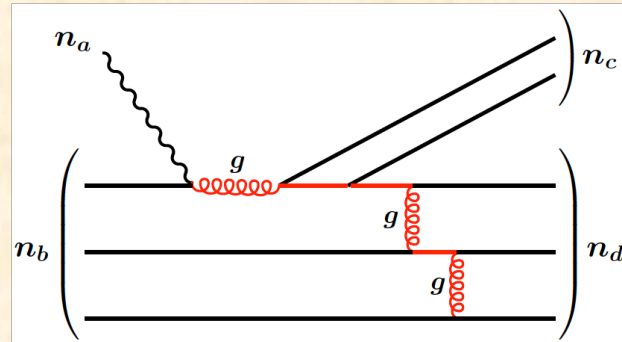
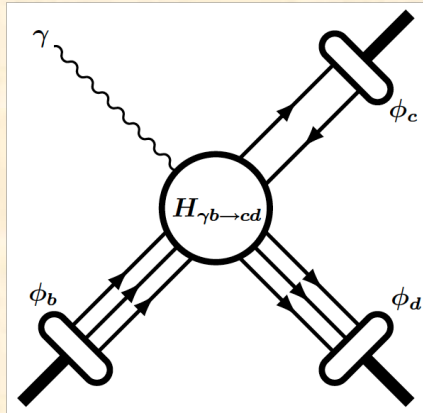
$$\frac{d\sigma_{ab \rightarrow cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10 \text{ or } 12$$

Our prediction at high energies

M. Niiyama (Kyoto Univ),
J-PARC proposal,
under consideration



JLab hyperon productions



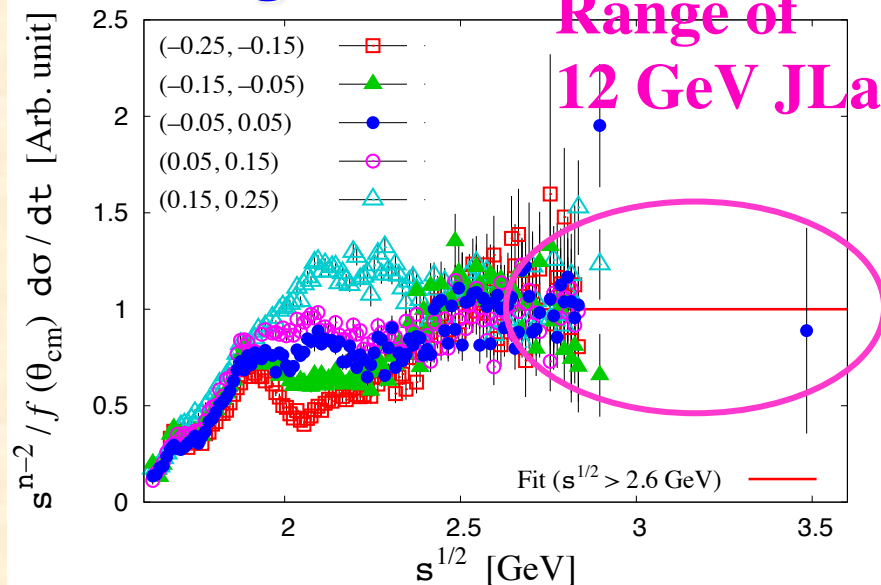
5 bins: $-0.25 < \cos \theta_{\text{cm}} < -0.15$, ..., $0.15 < \cos \theta_{\text{cm}} < 0.25$

4 bins: $-0.20 < \cos \theta_{\text{cm}} < -0.10$, ..., $0.10 < \cos \theta_{\text{cm}} < 0.20$

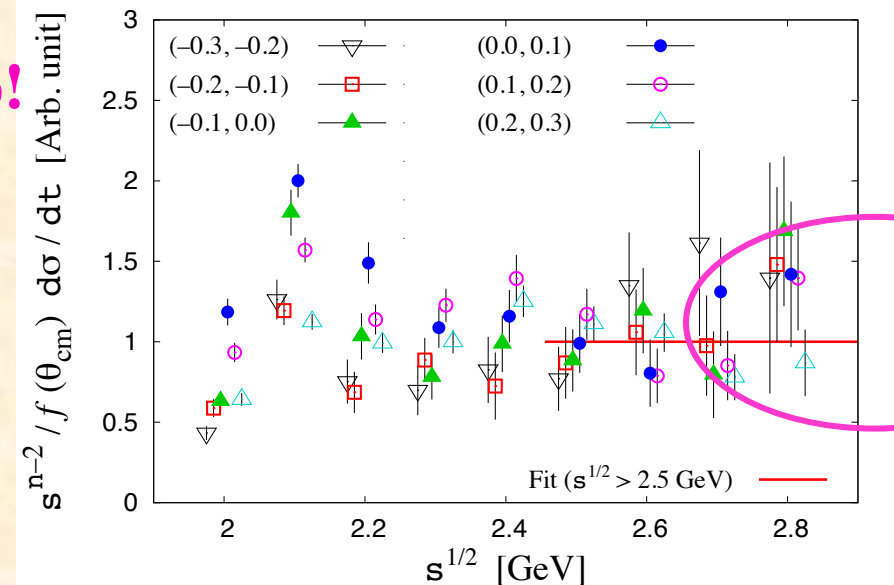
...

1 bin: $-0.05 < \cos \theta_{\text{cm}} < +0.05$

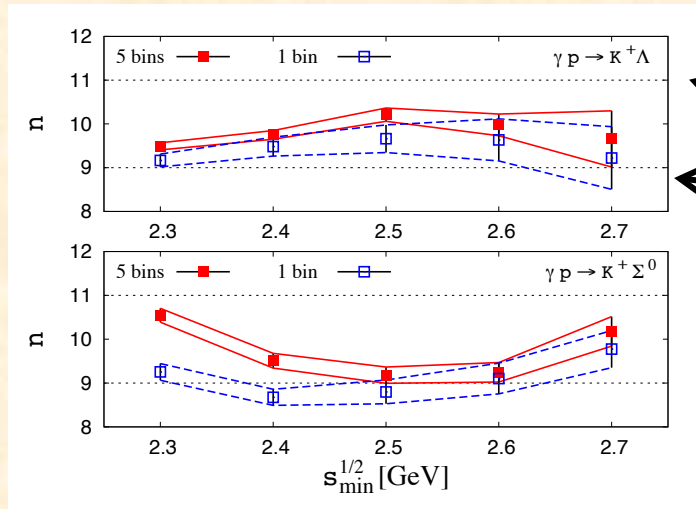
ground Λ



$\Lambda(1405)$

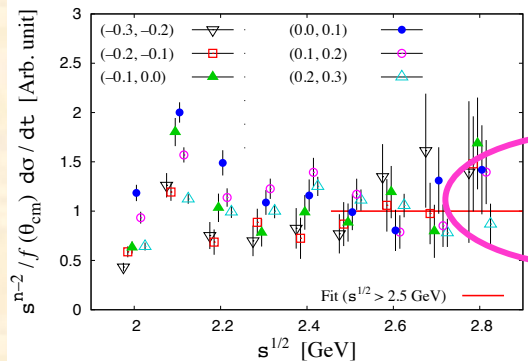
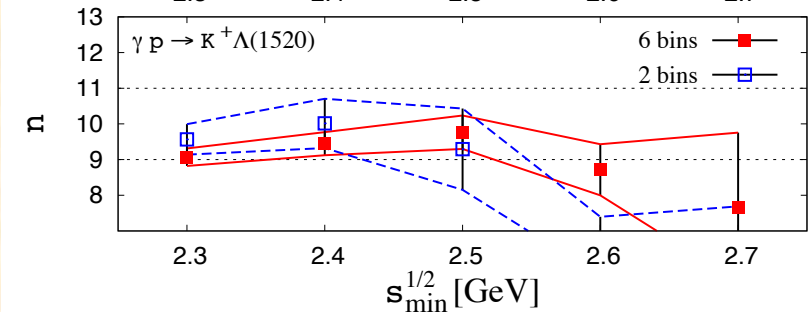
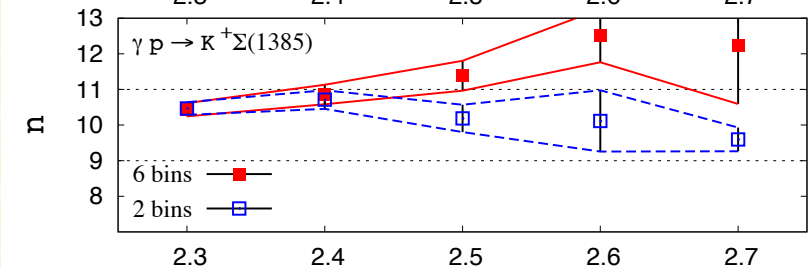
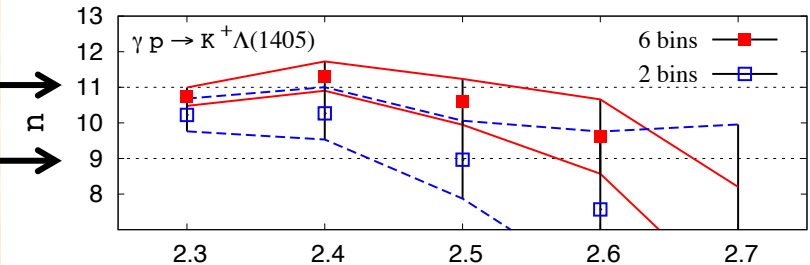


JLab hyperon productions including $\Lambda(1405)$



$n_\Lambda = 5$

$n_\Lambda = 3$



- Λ , $\Lambda(1520)$ and Σ seem to be consistent with ordinary baryons with $n = 3$.
- $\Lambda(1405)$ looks penta-quark at low energies but $n \sim 3$ at high energies???
- $\Sigma(1385)$: $n = 5$???

→ In order to clarify the nature of $\Lambda(1405)$ [$qqq, \bar{K}N, qqqq\bar{q}$], the JLab 12-GeV experiment plays an important role!

Summary on exotic hadron structure by hard exclusive processes

- We propose to use hard exclusive production of exotic hadrons for probing internal quark-gluon structure by the constituent counting rule, $\frac{d\sigma}{dt} = \frac{\text{const}}{s^{n-2}}$.
- As an example, $\pi^- + p \rightarrow K^0 + \Lambda(1405)$ is studied together with $\pi^- + p \rightarrow K^0 + \Lambda$ as a reference of an ordinary hadron.
- $\gamma + p \rightarrow K^+ + \Lambda(1405)$ is studied.
 $\Lambda(1405)$ = pentaquark at low energies
= 3-quark baryon at high energies ???
→ Measurements of extended kinematical range are necessary (12 GeV JLab).
- Exclusive processes of exotic hadrons can be investigated at many facilities in the world.
For example, J-PARC, LEP, JLab, COMPASS,
in general any hadron facilities like GSI, Fermilab, RHIC, LHC, ...

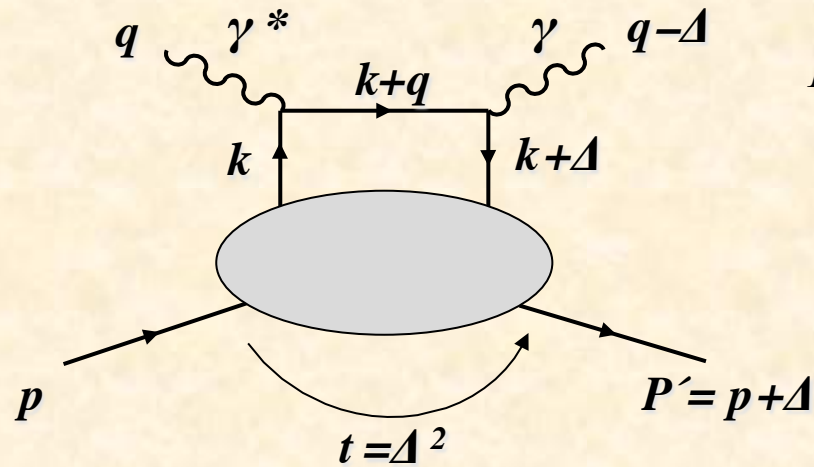
Generalized Parton Distributions (GPDs)

and J-PARC project

GPDs in the ERBL region at hadron facilities

**S. Kumano, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003.**

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

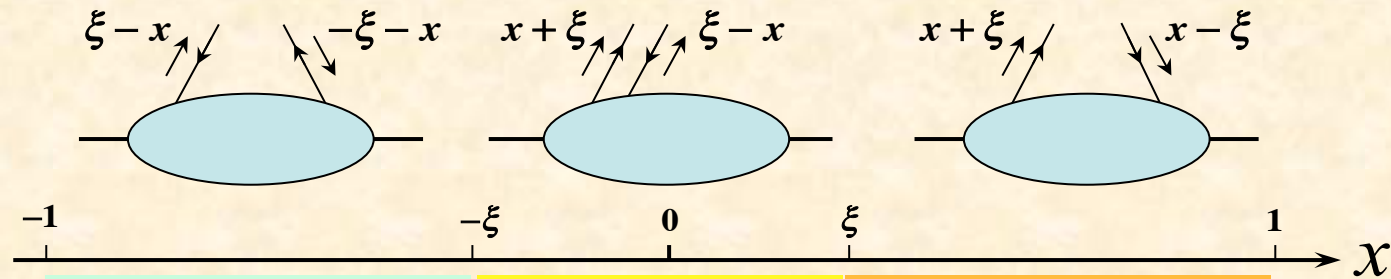
Dirac and Pauli form factors F_1, F_2 $\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

GPDs in different x regions and GPDs at hadron facilities



$-1 < x < -\xi$ ($x + \xi < 0, x - \xi < 0$)

$\xi < x < 1$ ($x + \xi > 0, x - \xi > 0$)

$-\xi < x < \xi$ ($x + \xi > 0, x - \xi < 0$)

Quark distribution

Emission of quark with momentum fraction $x + \xi$
Absorption of quark with momentum fraction $x - \xi$

$q\bar{q}$ (meson)-like distribution amplitude

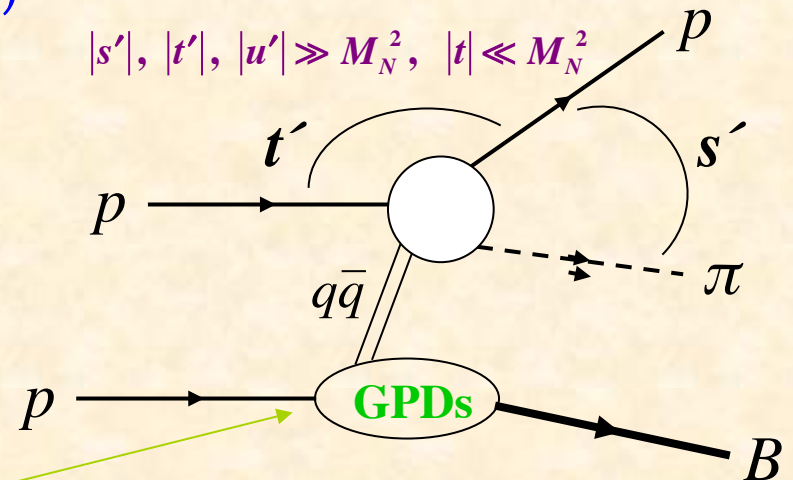
Emission of quark with momentum fraction $x + \xi$
Emission of antiquark with momentum fraction $\xi - x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi - x$
Absorption of antiquark with momentum fraction $-\xi - x$

Consider a hard reaction with

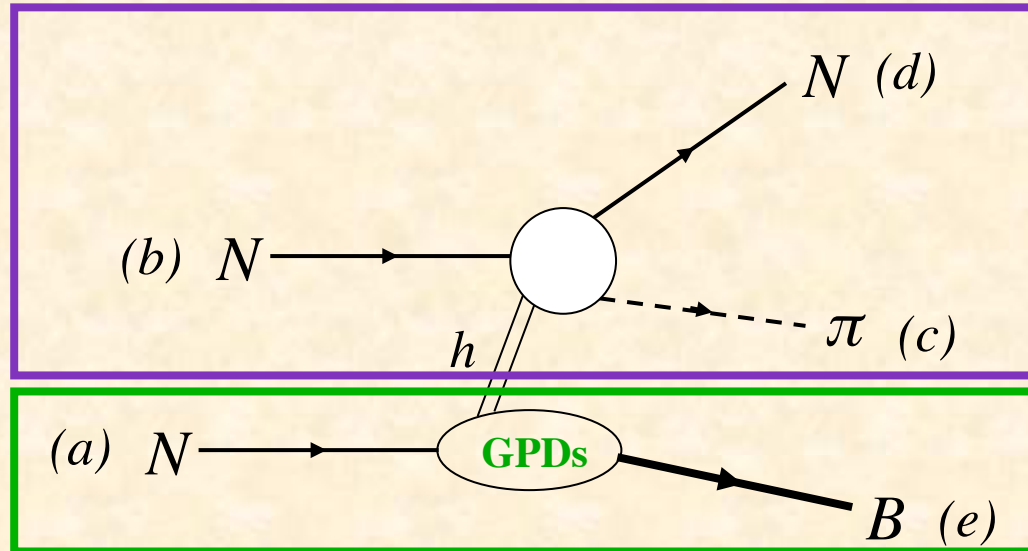
$|s'|, |t'|, |u'| \gg M_N^2, |t| \ll M_N^2$



GPDs at J-PARC: S. Kumano, M. Strikman, and K. Sudoh, PRD 80 (2009) 074003.

Efremov-Radyushkin
-Brodsky-Lepage (ERBL) region

Cross section estimates



$\frac{d\sigma(s',t')}{dt'}$ so as to explain
AGS experimental data on
 $\pi + p \rightarrow \pi + p$, $\pi + p \rightarrow \rho + p$

This part is expressed by GPDs.

Purposes of our studies:

- (1) The ultimate purpose is to extract the GPDs in the ERBL region by measurements at hadron facilities in addition to lepton ones.
- (2) Since our work is the first one to point out the GPD studies at hadron reactions, we estimate the order of magnitude of cross sections simply by using meson-pole expressions of the GPDs.
→ For experimental feasibility studies.

Cross section estimate (ξ dependence)

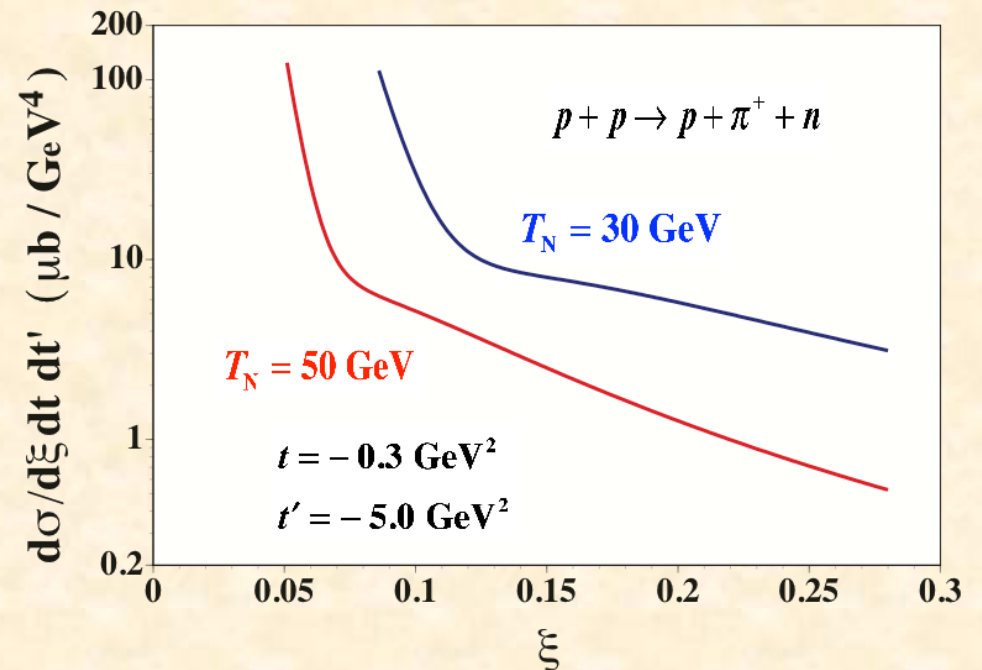
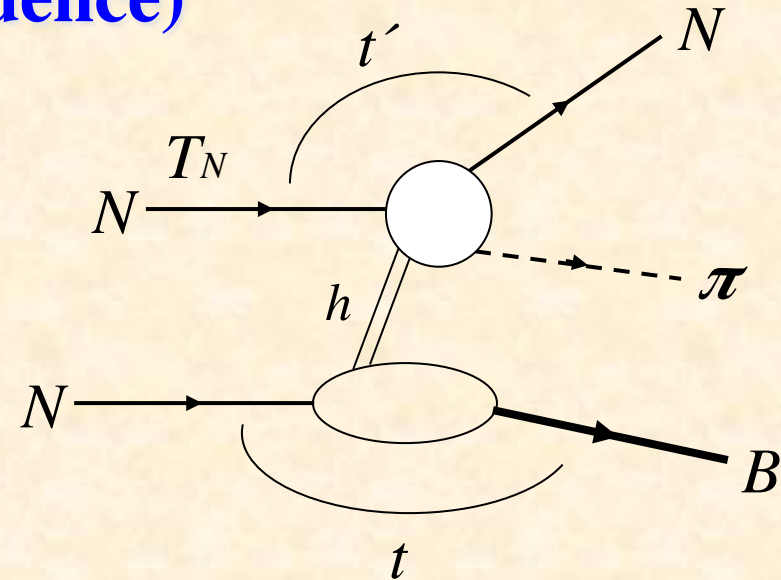
Skewness parameter: $\xi = \frac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\frac{d\sigma}{d\xi dt dt'} \left(\frac{\mu\text{b}}{\text{GeV}^2} \right)$ as a function of ξ

at fixed $T_N = 30$ (50) GeV,

$t = -0.3 \text{ GeV}^2, \quad t' = -5 \text{ GeV}^2.$

At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π - and ρ -like intermediate states.



GPDs for exotic hadrons at hadron facilities

**H. Kawamura and S. Kumano
Phys. Rev. D 89 (2014) 054007.**

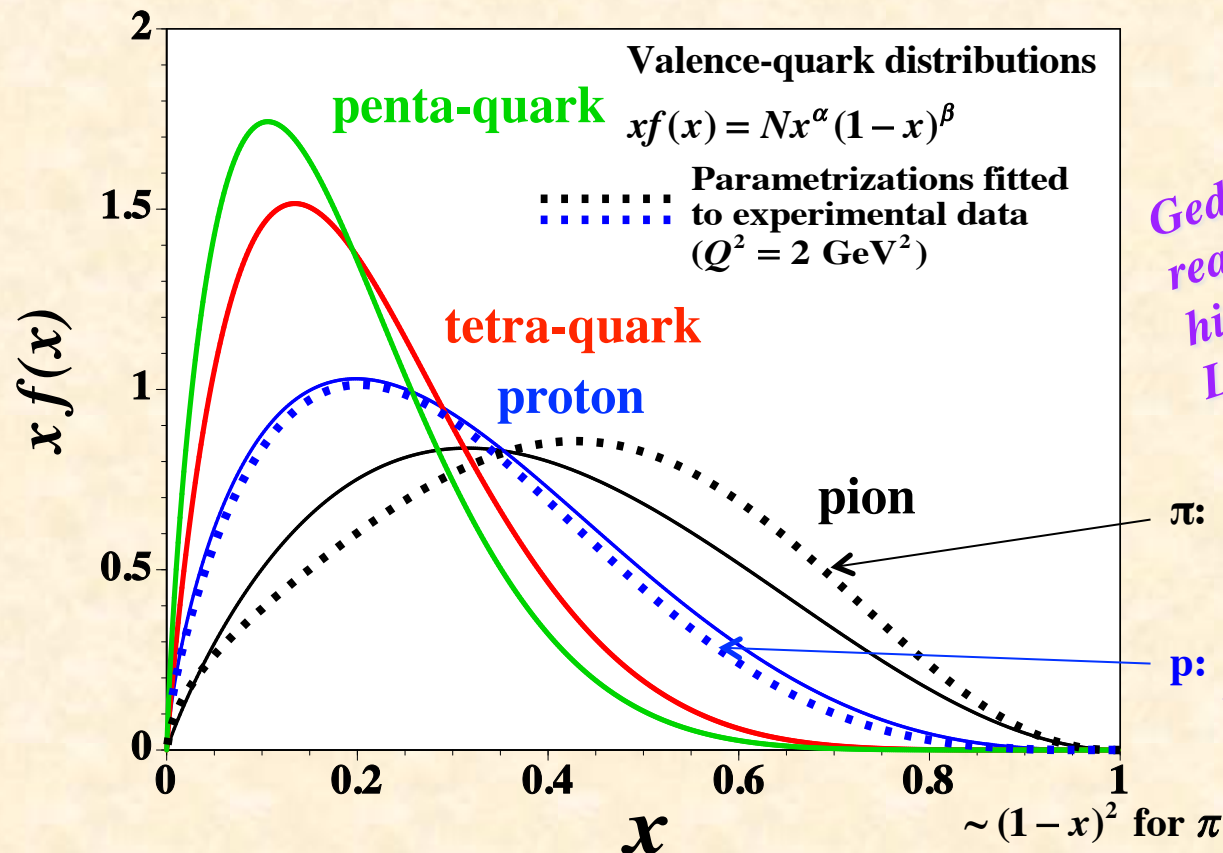
Simple function of GPDs

$$H_q^h(x, t) = f(x)F(t, x)$$

M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent counting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx x f(x)$



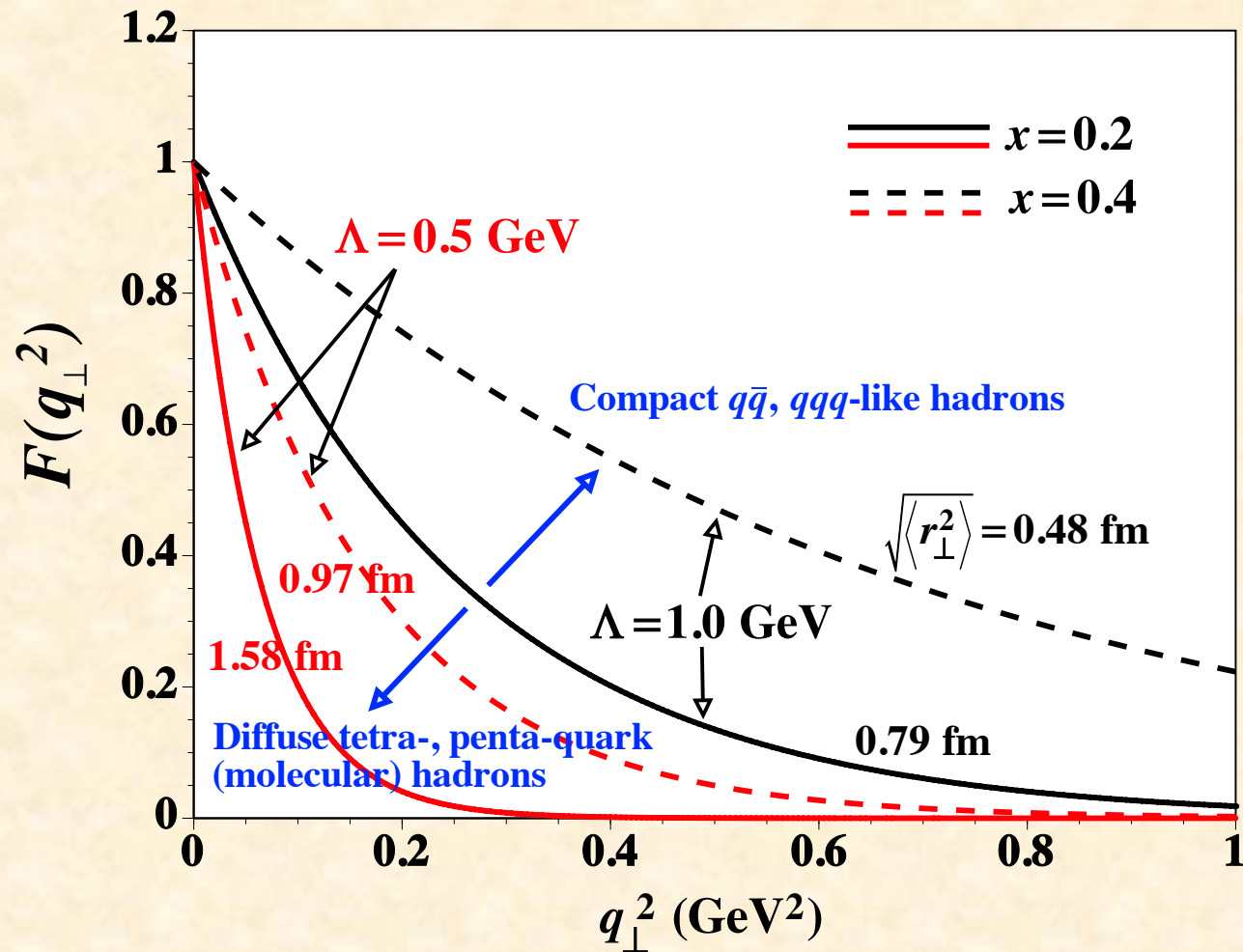
*Gedankenexperiment, but ...
 read our paper for studying exotics in
 high-energy processes at KEK-B,
 Linear Collider,*

π : M. Aicher, A. Schafer, W. Vogelsang,
PRL 105 (2010) 252003.

p : A. D. Martin, R. G. Roberts,
W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$

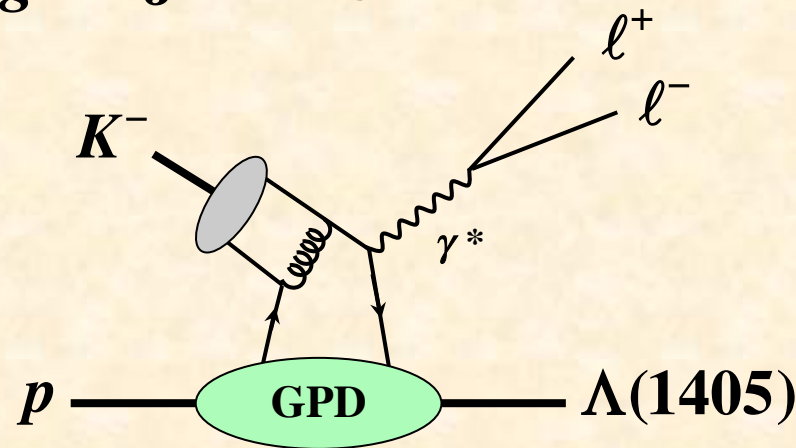


GPDs for exotic hadrons

Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ **Transition GPDs**

e.g. at J-PARC



$$K^-(\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

Comments on J-PARC project

**T. Sawada, W.-C. Chang, S. Kumano,
J.-C. Peng, S. Sawada, and K. Tanaka,
Phys. Rev. D93 (2016) 114034.**

**S. Kumano, Int. J. Mod. Phys.:
Conf. Series, 40 (2016) 1660009**

Aerial photograph



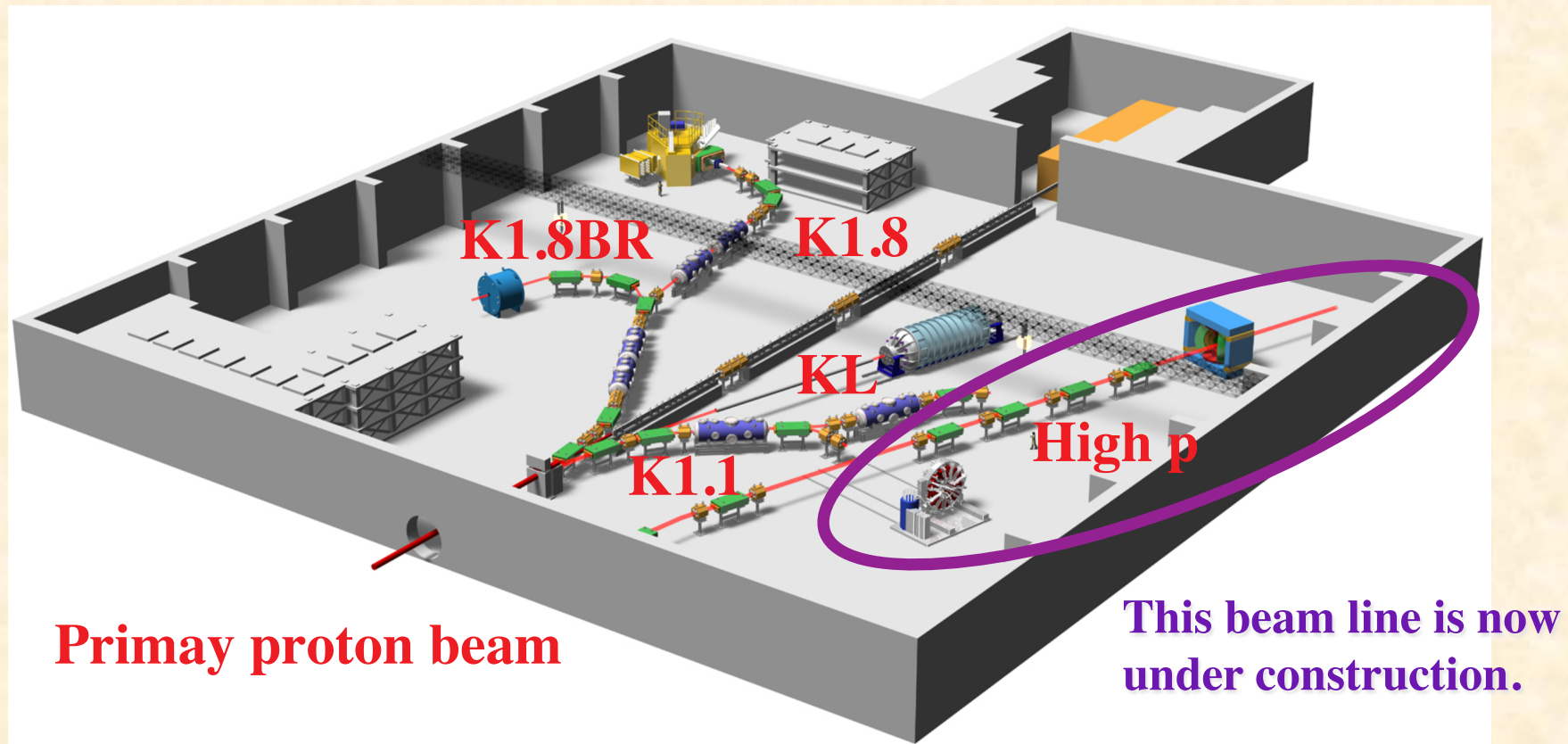
**KEK Tokai campus,
Theory activities**

**Neutrino
facility**

**Hadron
facility**

Hadron facility

Workshops on high-momentum beamline physics,
<http://www-conf.kek.jp/hadron1/j-parc-hm-2013/>
<http://research.kek.jp/group/hadron10/j-parc-hm-2015/>.



- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV

Toward a new proposal at J-PARC

T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.

Exclusive Drell-Yan: $\pi^- + p \rightarrow \mu^+ \mu^- + n$

PHYSICAL REVIEW D 93, 114034 (2016)

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC

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and J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies,
KEK, 203-1, Shirakata, Tokai, Ibaraki 319-1106, Japan*

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Shinya Sawada[¶]

High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

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*Department of Physics, Juntendo University, Inzai, Chiba 270-1695, Japan and J-PARC Branch,
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KEK, 203-1, Shirakata, Tokai, Ibaraki 319-1106, Japan
(Received 15 May 2016; published 29 June 2016)*

Generalized parton distributions (GPDs) encoding multidimensional information of hadron partonic structure appear as the building blocks in a factorized description of hard exclusive reactions. The nucleon GPDs have been accessed by deeply virtual Compton scattering and deeply virtual meson production with lepton beam. A complementary probe with hadron beam is the exclusive pion-induced Drell-Yan process. In this paper, we discuss recent theoretical advances on describing this process in terms of nucleon GPDs and pion distribution amplitudes. Furthermore, we address the feasibility of measuring the exclusive pion-induced Drell-Yan process $\pi^- p \rightarrow \mu^+ \mu^- n$ via a spectrometer at the High Momentum Beamline being constructed at J-PARC in Japan. Realization of such measurement at J-PARC will provide a new test of perturbative QCD descriptions of a novel class of hard exclusive reactions. It will also offer the possibility of experimentally accessing nucleon GPDs at large timelike virtuality.

Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$

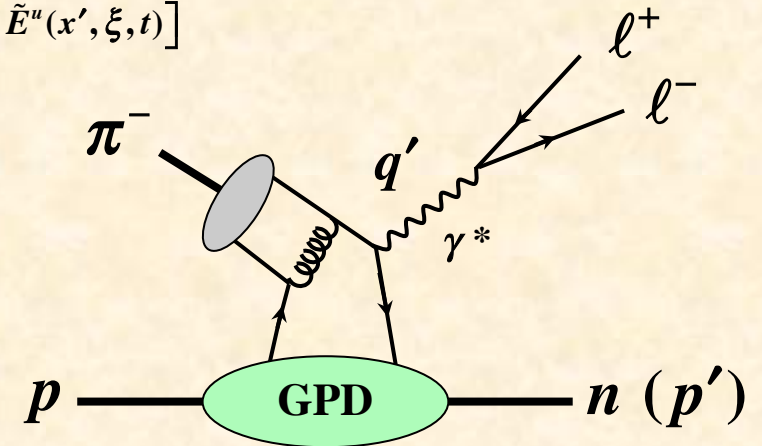
$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \simeq \frac{Q'^2}{s - m_N^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_s q(z/2) | p(p) \rangle_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_s u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_s \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_s q_u(z/2) | p(p) \rangle_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_s u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_s \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

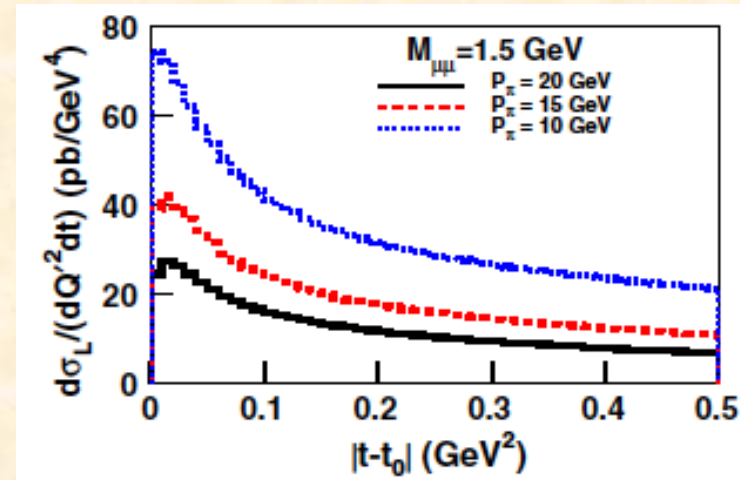
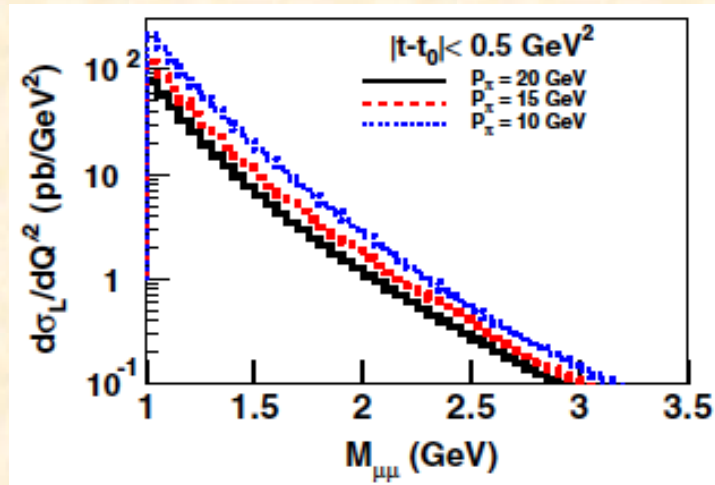


$$\pi^- (\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^* (\rightarrow \ell^+ \ell^-)$$

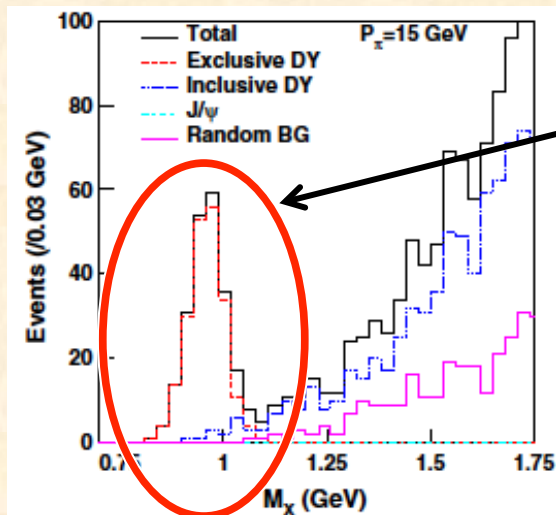
Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

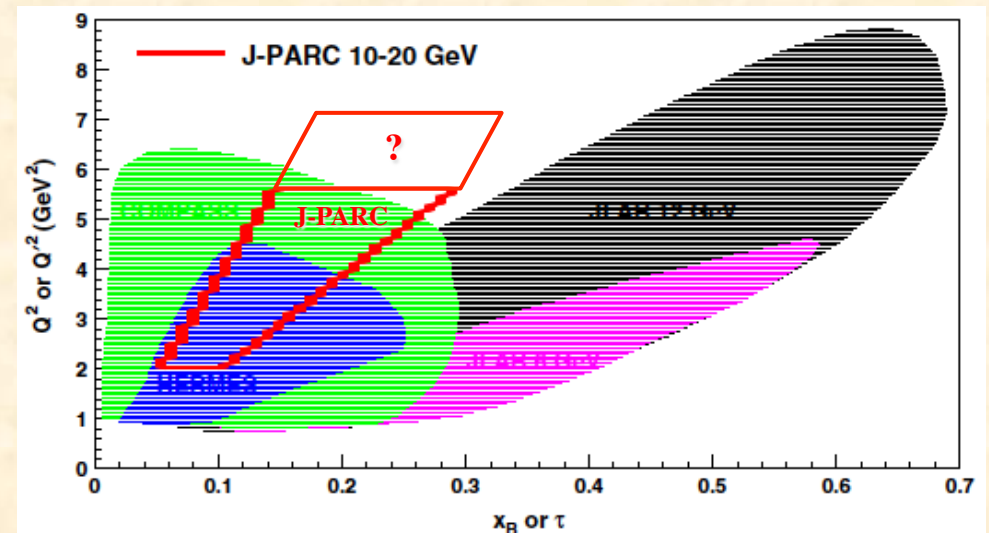
$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



Missing mass



**Exclusive
Drell-Yan**



Generalized Distribution Amplitudes (GDAs)

and KEKB project

**H. Kawamura and S. Kumano,
Phys. Rev. D 89 (2014) 054007.**

**S. Kumano and Q.-T. Song,
Research in progress.**

GPDs for exotic hadrons !?

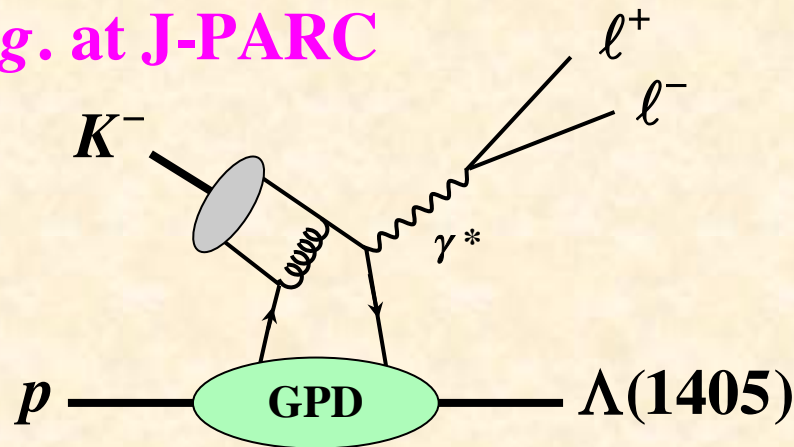
Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or

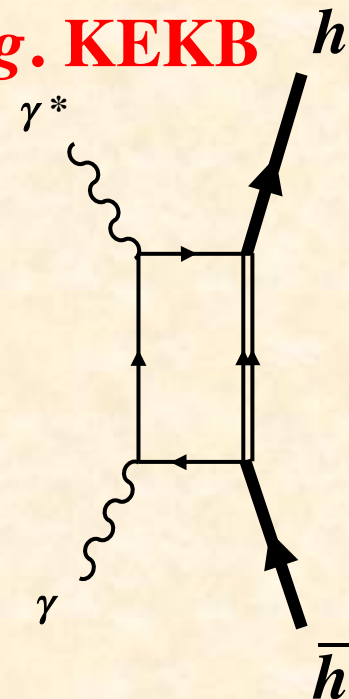
→ $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

e.g. at J-PARC



$$K^- (\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

e.g. KEKB



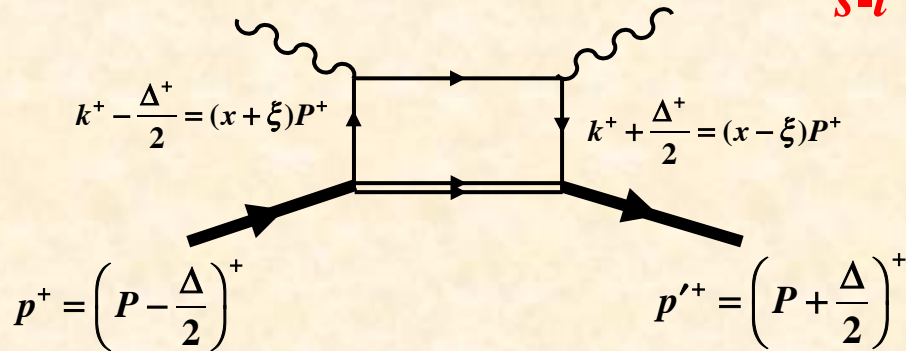
GPD $H_q^h(x, \xi, t)$ and GDA $\Phi_q^{h\bar{h}}(z, \zeta, W^2)$

$$\text{GPD: } H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$$

$$\text{GDA: } \Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$$

$$\text{DA: } \Phi_q^h(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$$

$H_q^h(x, \xi, t)$



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

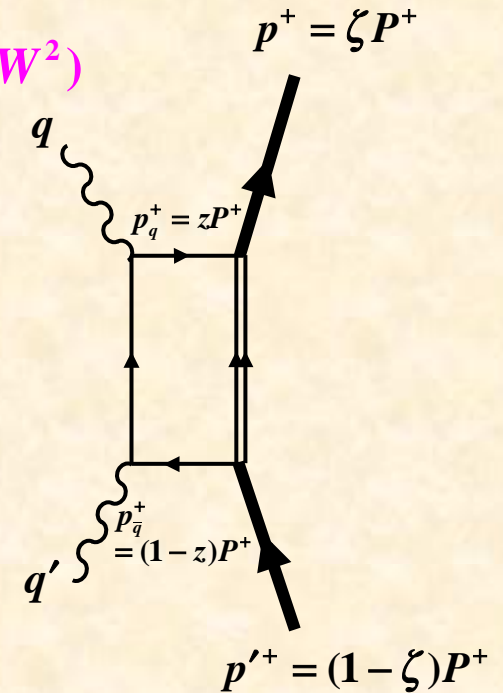
Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

$\longleftrightarrow$
 s - t crossing

$\Phi_q^{h\bar{h}}(z, \zeta, W^2)$



Bjorken variable for γ^* : $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for h in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $e^+ + e^- \rightarrow e^+ + e^- + h + \bar{h}$

$$\frac{d\sigma_{ee \rightarrow ee\pi\pi}}{dQ^2 dW^2 d(\cos\theta) d\phi dx_2} = N(x_2) \frac{d\sigma_{e\gamma \rightarrow e\pi\pi}}{dQ^2 dW^2 d(\cos\theta) d\phi}, \quad x_2 = \frac{s_{e\gamma}}{s_{ee}}$$

Cross section for $e + \gamma \rightarrow e' + h + \bar{h}$

$$d\sigma = \frac{1}{4\sqrt{(k \cdot q')^2 - k^2 q'^2}} \bar{\sum}_{\lambda_\gamma, \lambda_e, \lambda'_e} |M(e\gamma \rightarrow e'h\bar{h})|^2 d\Phi_3$$

$$M(e\gamma \rightarrow e'h\bar{h}) = \bar{u}(k')(ie\gamma^\mu)u(k) \frac{ig^{\mu\nu}}{q^2} T_{\nu\rho} \varepsilon^\rho(q')$$

$$T_{\mu\nu} = i \int d^4\xi e^{-i\xi \cdot q} \langle h(p)\bar{h}(p') | T J_\mu^{em}(\xi) J_\nu^{em}(0) | 0 \rangle = -g_{T\mu\nu} e^2 \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

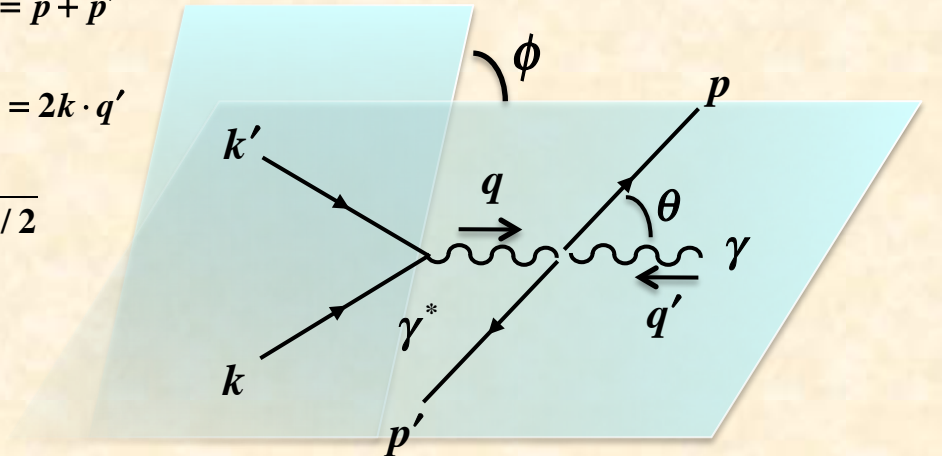
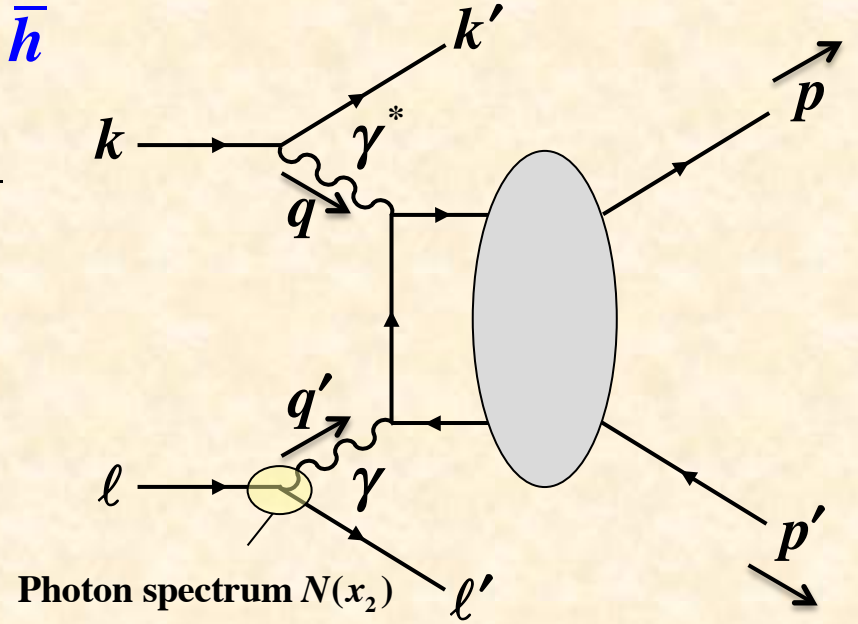
$$\Phi_q^{h\bar{h}}(z, \zeta, W^2) = \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle h(p)\bar{h}(p') | \bar{q}(x^-) \gamma^+ q(0) | 0 \rangle_{x^+=0, \vec{x}_\perp=0}, \quad P = p + p'$$

$$\frac{d\sigma_{e\gamma \rightarrow e\pi\pi}}{dQ^2 dW^2 d(\cos\theta) d\phi} = \frac{\alpha^3}{16\pi} \frac{\beta}{s_{e\gamma}^2} \frac{1}{Q^2(1-\varepsilon)} |A_{(+,+)}(\zeta, W^2)|^2, \quad s_{e\gamma} = (k + q')^2 = 2k \cdot q'$$

$$\text{longitudinal-transverse polarization ratio of } \gamma^*: \quad \varepsilon = \frac{1-y}{1-y+y^2/2}$$

$$A_{(+,+)} = \varepsilon_\mu^{(+)}(q) \varepsilon_\nu^{(+)}(q') T^{\mu\nu} / e^2 = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{hh}(z, \zeta, W^2)$$

$$\text{A simple form: } \Phi_q^{h\bar{h}}(z, \zeta, W^2) = N_h z^\alpha (1-z)^\beta (2z-1) \zeta (1-\zeta) F_{h(q)}(s)$$



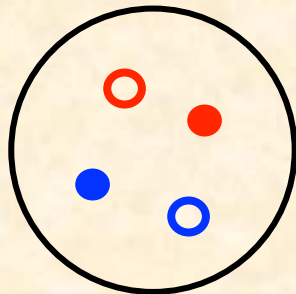
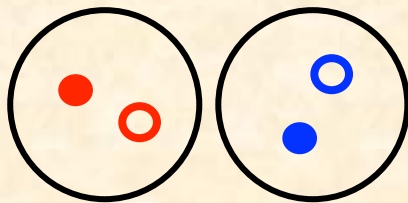
Cross section: form factor dependence

$$\Phi_q^{h\bar{h}(I=0)}(z, \zeta, W^2) \propto F_h(W^2)$$

Ordinal $q\bar{q}$



Molecule $K\bar{K}$
or tetra-quark $qq\bar{q}\bar{q}$

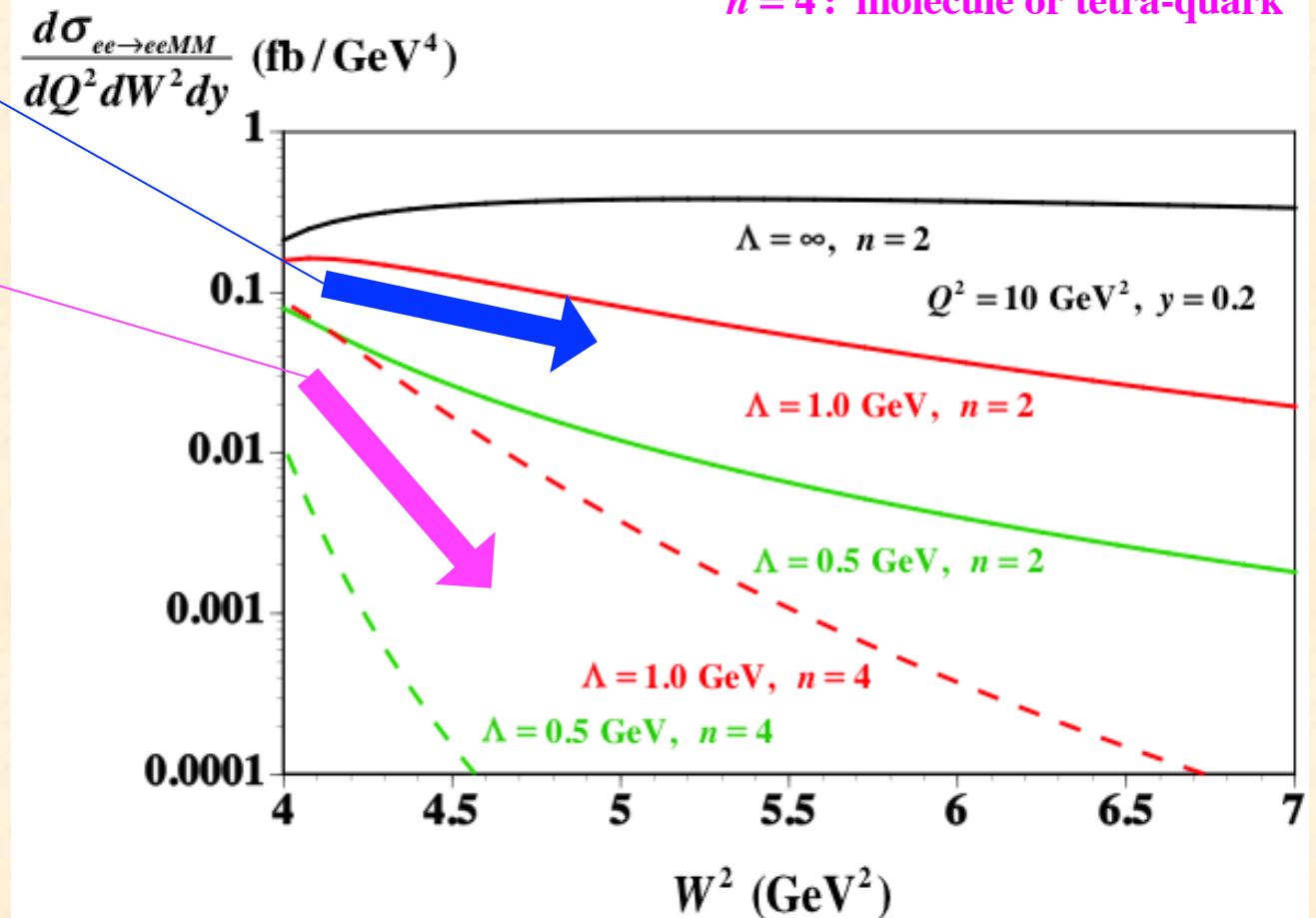


$$F_h(W^2) = \frac{1}{[1 + (W^2 - 4m_h^2) / \Lambda^2]^{n-1}}$$

Constituent-counting rule

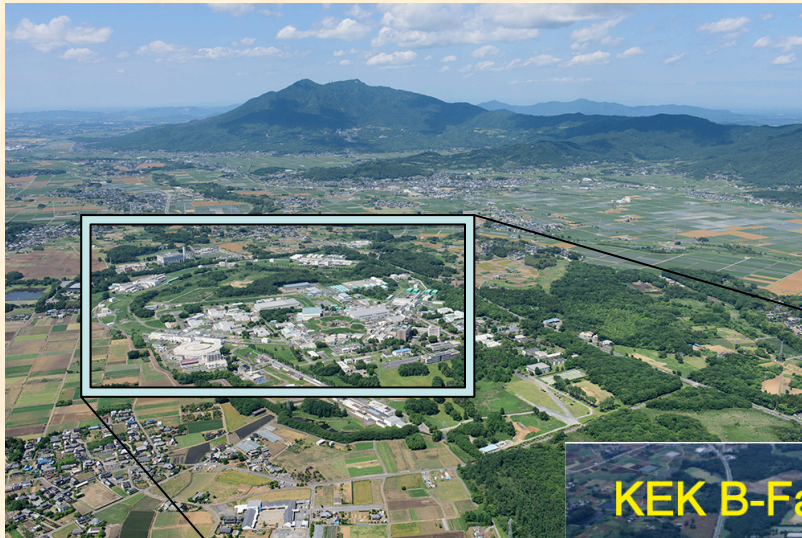
$n = 2$: ordinary meson

$n = 4$: molecule or tetra-quark

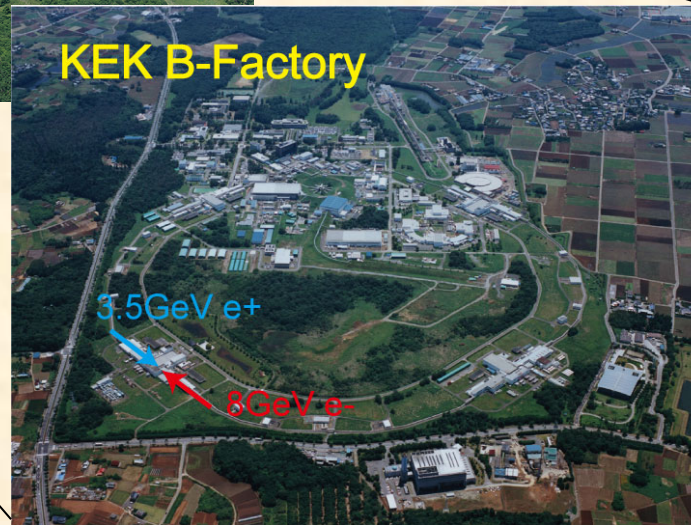


Experimental studies of GDAs in future

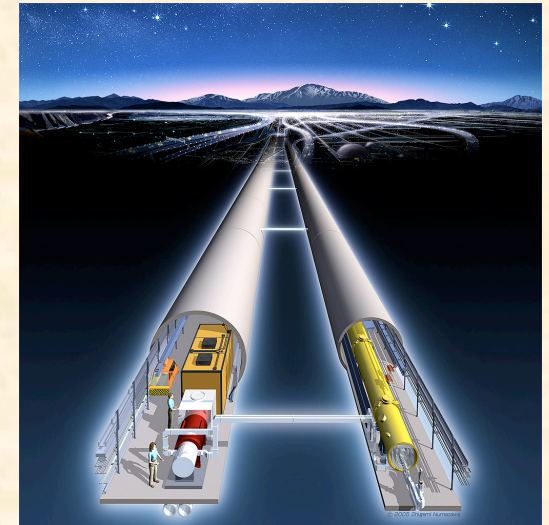
$\gamma\gamma \rightarrow h\bar{h}$ for internal structure of exotic hadron candidate h



KEK B-factory



Linear Collider ?



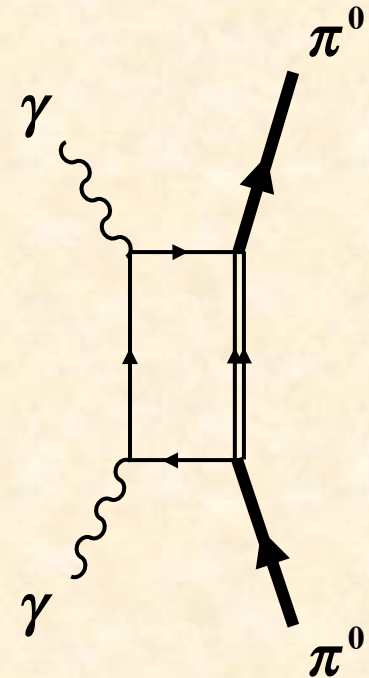
KEKB-Belle measurement (2016)

M. Masuda *et al.*, Phys. Rev. D 93 (2016) 032003 (arXiv:1508.06757).

PHYSICAL REVIEW D 93, 032003 (2016)

Study of π^0 pair production in single-tag two-photon collisions

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(The Belle Collaboration)



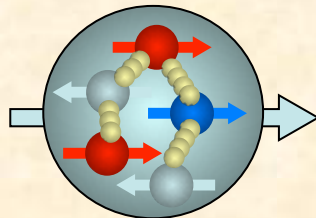
Research in progress to extract $\Phi_q^{\pi\pi}(z, \zeta, W^2) \dots$

Summary

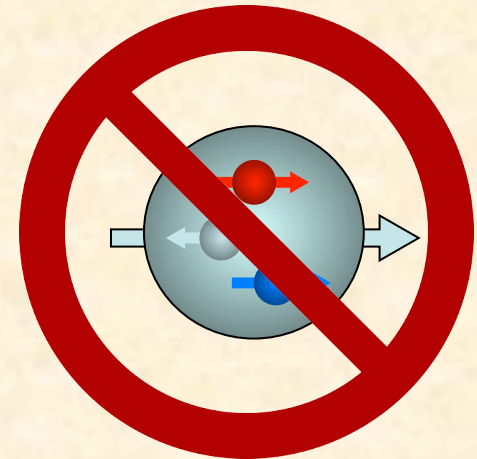
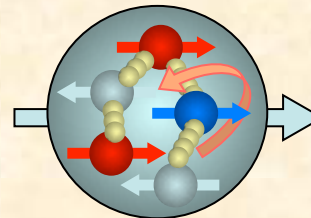
Origin of nucleon spin ...



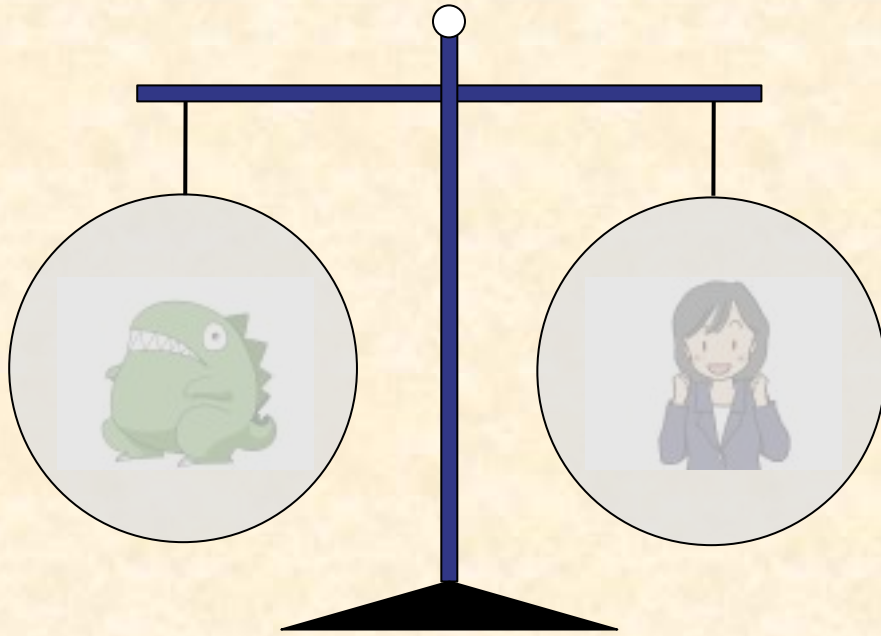
By the tomography, we determine



or



Search for exotic hadrons ...



It is difficult to determine whether or not a hadron is exotic by low-energy observables, masses, decay widths, ...

(Already, history of a half century)



By the tomography, we determine



or



.

Summary

**Hadron tomography studies are important
for solving the origin of the nucleon spin,
for probing internal structure of exotic hadrons.**

Constituent-counting rule for exotic hadrons

High energies = Quark and gluon degrees of freedom

**It is appropriate to use high-energy processes for determination
of internal configurations for exotic-hadron candidates.**

GPDs at J-PARC

**GPDs can be investigated by not only DVCS at lepton facilities
but also exclusive reactions at hadron facilities.**

GDAs at KEKB

3D structure of hadrons can be investigated by GDAs ($s \leftrightarrow t$).

Related experimental projects

**RHIC, Fermilab, CERN-COMPASS, JLab, BES, Liner Collider,
GSI, EIC, LHeC, ...**

The End

The End