Design of Dipole & Quadrupole Magnets

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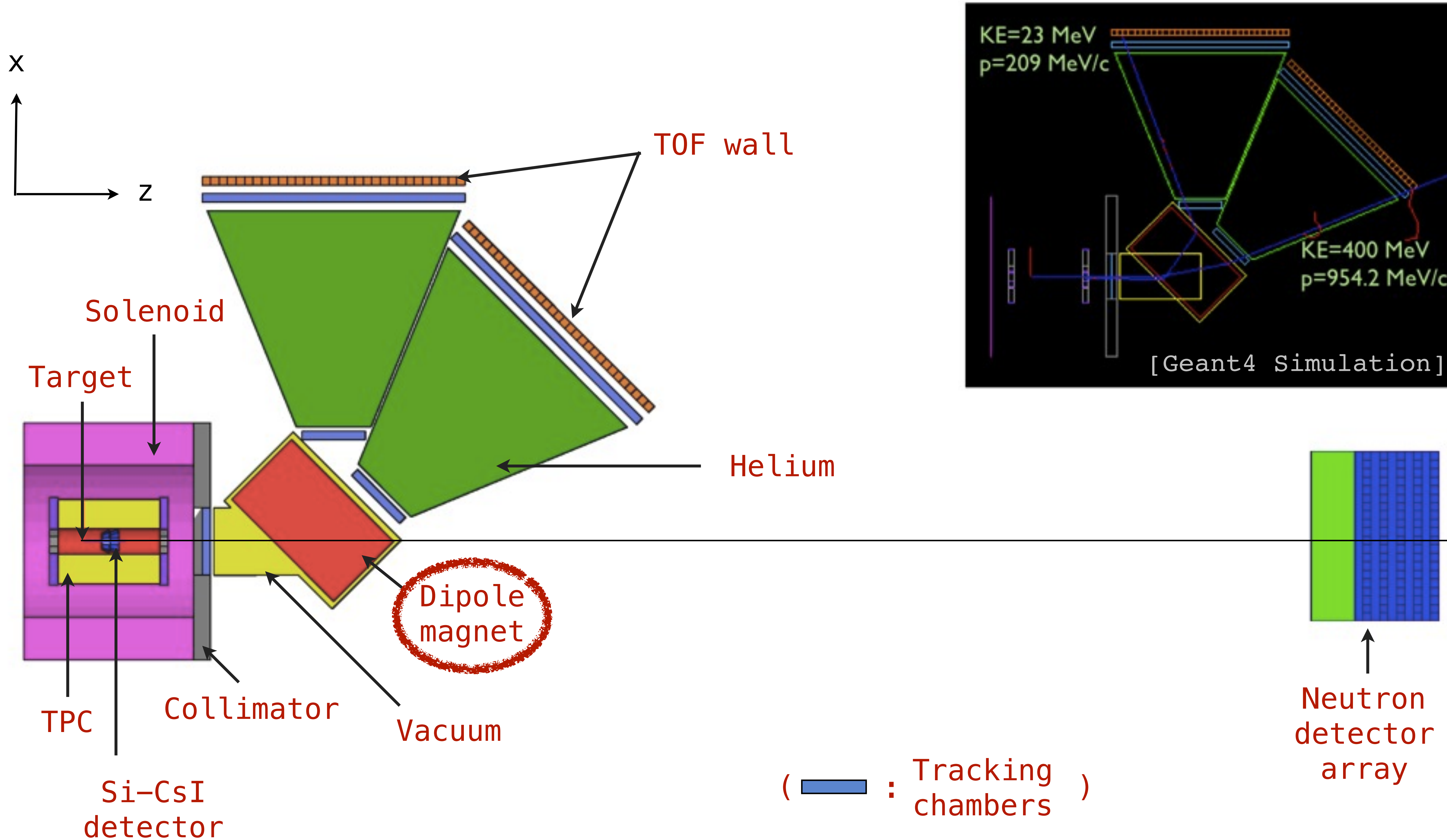
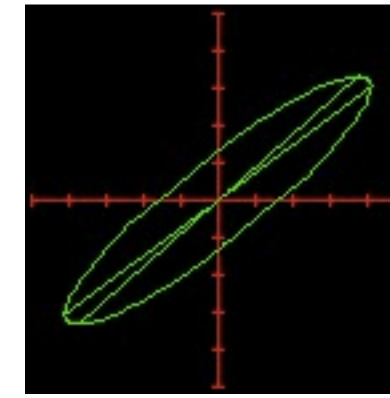
Design of LAMPS$_H$



Simulation Programs



- **TRANSPORT**

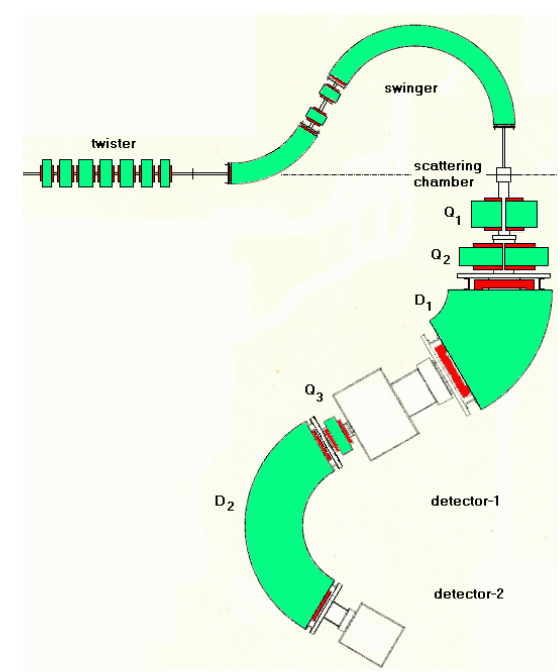
  http://aea.web.psi.ch/Urs_Rohrer/MyWeb/trans.htm

  : 1st & 2nd order matrix multiplication computer program for the design of static-magnetic beam transport system.

- **K-trace program**

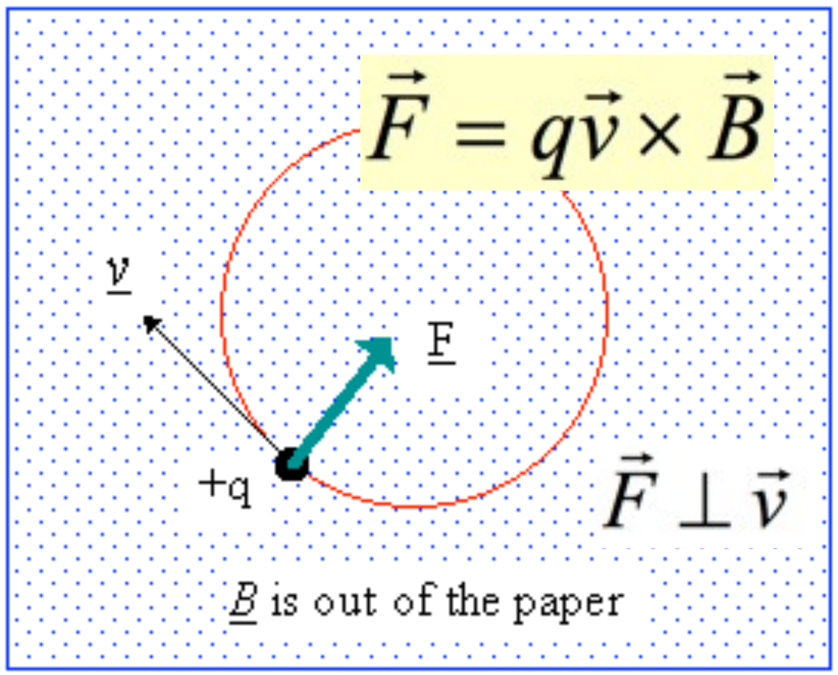
  http://www.ne.jp/asahi/kato/jsa/magnet/magnet.html

  : Magnetic spectrometer simulation program for nuclear reaction experiments



Lorentz Force

- **Charged particle moving in Magnetic field**



integer

$$F = q|e|vB = \gamma\frac{mv^2}{\rho}$$

curvature radius

- **Magnetic Rigidity [Tm]**

$$\boxed{B\rho} = \frac{\gamma mv}{q|e|} = \frac{p}{q|e|} = \frac{\boxed{p}}{0.3q}$$

momentum [GeV/c]

- Position sensitive detector ➪ gives momentum information
- Energy information from position counter
  ➪ much better than that of energy counter

MAGNETIC SPECTROMETER

- **Magnetic Spectrometer (for nuclear research)**

: select and detect the charged particles emitted in a nuclear reaction (at forward scattering angles)

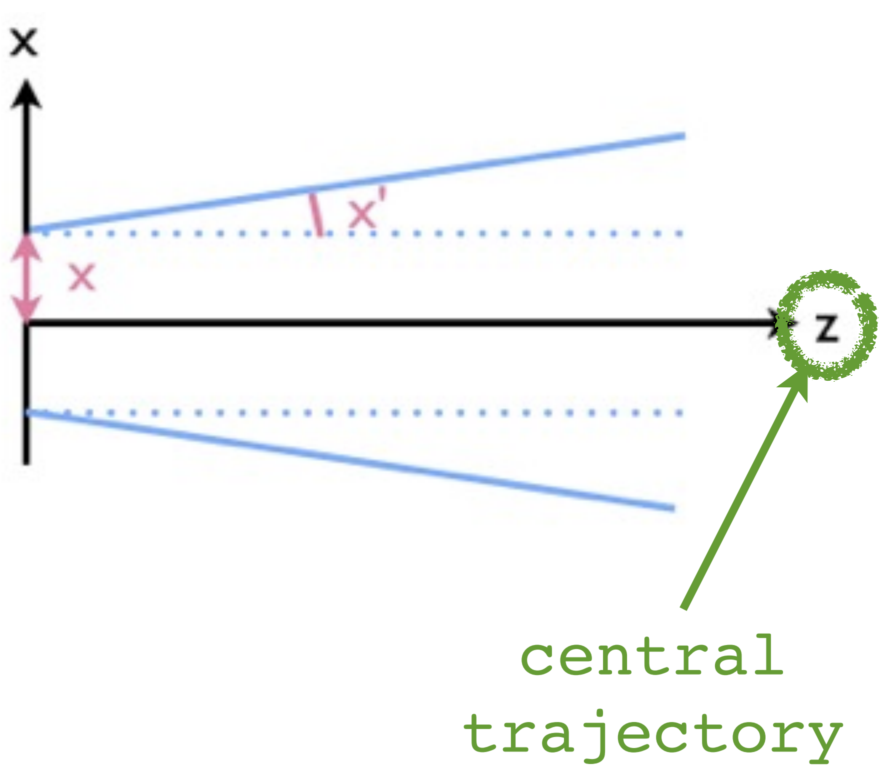
: require large acceptance detection system

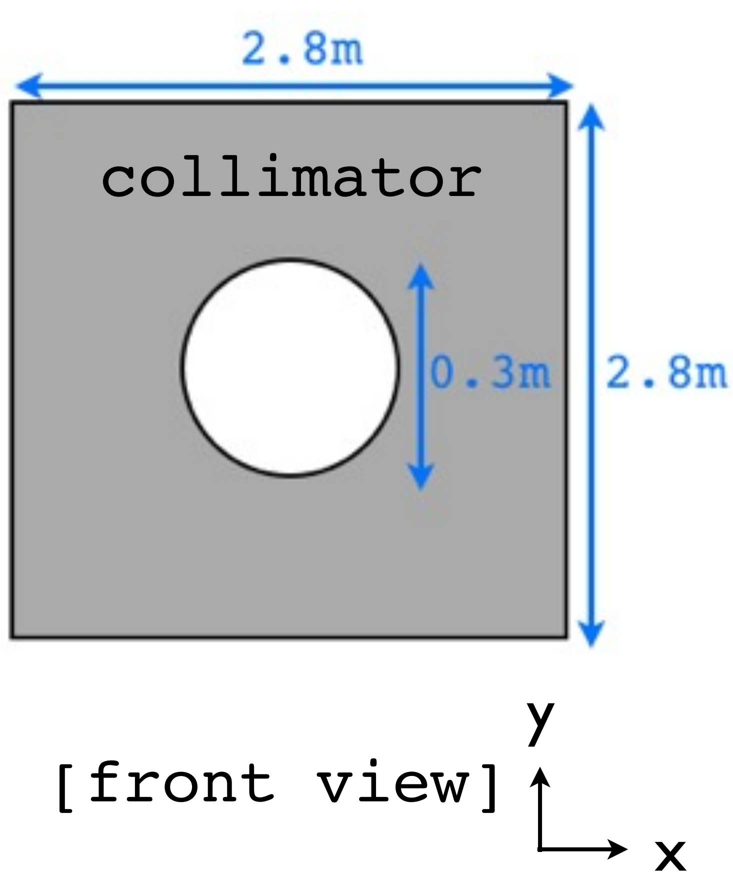
- **large momentum acceptance**
  - ➪ inversely proportional to dispersion(resolution)
- **larger angular acceptance**
  - ➪ increasing the aberration
  - ➪ deteriorating the momentum resolution

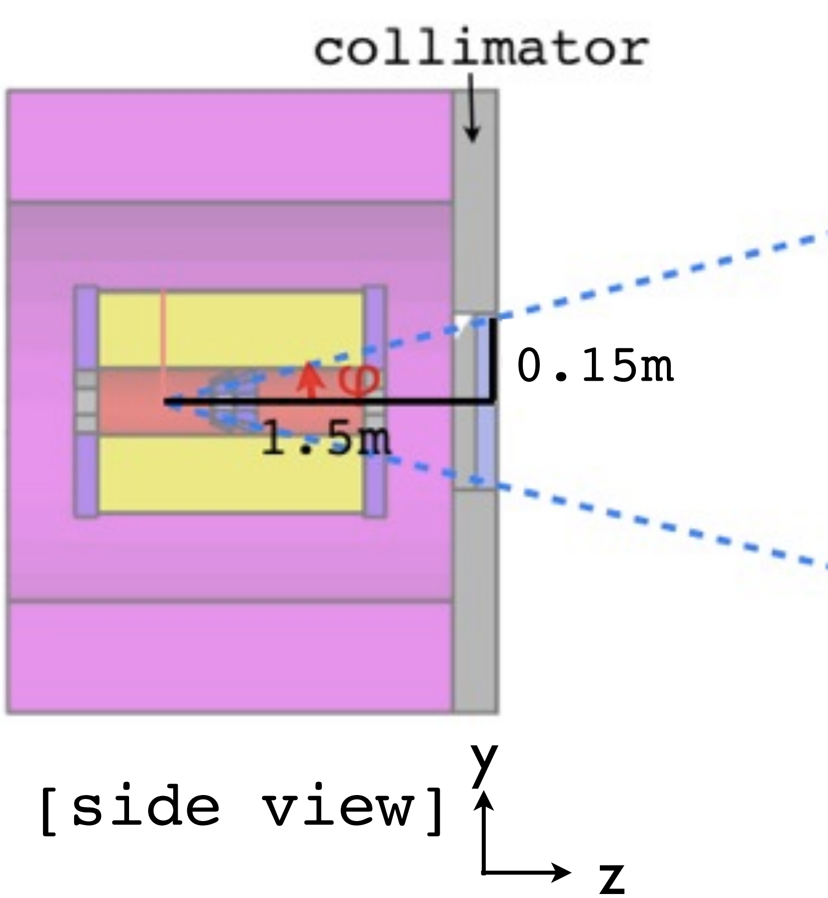
Merits	Demerits
• high resolution • particle identification • background reduction • tough	• expensive • heavy • power consumption

Beam Condition

- x : 1/2 of the horizontal beam extent = 1mm : rndm
- y : 1/2 of the vertical beam extent = 1mm : rndm
- θ : 1/2 of the horizontal beam divergence = 99.7mr
- φ : 1/2 of the horizontal beam divergence = 99.7mr
- δ : 1/2 of the momentum spread (Δp/p) = 0.1% : rndm
- p : the momentum of the central trajectory = 10-400MeV/c ?





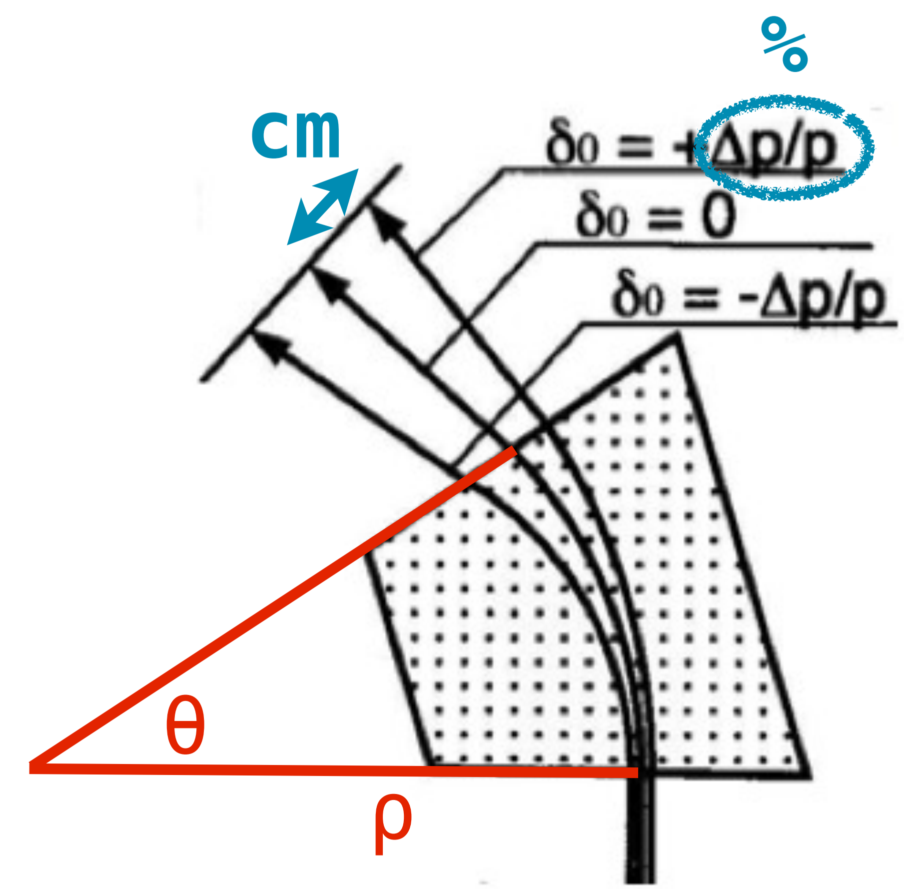


Dipole Magnet

- Change the beam direction
- Momentum separation => Dispersion [cm/%]

$$\boxed{D = \rho(1 - \cos\theta)}$$

ρ : radius of central trajectory (=1/h)
θ : deflection angle



- L = ρθ
  : effective path length
  of the central trajectory

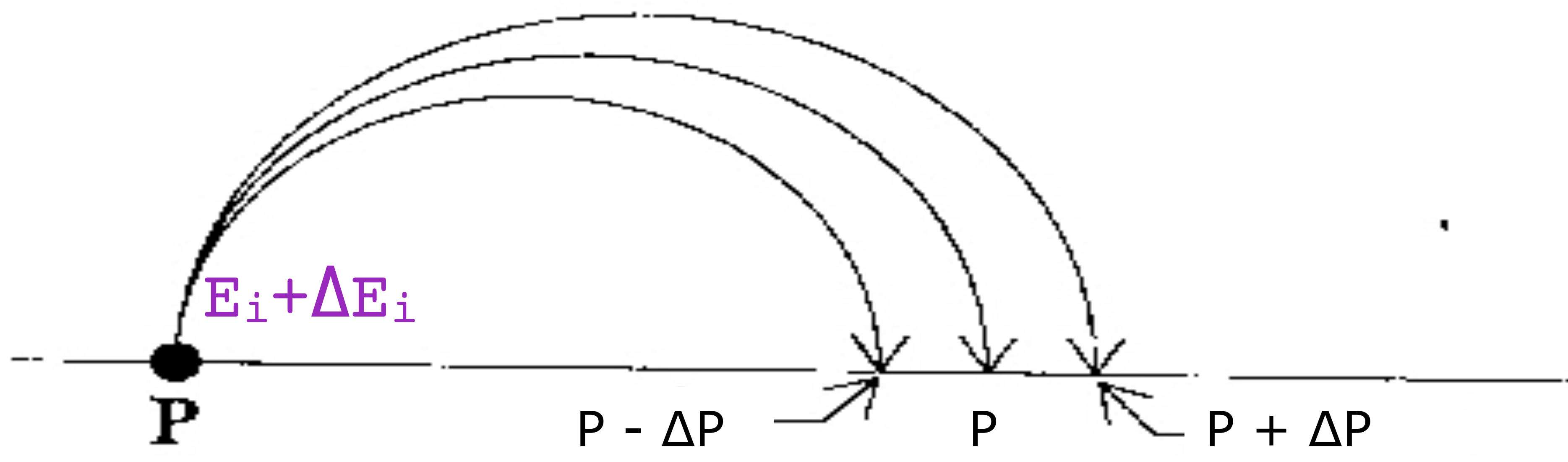
⇨ ρ & θ larger (L larger) ⇨ D larger!!

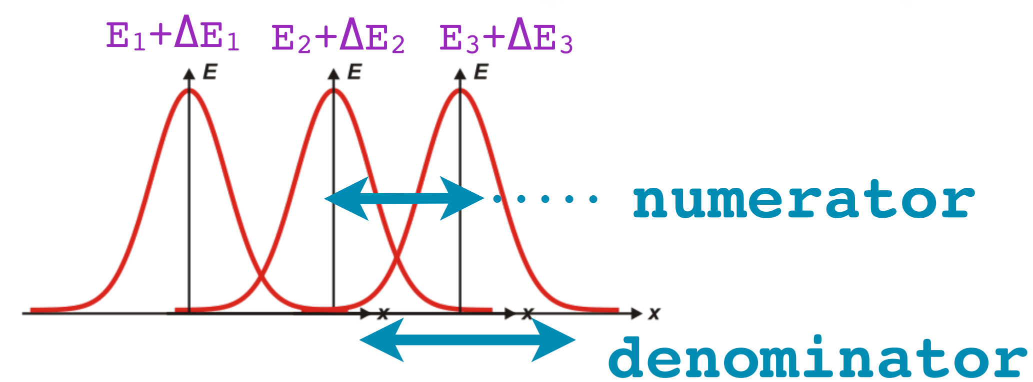
Resolving Power

- **Resolving power R**

: directly related to <u>the momentum resolution</u>

$$R = \frac{D}{2x_0 (x|x)}$$





* D : dispersion (R_{16})

* (x|x) : magnification M_x (R_{11})

$$* \ R = \frac{\text{separation at counter}}{\text{beam image size at counter}} = \frac{\text{dispersion}}{(\text{beam size})(\text{magnification}) + (\text{aberration})}$$

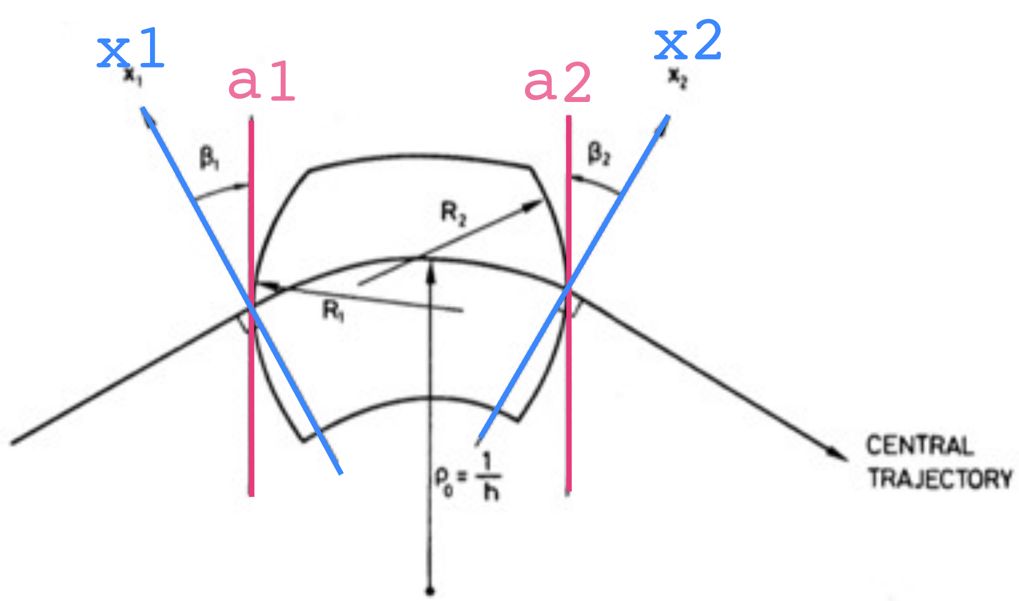
Edge Effect

- **Shim angle (additional focusing)**

  β : angle of rotation of pole face

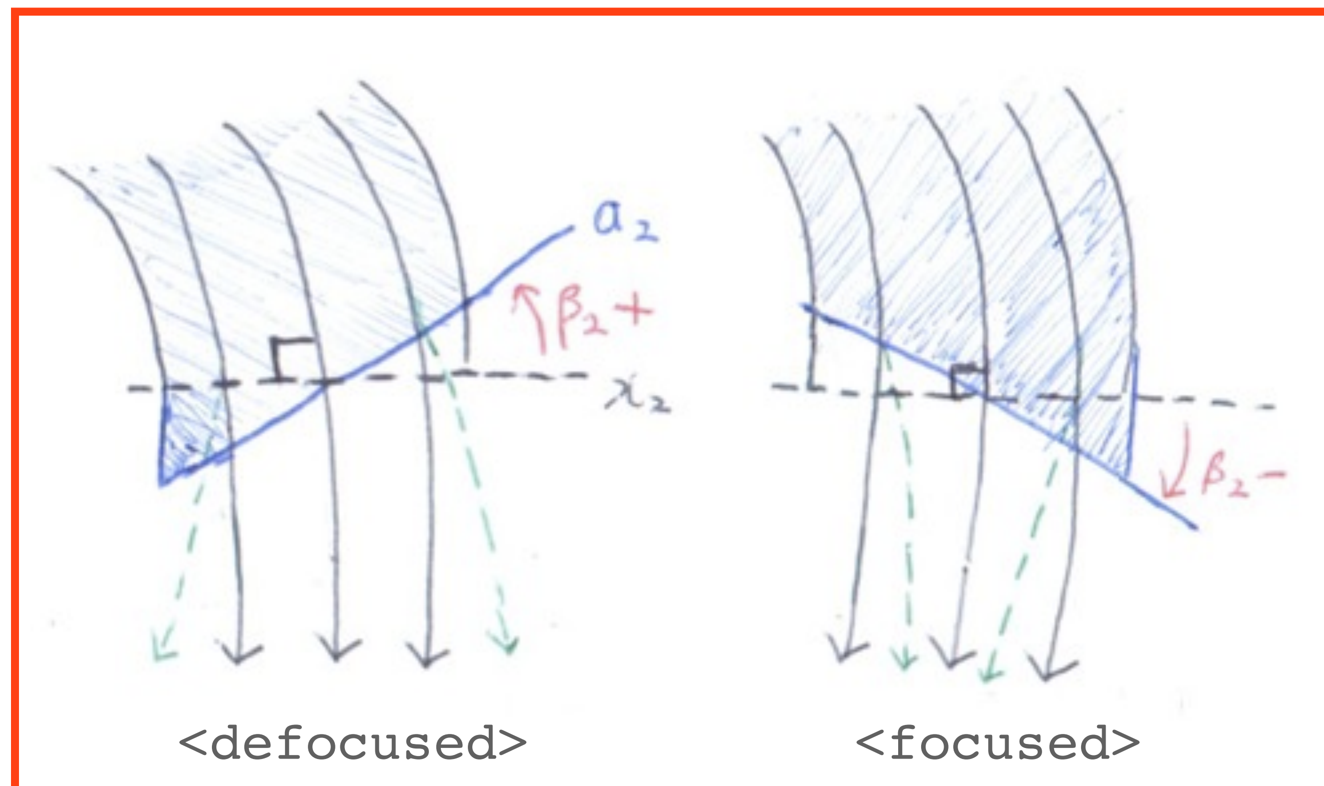
  positive value - focus in vertical(y)

  negative value - focus in horizontal(x)



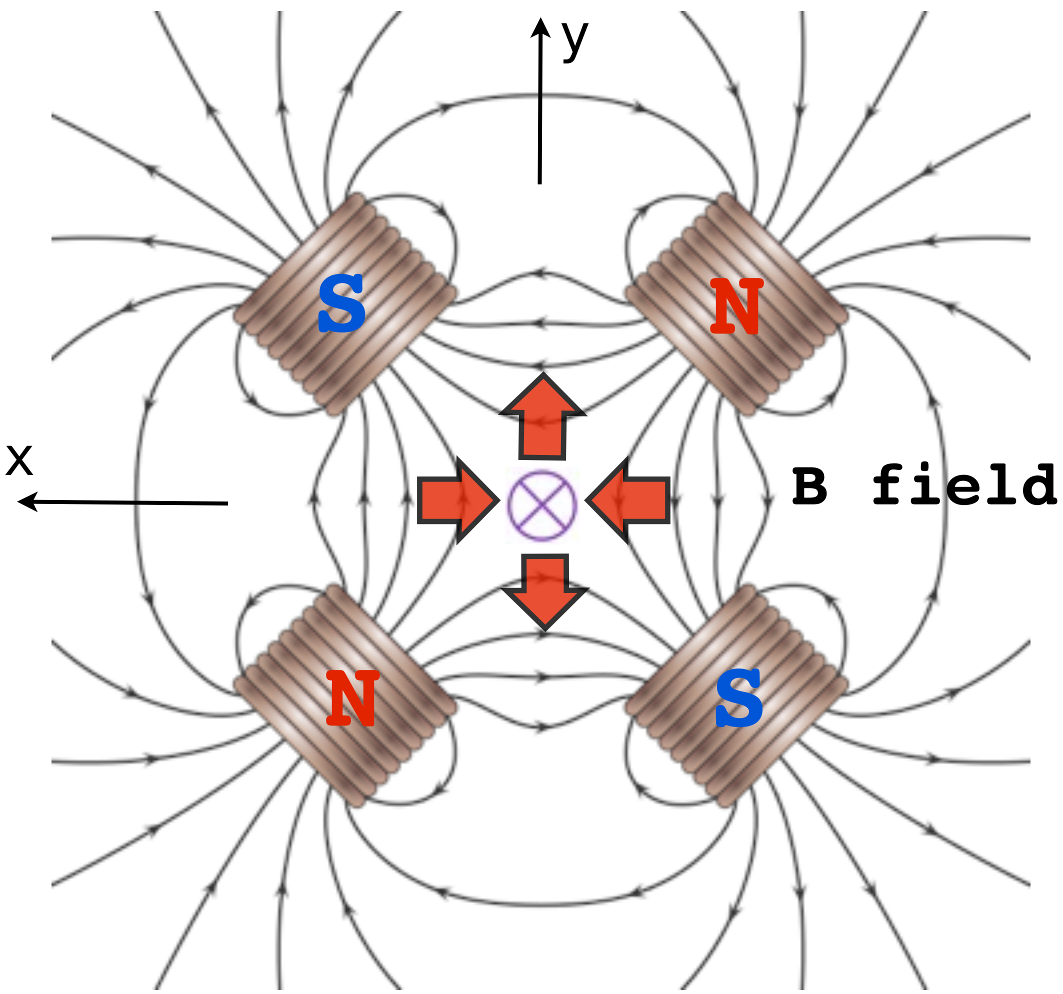
x1, x2 : perpendicular to central trajectory

a1, a2 : actual pole face of dipole magnet

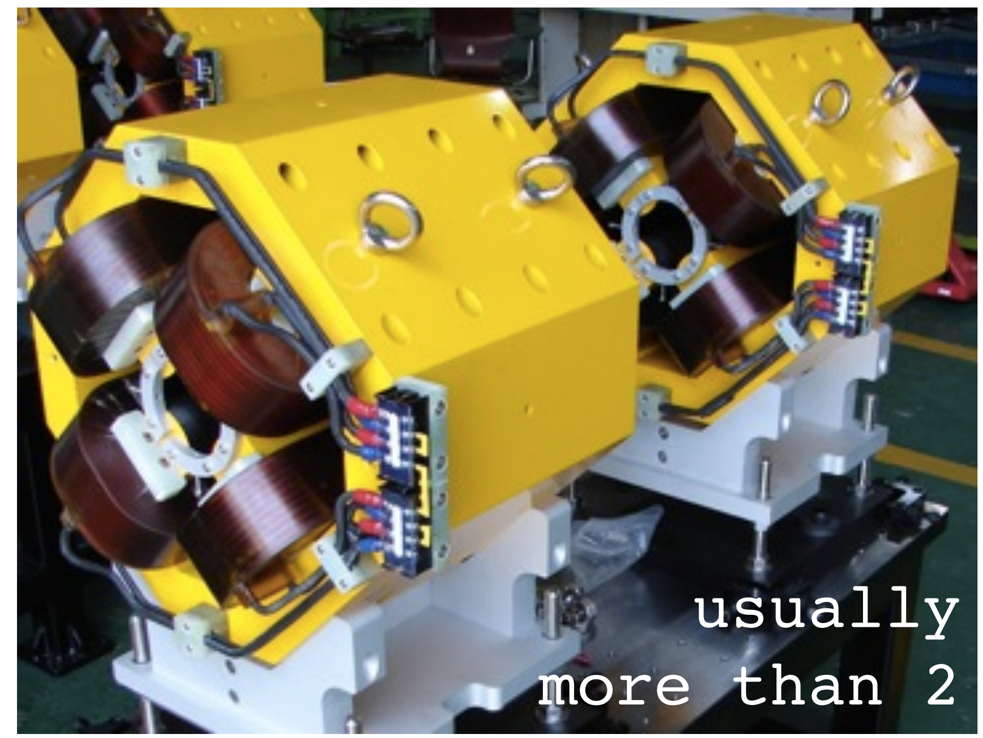


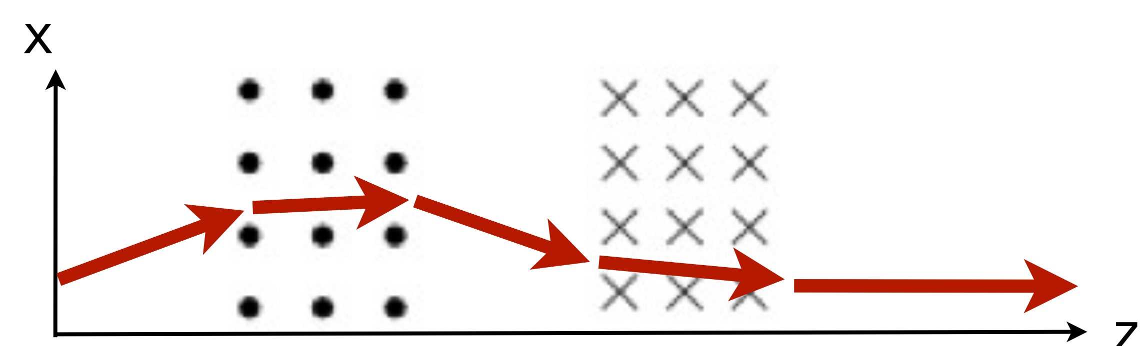
Quadrupole Magnet

- horizontally(x)-focused & vertically(y)-defocused. (vice versa)
- We can <u>not</u> focus in both directions simultaneously.



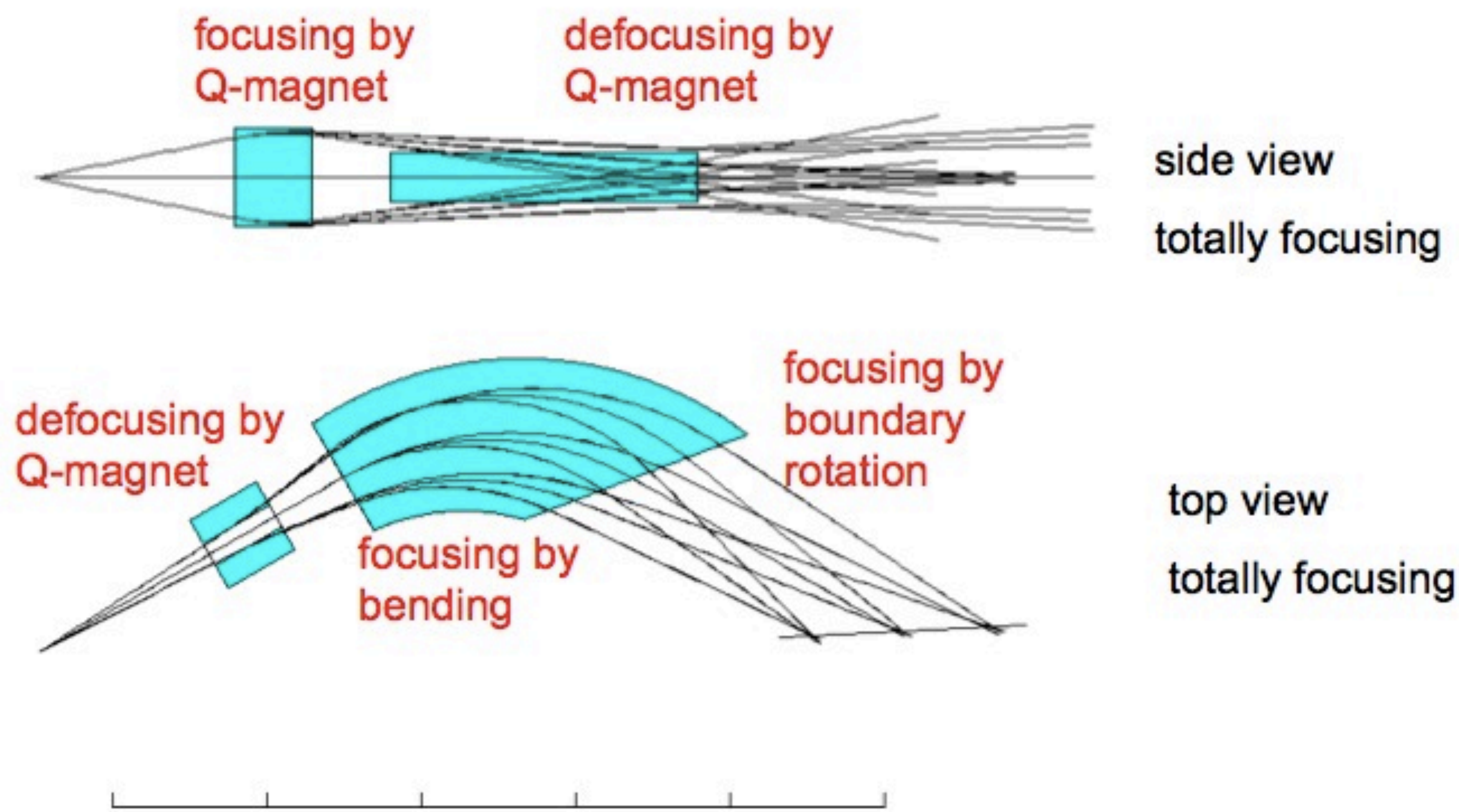
⊗ : positively charged ptl
➡ : Lorentz force





⇨ *increase angular acceptance!*

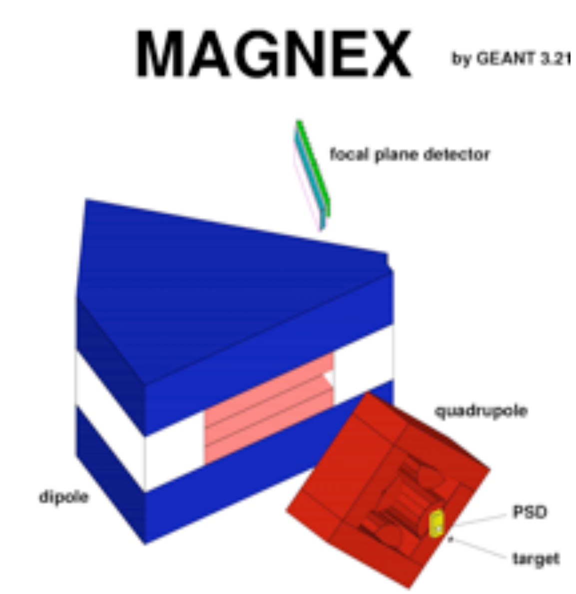
QD spectrometer



- **MAGNEX spectrometer**

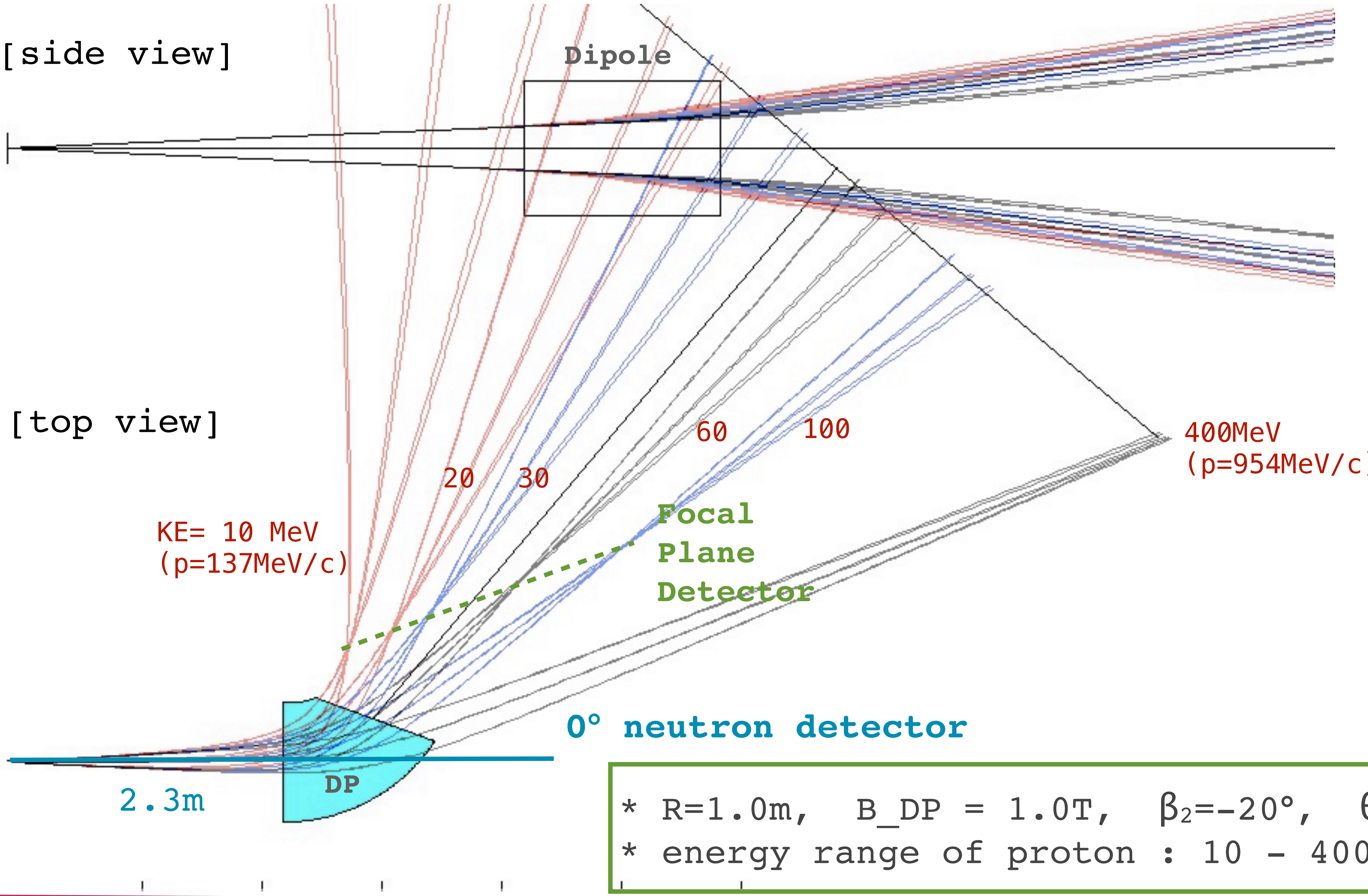
  http://www.lns.infn.it/magnex/magnex.htm

: A large acceptance MAGNetic spectrometer for EXcyt beams.
: INFN - Laboratori Nazionali del Sud, Catania, Italy
: Study of nuclear structure and reaction dynamics



One Dipole System

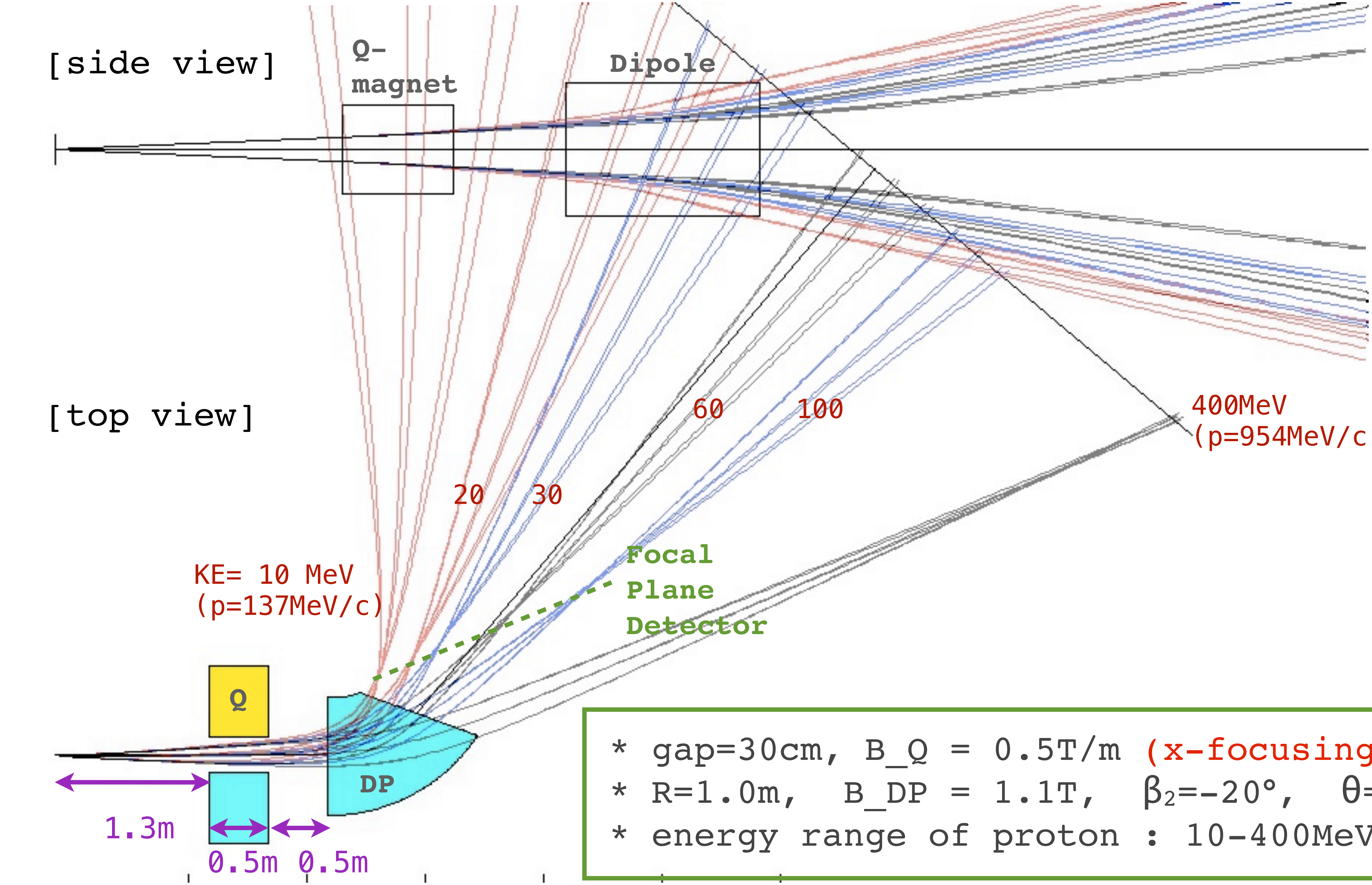
10-400MeV



* R=1.0m, B_DP = 1.0T, β_2=-20°, θ=50°
* energy range of proton : 10 - 400 MeV

10-400MeV

QD System

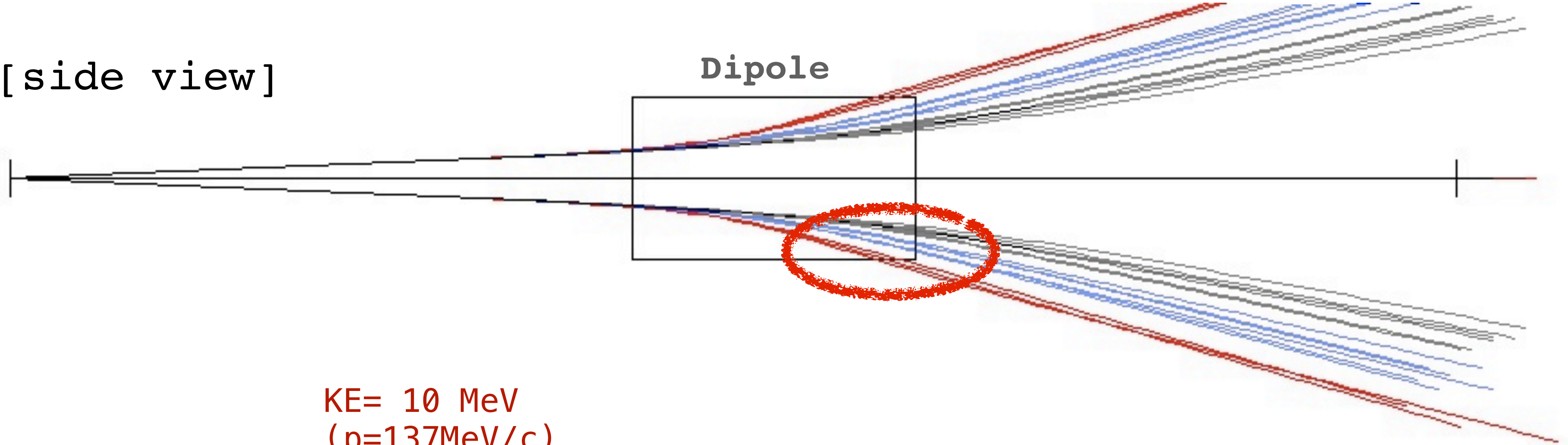


* gap=30cm, B_Q = 0.5T/m (x-focusing)
* R=1.0m, B_DP = 1.1T, β_2=-20°, θ=50°
* energy range of proton : 10-400MeV

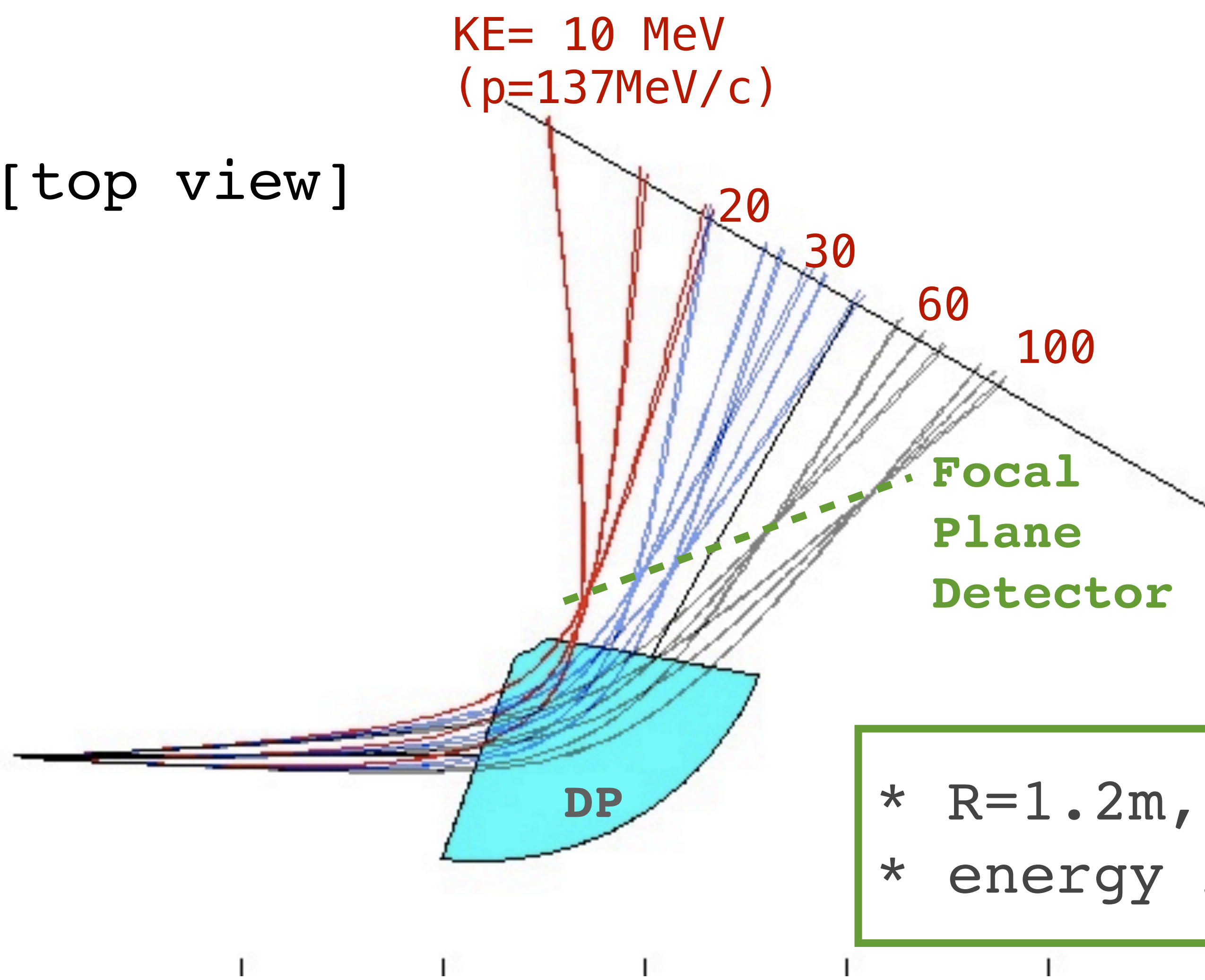
10-400MeV

One Dipole System

[side view]



[top view]

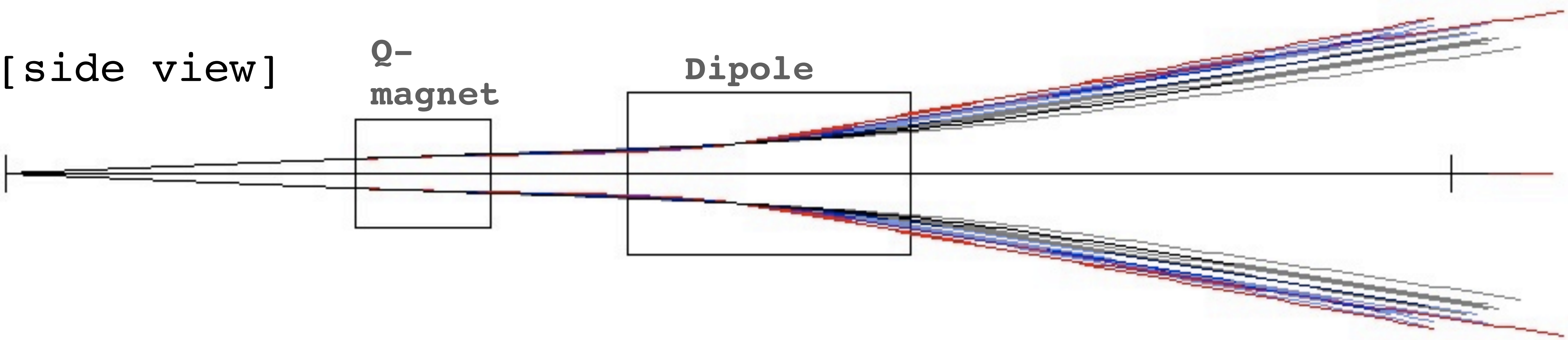


* R=1.2m, B_DP=1.0T, β_1=-20°, β_2=-20°, θ=65°
* energy range of proton : 10-100MeV

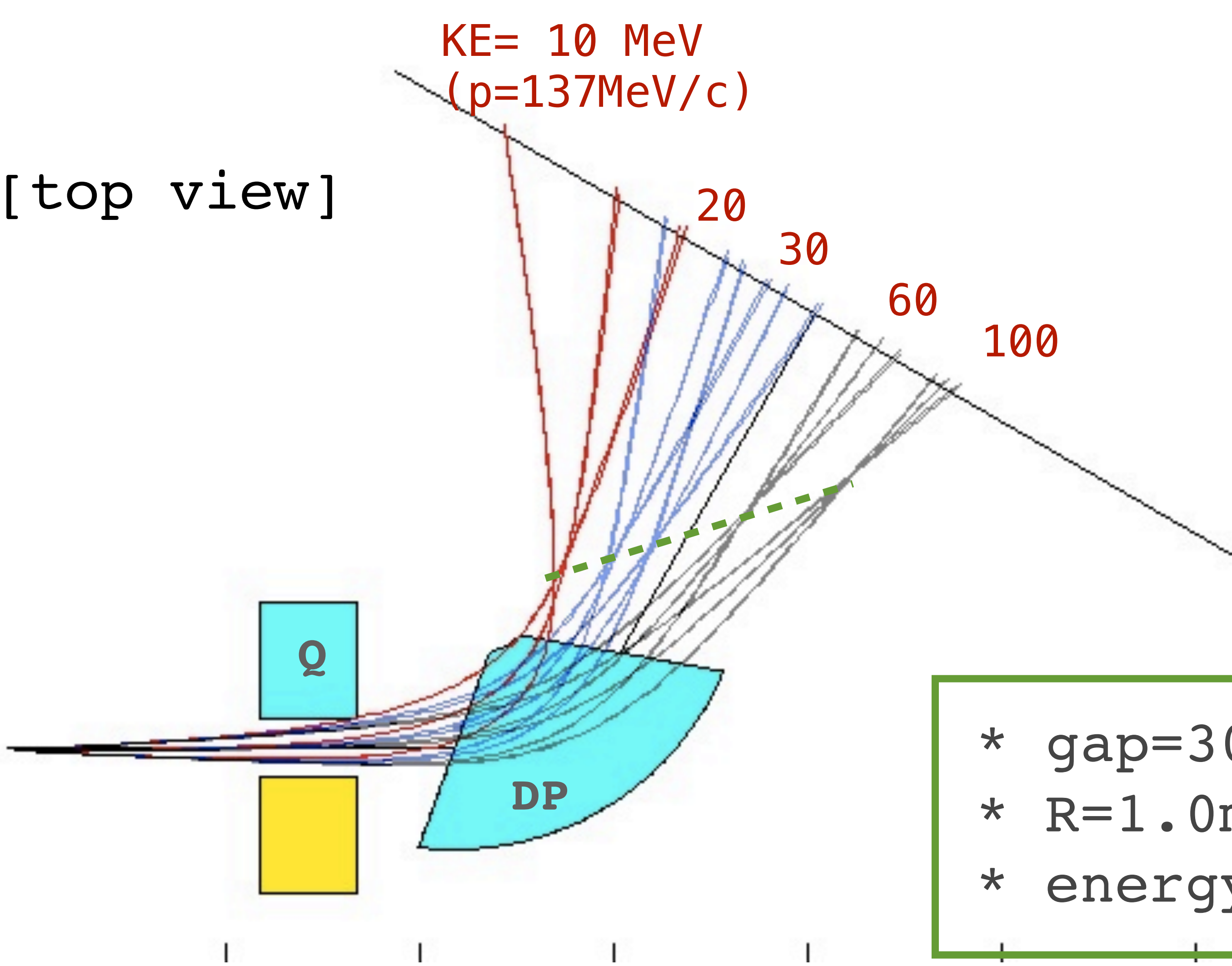
10-100MeV

QD System

[side view]



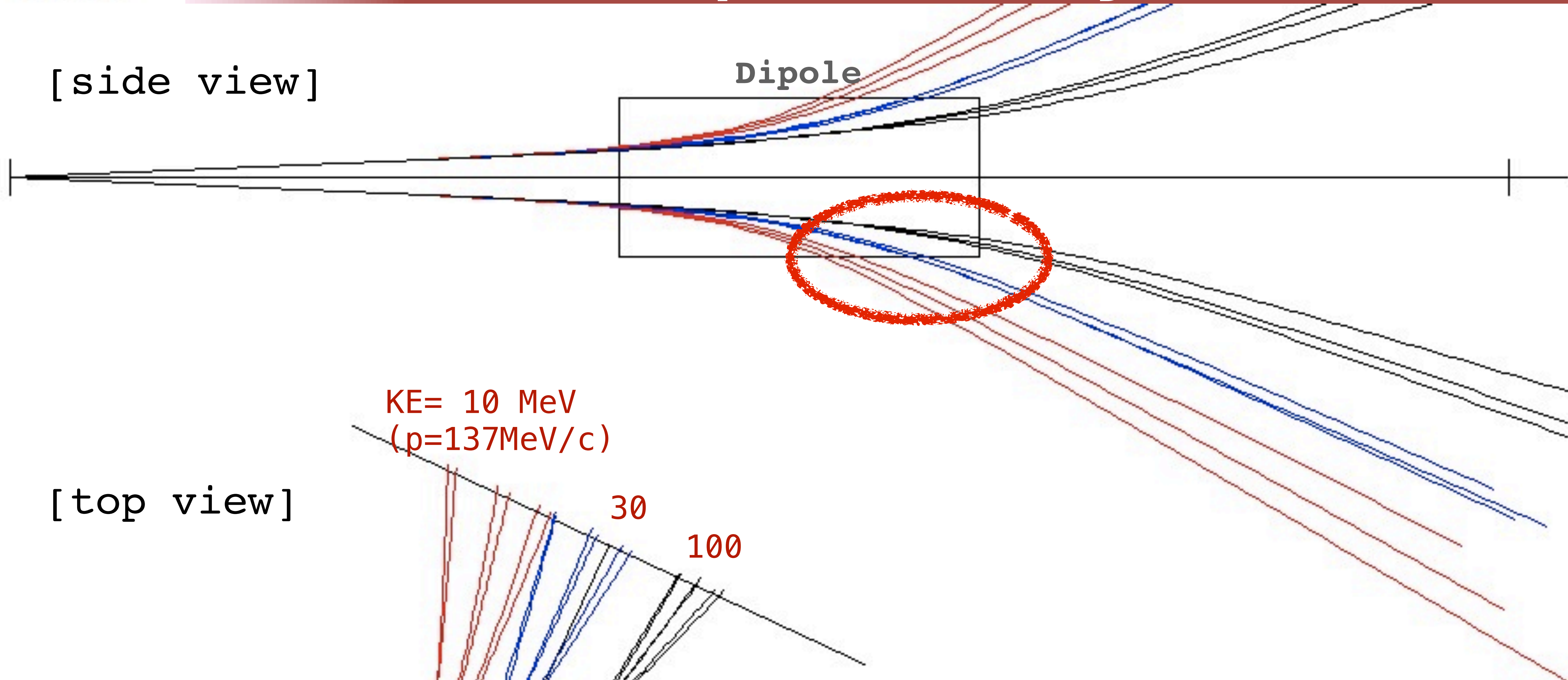
[top view]

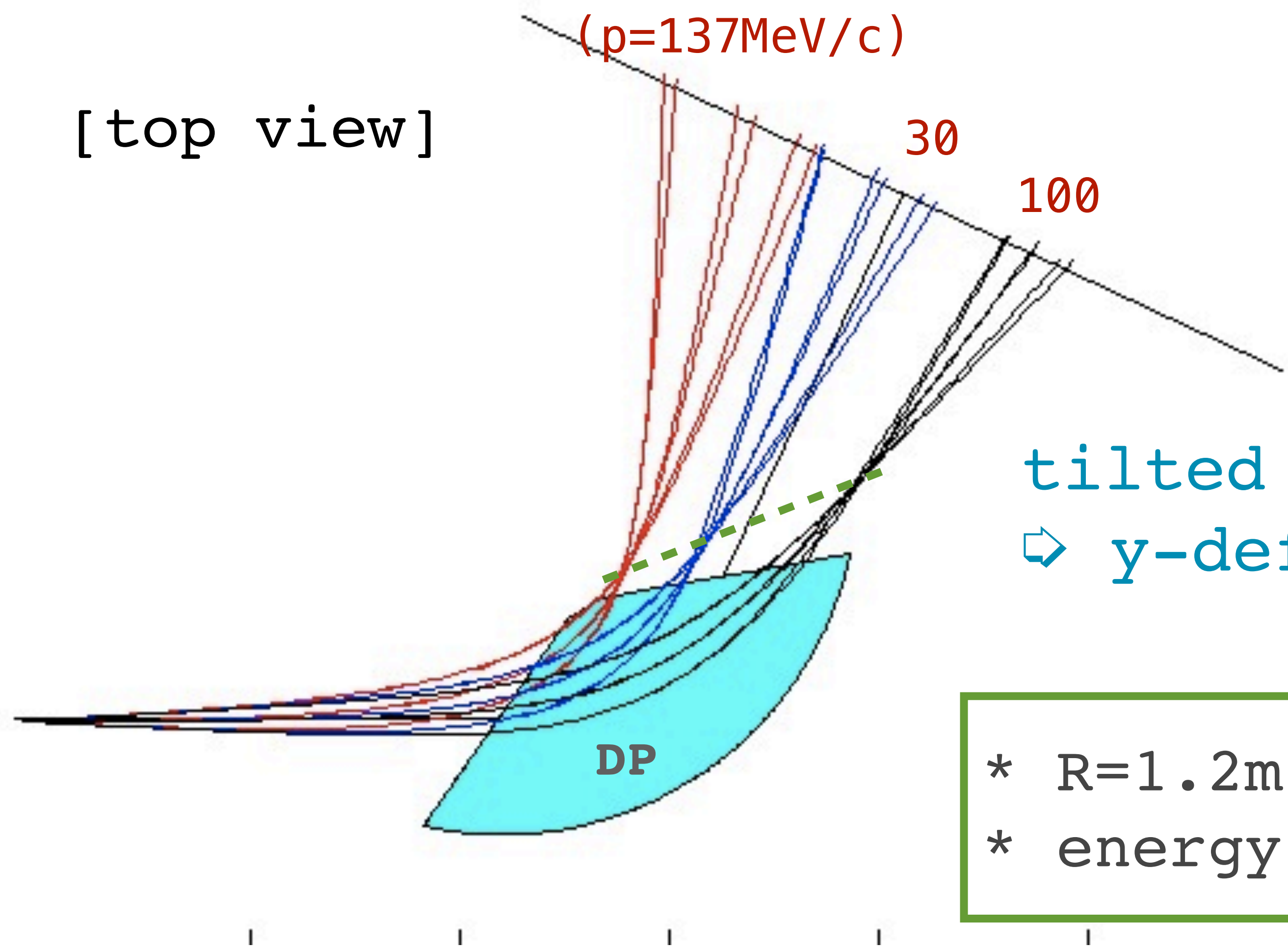


* gap=30cm, B_Q = 0.4T/m (y-focusing)
* R=1.0m, B_DP = 1.1T, $\beta_1=\beta_2=-20°$, $\theta=50°$
* energy range of proton : 10-100MeV

large shim angle

One Dipole System





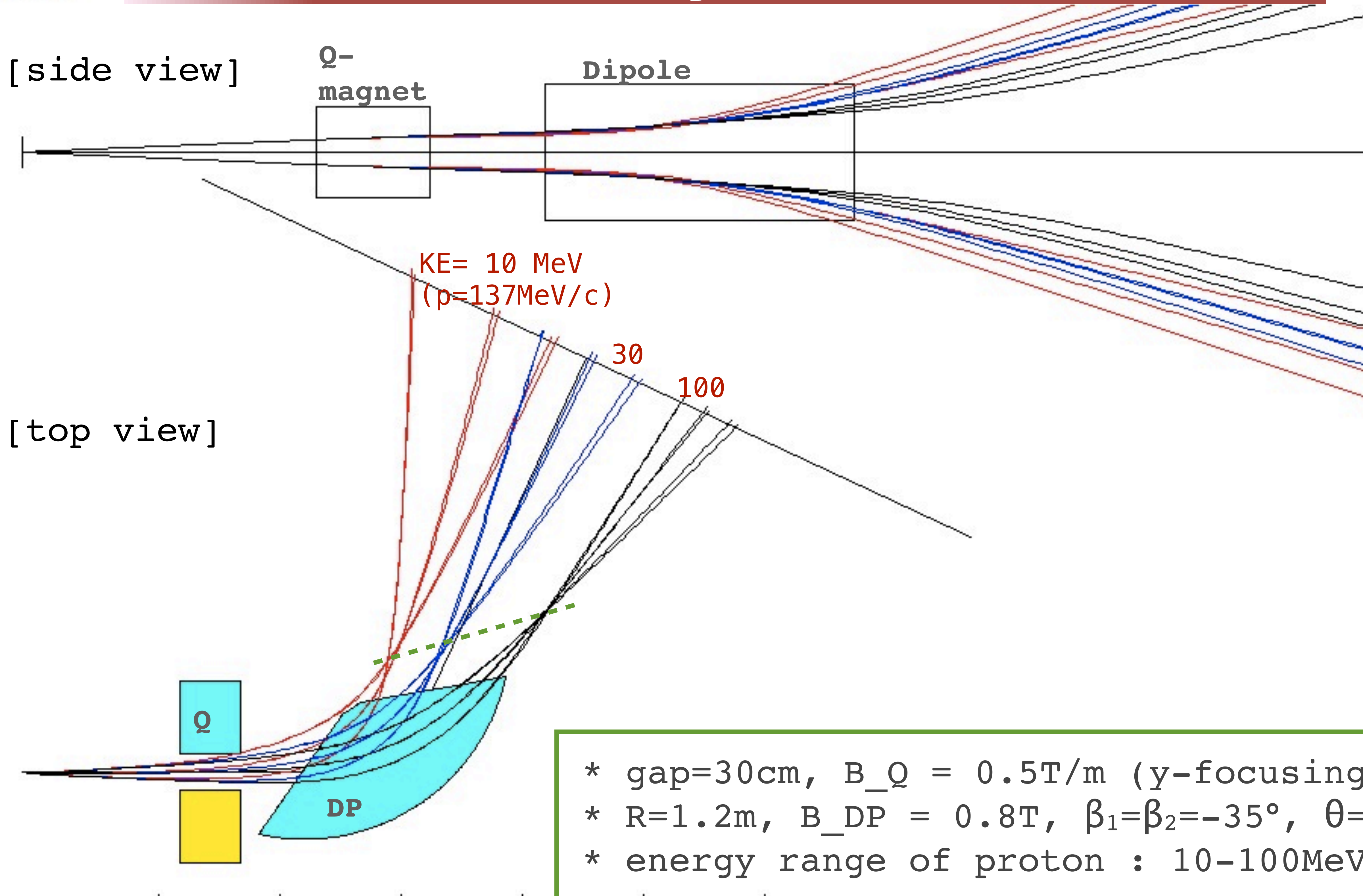
tilted angle smaller

➪ y-defocusing effect larger

* R=1.2m, B_DP=0.9T, β_1=-35°, β_2=-35°, θ=65°
* energy range of proton : 10-100MeV

large shim angle

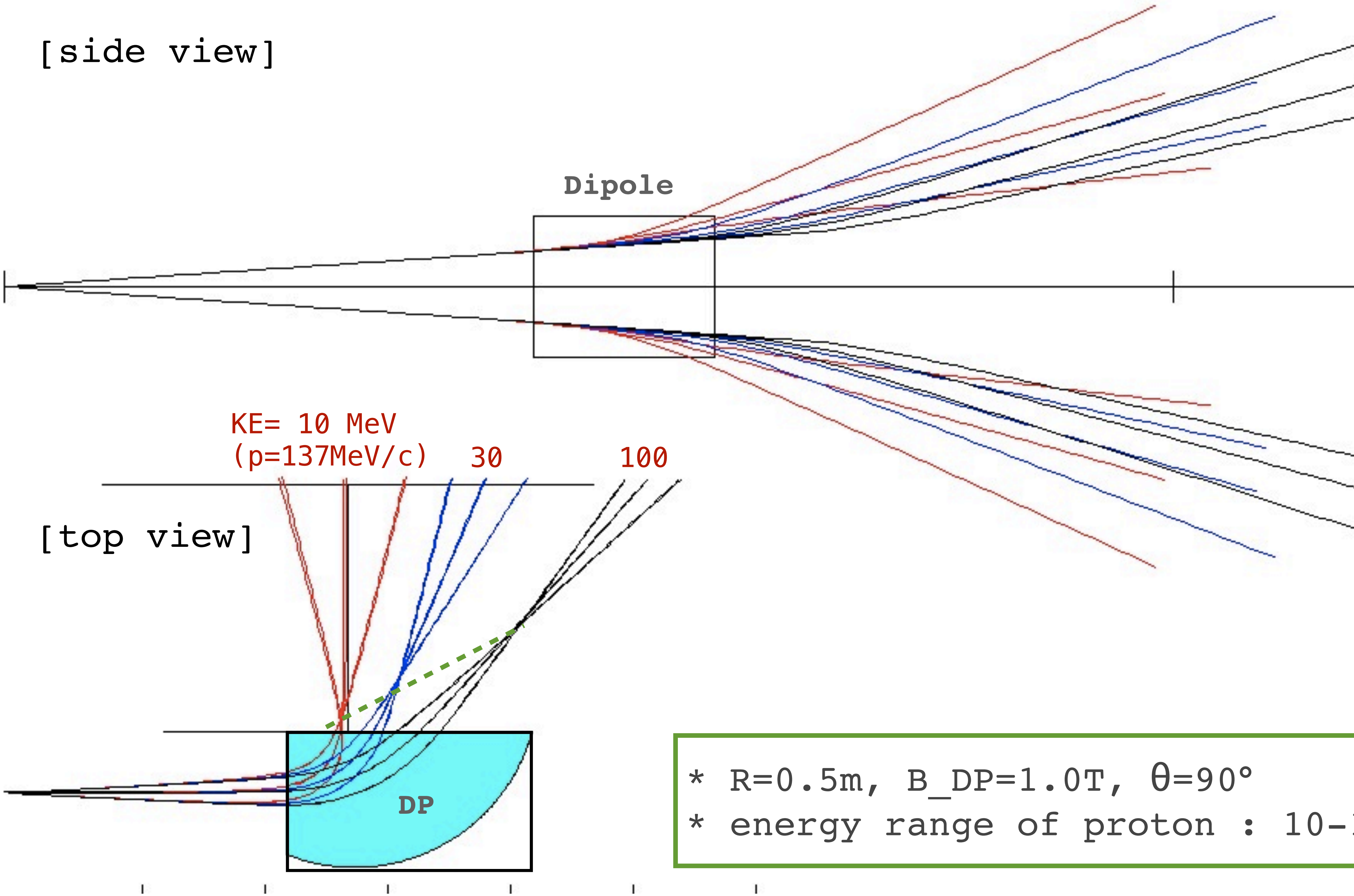
QD System



* gap=30cm, B_Q = 0.5T/m (y-focusing)
* R=1.2m, B_DP = 0.8T, $\beta_1=\beta_2=-35°$, $\theta=65°$
* energy range of proton : 10-100MeV

square-shaped

One Dipole System

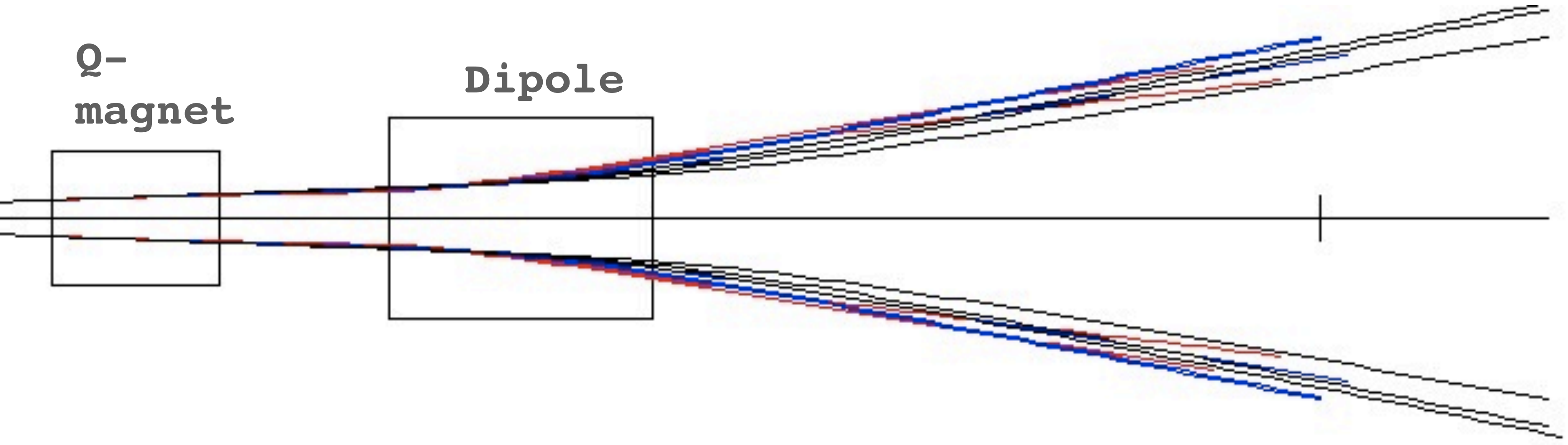


* R=0.5m, B_DP=1.0T, θ=90°
* energy range of proton : 10-100MeV

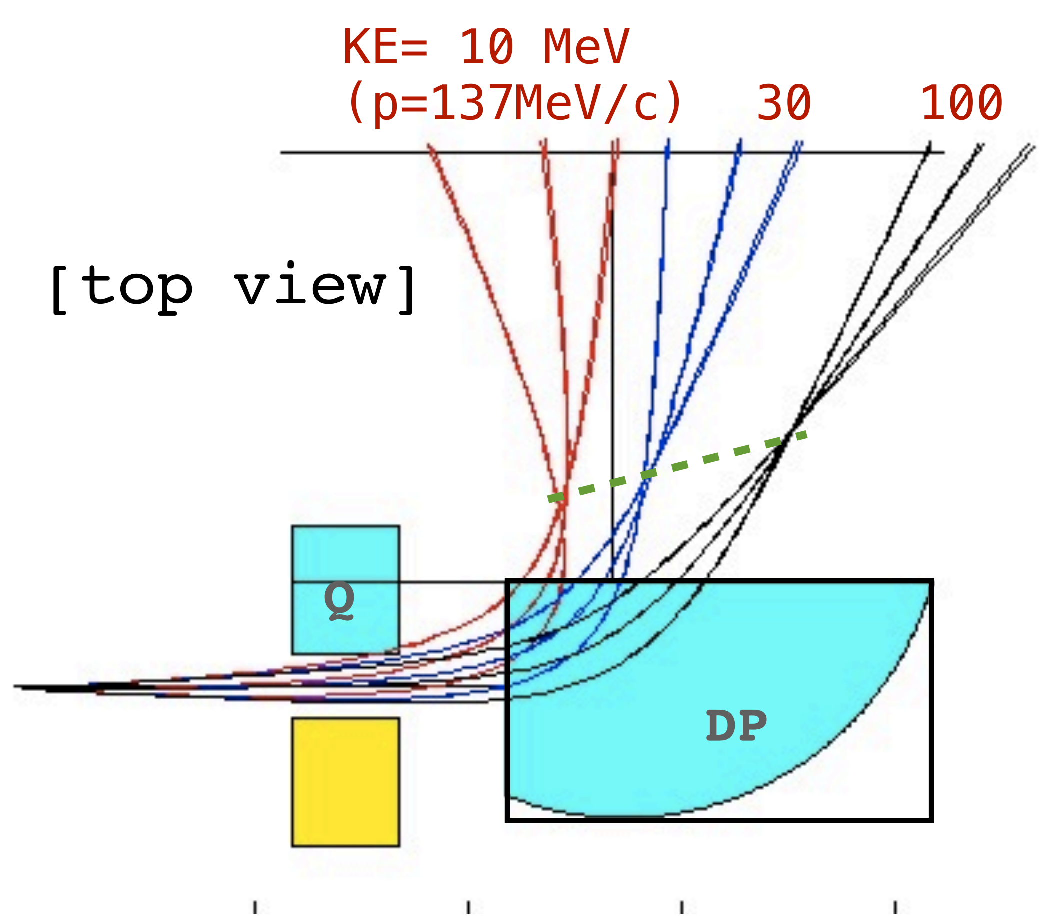
square-shaped

One Dipole System

[side view]



[top view]



* gap=30cm, B_Q = 0.3T/m (y-focusing)
* R=0.5m, B_DP=1.0T, θ=90°
* energy range of proton : 10-100MeV

SUMMARY & FUTURE PLANS

- **smaller momentum acceptance**

  : KE range 10-100MeV is still too broad

  : up to 50MeV or even smaller (max ~30%?)

- **various designs**

  : Not only D, QD

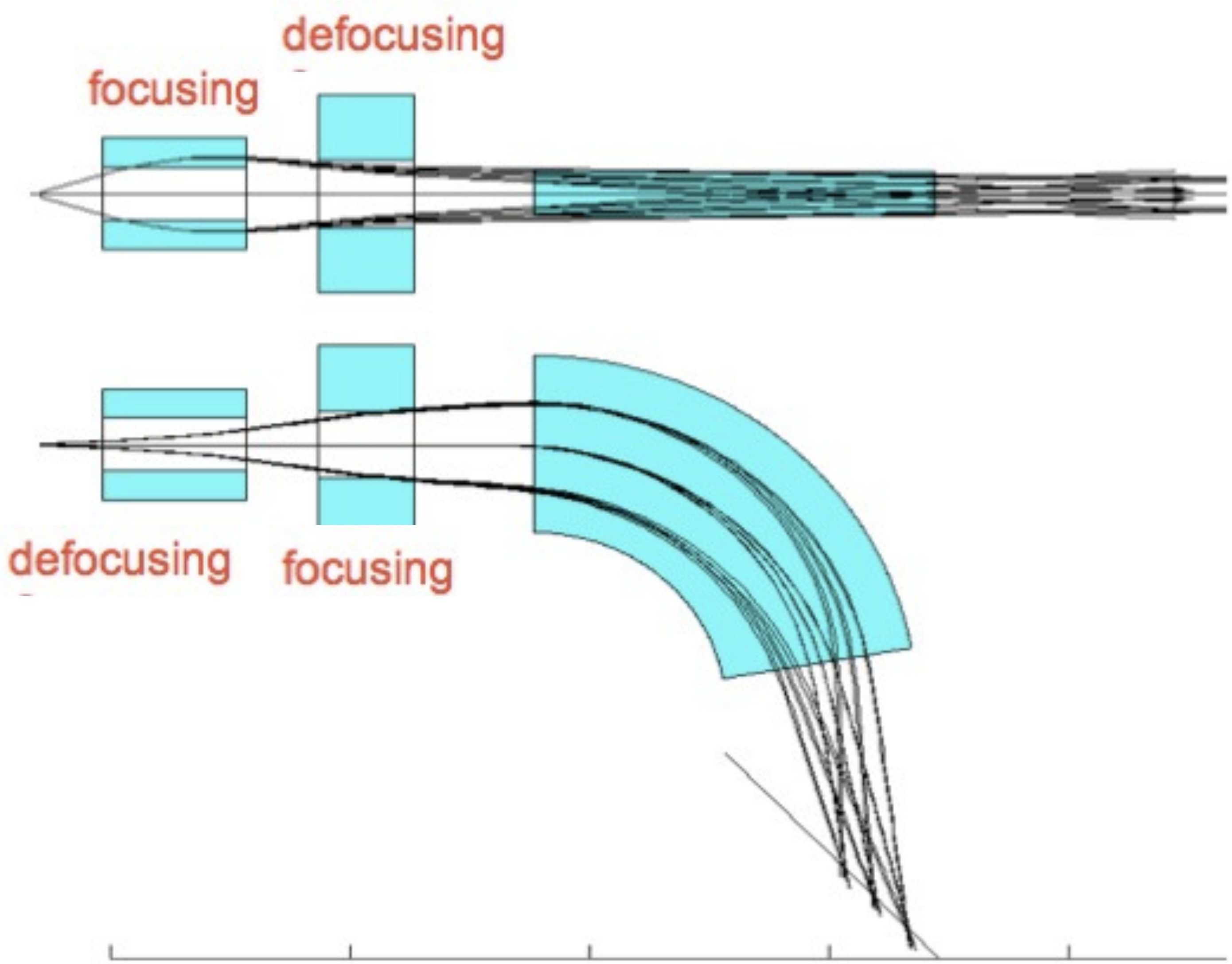
  : QQD, QQDD, QDD etc. (next slides)

- **more precise calculation & simulation**

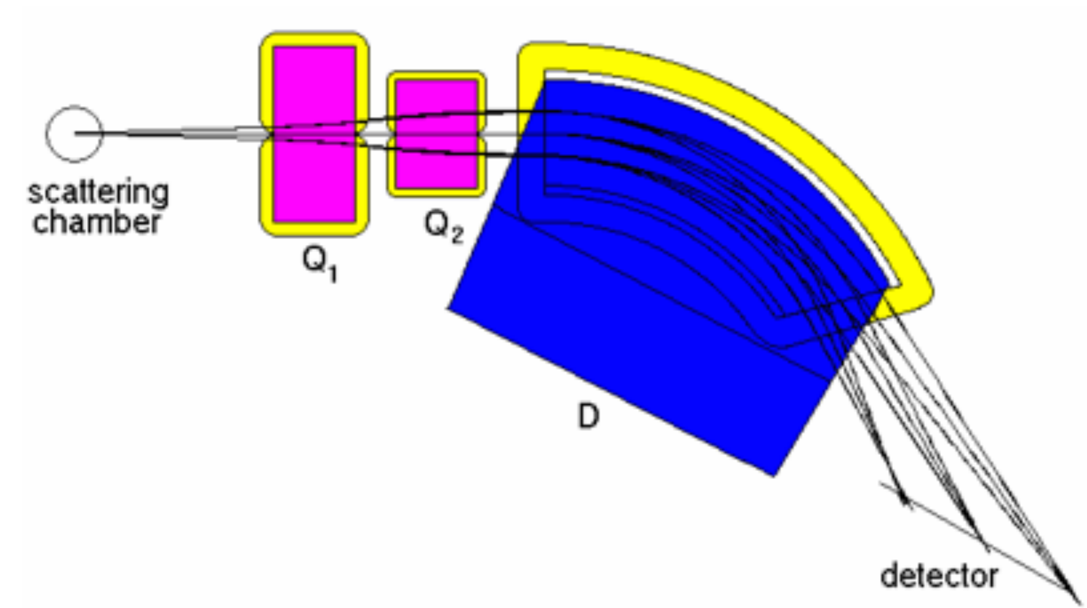
  : using other programs such as GICOSY, Geant4, etc.

  : solenoid effect for low momentum particles

QQD spectrometer

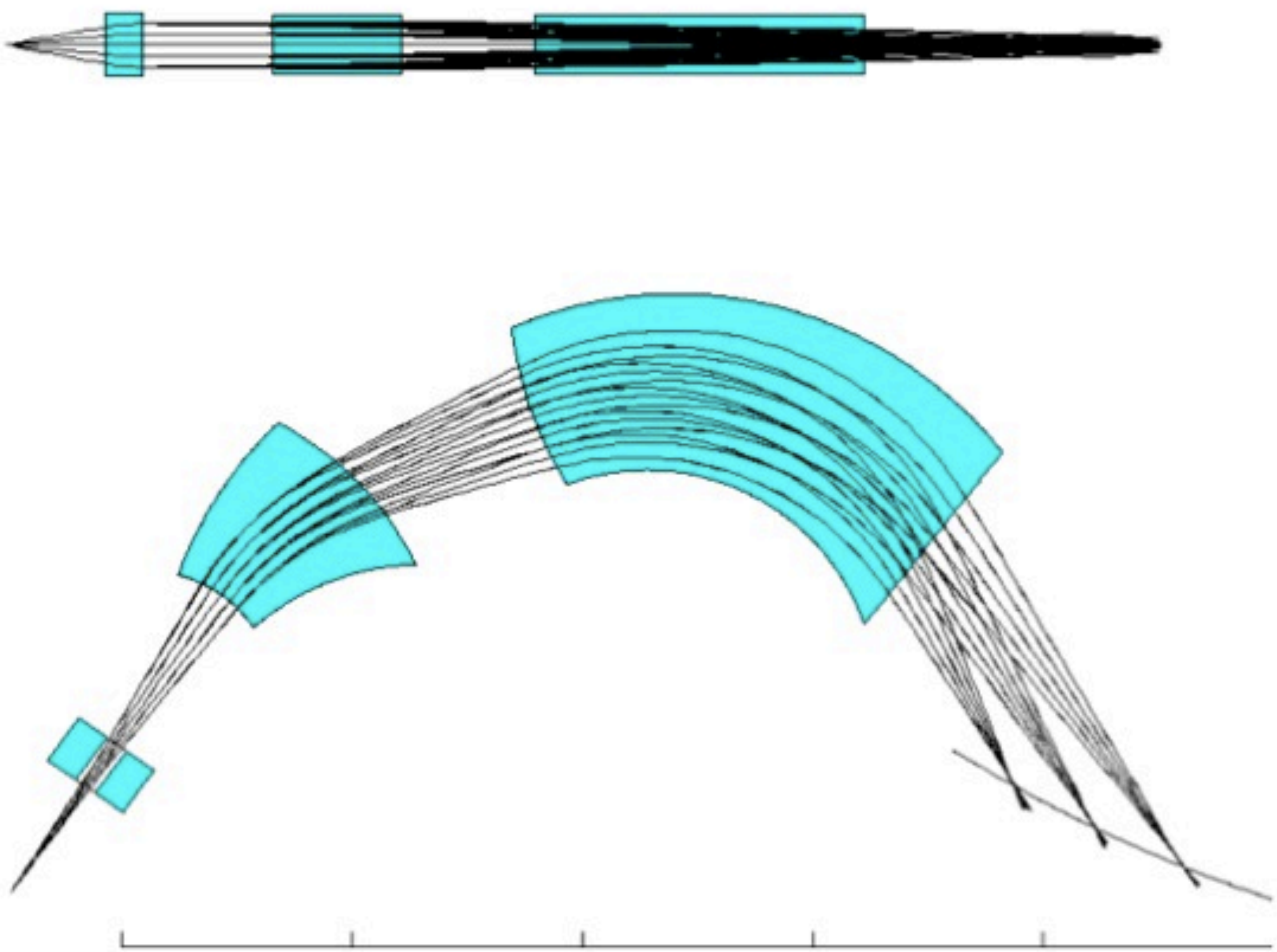


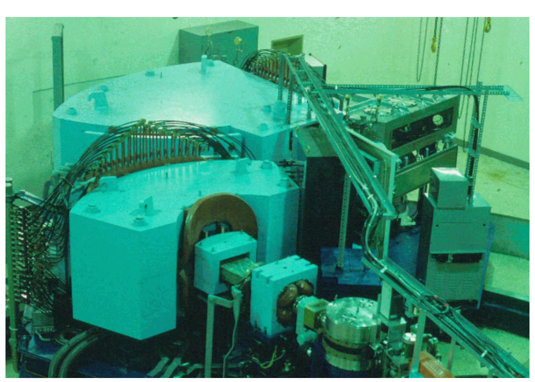
Big-Bite Spectrometer at KVI, University of Groningen, The Netherlands



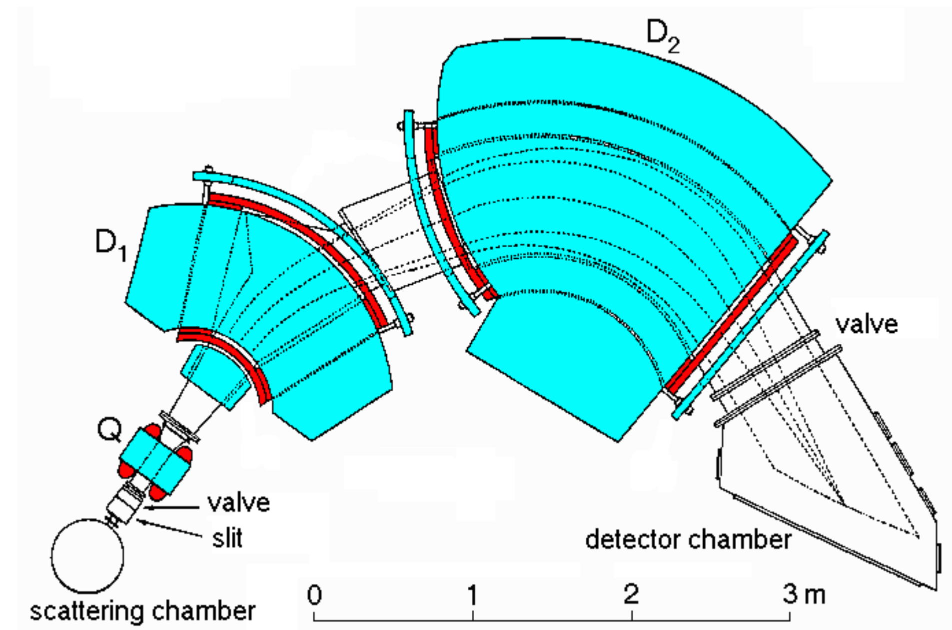
モード(mode)	大立体角 large solid angle	広帯域 broad range
最大立体角(maximum solid angle)	14 msr	7 msr
エネルギー帯域(energy range)	13 %	50 %
分散(dispersion)	2.6 m	2.6 m
横倍率(horizontal magnification)	-0.48	-0.45
縦倍率(vertical magnification)	-11.0	-5.7
運動量分解能(momentum resolution)	1/2500	1/2500
軌道半径(orbit radius)	2.2 m	2.2 m
最大磁場(maximum field)	1.4 T	1.4 T

QDD spectrometer





Spectrometer at INS SF cyclotron



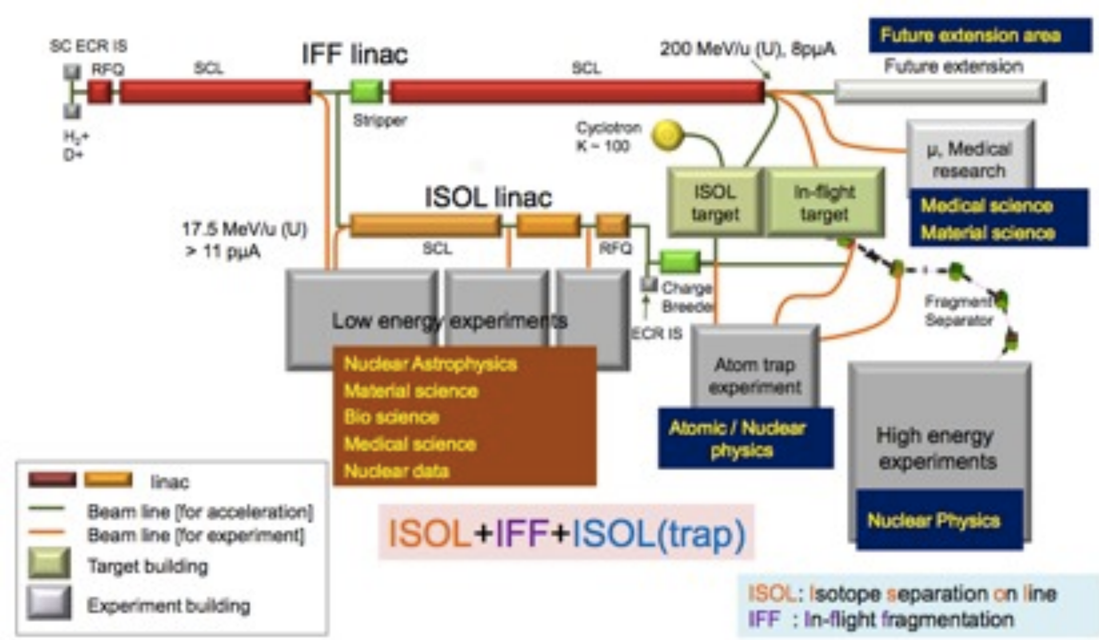
分散(dispersion)	3.9 m
横倍率(horizontal magnification)	-0.4
縦倍率(vertical magnification)	-4.4
運動量分解能(first-order momentum resolution)	1/10000
最大立体角(maximum solid angle)	6.4 msr
軌道半径(orbit radius)	1.3 - 1.5 m
エネルギー帯域(energy range)	30 %
四極電磁石磁極間隙(pole gap of quadrupole magnet)	10.6 cm
双極電磁石磁極間隙(pole gap of dipole magnet)	10 cm
最大磁場(maximum field)	1.27 T
電磁石重量(magnet weight)	38 ton

BACK-UP

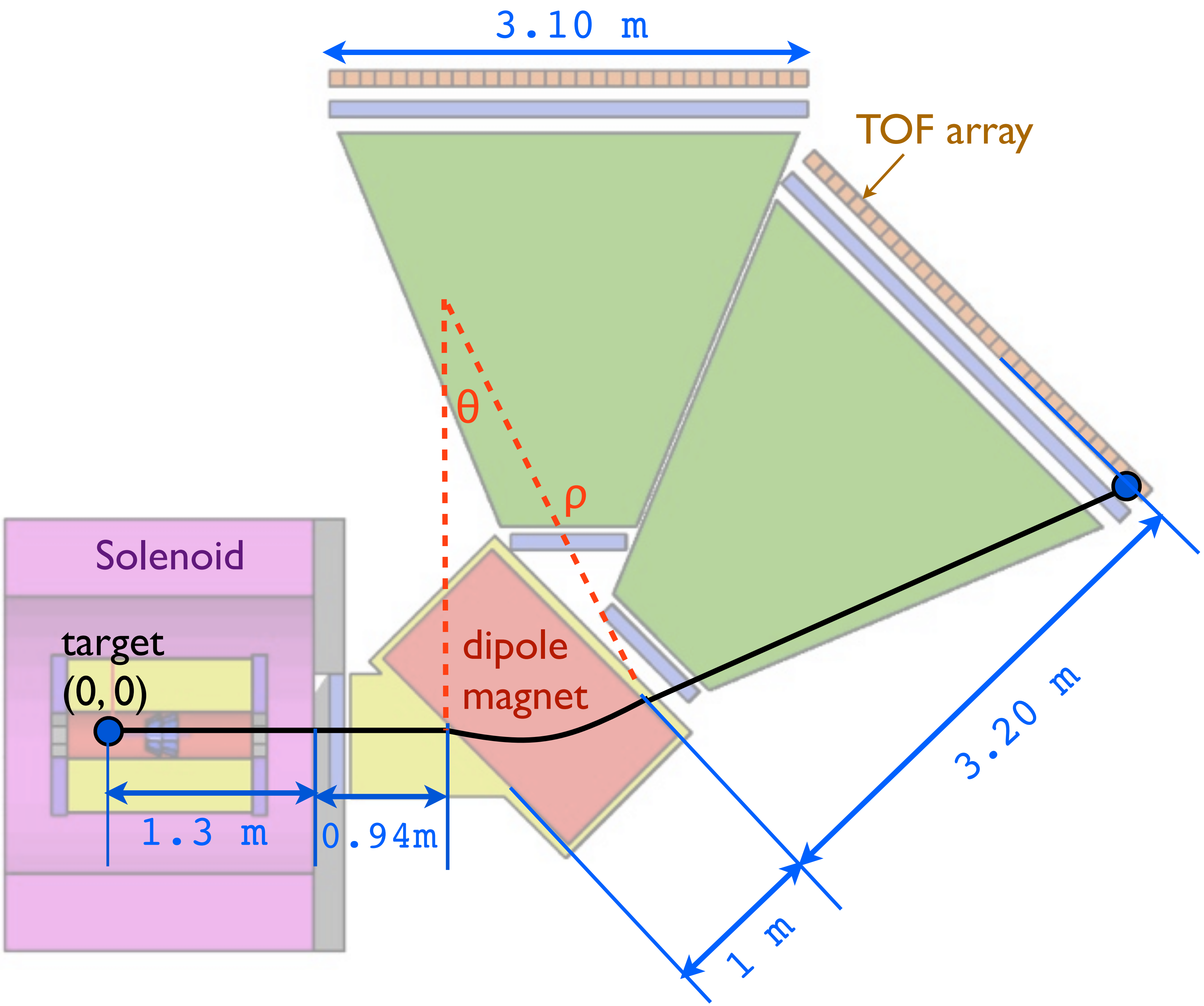
INTRODUCTION

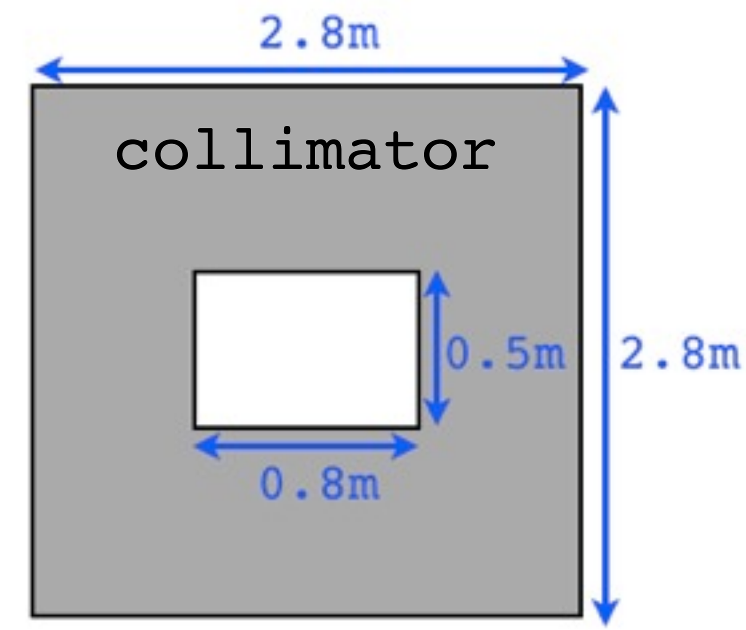
- **RAON (Korea Rare Isotope Accelerator)**
  : A future multi-purpose accelerator for the basic and applied sciences.

- **LAMPS (Large Acceptance MultiPurpose Spectrometer)**
  : primary purpose ⇒ nuclear symmetry energy
  : versatile for nuclear structure and nuclear astrophysics.



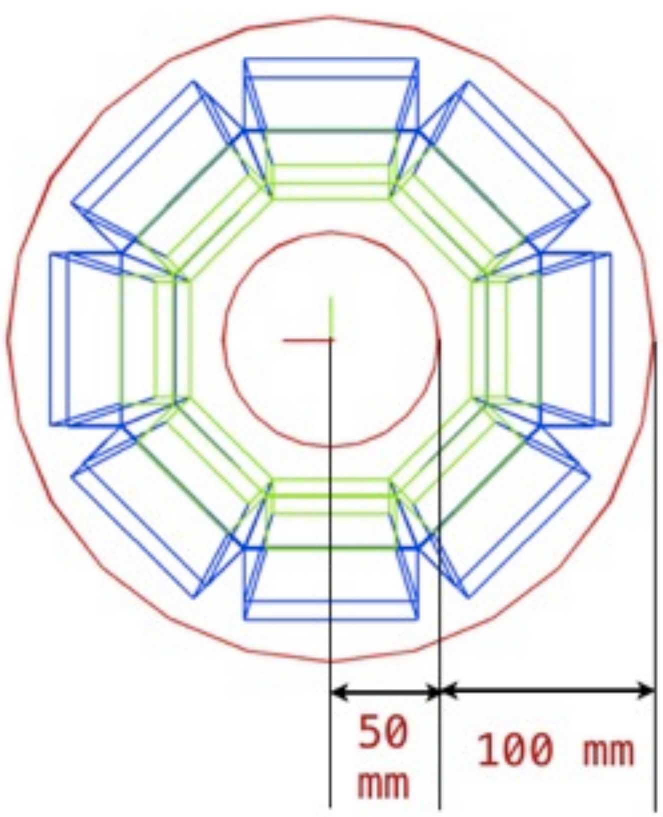
scale of LAMPS$_H$

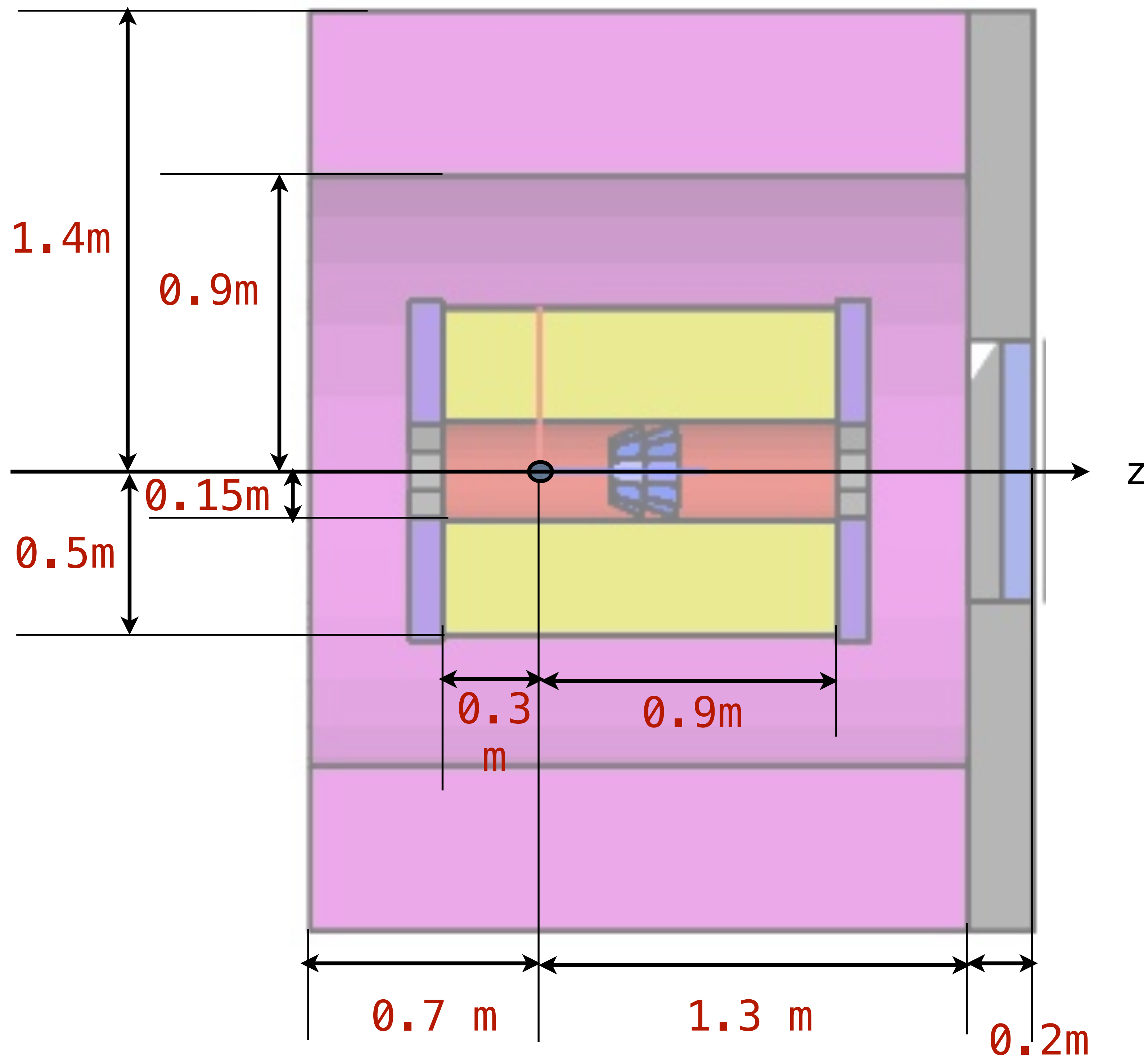




scale

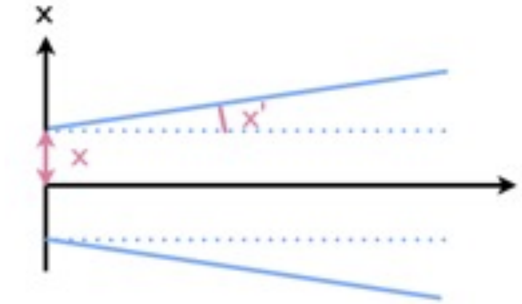
* TPC inner R = 0.15 m
* TPC outer R = 0.5 m
* Solenoid inner R = 0.9m
* Solenoid outer R = 1.4 m
* Si-CsI inner R = 0.05 m
* Si-CsI outer R = 0.15 m





Beam Optics

- **Matrix approach**



- **BEAM (charged particles)**

: expressed as a column vector X
: X(0) initial vector
: X(1) final vector

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{pmatrix}$$

x : the horizontal beam extent (cm)
x′ : the horizontal beam divergence (mrad)
y : the vertical beam extent (cm)
y′ : the vertical beam divergence (mrad)
ℓ : the longitudinal beam extent (cm)
δ : the momentum spread (%) ΔP/P

- **ELEMENTS**

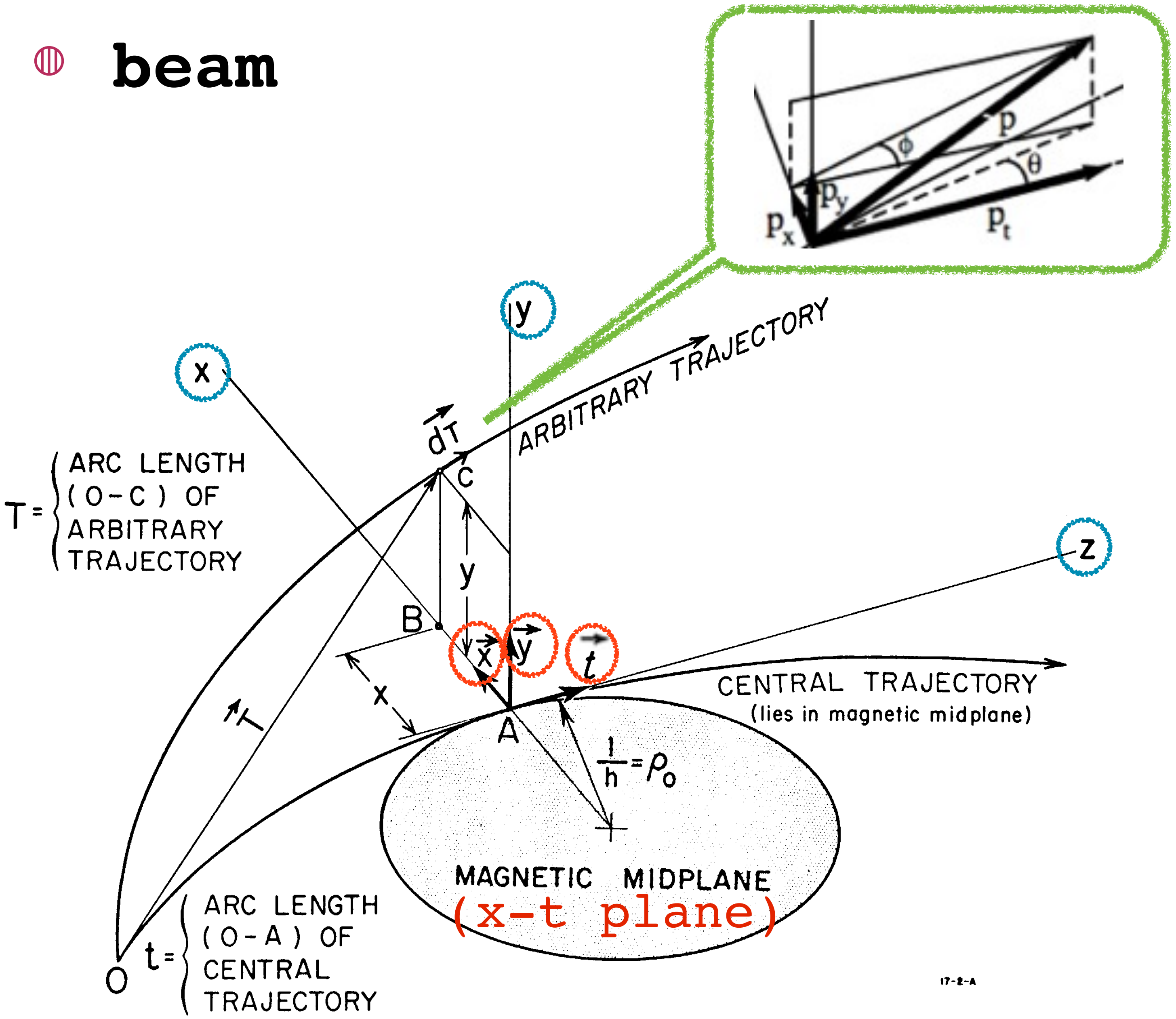
: Q-magnet, Dipole magnet, drift space, etc.
: expressed as a 6*6 Transfer Matrix R

$$X(1) = R(Ln)R(Mn)\cdots R(L4)R(M2)R(L3)R(Q2)R(L2)R(M1)R(L1)X(0)$$

$$X(1) = RX(0)$$

Coordinate

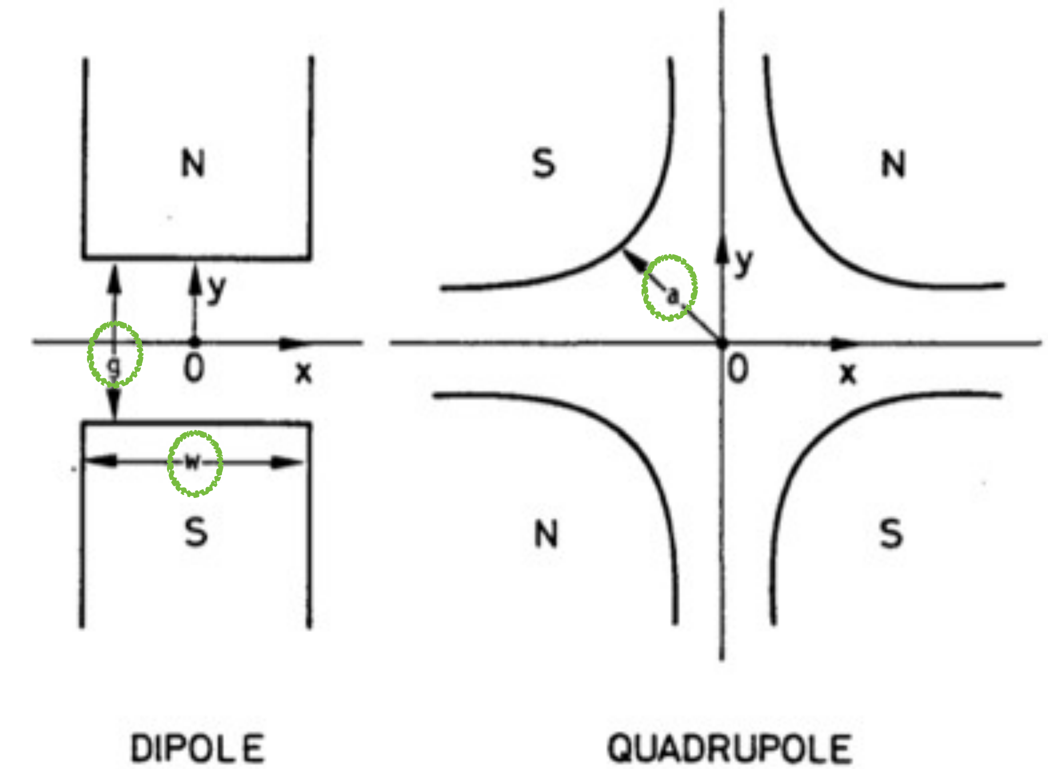
beam



○ : laboratory frame

○ : optical reference frame

Magnet



w : width

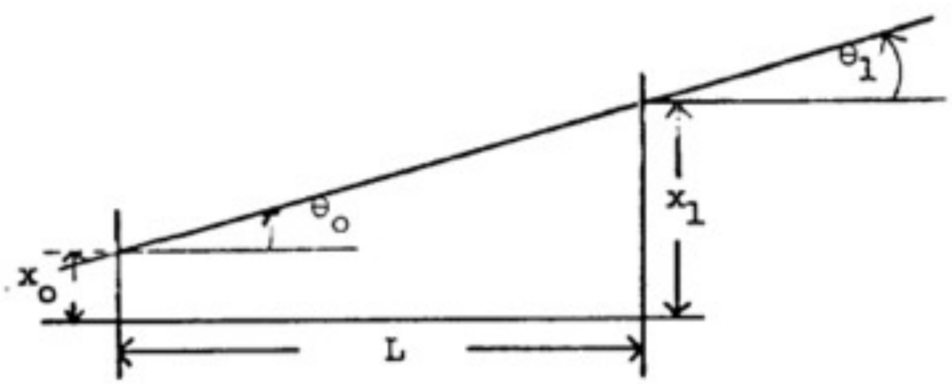
g : gap

a : aperture radius

Drift Space

- **Drift Space**

: field free region between elements (=free space)



$$\begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

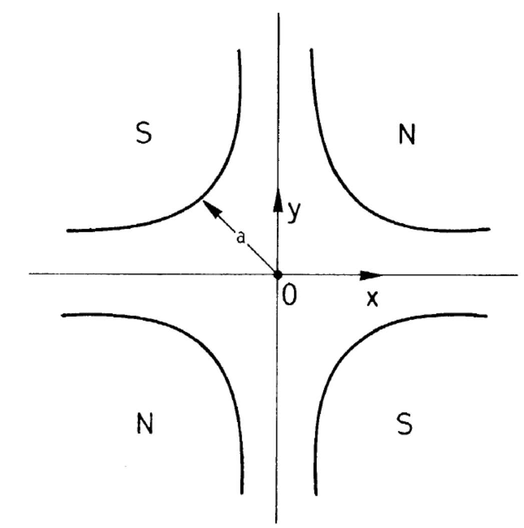
$$\begin{cases} x = x_0 + L x_0' \\ x' = x_0' \end{cases}$$

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

$$\begin{cases} y = y_0 + L y_0' \\ y' = y_0' \end{cases}$$

Q–Magnet

$$\begin{pmatrix} \cos k_q L & \frac{1}{k_q}\sin k_q L & 0 & 0 & 0 & 0 \\ -k_q \sin kqL & \cos k_q L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh k_q L & \frac{1}{k_q}\sinh k_q L & 0 & 0 \\ 0 & 0 & k_q \sinh k_q L & \cosh k_q L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



QUADRUPOLE

$$k_q{}^2 = \frac{B_0}{a}\frac{1}{B\rho_0}$$

L : effective length of the central trajectory

a : the radius of the aperture

B_0 : the field at radius a (pole tip)

positive - focus in horizontal

negative - focus in vertical

Dipole Magnet

$$\begin{bmatrix} \cos k_x L & \frac{1}{k_x}\sin k_x L & 0 & 0 & 0 & \boxed{\frac{h}{k_x^2}\left(1-\cos k_x L\right)} \\ -k_x \sin k_x L & \cos k_x L & 0 & 0 & 0 & \frac{h}{k_x}\sin k_x L \\ 0 & 0 & \cos k_y L & \frac{1}{k_y}\sin k_y L & 0 & 0 \\ 0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\ -\frac{h}{k_x}\sin k_x L & -\frac{h}{k_x^2}\left(1-\cos k_x L\right) & 0 & 0 & 1 & -\frac{h^2}{k_x^3}\left(k_x L - \sin k_x L\right) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

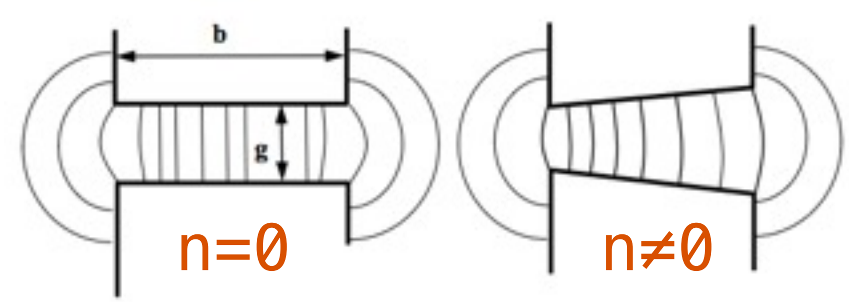
$$k_x^2 = (1-n)h^2$$

$$k_y^2 = nh^2$$

cf)

n : n-value (related to field gradient)

$$B_y(x,0,t) = B_y(0,0,t)(1 - nhx + \ldots)$$



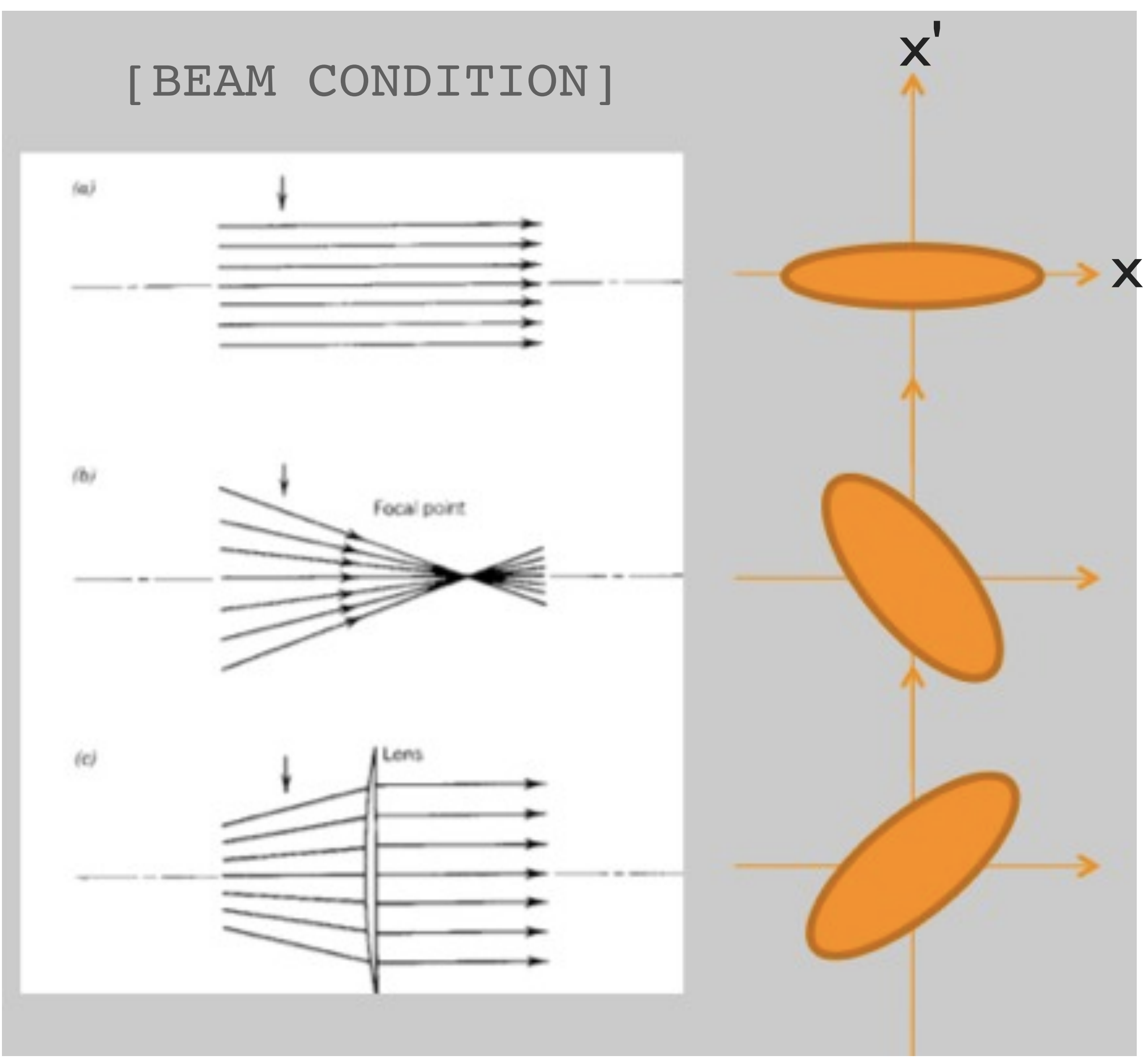
Phase Space Motion

- **Beam Emittance**

: avg. spread of particle coordinates in phase space

- **Liouville's theorem**

: total emittance doesn't change through the beam line.

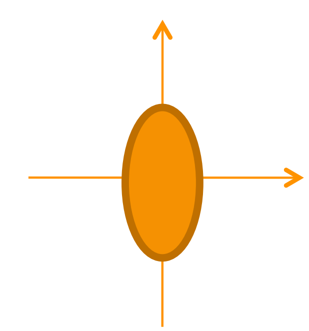


$$R_x = \begin{pmatrix} (x/x) & (x/x') & D \\ (x'/x) & (x'/x') & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} (y/y) & (y/y') \\ (y'/y) & (y'/y') \end{pmatrix}$$

- For focal points

=> Standing ellipse!



Matrix

$$X(1) = RX(0)$$

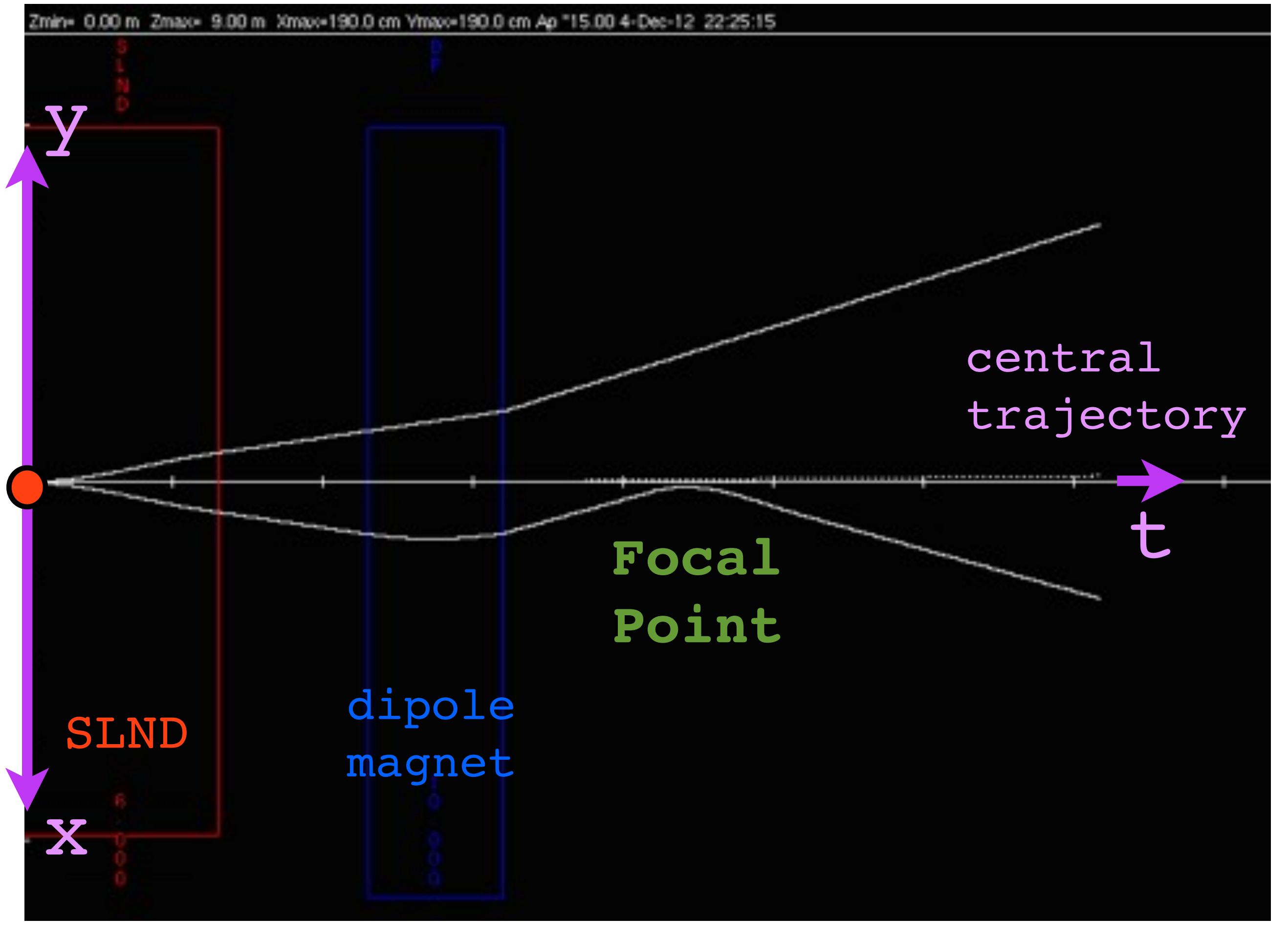
$$\begin{pmatrix} x_1 \\ x'_1 \\ y_1 \\ y'_1 \\ l_1 \\ \delta_1 \end{pmatrix} = \begin{pmatrix} (x|x) & (x|x') & (x|y) & (x|y') & (x|l) & \overset{=D}{\boxed{(x|\delta)}} \\ (x'|x) & (x'|x') & (x'|y) & (x'|y') & (x'|l) & (x'|\delta) \\ (y|x) & (y|x') & (y|y) & (y|y') & (y|l) & (y|\delta) \\ (y'|x) & (y'|x') & (y'|y) & (y'|y') & (y'|l) & (y'|\delta) \\ (l|x) & (l|x') & (l|y) & (l|y') & (l|l) & (l|\delta) \\ (\delta|x) & (\delta|x') & (\delta|y) & (\delta|y') & (\delta|l) & \underset{=1}{\boxed{(\delta|\delta)}} \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ l_0 \\ \delta_0 \end{pmatrix}$$

- assume ℓ=0
- x & y independent to each other
- $\delta_1=\delta_0$ and $(x|\delta)=D$

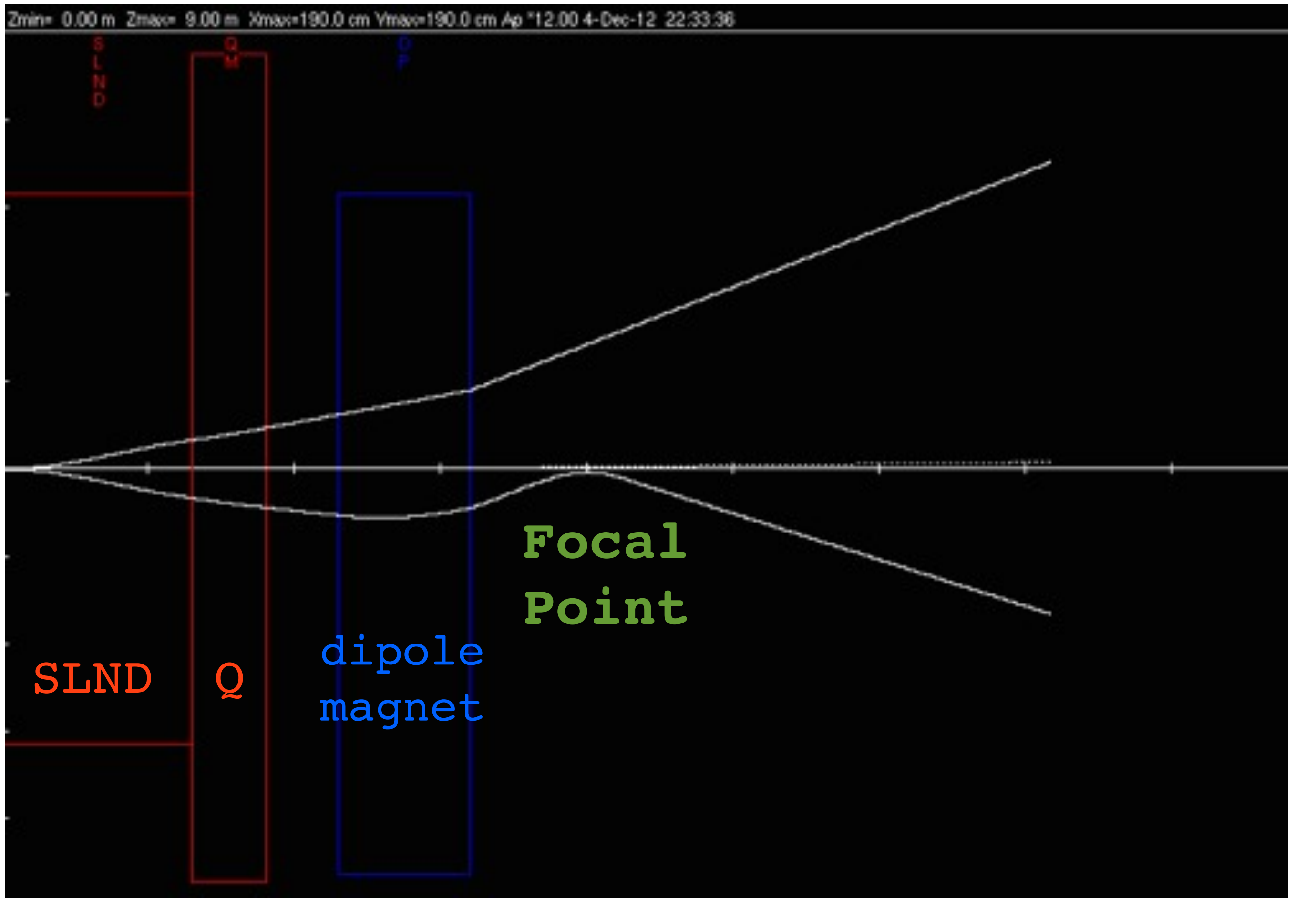
by TRANSPORT

In Optical frame

<One Dipole System>



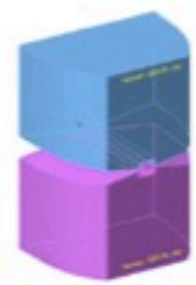
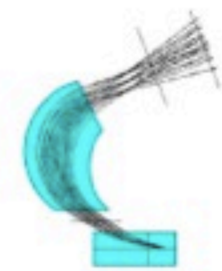
<Q-D System>



* kinetic energy of proton : 50 MeV

by K-Trace

references

Type			
proposed by	I. A. Koop	S. Kato	S. Kato
ϕ acceptance	300 mr	300 mr	1200 mr
θ acceptance	?	100 mr	150 mr
solid angle	?	30 msr	180 msr
$\theta_{min} \sim \theta_{max}$	$(40^{\circ} \sim 80^{\circ})$?	$15^{\circ} \sim 52^{\circ}$	$(50^{\circ} \sim 120^{\circ})$
collision length	?	30 cm	5~? cm
momentum bite	25 %	20 %	20 %
Q magnet	super-conducting	normal conducting	super-conducting
D gap	25 cm	16 cm	30 cm
median plane	vertical	horizontal	horizontal