Low-energy exclusive $e^+e^- \to$ hadrons cross sections and g-2 of the muon

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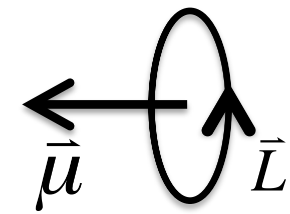
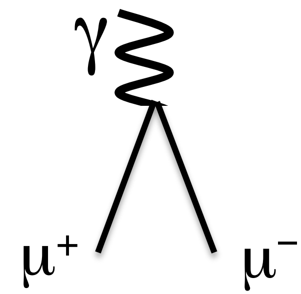
g_μ - 2 in the standard model

- Gyromagnetic ratio g: $$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$



→ The relationship between angular momentum L (or s) and magnetic moment μ



- Dirac (tree-level) result for a charged lepton l:
  g_l = 2 (exactly)

- Radiative corrections alter the prediction: $g_l = 2\,(1 + a_l)$, introducing sensitivity to new physics through loops

- The "anomalous" moment $$a_l = \frac{g_l - 2}{2}$$

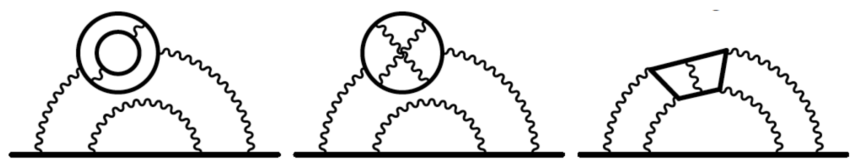
The muon anomaly a_μ is much more sensitive to virtual heavy particle production in loops than the electron anomaly a_e: the relative virtual terms scale like $(m_\mu/m_e)^2 \approx 43{,}000$

g_μ - 2 in the standard model

In the standard model, $a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadronic}$

$$a_\mu^{QED} = 116\,584\,718.951 \pm 0.080 \times 10^{-11}$$

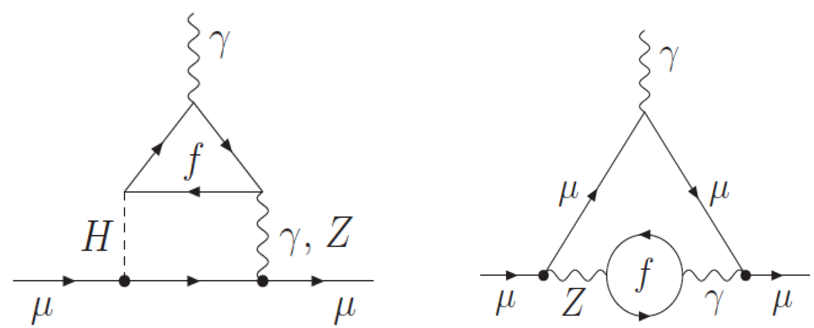
Aoyama, Hayakawa, Kinoshita, Nio; PRL 109 (2012) 111808



Calculations up to 5th order in α_{QED}

$$a_\mu^{EW} = 153.6 \pm 1.0 \times 10^{-11}$$

Calculations up to 3rd order in the weak coupling strength

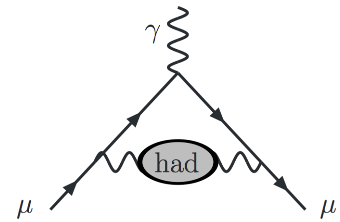


Gnendiger et al., PRD88 (2013) 053005

g_μ - 2 in the standard model

$$a_\mu^{hadronic} = a_\mu^{had,LO-VP} + a_\mu^{had,NLO-VP} + a_\mu^{had,LbLS}$$

Leading-order hadronic vacuum polarization



$$a_\mu^{had,LO-VP} = 6923 \pm 42 \times 10^{-11}$$

Davier et al., EPJC71 (2011) 1515

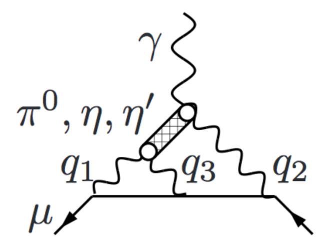
[$6949 \pm 43 \times 10^{-11}$: Hagiwara et al., J. Phys. G38 (2011) 085003]

Higher-order hadronic vacuum polarization

$$a_\mu^{had,HO-VP} = -98.4 \pm 0.7 \times 10^{-11}$$

Hagiwara et al., J. Phys. G38 (2011) 085003

Light-by-light scattering



$$a_\mu^{had,LbLS} = 105 \pm 26$$

Prades et al., arXiv:0901.0306 (2009)

Diagrams from Jegerlehner and Nyffeler, Phys. Rept. 477 (2009) 1

g_μ - 2 in the standard model

Summary: individual SM contributions:

a_μ^{QED}	116584718.951 ± 0.080
a_μ^{EW}	153.6 ± 1.0
$a_\mu^{had,LO-VP}$	6923 ± 42
$a_\mu^{had,HO-VP}$	-98.4 ± 0.7
$a_\mu^{had,LbLs}$	105 ± 26

all in units of 10^{-11}

Total SM prediction compared to measurement

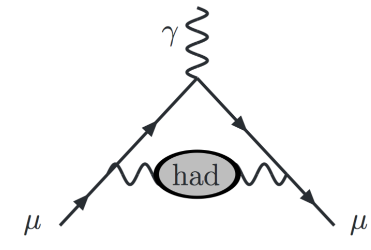
$a_\mu^{total-SM}$	116591802 ± 49
$a_\mu^{BNL-E821}$	116592089 ± 63
Data - SM	287 ± 80

Davier et al., EPJC71 (2011) 1515

- ~ 3.5 σ difference between data and SM prediction
- Uncertainty in the SM prediction dominated by LO-VP term

LO hadronic vacuum polarization term $a_\mu^{had,LO\text{-}VP}$

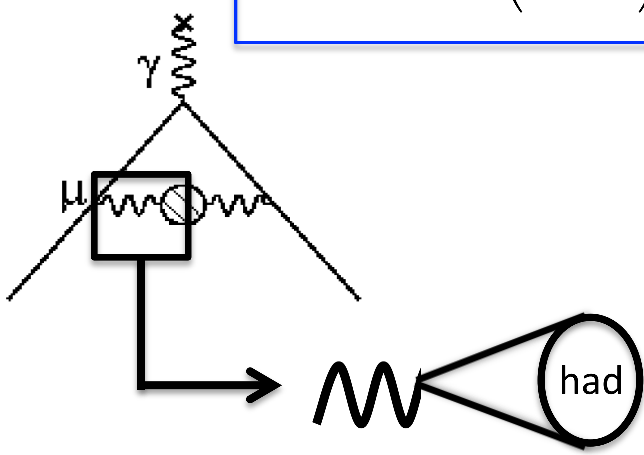
- Energy scale too low for perturbative calculations
- Lattice calculations not yet sufficiently precise
  [M Della Morte et al., JHEP 1203 (2012) 055]
- The most precise result for $a_\mu^{had,LO\text{-}VP}$ obtained from low-energy $e^+e^- \rightarrow$ *hadrons* data and an integral over a dispersion relation



$$a_\mu^{had,LO-VP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{K(s) R_{had}}{s^2} ds \qquad R_{had} = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$K(s) = \text{kinematic factor}$$

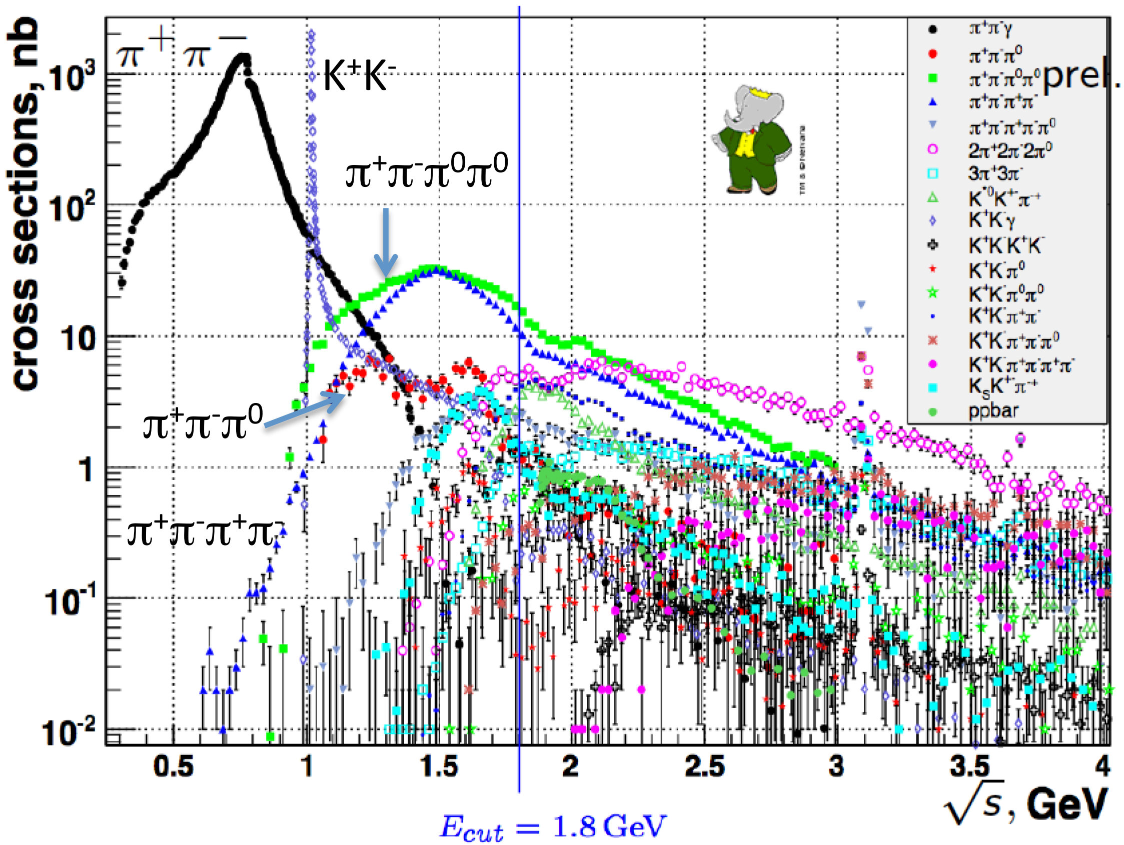
see Phys. Rep. 477 (2009) 1



$e^+e^- \rightarrow$ hadrons production through a photon coupling

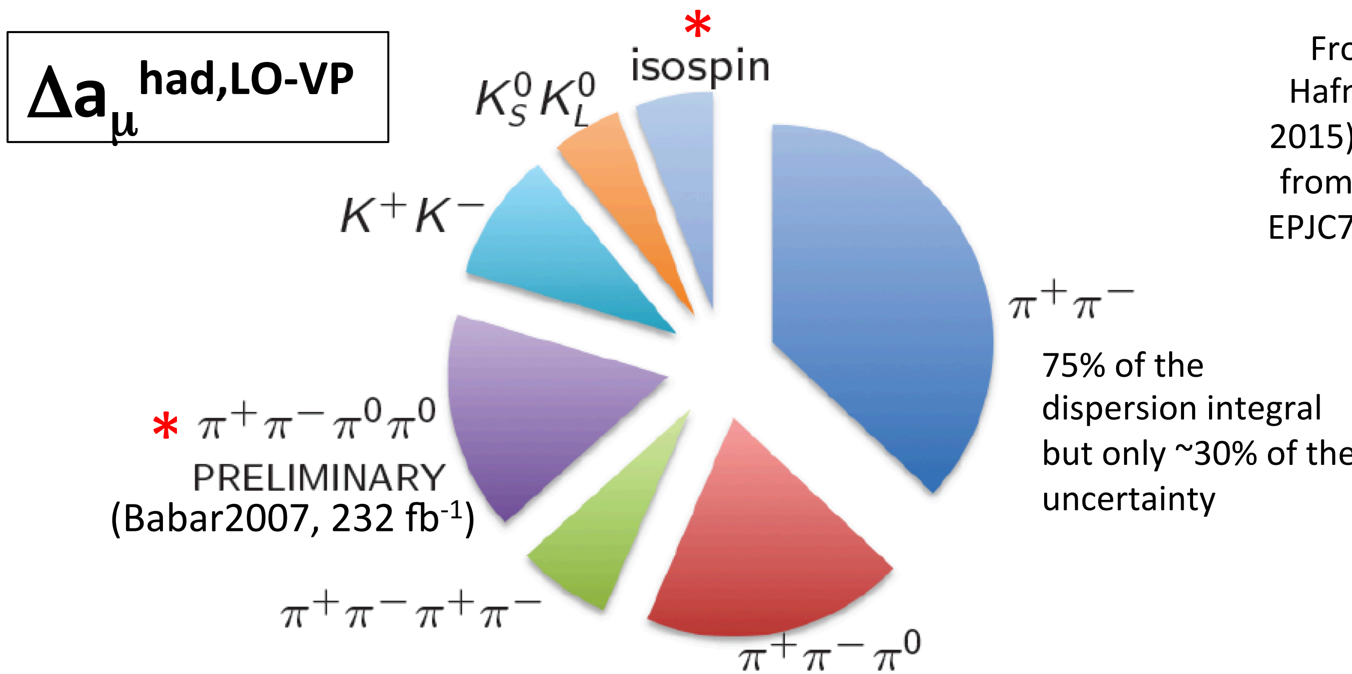
- $1/s^2$ term → low-energy contributions dominate
- Need precise measurements of R_{had} at low $\sqrt{s}$
- Use sum of exclusive channels for E < 2 GeV: background to inclusive channel from $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ events very large & inclusive detection efficiency not precise below 2 GeV [generic MC (Jetset) doesn't work]
- Perturbative calculation or inclusive $\sigma(e^+e^- \rightarrow$ *hadrons*) data used for E > 2 GeV

LO hadronic vacuum polarization term $a_\mu^{had,LO-VP}$



- Contribution of the $\pi^+\pi^-$ state to the dispersion integral: 75%
- The 3π, 4π, and KK channels the next most important
- Final states with kaons, e.g., $KK\pi$, $KK\pi\pi$, important above the ϕ mass

Relative contributions of different channels to the $\underline{\text{uncertainty}}$ in $a_\mu^{had,LO-VP}$



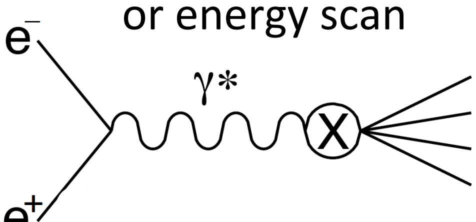
From Andreas Hafner (Moriond 2015), using results from Davier et al., EPJC71 (2011) 1515

- "Isospin" refers to processes with unmeasured cross sections estimated from isospin relations: largest contributions are $KK\pi$ and $KK\pi\pi$
- * New results (preliminary) shown here on $\pi^+\pi^-\pi^0\pi^0$ and for $KK\pi$, $KK\pi\pi$ states
- Recently published Babar results on K^+K^-, K_SK_L not yet accounted for in $a_\mu^{had,LO-VP}$

Partial list of experiments: R_{had} at $\sqrt{s}$ < 2 GeV

Experiment	Collider	Location	Beam energy
KLOE	DAϕNE	Frascati	1.02 GeV
DM1. DM2	DCI	Orsay	1.35-2.4 GeV
CMD2, SND	VEPP-2M	Novosibirsk	0.4-1.4 GeV
CMD3, SND	VEPP-2000	Novosibirsk	0.3-2 GeV
BESIII	BEPC-II	IHEP, Beijing	2-4.6 GeV
CLEO-c	CESR	Cornell	3.67-4.17 GeV
Babar	PEP-II	SLAC	10.6 GeV

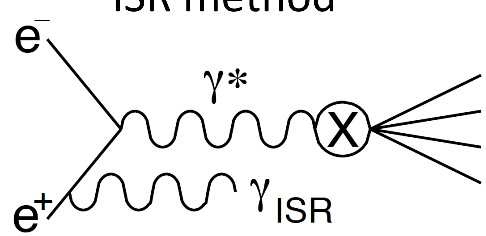
Single fixed energy
or energy scan



CMD3, SND

or

ISR method



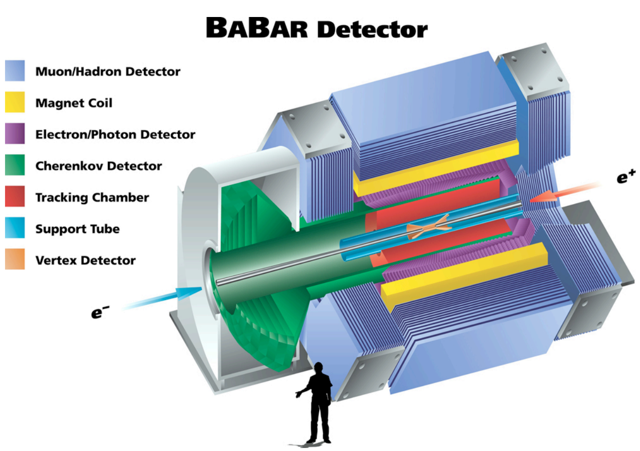
KLOE, BESIII, Babar

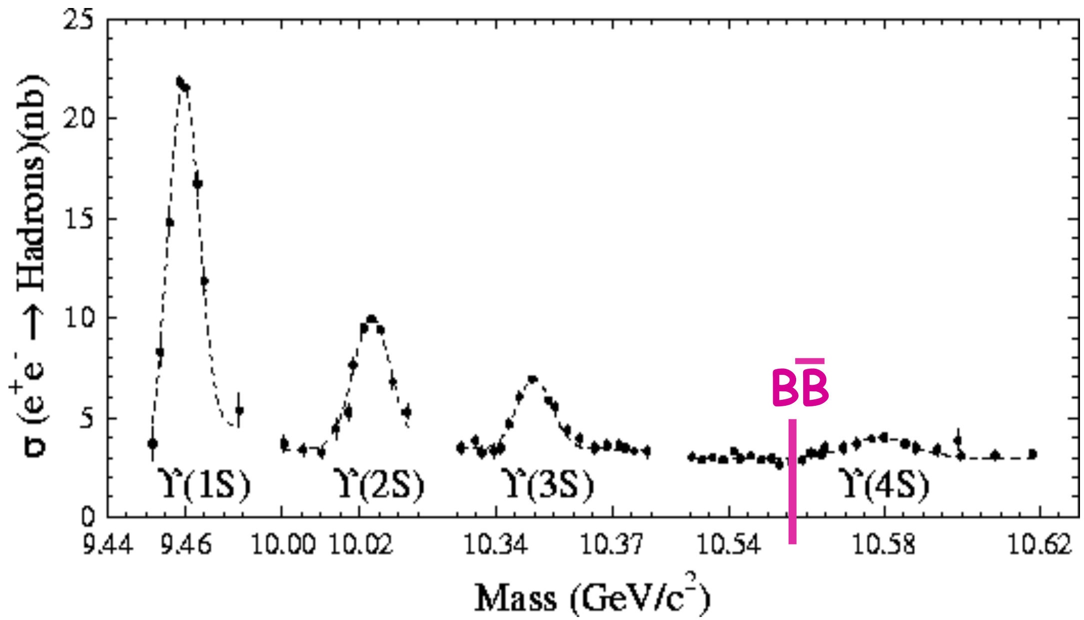
Measure σ ($e^+e^- \rightarrow X$)
versus $m_{\gamma*}$ = m_X = $\sqrt{s'}$

The Babar experiment

- PEP-II rings: asymmetric e^+e^- collider @ SLAC 9 GeV e^- and 3.1 GeV e^+
- Collected data 1999-2008; data analysis still active (~10 new papers in 2016)

Υ(4S): 10.58 GeV	424 fb^{-1}
Υ(3S)	28 fb^{-1}
Υ(2S)	14 fb^{-1}
Other (mostly off-resonant at 10.54 GeV)	~60 fb^{-1}





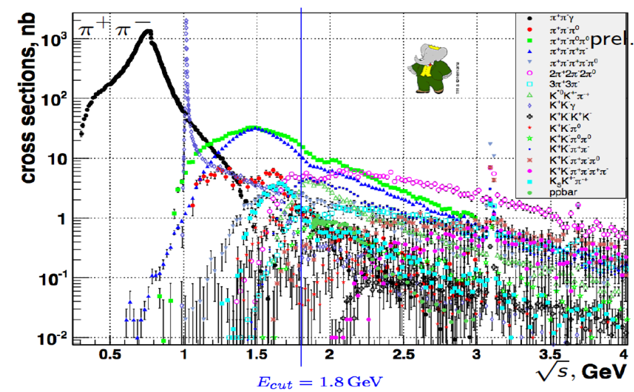
465 x 10^6 Υ(4S) → $B\bar{B}$ events ;
650 x 10^6 $e^+e^- \to c\bar{c}$ events ;
430 x 10^6 $e^+e^- \to \tau^+\tau^-$ events

The Babar ISR $e^+e^- \rightarrow$ hadrons program

A long-term project nearing completion

Published results on low-energy exclusive cross sections:

* $\pi^+\pi^-$ (232 fb^{-1})	PRL 103 (2009) 231801
* $\pi^+\pi^-\pi^0$ (89 fb^{-1})	PRD 70 (2004) 072004
$2(\pi^+\pi^-)$	PRD 85 (2012) 112009
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$	PRD 86 (2012) 012008
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$	PRD 76 (2007) 092005
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$	PRD 73 (2006) 052003
$K^+K^-\eta$, $K^+K^-\pi^0$, $K_SK^+\pi^-$	PRD 77 (2008) 092002
$p\bar{p}$	PRD 87 (2013) 092005 PRD 88 (2013) 072009
$\Lambda\Lambda$, $\Lambda\Sigma^0$, $\Sigma^0\Sigma^0$	PRD 76 (2007) 092006
K^+K^-, tagged ISR	PRD 88 (2013) 032013
K^+K^-, untagged ISR	PRD 92 (2015) 072008
K_SK_L, $K_SK_{S/L}\pi^+\pi^-$, $K_SK_SK^+K^-$	PRD 89 (2014) 092002



Four new studies (preliminary) discussed here:

1) $\pi^+\pi^-\pi^0\pi^0$
2) $K_SK_L\pi^0$, $K_SK_L\eta$, $K_SK_L\pi^0\pi^0$;
3) $K_SK^+\pi^-\pi^0$, $K_SK^+\pi^-\eta$;
4) $\pi^+\pi^-\eta$.

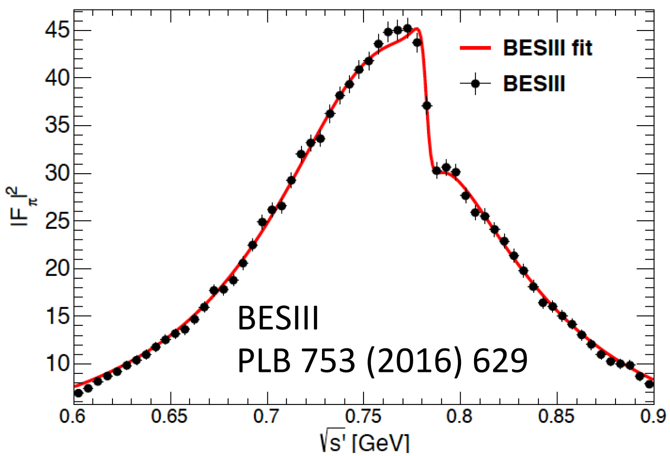
(items 2–4:) Previously unmeasured

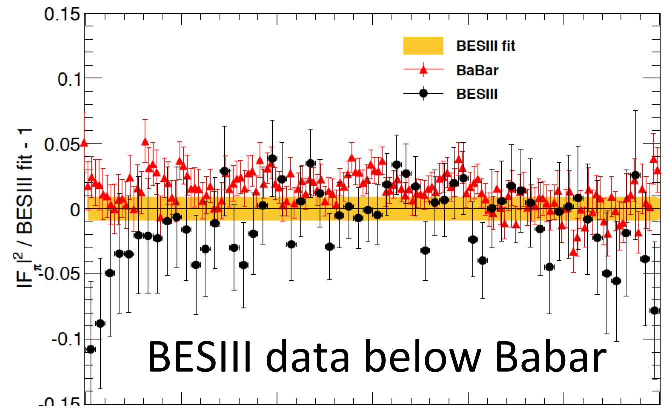
Essentially the complete set of significant exclusive channels

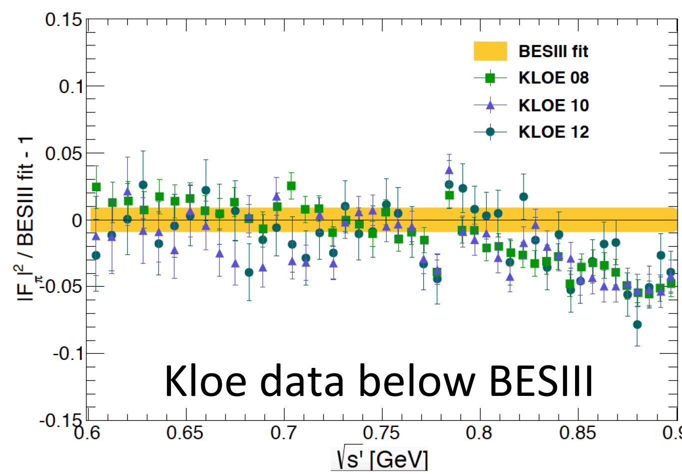
* The $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ studies being updated to include the full ~490 fb^{-1} Babar data sample

Current status of published results: $e^+e^- \to \pi^+\pi^-$

Most precise published $e^+e^- \to \pi^+\pi^-$ measurements: Babar (2009), KLOE (2009, 2011, 2013), and BESIII (2016)







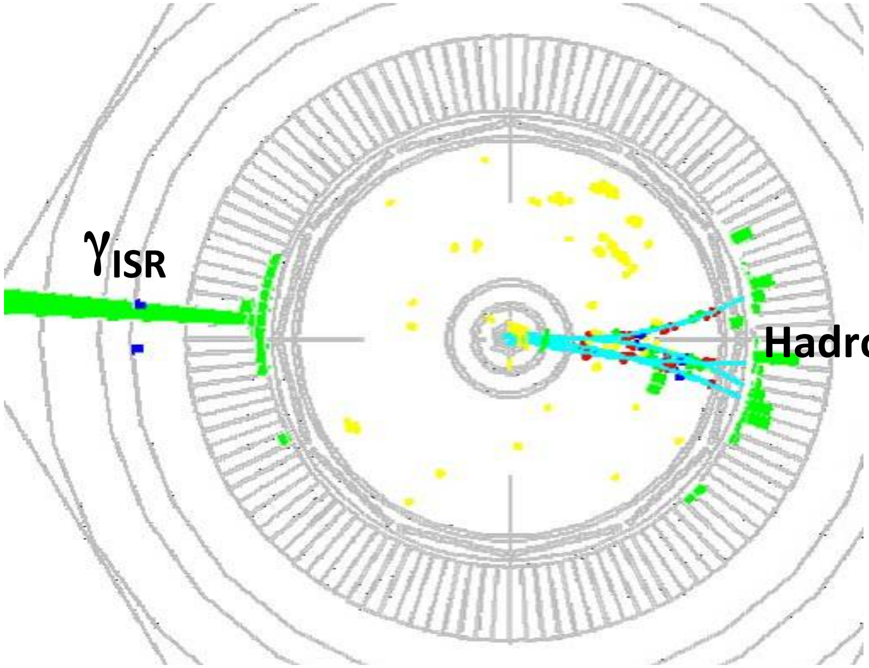
- Each of the three experiments claims ~1% precision in the measurement of the $\pi^+\pi^-$ cross section.
- However, the Babar and Kloe results differ by 3-6%, with BESIII in between
- May need additional studies from BESIII, CMD3, and Babar

The Babar ISR $e^+e^- \to$ hadrons program

Babar tagged ISR analyses

- e^+e^- collisions at ~10.6 GeV
- ≥ 1 photon identified in the detector with E* > 3 GeV
  (* = CM frame)
- ISR photon γ_{ISR} = photon with highest E*
- Boost of the recoil system provides good efficiency for soft particles, allowing measurements of cross sections down to the production threshold

Generic Babar ISR event

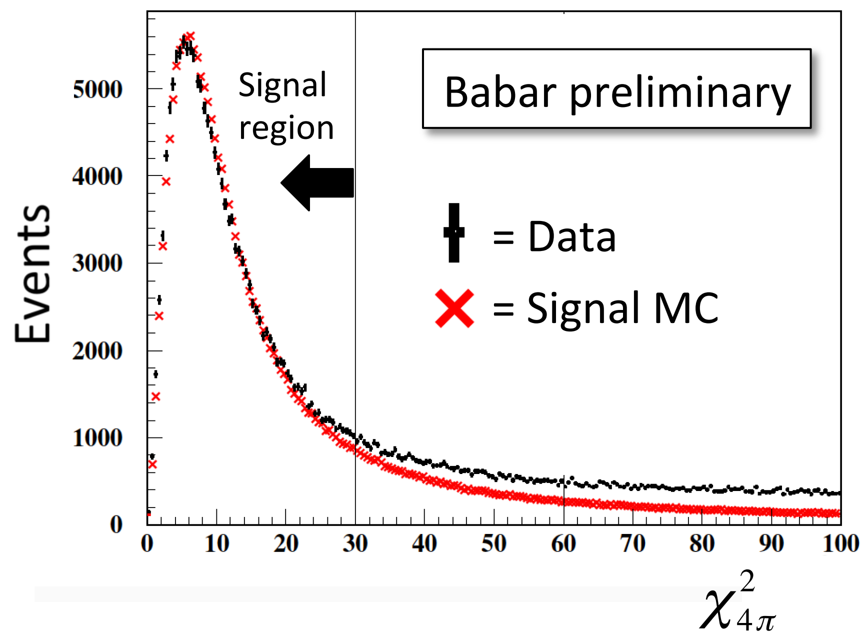


- Event acceptance larger for the ISR method than for the energy scan method since event is pointed into the detector and not down the beam pipe
- Can access a wide range of energy in a single experiment: from threshold to ~5 GeV

(I) $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ cross section

One of the least known cross sections important for g-2 of the muon

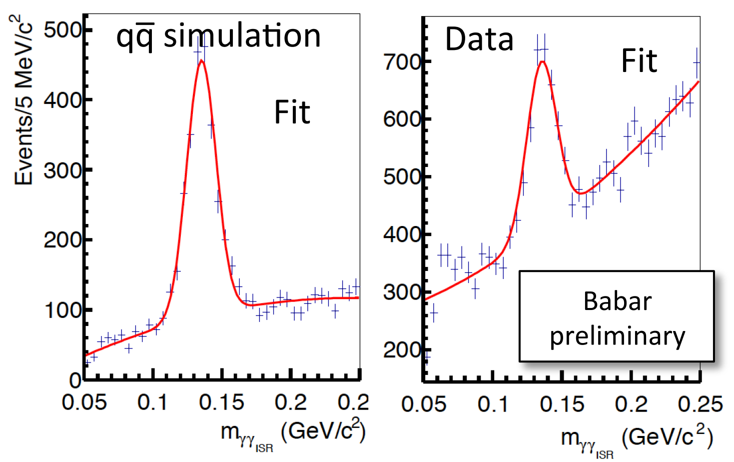
- Exactly 2 tracks, p > 100 MeV, opposite charge, $d_0 < 1.5$ cm, $z_0 < 2.5$ cm
- ISR photon candidate: highest energy photon, with E > 3 GeV in CM frame
- ≥ 4 other photons, E > 50 MeV
- Perform kinematic fit to the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\gamma$ hypothesis, constraining two 2γ combinations to the π^0 mass
- Select the overall combination of four photons yielding the smallest $\chi_{4\pi\gamma}^2$, requiring $\chi_{4\pi\gamma}^2 < 30$
- Difference between the $\chi_{4\pi\gamma}^2$ distributions of data and signal MC due to background in the former

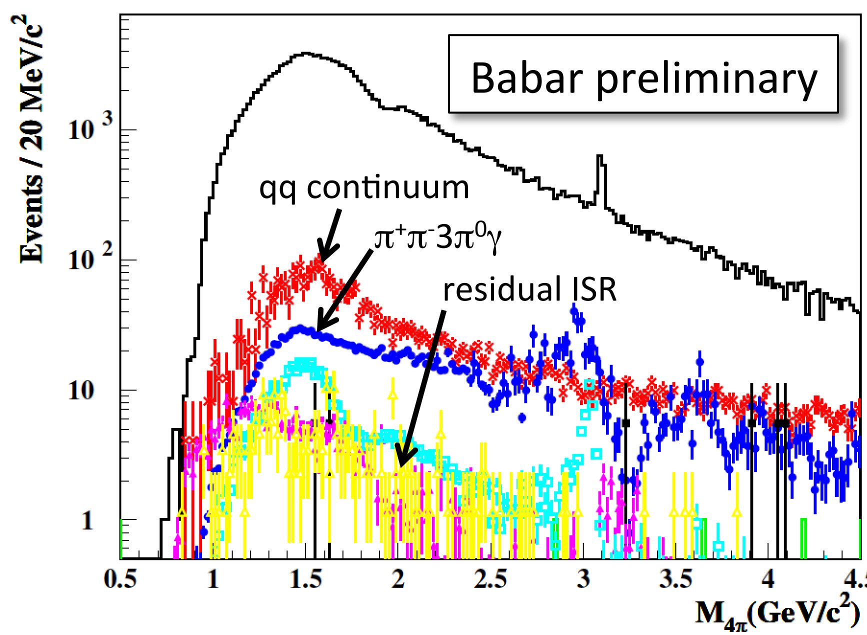


(I) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section

- ISR background from $\pi^+\pi^-\pi^0\gamma$, $\pi^+\pi^-\pi^0\eta\gamma$, $\pi^+\pi^-2\eta\gamma$ suppressed by rejecting events with kinematic fits consistent with those hypotheses; e.g., require $\chi_{3\pi\gamma}{}^2 > 25$
- Largest remaining background is from non-ISR $q\bar{q}$ continuum events, arising from the misidentification of a photon from π^0 decay as an ISR photon
- Subtract $q\bar{q}$ background using simulation normalized to the π^0 peak from $\gamma_{ISR}\gamma$ combinations:
  - γ_{ISR} = the selected ISR photon
  - γ = any other photon not assigned to a signal π^0 candidate

Example from $e^+e^- \to \pi^+\pi^-\eta\gamma$ analysis

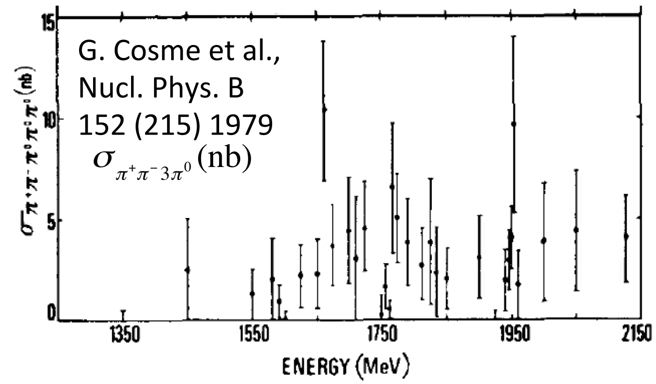


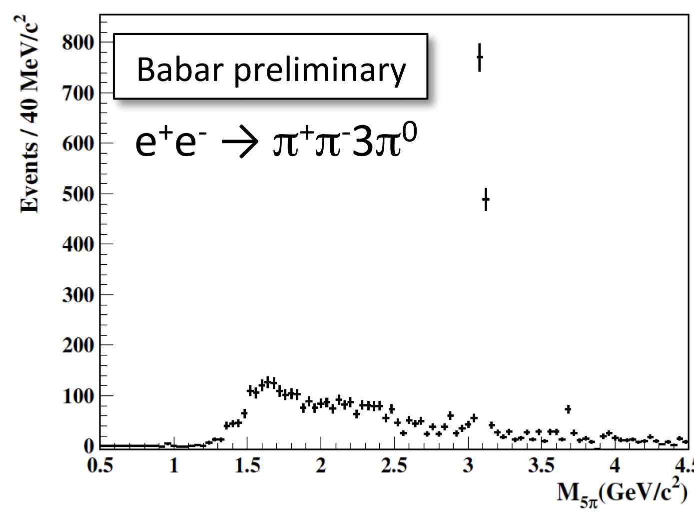


(I) $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ cross section

- Background from residual ISR processes ($K_SK^+\pi^-\gamma$, $K^+K^-\pi^0\pi^0\gamma$, $\pi^+\pi^-\pi^0\gamma$): use the existing measurements to correct simulation (rate and shape) & subtract

- Largest ISR background is from $\pi^+\pi^-3\pi^0\gamma$ events
  - Cross section not well measured; perform new measurement to obtain reliable background estimate
  - Perform kinematic fit under the $\pi^+\pi^-3\pi^0\gamma$ hypothesis; require $\chi_{5\pi\gamma}^2 < 25$
  - Subtract $q\bar{q}$ background using simulation normalized using π^0 peak in $\gamma_{ISR}\gamma$
  - Subtract residual ISR background using data sideband in $\chi_{5\pi\gamma}^2$
  - Determine relative contributions of $\omega\pi^0\pi^0\gamma$ ($\omega \rightarrow \pi^+\pi^-\pi^0$), $\eta\pi^+\pi^-$ ($\eta \rightarrow 3\pi^0$), and non-resonant channels; reweight simulation accordingly to evaluate efficiency





(I) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section

- Cross section corrected for detector acceptance and resolution
  [AfkQed & Phokhara MC]

$$\frac{\mathrm{d}\sigma_{2\pi 2\pi^0\gamma}(M)}{\mathrm{d}M} = \frac{\mathrm{d}N_{2\pi 2\pi^0\gamma}(M)}{\mathrm{d}M} \cdot \frac{1}{\epsilon(M)\mathcal{L}_{\mathrm{tot}}(1+\delta)}$$

δ = radiative correction, including FSR

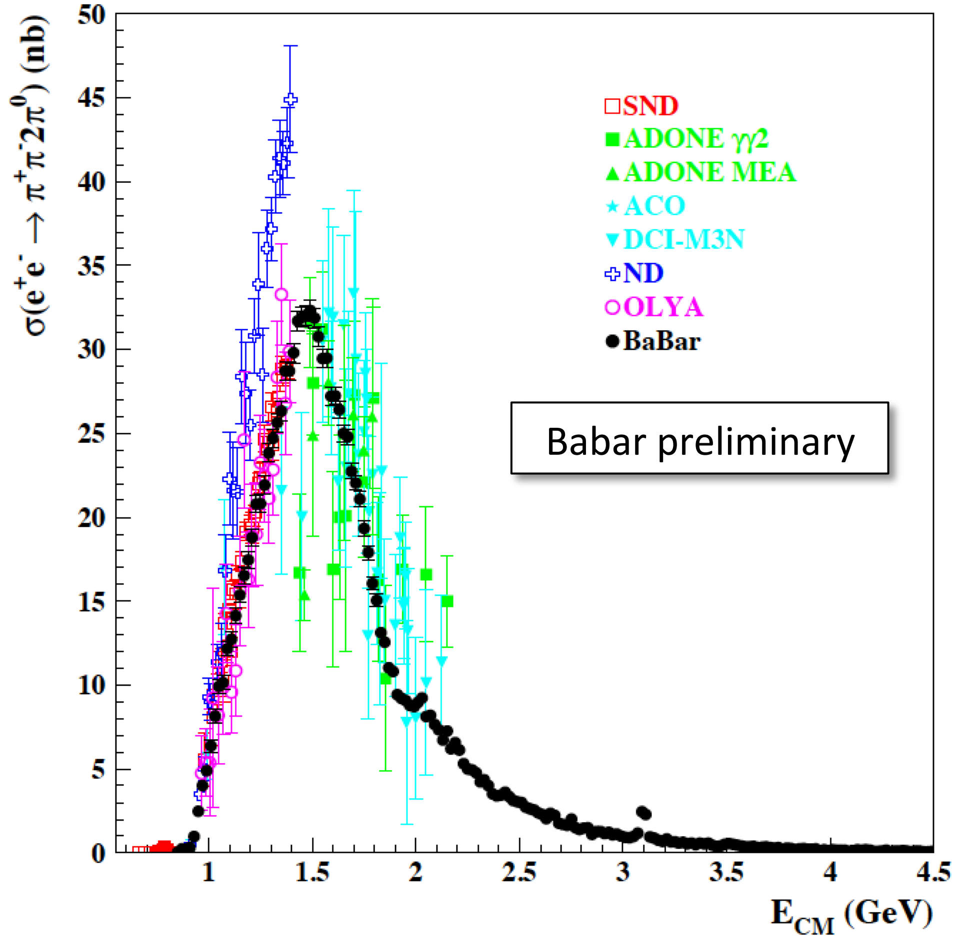
- Interpret in terms of nonradiative cross section $\sigma_{\pi^+\pi^-2\pi^0}$

$$\frac{\mathrm{d}\sigma_{\pi^+\pi^-2\pi^0\gamma}(M)}{\mathrm{d}M} = \frac{2M}{s} \cdot W(s,x,C) \cdot \sigma_{\pi^+\pi^-2\pi^0}(M)$$

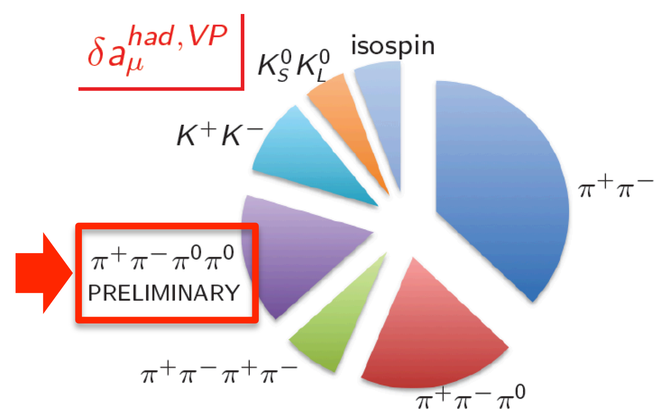
$W(s,x,C)$ = radiator function, the probability for the initial e^+ or e^- to radiate a photon within the CM polar angle range $|\cos(\theta_\gamma)| < C$, thus lowering the effective annihilation energy from $\sqrt{s}$ to M

$x = 2\,E_\gamma/\sqrt{s}$; E_γ measured in CM

(I) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section

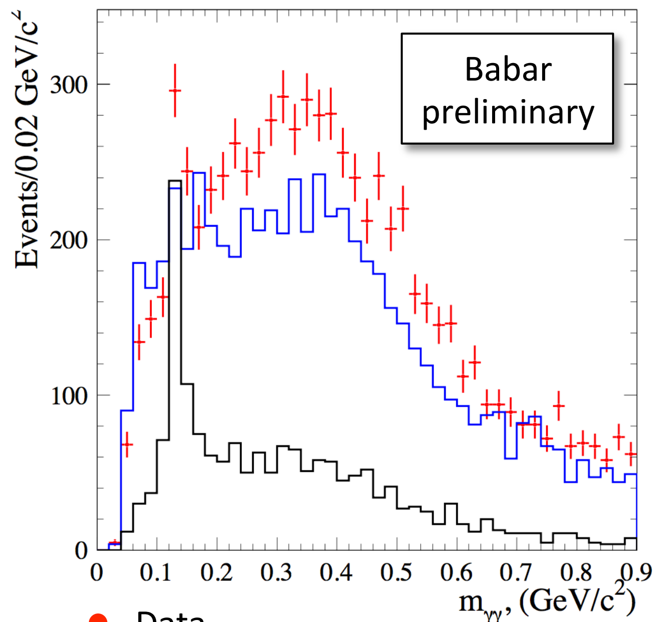


- Babar results far more precise & covers wider energy range
- Contribution to a_μ for $1.02<E_{CM}<1.8$ GeV measured to be [175 ± 6 (stat+syst)] x 10^{-11} (3.4% precision)
- Previous result, including the preliminary Babar data from 2007, is [180 ± 12 (stat+syst)] x 10^{-11} (6.7% precision)



(II) First measurements of the $e^+e^- \rightarrow K_S K^+ \pi^- \pi^0$ and $K_S K^+ \pi^- \eta$ cross sections

- ISR photon = highest energy γ with CM $E_\gamma > 3$ GeV
- At least one $K_S \rightarrow \pi^+\pi^-$ candidate consistent with interaction point
- At least two additional photons, with $m_{\gamma\gamma}$ consistent with m_{π^0} or m_η
- Two oppositely charged tracks, one identified as a pion & one as a kaon
- Background from non-ISR $q\bar{q}$ events (primarily $e^+e^- \rightarrow K_S K^+ \pi^- \pi^0 \pi^0$ and $K_S K^+ \pi^- \eta \pi^0$) evaluated from simulation normalized to data using the $\gamma_{ISR}\gamma\ \pi^0$ peak
- Background from ISR $e^+e^- \rightarrow K_S K^+ \pi^-$ and $K_S K^+ \pi^- \pi^0 \pi^0$ or $K_S K^+ \pi^- \eta \pi^0$ [one more or one less π^0] events evaluated from data sidebands

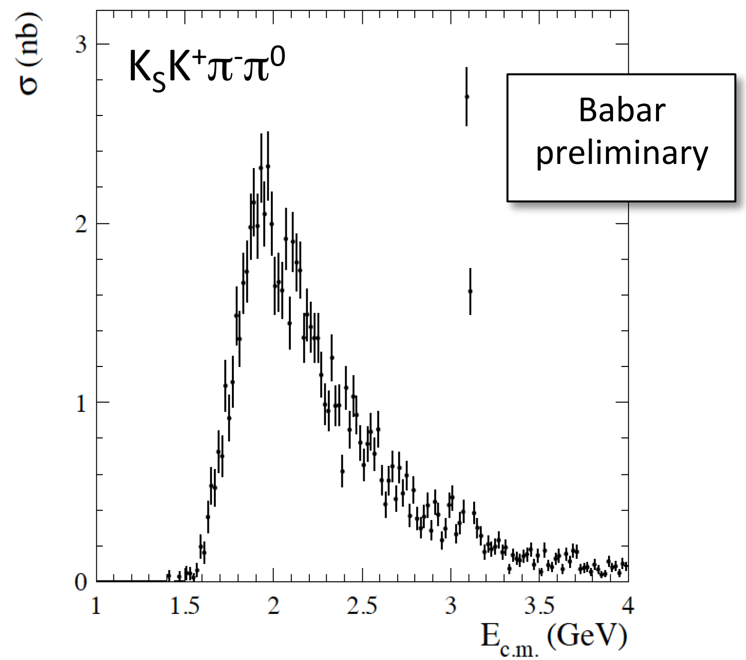


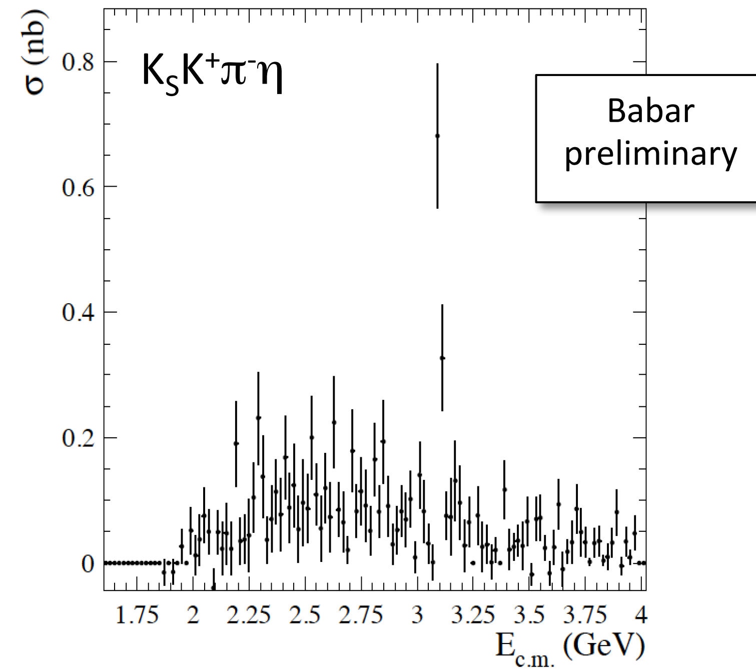
Data

$e^+e^- \rightarrow K_S K^+ \pi^- \pi^0 \gamma$ (signal) MC

$e^+e^- \rightarrow K_S K^+ \pi^- \pi^0 \pi^0$ (background) MC

(II) First measurements of the $e^+e^- \to K_S K^+ \pi^- \pi^0$ and $K_S K^+ \pi^- \eta$ cross sections





J/ψ branching fractions (preliminary)

$$B_{J/\varphi \to K_S K^+ \pi^- \pi^0} = (5.7 \pm 0.3 \pm 0.4) \times 10^{-3}$$

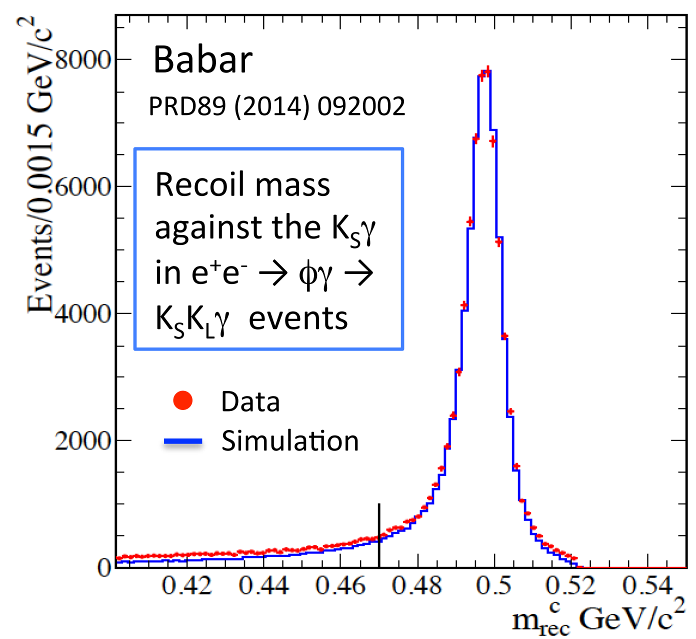
First observation

$$B_{J/\varphi \to K_S K^+ \pi^- \eta} = (1.30 \pm 0.25 \pm 0.07) \times 10^{-3}$$

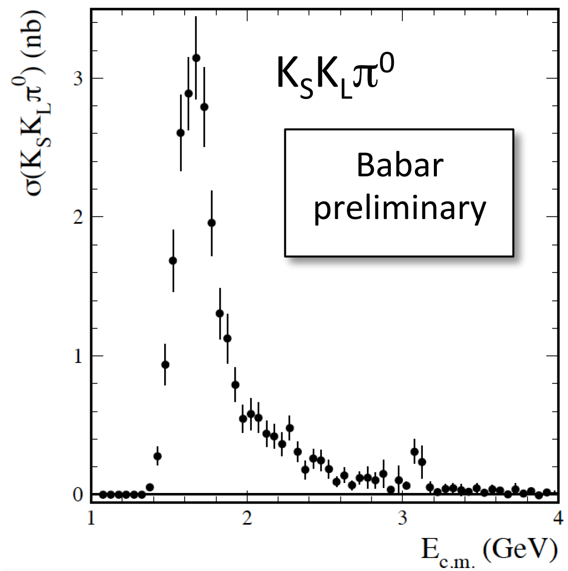
BES result: $(2.2 \pm 0.4) \times 10^{-3}$
[PRD 77 (2008) 032005]
(Babar result around 2σ lower)

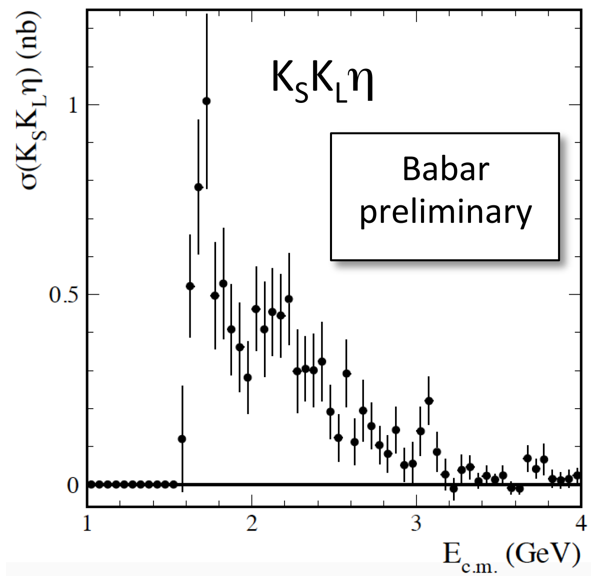
(III) First measurements of the $e^+e^- \rightarrow K_SK_L\pi^0$, $K_SK_L\eta$, and $K_SK_L\pi^0\pi^0$ cross section

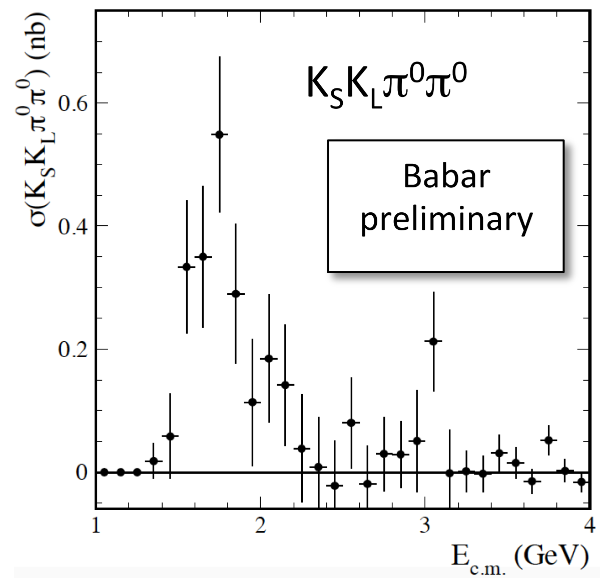
- ISR photon = highest energy γ with CM $E_\gamma > 3$ GeV
- At least one $K_S \rightarrow \pi^+\pi^-$ candidate consistent with interaction point
- At least two additional photons, with $m_{\gamma\gamma}$ consistent with m_{π^0} or m_η
- K_L candidates identified as isolated calorimeter clusters with E > 0.2 GeV; detection efficiency as a function of the K_L energy and direction measured in data from $e^+e^- \rightarrow \phi\gamma \rightarrow K_SK_L\gamma$ events
- Perform fit to all the signal processes ; retain $K_SK_L\pi^0$, $K_SK_L\eta$, and $K_SK_L\pi^0\pi^0$ candiate combinations with the lowest respective χ^2
- Background suppression and subtraction similar to that described for studies described previously



(III) First measurements of the $e^+e^- \rightarrow K_S K_L \pi^0$, $K_S K_L \eta$, and $K_S K_L \pi^0\pi^0$ cross section







First observations (all preliminary) of

$$B_{J/\varphi \rightarrow K_S K_L \pi^0} = (2.06 \pm 0.24 \pm 0.10) \times 10^{-3}$$

$$B_{J/\varphi \rightarrow K_S K_L \eta} = (1.45 \pm 0.32 \pm 0.08) \times 10^{-3}$$

$$B_{J/\varphi \rightarrow K_S K_L \pi^0 \pi^0} = (1.86 \pm 0.43 \pm 0.10) \times 10^{-3}$$

a_μ for $KK\pi\pi$

Use the results presented above for

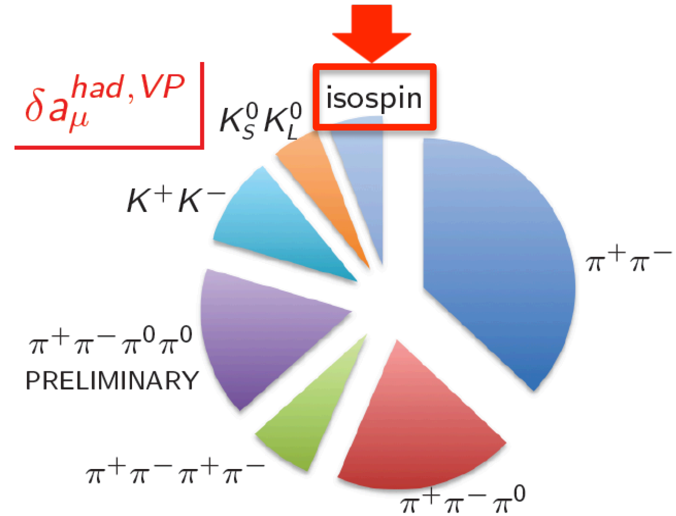
(II) $e^+e^- \to K_S K^+\pi^-\pi^0$ and

(III) $e^+e^- \to K_S K_L \pi^0\pi^0$

combined with our already published results for

- $e^+e^- \to K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$ [PRD 86 (2012) 012008];
- $e^+e^- \to K_S K_S \pi^+\pi^-$, $K_S K_L \pi^+\pi^-$ [PRD 89 (2014) 092002]

(including study of the intermediate states for all channels)

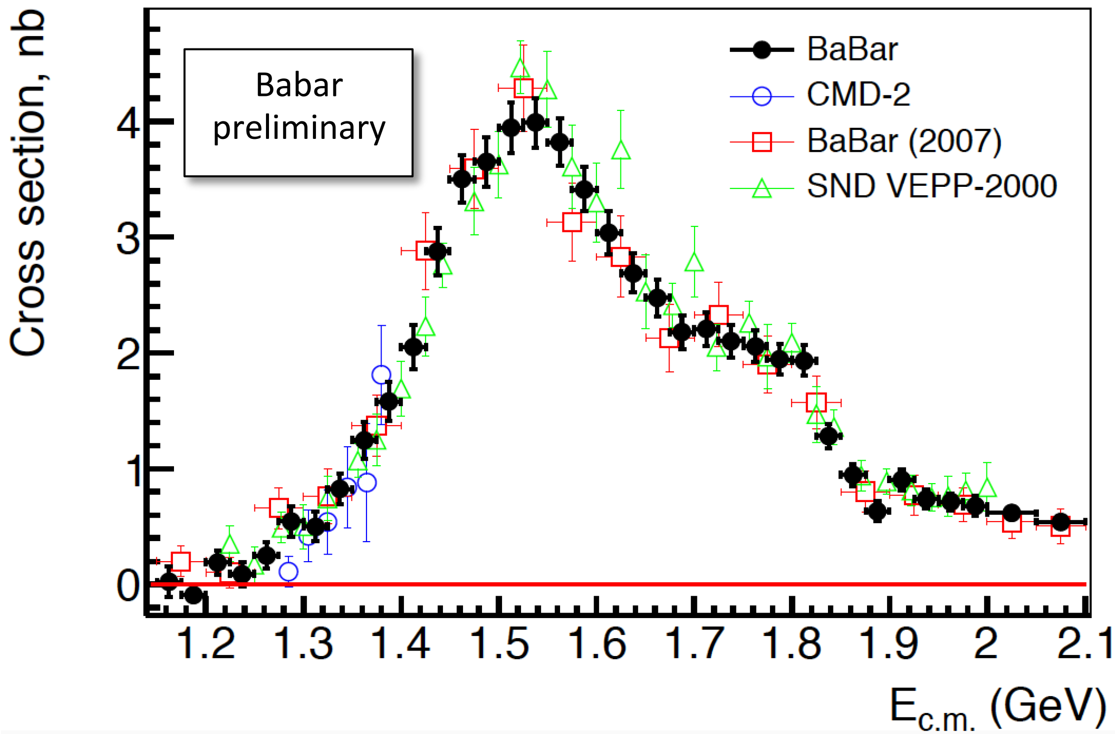


to calculate the contribution of $KK\pi\pi$ states to a_μ with a <u>much reduced reliance</u> on isospin relations to cover unmeasured channels

- Contribution to a_μ for $E_{CM} < 1.8$ GeV is [8.5 ± 0.5 (stat+syst)] x 10^{-11} (6% precision)

- Previous result was [13.5 ± 3.9 (stat+syst)] x 10^{-11} (30% precision)

(IV) Measurement of the $e^+e^- \rightarrow \pi^+\pi^-\eta$ cross section



- Provides a sizable contribution to the total hadronic cross section in the region relevant for a_μ
- Based on the final Babar data set (469 fb^{-1}) and the $\eta \rightarrow \gamma\gamma$ decay mode
- Complements and improves the precision of the Babar result from 2007 [PRD 76 (2007) 092005], based on 232 fb^{-1} and the $\eta \rightarrow \pi^+\pi^-\pi^0$ decay mode

Summary

- Precise low-energy e^+e^- hadronic cross section data needed to obtain an accurate SM prediction for $a_\mu^{had,LO-VP}$
- New results on from Babar reduce the respective uncertainty in $a_\mu^{had,LO-Vp}$ due to
    - $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ from around 7% to around 3% [175 ± 6 (stat+syst)] x 10^{-11}
    - $e^+e^- \rightarrow KK\pi\pi$ from around 30% to around 6% [8.5 ± 0.5 (stat+syst)] x 10^{-11}
- With the new data, including recently published results on $e^+e^- \rightarrow K^+K^-$ and K_SK_L from Babar, can perhaps reduce the uncertainty in the SM prediction for $a_\mu^{had,LO-VP}$ by up to 50% in the next few years [Blum et al., arXiv:1311.2198 (2013)]
- Besides the implications for $a_\mu^{had,LO-VP}$, the Babar ISR program has provided new tests of QCD, a wealth of information about low-mass resonances, and first observations of cross sections, and J/ψ and $\psi(2S)$ branching fractions