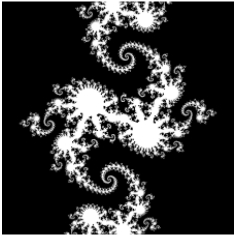
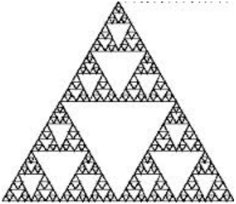
Fractal Aspects of Hadronic Interaction

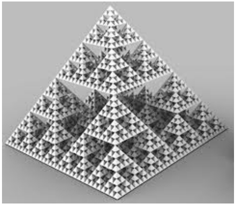
Airton Deppman

ISMD2016 - Jeju Island - S. Korea (Aug 29 - Sep 04 , 2016)

What are fractals?







Complex patterns obtained from simple rules repeated many times

Intermittency

Normalized Moments:

R.Hwa
PRD41 (1990) 1456

$$C_q = \sum_{k_0}^{\infty} k^q P_k / \left(\sum_{k_0}^{\infty} k P_k \right)^q = \delta^{\tau(q)}$$

$$P_k^q = (Q_k/N)^q = \delta^{\alpha_q}$$

Q_k = number of events with k particles in the bin with width δ

N = total number of events

$$\tau(q) = q\alpha_q - f(\alpha_q) = (q-1)D_q$$

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fractal spectrum $\tau(q) = q\alpha_q - f(\alpha_q) = (q-1)D_q$ fractal dimension

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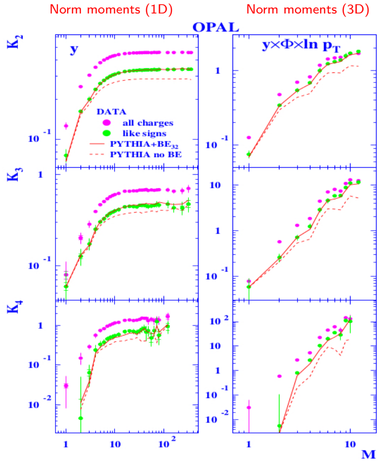
Self-similarity $\rightarrow$
- Intermittency
- Exponential growth of cummulants (integrated correlation)

N.G. Antoniou et al. PRC93, 014908 (2016)

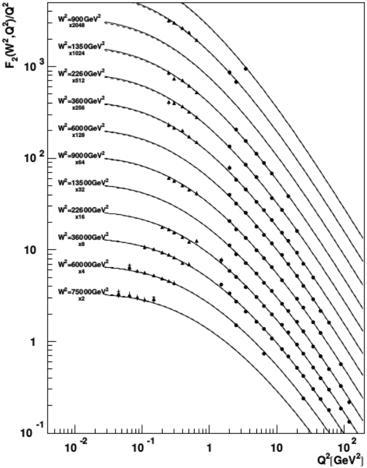
Intermittency data analysis

E. Sarkisyan: arXiv: hep-ex/0209079



Parton Distribution Function

T. Lastovicka EPJC 24(2002) 529

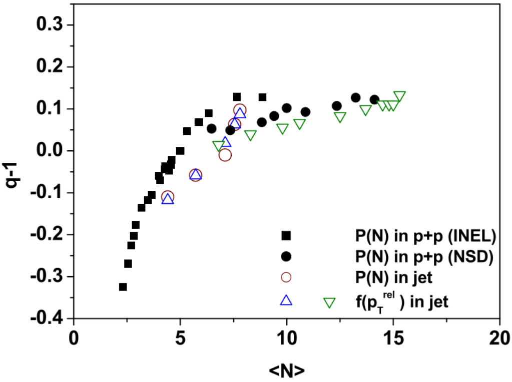


	$\mathcal{D}_0$	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	Q_0^2 [GeV^2]
all fit	0.339	0.073	1.013	−1.287	0.062
	±0.145	±0.001	±0.01	±0.01	±0.01
$\mathcal{D}_2$ fixed	0.523	0.074	*1*	−1.282	0.051
	±0.014	±0.001	const.	±0.01	±0.002

$$logf_i(x, Q^2) = D_1 log(1/x) log(1 + Q^2/Q_o^2) + D_2 log(1/x) + D_3 log(1 + Q^2/Q_o^2) + D_o^i$$

Self-similarity in experimental data

Wilk & Wlodarczyk PLB 727 (2013) 163–167



Fireball and hadron definitions

Hagedorn's defintion for fireball

> A fireball is:
>
> ⟶ *a statistical equilibrium (hadronic black-body radiation) of an undetermined number of all kinds of fireballs, each of which, in turn, is considered to be* —

> The model we wish to focus on in this paper is the *bootstrap model of hadrons*, in which the hadrons are assumed to be compounds of hadrons. The model can be represented schematically by

Frautischi's defintion for hadrons:

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From this definition Hagedorn developed the thermodynamics of fireballs

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Frautischi's defintion for hadrons:

Early consequences of Hagedorn's theory

T_H limiting value (Hagedorn's temperature)

$\rho(m) \propto m^{-5/2} e^{-\beta_o m}$ (hadron mass spectrum)

Exponential behaviour of high p_T distribution

T_H as a critical temperature (quark-gluon plasma)

HRG models - sucessful in describing many features of HEP

Early consequences of Hagedorn's theory

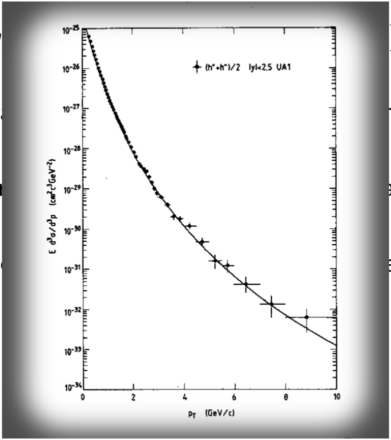
T_H limiting value (Hagedorn's temperature)

$\rho(m) \propto m$ …)

Exponenti… n

T_H as a cr… asma)

HRG mod… eatures of HEP



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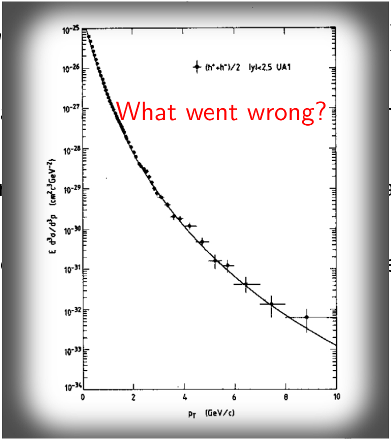


Thermofractal - definition

1. The total energy is given by

$$U = F + E\,,$$

   The number of subsystem in N for all thermofractals.
2. $\langle E\rangle/\langle F\rangle$ is constant for all the subsystems. $E/F \rightarrow \tilde{P}(E/F)$.
3. At some point n of the hierarchy of subsystems the phase space is so narrow that one can consider

$$\tilde{P}(E_n)dE_n = \rho dE_n\,,$$

   with ρ being independent of the energy E_n.

Thermofractal - Thermodynamics

For an ideal gas of elementary particles (Landau):

$$P(U)dU = (kT)^{-\frac{3N}{2}} U^{\frac{3N}{2}-1} \exp\left(-\frac{U}{kT}\right) dU\,,$$

Define for a thermofractal:

$$P(U)dU = A\exp(-\alpha F/kT)DFDE$$

with

$$\alpha = 1 + \frac{\varepsilon}{NkT}$$

and

$$\varepsilon = \frac{E}{F}kT\,.$$

$$DF = F^{\frac{3N}{2}-1}dF$$

and for the internal energy it is possible to write

$$DE = \tilde{P}(E)dE\,,$$

Thermofractal - Thermodynamical potential

The thermodynamical potential is given by

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{\alpha F}{kT}\right) dF \tilde{P}(\varepsilon) d\varepsilon \,.$$

which, after integration on F results in

$$\Omega = A \int_0^\infty \left[1 + \frac{\varepsilon}{NkT}\right]^{-3N/2} \tilde{P}(\varepsilon) d\varepsilon \,.$$

Second property of thermofractals (self-affine solution):

$$\ln P(U) \propto: \ln \tilde{P}(\varepsilon)$$

$$\tilde{P}(\varepsilon) = A \left[1 + \frac{\varepsilon}{NkT}\right]^{-3Nn/2}$$

Thermofractal and Tsallis

Second property of thermofractals (self-similar solution):

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{\alpha F}{kT}\right) dF [\tilde{P}(\varepsilon)]^\nu d\varepsilon \,.$$

$$P(U) := \tilde{P}(\varepsilon)$$

$$\tilde{P}(\varepsilon) = A\left[1 + \frac{\varepsilon}{NkT}\right]^{-\frac{3N}{2}\frac{1}{1-\nu}}$$

Introducing the index q by

$$q - 1 = \frac{2}{3N}(1-\nu)$$

and the effective temperature

$$\tau = \frac{2(1-\nu)}{3} T$$

$$\tilde{P}(\varepsilon) = A\left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{-\frac{1}{q-1}} ,$$

For an ideal gas of thermofractals Tsallis statistics must be used!

Nonextensive self-consistent theory

$$Z_q(V_o,T)=\int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}}dE$$

and

$$\ln[1+Z_q(V_o,T)]=\frac{V_o}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^\infty dm\int_0^\infty dp\,p^2\rho(n;m) \times[1+(q-1)\beta\sqrt{p^2+m^2}]^{-\frac{nq}{(q-1)}},$$

Self-consistency principle:

$$Z_q(V_o,T)=\int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}}dE =\exp\left\{\frac{V_o}{2\pi^2\beta^{3/2}}\int_0^\infty dm\,m^{3/2}\rho(m)[1+(q-1)\beta m]^{-\frac{1}{q-1}}\right\}-1$$

Weak constraint:

$$\ln[\sigma(E)]=\ln[\rho(m)]$$

Self-consistency solution

Self-consistency is obtained if

$$\rho(m) = \frac{\gamma}{m^{5/2}}[1 + (q_o - 1)\beta_o m]^{\frac{1}{q_o - 1}}$$

and

$$\sigma(E) = bE^a\left[1 + (q_o - 1)\beta_o E\right]^{\frac{1}{q_o - 1}}$$

Partition function:

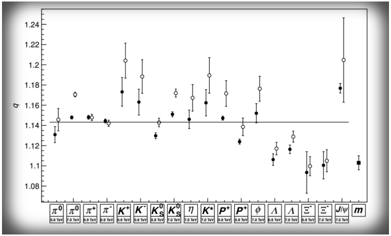
$$Z_q(V_o, T) \to b\Gamma(a+1)\left(\frac{1}{\beta - \beta_o}\right)^{a+1}$$

with

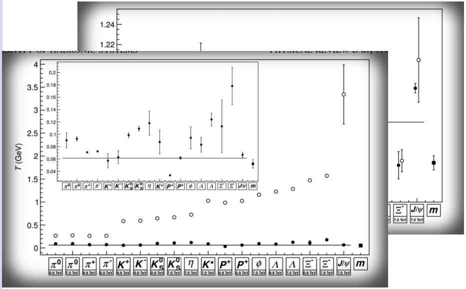
$$a + 1 = \alpha = \frac{\gamma V_o}{2\pi^2 \beta^{3/2}}$$

Limiting temperature: β_o and entropic index: q_o.

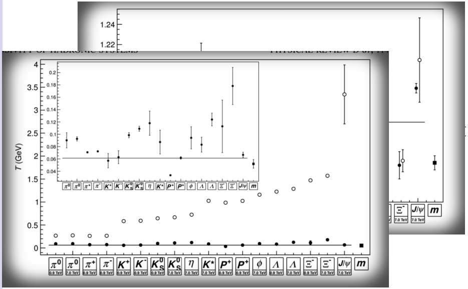
Experimental analyses

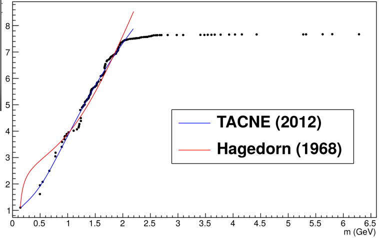


Experimental analyses



Experimental analyses





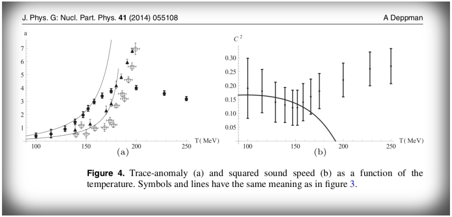
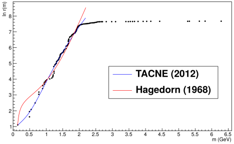

J. Phys. G: Nucl. Part. Phys. **41** (2014) 055108 A Deppman

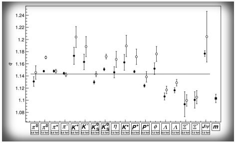
Figure 4. Trace-anomaly (a) and squared sound speed (b) as a function of the temperature. Symbols and lines have the same meaning as in figure 3.

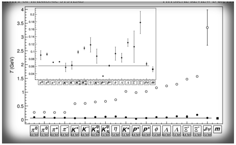
Unified description of different properties

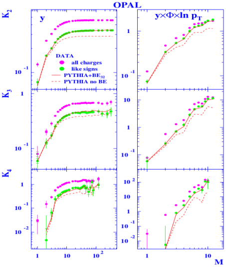
$$D = 1 + \frac{\log N'}{\log R} \qquad N = \frac{1}{(q-1)}\frac{\tau}{T}$$

$$R = \frac{(q-1)N/N'}{3-2q+(q-1)N} \qquad N' = N + 2/3$$









Unified description of different properties

with only two free parameters

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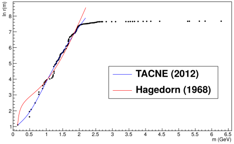
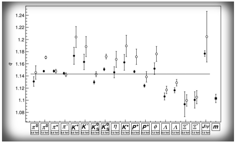
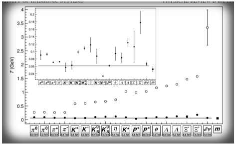
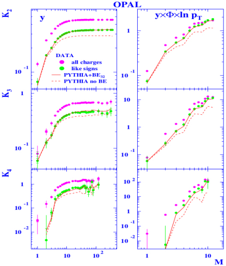








Partition function for a nonextensive ideal gas

Nonextensive thermodynamics for hadronic matter with finite chemical potentials

Eugenio Megías,[1] Débora P. Menezes,[2,3] and Airton Deppman[4]

[1] *Grup de Física Teòrica and IFAE, Departament de Física, Universitat Autònoma de Barcelona, Bellaterra E-08193 Barcelona, Spain*
email: emegias@ifae.es

[2] *Departamento de Física, CFM, Universidade Federal de Santa Catarina, CP 476, CEP 88.040-900 Florianópolis - SC - Brazil*

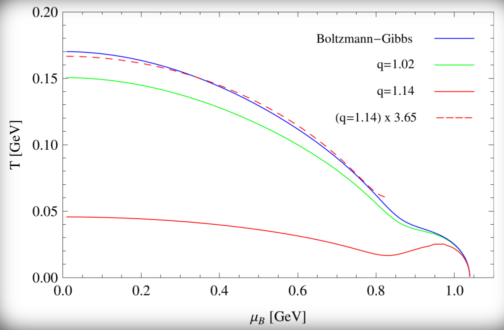
[3] *Departamento de Física Aplicada, Universidad de Alicante, Ap. Correus 99, E-03080, Alicante, Spain*
email: debora.p.m@ufsc.br

[4] *Instituto de Física, Universidade de São Paulo - Rua do Matão Travessa R Nr.187 CEP 05508-090 Cidade Universitária, São Paulo - Brasil*
email: deppman@if.usp.br

$$\log \Xi_q(V,T,\mu) = -\xi V \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \log_q^{(-r)} \left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)} \right), \quad (6)$$

EOS than that provided by the Boltzmann-Gibbs

PACS numbers: 05.70.Ce,95.30.Tg,26.60.-c



Hadronic Fractal Dimension

$q = 1.14$ and $\tau/T = 0.32$

$N = 2.3$ and $N' = 1.7$

$R = 0.104$ and $D = 0.69$

Intermittency in rapidity distribution for pp: $D = 0.43 - 0.65$

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Intermittency in rapidity distribution for pp: $D = 0.43 - 0.65$

PDF from NESCT coming soon

Microscopic origins of S_q

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}} \exp\left(-\frac{F}{kT}\right) dF \left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{\nu/(q-1)} d\varepsilon$$

$$\frac{\nu}{q-1} = \frac{1}{q-1} - \frac{3N}{2}$$

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{F}{kT}\right) dF \left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{1/(q-1)} d\varepsilon$$

$$\Omega_o = \int_0^\infty \int_0^\infty \exp\left(-\frac{F}{kT}\right) F^{3N/2} dF$$

$$\Omega = \Omega_o - \int_0^\infty A \exp\left(-\frac{F}{kT}\right) F^{\frac{3N}{2}-1} \times \left[1 - \int_0^\infty \exp\left(-(q-1)\frac{\varepsilon}{Nk\tau}\frac{F}{kT}\right)\left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{-\nu/(q-1)} d\varepsilon\right] dF$$

Microscopic origins of S_q

Dashen, Ma, Bernstein (PR 187 1969):

$$\Omega = \Omega_o - \frac{1}{4\pi\beta i}\int_0^\infty \exp(-E/kT)\left(TrS^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right)_C$$

Therefore:

$$\left(TrS^{-1}\frac{\partial}{\partial E}S\right)_C = 1-\int_0^\infty \exp\left(-\frac{(q-1)\varepsilon}{Nk\tau}\frac{F}{kT}\right)\left[1+(q-1)\frac{\varepsilon}{k\tau}\right]^{-\frac{\nu}{q-1}} d\varepsilon$$

Conclusions

1) Thermofractal structure + NESCT $\rightarrow$ unified description of p_T distribution, hadron mass spectrum, intermittency.
2) The parameters T_o and q_o are the only free parameters that needs to be obtained from experimental data.
3) It is possible that PartonDistribution Functions can be connected with the thermodynamical theory as well.
4) Contribution to the understanding of nonperturbative QCD through *S-matrix* connection.

Thank you