Photon from the Color Glass Condensate in the pA collision

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arXiv:1602.01989

in collaboration with:

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Motivation

- photon $\rightarrow$ clean probes

- in AA $\rightarrow$ early stage, QGP, hadron phase
- in pA $\rightarrow$ initial state

- **goal of this work**

  saturation effects in photon spectrum

Color Glass Condensate

- universal form of matter at

$$x \ll 1 \,, \quad Q^2 = \text{fixed}$$

→ **saturation scale** Q_s^2

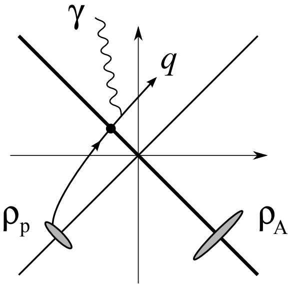
$$\frac{\alpha_s}{Q_s^2}\frac{xf_g(x,Q_s^2)}{\pi R^2} \sim 1$$

→ large gluon occupation number

→ classical color fields

Photon in pA

- valence quark bremsstrahung $O(\alpha)$



- Gelis – Jalilian-Marian formula

$$\frac{1}{\pi R_A^2}\frac{d\sigma^{q\to q\gamma}}{d^2k_\perp} = \frac{\alpha}{\pi}\frac{1}{k_\perp^2}\int_0^1 dz\frac{1+(1-z)^2}{z}\int_{l_\perp} C(\mathbf{l}_\perp)\frac{l_\perp^2}{(\mathbf{l}_\perp - \mathbf{k}_\perp/z)^2}$$

color dipole: $\int_{x_\perp} e^{i\mathbf{l}_\perp\cdot\mathbf{x}_\perp}\langle U(0)U^\dagger(\mathbf{x}_\perp)\rangle$

Gelis, Jalilian-Marian, Phys. Rev. D **66** (2002) 014021

Photon in pA

- but: high energy (small x)
  $\rightarrow$ gluon component of the proton wave function becomes dominant

  $\rightarrow$ **new emission processes**

Power counting

- proton: gluons more abundant than quarks

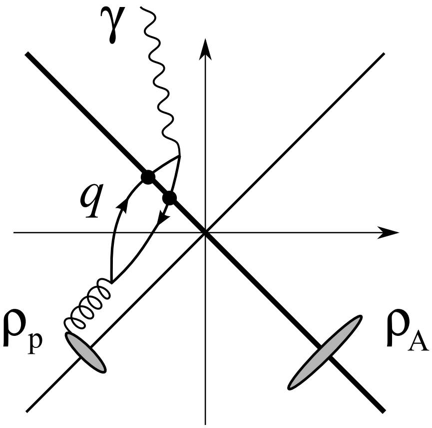
$$f_q \ll f_g$$

- nucleus dense, proton dilute

$$\rho_p \ll \rho_A$$

Photon in pA

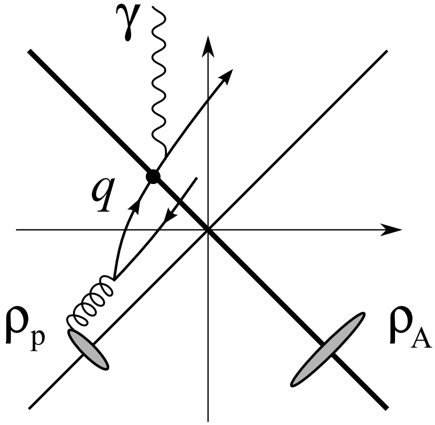
- annihilation $O(\alpha\alpha_s)$



SB, Fukushima, arXiv:1602.01989

Photon in pA

- bremsstrahlung from produced $\bar{q}q$ $O(\alpha\alpha_s)$



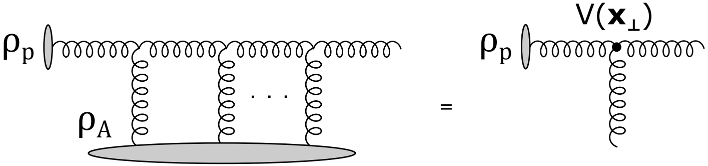
SB, Fukushima, arXiv:1602.01989

pA CGC Feynman rules

- LC gauge: $n^\mu \mathcal{A}_\mu = 0$, $n^\mu \equiv \delta^{\mu +}$

1. background gluon field

$$[D_\mu, \mathcal{F}^{\mu\nu}] = g\delta^{\nu +}\delta(x^-)\rho_p(\boldsymbol{x}_\perp) + g\delta^{\nu -}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$



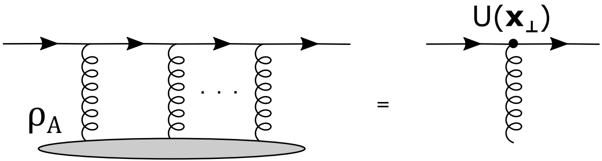
Gelis, Mehtar-Tani, Phys. Rev. D **73** (2006) 034019
Fukushima and Hidaka, Nucl. Phys. A **813** (2008) 171

pA CGC Feynman rules

2. quark propagator

$$S_{(0)}(x,y) = S_F(x-y)$$
$$+ i\theta(x^+)\theta(-y^+)\int d^4z\delta(z^+)(U(\mathbf{z}_\perp)-1)S_F(x-z)\not{n}S_F(z-y)$$
$$- i\theta(-x^+)\theta(y^+)\int d^4z\delta(z^+)(U(\mathbf{z}_\perp)-1)^\dagger S_F(x-z)\not{n}S_F(z-y)$$

multiple scattering effect: $\mathcal{P}_{x^+}\exp\left[ig^2\int_{-\infty}^{\infty}dx^+\mathcal{A}^-_{(0)}(x)\right]$



Baltz, McLerran, Phys. Rev. C **58** (1998) 1679

Annihilation - amplitude

$$\mathcal{M}_\lambda(\boldsymbol{k}) = eg \int_{xy} e^{ik\cdot x}\,\mathrm{Tr}\big[\not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x,y) \not{A}_{(1)}(y) S_{(0)}(y,x)\big]$$

Annihilation - amplitude

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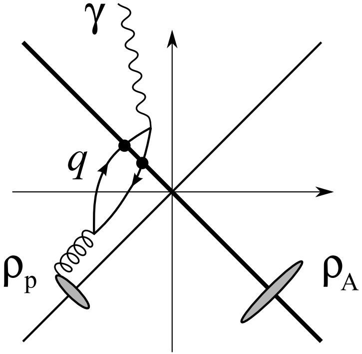

Annihilation - rate

$$\frac{1}{\pi R_A^2}\frac{dN}{d^2\mathbf{k}_\perp dy} = \frac{\alpha\,\alpha_s}{16\pi^8}\frac{N_c}{N_c^2-1}\int_{xx'}\int_{\mathbf{u}_\perp\mathbf{u}'_\perp\mathbf{w}_\perp} e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}$$
$$\times\, S\left(\mathbf{u}_\perp - \frac{\mathbf{v}_\perp}{2}, \mathbf{u}_\perp + \frac{\mathbf{v}_\perp}{2}, \mathbf{u}'_\perp - \frac{\mathbf{v}'_\perp}{2}, \mathbf{u}'_\perp + \frac{\mathbf{v}'_\perp}{2}\right)$$
$$\times \int_{\mathbf{l}_\perp} e^{i\mathbf{l}_\perp\cdot\mathbf{r}_\perp}\varphi_p(l_\perp)\left(\hat{\mathbf{u}}_\perp\cdot\hat{\mathbf{u}}'_\perp\Psi_1\Psi_1'^* + \Psi_2\Psi_2'^* + 2\hat{\mathbf{u}}_\perp\cdot\hat{\mathbf{l}}_\perp\Psi_1\Psi_2'^*\right)$$

- unintegrated gluon distribution

$$g^2\left\langle\rho_p^a(\mathbf{l}_\perp)\rho_p^{a'}(\mathbf{l}_\perp)\right\rangle \equiv \frac{\delta^{aa'}}{\pi(N_c^2-1)} l_\perp^2\,\varphi_p(l_\perp)$$

- inelastic quadrupole

$$S(\mathbf{y}_\perp,\mathbf{z}_\perp,\mathbf{y}'_\perp,\mathbf{z}'_\perp) \equiv \frac{1}{N_c}\left\langle \mathrm{Tr}_c\left[U(\mathbf{y}_\perp)T_F^a U^\dagger(\mathbf{z}_\perp)\right]\mathrm{Tr}_c\left[U(\mathbf{z}'_\perp)T_F^a U^\dagger(\mathbf{y}'_\perp)\right]\right\rangle$$

SB, Fukushima, arXiv:1602.01989

Remarks

✓ photon Ward identity

✓ Furry theorem (vanishing of $gg \to \gamma$)

✓ UV finite (lowest order is $ggg \to \gamma$)

- chiral limit

✓ collinear factorization on the proton side

Brems from produced $\bar{q}q$ - amplitude

$$\mathcal{M}_\lambda(\mathbf{k},\mathbf{q},\mathbf{p}) = ieg\int_{xyz} e^{ik\cdot x+iq\cdot y+ip\cdot z}\,\bar{u}(\mathbf{q})(i\vec{\not\partial}_y - m)$$
$$\times\Big\{S_{(0)}(y,w)\not{\mathcal{A}}_{(1)}(w)S_{(0)}(w,x)\not\epsilon_\lambda(\mathbf{k})S_{(0)}(x,z)$$
$$+ S_{(0)}(y,x)\not\epsilon_\lambda(\mathbf{k})S_{(0)}(x,w)\not{\mathcal{A}}_{(1)}(w)S_{(0)}(w,z)\Big\}(i\overleftarrow{\not\partial}_z + m)v(\mathbf{p})$$

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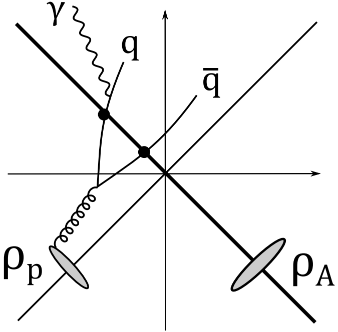

SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

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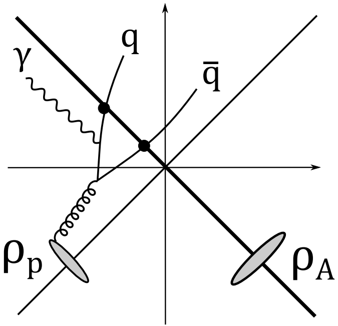
$$+\,S_{(0)}(y,x)\not\epsilon_\lambda(\mathbf{k})S_{(0)}(x,w)\not{\mathcal{A}}_{(1)}(w)S_{(0)}(w,z)\Big\}(i\overleftarrow{\not\partial}_z + m)v(\mathbf{p})$$



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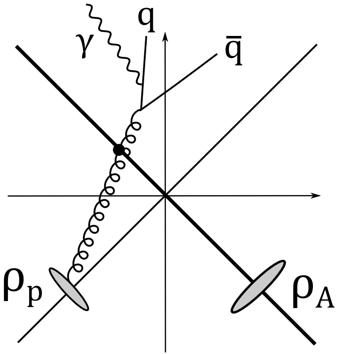
$$\times\Big\{S_{(0)}(y,w)\not{\mathcal{A}}_{(1)}(w)S_{(0)}(w,x)\not{\epsilon}_\lambda(\mathbf{k})S_{(0)}(x,z)$$
$$+ S_{(0)}(y,x)\not{\epsilon}_\lambda(\mathbf{k})S_{(0)}(x,w)\not{\mathcal{A}}_{(1)}(w)S_{(0)}(w,z)\Big\}(i\overleftarrow{\not{\partial}}_z + m)v(\mathbf{p})$$



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Brems from produced $\bar{q}q$ - rate

$$
\begin{aligned}
&\frac{1}{\pi R_A^2}\frac{d\sigma}{d^2\mathbf{k}_\perp dy\, d^2\mathbf{q}_\perp dy_q\, d^2\mathbf{p}_\perp dy_p} = -\frac{\alpha\alpha_s N_c}{256\pi^8(N_c^2-1)}\int_{\mathbf{l}_\perp \mathbf{s}_\perp \mathbf{s}'_\perp}\frac{\varphi_p(l_\perp)}{l_\perp^2}\\
&\times\int_{\mathbf{u}_\perp \mathbf{u}'_\perp \mathbf{w}_\perp} e^{i\left(\mathbf{s}_\perp+\frac{\mathbf{l}_\perp-\mathbf{Q}_\perp}{2}\right)\cdot\mathbf{u}_\perp} e^{-i\left(\mathbf{s}'_\perp+\frac{\mathbf{l}_\perp-\mathbf{Q}_\perp}{2}\right)\cdot\mathbf{u}'_\perp} e^{i(\mathbf{l}_\perp-\mathbf{Q}_\perp)\cdot\mathbf{w}_\perp}\\
&\times C\left(\mathbf{u}_\perp-\frac{\mathbf{v}_\perp}{2},\mathbf{u}_\perp+\frac{\mathbf{v}_\perp}{2},\mathbf{u}'_\perp-\frac{\mathbf{v}'_\perp}{2},\mathbf{u}'_\perp+\frac{\mathbf{v}'_\perp}{2}\right)\\
&\times\Big\{C_\mu(Q,\mathbf{l}_\perp,\mathbf{Q}_\perp-\mathbf{l}_\perp)C_{\mu'}(Q,\mathbf{l}_\perp,\mathbf{Q}_\perp-\mathbf{l}_\perp)\mathrm{Tr}_D\left[(\not{q}+m)T_g^{\mu\nu}(\not{p}-m)\overline{T}^{\mu'}_{g\nu}\right]\\
&+\frac{l_i}{Q^+}\frac{l_{i'}}{Q^+}\mathrm{Tr}_D\left[(\not{q}+m)T^{i\nu}_{\bar{q}q}(\mathbf{l}_\perp,\mathbf{s}_\perp)(\not{p}-m)\overline{T}^{i'}_{\bar{q}q\nu}(\mathbf{l}_\perp,\mathbf{s}'_\perp)\right]\\
&+\frac{l_{i'}}{Q^+}C_\mu(Q,\mathbf{l}_\perp,\mathbf{Q}_\perp-\mathbf{l}_\perp)\mathrm{Tr}_D\left[(\not{q}+m)T_g^{\mu\nu}(\not{p}-m)\overline{T}^{i'}_{\bar{q}q\nu}(\mathbf{l}_\perp,\mathbf{s}'_\perp)\right]+\text{h. c.}\Big\}
\end{aligned}
$$

- same Wilson line product as in $\bar{q}q$ production

$$
C(\mathbf{y}_\perp,\mathbf{z}_\perp,\mathbf{y}'_\perp,\mathbf{z}'_\perp)\equiv\frac{1}{N_c}\left\langle\mathrm{Tr}_c\left[U(\mathbf{y}_\perp)T_F^a U^\dagger(\mathbf{z}_\perp)U(\mathbf{z}'_\perp)T_F^a U^\dagger(\mathbf{y}'_\perp)\right]\right\rangle
$$

SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

Remarks

- ✓ photon Ward identity
- ✓ soft photon theorem
- ✓ collinear factorization

Color average

$$\left\langle \mathcal{O}[\rho_p, \rho_A] \right\rangle = \int [d\rho_p][d\rho_A] W_p[x_p; \rho_p] W_A[x_A; \rho_A] \mathcal{O}[\rho_p, \rho_A]$$

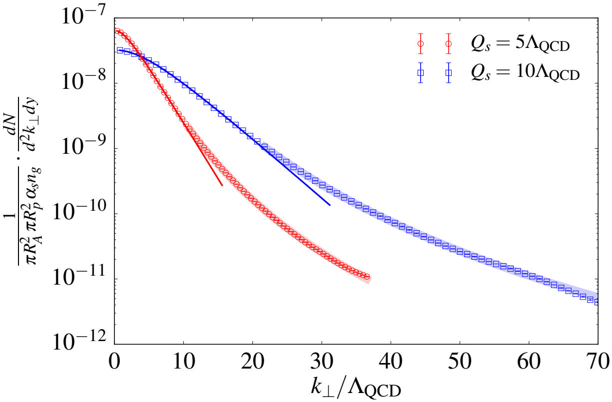
- McLerran-Venugopalan model

$$\left\langle \rho_A^a(\mathbf{x}_\perp) \rho_A^b(\mathbf{y}_\perp) \right\rangle = g^2 \delta^{ab} \mu_A^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

$$Q_s^2 \equiv \frac{N_c^2 - 1}{4N_c} g^4 \mu_A^2$$

- reasonable for x $\sim 10^{-2}$
- x evolution $\rightarrow$ JIMWLK

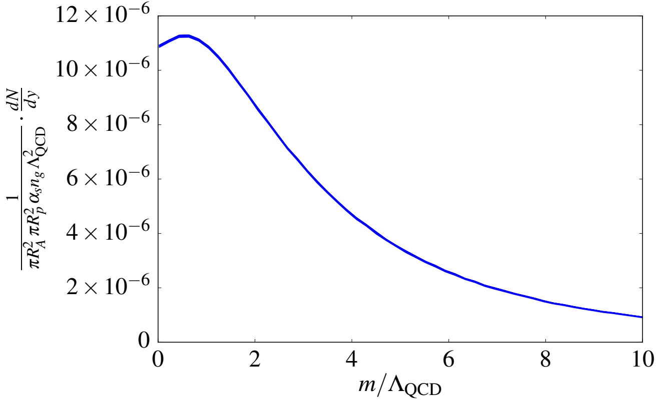
Annihilation - photon spectrum



- single flavor, chiral limit
- thin lines: $\exp\left(-\sqrt{k_\perp^2 + (0.5Q_s)^2}/0.5Q_s\right)$
- thick lines: $(\log(k_\perp/Q_s))^{1.5}/k_\perp^{5.6}$

SB, Fukushima, arXiv:1602.01989

Annihilation - mass dependence



- fit $(\log(m/\Lambda_{QCD}))^{1.8}/m^{2.6}$
- mass corrections important

SB, Fukushima, arXiv:1602.01989

Conclusions and outlook

✓ complete analytical result at $O(\alpha\alpha_s)$

✓ numerical evaluation of the annihilation diagram

- sensitivity to quadrupole gluon correlators

- phenomenological applications

  bremsstrahlung - numerical evaluation

  small x evolution

Backup slides

Gluon field

- expansion in powers of proton density

$$\mathcal{A}^{\mu} = \mathcal{A}^{\mu}_{(0)} + \mathcal{A}^{\mu}_{(1)}$$

$$\mathcal{A}^{\mu}_{(0)} = -gn^{\mu}\delta(x^{+})\frac{1}{\partial_{\perp}^{2}}\rho_{A}(\boldsymbol{x}_{\perp})$$

$$\mathcal{A}^{\mu}_{(1)}(x) = \mathcal{A}^{\mu}_{(1<)}(x) + \mathcal{A}^{\mu}_{(1>)}(x)$$

$$\mathcal{A}^{+}_{(1<)}(p) = 0 \quad \mathcal{A}^{-}_{(1<)}(p) = 0 \quad -p^{2}\mathcal{A}^{i}_{(1<)}(p) = -igp^{i}\frac{l^{2}}{(p^{+}+i\epsilon)(p^{-}-i\epsilon)}\frac{\rho_{p}(\mathbf{p}_{\perp})}{p_{\perp}^{2}}$$

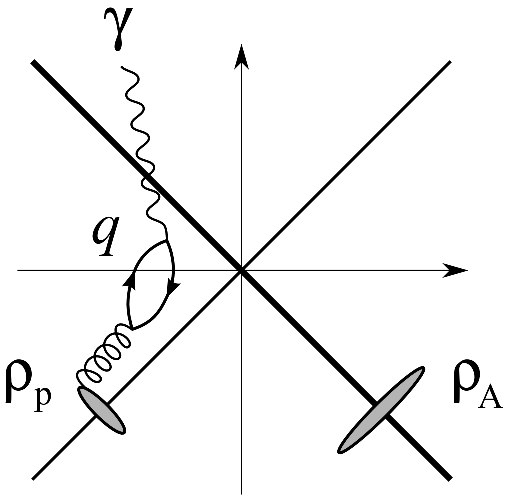
$$-p^{2}\mathcal{A}^{\mu}_{(1>)}(p) = -ig\int\frac{d^{2}\mathbf{q}_{\perp}}{(2\pi)^{2}}C^{\mu}(p;\mathbf{q}_{\perp},\mathbf{p}_{\perp}-\mathbf{q}_{\perp})V(\mathbf{p}_{\perp}-\mathbf{q}_{\perp})\frac{\rho_{p}(\mathbf{q}_{\perp})}{q_{\perp}^{2}}$$

$$C^{+}(p;\mathbf{q}_{\perp},\mathbf{p}_{\perp}-\mathbf{q}_{\perp}) = 0$$

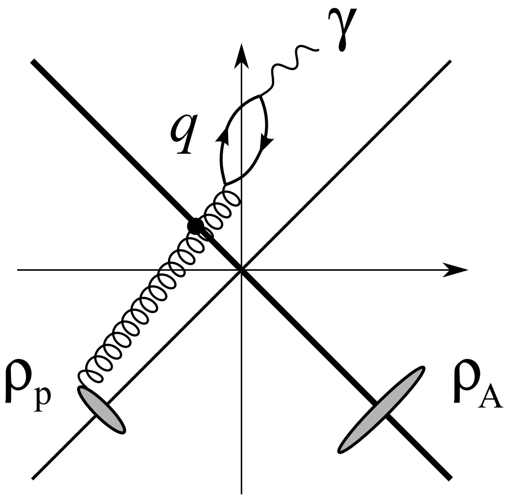
$$C^{-}(p;\mathbf{q}_{\perp},\mathbf{p}_{\perp}-\mathbf{q}_{\perp}) = \frac{-2\mathbf{q}_{\perp}\cdot(\mathbf{p}_{\perp}-\mathbf{q}_{\perp})}{p^{+}+i\epsilon}$$

$$C^{i}(p;\mathbf{q}_{\perp},\mathbf{p}_{\perp}-\mathbf{q}_{\perp}) = \frac{p^{i}\,q_{\perp}^{2}}{(p^{+}+i\epsilon)(p^{-}+i\epsilon)} - 2q^{i}$$

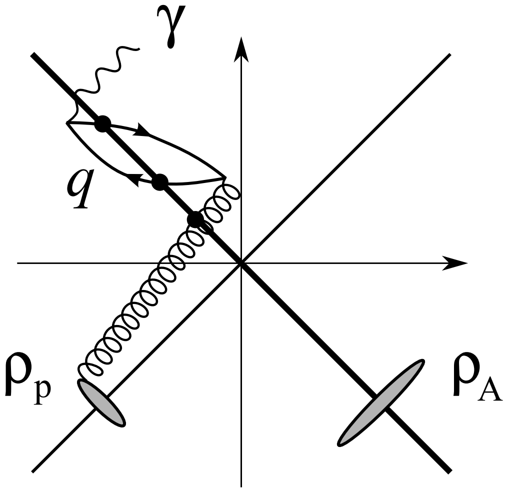
Annihilation - diagrams



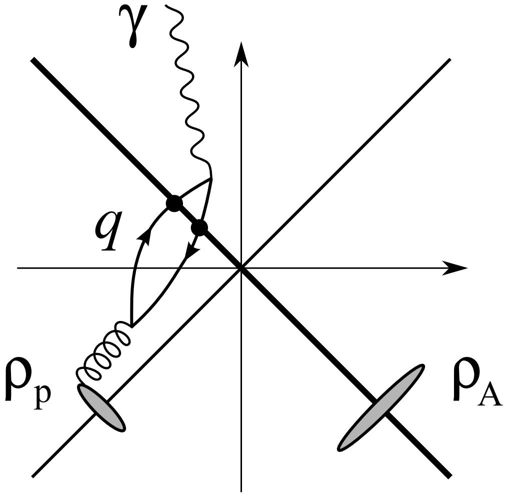
Annihilation - diagrams



Annihilation - diagrams



Annihilation - diagrams



Annihilation - amplitude

- after light-cone integrations

$$\mathcal{M}_\lambda(\mathbf{k}) = \frac{eg^2}{4\pi^3}\int_0^1 dx \int_{\mathbf{u}_\perp \mathbf{v}_\perp}\int_{\mathbf{l}_\perp} e^{-i(\mathbf{k}_\perp - \mathbf{l}_\perp)\cdot[\mathbf{v}_\perp + a(x)\mathbf{u}_\perp]}$$
$$\times \frac{\rho_p^a(\mathbf{l}_\perp)}{l_\perp^2}\mathrm{Tr}_c\left[U\left(\mathbf{v}_\perp + \frac{\mathbf{u}_\perp}{2}\right)T_F^a U^\dagger\left(\mathbf{v}_\perp - \frac{\mathbf{u}_\perp}{2}\right)\right]$$
$$\times \left[\hat{u}_\lambda l_\perp \Psi_1(\mathbf{l}_\perp, \mathbf{u}_\perp, x) + l_\lambda \Psi_2(\mathbf{l}_\perp, \mathbf{u}_\perp, x)\right]$$

$$\Psi_1(\mathbf{l}_\perp, \mathbf{u}_\perp, x) \equiv -4ia(x)b(x)l_\perp K_0(m_l(x)u_\perp)mK_1(mu_\perp) + 4b(x)\hat{\mathbf{l}}_\perp \cdot \hat{\mathbf{u}}_\perp m_l(x)K_1(m_l(x)u_\perp)mK_1(mu_\perp)$$
$$\Psi_2(\mathbf{l}_\perp, \mathbf{u}_\perp, x) \equiv mK_1(mu_\perp)m_l(x)K_1(m_l(x)u_\perp) + m^2 K_0(mu_\perp)K_0(m_l(x)u_\perp)$$

Brems from produced $\bar{q}q$ - amplitude

- after light-cone integrations

$$\mathcal{M}_\lambda(\mathbf{k},\mathbf{q},\mathbf{p}) = eg^2 \int_{\mathbf{l}_\perp \mathbf{s}_\perp} \frac{\rho_p^a(\mathbf{l}_\perp)}{l_\perp^2} \int_{\mathbf{x}_\perp \mathbf{y}_\perp} e^{-i\mathbf{s}_\perp\cdot\mathbf{x}_\perp} e^{i(\mathbf{l}_\perp+\mathbf{s}_\perp-\mathbf{Q}_\perp)\cdot\mathbf{y}_\perp} \times \bar{u}(\mathbf{q})\Big\{ T_g^{\mu\nu} C_\mu(Q,\mathbf{l}_\perp,\mathbf{Q}_\perp-\mathbf{l}_\perp)\epsilon_{\lambda\nu}(\mathbf{k}) T_F^b V^{ba}(\mathbf{y}_\perp) + T_{\bar{q}q}^{i\nu}(\mathbf{l}_\perp,\mathbf{s}_\perp)\frac{l_i}{Q^+}\epsilon_{\lambda\nu}(\mathbf{k}) U(\mathbf{x}_\perp) T_F^a U^\dagger(\mathbf{y}_\perp)\Big\} v(\mathbf{p})$$

Brems from produced $\bar{q}q$ - amplitude

$$T_g^{\mu\nu} \equiv T_1^{\mu\nu} + T_7^{\mu\nu}$$
$$T_{\bar{q}q}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv T_5^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_6^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_{11}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_{12}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp)$$

$$T_1^{\mu\nu} \equiv -\frac{1}{Q^2}\gamma^\mu S_F(-k-p)\gamma^\nu$$

$$T_5^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv \frac{\not{n}(\not{q} - \not{s} + m)\gamma^\mu(\not{q} - \not{l} - \not{s} + m)\not{n} S_F(-k-p)\gamma^\nu}{2(k^+ + p^+)\omega_{q-s}^2 + 2q^+\omega_{q-l-s}^2}$$

$$T_6^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv -2q^+ \frac{\not{n}(\not{q} - \not{s} + m)\gamma^\mu(\not{q} - \not{l} - \not{s} + m)\gamma^\nu(\not{k} + \not{q} - \not{l} - \not{s} + m)\not{n}}{[2(k^+ + p^+)\omega_{q-s}^2 + 2q^+\omega_{q-l-s}^2][2q^+2p^+k^- + 2p^+\omega_{q-s}^2 + 2q^+\omega_{k+q-l-s}^2]}$$

$$T_7^{\mu\nu} \equiv -\frac{1}{Q^2}\gamma^\nu S_F(k+q)\gamma^\mu$$

$$T_{11}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv \frac{\gamma^\nu S_F(k+q)\not{n}(\not{k} + \not{q} - \not{s} + m)\gamma^\mu(\not{k} + \not{q} - \not{l} - \not{s} + m)\not{n}}{2p^+\omega_{k+q-s}^2 + 2(k^+ + q^+)\omega_{k+q-l-s}^2}$$

$$T_{12}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv -2p^+ \frac{\not{n}(\not{q} - \not{s} + m)\gamma^\nu(\not{k} + \not{q} - \not{s} + m)\gamma^\mu(\not{k} + \not{q} - \not{l} - \not{s} + m)\not{n}}{[2p^+\omega_{k+q-s}^2 + 2(k^+ + q^+)\omega_{k+q-l-s}^2][2q^+2p^+k^- + 2p^+\omega_{q-s}^2 + 2q^+\omega_{k+q-l-s}^2]}$$

Annihilation - color average

$$S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) = \frac{1}{N_c} \frac{(N_c^2 - 1)(\beta - \alpha)}{\sqrt{N_c^2(\alpha - \gamma)^2 - 4(\alpha - \beta)(\beta - \gamma)}}$$

$$\times \exp\left[-\frac{2\beta + (N_c^2 - 2)(\alpha + \gamma)}{N_c^2 - 1}\right] \sinh\left[\frac{N_c}{N_c^2 - 1}\sqrt{N_c^2(\alpha - \gamma)^2 - 4(\alpha - \beta)(\beta - \gamma)}\right]$$

$$2\alpha(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\boldsymbol{y}_\perp - \boldsymbol{z}'_\perp) + B_2(\boldsymbol{z}_\perp - \boldsymbol{y}'_\perp)$$
$$2\beta(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp) + B_2(\boldsymbol{z}_\perp - \boldsymbol{z}'_\perp)$$
$$2\gamma(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\boldsymbol{y}_\perp - \boldsymbol{z}_\perp) + B_2(\boldsymbol{y}'_\perp - \boldsymbol{z}'_\perp)$$

$$B_2(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv Q_s^2 \int d^2\mathbf{z}_\perp [G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)]$$