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Effects of diffraction in *pp* **and** *pA* **collisions**

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Work in coll. with L. Lönnblad, C. Bierlich and others

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- \mathbf{h} 5. Collisions with nuclei Glauber model and Gribov corrections Wounded nucleons and centrality determination Final states in *pA* collisions

6. Conclusions

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1. The role of pertubation theory

DIS at Hera: High parton density at small *x* grows ∼ 1/*x*1.³ Predicted by pert. BFKL pomeron

pp coll.: Models based on multiple pert. parton-parton subcollisions very successful at high energies

PYTHIA (Sjöstrand–van Zijl 1987) HERWIG also dominated by semihard subcollisions

May be understood from unitarity constraints:

Saturation \Rightarrow suppression of partons with $k_\perp < Q_s^2$ (*cf.* CGC:

When perturbative physics dominates, can the result be calculated from basic principles, without input pdf's?

2. Unitarity constraints and Eikonal approximation

Saturation most easily described in impact parameter space Rescattering ∼ convolution in **k**⊥-space → product in **b**-space

Re A^{pp} small ⇒ interaction driven by absorption Absorption probability in Born approx. = $2F(b) \Rightarrow$

 $d\sigma_{inel}/d^2b = 1 - e^{-2F(b)}$

If NO fluctuations:

Optical theorem \Rightarrow Im $A_{el} \equiv T(b) = 1 - e^{-F}$

 $d\sigma_{el}/d^2b = T^2 = (1 - e^{-F})^2$ $d\sigma_{tot}/d^2b = 2T = 2(1 - e^{-F})$

With fluctuations: *e*−*^F* → ⟨*e*−*^F* ⟩

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3. BFKL evolution in impact param. space

a) Mueller's Dipol model:

BFKL evolution in transverse coordinate space Gluon emission: dipole splits in two dipoles:

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Dipole-dipole scattering

Single gluon exhange \Rightarrow Colour reconnection between projectile and target

Reproduces LL BFKL evolution

BFKL stochastic process with independent subcollisions: Multiple subcollisions handled in eikonal approximation

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b) The Lund cascade model, DIPSY

(E. Avsar, GG, L. Lönnblad, Ch. Flensburg)

Based on Mueller's dipole model in transverse space Includes also:

- **IMPORTANT NON-leading effects in BFKL evol.**
- \triangleright Saturation from pomeron loops in the evolution
- ! Confinement ⇒ *t*-channel unitarity
- \triangleright MC DIPSY; includes also fluctuations and correlations
- \triangleright Applicable to collisions between electrons, protons, and nuclei

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Dipole evolution [DIPSY pp results](#page-7-0) Collisions with nuclei

Results *pp* **scattering**

Total and elastic cross sections

Underlying event: *N_{ch}* in transv. region *vs p*^{*lead*}</sup> Atlas data 7 TeV

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4. Diffractive excitation

Example: A photon in an optically active medium:

Righthanded and lefthanded photons move with different velocity; they propagate as particles with different mass.

Study a beam of righthanded photons hitting a polarized target, which absorbs photons linearly polarized in the *x*-direction.

The diffractively scattered beam is now a mixture of right- and lefthanded photons.

If the righthanded photons have lower mass:

 $\frac{m}{\sqrt{2}}$ The diffractive beam contains also photons excited to a state with higher mass

pp coll.: Diffract. exc. is large $\approx 1/3$ σ_{inel} [at L](#page-7-0)[H](#page-9-0)[C](#page-7-0)

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Good–Walker formalism:

Projectile with a substructure:

Mass eigenstates Ψ*^k* can differ from eigenstates of diffraction Φ*ⁿ* (eigenvalues *Tn*)

Elastic amplitude = $\langle \Psi_{in} | T(b) | \Psi_{in} \rangle$

 \Rightarrow $d\sigma_{el}/d^2b = \langle T(b) \rangle^2$

Total diffractive cross section (incl. elastic):

$$
d\sigma_{diff\,tot}/d^2b = \sum_k \langle \Psi_{in} | T | \Psi_k \rangle \langle \Psi_k | T | \Psi_{in} \rangle = \langle T(b)^2 \rangle
$$

Diffractive excitation determined by the fluctuations:

$$
d\sigma_{\text{diff ex}}/d^2b = d\sigma_{\text{diff}} - d\sigma_{\text{el}} = \langle T^2 \rangle - \langle T \rangle^2
$$

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Scattering against a fluctuating target

Total diffractive excitation:

 $d\sigma_{tot.\textit{diffr}.\textit{exc.}}/d^2b = \langle~\mathcal{T}^2~\rangle_{\rho,t} - \langle~\mathcal{T} \rangle_{\rho,t}^2$ $d\sigma_{el}/d^2b = \langle T \rangle_{\rho,t}^2$

Averageing over target states before squaring

 \Rightarrow elastic interaction for the target.

Subtract $\sigma_{el} \rightarrow$ single diffr. excit.:

$$
d\sigma_{SD,p}/d^2b = \langle \langle T \rangle_t^2 \rangle_p - \langle T \rangle_{p,t}^2
$$

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$$
d\sigma_{SD,t}/d^2b = \langle \langle T \rangle_p^2 \rangle_t - \langle T \rangle_{p,t}^2
$$

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$$
d\sigma_{DD}/d^2b = \langle T^2 \rangle_{p,t} - \langle \langle T \rangle_t^2 \rangle_p - \langle \langle T \rangle_p^2 \rangle_t + \langle T \rangle_{p,t}^2.
$$

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Relation Good–Walker *vs* **triple-pomeron**

Diffractive excitation in *pp* coll. commonly described by Mueller's triple-pomeron formalism

Stochastic nature of the BFKL cascade ⇒ Good–Walker and Triple-pomeron describe the same dynamics (PL B718 (2013) 1054)

But: Saturation is easier treated in the Good–Walker formalism; in particular for collisions with nuclei

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Fluctuations and diffraction

What are the diffractive eigenstates?

Parton cascades, which can come on shell through interaction with the target.

BFKL dynamics \Rightarrow Large fluctuations,

Continuous distrib. up to high masses

(Also Miettinen–Pumplin (1978), Hatta *et al.* (2006))

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Dipole evolution [DIPSY pp results](#page-7-0) Collisions with nuclei

Single diffraction in *pp*

Inclusive $M_X < M_X^{(cut)}$ 1.8 TeV, Shaded: CDF

Final state

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Note: Calculated directly fron pert. QCD Tuned only to total and elastic cross sections

5. Collisions with nuclei

Initial state:

DIPSY gives full partonic picture, dense gluon soup.

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Ex.: Pb − Pb 200 GeV/N
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Accounts for:

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saturation within the cascades,

correlations and fluctuations in partonic state,

finite size effects

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Understanding the initial state essential for interpretation of collective final state effects

Models for initial state in *AA* collisions can be tested in *pA*

Study coherence effects in total, elastic, and diffractive cross sections

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Glauber model

The Glauber model is frequently used in analyses of experimental data, for estimating centrality or the initial state

E.g. # "wounded" nucleons and # binary *NN* collisions

Study a projectile proton at impact param. **b**, hitting a nucleus with nucleon positions \mathbf{b}_{ν} ($\nu = 1, \ldots, A$)

In **b**-space rescattering is given by a product:

- \Rightarrow *S*-matrix factorizes: $S^{(\rho A)}(\mathbf{b}) = \prod_{\nu=1}^{A} S^{(\rho \rho, \nu)}(\mathbf{b} \mathbf{b}_{\nu})$
- \Rightarrow Elastic amplitude:

 $T^{(pA)}(\mathbf{b}) = 1 - \prod_{\nu=1}^{A} S^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_{\nu}) = 1 - \prod_{\nu} \{1 - T^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_{\nu})\}$

Gribov corrections

A proton may fluctuate between different diffractive eigenstates

- ⇒ diffractive excitation
- The projectile is frozen in the same state, *k*, during the passage through the nucleus
- The target nucleons are in different, uncorrelated states *l*ν.
- ⇒ Elastic *pA* scattering amplitude:

 $\langle \ T^{(\rho\mathsf{A})}(\mathsf{b}) \ \rangle = \mathsf{1} - \langle \ \langle \prod_{\nu} \ \langle \, \{ \mathsf{1} - T^{(\rho\mathsf{p},\nu)}_{k,l_{\nu}}(\mathsf{b}-\mathsf{b}_{\nu}) \} \ \rangle_{l_{\nu}} \ \rangle_{\mathsf{b}_{\nu}} \ \rangle_{k_{\nu}}$

 ${\cal A}$ $d\sigma^{pA}_{tot}/d^2b = 2\,\langle\, {\cal T}^{(pA)}({\bf b})\rangle, \quad d\sigma^{pA}_{el}/d^2b = \langle\, {\cal T}^{(pA)}({\bf b})\rangle^2$

Note: High powers of *pp* amplitudes

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Specification of wounded nucleons

Wounded nucleon model (Białas *et al.*)

$$
+ N_{w}^{t} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \rightarrow y
$$

Central particle density
$$
\frac{dN^{pA}}{d\eta} \approx \frac{1+N_w^t}{2} \frac{dN^{pp}}{d\eta}
$$

Should diffractively excited target nucleons count as wounded?

/home/shakespeare/people/leif/personal/lib/te Yes, for forward observables and centrally if $\sigma_{SD}/\mathit{dM}_X^2 \sim \mathit{dM}_X^2/ (M_X^2)^{1+\epsilon},$ with $\epsilon \approx 0$ No for central observables, if ϵ is large (\sim [0](#page-17-0).2[\)](#page-19-0)

Wounded nucleon cross sections

a) Wounded nucleons = absorbed nucleons (inel. non diffr.)

Absorption probability: $d\sigma_{abs}/d^2b = 1 - S^2$

*S*² also factorizes

Absorptive cross section:

 $d\sigma_{\sf abs}^{\sf pA}/d^2b = \langle 1 - \prod_\nu (S^{({\sf p}{\sf p},\nu)})^2\rangle =$ $= 1 - \langle \prod_{\nu} \langle \left\{ 1- 7^{(\rho\rho,\nu)} (\mathbf{b}-\mathbf{b}_{\nu}) \right\}^2 \rangle_{l_{\nu}} \rangle_{k}$

Involves target average of $T^{(pp)}$ squared: $({\langle T^{(pp)2} \rangle}_{tara})^n$

b) Inclusively wounded nucleons $d\sigma_{\textit{winc}}^{\textit{pA}}/d^2b = 1 - \langle \prod_{\nu}\{1 - \langle \mathcal{T}^{(\textit{pp},\nu)}(\mathbf{b}-\mathbf{b}_{\nu})\rangle^2_{l_{\nu}}\}\rangle_k$

Only first power of *T* in target average

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Simple approximations Frequently used in exp. analyses (i) Black disc model: $T^{pp}(b) = \theta(R - b)$ Single parameter R \Rightarrow $\sigma_{\text{abs}}^{\text{pp}} = \sigma_{\text{el}}^{\text{pp}} = \sigma_{\text{tot}}^{\text{pp}}/2$ \Rightarrow Diffraction completely negected *R* reprod. $\sigma_{inel,tot}^{pp}$ \Rightarrow σ_{tot}^{pA} overestimated *R* chosen to reproduce σ_{tot}^{pp} \Rightarrow σ_{inel}^{pA} underestimated (ii) Gray disc model: Projectile absorbed with prob. *a*, for *b* < *R*

Somewhat better, but not possible to distinguish SD*target* , SD*proj*, and DD

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pPb **cross sections at 5 TeV (barn)**

Different approximations give very different results

(GG, L Lönnblad, A Ster, T Csörgő, JHEP 1510 (2015) 022, 1506.09095)

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The model by Strikman and coworkers

Blättel *et al.* 1993, sometimes called the GG model; often used in LHC exp.

Accounts for a fluctuating projectile

(but not fluctuating target nucleons)

Notation:

Fluctuating *pp* total cross section, averaged over target states:

 $\hat{\sigma}_{\textit{tot}} = 2 \int d^2b \, \langle \; T^{\textit{(pp)}}(b) \, \rangle_{\textit{targ}}$

Average also over projectile states $\Rightarrow \sigma_{tot}^{(pp)}=\langle \hat{\sigma}_{tot}\rangle_{proj}$

$$
\text{Ansatz:} \quad \frac{dP}{d\hat{\sigma}_{tot}} = \rho \frac{\hat{\sigma}_{tot}}{\hat{\sigma}_{tot} + \sigma_0^{tot}} \exp\left\{-\frac{(\hat{\sigma}_{tot}/\sigma_0^{tot} - 1)^2}{\Omega^2}\right\}
$$

 $Ω$ is a parameter determining the fluctuations, related to $\sigma_{SD}^{(PP)}$

 σ_0^{tot} σ_0^{tot} σ_0^{tot} is fixed from $\sigma_{tot}^{(pp)}$; ρ is a normalization co[ns](#page-21-0)t[an](#page-23-0)t[.](#page-22-0)

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Wounded nucleon cross section

1) Absorptive cross section

$$
\begin{aligned} \text{Def. } \hat{\sigma}_{\text{abs}} &= \int d^2b \, \langle \, \{ 2\, \mathcal{T}^{(pp)}(b) - \mathcal{T}^{(pp)2}(b) \} \, \rangle_{\text{targ}} \qquad (*) \\ \sigma_{\text{abs}}^{(pp)} &= \langle \hat{\sigma}_{\text{abs}} \rangle_{\text{proj}} \end{aligned}
$$

The same form is used, but Ω need not be the same:

$$
\frac{dP}{d\hat{\sigma}_{\text{abs}}} = \rho' \frac{\hat{\sigma}_{\text{abs}}}{\hat{\sigma}_{\text{abs}} + \sigma_0^{\text{abs}}} \exp\left\{-\frac{(\hat{\sigma}_{\text{abs}}/\sigma_0^{\text{abs}} - 1)^2}{\Omega^2}\right\}
$$

Note: σ_0^{abs} ought to be adjusted to reproduce $\sigma_{inel\,ND}^{pp}$, but is often tuned to $\sigma_{inel\ tot}^{pp}$!

2) Wounded incl. target exc. $\Rightarrow \hat{\sigma}_{\text{abs}} \rightarrow \hat{\sigma}_{\text{w}}$ where $\langle T^{(pp)2}(b)\rangle_{targ} \to \langle T^{(pp)}(b)\rangle_{targ}^2$ in eq. (*)

DIPSY results compared with GG

(tuned to the DIPSY cross sections)

Prob. distrib. in *total* and *wounded_{incl}* cross sects.

DIPSY have larger tails to large cross sections.

Well fitted by the GG formalism with a log-normal distrib.

(C. Bierlich, GG, L. Lönnblad, arXiv:1607.04434)

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Final states

Old Fritiof model inspired by the wounded nucleon idea. Worked well in fixed target region

Model "FritiofP8"

Secondary absorbed nucleons generated by PYTHIA8

Diffr. target nucl.: similar $\,$ contrib. if $d\sigma/dM_X^2 \sim 1/M_X^2$

Compared with model called "Absorptive"

assuming all wounded target nucleons contribute like a *pp* collision at full energy. (*Cf* G-Pythia used by ALICE)

Expected to be better for high *p*⊥ particles originating from hard parton interactions.

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Results

Comparisons to ATLAS data

∑ *E*_⊥ in forward (nucleus) direction

"FritiofP8" (- -) agrees well with data. "Absorptive" (-) too high.

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η distrib. for central collisions *p*⊥ distrib. at central rapidity

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FritiofP8 works well for *p*⊥-integrated data and for low *p*⊥; Absorptive better for higher *p*⊥

7. Conclusions

Saturation suppresses low-*p*⊥ gluons \Rightarrow high energy hadronic collisions dominantly perturbative

The DIPSY dipole cascade model is based on BFKL dynamics with non-leading corrections and saturation.

Includes correlations and fluctuations

Gives a fair description of DIS and *pp* data, with no input pdf's

Can give the initial condition in *pA* and *AA* collisions

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pA scattering intermediate step between *pp* and *AA*

Glauber model frequently used in experimental analyses Gribov pointed out importance of diffractive scattering Frequently not taken into account or treated in an improper way

Diffractively excited target nucleons are a significant fraction Contributes more or less depending on the observable

For observables not obtaining contribution from diffractive target nucleons, the distribution in the GG model should be normalized to $\sigma_{inel,ND} \approx 2/3 \sigma_{inel}$

Simple model based on Fritiof works rather well for min bias final states in *pA*

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DIPSY pp results[®] [Glauber](#page-16-0)

Extra slides

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pA **collisions**

Test: DIPSY agrees with CMS and LHCb inelastic cross section

pPb inelastic cross sections

(GG, L. Lönnblad, A. Ster, T. Csörgő, arXiv:[150](#page-30-0)[6.](#page-32-0)[0](#page-30-0)[90](#page-31-0)[95](#page-32-0)[\)](#page-15-0)

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Results for *pPb* at 5 TeV

GG, L Lönnblad, A Ster, T Csörgő, JHEP 1510 (2015) 022

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γ[∗]*A* collisions

(Note: γ[∗] → *qq*¯ frozen during passage through nucleus)

 $\gamma^* p$ scaling closer to $\sim A \sigma_{tot}^{\gamma^*}.$

More transparent (and more so for high *Q*2) ⇒ dynamic effects more visible

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Saturation within evolution

Multiple interactions ⇒ colour loops ∼ pomeron loops

Gluon scattering is colour suppressed *cf* to gluon emission ⇒ Loop formation related to identical colours.

Multiple interaction in one frame \Rightarrow colour loop within evolution in another fra[me](#page-33-0)

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Colour loop formation in a different frame

Same colour \Rightarrow quadrupole

May be better described by recoupled smaller dipoles

 \Rightarrow smaller cross section: fixed resolution \Rightarrow effective $2 \rightarrow 1$ and $2 \rightarrow 0$ transitions

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Is a form of colour reconnection

Not included in Mueller's model or in BK equation

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Final states

Comparisons to ATLAS data at 7 TeV

Min bias **Nights** Underlying event

vs p^{lead}

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Charged particles *Nch* in transv. region η -distrib. ρ_{τ} -distrib.

Charged particle *η* at 7 TeV, track $p_\perp > 500$ MeV, for $N_{\rm ch} \geq 6$ Charged particle *p*[⊥] at 7 TeV, track *p*[⊥] > 500 MeV, for *N*ch ≥ 6 Transverse $N_{\rm chg}$ density vs. $p_{\perp}^{\rm trk1}, \sqrt{s} = 7$ TeV / N_o d N_a/dn 1/*N*ev d*N*ch/d*η* 4 1/*N*ev 1/2*πp*⊥ d*σ*/d*η*d*p*⊥ ⟨d2*N*chg/d*η*d*φ*⟩ ŧ 1 - Data 1.2 ÷ 3.5 - Rope ÷ 10−¹ - String 1 $2\pi p$ 10−² 2.5 0.8 2 10−³ 0.6 ATLAS data 1.5 Data Rope 10−⁴ **DIPSY** 0.4 Pythia8 1 String 10−⁵ 0.2 0.5 10−⁶ 0 1.4 0 1.4 1.4 1.2 MC/Data 1.2 MC/Data MC/data 1.2 1 0.8 0.8 0.8 0.6 0.6 0.6 /home/shakespeare/people/leif/personal/lib/te -2 -1 0 1 2 $10¹$ 2 4 6 8 10 12 14 16 18 20 *p*[⊥] (leading track) [GeV] *p*⊥ [GeV] *η*

Impact parameter profile

Saturation \Rightarrow Fluctuations suppressed in central collisions Diffr. excit. largest in a circular ring,

expanding to larger radius at higher energy

Factorization broken between *pp* and DIS

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Exclusive final states in diffraction

- If gap events are analogous to diffraction in optics \Rightarrow
- Diffractive excitation fundamentally a quantum effect
- Different contributions interfere destructively, no probabilistic picture
- Still, different components can be calculated in a MC, added with proper signs, and squared
- Possible because opt. th. \Rightarrow all contributions real
- (JHEP 1212 (2012) 115, arXiv:1210.2407)

(Makes it also possible to take Fourier transform and get *dσ/dt*. JHEP 1010, 014, arXiv:1004.5502)

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Early results for DIS and *pp*

H1: $W = 120$, $Q^2 = 24$ dn_{ch}/dn in 2 M_X -bins

UA4: $W = 546$ GeV $\langle M_x \rangle$ = 140 GeV

Too hard in proton fragmentation end. Due to lack of quarks in proton wavefunction

Has to be added in future improvements

Note: Based purely on fundamental QCD dynamics

(JHEP 1212 (2012) 115, arXiv:1210.2407) **[Effects of diffraction](#page-0-0)** 40 Gösta Gustafson Lund University Lund University

For a deuteron target this involves the averages

 $\langle \langle T^{(pp)} \rangle_{\text{targ}} \rangle_{\text{proj}}$ and $\langle \langle T^{(pp)} \rangle_{\text{targ}}^2 \rangle_{\text{proj}}$ Note: The second average can be determined from single diffractive excitation in *pp* scattering, once the space distribution in the deuteron is known

Heavier targets involve higher moments ⟨ ⟨ *T*(*pp*) ⟩*ⁿ targ* ⟩*proj*

Can be calculated if the full distribution *dP*/*d* ⟨*T*(*pp*) (*b*)⟩*targ* is known, for all possible projectile states

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Wounded nucleon distributions

a) Wounded nucleons = absorbed nucleons (not incl. diffr.) $(S^{(pp,\nu)})^2$ = probability that target nucleon ν is not absorbed Prob. for target nucleon ν to be absorbed by a proj. in state k : $\langle (S_{k,l_{\nu}}^{\rho\rho,\nu})^2 \, \rangle_{l_{\nu}} = \langle \, 2 \, T_{k,l_{\nu}}^{\rho\rho,\nu} - (\, T_{k,l_{\nu}}^{\rho\rho,\nu})^2 \, \rangle_{l_{\nu}}$

b) Inclusively wounded nucleons

Prob. for target nucleon ν to be inclusively wounded:

 $\langle (S_{k,l\nu}^{\rho\rho,\nu})^2 \rangle_{k,l\nu} = \langle 2\mathcal{T}_{k,l\nu}^{\rho\rho,\nu} \rangle_{l\nu}^2 - \langle \mathcal{T}_{k,l\nu}^{\rho\rho,\nu} \rangle_{l\nu}^2$

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