Multiple scattering aspects of gluon TMDs

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university of groningen

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Asia/Seoul timezone

Current registrants (1)

name	institution	city	country/region
BOER, Daniel	University of Groningen	Groningen	NETHERLANDS

Outline

- Color flow in high energy scattering processes
- Effects in polarized proton collisions
- Effects in unpolarized proton collisions
- The small-x limit: the polarization of the Color Glass Condensate

Color flow in high energy scattering processes

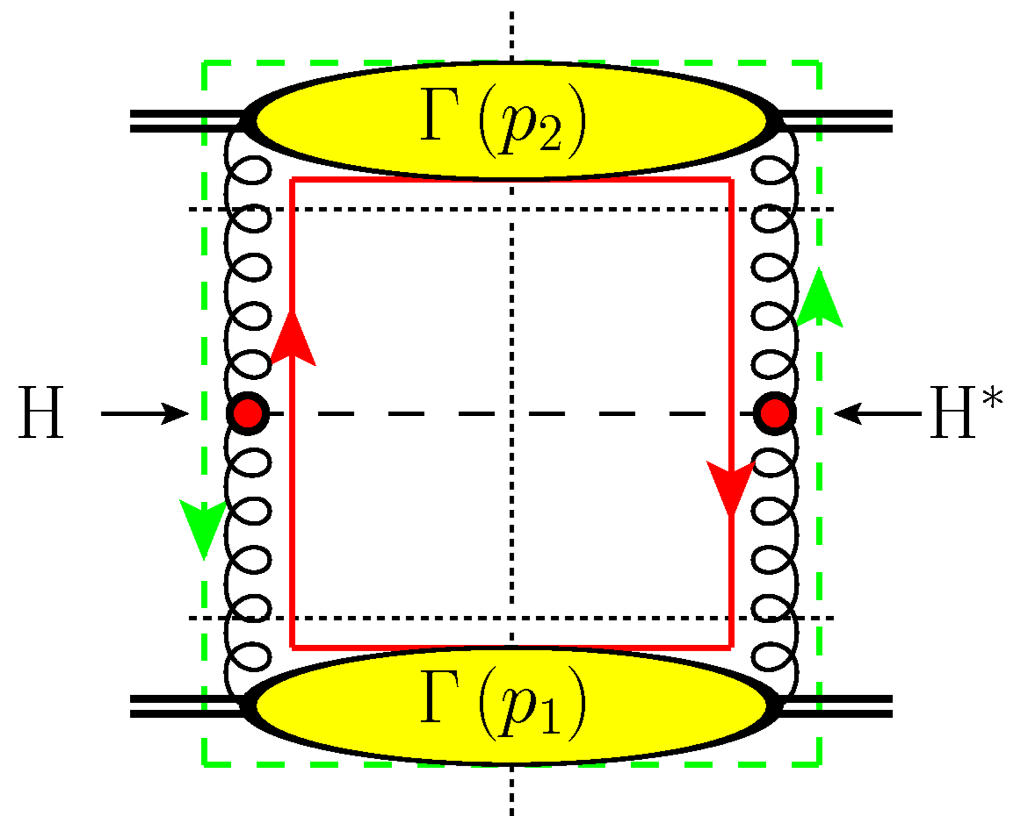
Factorization and color flow

Theoretical description of high-energy scattering cross sections is based on **factorization** in perturbative partonic hard scattering factors (H) and nonperturbative hadronic correlators (Φ,Γ,Δ), i.e. parton distributions

Higgs production: $pp \to HX$

Color treatment is simple at high energies: separate traces, not dependent on kinematics

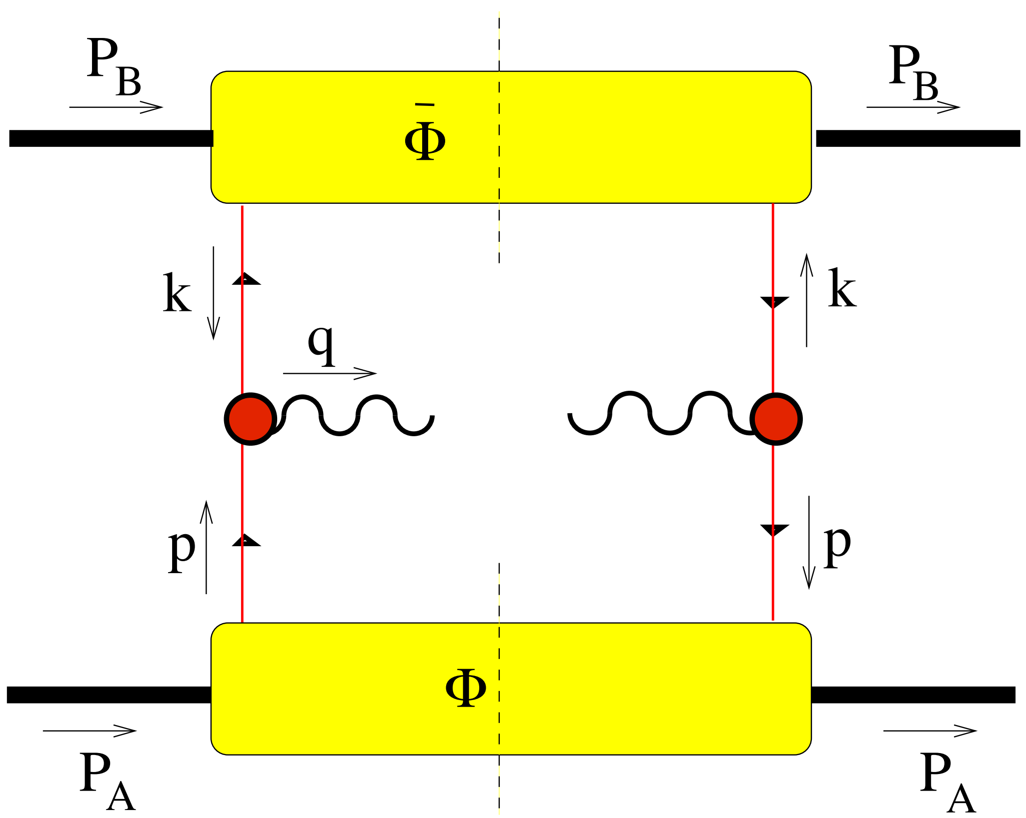
But in the actual process there are no colored final states and there are many soft gluons exchanged to balance the color



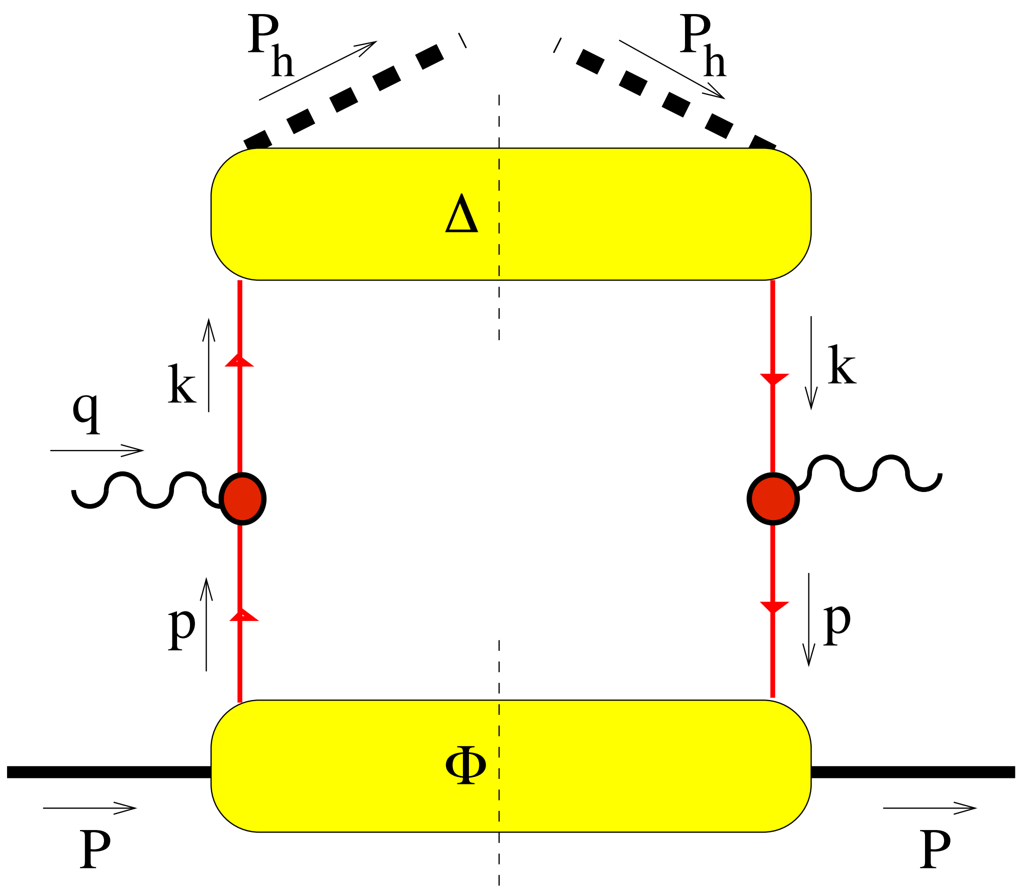
This cartoon version of the color flow works fine in most cases, when collinear factorization applies

Factorization in terms of correlators

Similarly, one would expect that the following two processes involve the same color trace and the dynamics is unaffected by the color flow



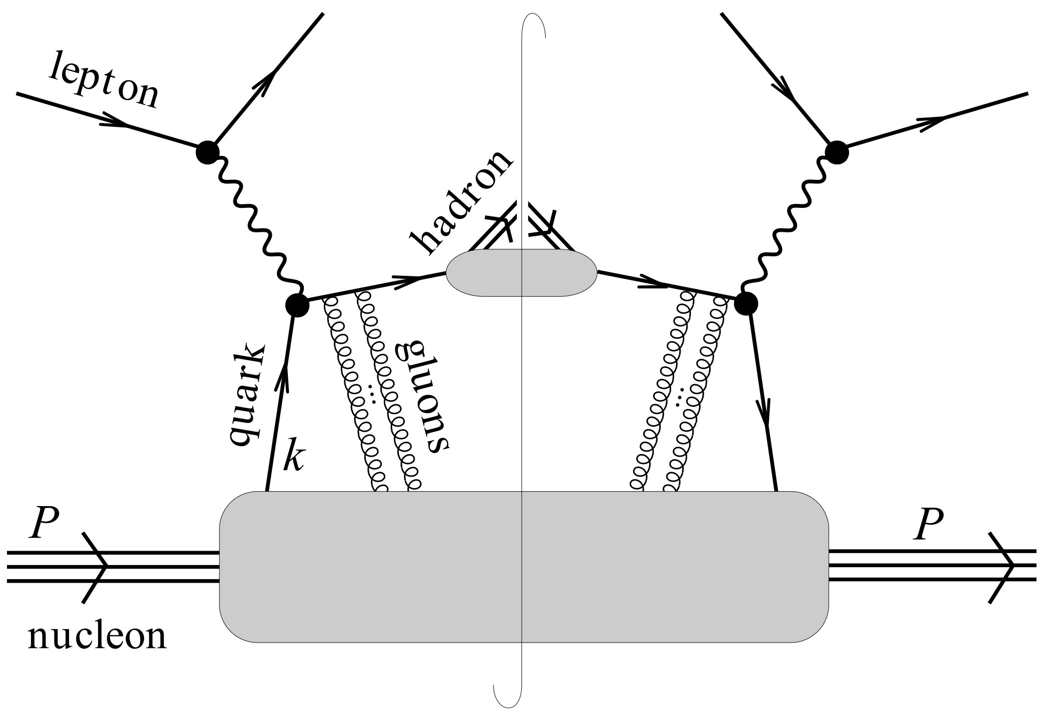
$pp \to \gamma^* X$ (Drell-Yan)



$\gamma^* p \to h\, X$ (SIDIS)

However, this is not always the case, e.g. for certain differential cross sections, that are sensitive to the transverse momentum of the partons

Gauge invariance of correlators



summation of all gluon exchanges leads to *path-ordered exponentials* in the operators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu \, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

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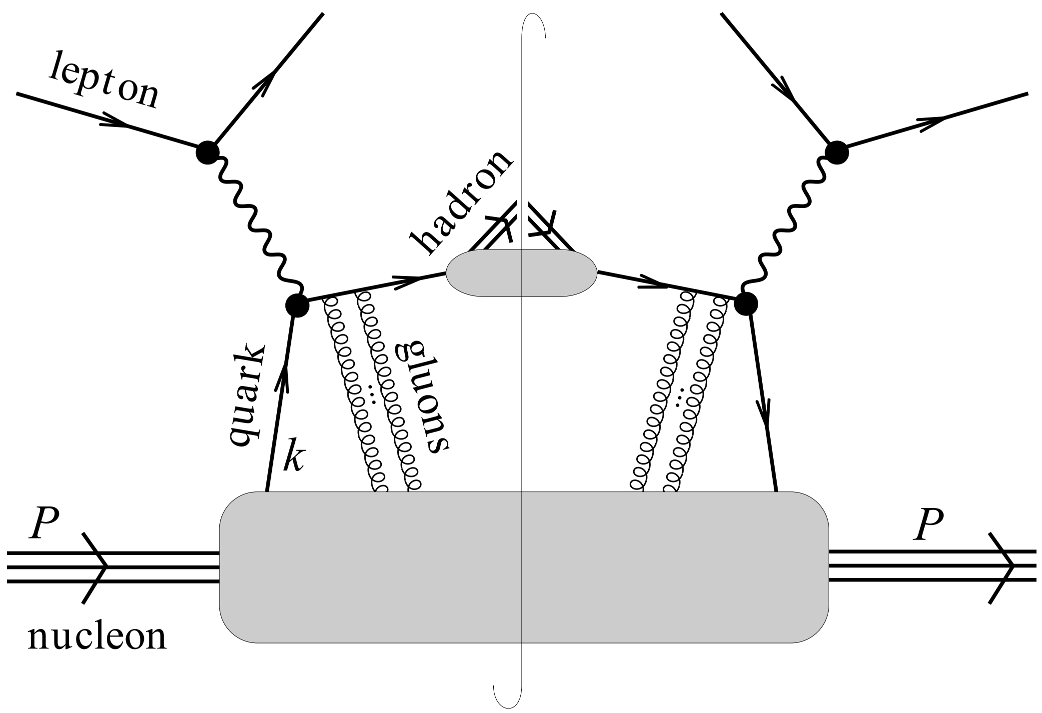

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The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003; Boer, Mulders & Pijlman, 2003]

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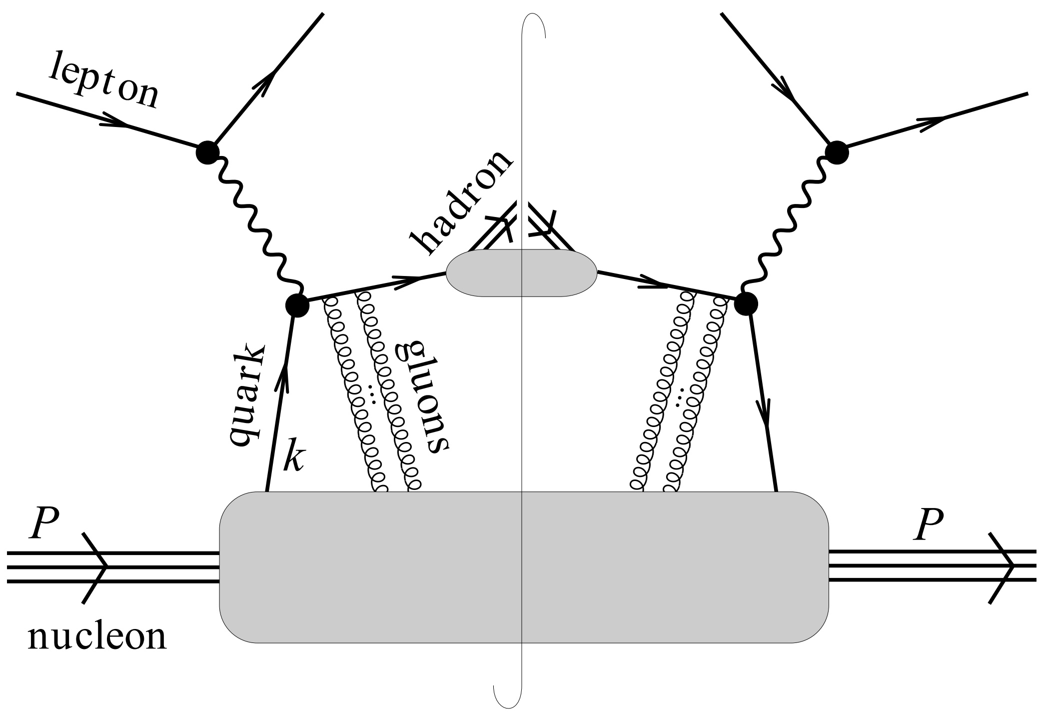

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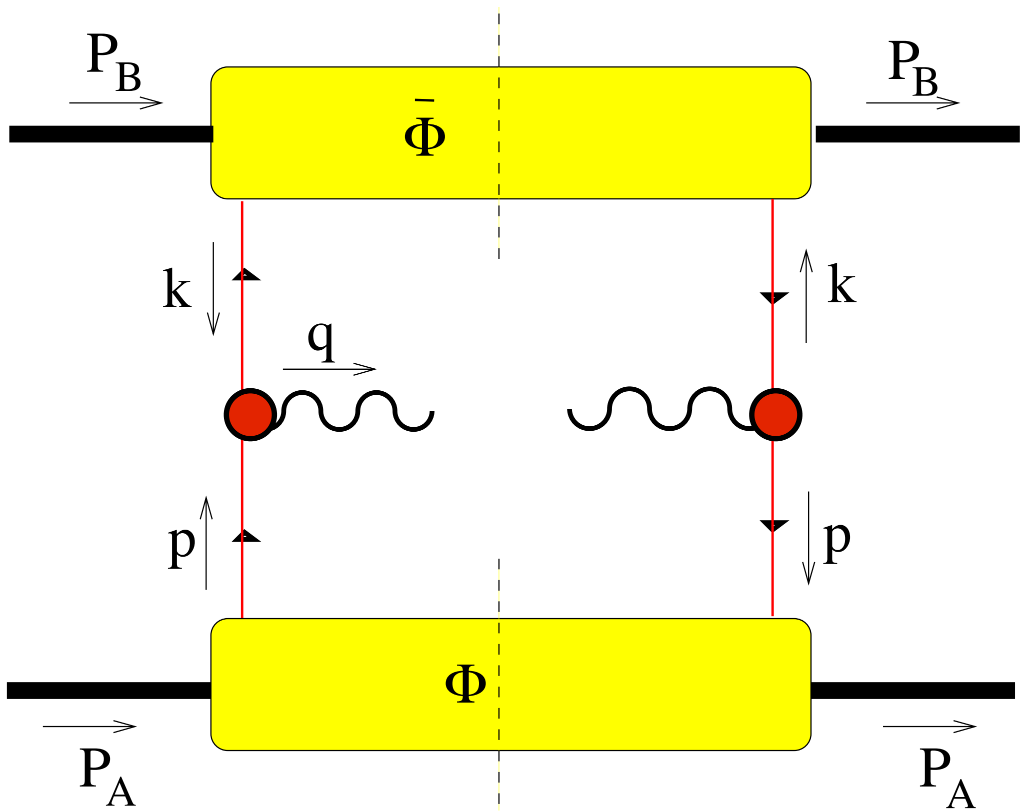
This does not automatically imply that the *gauge links* affect observables, but it turns out that they do in certain cases sensitive to the transverse momentum

In that case the gauge link path has extent ξ_T in the transverse direction (ξ_T is conjugate to k_T) which can be located at different places along the lightcone

Process dependence of gauge links

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing

[Belitsky, Ji & Yuan '03]



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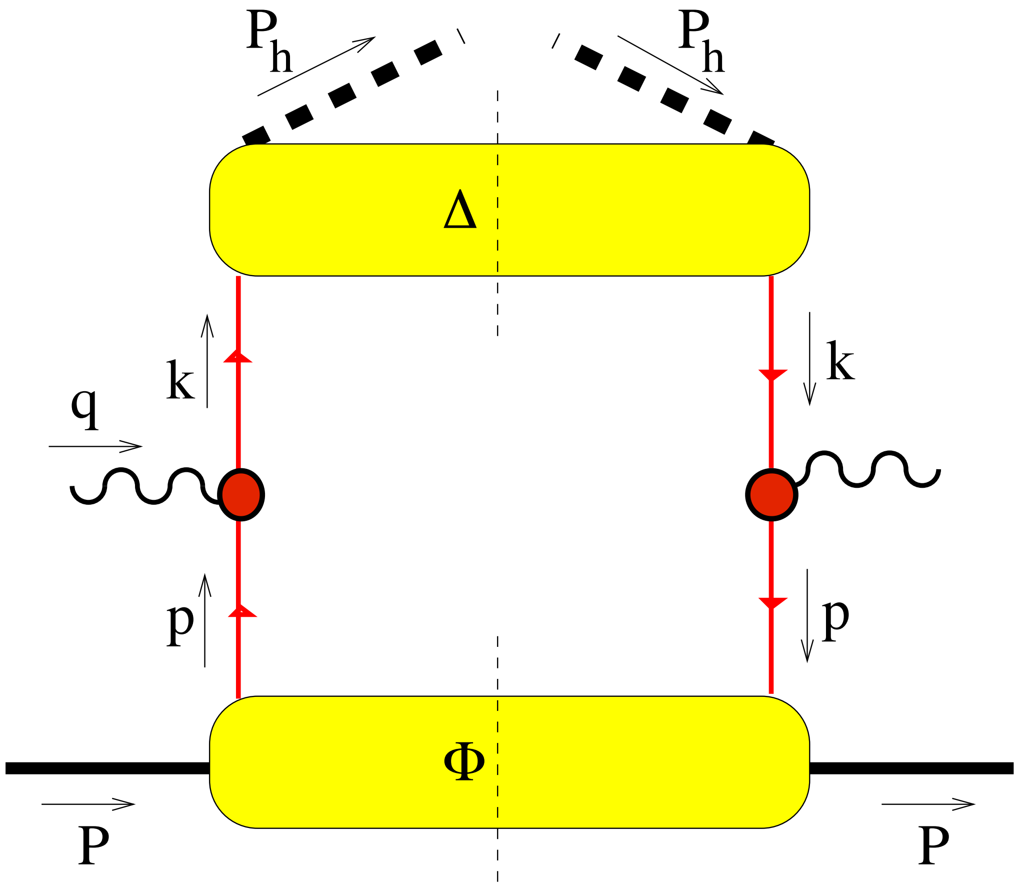

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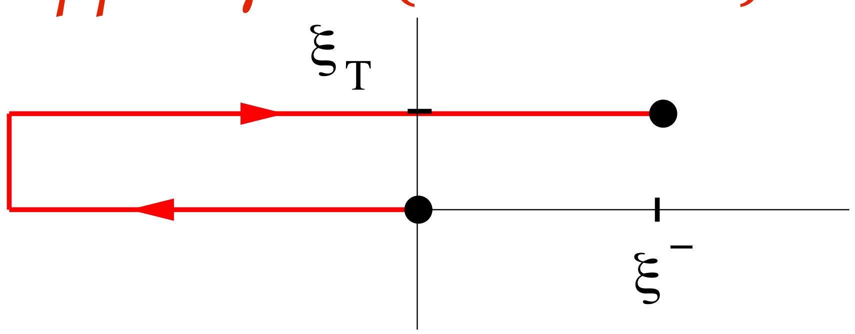

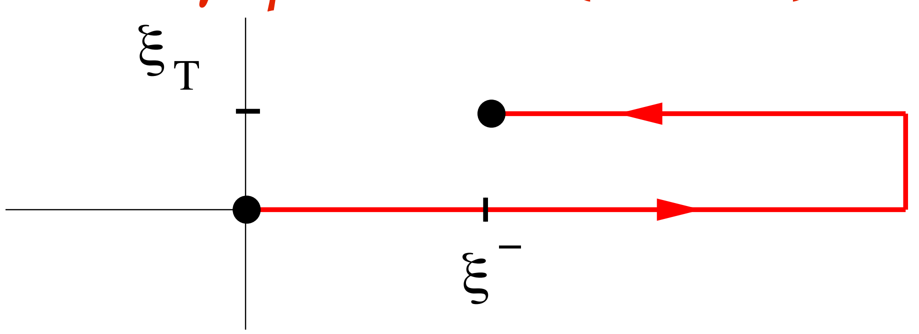

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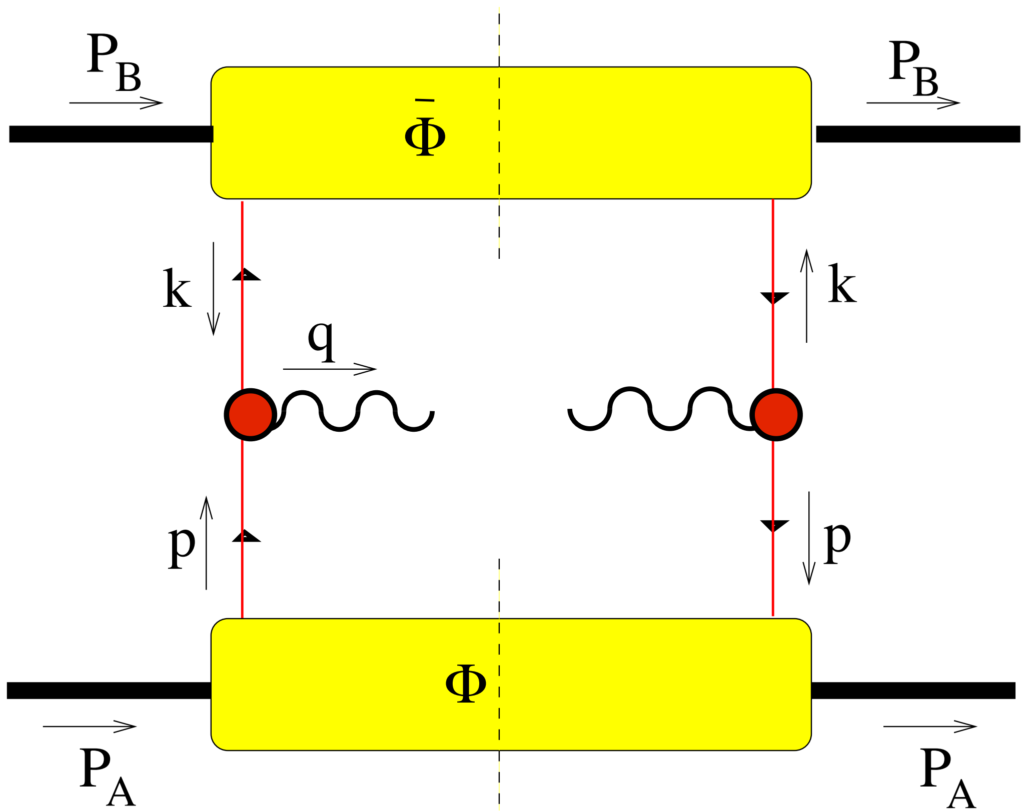

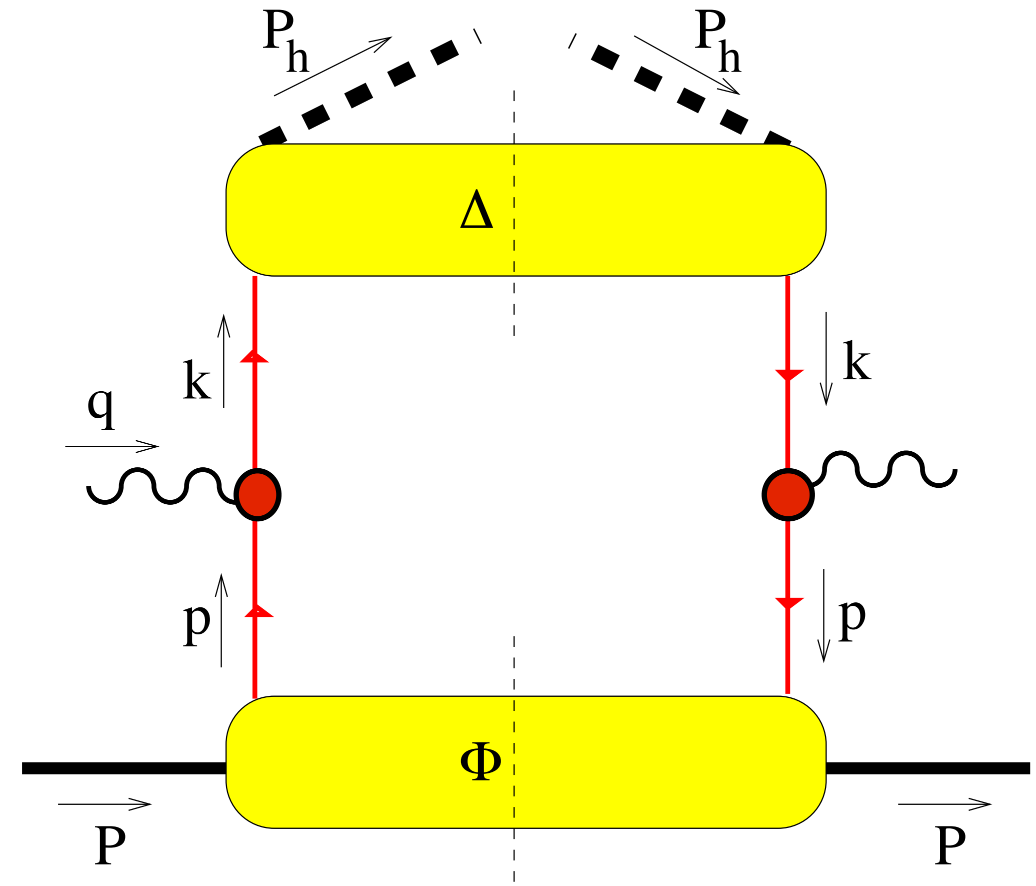

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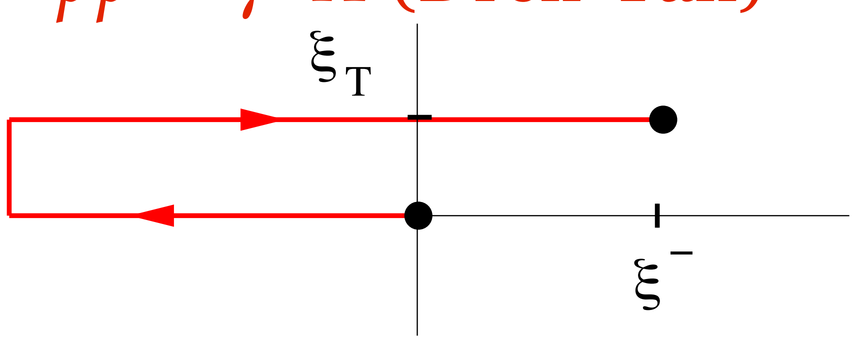
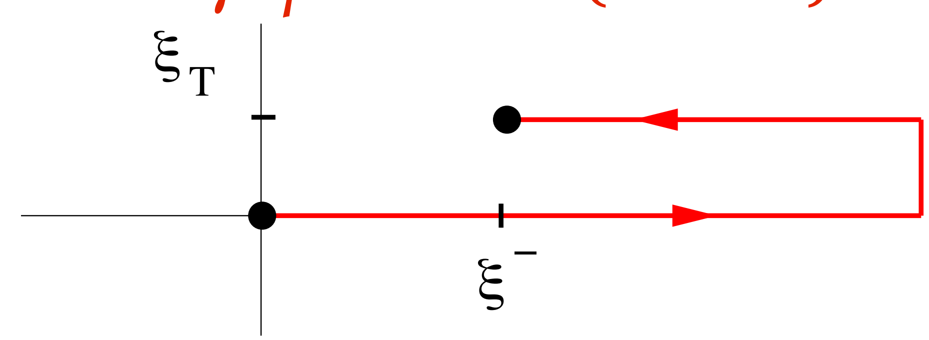
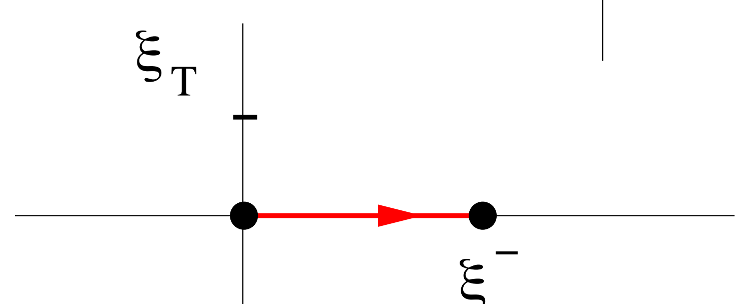




$$\int dk_T \longrightarrow \xi_T = 0 \longrightarrow$$



the same in both cases

Transverse Momentum of Partons

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

The quark correlator is parametrized in terms of t*ransverse momentum dependent* parton distributions (TMDs)

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The transverse momentum dependence can be correlated with the spin, e.g.

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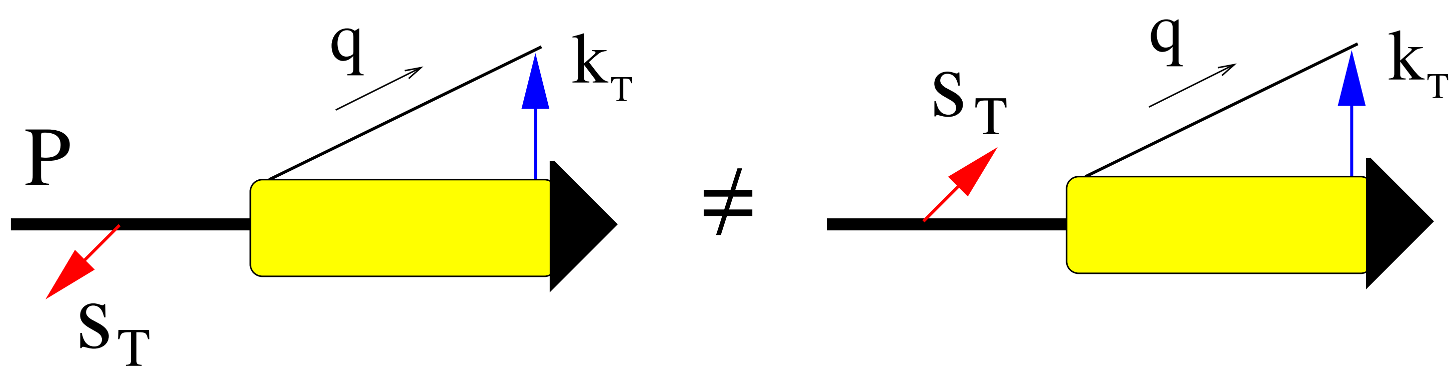

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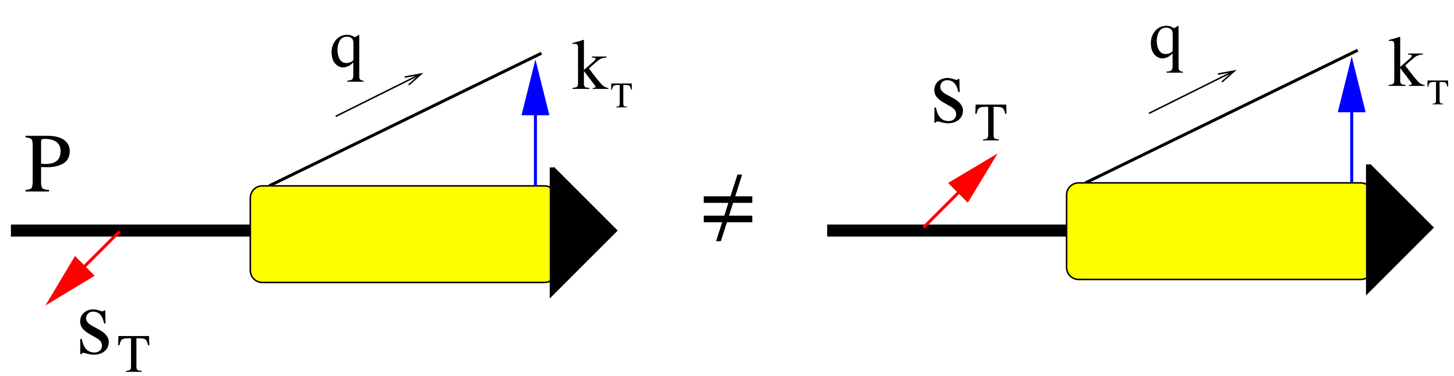

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Quark correlator:

Sivers function

$$\Phi(x,\boldsymbol{k}_T) = \frac{M}{2}\left\{ f_1(x,\boldsymbol{k}_T^2)\,\frac{\not{P}}{M} + f_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{P}}{M} \right.$$

$$\left. + h_{1T}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{S}_T\,\not{P}}{M} + h_{1s}^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{k}_T\,\not{P}}{M^2} + h_1^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{i\,\not{k}_T\,\not{P}}{M^2} \right\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

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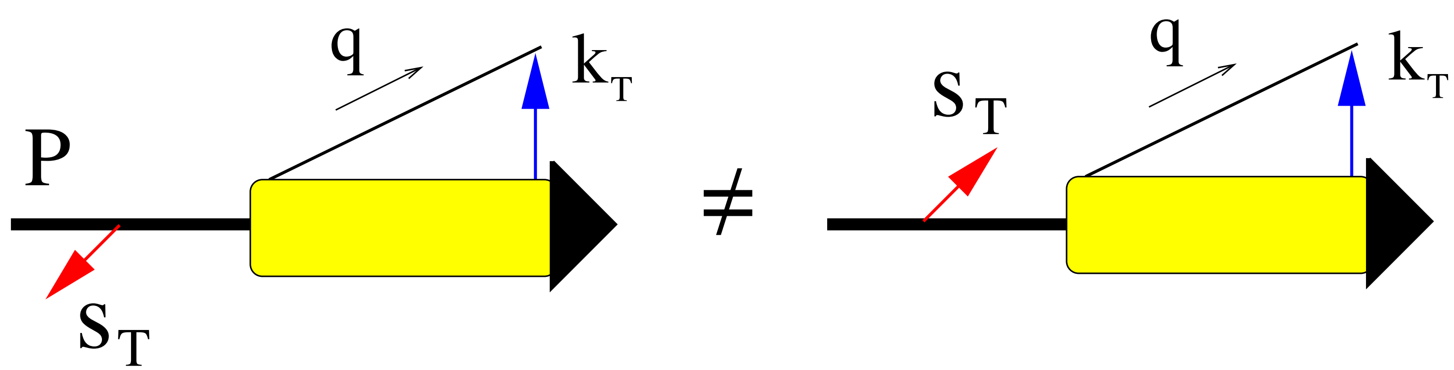

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k_T-odd

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Sivers TMD

The proper theoretical definition of the Sivers TMD is *not unique*

$$P\cdot(\mathbf{k}_T \times \mathbf{S}_T)\, f_{1T}^{\perp[\mathcal{C}]}(x,\mathbf{k}_T^2) \propto \;\text{F.T.}\,\langle P,S_T|\bar{\psi}(0)\,\mathcal{L}_{\mathcal{C}[0,\xi]}\,\gamma^+\,\psi(\xi)|P,S_T\rangle\big|_{\xi=(\xi^-,0^+,\xi_T)}$$

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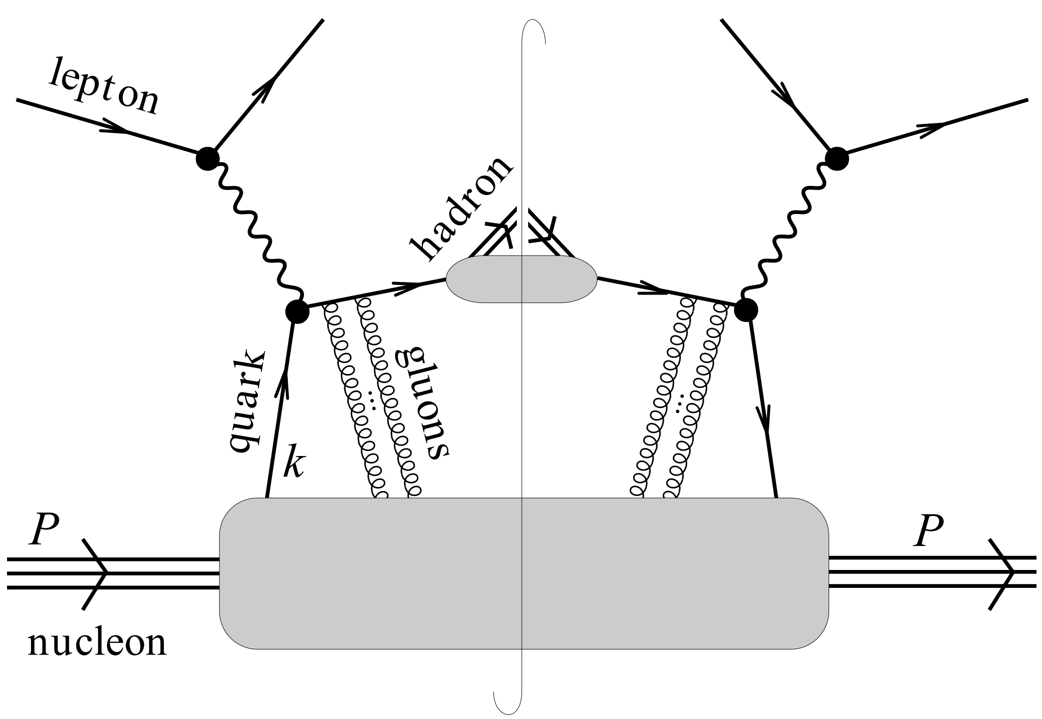
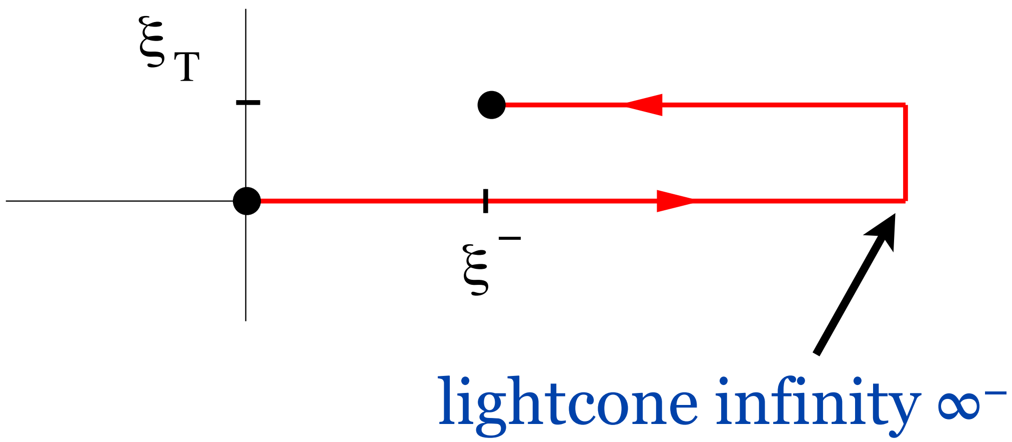
$$e\,p \to e'\,h\,X$$



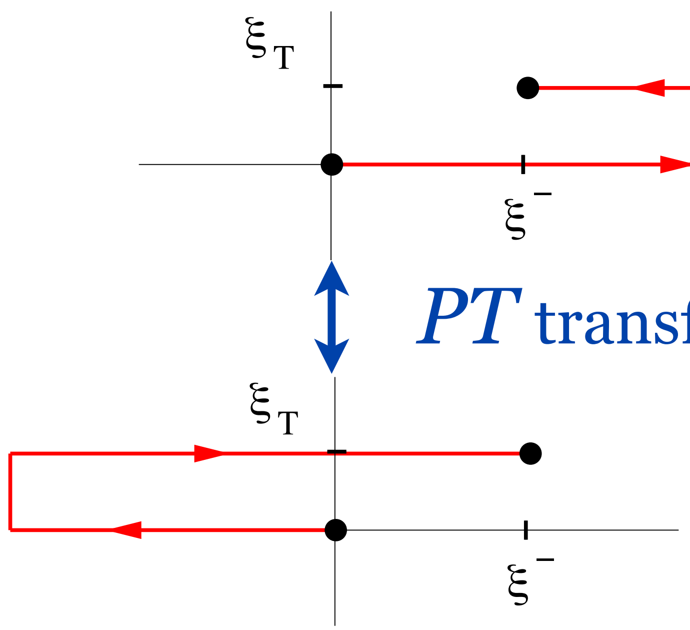


$$k \approx xP + k_T$$

$$P^\mu \approx P^+$$

Sivers TMD

The future and past pointing gauge links are related by a combination of a P and T transformation and there happen to be parton distributions that are odd under this transformation, such as the Sivers TMD



Sivers effect is <u>odd</u> under + ↔ −

PT transformation: $$f_{1T}^{\perp[+]}(x, p_T^2) = -f_{1T}^{\perp[-]}(x, p_T^2)$$

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \overset{T}{\rightarrow} -x^{\mp} \overset{P}{\rightarrow} -x^{\pm}$$

[Collins '02]

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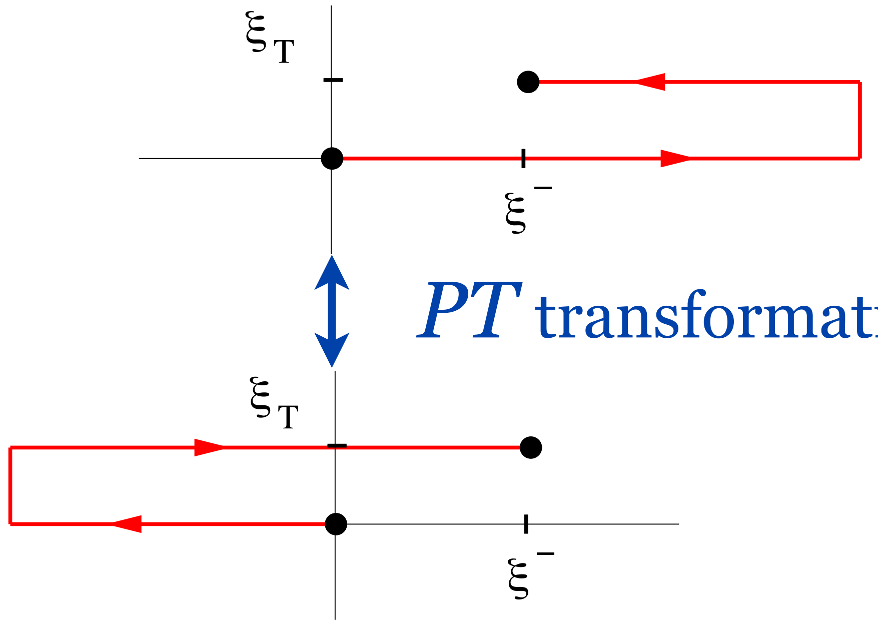

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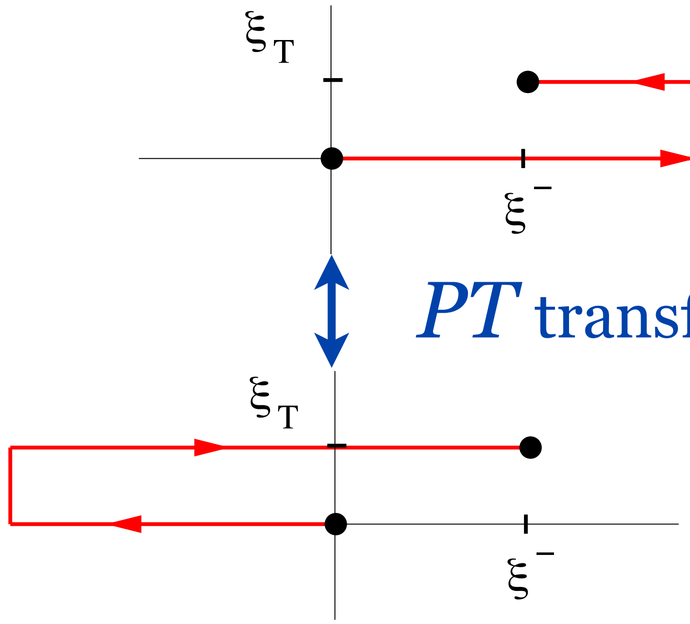

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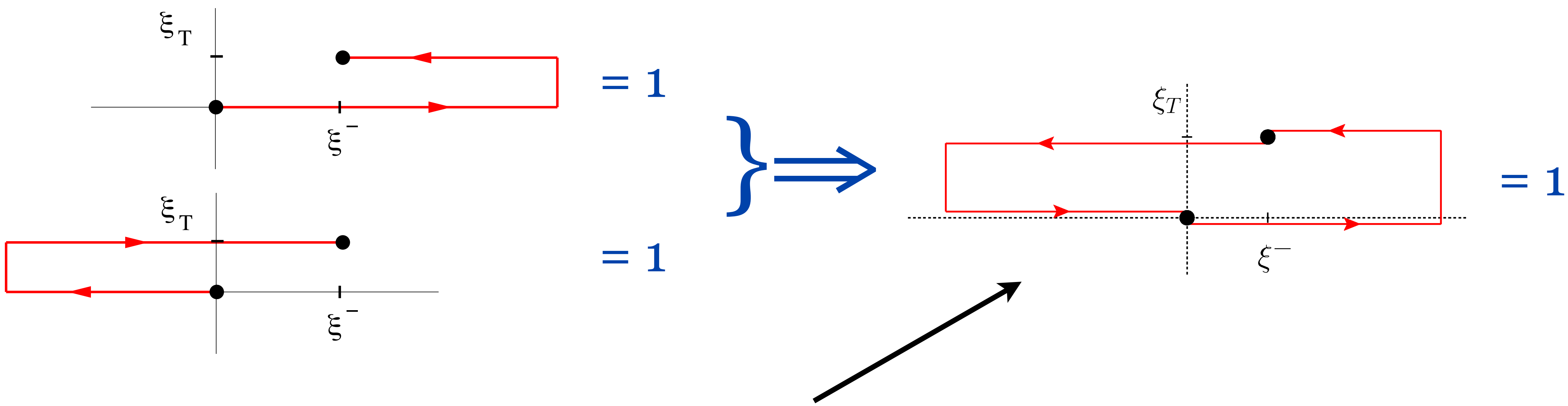
Initially it was thought that the path of the gauge link is irrelevant, because a gauge can always be chosen such that it is unity. For example, for the future pointing staple link:

Lightcone gauge (A^+=0) with advanced boundary condition ($A_T(\infty^-, \xi_T)$=0)

But now the path/process dependence is in the gauge condition

Contour gauge

Imposing lightcone gauge (A^+=0) with advanced *and* retarded boundary condition ($A_T(\pm\infty^-,\xi_T)$=0) is not allowed, as it overfixes the gauge



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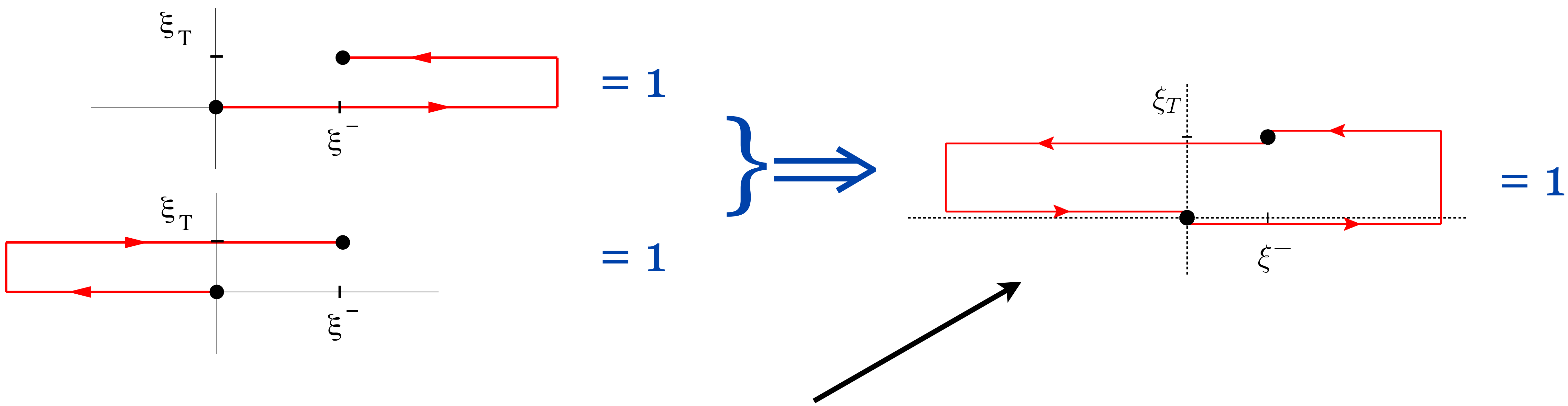

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Contour gauge:

$$P\exp\left(ig_s\int_{x_0}^{x} ds_\mu A^\mu(s)\right)=1$$

Contour gauge can be chosen for each x for just one non-self-intersecting path from x_0 to x, otherwise one forces $F^{\mu\nu}$ to be zero in certain regions, which is not allowed

S.V. Ivanov, G.P. Korchemsky & A.V. Radyushkin, Sov. J. Nucl. Phys. 44 (1986) 145

Process dependence of Sivers TMD

Time reversal invariance relates the Sivers functions of SIDIS and Drell-Yan

This is a *calculable* process dependence, which yields the relation [Collins '02]:

$$f_{1T}^{\perp [\mathrm{SIDIS}]} = -f_{1T}^{\perp [\mathrm{DY}]} \qquad \text{to be tested}$$

The Sivers effect in SIDIS has been clearly observed by HERMES at DESY (PRL 2009) & COMPASS at CERN (PLB 2010)

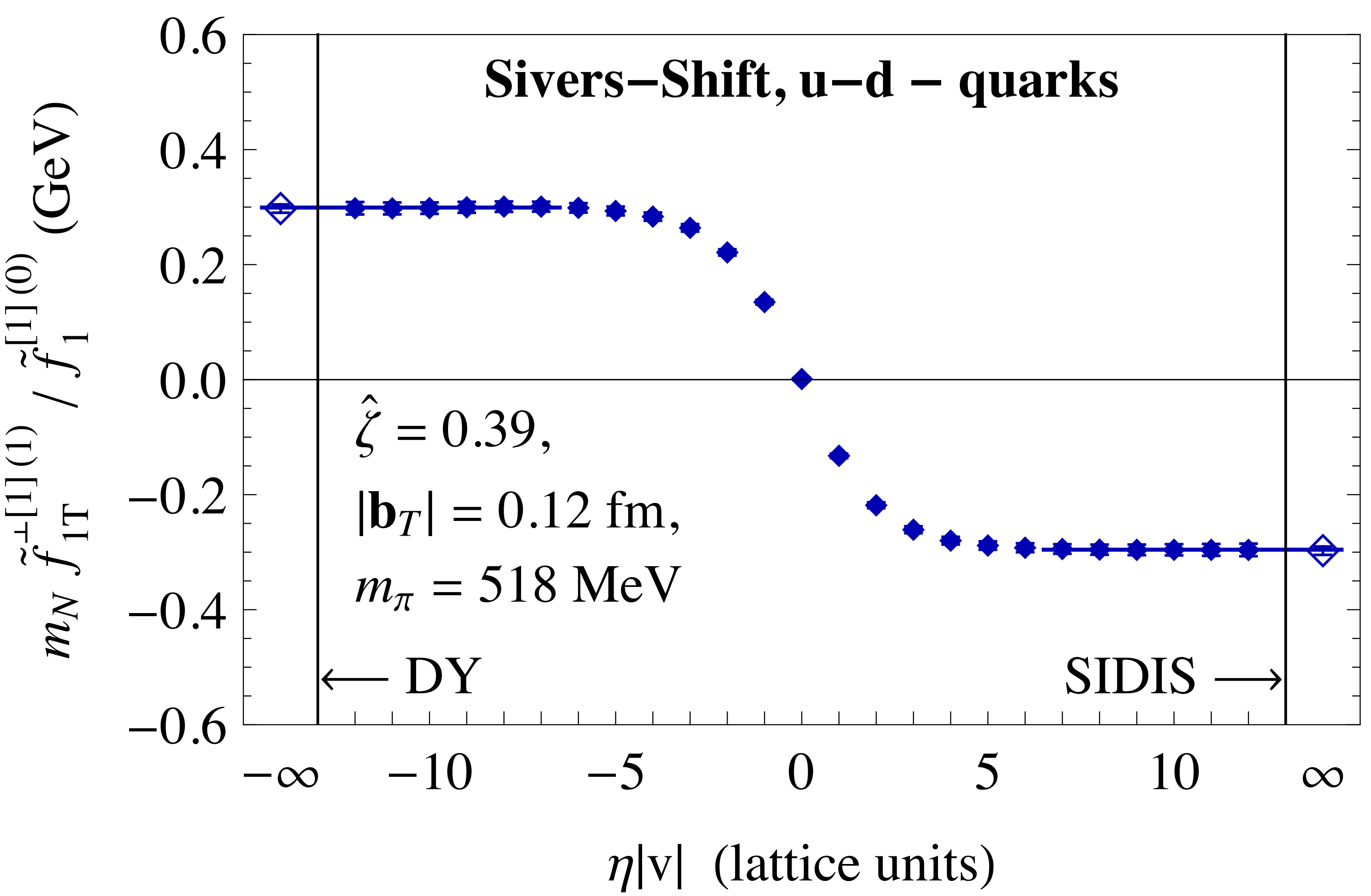
The corresponding DY experiments are in progress at CERN (COMPASS), Fermilab (SeaQuest) & RHIC (W-boson production rather) & planned at NICA (Dubna)

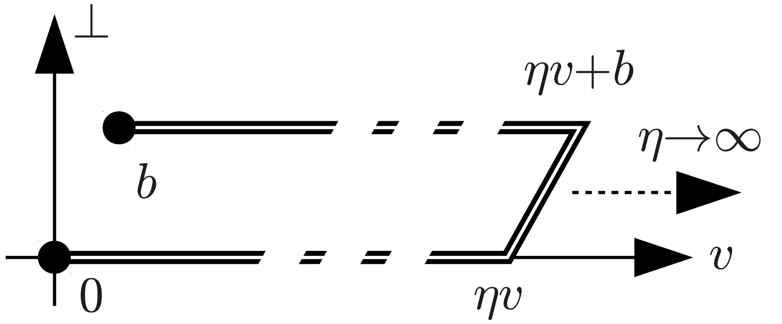
Not just a test of this relation, but of TMD factorization formalism in essence

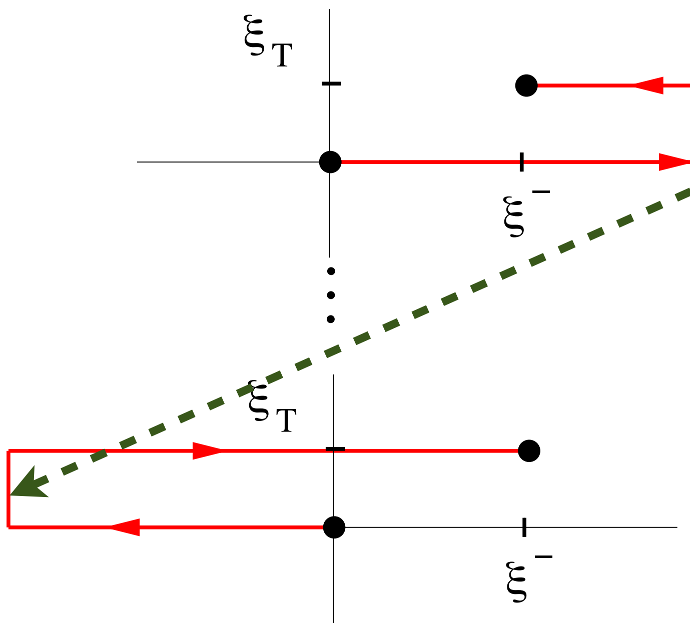
Sivers function on the lattice

By taking specific x and k_T integrals one can define the "Sivers shift" <k_T x S_T>(n,b_T): the average transverse momentum shift orthogonal to transverse spin S_T
[Boer, Gamberg, Musch, Prokudin, 2011]

This well-defined quantity can be evaluated on the lattice
[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]







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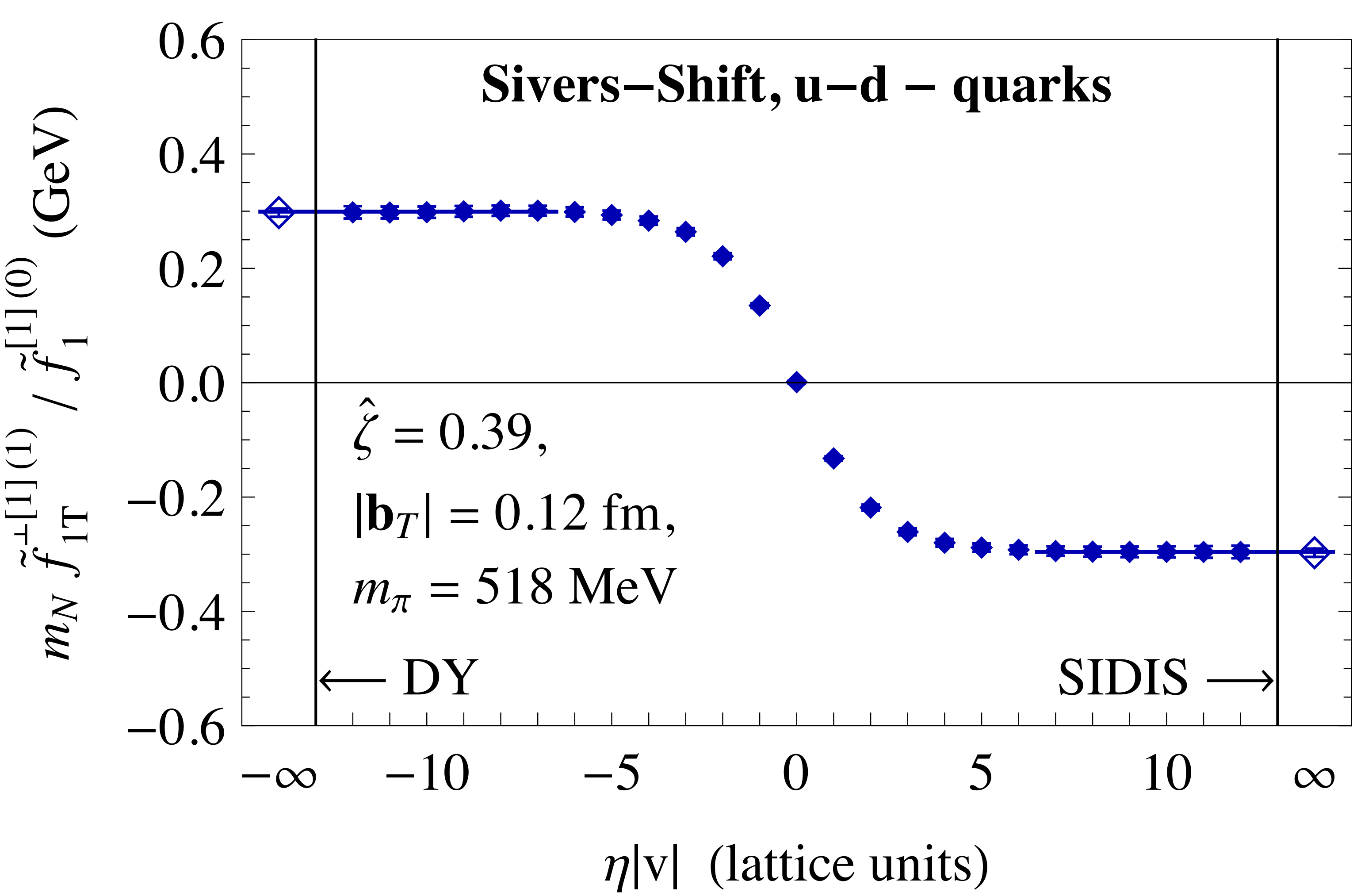

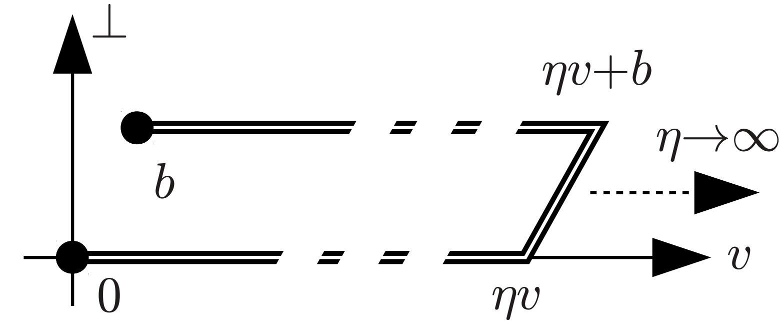

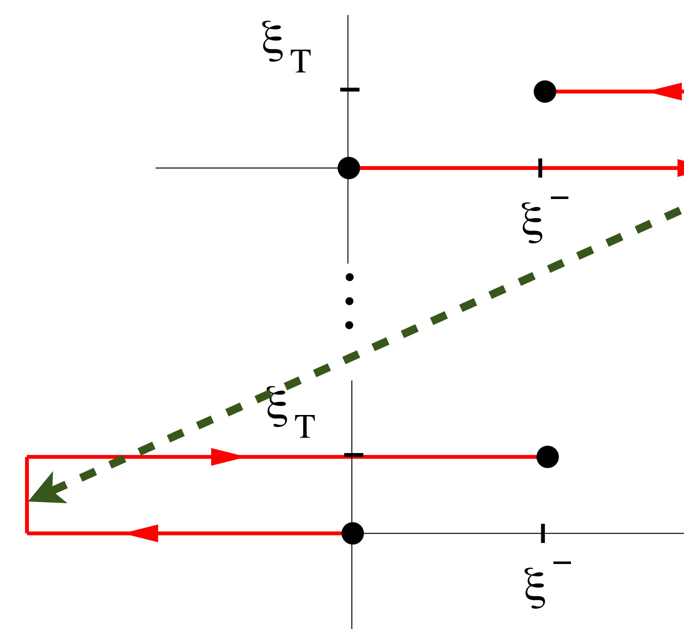

This is the first `first-principle' demonstration that the Sivers function is nonzero for staple-like links. It clearly corroborates the sign change relation (as it should)

Gluon Sivers effect

There is also a Sivers effect for gluons

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$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

Gluon TMDs depend on two path-dependent gauge links

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For transversely polarized protons:

gluon Sivers function

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[Mulders, Rodrigues '01]

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These processes probe 2 distinct, **independent** gluon Sivers functions

Related to antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Process dependence of **gluon** Sivers TMD

$e\,p^{\uparrow} \to e'\,Q\bar{Q}\,X$ This process probes a gluon correlator with two + links

$p^{\uparrow}\,p \to \gamma\,\gamma\,X$ This process probes a gluon correlator with two − links

$p^{\uparrow}\,p \to \gamma\,\text{jet}\,X$ This process probes a gluon correlator with a + and − link

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Conclusion: despite the process dependence of the gluon Sivers TMD, one can still know if and how the different processes are related

Predictability is not automatically lost because of nonuniversality

Single spin asymmetry studies at various colliders can thus be either related or complementary, depending on the processes considered

D.B., Lorcé, Pisano & Zhou, arXiv:1504.04332

Process dependence of gluon TMDs

Is this TMD nonuniversality a polarization issue only? No!

This process dependence is also present for the *unpolarized* gluon TMD, as was first realized in a small-x context

Dominguez, Marquet, Xiao, Yuan, 2011

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Dominguez, Marquet, Xiao, Yuan, 2011

Kharzeev, Kovchegov & Tuchin (2003): ``A tale of two gluon distributions"
They noted there are 2 distinct but equally valid definitions for the small-x gluon distribution: the Weizsäcker-Williams (WW) and the dipole (DP) distribution

KKT: “cannot offer any simple physical explanation of this paradox”

The explanation turns out to be in the process dependence of the gluon distribution, in other words, its sensitivity to the initial and/or final state interactions (ISI/FSI) in a process

The difference between the WW and DP distributions would disappear without ISI/FSI

WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$ [+,+]

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$ [+,−]

For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

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For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')} \left\langle \mathrm{Tr}\left[\partial_i U(v)\right] U^\dagger(v') \left[\partial_i U(v')\right] U^\dagger(v)\right\rangle_{x_g} \qquad \text{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp} \frac{1}{N_c} \left\langle \mathrm{Tr} U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \qquad \text{DP}$$

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Different processes probe one or the other or a mixture, so this can be tested

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Different processes probe one or the other or a mixture, so this can be tested

Higgs production in pp and pA collisions probes the [−,−] or WW gluon TMD

Transverse Momentum of Gluons

In addition, for *unpolarized* protons there is another type of gluon distribution

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \int \frac{d(\xi\cdot P)\, d^2\xi_T}{(xP\cdot n)^2(2\pi)^3} e^{i(xP+k_T)\cdot\xi} \mathrm{Tr}_c\Big[\langle P|F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\, F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}|P\rangle\Big]_{\xi\cdot P'=0}$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{1}{2x}\left\{-g_T^{\mu\nu} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon distribution function (f_1^g)

linearly polarized gluon distribution ($h_1^{\perp\, g}$)

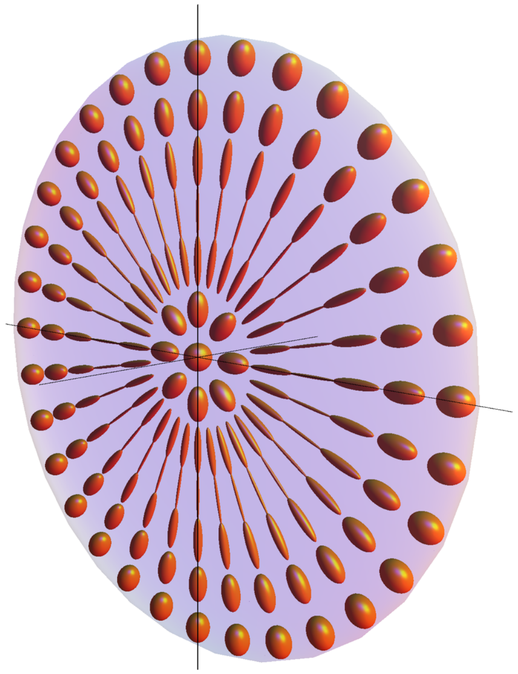
Gluons inside unpolarized protons can be polarized!

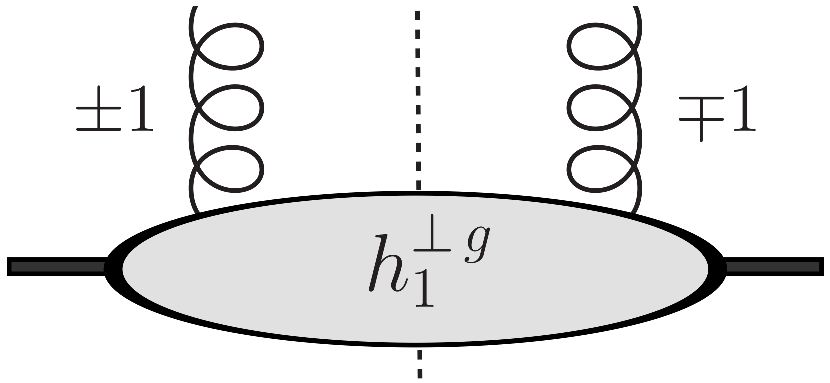
[Mulders, Rodrigues '01]

Gluon polarization inside unpolarized protons

Gluons inside unpolarized protons can be polarized! [Mulders, Rodrigues '01]

Linear polarization of gluons



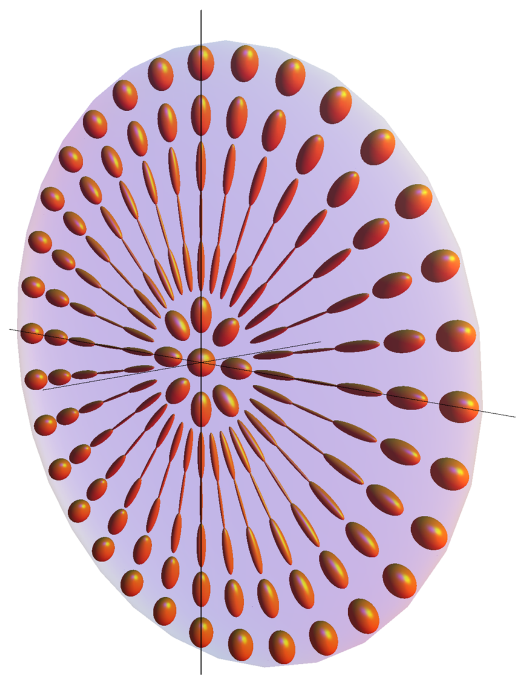


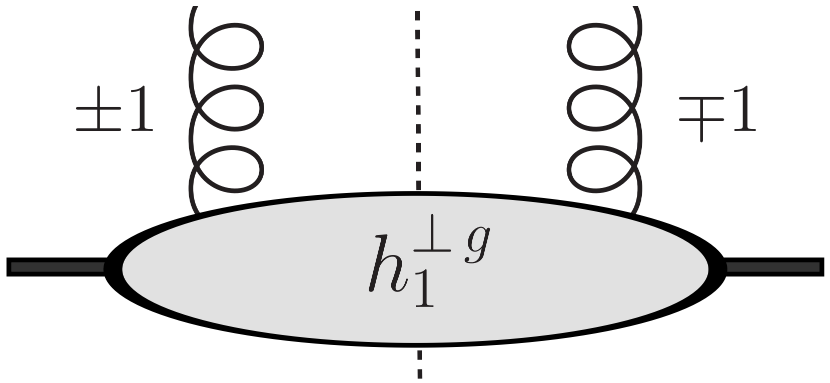
an interference between
±1 helicity gluon states

Gluon polarization inside unpolarized protons

Gluons inside unpolarized protons can be polarized! [Mulders, Rodrigues '01]

Linear polarization of gluons





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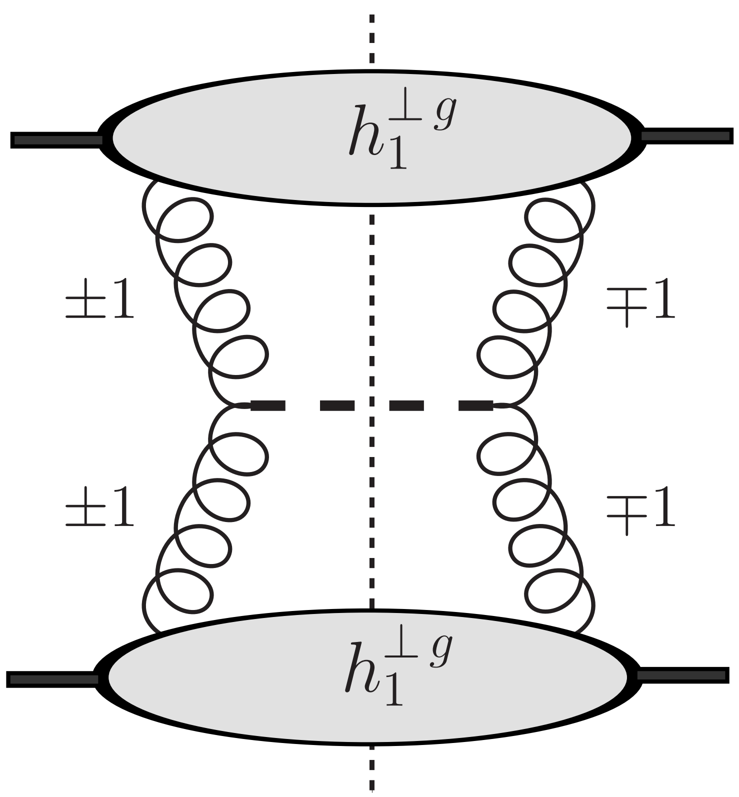


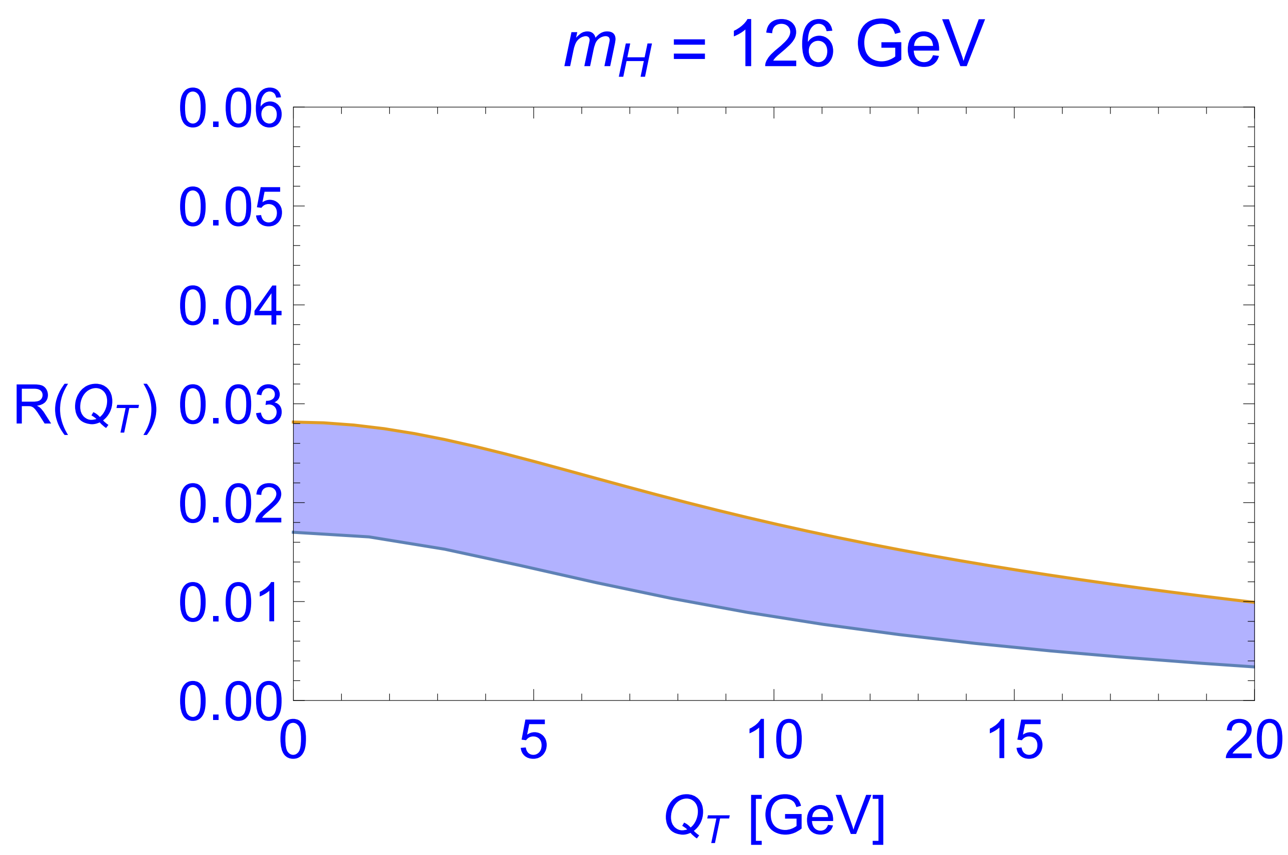
Affects Higgs production at the LHC

Boer, Den Dunnen, Pisano, Schlegel, Vogelsang, PRL 2012

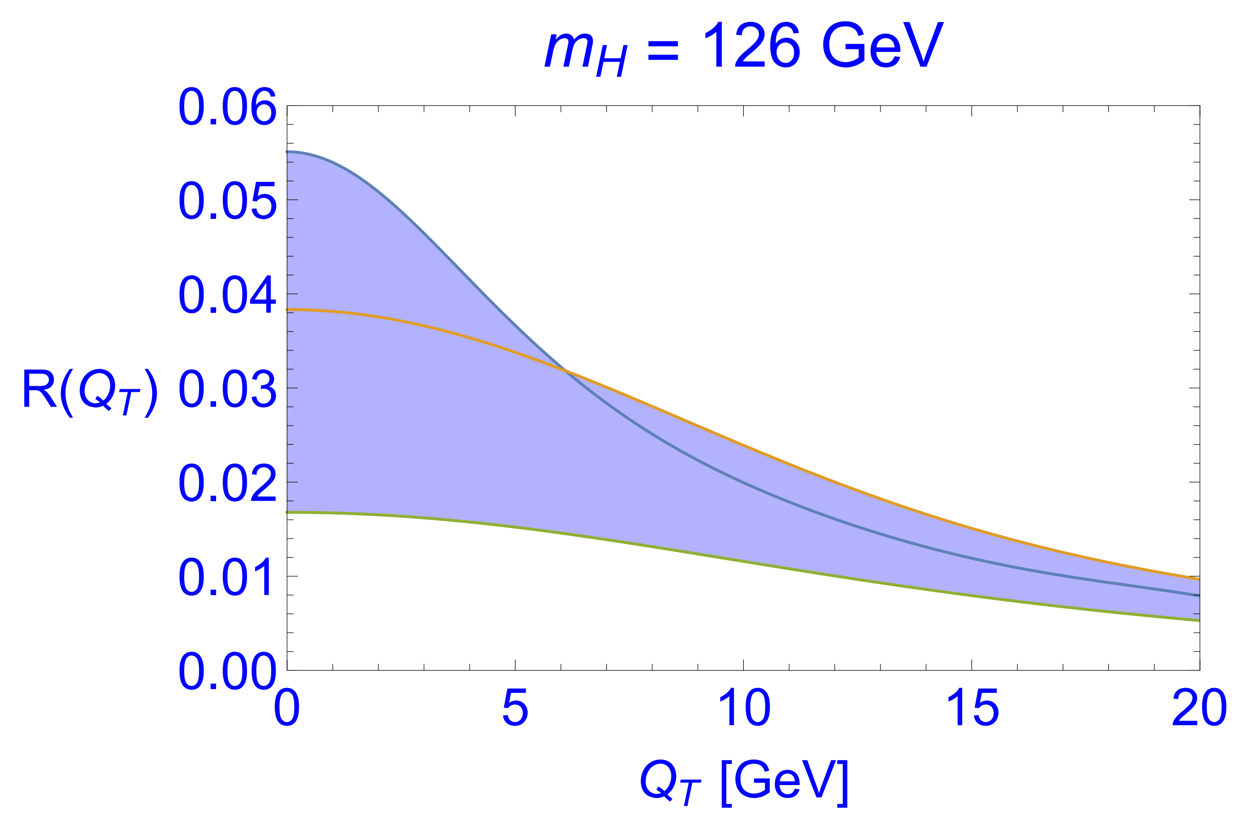
The LHC is actually a *polarized* gluon collider

It remains to be seen whether this can be exploited

Range of predictions



Boer & den Dunnen, 2014

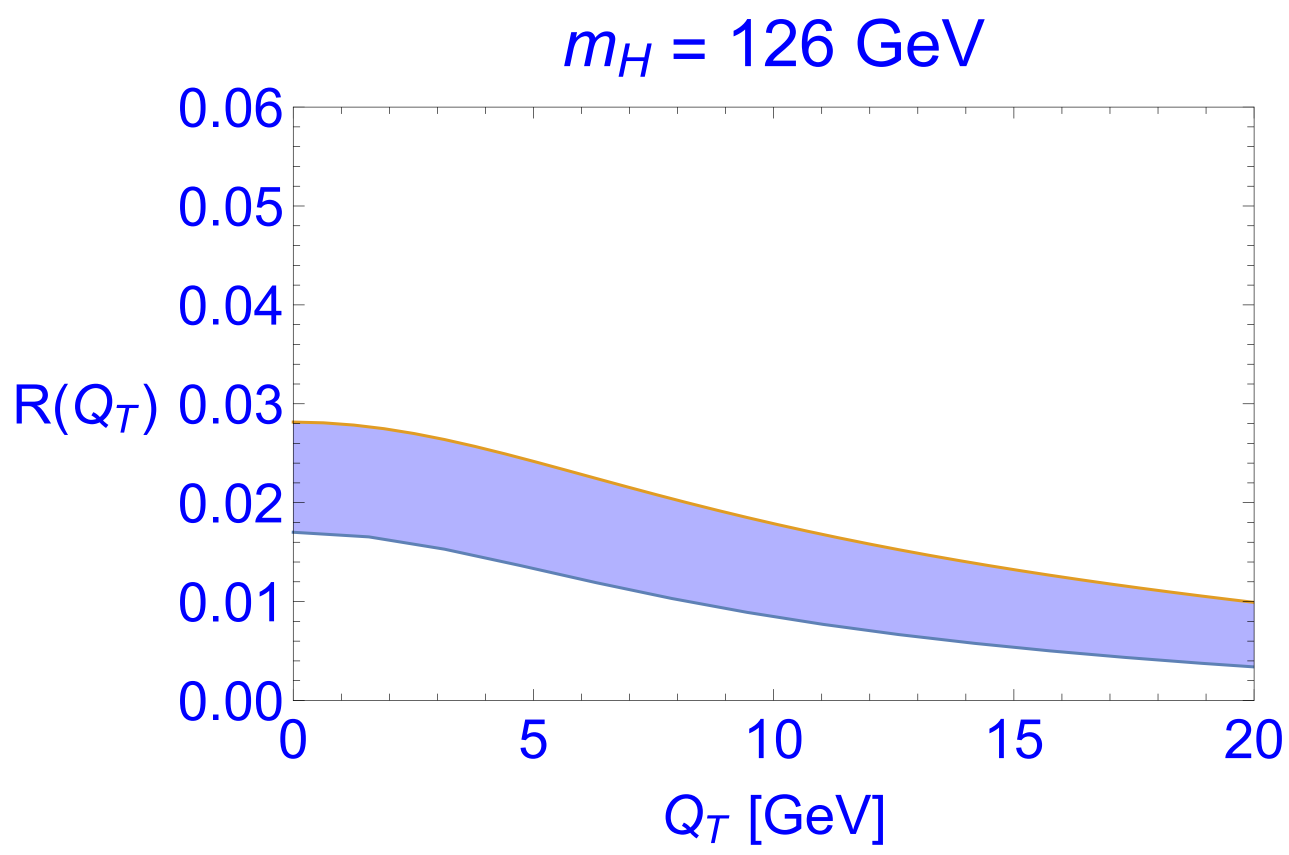


Echevarria, Kasemets, Mulders, Pisano, 2015

Left: variation of the nonperturbative input and of the large Q_T behavior

Right: variation of the nonperturbative input and the renormalization scale

Range of predictions



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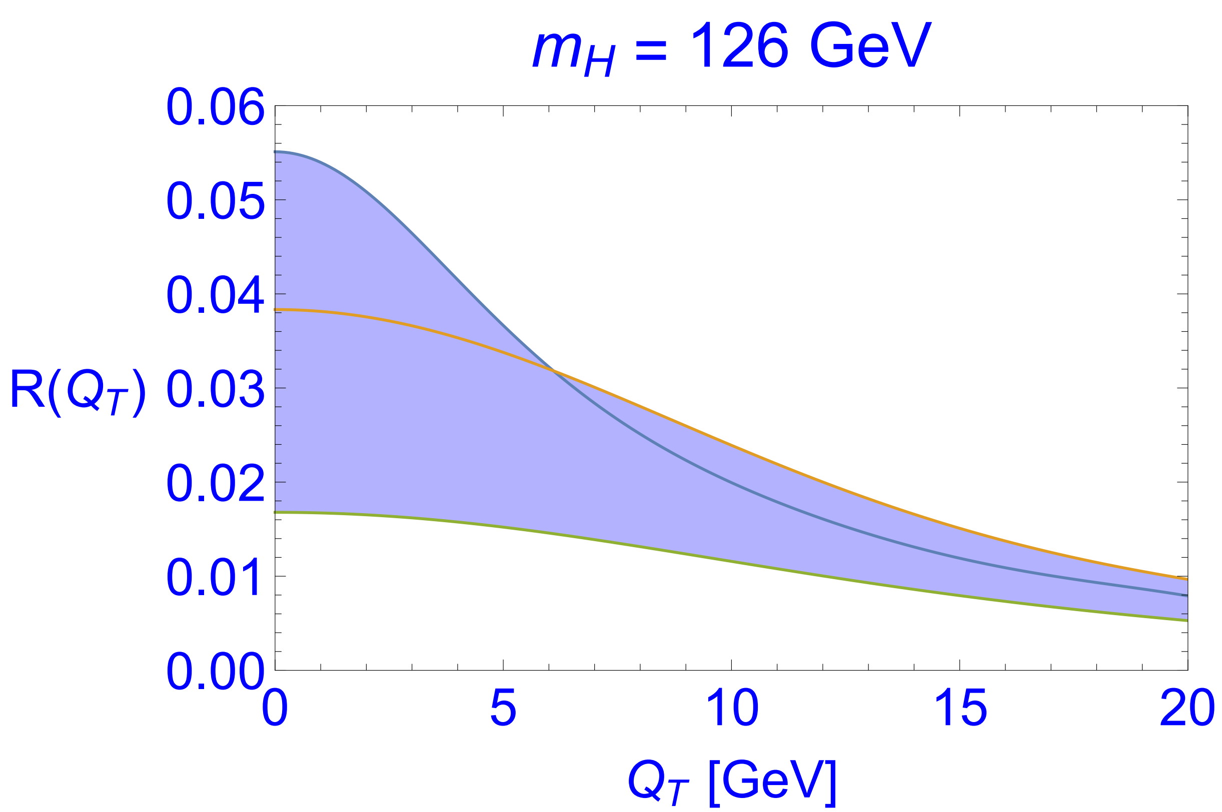

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Left: variation of the nonperturbative input and of the large Q_T behavior

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Conclusions:

- effect of linear gluon polarization in Higgs production on the order of 2-5%
- extraction of $h_1^{\perp g}$ from Higgs production may be too challenging

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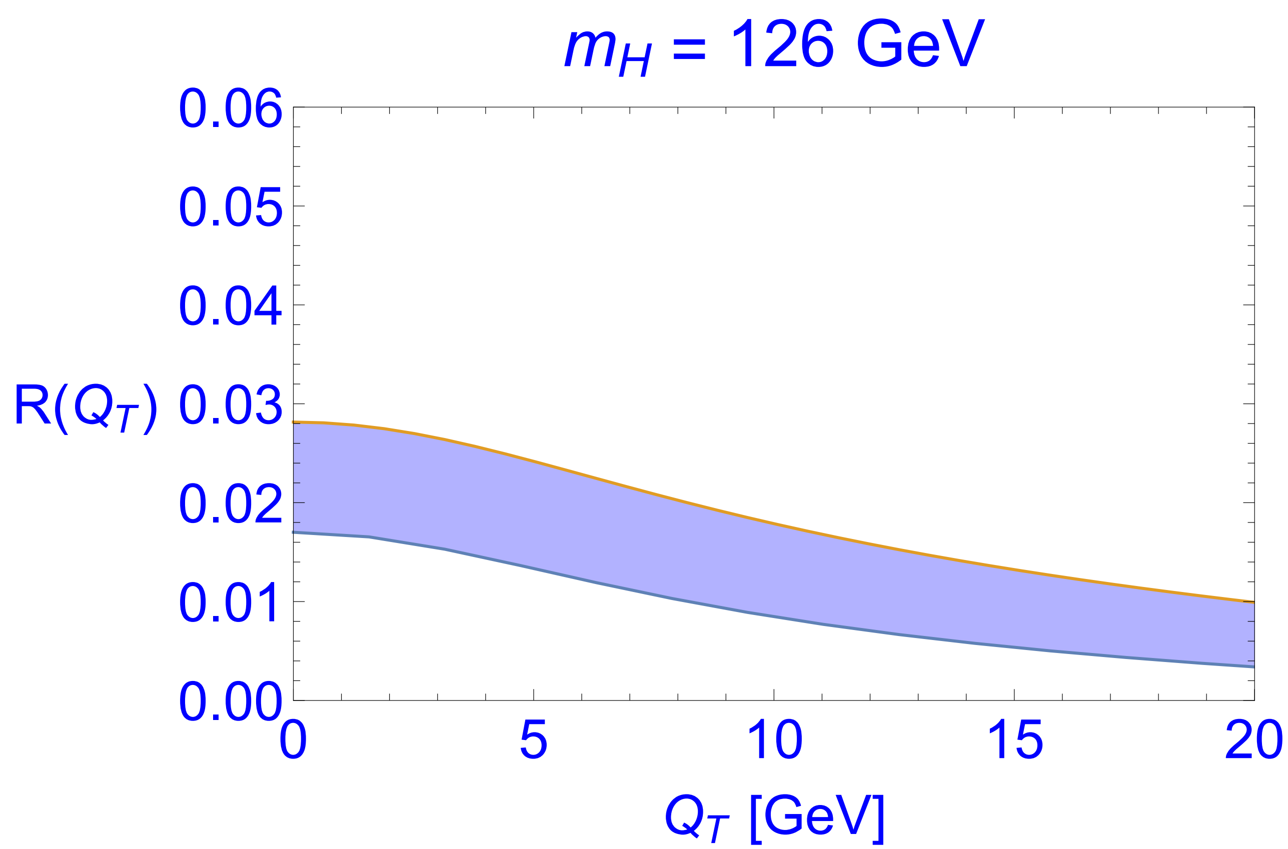

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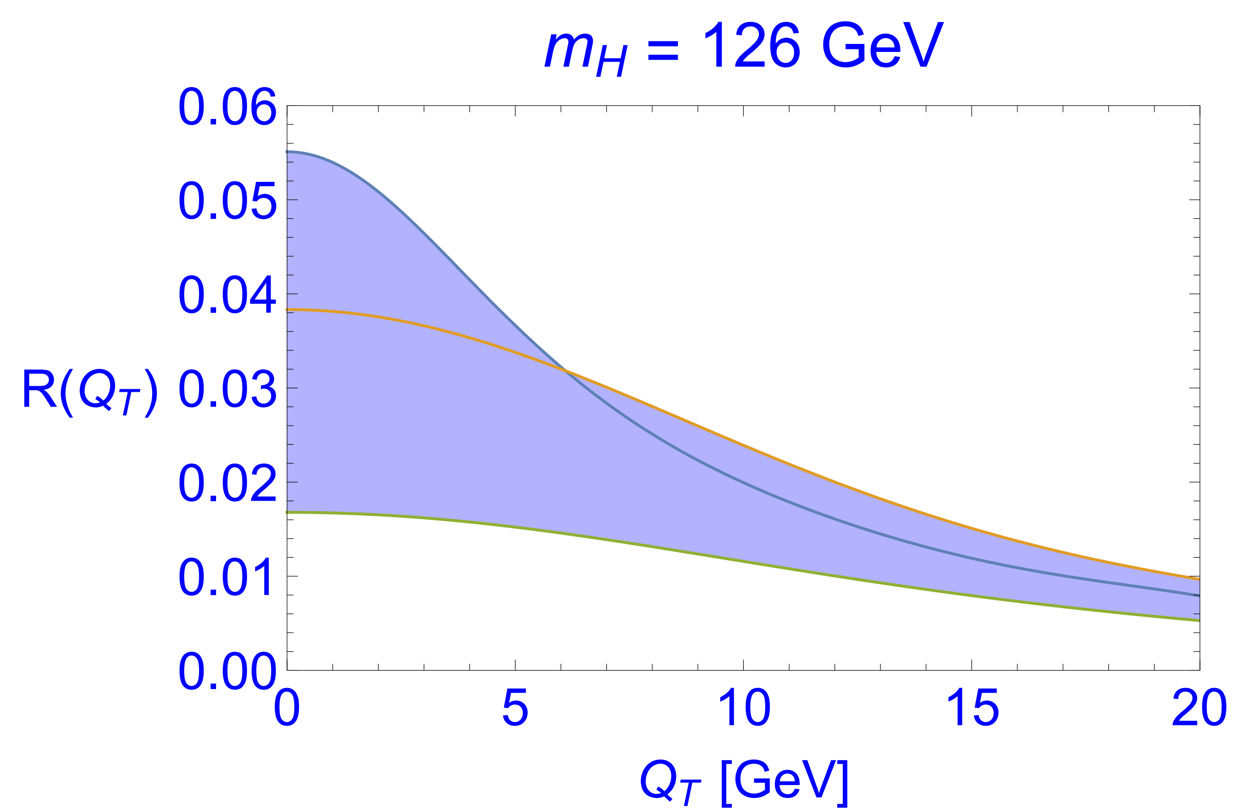

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- effect of linear gluon polarization in Higgs production on the order of 2-5%
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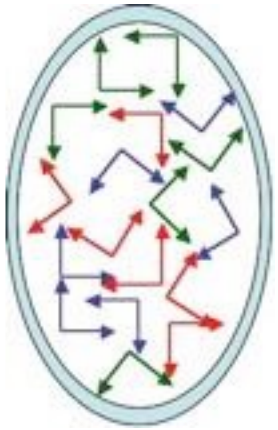
Effects larger at smaller Q ($0^{\pm +}$ quarkonia) and at small x (plots are for x ~ 0.016)

Polarization of the CGC



Evolution: $h_1^{\perp g}$ has the same $1/x$ growth as f_1

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

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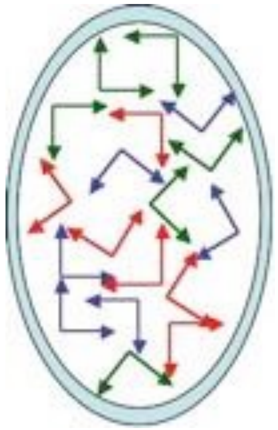

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CGC framework calculations show the CGC gluons are in fact linearly polarized

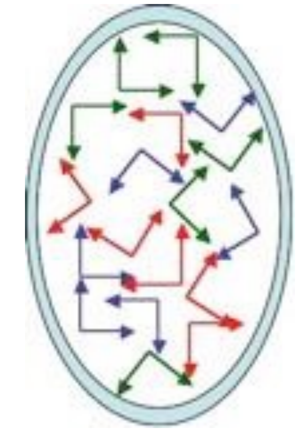
$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$x h_{1,DP}^{\perp g}(x, k_\perp) = 2 x f_{1,DP}^{g}(x, k_\perp)$$

Metz, Zhou '11

The CGC can be 100% polarized, but its observable effects depend on the process

Polarization of the CGC



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Metz, Zhou '11

The CGC can be 100% polarized, but its observable effects depend on the process

pp → H X and pp → $\eta_{c/b}$ X or $\chi_{c/b0}$ X probe [−,−] = WW [D.B., Pisano, 2012]

pp → γ^* + jet + X probes [+,−] = DP [Jian Zhou. 2016]

For this purpose pA collisions are even better suited of course

Linear gluon polarization at EIC

WW $h_1^{\perp g}$ also accessible in dijet production in eA collisions at a high-energy EIC

[Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

The WW $h_1^{\perp g}$ is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1\,WW}^{\perp\,g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$

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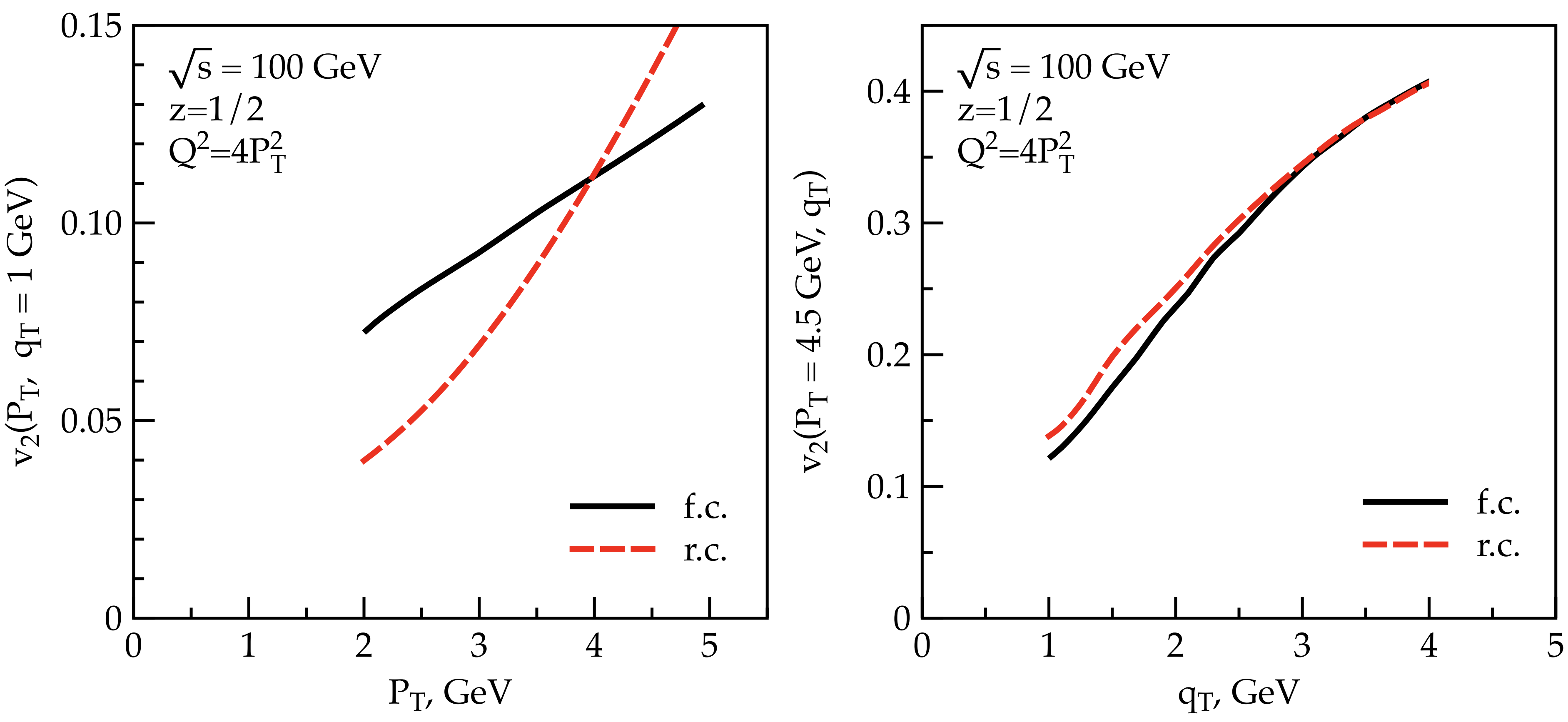

Large effects are found
Dumitru, Lappi, Skokov, 2015

v_2 in eA collisions

Gluon Sivers effect at small x

	DIS	DY	SIDIS	$p^\uparrow A \to h\, X$	$p^\uparrow A \to \gamma^{(*)}\, \text{jet}\, X$	Dijet in DIS	Dijet in $p^\uparrow A$
$f_{1T}^{\perp\, g\, [+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$
$f_{1T}^{\perp\, g\, [+,-]}$ (DP)	$\times$	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$

EIC (SIDIS) — EIC (Dijet in DIS)

↑ backward hadron production ($p^\uparrow A \to h\, X$)

At small x the WW Sivers function is suppressed by a factor of x compared to the unpolarized gluon function

The DP-type Sivers function is not suppressed and can be probed in pA collisions

Gluon Sivers effect at small x

	DIS	DY	SIDIS	$p^\uparrow A \to h X$	$p^\uparrow A \to \gamma^{(*)}$ jet X	Dijet in DIS	Dijet in $p^\uparrow A$
$f_{1T}^{\perp g\,[+,+]}$ (WW)	×	×	×	×	×	√	√
$f_{1T}^{\perp g\,[+,-]}$ (DP)	×	√	√	√	√	×	√

EIC (SIDIS) — backward hadron production ($p^\uparrow A \to h X$) — EIC (Dijet in DIS)

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The DP-type Sivers function is not suppressed and can be probed in pA collisions

The DP-type Sivers function turns out to be the ***spin-dependent odderon*** [Zhou, 2013]

$$\Gamma_{(d)}^{(T-\mathrm{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.} \langle P, S_T | \mathrm{Tr}\left[U^{[\square]}(0_T, y_T) - U^{[\square]\dagger}(0_T, y_T)\right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, Zhou, PRL 2016

a single Wilson loop matrix element

$$U^{[\square]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

Gluon Sivers effect at small x

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EIC (SIDIS) — EIC (Dijet in DIS)

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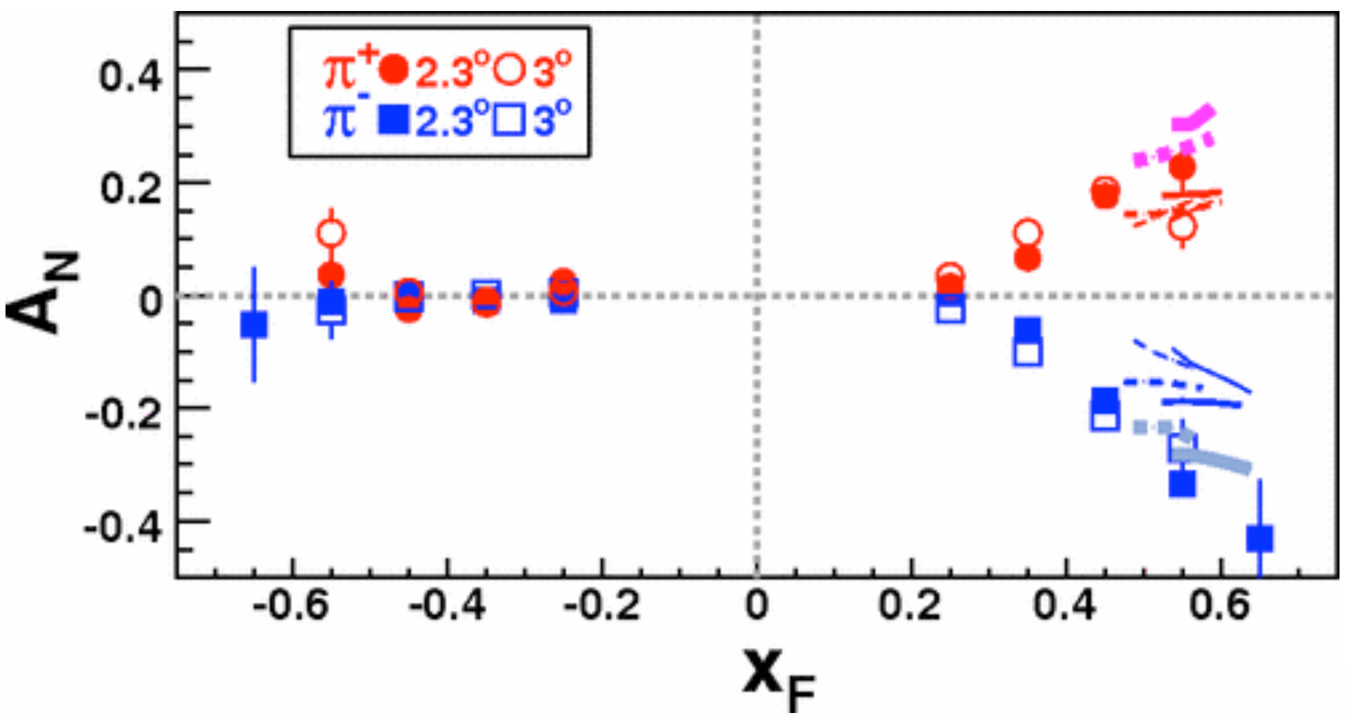
D.B., Echevarria, Mulders, Zhou, PRL 2016

a single Wilson loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

The imaginary part of the Wilson loop determines the gluonic single spin asymmetry
It is the only contribution, as opposed to the many contributions at larger x

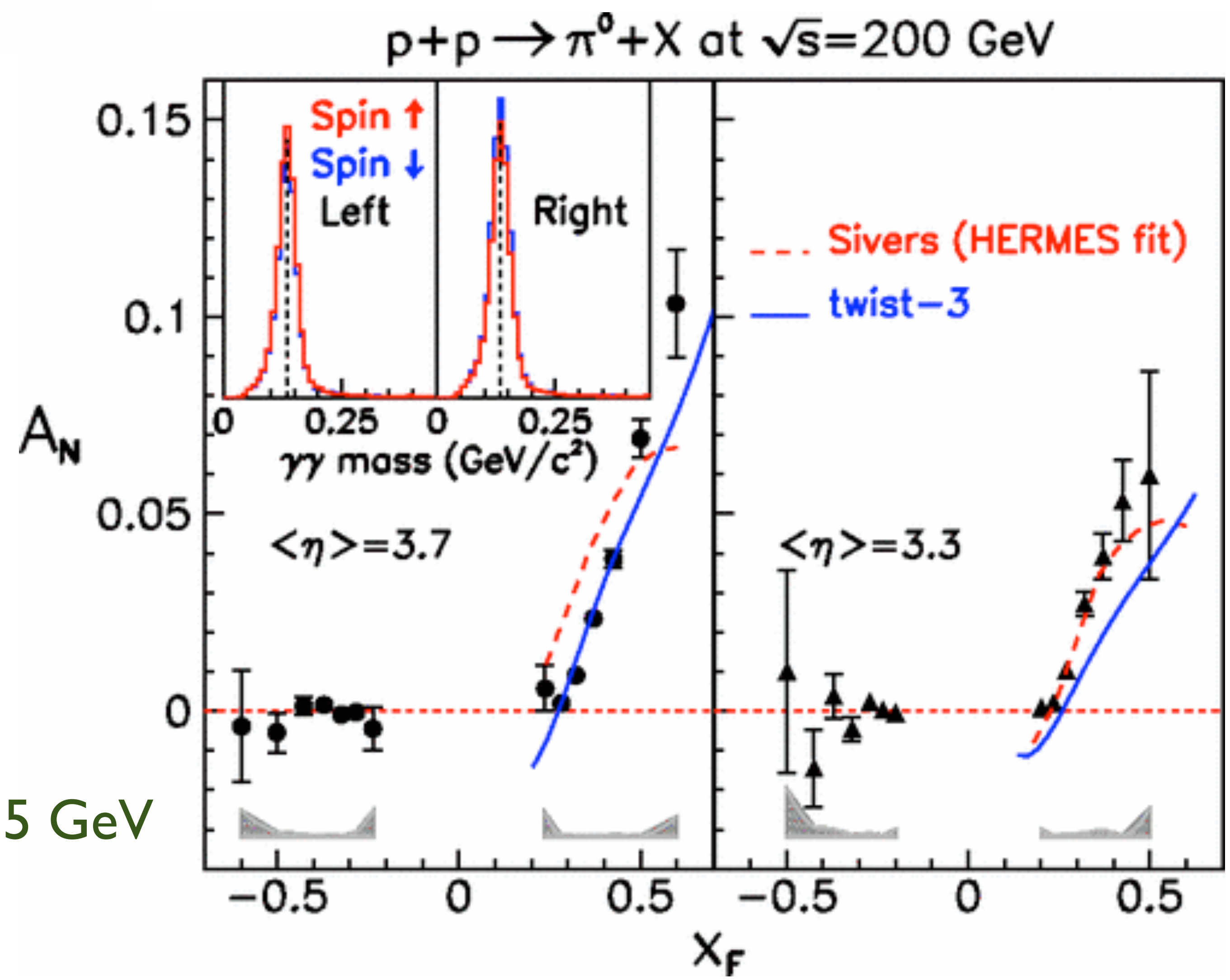
$p^{\uparrow}p \to h^{\pm}X$ at $x_F < 0$



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

spin-dependent odderon is C-odd,
whereas gg in the CS state is C-even

expect smaller asymmetries
in neutral pion and jet production



STAR, 2008
$\sqrt{s}$ = 200 GeV
p_T between 1 and 3.5 GeV

Conclusions

Conclusions

High energy scattering processes sensitive to the transverse momentum of partons involve highly nonlocal gauge links that track/reflect the color flow

This unexpectedly affects observables, such as transverse spin asymmetries (sign change between processes) or the Higgs transverse momentum distribution

The corresponding nonlocal quantities, such as the Sivers function, are amenable to lattice calculations, which indicates a substantial magnitude (as do the data)

Also unpolarized protons and spin-0 hadrons have a nontrivial spin structure

The state of highest gluon density can be maximally polarized
The color flow determines how much this polarization affects observables

Back-up slides

Gluon Sivers effect

There is also a Sivers effect for gluons

Gluon Sivers effect

There is also a Sivers effect for gluons

$$\Gamma_g^{\mu\nu\,[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

Gluon TMDs depend on two path-dependent gauge links

Gluon Sivers effect

There is also a Sivers effect for gluons

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Gluon TMDs depend on two path-dependent gauge links

For transversely polarized protons:

gluon Sivers function

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There is also a Sivers effect for gluons

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These processes probe 2 distinct, **independent** gluon Sivers functions

Related to antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Process dependence of **gluon** Sivers TMD

$$e\, p^{\uparrow} \to e'\, Q\bar{Q}\, X$$

$$\gamma^*\, g \to Q\bar{Q}$$

This subprocess probes a gluon correlator with two + links (both future pointing)

Qiu, Schlegel, Vogelsang, 2011

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A sign-change relation for gluon Sivers functions

Linear gluon polarization at small x

Like any TMD $h_1^{\perp g}$ distributions has a link/process dependence. In the CGC framework:

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

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The perturbative tail of $h_1^{\perp g}$ has a $1/x$ growth, which keeps up with f_1:

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

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There is no reason to expect $h_1^{\perp g}$ to be small, especially at small x

MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x,q_\perp) \overset{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x,q_\perp)$$

Processes involving $G^{(1)}$ (WW) [+,+] in the MV model can be expressed in terms of $G^{(2)} \sim C(k_\perp)$

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$$\frac{d\sigma_T}{dy\, dk_\perp} = \pi R^2 \frac{2N_c(Z\alpha)^2}{3\pi^3} \ln\left(\frac{\gamma}{2mR}\right) k_\perp C(k_\perp) \times \left\{1 + \frac{4(k_\perp^2 - m^2)}{k_\perp\sqrt{k_\perp^2+4m^2}} \operatorname{arcth} \frac{k_\perp}{\sqrt{k_\perp^2+4m^2}}\right\}$$

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Higgs production in pp and pA collisions probes the WW gluon distribution

For dijet in pA the result requires large N_c, otherwise 4 additional functions appear

Finite N_c: Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

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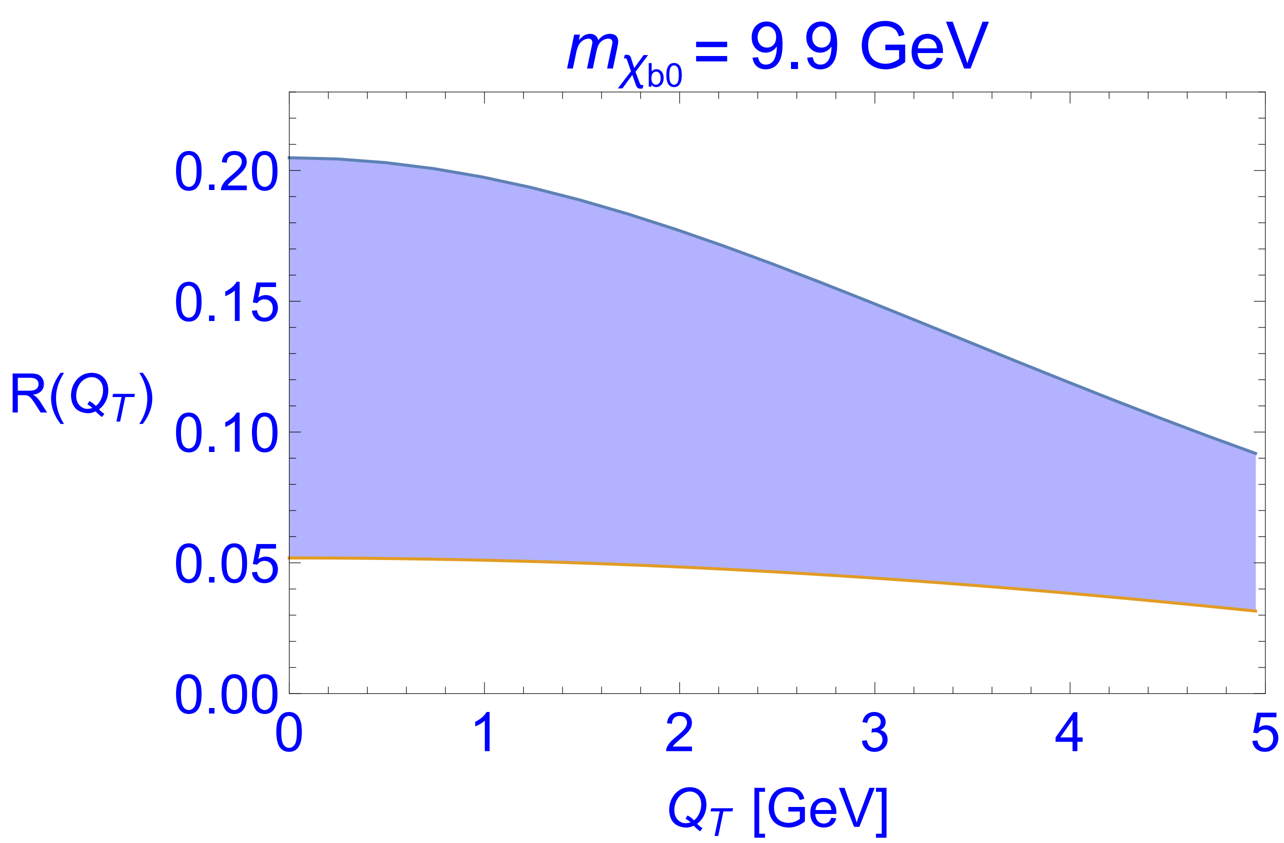
$0^{\pm+}$ quarkonium production allows to measure the polarization of the CGC using the angular independent p_T distribution

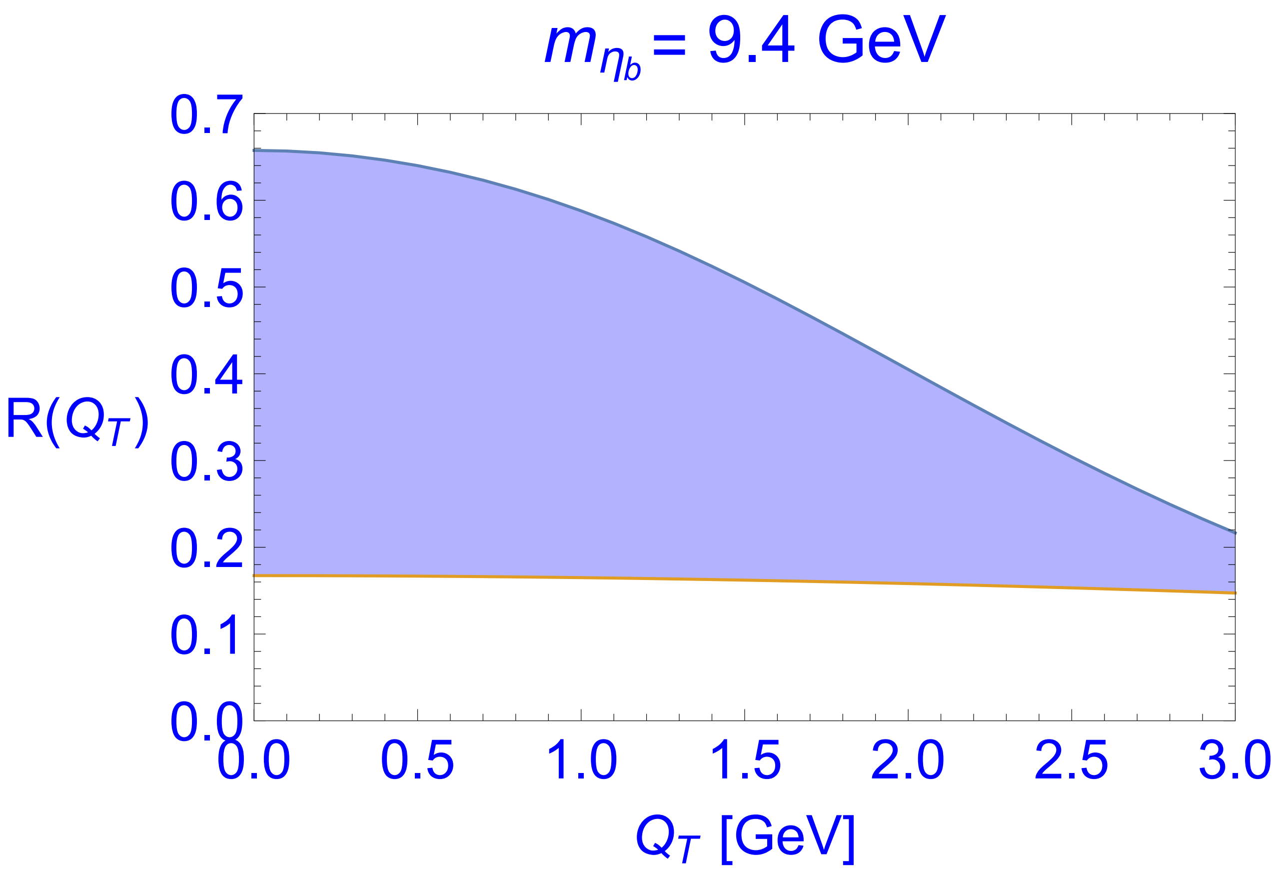
Bottomonium production

More promising may be C-even (pseudo-)scalar quarkonium production

Boer & Pisano, 2012

The range of predictions for bottomonium production:





Boer & den Dunnen, 2014

Echevarria, Kasemets, Mulders, Pisano, 2015

Conclusion: very large theoretical uncertainties in quarkonium production (more sensitive to unknown nonperturbative part than Higgs production), but larger effects

Electron-Ion Collider

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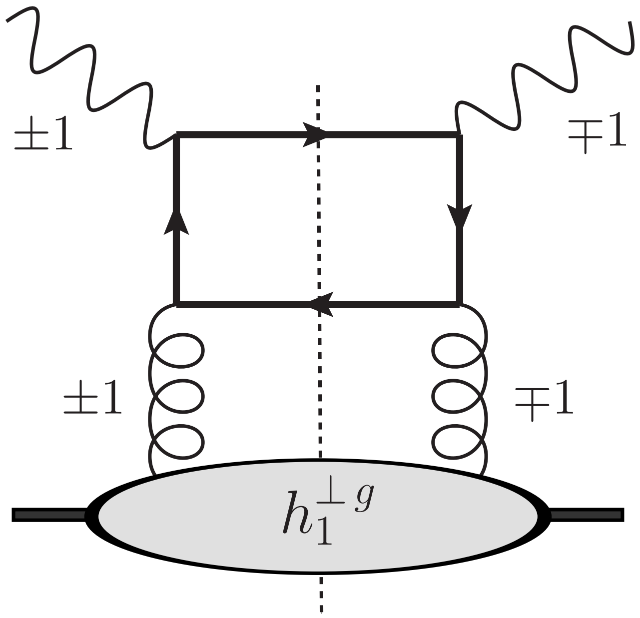

Best measured at a future Electron-Ion Collider (USA) or LHeC (CERN)

Electron-Ion Collider

- effect of linear gluon polarization in Higgs production on the order of 2-5%
- extraction of $h_1^{\perp g}$ from Higgs production may be too challenging

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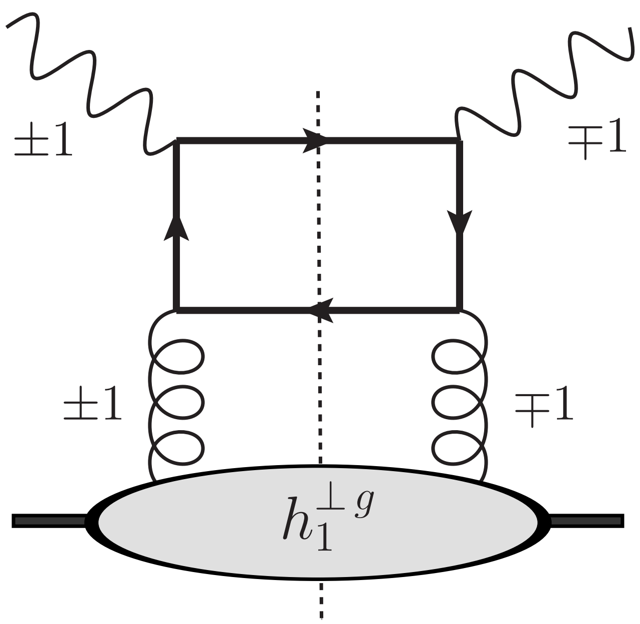

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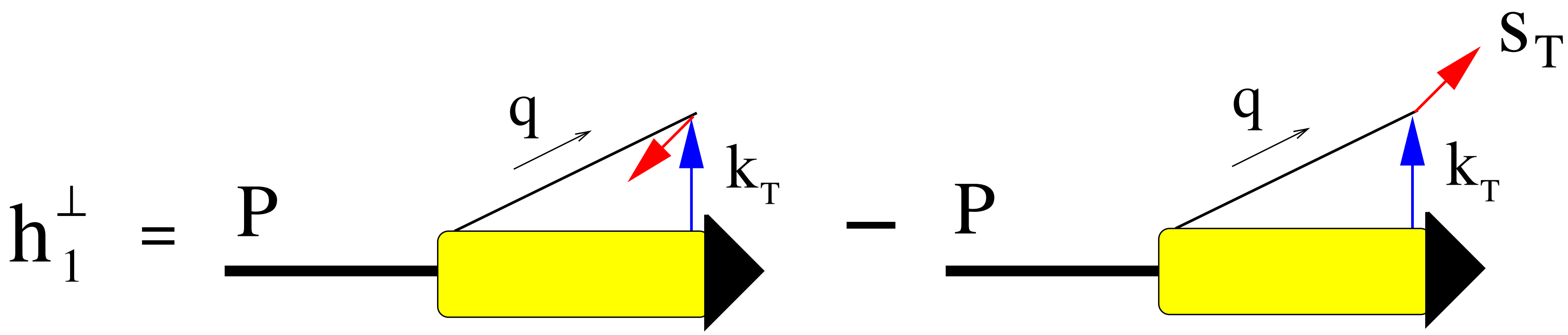

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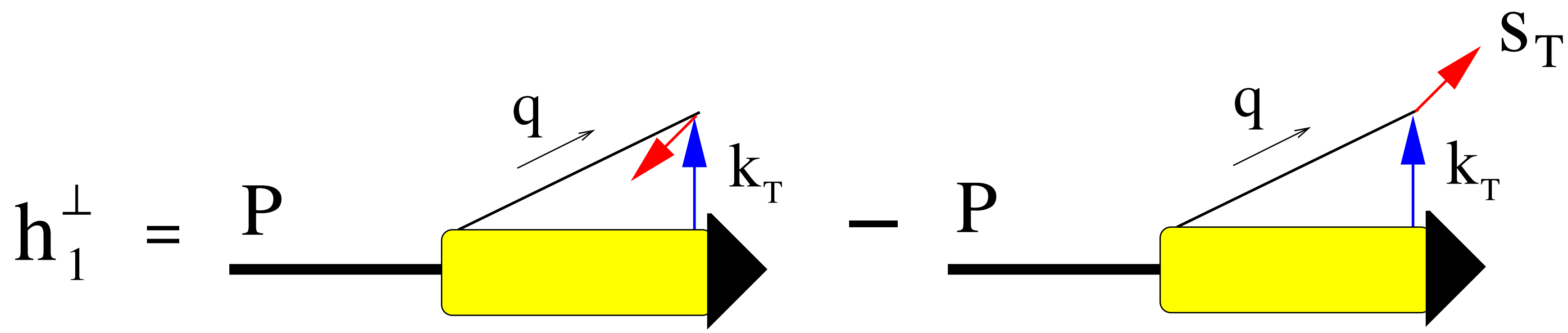

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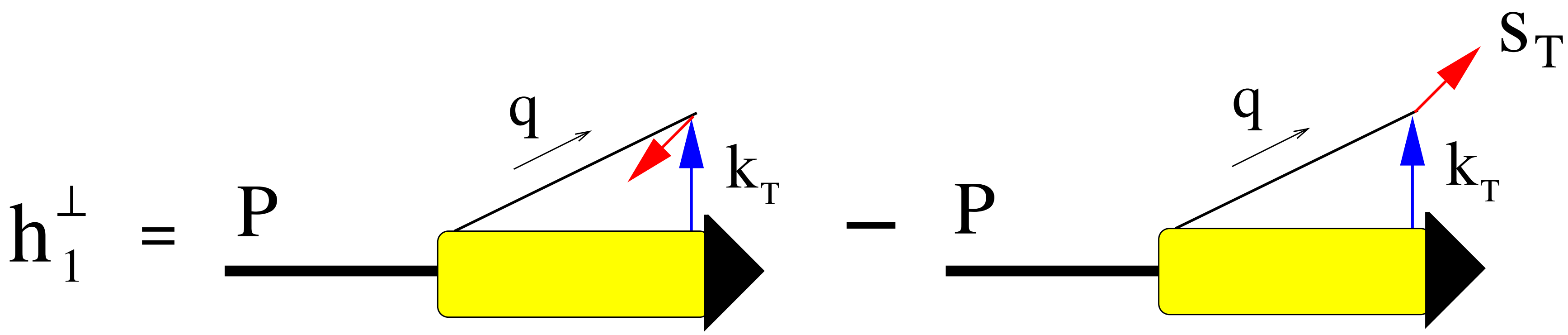

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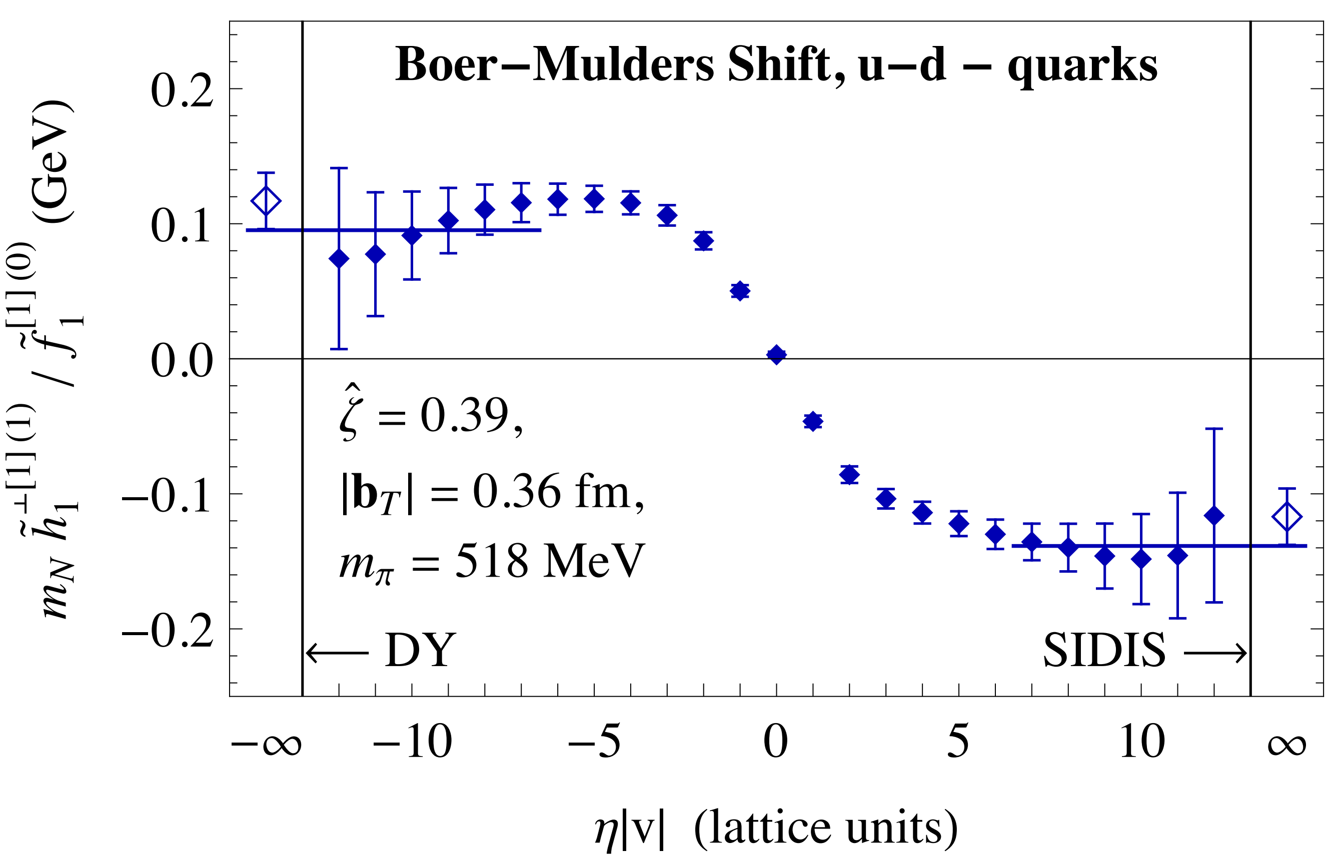

$\langle k_T \times s_T \rangle(n, b_T)$, the average transverse momentum shift orthogonal to a given transverse quark polarization is clearly nonzero inside an unpolarized proton [Musch *et al.*, '11, Engelhardt *et al.* '14]

Conclusion: unpolarized protons have an asymmetric quark spin structure which would vanish without gauge links