





# 포화 그리고 기하학적 스케일링

미할 프라샬로비츠  
야게로난 대학, 크라코프, 폴란드

제주도, 1.9.2016.



# Saturation and geometrical scaling

Michał Prasałowicz

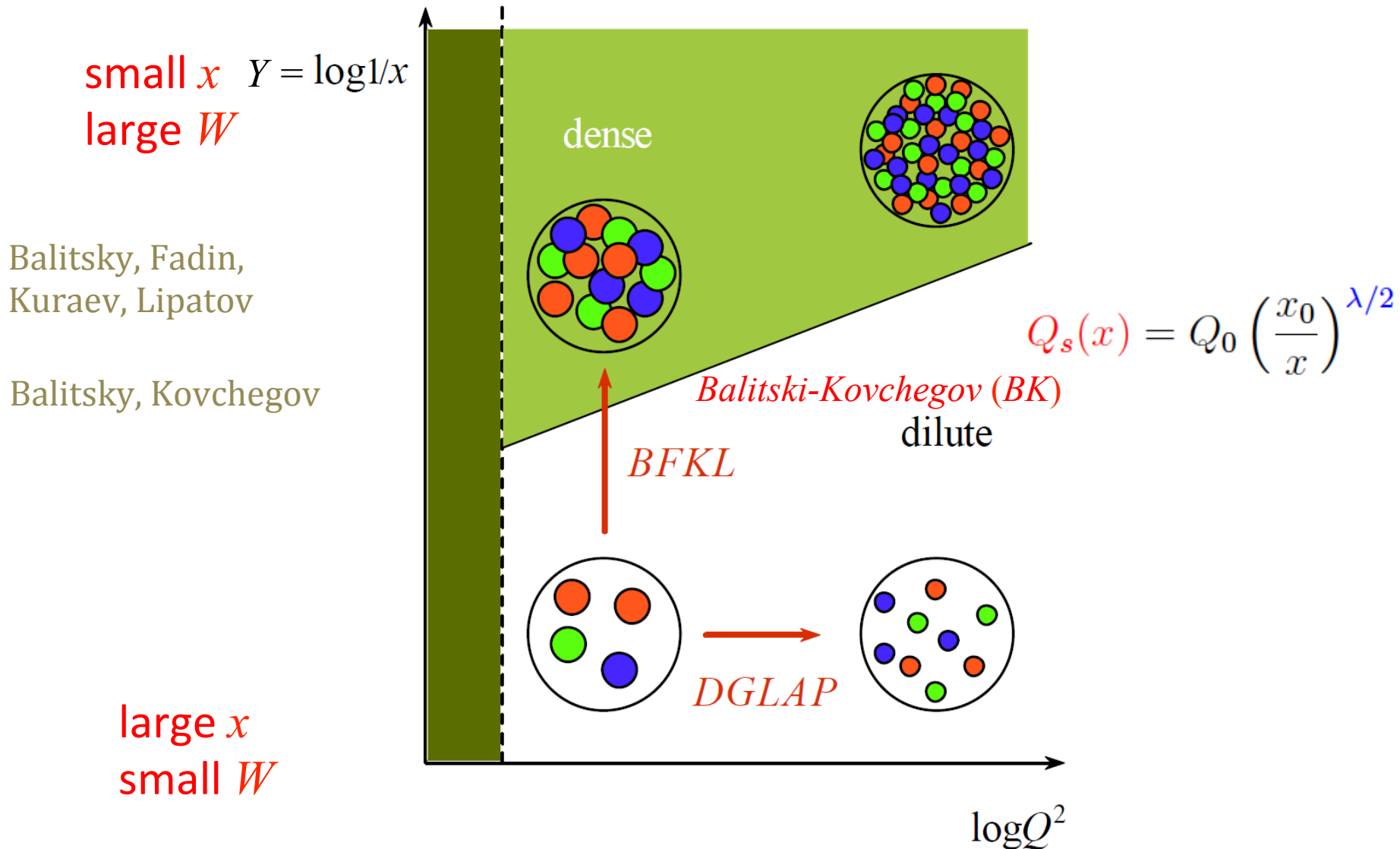
M. Smoluchowski Inst. of Physics

Jagiellonian University, Kraków, Poland

Jeju Island 1.9.2016.



# DGLAP vs BFKL Evolution

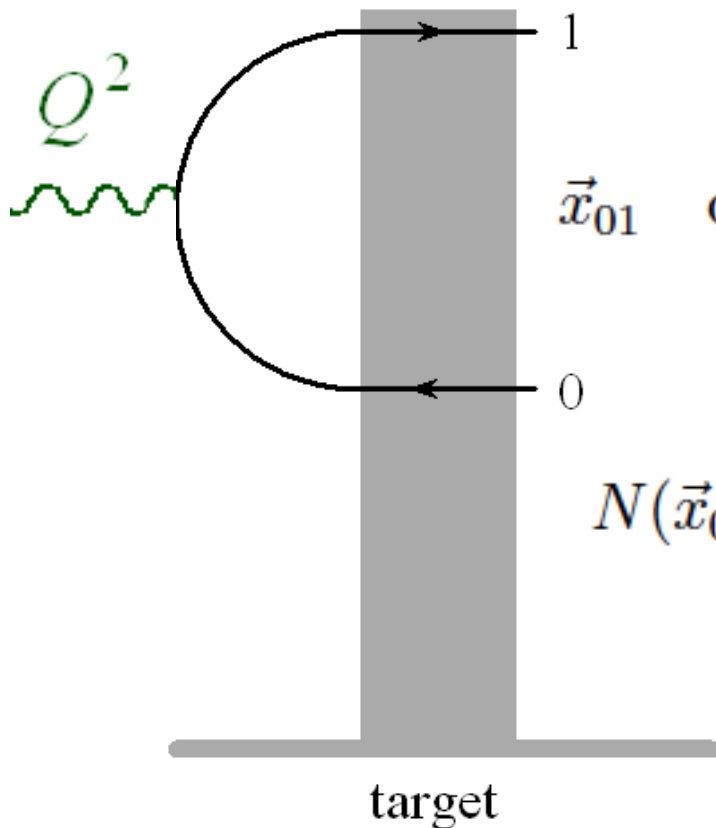




# Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller

A.H. Mueller and J.-w. Qiu,  
Nucl. Phys. B 268 (1986) 427



$\vec{x}_{01}$  dipole transverse size  $Y = \log 1/x$

$N(\vec{x}_{01}, Y)$  dipole-target forward amplitude



# BK Equation

in terms of a Fourier transform: 
$$N(x, Y) = x^2 \int \frac{d^2 \vec{k}}{2\pi} e^{i\vec{k} \cdot \vec{x}} \tilde{N}(k, Y)$$

$$\frac{\partial}{\partial Y} \tilde{N}(k, Y) = \bar{\alpha}_s \chi(-\partial/\partial \ln k^2) \tilde{N}(k, Y) - \bar{\alpha}_s \tilde{N}^2(k, Y)$$

here  $\chi$  is a BFKL characteristic function

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$$

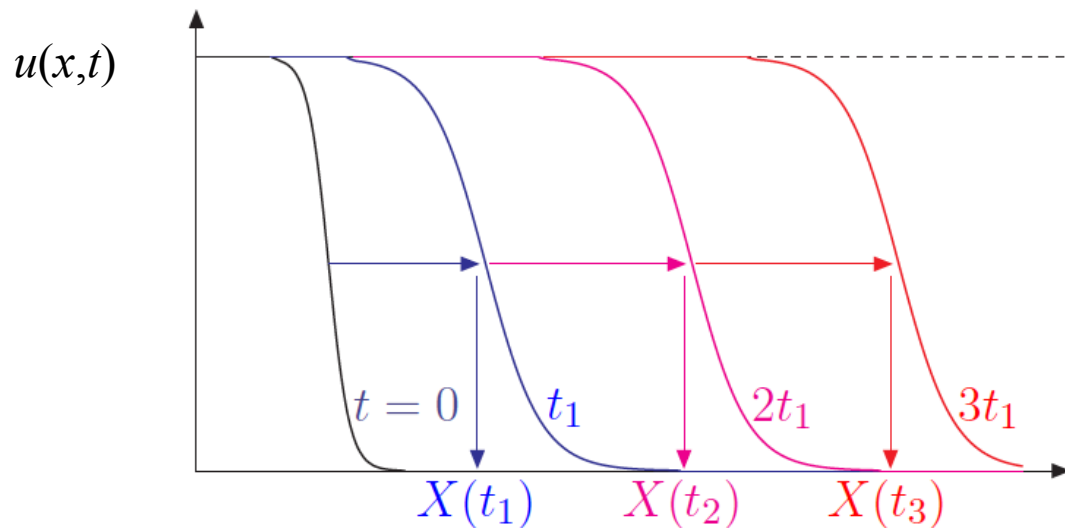
there exists a theorem from the '30 (Fisher, Kolomogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions



S. Munier, R.B. Peschanski  
PRL 91 (2003) 232001  
PRD 69 (2004) 034008

# Travelling waves

identify time :  $t = Y$ , position :  $x = \ln k^2$



Asymptotic solution:  
travelling wave  
 $u(x,t) = u(x - v_c t)$

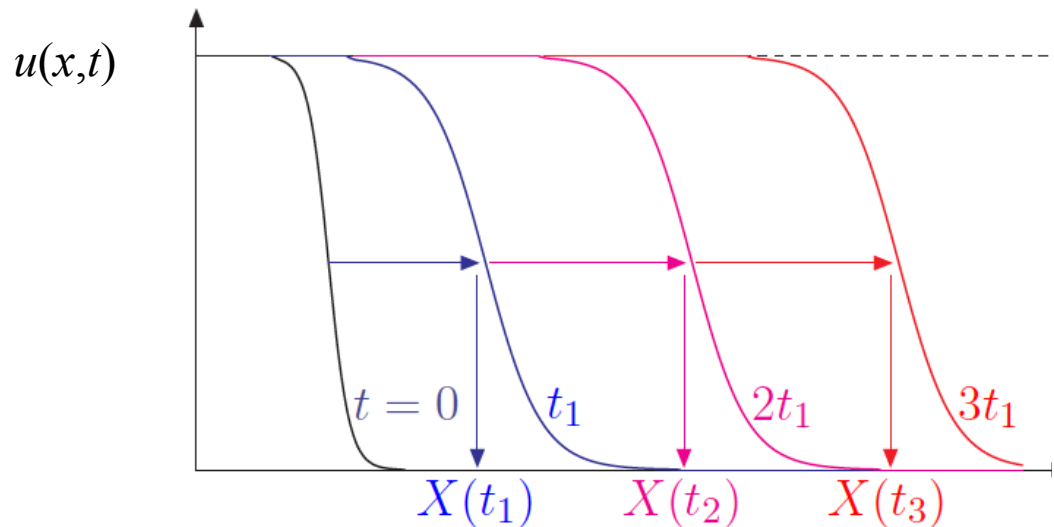
Position:  $X(t) = X_0 + v_c t$  © G. Soyez



# Travelling waves

identify

time :  $t = Y,$  position :  $x = \ln k^2$



Asymptotic solution:  
travelling wave

$$u(x, t) = u(x - v_c t)$$

$$x - v_c t = \log \left( \frac{k^2}{k_0^2} \right) - v_c \log \left( \frac{1}{x} \right)$$

$$= \log \left[ k^2 \times \frac{1}{k_0^2} \left( \frac{1}{x} \right)^{-v_c} \right]$$

$$= \log \left( \frac{k^2}{Q_{\text{sat}}^2(x)} \right)$$

Position:  $X(t) = X_0 + v_c t$  © G. Soyez

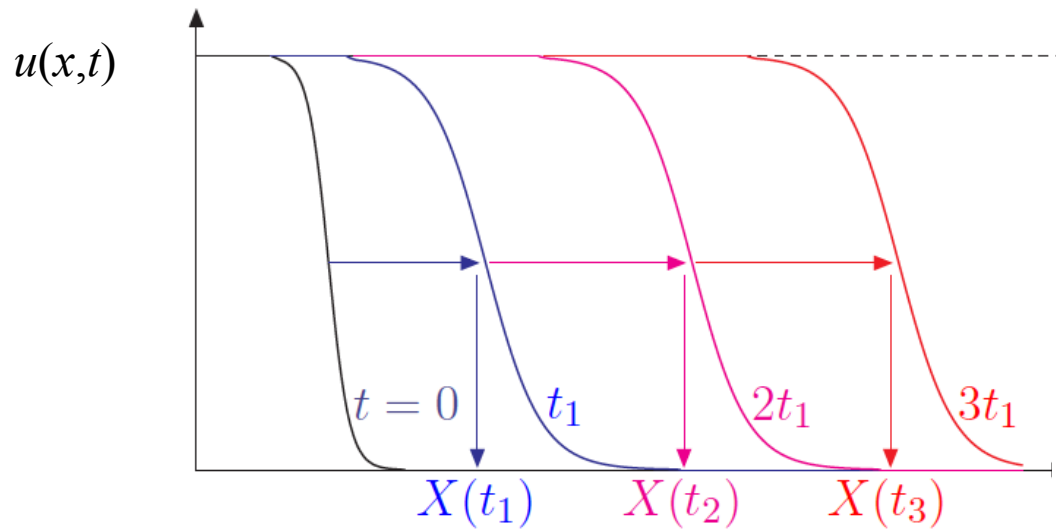




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$$= \log \left( \frac{k^2}{Q_{\text{sat}}^2(x)} \right)$$

Position:  $X(t) = X_0 + v_c t$  © G. Soyez

$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda$$



# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$



# Travelling waves in QCD imply Geometrical Scaling

$$f(x, k^2) = \mathcal{F} \left( \frac{k^2}{Q_s^2(x)} \right)$$

$$Q_s(x) = Q_0 \left( \frac{x_0}{x} \right)^{\lambda/2}$$



# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges



# Deep Inelastic Scattering



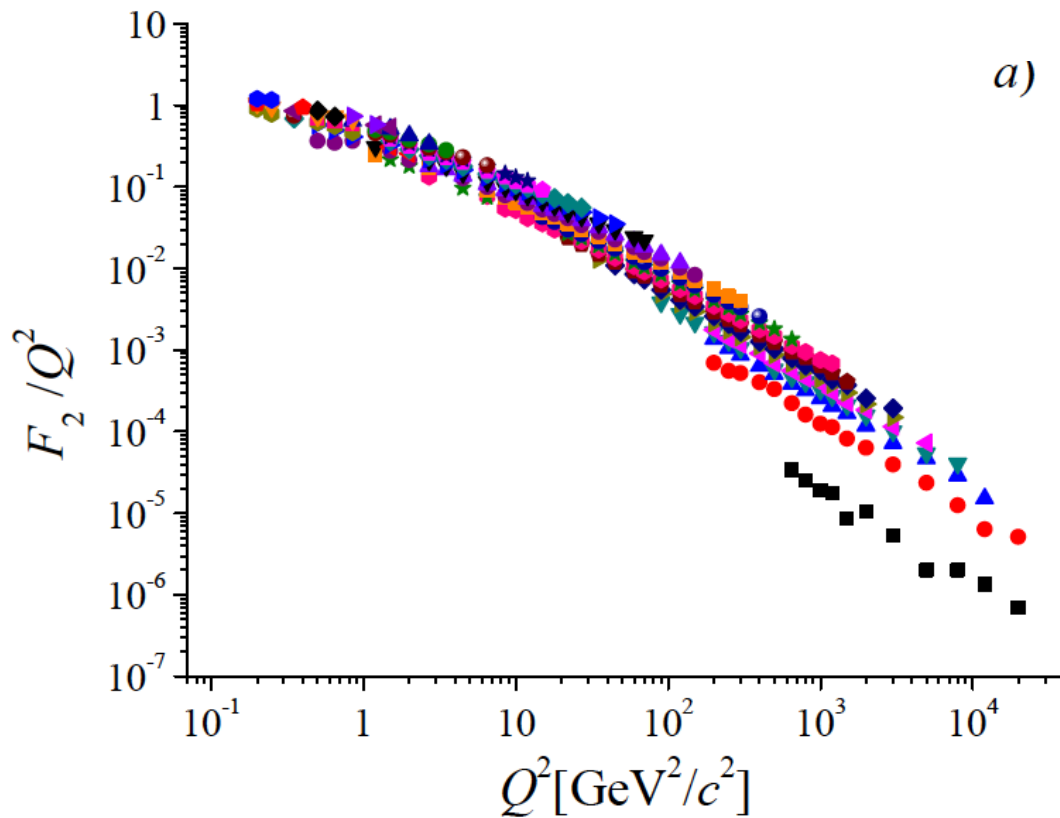
# Saturation scale: energy and $x$ dependence

$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$

a)

A.M. Stasto, K. J. Golec-Biernat,  
J. Kwiecinski  
PRL 86 (2001) 596-599

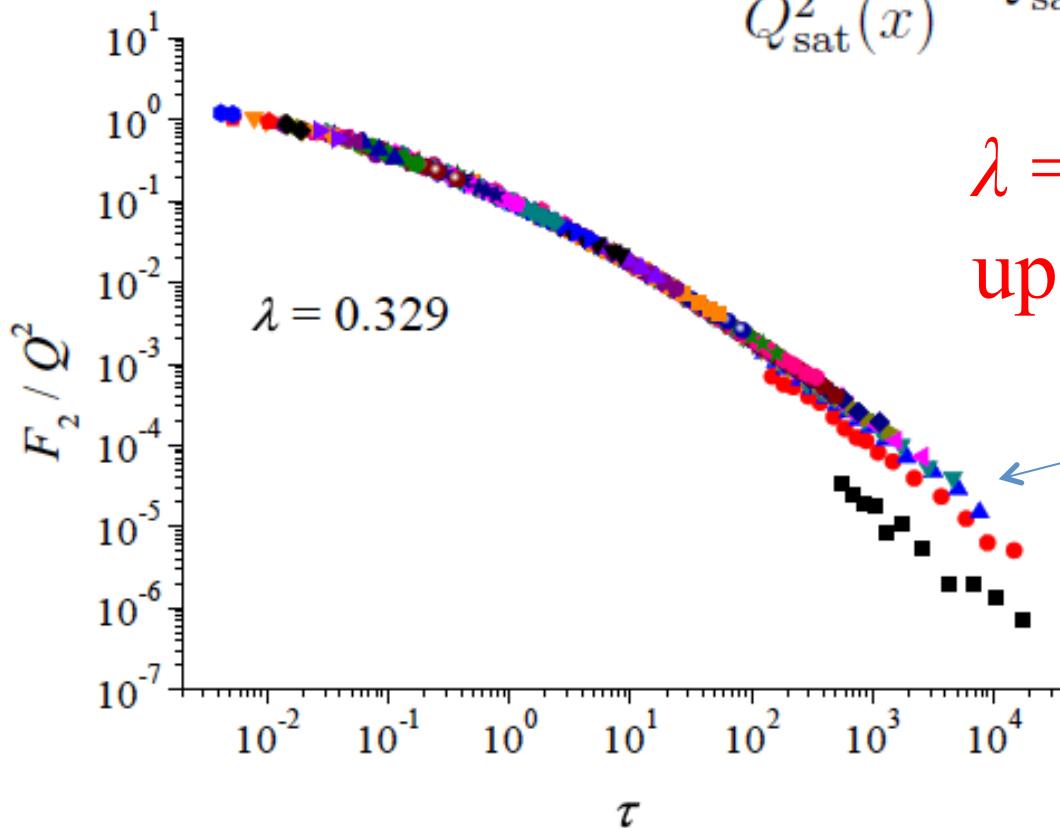
M.Praszalowicz and T.Stebel  
JHEP 1303, 090 (2013)  
arXiv:1211.5305 [hep-ph]  
and  
JHEP 1304, 169 (2013)  
arXiv:1302.4227 [hep-ph]





# Saturation scale: energy and $x$ dependence

$$\tau = \frac{Q^2}{Q_{\text{sat}}^2(x)} \quad Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$   
up to  $x = 0.08$  (!)

large  $x$

more "sophisticated" scaling  
variables do not work well

# Workshop on QCD and Diffraction

*saturation 1000+*

5-7 December 2016  
Cracow, Poland

[qcdworkshop.ifj.edu.pl](http://qcdworkshop.ifj.edu.pl)

## Organising Committee:

Wojciech Broniowski    Christophe Royon  
Janusz Chwastowski    Anna Staśto  
Krzysztof Kutak        Rafał Staszewski  
Michał Praszalowicz







# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$
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- GS works well in DIS up to relatively large  $x \sim 0.08$  with  $\lambda \sim 0.33$



# proton-proton @ LHC

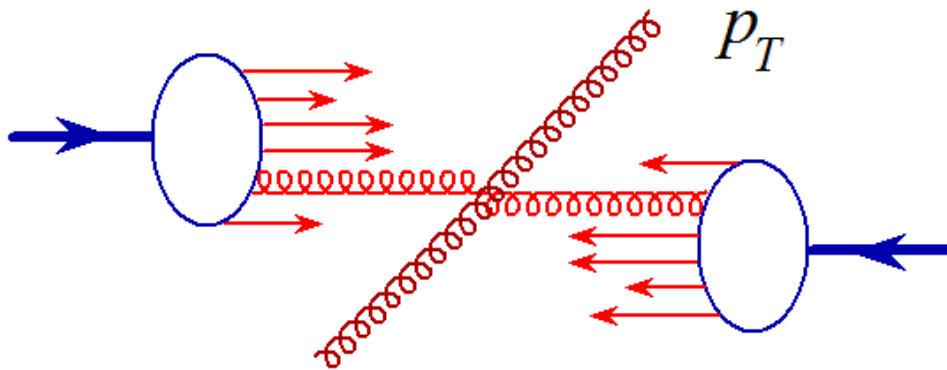


# Basics of geometrical scaling

Gribov, Levin Ryskin, *High  $p_T$  Hadrons In The Pionization Region In QCD.*  
Phys.Lett.B100:173-176,1981.

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2p_T^2} \int d^2\vec{k}_T \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2)$$

$$x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}$$



gluon distribution  $xG(x, Q^2) = \int^{Q^2} dk_T^2 \varphi(x, k_T^2)$  unintegrated glue

Kharzeev, Levin  
Phys.Lett.B523:79-87,2001.



# Basics of geometrical scaling

gluon distribution  $xG(x, Q^2) = \int^{Q^2} dk_{\perp}^2 \varphi(x, k_{\perp}^2)$  unintegrated glue

Golec-Biernat – Wuesthoff (DIS)

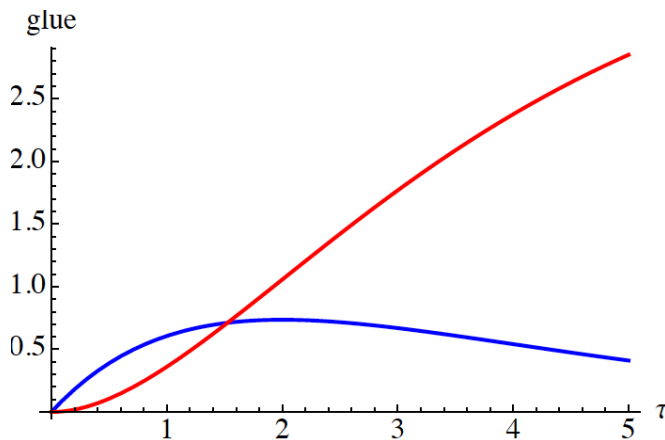
Kharzeev – Levin (AA)

$$\varphi(x, k_{\perp}^2) = S_{\perp} \frac{3}{4\pi^2} \frac{k_{\perp}^2}{Q_s(x)^2} \exp(-k_{\perp}^2/Q_s(x)^2)$$

$$S_{\perp} = \sigma_0$$

$$\varphi(x, k_{\perp}^2) = S_{\perp} \begin{cases} 1 & \text{for } k_{\perp}^2 < Q_s(x)^2 \\ Q_s(x)^2/k_{\perp}^2 & \text{for } Q_s(x)^2 < k_{\perp}^2 \end{cases}$$

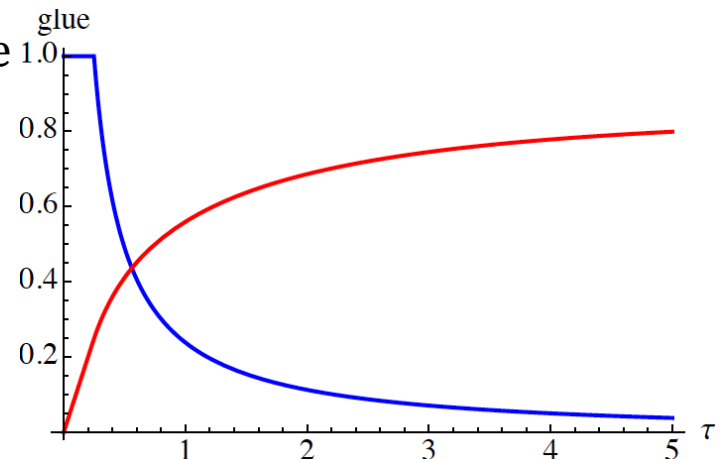
$S_{\perp}$  is the transverse size given by geometry



scaling variable

$$\tau = \frac{p_{\perp}^2}{Q_s^2(x)}$$

Michał Praszalowicz





# Basics of geometrical scaling

for  $y \sim 0$  (central rapidity) *i.e.* for  $x_1 \sim x_2 = x$  and for symmetric systems

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_1\left(\frac{\vec{k}_T^2}{Q_s^2(x)}\right) \varphi_2\left(\frac{(\vec{k} - \vec{p})^2}{Q_s^2(x)}\right)$$



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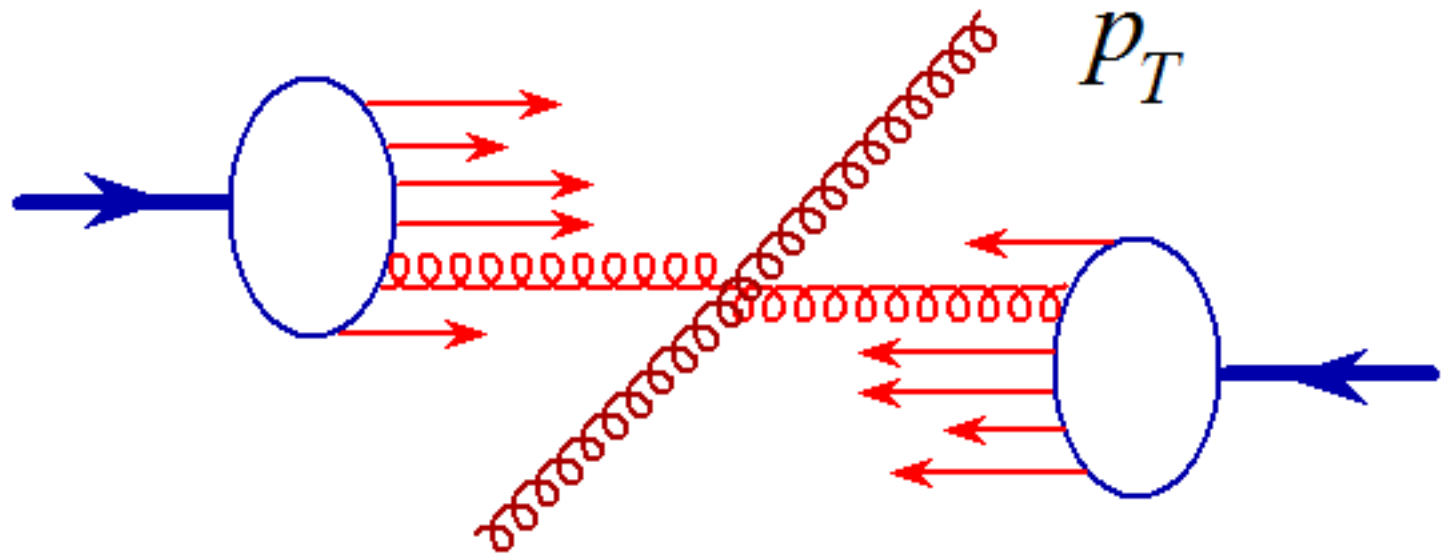
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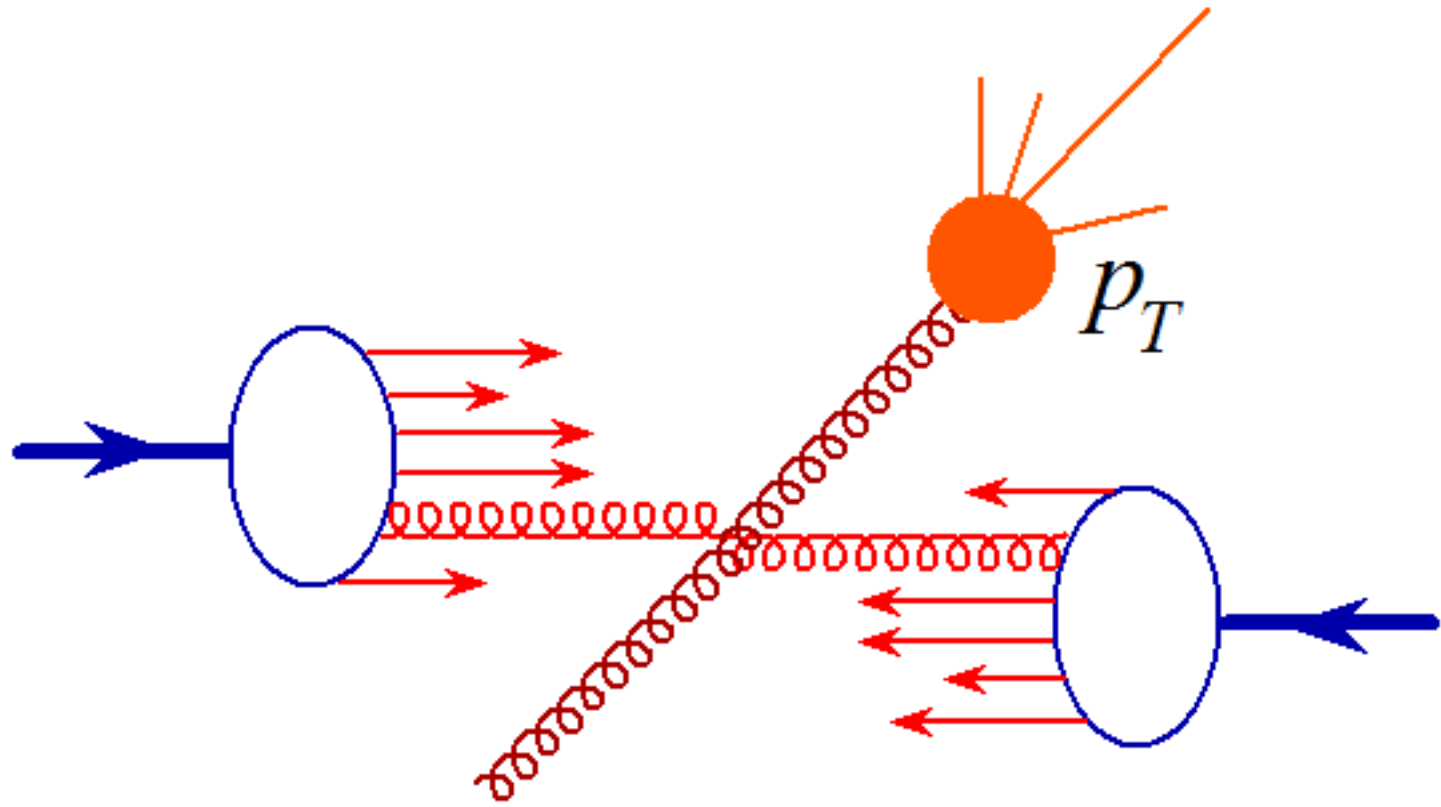
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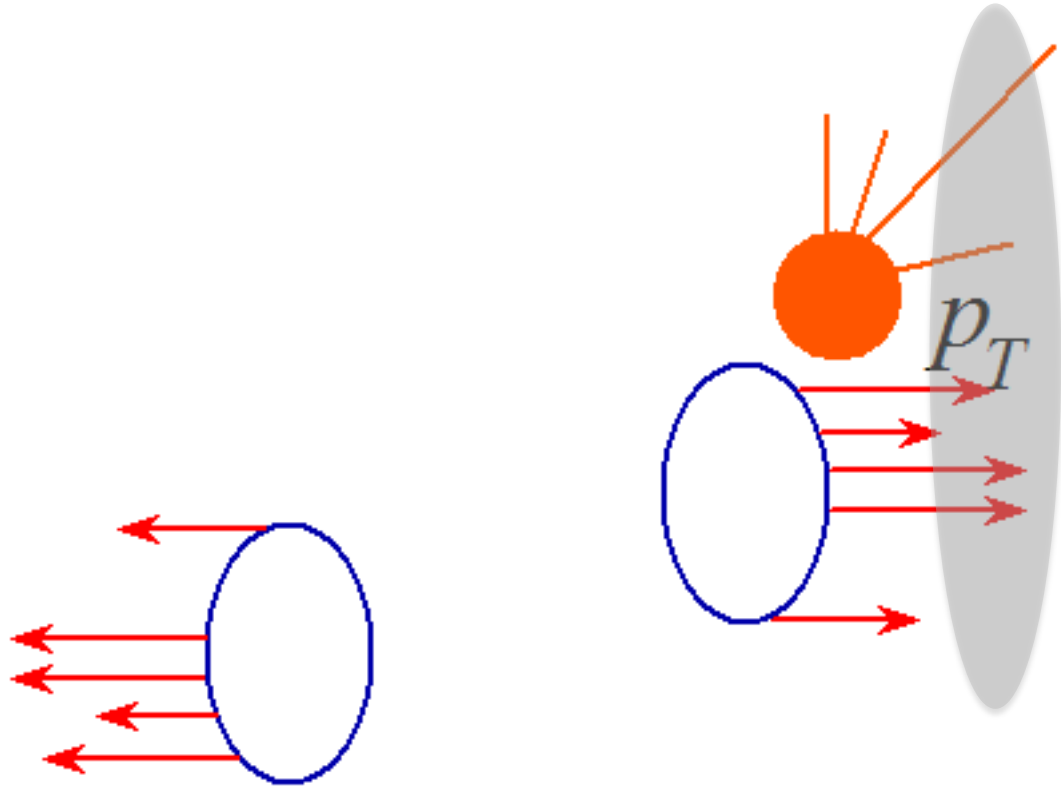
$$\frac{d\sigma}{dyd^2p_T} = S_{\perp}^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

parton – hadron duality:  
power-like growth of  
particle multiplicity











# Geometrical scaling of $p_T$ distributions

L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011

M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566

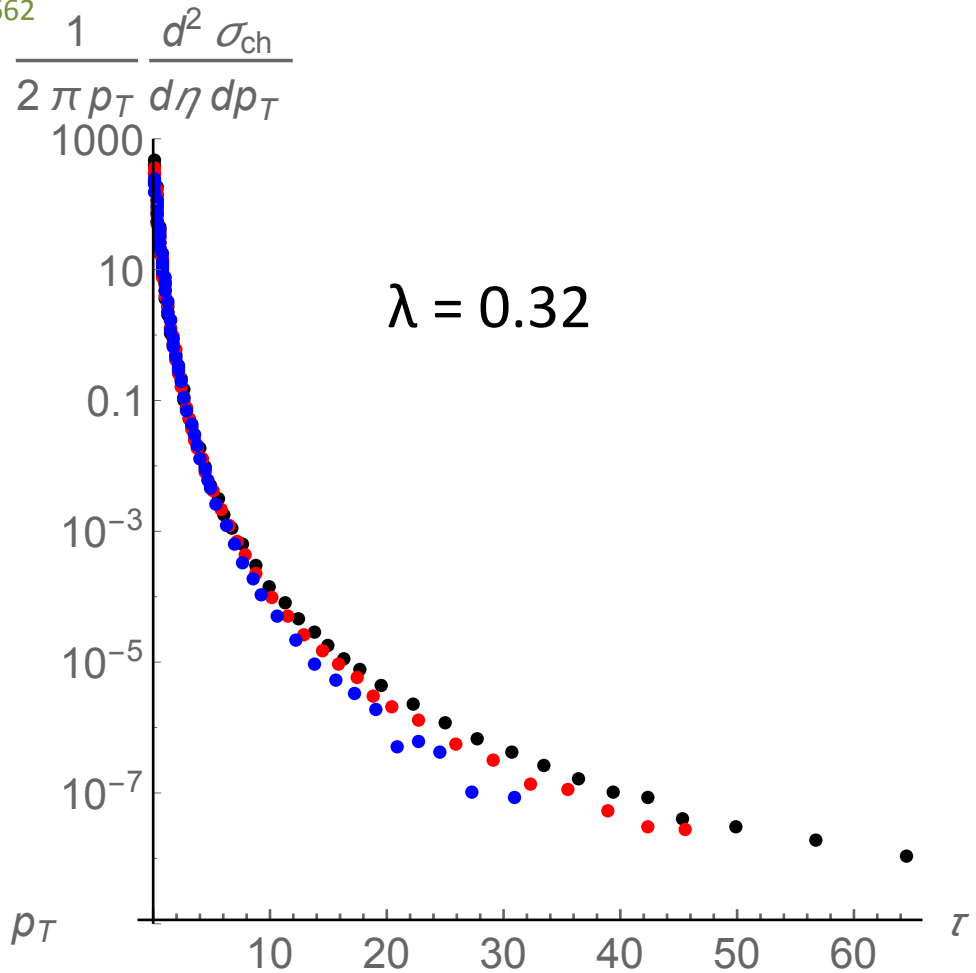
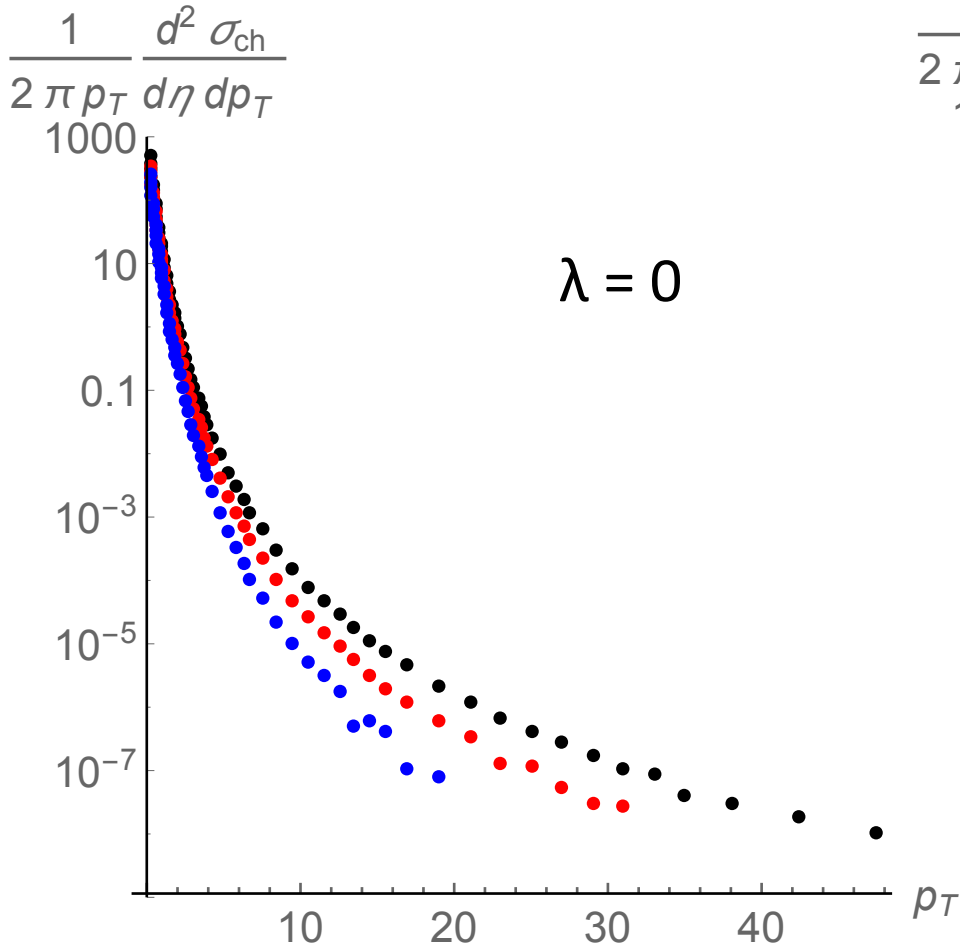
Phys.Rev. D87 (2013) 071502(R)

$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$



# Cross-section scaling in pp

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662

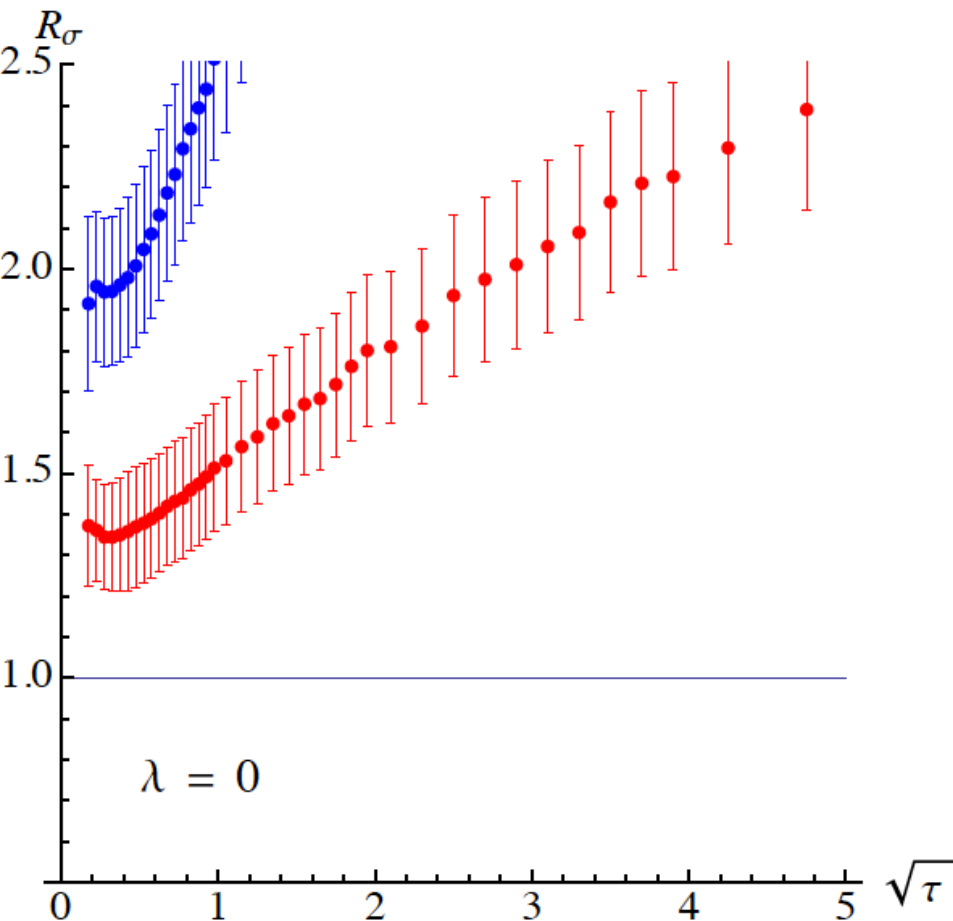


ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662



# Determination of lambda

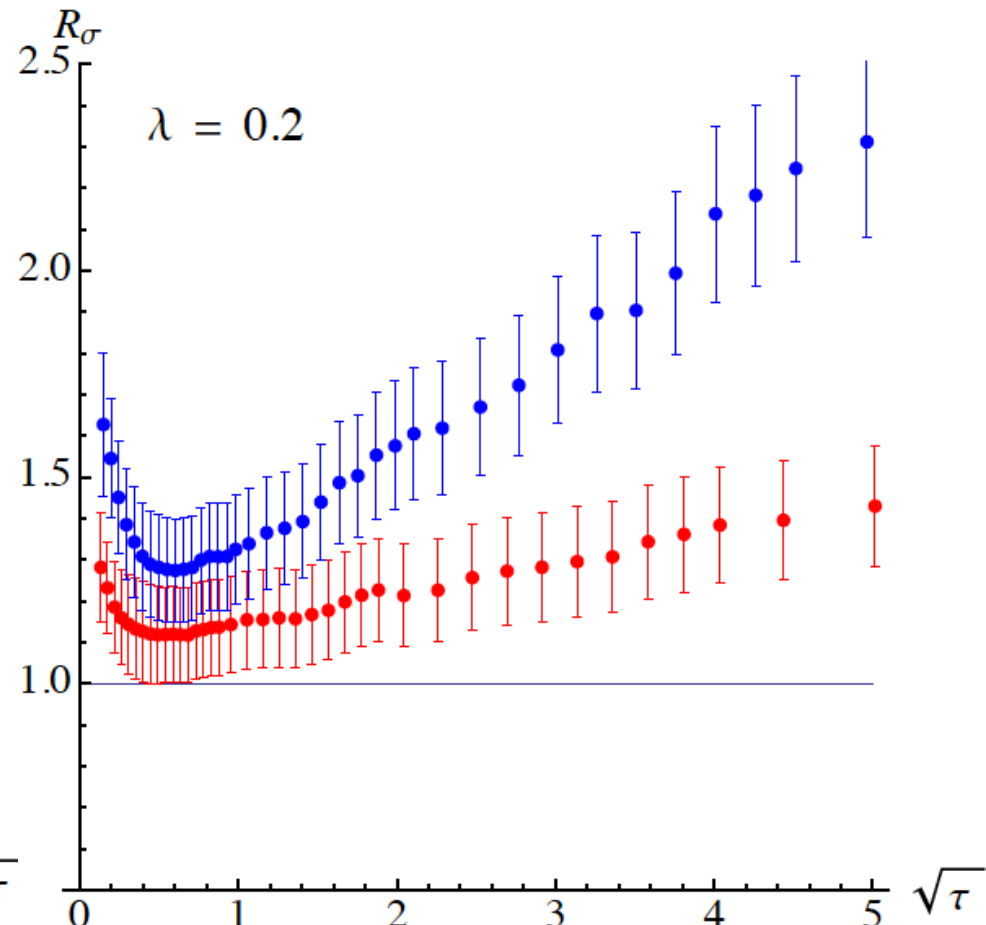
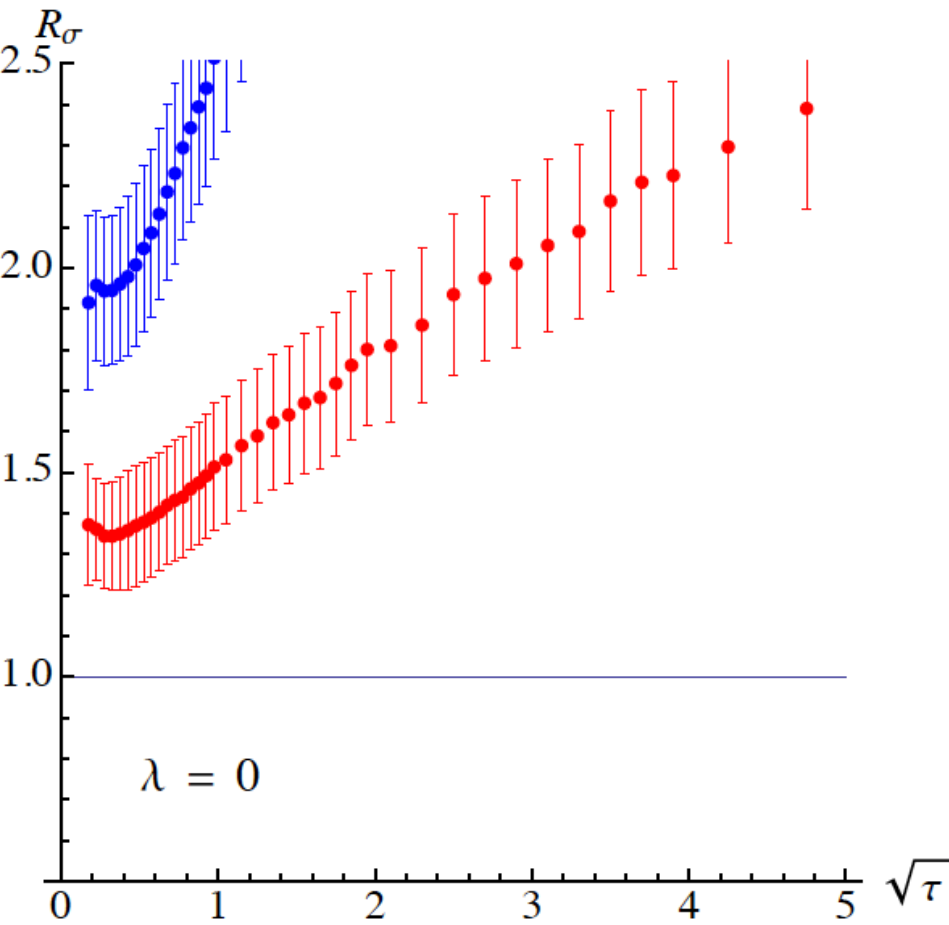
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# Determination of lambda

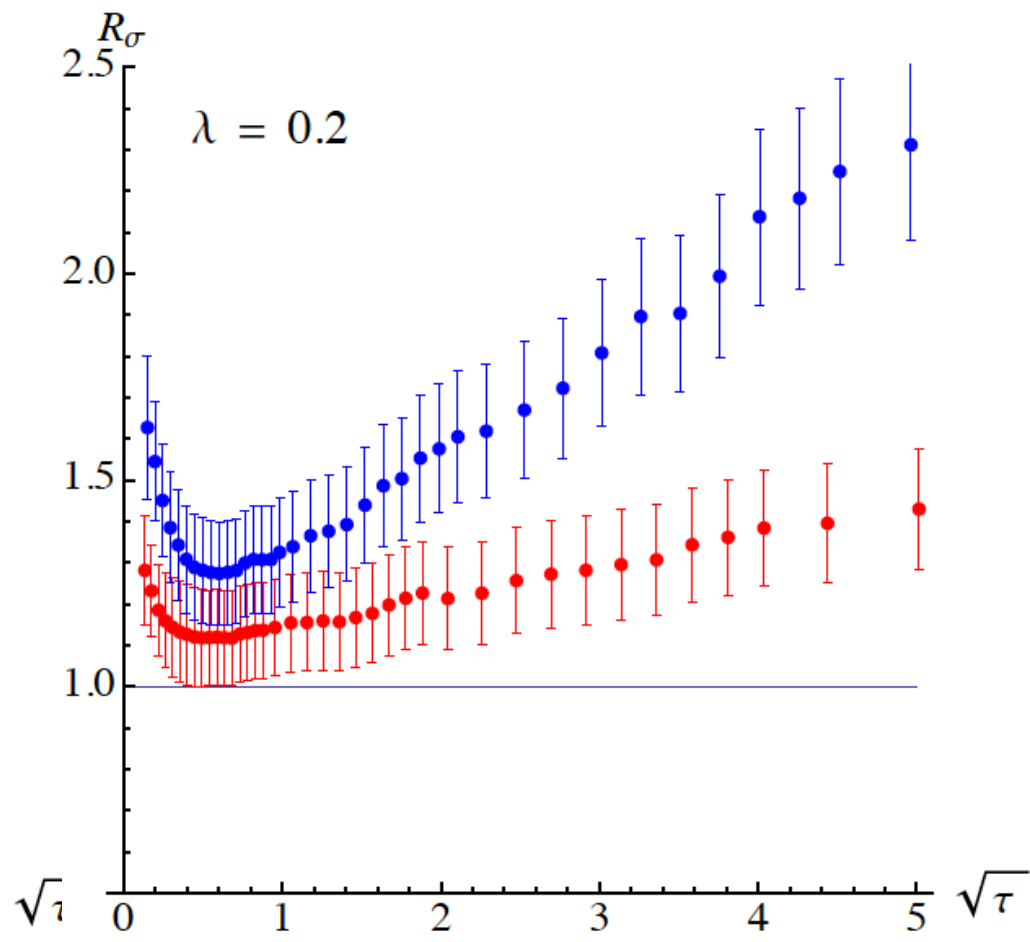
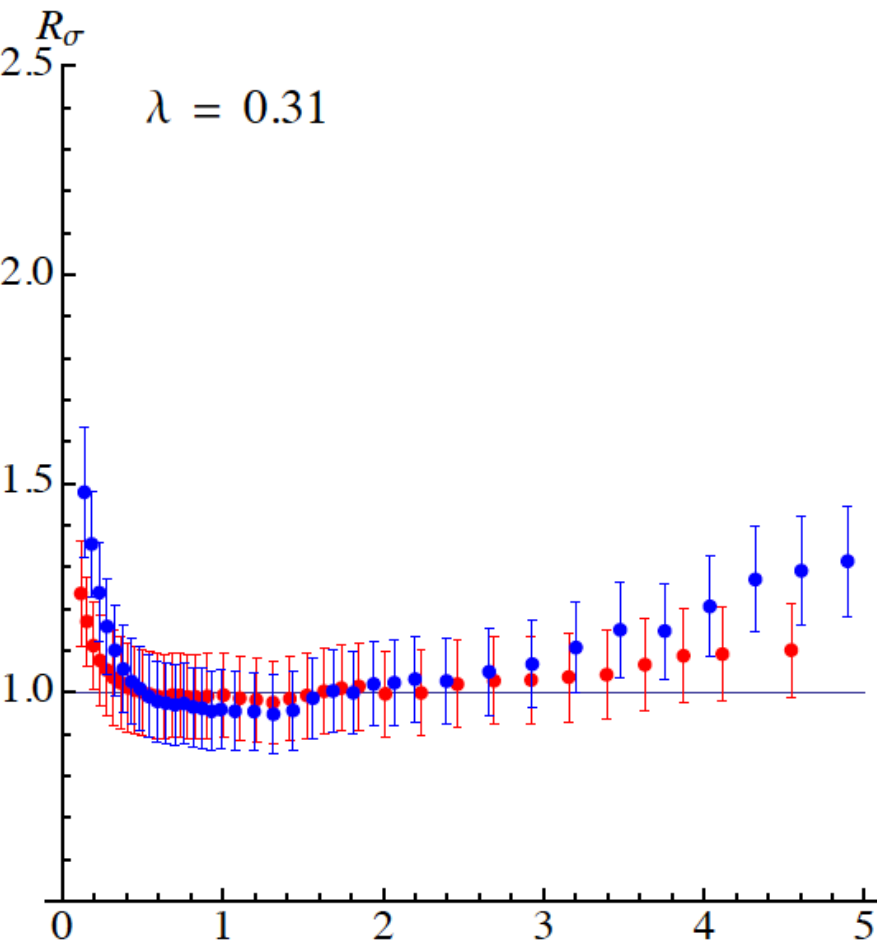
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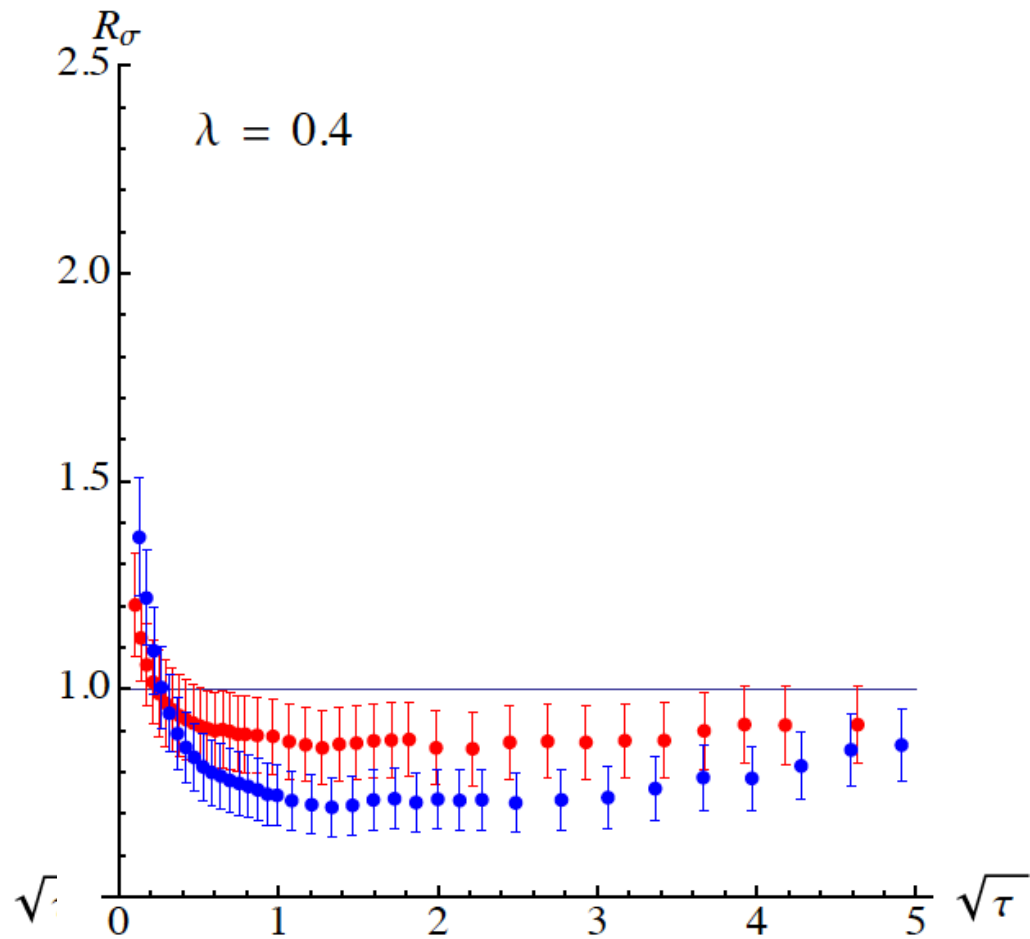
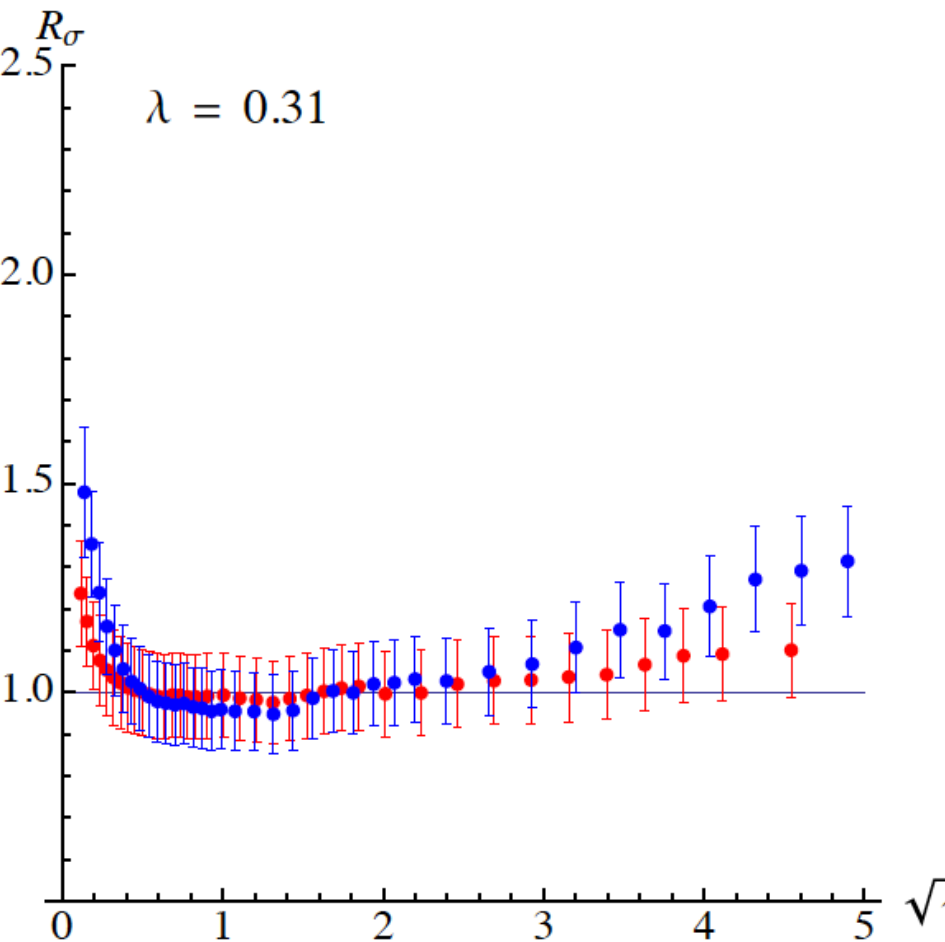
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# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$
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- GS works well in DIS up to relatively large  $x \sim 0.08$  with  $\lambda \sim 0.33$
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# Basics of geometrical scaling

for  $y \sim 0$  (central rapidity) *i.e.* for  $x_1 \sim x_2 = x$  and for symmetric systems

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s Q_s^2(x)}{2 p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_1 \left( \frac{\vec{k}_T^2}{Q_s^2(x)} \right) \varphi_2 \left( \frac{(\vec{k} - \vec{p})^2}{Q_s^2(x)} \right)$$

$$\frac{d\sigma}{dyd^2p_T} = S_{\perp}^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau$$
$$\bar{Q}_s(W) = Q_0 \left( \frac{W}{Q_0} \right)^{\lambda/(2+\lambda)}$$



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$$\frac{d\sigma}{dy} = S_{\perp}^2 \int \mathcal{F}(\tau) d^2p_T = S_{\perp}^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \dots d\tau = \frac{1}{\kappa} S_{\perp}^2 \bar{Q}_s^2(W)$$



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$$\frac{d\sigma}{dy} = S_{\perp} \frac{dN}{dy} = \frac{1}{\kappa} S_{\perp}^2 \bar{Q}_s^2(W) \rightarrow \bar{Q}_s^2(W) = \frac{\kappa}{S_{\perp}} \frac{dN}{dy}$$





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$$\frac{d\sigma}{dy} = \sigma^{\text{MB}}(W) \frac{dN}{dy} = \frac{1}{\kappa} S_{\perp}^2 \bar{Q}_s^2(W) \rightarrow \left[ S_{\perp} \frac{\bar{Q}_s^2(W)}{\sigma^{\text{MB}}(W)} \right] = \frac{\kappa}{S_{\perp}} \frac{dN}{dy}$$

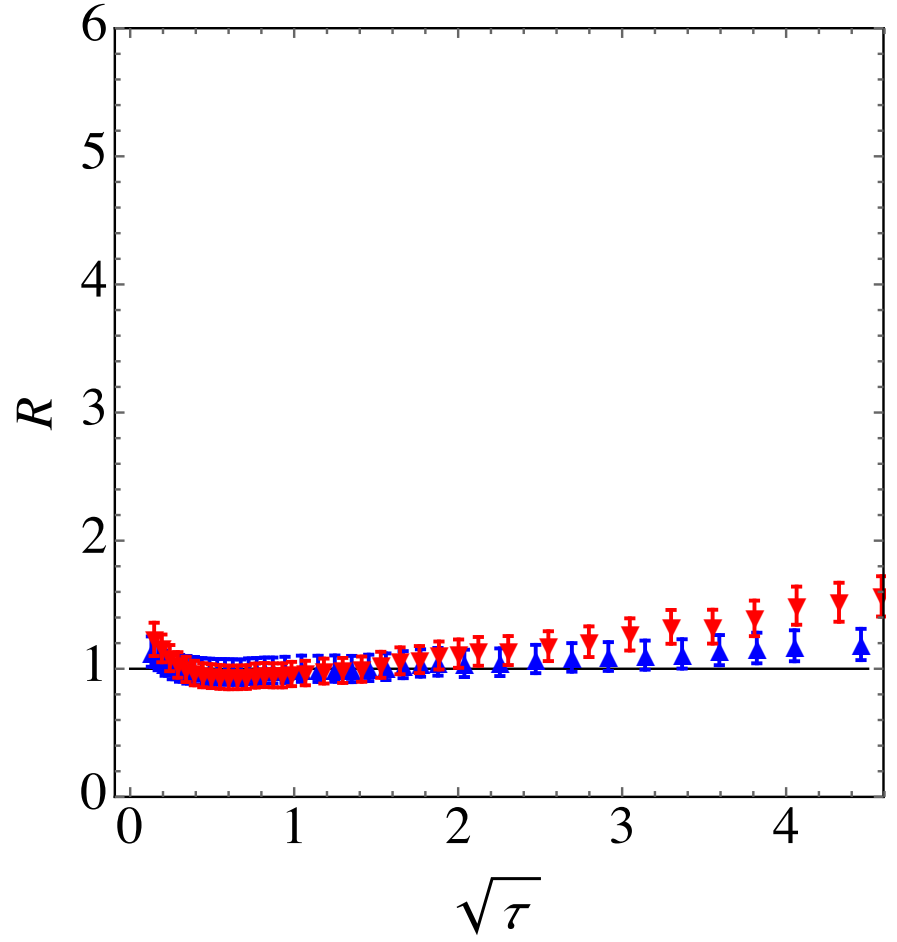
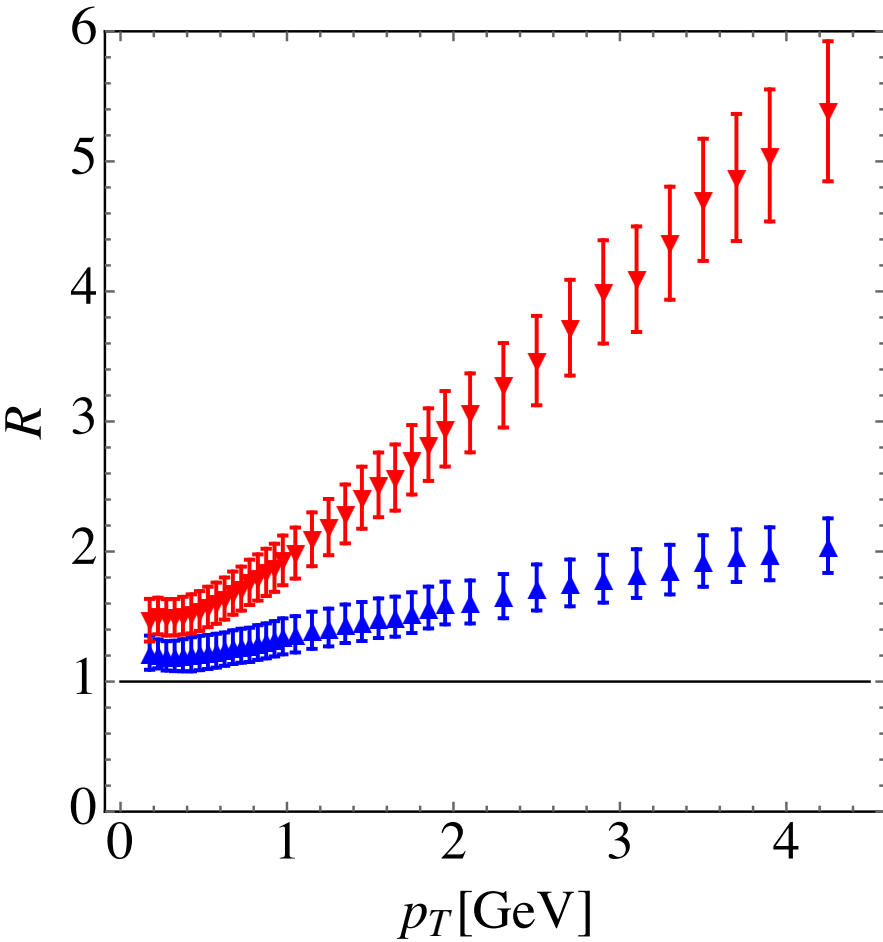




# Determination of lambda

$$\frac{dN_{\text{ch}}}{dyd^2p_T} = S_{\perp} \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^{\lambda}$$

$\lambda = 0$   $\lambda = 0.22$





# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large  $x \sim 0.08$  with  $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with  $\lambda \sim 0.22$  (!)



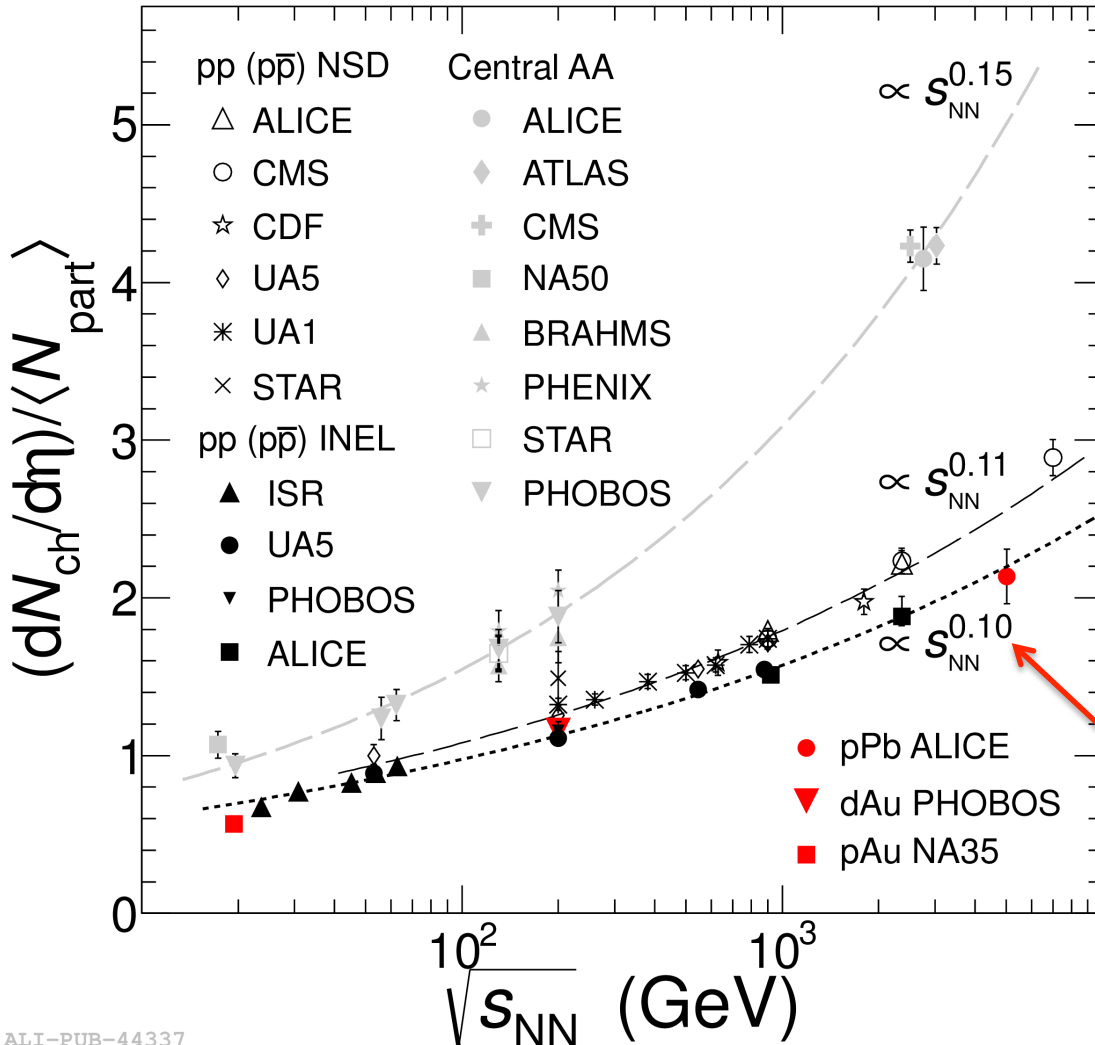
**continue  
with multiplicity scaling...**





# Power-like growth of multiplicity

[http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger\\_1.pdf](http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf)



plot: P. Braun-Munzinger,  
54 Cracow School of  
Theoretical Physics  
(from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_{\perp} \bar{Q}_s^2(W)$$

$$\sim S_{\perp} Q_0^2 \left( \frac{W^2}{Q_0^2} \right)^{\lambda/(2+\lambda)}$$

transverse area is  
energy independent

$$\lambda/(2 + \lambda) \simeq 0.099$$



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- In pp GS works for multiplicity distributions with  $\lambda \sim 0.22$  (!)
- As a consequence total multiplicity grows with energy as  $s^{0.1}$



# Application to pA scattering at the LHC



# Color Glass Condensate in pPB

stolen from Bozek, Bzdak, Skokov

$$\frac{dN}{dy} = S_{\perp} Q_p^2 \left( 2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

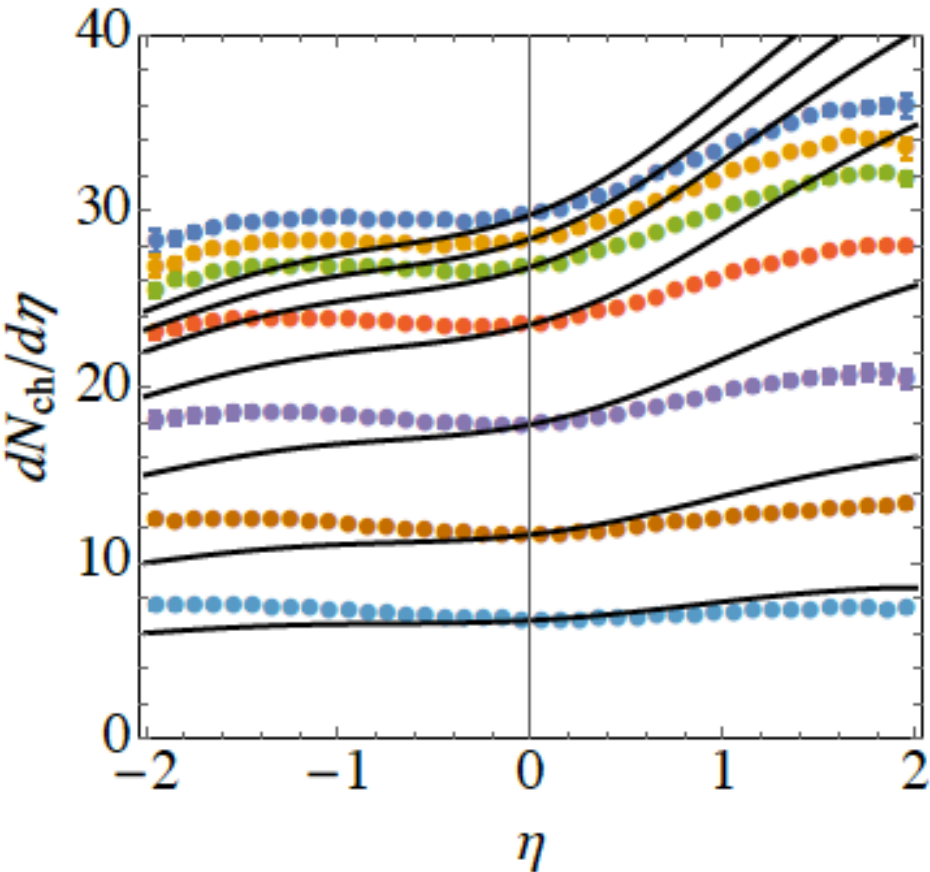
$$Q_p^2(W, y) = Q_0^2 \left( \frac{W}{W_0} \right)^{\lambda} \exp(\lambda y),$$

$$Q_A^2(W, y) = Q_0^2 N_{\text{part}} \left( \frac{W}{W_0} \right)^{\lambda} \exp(-\lambda y)$$

$$\lambda = 0.32$$



# Multiplicity for pPb



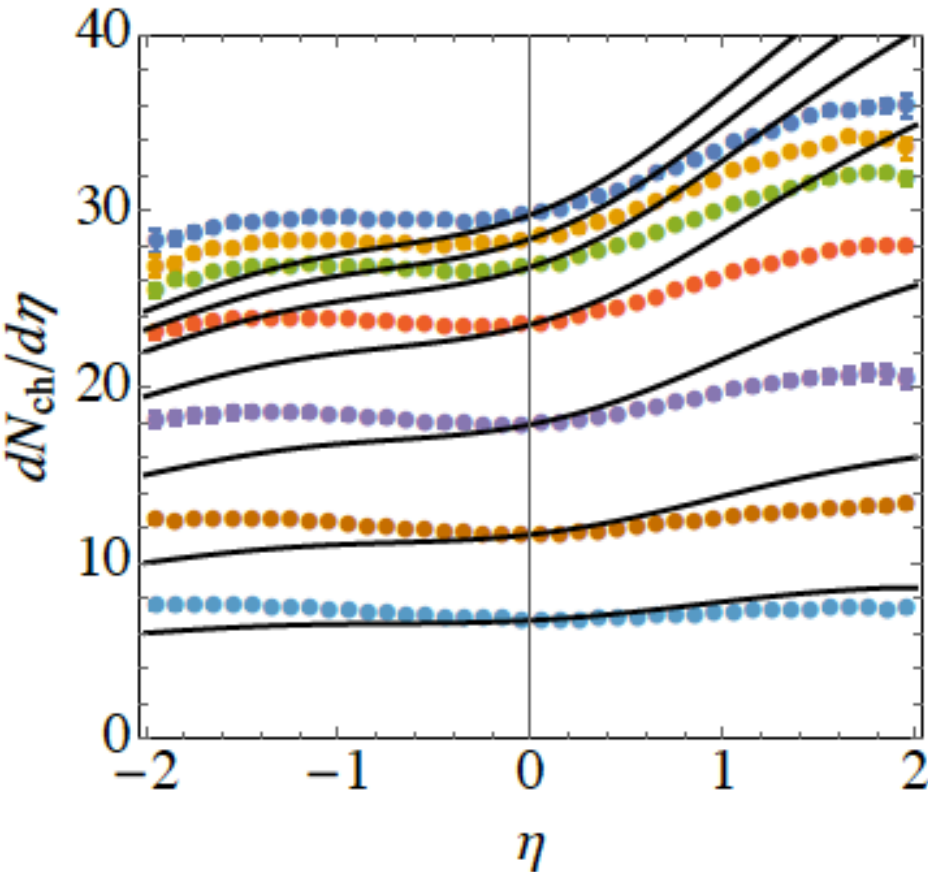
$$\frac{dN_{\text{ch}}}{dy} = S_{\perp} Q_p^2 \left( 2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

ZNA method

J. Adam *et al.* [ALICE Collaboration], Phys. Rev. C **91** (2015) 064905.



# Multiplicity for pPb



$$\frac{dN_{\text{ch}}}{dy} = S_{\perp} Q_p^2 \left( 2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

Allow  $Q_p$  to fluctuate:

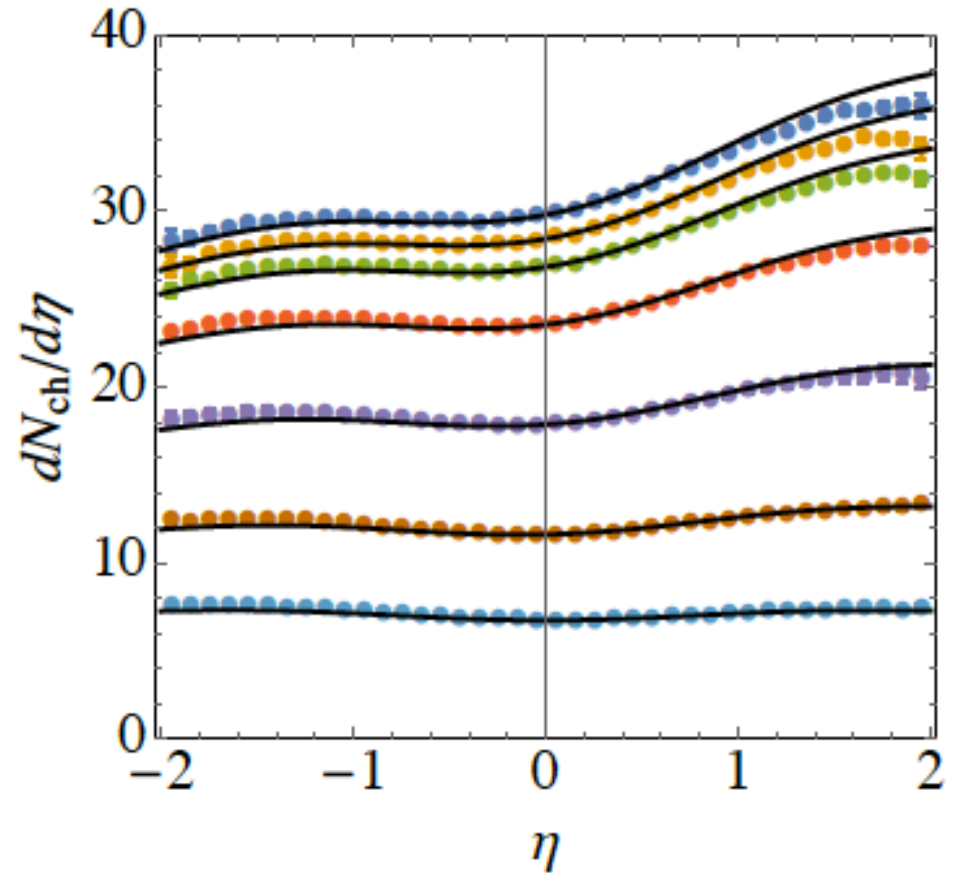
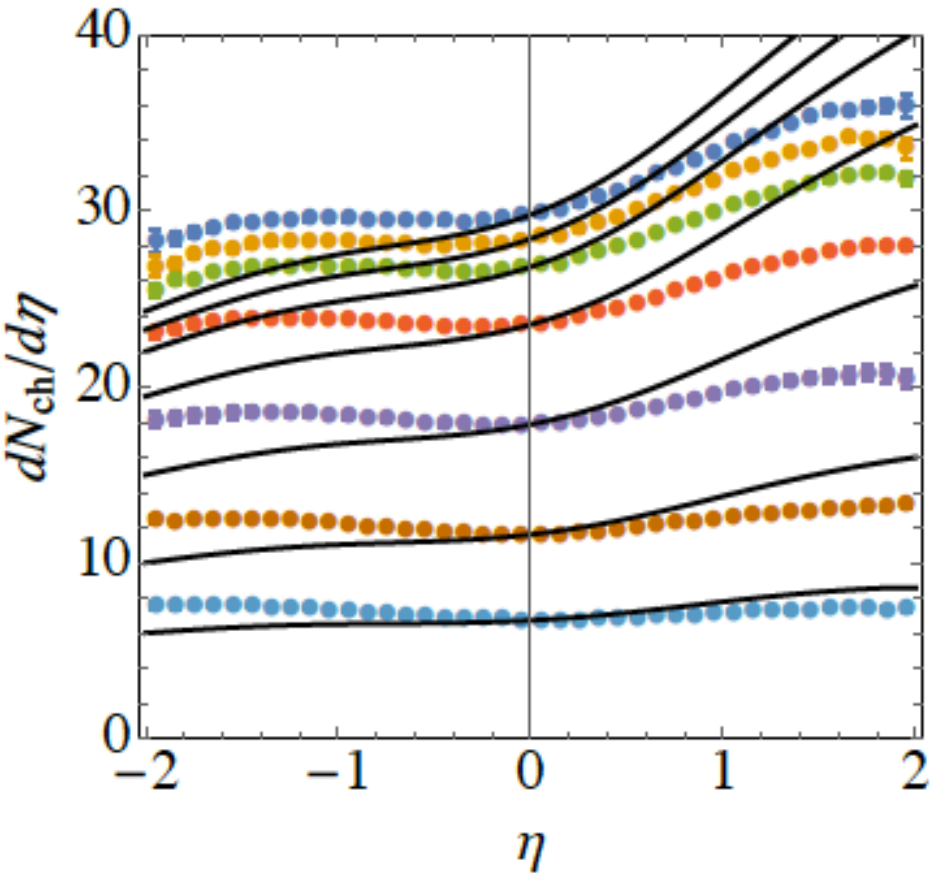
$$\frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{(\ln Q_p^2 / Q_0^2 - \ln \bar{Q}_p^2 / Q_0^2)^2}{2\sigma^2} \right)$$

E. Iancu, A. H. Mueller, S. Munier  
Phys. Lett. B **606** (2005) 342

J. Adam *et al.* [ALICE Collaboration], Phys. Rev. C **91** (2015) 064905.



# Fluctuations of $Q_{\text{sat}}$ in pPb



rather large  $\sigma \sim 1.55$



# Conclusions

- Nonlinear BK equation generates saturation scale  $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large  $x \sim 0.08$  with  $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with  $\lambda \sim 0.22$  (!)
- As a consequence total multiplicity grows with energy as  $s^{0.1}$
- Fluctuations of the saturation scale may explain  $dN/dy$





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# Not discussed

- Consequences of GS for  $F_L$
- Scaling violations in pp due to  $y \neq 0$
- Scaling violations in pp due to  $\lambda(Q^2)$
- Scaling in pp for identified particles
- Connection with Tsallis distribution
- $\langle p_T \rangle(N)$  for identified particles
- geometrical scaling predicts energy dependence of  $\langle p_T \rangle$
- $\langle p_T \rangle(N_{ch})$  difficult to describe by untuned MonteCarlos
- scaling of  $\langle p_T \rangle(N_{ch})$  induced by energy dependence of  $Q_{sat} + CGC$
- GS in heavy ion collisions – scaling with energy and  $N_{part}$
- $\frac{11}{E}$



감사합니다!



# Workshop on QCD and Diffraction

*saturation 1000+*

5-7 December 2016  
Cracow, Poland

[qcdworkshop.ifj.edu.pl](http://qcdworkshop.ifj.edu.pl)

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