

Photon from the Color Glass Condensate in the pA collision

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arXiv:1602.01989

in collaboration with:

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Motivation

- photon \rightarrow clean probes
- in AA \rightarrow early stage, QGP, hadron phase
- in pA \rightarrow initial state
- **goal of this work**
saturation effects in photon spectrum

Color Glass Condensate

- **universal** form of matter at

$$x \ll 1, \quad Q^2 = \text{fixed}$$

→ **saturation scale** Q_s^2

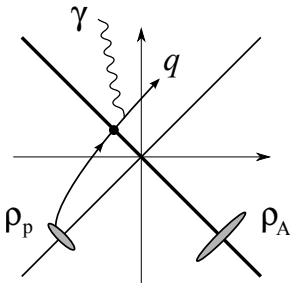
$$\frac{\alpha_s}{Q_s^2} \frac{x f_g(x, Q_s^2)}{\pi R^2} \sim 1$$

→ **large gluon occupation number**

→ **classical color fields**

Photon in pA

- valence quark bremsstrahlung $O(\alpha)$



- Gelis – Jalilian-Marian formula

$$\frac{1}{\pi R_A^2} \frac{d\sigma^{q \rightarrow q\gamma}}{d^2 k_\perp} = \frac{\alpha}{\pi} \frac{1}{k_\perp^2} \int_0^1 dz \frac{1 + (1-z)^2}{z} \int_{l_\perp} \boxed{C(l_\perp)} \frac{l_\perp^2}{(l_\perp - \mathbf{k}_\perp/z)^2}$$

color dipole: $\int_{x_\perp} e^{i l_\perp \cdot x_\perp} \langle U(0) U^\dagger(x_\perp) \rangle$

Photon in pA

- but: high energy (small x)
 - gluon component of the proton wave function becomes dominant
 - **new emission processes**

Power counting

- proton: gluons more abundant than quarks

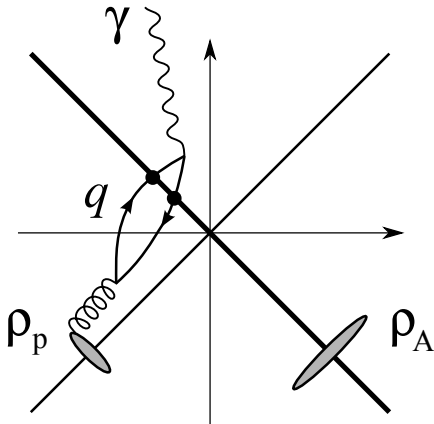
$$f_q \ll f_g$$

- nucleus dense, proton dilute

$$\rho_p \ll \rho_A$$

Photon in pA

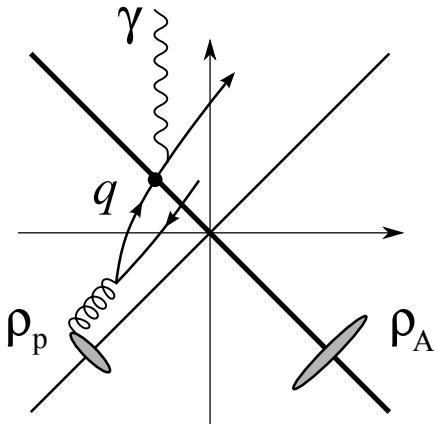
- annihilation $O(\alpha\alpha_s)$



SB, Fukushima, arXiv:1602.01989

Photon in pA

- bremsstrahlung from produced $\bar{q}q$ $O(\alpha\alpha_s)$



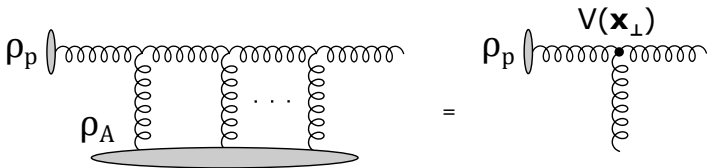
SB, Fukushima, arXiv:1602.01989

pA CGC Feynman rules

- **LC gauge:** $n^\mu \mathcal{A}_\mu = 0$, $n^\mu \equiv \delta^{\mu+}$

1. background gluon field

$$[D_\mu, \mathcal{F}^{\mu\nu}] = g\delta^{\nu+}\delta(x^-)\rho_p(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_A(\mathbf{x}_\perp)$$



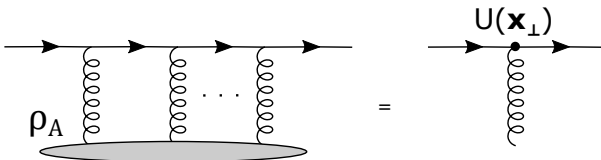
Gelis, Mehtar-Tani, Phys. Rev. D **73** (2006) 034019
Fukushima and Hidaka, Nucl. Phys. A **813** (2008) 171

pA CGC Feynman rules

2. quark propagator

$$\begin{aligned}
 S_{(0)}(x, y) &= S_F(x - y) \\
 &+ i\theta(x^+)\theta(-y^+) \int d^4z \delta(z^+) (U(\mathbf{z}_\perp) - 1) S_F(x - z) \not{n} S_F(z - y) \\
 &- i\theta(-x^+)\theta(y^+) \int d^4z \delta(z^+) (U(\mathbf{z}_\perp) - 1)^\dagger S_F(x - z) \not{n} S_F(z - y)
 \end{aligned}$$

multiple scattering effect: $\mathcal{P}_{x^+} \exp \left[ig^2 \int_{-\infty}^{\infty} dx^+ \mathcal{A}_{(0)}^-(x) \right]$



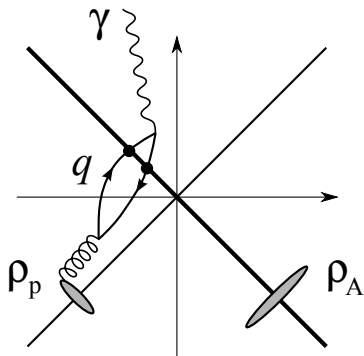
Baltz, McLerran, Phys. Rev. C **58** (1998) 1679

Annihilation - amplitude

$$\mathcal{M}_\lambda(\mathbf{k}) = eg \int_{xy} e^{ik \cdot x} \text{Tr}[\not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, y) \mathcal{A}_{(1)}(y) S_{(0)}(y, x)]$$

Annihilation - amplitude

$$\mathcal{M}_\lambda(\mathbf{k}) = eg \int_{xy} e^{ik \cdot x} \text{Tr}[\not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, y) \not{A}_{(1)}(y) S_{(0)}(y, x)]$$



Annihilation - rate

$$\begin{aligned} \frac{1}{\pi R_A^2} \frac{dN}{d^2\mathbf{k}_\perp dy} &= \frac{\alpha \alpha_s}{16\pi^8} \frac{N_c}{N_c^2 - 1} \int_{xx'} \int_{\mathbf{u}_\perp \mathbf{u}'_\perp \mathbf{w}_\perp} e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \\ &\times S\left(\mathbf{u}_\perp - \frac{\mathbf{v}_\perp}{2}, \mathbf{u}_\perp + \frac{\mathbf{v}_\perp}{2}, \mathbf{u}'_\perp - \frac{\mathbf{v}'_\perp}{2}, \mathbf{u}'_\perp + \frac{\mathbf{v}'_\perp}{2}\right) \\ &\times \int_{I_\perp} e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp} \varphi_p(I_\perp) (\hat{\mathbf{u}}_\perp \cdot \hat{\mathbf{u}}'_\perp \Psi_1 \Psi_1^* + \Psi_2 \Psi_2^* + 2\hat{\mathbf{u}}_\perp \cdot \hat{\mathbf{l}}_\perp \Psi_1 \Psi_2^*) \end{aligned}$$

- unintegrated gluon distribution

$$g^2 \langle \rho_p^a(\mathbf{l}_\perp) \rho_p^{a'}(\mathbf{l}_\perp) \rangle \equiv \frac{\delta^{aa'}}{\pi(N_c^2 - 1)} l_\perp^2 \varphi_p(I_\perp)$$

- inelastic quadrupole

$$S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv \frac{1}{N_c} \left\langle \text{Tr}_c [U(\mathbf{y}_\perp) T_F^a U^\dagger(\mathbf{z}_\perp)] \text{Tr}_c [U(\mathbf{z}'_\perp) T_F^a U^\dagger(\mathbf{y}'_\perp)] \right\rangle$$

Remarks

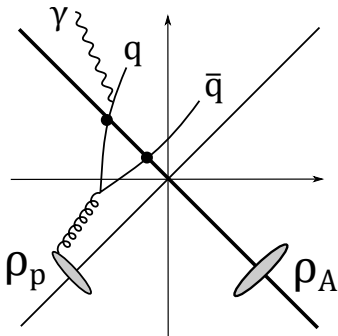
- ✓ photon Ward identity
- ✓ Furry theorem (vanishing of $gg \rightarrow \gamma$)
- ✓ UV finite (lowest order is $ggg \rightarrow \gamma$)
 - chiral limit
- ✓ collinear factorization on the proton side

Brems from produced $\bar{q}q$ - amplitude

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{ik \cdot x + iq \cdot y + ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\not{\partial}}_y - m) \\ &\times \left\{ S_{(0)}(y, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, z) \right. \\ &\left. + S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, z) \right\} (i\vec{\not{\partial}}_z + m) v(\mathbf{p}) \end{aligned}$$

Brems from produced $\bar{q}q$ - amplitude

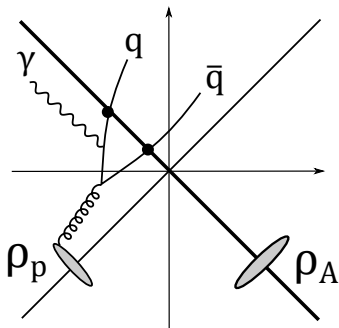
$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{ik \cdot x + iq \cdot y + ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\not{\partial}}_y - m) \\ &\times \left\{ S_{(0)}(y, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, z) \right. \\ &\left. + S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, z) \right\} (i\vec{\not{\partial}}_z + m) v(\mathbf{p}) \end{aligned}$$



SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

Brems from produced $\bar{q}q$ - amplitude

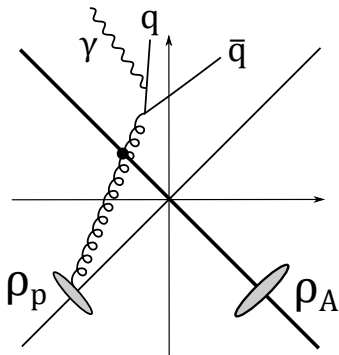
$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{ik \cdot x + iq \cdot y + ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\not{\partial}}_y - m) \\ &\times \left\{ S_{(0)}(y, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, z) \right. \\ &\left. + S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, z) \right\} (i\vec{\not{\partial}}_z + m) v(\mathbf{p}) \end{aligned}$$



SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

Brems from produced $\bar{q}q$ - amplitude

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{ik \cdot x + iq \cdot y + ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\partial}_y - m) \\ &\times \left\{ S_{(0)}(y, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, z) \right. \\ &\left. + S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, z) \right\} (i\vec{\partial}_z + m) v(\mathbf{p}) \end{aligned}$$



SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

Brems from produced $\bar{q}q$ - rate

$$\begin{aligned}
 & \frac{1}{\pi R_A^2} \frac{d\sigma}{d^2\mathbf{k}_\perp dy d^2\mathbf{q}_\perp dy_q d^2\mathbf{p}_\perp dy_p} = -\frac{\alpha\alpha_s N_c}{256\pi^8(N_c^2 - 1)} \int_{\mathbf{l}_\perp \mathbf{s}_\perp \mathbf{s}'_\perp} \frac{\varphi_p(\mathbf{l}_\perp)}{l_\perp^2} \\
 & \times \int_{\mathbf{u}_\perp \mathbf{u}'_\perp \mathbf{w}_\perp} e^{i(\mathbf{s}_\perp + \frac{\mathbf{l}_\perp - \mathbf{Q}_\perp}{2}) \cdot \mathbf{u}_\perp} e^{-i(\mathbf{s}'_\perp + \frac{\mathbf{l}_\perp - \mathbf{Q}_\perp}{2}) \cdot \mathbf{u}'_\perp} e^{i(\mathbf{l}_\perp - \mathbf{Q}_\perp) \cdot \mathbf{w}_\perp} \\
 & \times C\left(\mathbf{u}_\perp - \frac{\mathbf{v}_\perp}{2}, \mathbf{u}_\perp + \frac{\mathbf{v}_\perp}{2}, \mathbf{u}'_\perp - \frac{\mathbf{v}'_\perp}{2}, \mathbf{u}'_\perp + \frac{\mathbf{v}'_\perp}{2}\right) \\
 & \times \left\{ C_\mu(Q, \mathbf{l}_\perp, \mathbf{Q}_\perp - \mathbf{l}_\perp) C_{\mu'}(Q, \mathbf{l}_\perp, \mathbf{Q}_\perp - \mathbf{l}_\perp) \text{Tr}_D [(\not{q} + m) T_g^{\mu\nu} (\not{p} - m) \bar{T}_{g\nu}^{\mu'}] \right. \\
 & + \frac{l_i}{Q^+} \frac{l'_i}{Q^+} \text{Tr}_D [(\not{q} + m) T_{\bar{q}q}^{i\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) (\not{p} - m) \bar{T}'_{\bar{q}q\nu}(\mathbf{l}_\perp, \mathbf{s}'_\perp)] \\
 & \left. + \frac{l'_i}{Q^+} C_\mu(Q, \mathbf{l}_\perp, \mathbf{Q}_\perp - \mathbf{l}_\perp) \text{Tr}_D [(\not{q} + m) T_g^{\mu\nu} (\not{p} - m) \bar{T}'_{\bar{q}q\nu}(\mathbf{l}_\perp, \mathbf{s}'_\perp)] + \text{h.c.} \right\}
 \end{aligned}$$

- same Wilson line product as in $\bar{q}q$ production

$$C(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv \frac{1}{N_c} \langle \text{Tr}_c [U(\mathbf{y}_\perp) T_F^a U^\dagger(\mathbf{z}_\perp) U(\mathbf{z}'_\perp) T_F^a U^\dagger(\mathbf{y}'_\perp)] \rangle$$

SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

Remarks

- ✓ photon Ward identity
- ✓ soft photon theorem
- ✓ collinear factorization

Color average

$$\langle \mathcal{O}[\rho_p, \rho_A] \rangle = \int [d\rho_p][d\rho_A] W_p[x_p; \rho_p] W_A[x_A; \rho_A] \mathcal{O}[\rho_p, \rho_A]$$

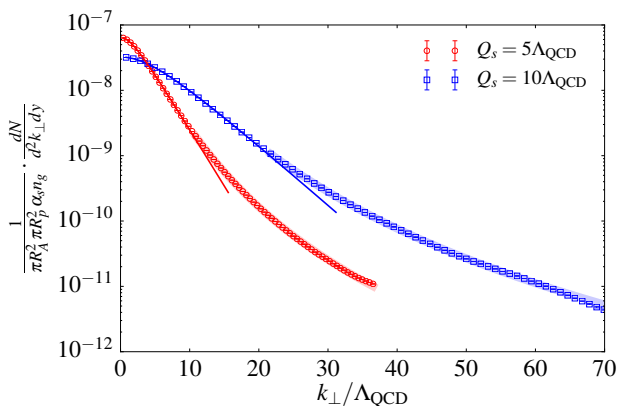
- McLerran-Venugopalan model

$$\langle \rho_A^a(\mathbf{x}_\perp) \rho_A^b(\mathbf{y}_\perp) \rangle = g^2 \delta^{ab} \mu_A^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

$$Q_s^2 \equiv \frac{N_c^2 - 1}{4N_c} g^4 \mu_A^2$$

- reasonable for $x \sim 10^{-2}$
- x evolution \rightarrow JIMWLK

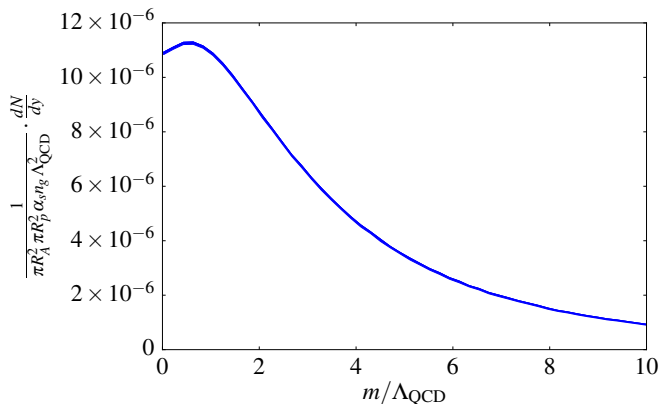
Annihilation - photon spectrum



- single flavor, chiral limit
- thin lines: $\exp(-\sqrt{k_{\perp}^2 + (0.5Q_s)^2}) / 0.5Q_s$
- thick lines: $(\log(k_{\perp}/Q_s))^{1.5} / k_{\perp}^{5.6}$

SB, Fukushima, arXiv:1602.01989

Annihilation - mass dependence



- fit $(\log(m/\Lambda_{\text{QCD}}))^{1.8}/m^{2.6}$
- mass corrections important

Conclusions and outlook

- ✓ complete analytical result at $O(\alpha\alpha_s)$
- ✓ numerical evaluation of the annihilation diagram
- sensitivity to quadrupole gluon correlators
- phenomenological applications
 - bremsstrahlung - numerical evaluation
 - small x evolution

Backup slides

Gluon field

- expansion in powers of proton density

$$\mathcal{A}^\mu = \mathcal{A}_{(0)}^\mu + \mathcal{A}_{(1)}^\mu$$

$$\mathcal{A}_{(0)}^\mu = -gn^\mu \delta(x^+) \frac{1}{\partial_\perp^2} \rho_A(\mathbf{x}_\perp)$$

$$\mathcal{A}_{(1)}^\mu(x) = \mathcal{A}_{(1<)}^\mu(x) + \mathcal{A}_{(1>)}^\mu(x)$$

$$\mathcal{A}_{(1<)}^+(p) = 0 \quad \mathcal{A}_{(1<)}^-(p) = 0 \quad -p^2 \mathcal{A}_{(1<)}^i(p) = -igp^i \frac{p^2}{(p^+ + i\epsilon)(p^- - i\epsilon)} \frac{\rho_p(\mathbf{p}_\perp)}{p_\perp^2}$$

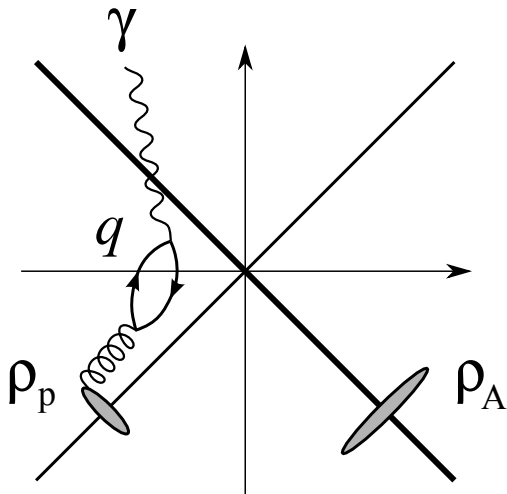
$$-p^2 \mathcal{A}_{(1>)}^\mu(p) = -ig \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} C^\mu(p; \mathbf{q}_\perp, \mathbf{p}_\perp - \mathbf{q}_\perp) V(\mathbf{p}_\perp - \mathbf{q}_\perp) \frac{\rho_p(\mathbf{q}_\perp)}{q_\perp^2}$$

$$C^+(p; \mathbf{q}_\perp, \mathbf{p}_\perp - \mathbf{q}_\perp) = 0$$

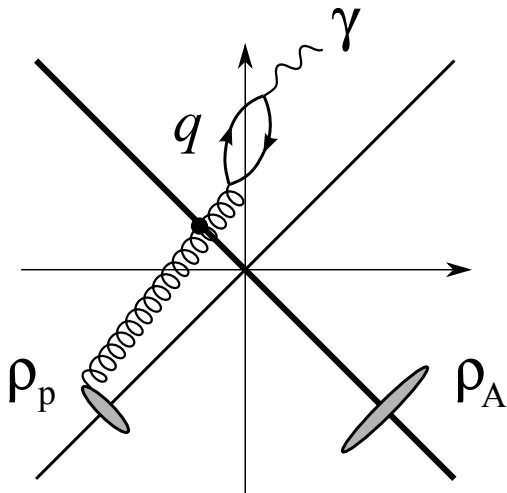
$$C^-(p; \mathbf{q}_\perp, \mathbf{p}_\perp - \mathbf{q}_\perp) = \frac{-2\mathbf{q}_\perp \cdot (\mathbf{p}_\perp - \mathbf{q}_\perp)}{p^+ + i\epsilon}$$

$$C^i(p; \mathbf{q}_\perp, \mathbf{p}_\perp - \mathbf{q}_\perp) = \frac{p^i q_\perp^2}{(p^+ + i\epsilon)(p^- + i\epsilon)} - 2q^i$$

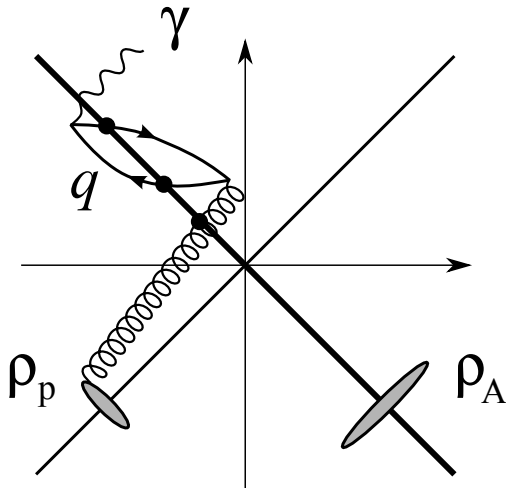
Annihilation - diagrams



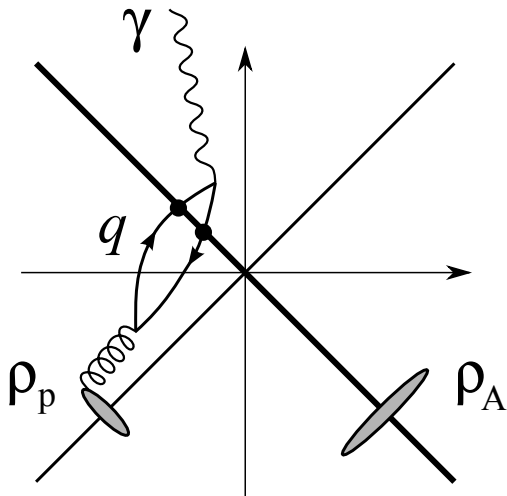
Annihilation - diagrams



Annihilation - diagrams



Annihilation - diagrams



Annihilation - amplitude

- after light-cone integrations

$$\begin{aligned}\mathcal{M}_\lambda(\mathbf{k}) &= \frac{eg^2}{4\pi^3} \int_0^1 dx \int_{\mathbf{u}_\perp \mathbf{v}_\perp} \int_{\mathbf{l}_\perp} e^{-i(\mathbf{k}_\perp - \mathbf{l}_\perp) \cdot [\mathbf{v}_\perp + a(x)\mathbf{u}_\perp]} \\ &\times \frac{\rho_p^a(\mathbf{l}_\perp)}{l_\perp^2} \text{Tr}_c \left[U\left(\mathbf{v}_\perp + \frac{\mathbf{u}_\perp}{2}\right) T_F^a U^\dagger\left(\mathbf{v}_\perp - \frac{\mathbf{u}_\perp}{2}\right) \right] \\ &\times \left[\hat{u}_\lambda l_\perp \Psi_1(\mathbf{l}_\perp, \mathbf{u}_\perp, x) + l_\lambda \Psi_2(\mathbf{l}_\perp, \mathbf{u}_\perp, x) \right]\end{aligned}$$

$$\begin{aligned}\Psi_1(\mathbf{l}_\perp, \mathbf{u}_\perp, x) &\equiv -4ia(x)b(x)l_\perp K_0(m_l(x)u_\perp)mK_1(mu_\perp) \\ &\quad + 4b(x)\hat{\mathbf{l}}_\perp \cdot \hat{\mathbf{u}}_\perp m_l(x)K_1(m_l(x)u_\perp)mK_1(mu_\perp) \\ \Psi_2(\mathbf{l}_\perp, \mathbf{u}_\perp, x) &\equiv mK_1(mu_\perp)m_l(x)K_1(m_l(x)u_\perp) \\ &\quad + m^2 K_0(mu_\perp)K_0(m_l(x)u_\perp)\end{aligned}$$

Brems from produced $\bar{q}q$ - amplitude

- after light-cone integrations

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) = & eg^2 \int_{\mathbf{l}_\perp \mathbf{s}_\perp} \frac{\rho_p^a(\mathbf{l}_\perp)}{l_\perp^2} \int_{\mathbf{x}_\perp \mathbf{y}_\perp} e^{-i\mathbf{s}_\perp \cdot \mathbf{x}_\perp} e^{i(\mathbf{l}_\perp + \mathbf{s}_\perp - \mathbf{Q}_\perp) \cdot \mathbf{y}_\perp} \\ & \times \bar{u}(\mathbf{q}) \left\{ T_g^{\mu\nu} C_\mu(Q, \mathbf{l}_\perp, \mathbf{Q}_\perp - \mathbf{l}_\perp) \epsilon_{\lambda\nu}(\mathbf{k}) T_F^b V^{ba}(\mathbf{y}_\perp) \right. \\ & \left. + T_{\bar{q}q}^{i\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \frac{l_i}{Q_+} \epsilon_{\lambda\nu}(\mathbf{k}) U(\mathbf{x}_\perp) T_F^a U^\dagger(\mathbf{y}_\perp) \right\} v(\mathbf{p}) \end{aligned}$$

Brems from produced $\bar{q}q$ - amplitude

$$T_g^{\mu\nu} \equiv T_1^{\mu\nu} + T_7^{\mu\nu}$$

$$T_{\bar{q}q}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv T_5^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_6^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_{11}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) + T_{12}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp)$$

$$T_1^{\mu\nu} \equiv -\frac{1}{Q^2} \gamma^\mu S_F(-k-p) \gamma^\nu$$

$$T_5^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv \frac{\not{\epsilon}(\not{\epsilon} - \not{s} + m) \gamma^\mu (\not{\epsilon} - \not{l} - \not{s} + m) \not{\epsilon} S_F(-k-p) \gamma^\nu}{2(k^+ + p^+) \omega_{q-s}^2 + 2q^+ \omega_{q-l-s}^2}$$

$$T_6^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv -2q^+ \frac{\not{\epsilon}(\not{\epsilon} - \not{s} + m) \gamma^\mu (\not{\epsilon} - \not{l} - \not{s} + m) \gamma^\nu (\not{k} + \not{\epsilon} - \not{l} - \not{s} + m) \not{\epsilon}}{[2(k^+ + p^+) \omega_{q-s}^2 + 2q^+ \omega_{q-l-s}^2][2q^+ 2p^+ k^- + 2p^+ \omega_{q-s}^2 + 2q^+ \omega_{k+q-l-s}^2]}$$

$$T_7^{\mu\nu} \equiv -\frac{1}{Q^2} \gamma^\nu S_F(k+q) \gamma^\mu$$

$$T_{11}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv \frac{\gamma^\nu S_F(k+q) \not{\epsilon} (\not{k} + \not{\epsilon} - \not{s} + m) \gamma^\mu (\not{k} + \not{\epsilon} - \not{l} - \not{s} + m) \not{\epsilon}}{2p^+ \omega_{k+q-s}^2 + 2(k^+ + q^+) \omega_{k+q-l-s}^2}$$

$$T_{12}^{\mu\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) \equiv -2p^+ \frac{\not{\epsilon}(\not{\epsilon} - \not{s} + m) \gamma^\nu (\not{k} + \not{\epsilon} - \not{s} + m) \gamma^\mu (\not{k} + \not{\epsilon} - \not{l} - \not{s} + m) \not{\epsilon}}{[2p^+ \omega_{k+q-s}^2 + 2(k^+ + q^+) \omega_{k+q-l-s}^2][2q^+ 2p^+ k^- + 2p^+ \omega_{q-s}^2 + 2q^+ \omega_{k+q-l-s}^2]}$$

Annihilation - color average

$$S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) = \frac{1}{N_c} \frac{(N_c^2 - 1)(\beta - \alpha)}{\sqrt{N_c^2(\alpha - \gamma)^2 - 4(\alpha - \beta)(\beta - \gamma)}} \\ \times \exp\left[-\frac{2\beta + (N_c^2 - 2)(\alpha + \gamma)}{N_c^2 - 1}\right] \sinh\left[\frac{N_c}{N_c^2 - 1} \sqrt{N_c^2(\alpha - \gamma)^2 - 4(\alpha - \beta)(\beta - \gamma)}\right]$$

$$2\alpha(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\mathbf{y}_\perp - \mathbf{z}'_\perp) + B_2(\mathbf{z}_\perp - \mathbf{y}'_\perp)$$

$$2\beta(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\mathbf{y}_\perp - \mathbf{y}'_\perp) + B_2(\mathbf{z}_\perp - \mathbf{z}'_\perp)$$

$$2\gamma(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv B_2(\mathbf{y}_\perp - \mathbf{z}_\perp) + B_2(\mathbf{y}'_\perp - \mathbf{z}'_\perp)$$

$$B_2(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv Q_s^2 \int d^2\mathbf{z}_\perp [G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)]$$